CHAPTER

1

Number and Algebra

Consumer arithmetic

This chapter reviews some important practical financial topics, such as investing and borrowing money, income tax and GST, inflation, depreciation, profits and losses, discounts and commissions. Formulas for compound interest and depreciation are introduced.

Everything in this chapter requires calculations with percentages. We are assuming that you are using a calculator, so we have made little attempt to set questions where the numbers work out nicely.

Nevertheless, you should always look over your work and check that the answers to your calculations are reasonable and sensible.

When the calculator displays numbers with many decimal places, you will need to round the answer in some way that is appropriate in the context of the question. This is an important skill in everyday life.

ICE-EM Mathematics 10 3ed ISBN 978-1-108-40434-1

Cambridge University Press
Updated June 2022

1A Review of percentages

We first review the calculation techniques involving percentages, which you have learned in previous years.

- To convert a percentage to a decimal, move the decimal point two places to the left. For example:

  $27\% = 0.27$

- To convert a percentage to a fraction, multiply by $\frac{1}{100}$. For example:

  $27\% = \frac{27}{100}$ and $2\frac{1}{2}\% = \frac{2\frac{1}{2}}{100} = \frac{5}{200} = \frac{1}{40}$

- To convert a decimal or a fraction to a percentage, multiply by 100%. For example:

  $$\begin{aligned}0.35 &= 0.35 \times 100\% \\ &= 35\%\end{aligned} \quad \text{and} \quad \begin{aligned}\frac{3}{5} &= \frac{3}{5} \times 100\% \\ &= 60\%\end{aligned}$$

- To find a percentage of a quantity, convert the percentage to a decimal or a fraction, and then multiply the quantity by it. For example:

  $$\begin{aligned}3.5\% \text{ of } 1250 &= 1250 \times 0.035 \\ &= 43.75\end{aligned} \quad \text{or} \quad \begin{aligned}3.5\% \text{ of } 1250 &= 1250 \times \frac{35}{1000} \\ &= 43\tfrac{3}{4}\end{aligned}$$

- To calculate the percentage that one quantity, a, is of another quantity, b:
  - first convert both quantities to the same unit of measurement
  - then form the fraction $\frac{a}{b}$ and multiply it by 100%.

  For example, to express 32 cm as a percentage of 2.4 m:

  First, write 2.4 m = 240 cm

  Then $\frac{32}{240} \times \frac{100}{1}\% = 13\frac{1}{3}\%$

  So 32 cm is $13\frac{1}{3}\%$ of 2.4 m

Finding the original amount

We now introduce another important method that will be used with percentages throughout this chapter.

- To find the original amount, given 12% of it, divide by 12%.

Example 1

Ken saves 12% of his after-tax salary every week. If he saves \$108 a week, what is his after-tax salary?

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Solution

Savings = after-tax salary × 12%

Reversing this:

$$\begin{aligned}\text{After-tax salary} &= \text{savings} \div 12\% \\ &= \text{savings} \div 0.12 \qquad \text{(Replace 12\% by 0.12.)} \\ &= 108 \div 0.12 \\ &= \$900\end{aligned}$$

This technique of writing the percentage factor on the right and reversing the process using division is needed in many practical situations. It will be applied throughout this chapter to commissions, profit and loss, income tax and interest.

Commission

A **commission** is a fee that is charged by an agent who sells goods or services on behalf of someone else. The person who owns the goods or services is called the vendor, and the commission charged is usually determined as a percentage of the selling price.

Example 2

The Dandy Bay Gallery charges a commission of 8.6% on the selling price.

a An antique vase was sold recently for $18 000. How much did the gallery receive, and how much was left for the vendor?

b The gallery received a commission of $215 for selling a painting. What was the selling price of the painting, and what did the vendor actually receive?

Solution

a

$$\begin{aligned}\text{Commission} &= 18\,000 \times 8.6\% \\ &= 18\,000 \times 0.086 \\ &= \$1548\end{aligned}$$

$$\begin{aligned}\text{Amount received by vendor} &= 18\,000 - 1548 \\ &= \$16\,452\end{aligned}$$

b

Commission = selling price × 8.6%

Reversing this:

$$\begin{aligned}\text{Selling price} &= \text{commission} \div 8.6\% \\ &= 215 \div 0.086 \\ &= \$2500\end{aligned}$$

$$\begin{aligned}\text{Amount received by vendor} &= 2500 - 215 \\ &= \$2285\end{aligned}$$

Profit and loss as percentages

Is an annual profit of \$20 000 a great performance or a modest performance? For a business with annual sales of \$100 000, such a profit would be considered very large. For a business with annual expenditure of \$100 000 000, however, it would be considered a very poor performance.

For this reason, it is often convenient to express profit and loss as percentages of the total costs.

Example 3

The owners of Budget Shoe Shop spent \$6 600 000 last year buying shoes and paying salaries and other expenses. They made a 2% profit on these costs.

a What was their profit last year?

b What was the total of their sales?

c In the previous year, their costs were \$5 225 000 and their sales were only \$5 145 000. What percentage loss did they make on their costs?

d Two years ago their costs were \$5 230 000 and their sales were \$6 125 000. What percentage profit did they make on their costs?

Solution

a
$$\begin{aligned}\text{Profit} &= 6\,600\,000 \times 2\% \\ &= 6\,600\,000 \times 0.02 \\ &= \$132\,000\end{aligned}$$

b
$$\begin{aligned}\text{Total sales} &= \text{total costs} + \text{profit} \\ &= 6\,600\,000 + 132\,000 \\ &= \$6\,732\,000\end{aligned}$$

c
$$\begin{aligned}\text{Last year, loss} &= \text{total costs} - \text{total sales} \\ &= 5\,225\,000 - 5\,145\,000 \\ &= \$80\,000\end{aligned}$$

$$\begin{aligned}\text{Percentage loss} &= \frac{80\,000}{5\,225\,000} \times \frac{100}{1}\% \\ &\approx 1.53\% \text{ (Correct to the nearest 0.01\%.)}\end{aligned}$$

$$\left[\begin{aligned}\text{Alternatively, profit} &= \text{total sales} - \text{total costs} \\ &= 5\,145\,000 - 5\,225\,000 \\ &= -\ \$80\,000 \\ \text{Percentage change} &\approx -1.53\% \\ &= 1.53\% \text{ loss}\end{aligned}\right]$$

d
$$\begin{aligned}\text{Profit} &= \text{total sales} - \text{total costs} \\ &= 6\,125\,000 - 5\,230\,000 \\ &= \$895\,000\end{aligned}$$

$$\begin{aligned}\text{Percentage profit} &= \frac{895\,000}{5\,230\,000} \times \frac{100}{1}\% \\ &\approx 17.11\% \text{ (Correct the nearest 0.01\%.)}\end{aligned}$$

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 Updated June 2022

Example 4

Andrew's paint shop made a profit of 6.4% on total costs last year. If the actual profit was $87 000, what were the total costs, and what were the total sales?

Solution

$$\text{Profit} = \text{costs} \times 6.4\%$$

$$\begin{aligned}\text{Reversing this, costs} &= \text{profit} \div 6.4\% \\ &= 87\,000 \div 0.064 \\ &= \$1\,359\,375\end{aligned}$$

$$\begin{aligned}\text{Hence, total sales} &= \text{profit} + \text{costs} \\ &= 87\,000 + 1\,359\,375 \\ &= \$1\,446\,375\end{aligned}$$

Income tax

Income tax rates are often **progressive**. This means that the more you earn, the higher the rate of tax you pay on each extra dollar earned.

Australian income tax rates are progressive, but they often change, so here is an example using the rates of the fictional nation of Plusionta, where taxation rates have not changed for many years.

Example 5

Income tax in the fictional nation of Plusionta is calculated as follows.

- There is no tax on the first $12 000 that a person earns in any one year.
- From $12 001 to $30 000, the tax rate is 15c for each dollar over $12 000.
- From $30 001 to $75 000, the tax rate is 25c for each dollar over $30 000.
- For incomes exceeding $75 000, the tax rate is 35c for each dollar over $75 000.

Find the income tax payable by a person whose taxable income for the year is:

a $10 500 **b** $26 734 **c** $72 000 **d** $455 000

Solution

a There is no tax.

b Tax on first $12 000 = $0

Tax on remaining $14 734

$= 14\,734 \times 0.15$

$= \$2210.10$

This is the total tax payable.

