

Rearrangement Proof of the Pythagorean Theorem

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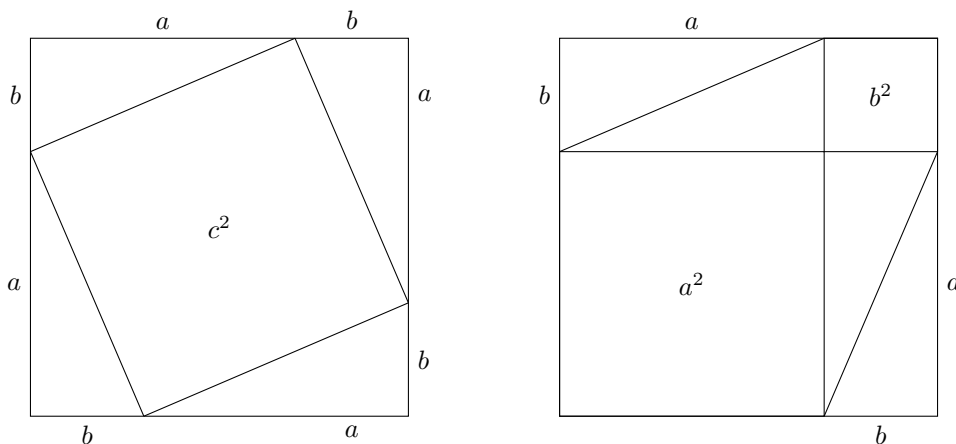
Theorem

For a right-angled triangle, the square of the length of the hypotenuse (c) is equal to the sum of the squares of the lengths of the other two sides (a and b). This can be represented as:

$$c^2 = a^2 + b^2$$

Proof

Consider two squares, each of side length $a + b$. We can place four congruent right-angled triangles inside each square, with side lengths a , b , and c .



In the first square, the total area covered by the triangles and the smaller square is $4 \times \frac{1}{2}ab + c^2 = 2ab + c^2$.

In the second square, after rearranging the triangles, two smaller squares appear: one of side length a and the other of side length b . Their combined area is $a^2 + b^2$.

From the two squares, it is clear that the area represented by c^2 in the first square is equal to the combined areas of a^2 and b^2 in the second square, thus proving the theorem.