CHAPTER

10

Review and problem-solving

ICE-EM Mathematics 10 3ed ISBN 978-1-108-40434-1
Cambridge University Press
Updated June 2022

10A Review

Chapter 1: Consumer arithmetic

1 The sum of $10 000 is borrowed for 5 years at 9% p.a. simple interest. How much interest is paid?

2 The sum of $7500 is borrowed at 7.5% p.a. simple interest and $1687.50 is paid in interest. For how many years has the money been borrowed?

3 The sum of $5600 is borrowed for 4 years and $1948.80 is paid in interest. Calculate the (per annum) rate of simple interest charged.

4 A department store is offering a 40% discount on all items in the store. Calculate the discounted price on the following items:

a a jacket with a marked price of $399

b a dress with a marked price of $120

5 A pair of shoes marked at $220 is sold for $176. What percentage discount has been allowed?

6 A music store is offering a 45% discount during a sale.

Calculate the original marked price of:

a a DVD that has a sale price of $13.20

b a boxed set of DVDs that has a sale price of $66

7 Calculate the missing entries.

	Original value	New value	Percentage change
a	120		10% decrease
b	90		15% increase
c	60		40% decrease
d		26	30% increase
e	500	375	
f	140	350	
g		203	20% decrease

8 Find the single percentage change that is equivalent to:

a a 20% increase followed by a 20% decrease

b a 10% increase followed by a 5% increase

c an 8% decrease followed by a 4% increase

d a 10% decrease followed by a 10% decrease

9 Due to market demands, the cost of petrol increases by 2%, 5% and 4% in three successive months. By what percentage has the cost of petrol increased over the three-month period?

ICE-EM Mathematics 10 3ed ISBN 978-1-108-40434-1

10 A quantity is increased by 10%. What further percentage change, applied to the increased value, is required to produce these changes?

a Increase of 32% **b** Increase of 15.5%

c Decrease of 12% **d** Decrease of 6.5%

11 Calculate the amount that an investment of \$30 000 will be worth if it is invested at 6.5% p.a. for 10 years compounded annually.

12 Calculate the amount that an investment of \$12 000 will be worth if it is invested at 8% p.a. for 6 years compounded:

a annually **b** quarterly (assume 2% per quarter) **c** monthly (assume $\frac{2}{3}$% per month)

Chapter 2: Review of surds

1 Simplify by collecting like surds:

a $2\sqrt{2}+3\sqrt{3}-\sqrt{3}+3\sqrt{2}$ **b** $5\sqrt{5}-3+2\sqrt{5}+7$

2 Simplify:

a $\sqrt{18}$ **b** $\sqrt{128}$ **c** $4\sqrt{72}$ **d** $3\sqrt{27}$

3 Simplify:

a $\sqrt{50}-3\sqrt{8}$ **b** $3\sqrt{12}+4\sqrt{75}$ **c** $\sqrt{147}+\sqrt{243}$

d $4\sqrt{63}-2\sqrt{28}$ **e** $3\sqrt{45}+\sqrt{72}+6\sqrt{8}-\sqrt{20}$ **f** $\frac{a}{\sqrt{a}}+\sqrt{a}$

4 Expand and simplify:

a $\sqrt{8}\left(\sqrt{6}-\sqrt{2}\right)$ **b** $\left(5+\sqrt{2}\right)\left(\sqrt{2}-3\right)$

c $\left(3\sqrt{2}-\sqrt{5}\right)\left(3\sqrt{2}+\sqrt{5}\right)$ **d** $\left(\sqrt{3}-\sqrt{2}\right)^2$

5 Simplify:

a $\sqrt{8}\times\sqrt{8}$ **b** $\left(\sqrt{a}-\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\right)$ **c** $\left(\sqrt{a}+\sqrt{b}\right)^2$