(continued over page)

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c Tax on first \$12 000 = \$0

Tax on next \$18 000

= 18 000 × 0.15

= \$2700

Tax on remaining \$42 000

= 42 000 × 0.25

= \$10 500

Total tax = 2700 + 10 500

= \$13 200

d Tax on first \$12 000 = \$0

Tax on next \$18 000 = \$2700

Tax on next \$45 000

= 45 000 × 0.25

= \$11 250

Tax on remaining \$380 000

= 380 000 × 0.35

= \$133 000

Total tax = 2700 + 11 250 + 133 000

= \$146 950

Simple interest

When money is lent by a bank or other lender, whoever borrows the money normally makes a payment, called **interest**, for the use of the money.

The amount of interest paid depends on:

- the **principal**, which is the amount of money borrowed
- the **rate** at which interest is charged
- the **time** for which the money is borrowed.

This section will deal only with **simple interest**. In simple interest transactions, interest is paid on only the original amount borrowed.

Conversely, if a person invests money in a bank or elsewhere, the bank pays the person interest because the bank uses the money to finance its own investments.

Formula for simple interest

Suppose that I borrow \$$P$ for T years at an interest rate of R per annum.

$$\begin{aligned}\text{Interest paid at the end of each year} &= P \times R \\ \text{Total interest, } \$I\text{, paid over } T \text{ years} &= P \times R \times T \\ &= PRT\end{aligned}$$

This gives us the well-known **simple interest formula**.

$I = PRT$ (Interest = principal × rate × time)

Note: The interest rate is normally given per year, so the time must also be written in years. In some books, R is written as $r\%$.

'Per annum' means 'per year'. It will sometimes be abbreviated to 'p.a.'.

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Example 6

Find the simple interest on $16 000 for eight years at 7.5% p.a.

Solution

$$\begin{aligned} I &= PRT \\ &= 16\,000 \times 7.5\% \times 8 \\ &= 16\,000 \times 0.075 \times 8 \\ &= 9600 \end{aligned}$$

Thus, the simple interest is $9600.

Reverse use of the simple interest formula

There are four pronumerals in the formula $I = PRT$. If any three are known, then substituting them into the simple interest equation allows the fourth to be found.

Example 7

John borrows $120 000 from his parents to put towards an apartment. His parents agree that John should only pay simple interest on what he borrows. Ten years later, John repays his parents $216 000, which includes simple interest on the loan. What was the interest rate?

Solution

$P = 120\,000$ and $T = 10$.

The total interest paid was $216 000 – $120 000 = $96 000, so $I = 96\,000$

$$\begin{aligned} I &= PRT \\ 96\,000 &= 120\,000 \times R \times 10 \\ R &= \frac{96\,000}{1\,200\,000} \times \frac{100}{1}\% \quad \text{(Interest rates are normally written as percentages.)} \\ &= 8\% \end{aligned}$$

The interest rate was 8%.

Simple interest formula

- Suppose that a principal $$P$ is invested for T years at an interest rate R p.a. Then the total interest $$I$ is given by:

  $I = PRT$

- If the interest rate R is given per year, the time T must be given in years.
- The formula has four pronumerals. If any three are known, the fourth can be found by substitution and solving the resulting equation.

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Exercise 1A

1 Express each percentage as a decimal.

a 56% **b** 8.2% **c** 12% **d** 3.75%

e 215% **f** 0.8% **g** $88\frac{1}{4}\%$ **h** $\frac{7}{8}\%$

2 Express each percentage as a fraction in lowest terms.

a 45% **b** 64% **c** $67\frac{1}{2}\%$ **d** $66\frac{2}{3}\%$

e 8.25% **f** 5.6% **g** 120% **h** 150%

i 7.25% **j** $12\frac{3}{4}\%$ **k** $\frac{1}{2}\%$ **l** 7.8%

3 Express each fraction or decimal as a percentage.

a $\frac{4}{5}$ **b** $\frac{7}{8}$ **c** $\frac{7}{16}$ **d** $1\frac{1}{2}$

e $\frac{9}{20}$ **f** $\frac{5}{3}$ **g** 0.46 **h** 0.025

i 1.4 **j** 1.125 **k** 0.000 75 **l** $2\frac{1}{4}$

4 Copy and complete this table.

	Percentage	Fraction	Decimal
a	64%		
b		$\frac{3}{5}$	
c			0.16
d	20.5%		
e			1.4
f		$\frac{5}{8}$	

5 Evaluate each amount, correct to two decimal places.

a 15% of 60 **b** 36% of 524

c 120% of 436 **d** 140.5% of 720

e 3.8% of 73 **f** 0.5% of 220

6 Evaluate each amount, correct to the nearest cent where necessary.

a 52% of \$50 **b** 24.2% of \$1050

c 110% of \$1590 **d** 0.30% of \$900

e $8\frac{1}{4}\%$ of \$2000 **f** $\frac{3}{4}\%$ of \$1060

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7 Find what percentage the first quantity is of the second quantity, correct to one decimal place.

a 9 km, 150 km
b \$5, \$400
c 28 kg, 600 kg
d 80 m, 50 m

8 Find what percentage the first quantity is of the second quantity, correct to two decimal places. You will first need to express both quantities in the same unit.

a 48 cents, \$10.00
b 3.4 cm, 2 m
c 28 hours, 4 weeks
d 250 m, 8 km
e 40 km, 1250 m
f 1 day, 2 years

9 There are 640 students at a primary school, 7% of whom have red hair. Calculate the number of students in the school who have red hair.

10 A sample of a certain alloy weighs 2.6 g.

a Aluminium makes up 58% of the alloy. What is the weight of the aluminium in the sample?
b The percentage of lead in the alloy is 0.28%. What is the weight of the lead in the sample?

11 A soccer match lasted 94 minutes (including injury time). If Team A was in possession for 65% of the match, for how many minutes and seconds was Team A in possession?

12 A football club with 15 000 members undertook a membership drive, and the membership increased by 110%.

a How many new members joined the club?
b What is the size of the club's membership now?

Example 1

13 Find the original quantity, given that:

a 5% of it is \$24
b 30% of it is 72 minutes
c 90% of it is 216 cm
d 7% of it is \$15.26
e 0.5% of it is 4 mm
f 15% of it is 56 mm

14 Sometimes customers pay a deposit on an item and then later pay the rest of the full price. Find the full price when a deposit of \$570 is 30% of the full price.

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Example 2 **15** Find the selling price if the commission and the commission rate are as given.

a Commission $46, rate 8%

b Commission $724, rate 5.6%

Example 3 **16** Find the percentage profit or loss on costs in these situations.

a Costs $26 000 and sales $52 000

b Costs $182 000 and sales $150 000

Example 4 **17** **a** A company made a profit of $28 000, which was a 5.4% profit on its costs. Find the costs and the total sales.

b A company made a loss of $750 000, which was a 6.5% loss on its costs. Find the costs and the total sales.

Example 5 **18** This question uses the income tax rates in the fictional nation of Plusionta. They are:

- There is no tax on the first $12 000 that a person earns in any one year.
- From $12 001 to $30 000, the tax rate is 15c for each dollar over $12 000.
- From $30 001 to $75 000, the tax rate is 25c for each dollar over $30 000.
- For incomes exceeding $75 000, the tax rate is 35c for each dollar over $75 000.

a Find the income tax payable on:

i $9000 **ii** $15 000 **iii** $38 000 **iv** $400 000

b What percentage of each person's income was paid in income tax in parts **i–iv** of part **a**?

c Find the income if the income tax on it was:

i $1580 **ii** $3860 **iii** $15 200 **iv** $15 000

Example 6 **19** $20 000 is invested at 8% p.a. simple interest for five years.

a How much interest will be earned each year?

b Use the formula $I = PRT$ to find how much interest will be earned over the five-year period.

20 Find the total simple interest earned in each investment.

a $4000 for three years at 6% p.a.

b $7500 for six years at 4.5% p.a.

Example 7 **21** Find the rate R in each simple interest investment.

a Interest of $7200 on $8000 for 12 years

b Interest of $3 400 000 on $12 500 000 for four years

22 Find the time T involved in each simple interest investment.

a Interest of $2500 on $1000 at 5% p.a. **b** Interest of $91 200 on $30 000 at 8% p.a.

23 Find the principal P in each simple interest investment.

a Interest of $4320 at 4.8% p.a. for six years

b Interest of $5020 at 6.75% p.a. for three years

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1B Percentage increase and decrease

When a quantity is increased or decreased, the change is often expressed as a percentage of the original amount.

