AMSI AUSTRALIAN MATHEMATICAL SCIENCES INSTITUTE

ICE-EM MATHEMATICS 10 &10A

THIRD EDITION

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INTERNATIONAL CENTRE OF EXCELLENCE FOR EDUCATION IN MATHEMATICS

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Preface

ICE-EM Mathematics Third Edition is a series of textbooks for students in years 5 to 10 throughout Australia who study the Australian Curriculum and its state variations.

The program and textbooks were developed in recognition of the importance of mathematics in modern society and the need to enhance the mathematical capabilities of Australian students. Students who use the series will have a strong foundation for work or further study.

Background

The International Centre of Excellence for Education in Mathematics (ICE-EM) was established in 2004 with the assistance of the Australian Government and is managed by the Australian Mathematical Sciences Institute (AMSI). The Centre originally published the series as part of a program to improve mathematics teaching and learning in Australia. In 2012, AMSI and Cambridge University Press collaborated to publish the Second Edition of the series to coincide with the introduction of the Australian Curriculum, and we now bring you the Third Edition.

The series

ICE-EM Mathematics Third Edition provides a progressive development from upper primary to middle secondary school. The writers of the series are some of Australia's most outstanding mathematics teachers and subject experts. The textbooks are clearly and carefully written, and contain background information, examples and worked problems.

For the Third Edition, the series has been carefully edited to present the content in a more streamlined way without compromising quality. There is now one book per year level and the flow of topics from chapter to chapter and from one year level to the next has been improved.

The year 10 textbook incorporates all material for the 10A course, and selected topics in earlier books carefully prepare students for this. *ICE-EM Mathematics Third Edition* provides excellent preparation for all of the Australian Curriculum's year 11 and 12 mathematics courses.

For the Third Edition, *ICE-EM Mathematics* now comes with an Interactive Textbook: a cutting-edge digital resource where all textbook material can be answered online (with students' working-out), additional quizzes and features are included at no extra cost. See 'The Interactive Textbook and Online Teaching Suite' on page xiii for more information.

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Author biographies

Peter Brown

Peter Brown studied Pure Mathematics and Ancient Greek at Newcastle University, and completed postgraduate degrees in each subject at the University of Sydney. He worked for nine years as a mathematics teacher in NSW State schools. Since 1990, he has taught Pure Mathematics at the School of Mathematics and Statistics at the University of New South Wales (UNSW). He was appointed Director of First Year Studies at UNSW from 2011 to 2015. He specialises in Number Theory and History of Mathematics and has published in both areas. Peter regularly speaks at teacher inservices, Talented Student days and Mathematics Olympiad Camps. In 2008 he received a UNSW Vice Chancellor's Teaching Award for educational leadership.

Michael Evans

Michael Evans has a PhD in Mathematics from Monash University and a Diploma of Education from La Trobe University. He currently holds the honorary position of Senior Consultant at the Australian Mathematical Sciences Institute at the University of Melbourne. He was Head of Mathematics at Scotch College, Melbourne, and has also taught in public schools and in recent years has returned to classroom teaching. He has been very involved with curriculum development at both state and national levels. In 1999, Michael was awarded an honorary Doctor of Laws by Monash University for his contribution to mathematics education, and in 2001 he received the Bernhard Neumann Award for contributions to mathematics enrichment in Australia.

Garth Gaudry

Garth Gaudry was Head of Mathematics at Flinders University before moving to UNSW, where he became Head of School. He was the inaugural Director of AMSI before he became the Director of AMSI's International Centre of Excellence for Education in Mathematics. Previous positions include membership of the South Australian Mathematics Subject Committee and the Eltis Committee appointed by the NSW Government to enquire into Outcomes and Profiles. He was a life member of the Australian Mathematical Society and Emeritus Professor of Mathematics, UNSW.

David Hunt

David Hunt graduated from the University of Sydney in 1967 with an Honours degree in Mathematics and Physics, then obtained a master's degree and a doctorate from the University of Warwick. He was appointed to a lectureship in Pure Mathematics at UNSW in early 1971, where he is currently an honorary Associate Professor. David has taught courses in Pure Mathematics from first year to master's level and was Director of First Year Studies in Mathematics for five years. Many of David's activities outside UNSW have centred on the Australian Mathematics Trust. These contributions as well as those to the International Mathematical Olympiad movement were recognised by the award of the Paul Erdos medal in 2016.

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Robert McLaren

Robert McLaren graduated from the University of Melbourne in 1978 with a Bachelor of Science (Hons) and a Diploma of Education. He commenced his teaching career in 1979 at The Geelong College and has taught at a number of Victorian Independent Schools throughout his career. He has been involved in textbook writing, curriculum development and VCE examination setting and marking during his teaching life. He has taught mathematics at all secondary levels and has a particular interest in problem solving. Robert is currently Vice Principal at Scotch College in Melbourne.