6 Rationalise the denominator and simplify:

a $\frac{2\sqrt{27}}{\sqrt{18}}$ **b** $\frac{2\sqrt{3}+3\sqrt{2}}{\sqrt{6}}$ **c** $\frac{\sqrt{3}+5}{\sqrt{3}}$

d $\frac{3\sqrt{5}-1}{\sqrt{5}}$ **e** $\frac{5\sqrt{12}+2\sqrt{10}}{\sqrt{5}}$ **f** $\frac{2\sqrt{6}-\sqrt{3}}{3\sqrt{3}}$

7 Express with a rational denominator in simplest form:

a $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$ **b** $\frac{\sqrt{3}+\sqrt{2}}{3\sqrt{3}-2\sqrt{2}}$ **c** $\frac{2\sqrt{2}+1}{\sqrt{6}-2}$ **d** $\frac{3\sqrt{10}+2}{2\sqrt{5}+\sqrt{2}}$

Chapter 3: Algebra review

1 Simplify:

a $6mn^2-7m+3mn^2+4m$ **b** $2x\times 3y-6x^2-6xy+3x\times 2x$

c $\frac{4a^2b^3}{2ab}+3a^2-\frac{6a^3b^2}{3ab^2}+5ab^2$ **d** $\frac{6p^3q^3}{pq}+\frac{2p^3q^2}{q^2}-\frac{14p^5q}{2p^2q}+\frac{12p^4q^2}{3p^2}$

ICE-EM Mathematics 10 3ed ISBN 978-1-108-40434-1 Cambridge University Press
 Updated June 2022

2 Expand and collect like terms for each expression.

a $3(a+2)+2(a-1)$
b $5(b+3)-3(b-2)$
c $2x(x+5)+4x(x-3)$
d $3y(y-1)-4y(2y-5)$
e $(2x+1)(x+5)$
f $(2a+7)(3a-2)$
g $(2y+3)(y+2)-(y-1)(y+3)$
h $(b+5)(3b+1)-(2b-3)(b-2)$

3 Expand and collect like terms for each expression.

a $3(x+3)(2x+5)$
b $2(2a+1)(3a-4)$
c $2(2y+1)(y+2)+3(y-2)(2y+3)$
d $5(b+2)(2b+1)-3(b-1)(b-3)$

4 Expand and collect like terms for each expression.

a $\frac{3}{4}(a+2)+\frac{1}{2}(a-1)$
b $5\left(\frac{b}{3}+\frac{1}{6}\right)-3\left(\frac{b}{6}-\frac{1}{2}\right)$
c $\frac{2}{3}x(x+5)+\frac{1}{4}x(x-3)$
d $\frac{2}{5}y\left(\frac{3}{4}y-1\right)-\frac{1}{2}y\left(2y-\frac{3}{5}\right)$
e $\left(\frac{2}{3}x+1\right)\left(x+\frac{1}{4}\right)$
f $\left(\frac{2}{3}y+3\right)\left(\frac{5}{6}y+2\right)-\left(y-\frac{1}{4}\right)\left(y+\frac{1}{3}\right)$

5 Solve:

a $2x-7=10$
b $5-3y=15$
c $5x+3=2x-8$
d $7y+5=5y-3$
e $\frac{3x+2}{5}=8$
f $\frac{5y-2}{4}+2=\frac{3y+2}{3}$
g $4(x-3)=3x+4$
h $\frac{2x-5}{3}-\frac{x-2}{5}=4$
i $3(2x-5)=2\left(4x+\frac{3}{2}\right)$
j $\frac{4(x+5)}{5}=1+\frac{2x-7}{2}$

6 A gardener has 60 m of garden edging, which she uses to set out a rectangular garden with width 5 m less than the length. Let x metres be the length of the garden.

a Find, in terms of x, the width of the garden.
b Hence, form an equation and solve it to find the length and width of the garden.

7 In an effort to catch a bus, I walked for 10 minutes and ran for 5 minutes. I know I can run 4 times as fast as I can walk. What was my running speed, in km/h, if I travelled a total of 3 km to catch the bus?

8 A completely filled car radiator with capacity 8 L contains a mixture of 40% antifreeze (by volume). If the radiator is partly drained and refilled with pure antifreeze, how many litres should be drained from the radiator so as to have a mixture of 70% antifreeze?