This section reviews a concise method of dealing with percentage increase and decrease. The method will be applied in various ways throughout the remaining sections of the chapter.

Percentage increase

The Shining Path Cleaning Company made a profit of $421 000 last year, and increased its profit this year by 23%.

We can find the new profit in one step by using the fact that the new profit is $100\% + 23\% = 123\%$ of the old profit.

$$\begin{aligned}\text{New profit} &= 421\,000 \times 123\% \\ &= 421\,000 \times 1.23 \\ &= \$517\,830\end{aligned}$$

When using a calculator, this is a simpler method than calculating the profit separately and adding it on. It will also allow us to handle repeated increases more easily and will make it simpler to reverse the process.

Percentage decrease

The same method can be used to calculate percentage decreases. For example, Grey Gully Station recently sold 41% of its 2288 head of cattle to the meatworks.

We can calculate how many head of cattle the station now has by using the fact that $100\% - 41\% = 59\%$ of its cattle remain.

$$\begin{aligned}\text{Number of remaining head of cattle} &= 2288 \times 59\% \\ &= 2288 \times 0.59 \\ &\approx 1350 \text{ (Correct to the nearest integer.)}\end{aligned}$$

Percentage increase and decrease

- To increase an amount by, say, 15%, multiply by $1 + 0.15 = 1.15$.
- To decrease an amount by, say, 15%, multiply by $1 - 0.15 = 0.85$.

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Finding the percentage increase or decrease

The method used in the following example is in keeping with the other methods covered in this chapter. It requires fewer calculations than finding the actual increase or decrease and then expressing that change as a percentage of the original amount. In all cases, subtracting the calculated percentage by 100% determines the percentage change.

Example 8

The water stored in the main Warrabimbie Dam has increased from 1677 gigalitres to 2043 gigalitres in three months. What percentage increase is this?

Solution

$$\begin{aligned}\frac{\text{New storage}}{\text{Old storage}} &= \frac{2043}{1677} \times \frac{100}{1}\% \\ &\approx 121.82\% \text{ (Correct to the nearest 0.01\%.)}\end{aligned}$$

Thus, the storage has increased by about $121.82\% - 100\% = 21.82\%$.

Reversing the process to find the original amount

Harry claims that his mathematics mark of 78 constitutes a 45% increase on his previous mathematics mark. What was his previous mark?

This mark is $100\% + 45\% = 145\%$ of the previous mark.

Hence, this mark $=$ previous mark $\times$ 1.45

$$\begin{aligned}\text{Reversing this, previous mark} &= \text{this mark} \div 1.45 \\ &= 78 \div 1.45 \\ &\approx 54 \text{ (Correct to the nearest mark.)}\end{aligned}$$

Thus, to find the original amount, we divide by 1.45, because dividing by 1.45 is the reverse process of multiplying by 1.45.

Exactly the same principle applies when an amount has been decreased by a percentage, as shown in the following example.

Example 9

The price of bananas has decreased by 70% over the last year to \$3 per kilogram. What was the price a year ago?

Solution

The new price is $100\% - 70\% = 30\%$ of the old price.

Hence, new price $=$ old price $\times$ 0.30

$$\begin{aligned}\text{Reversing this, old price} &= \text{new price} \div 0.30 \\ &= 3.00 \div 0.3 \\ &= \$10 \text{ per kilogram}\end{aligned}$$

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 Updated June 2022

Example 10

Ria has had mixed results with the shares that she bought three years ago. Shares in White Manufacturing rose 37% to \$14.56, but shares in Black Tile Distributors fell 28% to \$8.76. Find the prices she originally paid for these two shares, correct to the nearest cent.

Solution

White Manufacturing shares are now $100\% + 37\% = 137\%$ of their previous value.

$$\text{Thus, new value} = \text{original price} \times 1.37$$

$$\begin{aligned}\text{Reversing this, original price} &= 14.56 \div 1.37\\ &\approx \$10.63 \text{ (Correct to the nearest cent.)}\end{aligned}$$

Black Tile Distributors shares are now $100\% - 28\% = 72\%$ of their original value.

$$\text{Thus, new value} = \text{original price} \times 0.72$$

$$\begin{aligned}\text{Reversing this, original price} &= 8.76 \div 0.72\\ &\approx \$12.17 \text{ (Correct to the nearest cent.)}\end{aligned}$$

Finding the original amount

- To find the original amount after an increase of, say, 15%, divide the new amount by $1 + 0.15 = 1.15$.
- To find the original amount after a decrease of, say, 15%, divide the new amount by $1 - 0.15 = 0.85$.

Discounts

It is common for a shop to **discount** the price of an item. This can be done to sell stock of a slow-moving item more quickly, or simply to attract customers into the shop.

Discounts are normally expressed as a percentage of the original price.

Example 11

The Tie Knot Shop is expecting new stock and needs to make room on its shelves. It has discounted all its prices by 45% to try to sell some of its existing stock.

a What is the discounted price of a tie with an original price of \$90?

b What was the original price of a tie with a discounted price of \$90?

Solution

The discounted price of each item is $100\% - 45\% = 55\%$ of the old price.

a
$$\begin{aligned}\text{Discounted price} &= \text{original price} \times 0.55\\ &= 90 \times 0.55\\ &= \$49.50\end{aligned}$$

b
$$\begin{aligned}\text{Original price} &= \text{discounted price} \div 0.55\\ &= 90 \div 0.55\\ &\approx \$163.64\end{aligned}$$

(Correct to the nearest cent.)

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The GST

In 1999 the Australian Government introduced a Goods and Services Tax, or GST for short. This tax applies to nearly all goods and services in Australia.

The current rate of GST is 10% of the pre-tax price of the good or service.

- When GST applies, GST is added to the pre-tax price. This is easily done by multiplying by 1.10.
- Conversely, if a quoted price already includes the GST, the pre-tax price is obtained by dividing by 1.10.

Example 12

The current GST rate is 10% of the pre-tax price.

a The pre-tax price of a large fridge is $2150. What will the fridge cost after GST is added, and how much will be paid to the government?

b I recently paid $495 to have a tree pruned. What was the price before adding GST, and how much GST was paid to the government?

Solution

The after-tax price is 110% of the pre-tax price.

a After-tax price $= 2150 \times 1.10$
$= \$2365$

Tax $= 2365 - 2150$
$= \$215$

Alternatively, tax $= 2150 \times 0.1$
$= 215$

After-tax price $= \$2150 + \$215$
$= \$2365$

b Pre-tax price $= 495 \div 1.10$ (Divide by 1.10 to reverse the process.)
$= \$450$

Tax $= 495 - 450$
$= \$45$

Inflation

The prices of goods and services in Australia and other countries usually increase by a small amount every year. This gradual rise in prices is called **inflation**, and is measured by taking the average percentage increase in the prices of a large range of goods and services.

Other things, such as salaries and pensions, are often adjusted automatically every year to take account of inflation.

High rates of inflation are damaging to society, and governments generally try to keep inflation low.

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Cambridge University Press
Updated June 2022

Example 13

The economy in Espirito Santo is booming as a result of its mineral exports, but unfortunately, with a change of government, inflation has also taken hold. Last year inflation was 28%, meaning that on average, prices have increased by 28% over the last year.

a If the average winter electricity bill was \$460 last year, give an estimate of this year's bill, based on the inflation rate.

b If a new Hunter Flash station wagon now costs \$38 000, give an estimate of its cost a year ago, based on the inflation rate, correct to the nearest \$100.