Bill Pender

Bill Pender has a PhD in Pure Mathematics from Sydney University and a BA (Hons) in Early English from Macquarie University. After a year at Bonn University, he taught at Sydney Grammar School from 1975 to 2008, where he was Subject Master for many years. He has been involved in the development of NSW Mathematics syllabuses since the early 1990s, and was a foundation member of the Education Advisory Committee of AMSI. He has also lectured and tutored at Sydney University and at UNSW, and given various inservice courses. Bill is the lead author of the NSW calculus series *Cambridge Mathematics*.

Brian Woolacott

Brian Woolacott graduated from the University of Melbourne in 1978 with a Bachelor of Science and a Diploma of Education. In 1979 he started his teaching career at Scotch College, Melbourne, and during his career he has taught at all secondary levels. For 13 years, Brian was the Co-ordinator of Mathematics for Years 9 and 10, and during this time he was involved in co-authoring a number of textbooks for the Year 9 and 10 levels. Brian is currently the Dean of Studies at Scotch College.

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How to use this resource

The textbook

Each chapter in the textbook addresses a specific Australian Curriculum content strand and set of sub-strands. The exercises within chapters take an integrated approach to the concept of proficiency strands, rather than separating them out. Students are encouraged to develop and apply Understanding, Fluency, Problem-solving and Reasoning skills in every exercise.

The series places a strong emphasis on understanding basic ideas, along with mastering essential technical skills. Mental arithmetic and other mental processes are major focuses, as is the development of spatial intuition, logical reasoning and understanding of the concepts.

Problem-solving lies at the heart of mathematics, so *ICE-EM Mathematics* gives students a variety of different types of problems to work on, which help them develop their reasoning skills. Challenge exercises at the end of each chapter contain problems and investigations of varying difficulty that should catch the imagination and interest of students. Further, two 'Review and Problem-solving' chapters in each 7–10 textbook contain additional problems that cover new concepts for students who wish to explore the subject even further.

The Interactive Textbook and Online Teaching Suite

Included with the purchase of the textbook is the Interactive Textbook. This is the online version of the textbook and is accessed using the 16-character code on the inside cover of this book.

The Online Teaching Suite is the teacher version of the Interactive Textbook and contains all the support material for the series, including tests, worksheets, skillsheets, curriculum documentation and more.

For more information on the Interactive Textbook and Online Teaching Suite, see page xiii.

The Interactive Textbook and Online Teaching Suite are delivered on the *Cambridge HOTmaths* platform, providing access to a world-class Learning Management System for testing, task management and reporting. They do not provide access to the *Cambridge HOTmaths* stand-alone resource that you or your school may have used previously. For more information on this resource, contact Cambridge University Press.

AMSI's TIMES and SAM modules

The TIMES and SAM web resources were developed by the *ICE-EM Mathematics* author team at AMSI and are written around the structure of the Australian Curriculum. These resources have been mapped against your *ICE-EM Mathematics* book and are available to teachers and students via the AMSI icon on the Dashboard of the Interactive Textbook and Online Teaching Suite.

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The Interactive Textbook and the Online Teaching Suite

Interactive Textbook

The Interactive Textbook is the online version of the print textbook and comes included with purchase of the print textbook. It is accessed by first activating the code on the inside cover. It is easy to navigate and is a valuable accompaniment to the print textbook.

Students can show their working

All textbook questions can be answered online within the Interactive Textbook. Students can show their working for each question using either the Draw tool for handwriting (if they are using a device with a touch-screen), the Type tool for using their keyboard in conjunction with the pop-up symbol palette, or by importing a file using the Import tool.

Once a student has completed an exercise they can save their work and submit it to the teacher, who can then view the student's working and give feedback to the student, as they see appropriate.

Auto-marked quizzes

The Interactive Textbook also contains material not included in the textbook, such as a short auto-marked quiz for each section. The quiz contains 10 questions which increase in difficulty from question 1 to 10 and cover all proficiency strands. There is also space for the student to do their working underneath each quiz question. The auto-marked quizzes are a great way for students to track their progress through the course.

Additional material for Year 5 and 6

For Years 5 and 6, the end-of-chapter Challenge activities as well as a set of Blackline Masters are now located in the Interactive Textbook. These can be found in the 'More resources' section, accessed via the Dashboard, and can then easily be downloaded and printed.

Online Teaching Suite

The Online Teaching Suite is the teacher's version of the Interactive Textbook. Much more than a 'Teacher Edition', the Online Teaching Suite features the following:

- The ability to view students' working and give feedback – When a student has submitted their work online for an exercise, the teacher can view the student's work and can give feedback on each question.
- For Years 5 and 6, access to Chapter tests, Blackline Masters, Challenge exercises, curriculum support material, and more.
- For Years 7 to 10, access to Pre-tests, Chapter tests, Skillsheets, Homework sheets, curriculum support material, and more.
- A Learning Management System that combines task-management tools, a powerful test generator, and comprehensive student and whole-class reporting tools.

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