9 Solve each inequality.

a $3x-7>5$
b $4(2x-3)\geq 2x-1$
c $\frac{x-1}{4}-\frac{x-2}{5}<2$
d $\frac{x+5}{3}-\frac{3x+1}{2}\geq 4$
e $\frac{4(2-x)}{3}-2\geq\frac{22}{3}$
f $4(3-x)<3-3(4-x)$

10 The power used by a furnace, P watts, is related to the resistance of the wiring, R ohms, and the current, I amps, according to the formula $P=RI^2$. Find the power used by a furnace with wire resistance of 2.5×10^{-1} ohms that draws a current of 6.2×10^{3} amps.

ICE-EM Mathematics 10 3ed ISBN 978-1-108-40434-1

11 Given the relationship $s = ut + \frac{1}{2}at^2$:

a calculate s when $u = 20.8$, $t = 1.5$ and $a = 9.8$

b rearrange the formula to make a the subject

12 The formula for the time of swing, T seconds, of a pendulum is $T = 2\pi\sqrt{\frac{p}{g}}$, where p metres is the length of the pendulum and g is a constant related to gravity.

a Make g the subject of this formula.

b The time of swing is found to be 3 seconds when the length of the pendulum is 2.24 m. What is the value of g (correct to one decimal place)?

13 Make x the subject of each formula.

a $ax + b = c$ **b** $a(x + b) = c$ **c** $\frac{ax + b}{c} = d$

d $rx + b = tx + c$ **e** $\sqrt{\frac{x}{y}} = a$ **f** $\frac{1}{x} + \frac{1}{y} = \frac{1}{c}$

g $m = \sqrt{\frac{n - p}{x}}$ **h** $\frac{ax + b}{a^2} - \frac{x + 1}{b} = 0$ **i** $\frac{y - 3}{2} + 1 = \frac{x - 2}{3}$

14 Expand:

a $(x + 5)(x - 5)$ **b** $(x + 2)(x - 2)$ **c** $(3a + 1)(3a - 1)$

d $(5x + 2y)(5x - 2y)$ **e** $\left(\frac{1}{2}a + 1\right)\left(\frac{1}{2}a - 1\right)$ **f** $\left(\frac{1}{4}x + \frac{2}{3}y\right)\left(\frac{1}{4}x - \frac{2}{3}y\right)$

15 Factorise:

a $x^2 - 36$ **b** $a^2 - 64$ **c** $81b^2 - 1$ **d** $9x^2 - 4y^2$

16 Factorise:

a $x^2 - 18x$ **b** $3x^2 - 18x$ **c** $18b^2 - 50$ **d** $12b^2 - 27$

e $28x^2 - 63y^2$ **f** $54a^2 - 24b^2$ **g** $\frac{1}{4}x^2 - y^2$ **h** $\frac{3}{4}x^2 - \frac{12}{25}y^2$