Solution

We estimate this year's prices as $100\% + 28\% = 128\%$ of last year's prices.

a $$\begin{aligned}\text{Estimate of this year's bill} &= 460 \times 1.28\\ &\approx \$588.80\end{aligned}$$

b $$\begin{aligned}\text{Estimate of cost last year} &= 38\,000 \div 1.28\\ &\approx \$29\,700 \text{ (Correct to the nearest \$100.)}\end{aligned}$$

Exercise 1B

1 Increase each amount by the given percentage.

a \$570, 10% **b** \$9320, 5% **c** \$456, 6% **d** \$3120, 8%

2 Decrease each amount by the given percentage.

a \$9000, 10% **b** \$4560, 5% **c** \$826, 3% **d** \$9520, 4%

3 Traffic on all roads has increased by an average of 12% during the past 12 months. By multiplying by $112\% = 1.12$, estimate the number of vehicles now on a road where the number of vehicles a year ago was:

a 32 000 per day **b** 153 000 per day **c** 248 per day

4 Rainfall across Victoria has decreased over the last 10 years by about 38%. By multiplying by $62\% = 0.62$, estimate, correct to the nearest mm, the annual rainfall this year at a place where the rainfall 10 years ago was:

a 700 mm **b** 142 mm **c** 1268 mm

Example 8

5 The number of shops in different shopping centres in Borrington changed from 2011 to 2012, but by quite different percentage amounts. Find the percentage increase or decrease in the number of shops where the numbers during 2011 and 2012, respectively, were:

a 200 and 212 **b** 85 and 160 **c** 156 and 122 **d** 198 and 110

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Example 9 **6** **a** An amount is decreased by 10% and the new amount is \$567. What was the original amount?

b An amount is increased by 10% and the new amount is \$5676. What was the original amount?

7 Phoenix Finance Pty Ltd recently issued bonus shares that increased by 14% the number of shares held by each of the company's shareholders. By dividing by 114% = 1.14, find the original holding of a shareholder who now holds:

a 228 shares **b** 8321 shares **c** 77 682 shares

Example 10 **8** A research institute is trying to find out how much water Lake William had in it 8000 years ago. The lake now contains 7600 megalitres, but there are various conflicting theories about the percentage change over the last 8000 years. Find how much water the lake had 8000 years ago, correct to the nearest 10 megalitres, if in the last 8000 years the volume has:

a risen by 60% **b** fallen by 33% **c** risen by 312% **d** fallen by 88%

Example 11 **9** A clothing store is offering a 35% discount on all its summer stock. Find the discounted price of an item with a marked price of:

a \$80 **b** \$48 **c** \$680 **d** \$1.60

Example 11 **10** A furniture shop is offering a 55% discount at its end-of-year sale. Find the original price of an item with a discounted price of:

a \$1400 **b** \$327 **c** \$24.50

11 Mr Brown bought parcels of shares in June last year. He has a spreadsheet showing the value at which he bought his shares, the value at 31 December last year, and the percentage increase or decrease in their value. (Decreases are shown with a negative sign.) Unfortunately, a virus has corrupted one entry in each row of his spreadsheet. Help him fix his spreadsheet by calculating the missing values, correct to two decimal places.

Company	Value at purchase	Value at 31 December	Percentage increase
a	\$20 000		40%
b	\$14 268		–58%
c	\$3128.72		341.27%
d		\$80 000	15%
e		\$114 262	258.3%
f		\$32 516.24	–92.29%
g	\$50 000	\$52 000	
h	\$21 625	\$34 648	
i	\$48 372.11	\$40 072.11	

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Example 12

12 The GST is a tax on most goods and services, which is calculated at the rate of 10% of the pre-tax price.

a Find the after-tax price on goods or services with a pre-tax price of:

i \$2740 **ii** \$134 000 **iii** \$8.20

b Find the pre-tax price on goods or services with an after-tax price of:

i \$3927 **ii** \$426 877 **iii** \$3.19

c Find the after-tax price on goods or services on which the GST is:

i \$442 **ii** \$347 114 **iii** \$0.47

Example 13

13 **a** Prices have increased with inflation by an average of 3.4% since the same time last year. Estimate today's price for an item that one year ago cost:

i \$3000 **ii** \$24.15 **iii** \$361

b Estimate the price a year ago of an item that now costs:

i \$4200 **ii** \$14.30 **iii** \$76

(When finding your estimate, assume that the average increase applies to all items.)

14 **a** A table originally priced at \$370 was increased in price by 100%. What percentage discount will restore it to its original price?

b The number of daily passengers on the Jarrabine ferry-bus was 156, and in one year it increased by 25%. What percentage decrease next year would restore the number of passengers to its original value?

c Thien had savings of \$15 000, but he spent 45% of this last year. By what percentage of the new amount must he increase his savings to restore them to their original value?

d The profit of the Audry Goldfish Guild last year was \$3650, but this year it decreased by 42%. By what percentage must the profit increase next year to restore it to its original value?

15 **a** Find, correct to two decimal places, the percentage decrease necessary to restore a quantity to its original value if it has been increased by:

i 10% **ii** 18%

iii 360% **iv** 4.1%

b Find, correct to two decimal places, the percentage increase necessary to restore a quantity to its original value if it has been decreased by:

i 10% **ii** 18%

iii 80% **iv** 4.1%

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1C Repeated increases and decreases

Repeated increases

The method used in the last section becomes very useful when two or more successive increases or decreases are applied, because the original amount can simply be multiplied successively by two or more factors. Here is a typical example.

Example 14

Internet sales of Ferret Virus Guard have been increasing dramatically. Three years ago the number of registered users was 20 874. In the three years since then, the number of users rose by 8.1% in the first year, a further 46.4% in the second year, and then a further 112.8% in the third year.

a How many users were there at the end of the first year?

b How many users were there at the end of the second year?

c How many users are there now, at the end of the third year?

d What has the percentage increase in users been over the three years?

Solution

a After one year, the number of users was 108.1% of the original number.

$$\begin{aligned}\text{Hence, number after one year} &= 20\,874 \times 1.081 \\ &\approx 22\,565 \text{ (Correct to the nearest integer.)}\end{aligned}$$

b After two years, the number of users was 146.4% of the number after one year.

$$\begin{aligned}\text{Hence, number after two years} &= 20\,874 \times 1.081 \times 1.464 \\ &\approx 33\,035 \text{ (Correct to the nearest integer.)}\end{aligned}$$

c After three years, the number of users is 212.8% of the number after two years.

$$\begin{aligned}\text{Hence, number after three years} &= 20\,874 \times 1.081 \times 1.464 \times 2.128 \\ &\approx 70\,298 \text{ (Correct to the nearest integer.)}\end{aligned}$$

d
$$\begin{aligned}\text{Number after three years} &= \text{original number} \times 1.081 \times 1.464 \times 2.128 \\ &\approx \text{original number} \times 3.37\end{aligned}$$

This is about 337% of the original amount, or 337% − 100% = 237% more than it.
Hence, the number of users has increased by about 237% over the three years.

Note: We could have solved part **d** by calculating $\frac{70\,298}{20\,874} \approx 3.37$. It is best to keep the answer to **c** in your calculator for this calculation.

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Repeated decreases

The same method also applies to percentage decreases, as in Example 15.

Example 15

The median value of houses in the city of Winchester was reasonably stable for several years at \$340 000. After the local lead mine closed, however, the value of houses fell disastrously. One year later, the median house value had dropped by 58% and, over the next year, the median house value dropped a further 46%.

a What was the median house value one year later?

b What was the median house value two years later?

c What percentage of the original median house value was lost over the two years?

Solution

a One year later, the median value was $100\% - 58\% = 42\%$ of the original.

$$\begin{aligned}\text{Hence, median value after one year} &= 340\,000 \times 0.42\\ &= \$142\,800\end{aligned}$$

b Two years later, the value was $100\% - 46\% = 54\%$ of the value after one year.

$$\begin{aligned}\text{Hence, median value after two years} &= 340\,000 \times 0.42 \times 0.54\\ &= \$77\,112\end{aligned}$$

c
$$\begin{aligned}\text{Median value after two years} &= \text{original value} \times 0.42 \times 0.54\\ &\approx \text{original value} \times 0.23\end{aligned}$$

Hence, about 77% of the value was lost over the two years ($23\% - 100\% = -77\%$).

Combinations of increases and decreases

Some problems involve both increases and decreases. They can be solved in the same way.

Example 16

During July 2011, 34% more patients at St Michael's Hospital were treated than in June 2011. Monthly patient numbers then fell by 40% during August, rose by 30% during September, and finally fell by 24% during October 2011.

a What is the percentage increase or decrease over the four months, correct to the nearest 1%?

b If 5783 patients were treated during the month of June 2011, how many patients were treated during October 2011?

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Solution

a Final monthly number $= \text{original monthly number} \times 1.34 \times 0.60 \times 1.30 \times 0.76$

$\approx \text{original monthly number} \times 0.79$

$79\% - 100\% = -21\%$, so the number has decreased by about 21% over the four months.

b Final monthly number $= \text{original monthly number} \times 1.34 \times 0.60 \times 1.30 \times 0.76$

$= 5783 \times 1.34 \times 0.60 \times 1.30 \times 0.76$

≈ 4594 patients (Correct to the nearest integer.)

In the above example, you may notice that the sum of the percentages is $34\% - 40\% + 30\% - 24\% = 0\%$, but this is completely irrelevant to the problem.

Repeated increases and decreases

- To apply successive increases of, say, 15%, 24% and 38% to a quantity, multiply the quantity by $1.15 \times 1.24 \times 1.38$.
- To apply successive decreases of, say, 15%, 24% and 38% to a quantity, multiply the quantity by $0.85 \times 0.76 \times 0.62$.

Reversing the process to find the original amount

As we have already learned, division reverses the process to find the original amount, as in the following example.

Example 17

The cat population in Grahamsville grew by 145% over a decade, and then fell by 40% over the next decade.

a What was the percentage increase in the cat population over the 20 years?

b If the final population was 10 000, what was the original population 20 years earlier?

Solution

a After one decade, the population was $100\% + 145\% = 245\%$ of the original population.

After the second decade, the population was $100\% - 40\% = 60\%$ of the increased population.

Thus, final population $= \text{original population} \times (2.45 \times 0.60)$

$= \text{original population} \times 1.47$

So the population increased by $147\% - 100\% = 47\%$.

b Reversing this:

Original population $= \text{final population} \div (2.45 \times 0.60)$

$= 10\,000 \div (2.45 \times 0.60)$

$= 6803$ (Correct to the nearest integer.)

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Reversing repeated increases and decreases

- To find the original quantity after successive increases of, say, 15%, 24% and 38% to that quantity, divide the final quantity by $(1.15 \times 1.24 \times 1.38)$.
- To find the original quantity after successive decreases of, say, 15%, 24% and 38% to that quantity, divide the final quantity by $(0.85 \times 0.76 \times 0.62)$.

Successive equal percentage increases and decreases

When all the percentage changes are the same, we can use powers to reduce the number of calculations.

Example 18

The number of people visiting the Olympus Cinema each week has been decreasing by 4% per year for the last 10 years.

a What is the percentage decrease in weekly attendance over the 10 years?

b If there are 1250 visitors per week now, what was the weekly attendance 10 years ago?

Solution

Each year the weekly attendance is 96% of the previous year's weekly attendance.

a Final weekly attendance

$= \text{original weekly attendance} \times 0.96 \times 0.96 \times \ldots \times 0.96$

$= \text{original weekly attendance} \times (0.96)^{10}$

$\approx \text{original weekly attendance} \times 0.66$

Here the cinema's weekly attendance has decreased by about 34% over the 10 years.

b From part **a**:

Final weekly attendance $=$ original weekly attendance $\times (0.96)^{10}$

Reversing this:

$$\begin{aligned}\text{Original weekly attendance} &= \text{final weekly attendance} \div (0.96)^{10}\\ &= 1250 \div (0.96)^{10}\\ &\approx 1880 \text{ (Correct to the nearest integer.)}\end{aligned}$$

Exercise 1C

1 **a** Find the final value after \$10 000 is successively increased by 5%, 8% and 10%.

b Find the final value after \$10 000 is successively decreased by 8%, 7% and 6%.

c Find the final value after \$90 000 has been increased by 10% ten times. (Give your answer correct to the nearest cent.)

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Example 14

2 Three years ago, apples cost $2.80 per kg, but the price has increased by 8%, 15% and 10% in the past three successive years. Multiply by $1.08 \times 1.15 \times 1.1$ to find the price of apples now.

3 The dividend per share in Knowledge Bank Company has risen over the last four years by 32%, 112%, 155% and 8%, respectively. Find the total dividend received by a shareholder whose dividend four years ago was:

a $1000 **b** $12 472 **c** $16.64 **d** $512.21

4 Land rates in Crookwell Shire have risen by 6% every year for the last seven years.

a By what percentage have the land rates risen over the seven-year period?

b Find the rates now payable by a landowner whose rates seven years ago were:

i $1000 **ii** $17 268.24 **iii** $216.04

Example 15

5 Since a new 'Fitness for Freedom' program was introduced in a community, the number of people classified as overweight in that community has been falling. In four successive years, the number of overweight people fell by 4.8%, 7.1%, 10.5% and 6.2%, respectively. Find, correct to two decimal places, the percentage decrease over the four-year period.

Example 16

6 Calculate the total increase or decrease in a quantity when:

a it is increased by 20% and then decreased by 20%

b it is increased by 80% and then decreased by 80%

Example 17

7 The price of beans has been rising. The price has risen by 10%, 15% and 35% in three successive years, and they now cost $3.40 per kg. By dividing successively by 1.35, then by 1.15, and then by 1.10, find the:

a price one year ago **b** price two years ago **c** original price three years ago

Example 18

8 Shares in Value Radios have been falling by 18% per year for the last five years.

a Find the present worth of a parcel of shares with an original worth five years ago of:

i $1000 **ii** $24 000 **iii** $11 328 512

b By what percentage has the value fallen over the five-year period?

9 A particular strain of bacteria increases its population on a certain prepared Petri dish by 18% every hour. Calculate the size of the original population four hours ago if there are now:

a 10 000 bacteria **b** 1 000 000 bacteria **c** 120 000 bacteria

10 A potato is taken from boiling water at 100°C and placed in a fridge at 0°C. Every minute after this, the temperature of the potato drops by 16%.

a Find the temperature of the potato after:

i 4 minutes **ii** 8 minutes **iii** 20 minutes

b François measures the temperature of the potato and finds it to be 12°C. Find its temperature:

i 3 minutes ago **ii** 6 minutes ago **iii** 10 minutes ago

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11 Here is a table of the annual inflation rate in Australia for the years ending 30 June 2005 to 30 June 2010 (from the Reserve Bank of Australia website).

Year	2005	2006	2007	2008	2009	2010
Inflation rate	2.3%	2.7%	3.8%	2.3%	4.4%	1.8%

Calculate the percentage increase in prices, correct to one decimal place:

a over the whole six-year period **b** during the first four-year period

12 The radioactivity of any sample of the element iodine-131 decreases by 55% every seven days. Find the percentage reduction in radioactivity over each of the periods given below. (Give percentages correct to one decimal place.)

a 3 weeks **b** 10 weeks **c** 26 weeks

13 **a** A coat is discounted by 50%, and the resulting price is then increased by 50%. By what percentage is the price increased or decreased from its original value?

b The price of a coat is increased by 50%, and the resulting price is then decreased by 50%. By what percentage is the price increased or decreased from its original value?

c Can you explain the relationship between your answers to parts **a** and **b**?

1D Compound interest

With simple interest, the interest is always calculated on the original amount: the principal.

With **compound interest**, the interest is applied periodically to the balance of an account (whether it is an amount borrowed or an amount invested). For example, when interest is compounded annually (per annum), the interest is calculated on the balance at the end of each year. The interest earned in one year plus the previous balance becomes subject to interest calculations in the next year, and so on.

The first example below is done using the method of percentage increase developed in the previous two sections. After that, we will develop a general formula for compound interest.

Example 19

Siri has invested \$100 000 for five years with the St Michael Bank. The bank pays her interest at the rate of 6% p.a., compounded annually.

a How much will the investment be worth at the end of:

i one year? **ii** two years? **iii** five years?

b What is the percentage increase of her original investment at the end of five years?

c What is the total interest earned over the five years?

d What would the simple interest on the five-year investment have been, assuming the same interest rate of 6% p.a.?

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Solution

Each year the investment is worth 106% of its value the previous year.

a **i** Balance at the end of one year $= 100\,000 \times 1.06$
$= \$106\,000$

ii Balance at the end of two years $= 100\,000 \times 1.06 \times 1.06$
$= 100\,000 \times (1.06)^2$
$= \$112\,360$

iii Balance at the end of five years $= 100\,000 \times 1.06 \times 1.06 \times 1.06 \times 1.06 \times 1.06$
$= 100\,000 \times (1.06)^5$
$\approx \$133\,822.56$ (Correct to the nearest cent.)

b Final amount $=$ original amount $\times (1.06)^5$
$\approx$ original amount $\times 1.3382$

So the total increase over five years is about 33.82%.

c Total interest $\approx 133\,822.56 - 100\,000$
$\approx \$33\,822.56$

d Simple interest $= PRT$
$= 100\,000 \times 0.06 \times 5$
$= \$30\,000$

We will often use the word 'amount' for the 'balance' of the account.