17 Factorise:

a $x^2 + 5x + 6$ **b** $x^2 + 8x + 12$ **c** $x^2 - 3x + 2$

d $x^2 - 6x + 5$ **e** $x^2 - 9x + 18$ **f** $x^2 - 5x - 6$

g $x^2 - 3x - 10$ **h** $x^2 - 2x - 8$ **i** $x^2 - 4x - 21$

18 Factorise:

a $2x^2 + 7x + 6$ **b** $3x^2 + 19x + 6$ **c** $5x^2 + 19x + 12$

d $2x^2 - 5x + 2$ **e** $3x^2 - 13x + 10$ **f** $7x^2 - 23x + 18$

g $3x^2 - 7x - 6$ **h** $5x^2 - 6x - 8$ **i** $2x^2 - 11x - 21$

19 Factorise:

a $4x^2 + 8x + 3$ **b** $6x^2 + 13x + 6$ **c** $4x^2 + 19x + 12$

d $4x^2 - 16x + 15$ **e** $6x^2 - 19x + 10$ **f** $10x^2 - 27x + 18$

g $4x^2 - 4x - 15$ **h** $6x^2 - 11x - 10$ **i** $8x^2 - 2x - 15$

ICE-EM Mathematics 10 3ed ISBN 978-1-108-40434-1

20 Factorise:

a $2x^2 - 8x - 42$ **b** $5x^2 + 25x + 30$ **c** $4x^2 - 12x - 16$

d $3x^2 + 12x - 15$ **e** $6x^2 + 9x - 15$ **f** $8x^2 + 20x + 8$

g $10x^2 - 55x - 30$ **h** $-x^2 + 10x - 24$ **i** $-6x^2 - 14x - 8$

21 Express with a common denominator:

a $\frac{7x-1}{5} + \frac{3x-4}{7}$ **b** $\frac{2}{x+1} + \frac{3}{x+2}$ **c** $\frac{1}{x+2} - \frac{4}{x-3}$

d $\frac{5}{2x^2+3x} - \frac{1}{2x^2+5x+3}$ **e** $\frac{4}{x^2+x} - \frac{3}{x^2-1}$ **f** $\frac{3}{x^2-9} - \frac{2}{3+2x-x^2}$

22 Simplify:

a $\frac{x^2+4x+3}{x^2+x-6} \times \frac{x^2-4}{x^2+5x+4}$ **b** $\frac{x^2+7x+6}{x^2+x-2} \times \frac{x^2-x-6}{2x^2-5x-3}$

c $\frac{x^2+3x-4}{x^2+4x} \div \frac{x^2+x-2}{x+2}$ **d** $\frac{3x^2-3x}{2x^2+3x+1} \div \frac{9-9x}{2x^2+7x+3}$

Chapter 4: Lines and linear equations

1 Find the distance between each pair of points.

a (6, 4), (0, 0) **b** (3, 2), (5, 4) **c** (–3, 2), (2, 5) **d** (–4, –3), (–1, 2)

2 Find the midpoint of the interval AB, where:

a $A = (6, 4)$ and $B = (0, 0)$ **b** $A = (3, 2)$ and $B = (5, 4)$

c $A = (-3, 2)$ and $B = (2, 5)$ **d** $A = (-4, -3)$ and $B = (-1, 2)$

3 Find the gradient of the line that passes through each pair of points.

a (6, 4), (0, 0) **b** (3, 2), (5, 4) **c** (–3, 2), (2, 5) **d** (–4, –3), (–1, 2)

4 Write the gradient and y-intercept of each line.

a $y = 2x - 1$ **b** $y = x + 3$ **c** $y = -x + 7$

d $x + y = 4$ **e** $2x + 3y = 1$ **f** $3x - 4y = 2$

5 Find the gradient of a line that is:

i parallel **ii** perpendicular

to the line with equation:

a $y = 3x + 2$ **b** $y = 1 - 2x$ **c** $y = \frac{1}{2}x + 2$ **d** $y = -\frac{2}{3}x + 2$

6 Sketch the graph of each equation, and mark the intercepts.

a $y = 2x + 1$ **b** $y = 3 - 2x$ **c** $2x + 3y = 6$ **d** $3x - 4y = 12$

e $y = -2x$ **f** $y = 4x$ **g** $y = -4$ **h** $x = 3$

7 Find the equation of the line with:

a a gradient of 3 passing through (0, 2) **b** a gradient of 2 passing through (0, 1)

c a gradient of −1 passing through (0, −3) **d** a gradient of $-\frac{1}{2}$ passing through (0, 4)

ICE-EM Mathematics 10 3ed ISBN 978-1-108-40434-1

8 Find the equation of the line passing through the points:

a (2, 0) and (0, 3) **b** (2, 2) and (0, 1) **c** (1, 3) and (2, 3)

d (5, 3) and (5, –2) **e** (1, 1) and (2, 3) **f** (–2, 3) and (2, –1)