A formula for compound interest

It is not difficult to develop a formula for compound interest, provided that the interest rate is constant throughout the loan.

Suppose that a principal $\$P$ is invested for n units of time at an interest rate R per unit time, and that compound interest is paid. (The unit of time may be years, months, days or any other length of time.)

Let $\$A_n$ be the amount that the investment is worth after n units of time. That is, A_n is the balance of the account.

At the end of each unit of time, the amount increases by a factor of $1 + R$.

Thus, $A_1 = P(1 + R)$
and $A_2 = A_1(1 + R)$
$= P(1 + R) \times (1 + R) = P(1 + R)^2$
and $A_3 = A_2(1 + R)$
$= P(1 + R)^2 \times (1 + R) = P(1 + R)^3$

Continuing this process for n units of time gives:

$$A_n = P(1 + R)^n$$

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Compound interest

Suppose that a principal $$P$ is invested at an interest rate R per unit time, and that compound interest is paid. The amount $$A_n$ that the investment is worth after n units of time is:

$$A_n = P(1 + R)^n$$

Thus, in the previous example, the amount at the end of five years would be calculated as:

Substituting $P = 100\,000$, $R = 0.06$ and $n = 5$

$$\begin{aligned} A_5 &= 100\,000 \times (1.06)^5 \\ &\approx \$133\,822.56 \end{aligned}$$

Example 20

Wesley has retired, and he has invested $200 000.

a How much will his investment grow to after four years if he has invested the money at 0.5% per month compound interest?

b How much would it have grown to had Wesley invested the money at 6% p.a. compound interest for four years?

c How much would the investment grow to if he had invested it at 3% per six months compound interest for four years?

Solution

Each amount is calculated to the nearest cent.

a The money is invested for 48 months, and $R = 0.5\% = 0.005$ per annum.

$$\begin{aligned} A_n &= P(1 + R)^n \\ A_{48} &= 200\,000 \times (1.005)^{48} \\ &\approx \$254\,097.83 \end{aligned}$$

b The money is invested for four years, and $R = 6\% = 0.06$ per annum.

$$\begin{aligned} A_n &= P(1 + R)^n \\ A_4 &= 200\,000 \times (1.06)^4 \\ &\approx \$252\,495.39 \end{aligned}$$

c The money is invested for 8 periods of six months, and $R = 3\% = 0.03$ per six-month period.

$$\begin{aligned} A_n &= P(1 + R)^n \\ A_8 &= 200\,000 \times (1.03)^8 \\ &\approx \$253\,354.02 \end{aligned}$$

Note: The calculations of **a** and **b** in the above example show that 0.5% per month, compounded monthly, earns slightly more interest than 6% p.a., compounded annually. With compound interest, the more frequent the compounding, the greater the amount of interest.

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Compound interest on a loan

Exactly the same principles apply when someone borrows money from a bank and the bank charges compound interest on the loan. If no repayments are made, the amount owing compounds in the same way, and can grow quite rapidly.

Example 21

Assad is setting up a home renovation business and needs to borrow \$360 000 from a bank. The bank will charge him interest of 1% per month. Assad will pay the whole loan off all at once eight years later.

a How much will Assad owe the bank at the end of one year?

b How much will Assad owe the bank at the end of two years?

c How much will Assad owe the bank at the end of eight years?

d What is the percentage increase in the money owed at the end of eight years?

e What is the total interest that Assad will pay on the loan?

f What would the simple interest on the loan have been, assuming the same interest rate of 1% per month?

Solution

Each amount is calculated to the nearest cent.

The units of time are months, and $R = 1\% = 0.01$ per month.

a Amount owing at the end of one year $= P(1 + R)^n$

$$= 360\,000 \times (1.01)^{12}$$
$$\approx \$405\,657.01$$

b Amount owing at the end of two years $= 360\,000 \times (1.01)^{24}$

$$\approx \$457\,104.47$$

c Amount owing at the end of eight years $= 360\,000 \times (1.01)^{96}$

$$\approx \$935\,738.25$$

d Final amount $=$ original amount $\times (1.01)^{96}$

$\approx$ original amount $\times 2.60$

So the percentage increase over eight years is approximately 160%.

e Total interest $\approx 935\,738.25 - 360\,000$

$\approx \$575\,738.25$

f Simple interest $= PRT$

$= 360\,000 \times 0.01 \times 96$

$= \$345\,600$

Note: Making no repayments on a loan that is accruing compound interest can be a risky business practice because, as this example makes clear, the amount owing grows with increasing rapidity as time goes on. Similarly, not making regular payments on a credit card can be disastrous.

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Reversing the process to find the original amount

If we are given the final amount A_n, the interest rate R and the number n of units of time, we can substitute into the compound interest formula and solve the resulting equation to find the principal P.

Example 22

Carla wants to borrow money for six years to start a business, and then pay all the money back, with interest, at the end of that time. The bank will charge compound interest at a rate of 0.8% per month, and will limit her final debt, including interest, to \$1 000 000. What is the maximum amount that Carla can borrow?

Solution

The units of time are months, so $n = 72$ and $R = 0.8\% = 0.008$ per month.

Hence, $A_{72} = P \times (1.008)^{72}$

Substituting $A_{72} = 1\,000\,000$, the maximum amount that Carla can owe at the end of the loan:

$$1\,000\,000 = P \times (1.008)^{72}$$

$$P = 1\,000\,000 \div (1.008)^{72}$$

$$\approx \$563\,432.23$$

Carla can borrow a maximum of \$563 432.23.

Exercise 1D

Note: This exercise is based on the compound interest formula $A_n = P(1 + R)^n$. Remember that if an interest rate is given 'per annum' then it is assumed that the compounding occurs annually, and if it is given per month then the compounding occurs every month, and so on.

Example 19

1 Ming invested \$100 000 for five years at 7% p.a. interest, compounded annually.

- **a** Find the amount invested after one year.
- **b** Find the amount invested after two years.
- **c** Find the amount invested after five years.
- **d** Find the percentage increase in the investment over the five-year period, correct to two decimal places.
- **e** Find the total interest earned over the five years.
- **f** Find the simple interest on the principal of \$100 000 over the five years at the same annual interest rate.

2 The population of a town increases at a rate of 5.8% p.a. for 10 years, compounded annually. Initially, the population was 34 000.

- **a** What was the population at the end of the 10-year period?
- **b** What was the total percentage increase, correct to the nearest 1%?

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Example 20

3 A couple takes out a housing loan of $380 000 over a period of 25 years. They make no repayments during the 25-year period.

a **i** How much money would they owe if compound interest were payable at 6% p.a.?

ii What would the percentage increase in the debt be, correct to the nearest 1%?

b **i** How much money would they owe if compound interest were payable at 0.5% per month?

ii What would the percentage increase in the debt be, correct to the nearest 1%?

Example 21

4 Emmanuel has borrowed $300 000 for seven years at 9% p.a. interest, compounded annually, in order to start his carpentry business. He intends to pay the whole amount back, plus interest, at the end of the seven years.

a Find the amount owing after one year.

b Find the amount owing after seven years.

c Find the percentage increase in the debt over the seven-year period, correct to two decimal places.

d Find the total interest charged over the seven years.

e Find the simple interest on the principal of $300 000 over the seven-year period at the same annual interest rate.

5 **a** Find the compound interest on $1000 at 12% p.a. for 100 years.

b Find the compound interest on $1000 at 1% per month for 100 years.

c Find the simple interest on $1000 at 12% p.a. for 100 years.

6 A student borrows $20 000 from a bank for six years. Compound interest at 9% p.a. must be paid.

a How much money is owed to the bank at the end of the six-year period?

b How much of this amount is interest?

Example 22

7 Money borrowed at an interest rate of 8% p.a. grew to $100 000 in seven years. Find:

a the original amount invested

b the total percentage increase in the investment, correct to the nearest 1%

8 Emily wants to invest some money now so that it will grow to $250 000 in eight years' time. The compound interest rate is 0.5% per month.

a How much should she invest now?

b What will the total percentage increase be, correct to the nearest 1%?

9 A bank offers 0.7% per month compound interest. How much needs to be invested if the investment is to be worth $100 000 in:

a 10 years?

b 25 years?

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10 The population of the mountain town of Granite Peak has been growing at 7.4% every year and has now reached 80 000. Find the population:

a one year ago

b two years ago

c five years ago

d 10 years ago

11 Mr Brown has had further difficulties with the virus that attacked his spreadsheet entries. The spreadsheet calculated interest compounded annually on various amounts, at various interest rates, for various periods of time. Help him reconstruct the missing entries.

	Principal	Rate	Number of years	Final amount	Total interest
a	\$3000	7%	25		
b	\$3 000 000	5.2%	12		
c		7%	25	\$3000	
d		5.2%	12	\$3 000 000	

12 **a** Mr Yang invested \$90 000 at a compound interest rate of 6% p.a. for three years. The tax office wants to know exactly how much interest he earned each year. Calculate these figures for Mr Yang.

b Repeat these calculations with the rate of interest of 0.5% per month.

13 WestPlaza Holdings sold one of its shopping centres for \$20 000 000 and invested the money at a daily compound interest rate of 0.016%. How much interest did the company earn in the first year?

14 Find the percentage increase in each situation (correct to the nearest 0.01%).

a \$100 000 is borrowed at a compound interest rate of 0.01% per day for one year.

b \$1 000 000 is borrowed at a compound interest rate of 0.02% per day for one year.

15 Find the total percentage growth, correct to the nearest 0.1%, in a compound interest investment:

a at 15% p.a. for two years

b at 10% p.a. for three years

c at 6% p.a. for five years

d at 5% p.a. for six years

e at 3% p.a. for 10 years

f at 2% p.a. for 15 years

g What do you observe about these results?

16 A doctor took out a six-year loan to start a medical practice. For the first three years, he was charged compound interest at a rate of 9% p.a. For the second three-year period, he was charged compound interest at a rate of 13% p.a. Find the total percentage increase in the money owing, correct to the nearest 1%.

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1E Compound depreciation

Depreciation occurs when the value of an asset reduces as time passes. For example, a person may buy a car for \$50 000, but after six years the car will be worth a lot less, because the motor will be worn, the car will be out of date, the body and interior may have a few scratches, and so on.

Accountants usually make the assumption that an asset, such as a car, depreciates at the same rate every year. This rate is called the **depreciation rate**. In the following example, the depreciation rate is taken to be 20%.

In many situations, simple depreciation is used, but in other situations the depreciation is compounded. We will deal only with compound depreciation.

This first example is done using the methods of percentage decrease developed in Sections 1C and 1D. After that, we will develop a general formula for depreciation, as we did for compound interest.

Example 23

A person bought a car six years ago for \$50 000, and assumed that the value of the car would depreciate at 20% p.a.

a What value did the car have at the end of two years?

b What value does the car have now, after six years?

c What is the percentage decrease in value over the six-year period?

d What is the average reduction in value, in dollars p.a., on the car over the six-year period due to depreciation?

Solution

The value each year is taken to be $100\% - 20\% = 80\%$ of the value in the previous year.

a $$\begin{aligned}\text{Value at the end of two years} &= 50\,000 \times 0.80 \times 0.80\\ &= 50\,000 \times (0.80)^2\\ &= \$32\,000\end{aligned}$$

b $$\begin{aligned}\text{Value at the end of six years} &= 50\,000 \times 0.80 \times 0.80 \times 0.80 \times 0.80 \times 0.80 \times 0.80\\ &= 50\,000 \times (0.80)^6\\ &= \$13\,107.20\end{aligned}$$

c $$\begin{aligned}\text{Final value} &= \text{original value} \times (0.80)^6\\ &\approx \text{original value} \times 0.26\end{aligned}$$

Hence, the percentage decrease over six years is about $100\% - 26\% = 74\%$.

d $$\begin{aligned}\text{Depreciation over six years} &= 50\,000 - 13\,107.20\\ &= \$36\,892.80\end{aligned}$$

$$\begin{aligned}\text{Average reduction in value per year} &= \$36\,892.80 \div 6\\ &= \$6148.80 \text{ per year}\end{aligned}$$

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A formula for depreciation

We can develop a formula for depreciation, as we did for compound interest. The two formulas are very similar.

Suppose that an asset originally worth P depreciates at a rate, R, per unit time.

Let A_n be the value of the asset after n units of time.

At the end of each unit of time, the value decreases by a factor of $1 - R$.

$$\begin{aligned} \text{Thus,}\quad A_1 &= P(1 - R) \\ \text{and}\quad A_2 &= A_1(1 - R) \\ &= P(1 - R) \times (1 - R) = P(1 - R)^2 \\ \text{and}\quad A_3 &= A_2(1 - R) \\ &= P(1 - R)^2 \times (1 - R) = P(1 - R)^3 \end{aligned}$$

Continuing this process for n units of time gives:

$$A_n = P(1 - R)^n$$

Compound depreciation

Suppose that an asset with an original value P depreciates at a rate R per unit time. The value A_n of that asset after n units of time is:

$$A_n = P(1 - R)^n$$

Thus, in the previous example, the value at the end of six years would be calculated as:

Substituting $P = 50\,000$, $R = 0.8$ and $n = 6$

$$\begin{aligned} A_6 &= 50\,000 \times (0.8)^6 \\ &= \$13\,107.20 \end{aligned}$$

Example 24

A piece of machinery that cost \$560 000 depreciates at 30% p.a.

a What is its value after six years?

b What is the average reduction in value per year?

c If it had depreciated at 15% p.a., what would its value have been after six years?

Solution

a Here, $R = 0.3$, so $1 - R = 0.7$

$$\begin{aligned} A_n &= P(1 - R)^n \\ A_6 &= 560\,000 \times (0.7)^6 \\ &= \$65\,883.44 \end{aligned}$$

(continued over page)

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b Depreciation $= 560\,000 - 65\,883.44$

$= \$494\,116.56$

Average reduction per year $\approx 494\,117.56 \div 6$

$\approx \$82\,352.76$

c Here, $R = 0.15$, so $1 - R = 0.85$

$A_n = P(1 - R)^n$

$A_6 = 560\,000 \times (0.85)^6$

$\approx \$211\,203.73$

Reversing the process to find the original amount

Given the final depreciated value A_n and the depreciation rate R we can substitute into the depreciation formula and solve the resulting equation to find the original value of an item P.

Example 25

A company buys its staff new cars every four years. At the end of the four years, it offers to sell the cars to the staff on the assumption that they have depreciated at 22.5% p.a. The company is presently offering cars for sale at $9000 each.

a What did each car cost the company originally?

b What is the average reduction in value, in dollars p.a., on each car?

Solution

a Here, $R = 0.225$, so $1 - R = 0.775$

$A_n = P(1 - R)^n$

$9000 = P \times (0.775)^4$

$P = 9000 \div (0.775)^4$

$\approx \$24\,948$ (Correct to the nearest dollar.)

Each car originally cost the company about $24 948.

b Loss of value $\approx 24\,948 - 9000$

$= \$15\,948$

Average loss per year $\approx 15\,948 \div 4$

$= \$3987$

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Exercise 1E

Example 23, 24

1 The landlord of a large block of home units purchased washing machines for its units six years ago for \$600 000, and is assuming a depreciation rate of 30% p.a.

a Find the estimated value after one year.

b Find the estimated value after two years.

c Find the estimated value after six years.

d What is the percentage decrease in value over the six-year period?

e What is the average reduction in value, in dollars p.a., on the washing machines over the six-year period due to depreciation?

2 **a** A computer shop spent \$320 000 installing alarms at its premises. If it depreciated them at 20% p.a., find the estimated value after six years, and the percentage reduction of value over that period.

b The business borrowed the money to install the alarms, paying 8% p.a. compound interest for the six years. How much did it owe at the end of the six years?

3 A school bought a bus for \$90 000, depreciated it at 30% p.a., and sold it again five years later for \$20 000. Was the price that they obtained better or worse than the depreciated value, and by how much?

4 The Online Grocery spent \$4 540 000 buying computers for its offices, and depreciated them for taxation purposes at 40% p.a. Find the value of the computers at the end of each of the first four years, and the amount of the loss that the company could claim against its taxable income for each of those four years.

5 Sandra and Kevin each received \$80 000 from their parents. Sandra invested the money at 6.5% p.a. compounded annually, whereas Kevin bought a sports car that depreciated at a rate of 20% p.a. What were the values of their investments at the end of six years?

6 Taxis depreciate at 50% p.a., and other cars depreciate at 22.5% p.a.

a What is the total percentage reduction in value on each type of vehicle after six years?

b What is the difference in value after six years of a fleet of taxis and a fleet of other cars, if both fleets originally cost \$10 000 000?

Example 25

7 Mr Startit's 10-year-old car is worth \$6500, and has been depreciating at 22.5% p.a.

a By substituting into $A_n = P(1 - R)^n$, find how much (to the nearest dollar) it was estimated to be worth a year ago.

b How much, correct to the nearest dollar, was it estimated to be worth two years ago?

c How much, correct to the nearest dollar, was it estimated to be worth 10 years ago?

d What is the total percentage reduction in value on the car over the 10-year period?

e What was the average reduction in value in dollars per year over the 10-year period?

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8 Ms Rinoldis' seven-year-old car is worth $\$5600$, and has been depreciating at 22.5% p.a.

a How much, correct to the nearest dollar, was it worth four years ago?

b How much, correct to the nearest dollar, was it worth seven years ago?

c What is the total percentage reduction in value on the car over the seven-year period?

d What was the average reduction in value in dollars per year over the seven-year period?

e Ms Rinoldis, however, only bought the car four years ago, at its depreciated value at that time.

i What has been Ms Rinoldis' average loss in dollars over the four years she has owned the car?

ii What was the average loss in dollars over the first three years of the car's life?

9 I take a sealed glass container and remove 60% of the air. Then I remove 60% of the remaining air. I do this six times altogether. What percentage of the original air is left in the container?

10 The number of trees on Green Plateau fell by 5% every year for 10 years. Then the numbers rose by 5% every year for 20 years. What was the total percentage gain or loss of trees over the 30-year period?

11 **a** Find the total percentage decrease in an investment with a value that decreased at:

i 15% p.a. for two years

ii 10% p.a. for three years

iii 6% p.a. for five years

iv 5% p.a. for six years

v 3% p.a. for 10 years

vi 2% p.a. for 15 years

b What do you observe about these results?

12 A special depreciation ruling was obtained from the Taxation Office on a particular piece of scientific apparatus. For the first four years, it depreciates at 32% p.a., and for the second four years, it depreciates at 22% p.a. Find the total percentage decrease in value.

13 I take 500 mL of a liquid and dilute it with 100 mL of water. Then I take 500 mL of the mixture and again dilute it with 100 mL of water. I repeat this process 20 times in all. What percentage of the original liquid remains in the mixture at the end?

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Review exercise

1 Find the simple interest payable in each case.

a $10 000 borrowed for six years at 9% p.a.

b $3000 borrowed for 15 years at 6% p.a.

c $1500 borrowed for 40 years at 2.5% p.a.

2 What principal will earn $1000 simple interest at:

a 4% p.a. over five years?

b 10% p.a. over three years?

c 6% p.a. over eight years?

d 2.5% p.a. over 10 years?

3 At what rate of simple interest will:

a $10 000 grow to $14 000 over a five-year period?

b $8000 grow to $10 000 over a two-year period?

c $1500 grow to $2000 over a three-year period?

4 If a woman borrows $750 to buy a television and agrees to pay back $870 in one year's time, what annual rate of simple interest is she being charged?

5 An investor bought an antique table for $6000. He paid 5% as a deposit and borrowed the remainder from a bank for two years at 18% p.a. simple interest, payable monthly. How much interest does he have to pay each month?

6 Find the new value if:

a 60 is increased by 10%

b 50 is increased by 150%

c 80 is decreased by 20%

d 200 is increased by $12\frac{1}{2}\%$

e 400 is decreased by 2.5%

f 312 is decreased by $5\frac{1}{4}\%$

7 A clothing store offers a 15% discount on all its summer stock. How much will I need to pay in total if I buy a shirt with a marked price of $35, a pair of shorts with a marked price of $25, and a cotton sweater with a marked price of $50?

8 A general store in a country town adds 8% to the recommended retail price of all its stock due to transport costs. What will be the total charge if I buy a torch with a retail price of $9.50, a tin of coffee with a retail price of $11.30, and a hat with a retail price of $42.00?

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Cambridge University Press
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9 **a** What is the final value if:

i 90 is increased by 10%?

ii 120 is decreased by 20%?

iii 96 is increased by 4%?

iv 108 is decreased by 8%?

b Look carefully at the results of part **a i–iv**. Do they surprise you?

10 A computer store states that it will reduce the price of a computer by 10% each day until it is sold. The original price of the computer is $2500.

a What is the sale price after the first reduction?

b What is the sale price after the second reduction?

c What is the sale price after the third reduction?

d What single percentage decrease has the same effect as the three 10% reductions?

11 A manufacturer of suits can produce a suit at a cost of $250. When he sells it to a clothing store owner, he makes a 20% profit on the suit. To cover costs, the store owner increases the cost of the suit a further 30%.

a For what price does the store owner sell the suit?

b What is the total percentage increase in the cost of the suit?

12 The population of a country increased by 3%, 2.6% and 1.8% in three successive years. What was the total percentage increase in the country's population over the three-year period?

13 Find the balance if:

a $2000 is invested for 10 years at 8% p.a. compounded annually

b $5000 is invested for six years at 1% per month compound interest

c $500 is invested for 40 years at 3% per six-month period compound interest

14 A piece of machinery has an initial value of $25 000. Due to usage and age, its value depreciated by 8% each year. Find the value of the piece of machinery after:

a three years **b** five years **c** 10 years **d** n years

15 A new car is valued at $26 000. It is estimated to depreciate by 12% each year.

a Find its depreciated value after five years.

b Find its depreciated value after 10 years.

c Find how many years it takes for its depreciated value to fall below $11 000.

16 Jo bought a four-year old car for $20 880.25 at its correct depreciated value. If the car has a depreciation rate of 15% p.a., then find the value of the car when it was brand new.

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Challenge exercise

1 Waleed owns a portfolio of shares that he purchased for \$38 000. For the first four years, the portfolio appreciated in value at an average of 8% each year, but for the next four years, it depreciated in value at an average of 8% each year. Calculate:

a the value of the portfolio, correct to the nearest dollar, at the end of these eight years

b the equivalent simple interest rate of change in value per year, correct to two decimal places, over these eight years.

2 Two banks offer the following investment packages.

Bank A: 6.5% p.a. compounded annually, fixed for six years

Bank B: 5.3% p.a. compounded annually, fixed for eight years

a Which bank's package will yield the greater interest?

b If a customer invests \$10 000 in Bank A, how much would she have to invest with Bank B to produce the same amount produced by Bank A at the end of the investment period?

3 The Happy Pumpkin fruit shop sells grapes at a price 10% cheaper than the Akrivo Stafili fruit shop and 10% more expensive than the Costa fruit shop.

A customer buys \$50 worth of grapes from the Happy Pumpkin fruit shop. He obtains n kilograms of grapes for his \$50.

a What is the cost, in terms of n, of 1 kg of grapes from the Happy Pumpkin fruit shop?

b If he buys $\frac{n}{2}$ kilograms of grapes from the Akrivo Stafili fruit shop and $\frac{n}{2}$ kilograms of grapes from the Costa fruit shop, how much does he pay?

c If he buys $\frac{n}{4}$ kilograms of grapes from the Akrivo Stafili fruit shop and $\frac{3n}{4}$ kilograms of grapes from the Costa fruit shop, how much does he pay?

d If he buys $\frac{3n}{4}$ kilograms of grapes from the Akrivo Stafili fruit shop and $\frac{n}{4}$ kilograms of grapes from the Costa fruit shop, how much does he pay?

4 The population of a town decreases by 11% during 2012. What percentage increase is necessary during 2013 for the population to be restored to its population immediately before 2012?

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5 The length of a rectangle is increased by 12% and the width is decreased by 10%. What is the percentage change in the area?

6 A man earns a salary of \$2440 for working a 44-hour week. His weekly salary is increased by 12.5% and his hours are reduced by 10%. Find the percentage increase in his new hourly salary.

7 In a particular country in 2011, 12% of the population was unemployed and 88% was employed. In 2012, 10% of the unemployed people became employed and 10% of those employed became unemployed. What percentage of the population was employed at the end of 2012?

8 Andrea buys a house and she spends an extra 10% of what she paid for the house on repairs. She takes out a loan and pays 5% p.a. compound interest on the total amount spent (including repairs). Three years later she sells the house for \$565 100 and she gains 20% on the whole investment purchase price. How much did she pay for the house?

9 Anthony invests $\$P$ for 2 years at $r\%$ p.a. compound interest. At the end of the 2 years, Anthony receives his original $\$P$ and $\frac{\$P}{10}$ in interest. Find r, correct to two decimal places.

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