9 Solve each pair of simultaneous equations for x and y.

a $y = x + 1$
$x + 2y = 8$

b $y = 2x - 1$
$2x + 5y = 7$

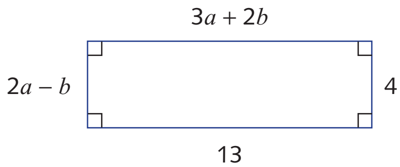
c $x + y = 1$
$3x + 2y = 8$

d $2x - 3y = -1$
$6x + 6y = 7$

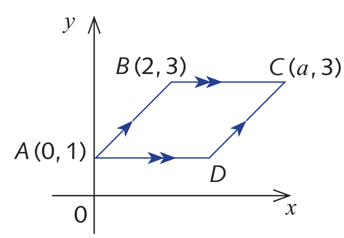
e $5x + 3y = 15$
$3x + 2y = 8$

f $2x - 3y = -10$
$3x + 2y = 7$

10 Find the values of a and b in the diagram shown.



11 $ABCD$ is a parallelogram, as shown opposite, where $a > 2$.



a If $a = 5$, find the length of BC.

b Find, in terms of a:

i the length BC

ii the coordinates of the point D

c i Find the gradient of the line AC in terms of a.

ii Find the gradient of the line BD in terms of a.

iii Show that when $a = 5$, the gradient of the line BD is –2.

d For $a = 5$, find:

i the gradient of the line AC

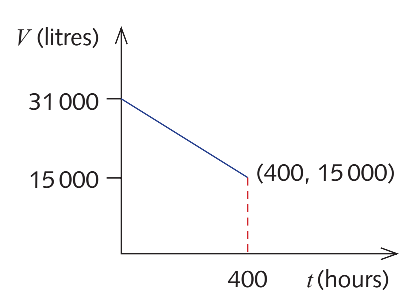
ii the equation of the line AC

iii algebraically, the coordinates of the intersection point of the line AC with the line BD, given that the equation of the line BD is $y = -2x + 7$

e i Find the length of AC in terms of a.

ii Find the exact value of a (as a surd in simplest form) so that $AC = 7$.

12 Water was leaking from a tank at a constant rate. The graph shows the volume of water (V litres) remaining in the tank after t hours.



a How many litres of water were initially in the tank?

b How many litres were leaking from the tank per hour?

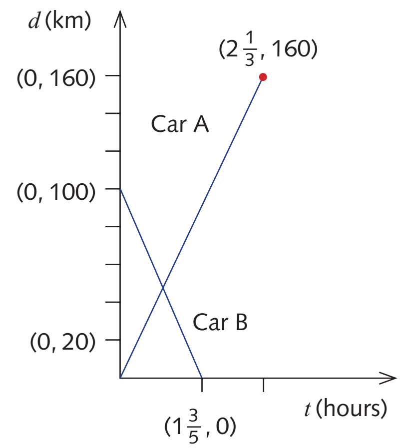
c Write a rule for finding the number of litres remaining (V) after t hours.

d When would the tank be empty if the leaking continued at this rate?

ICE-EM Mathematics 10 3ed ISBN 978-1-108-40434-1

13 The graph shown represents the trips of two cars, A and B, along the Hume Highway.

The vertical axis is the d axis, where d km is the distance from Melbourne along the Hume Highway. The horizontal axis is the t axis, where t hours is the time of travel. Assume that both cars started their trip at 9 a.m.



a Describe these aspects of each car's trip.

i Where did it start?

ii Where did it finish?

iii What was the time taken?

iv What was the average speed?

b Find the equation of the graph of each car's trip (in terms of d and t).

c Find the time at which they passed each other, giving your answer to the nearest minute.

Chapter 5: Quadratic equations

1 Solve:

a $x^2 = 16$ **b** $7x^2 = 28$ **c** $2x^2 - 98 = 0$

d $4x^2 - 25 = 0$ **e** $4x^2 - 1 = 0$ **f** $12x^2 - 75 = 0$

2 Solve:

a $4x^2 - 6x = 0$ **b** $27x^2 + 9x = 0$ **c** $5x^2 - 3x = 0$

d $18x^2 = 9x$ **e** $8x = 28x^2$ **f** $-3x^2 - 15x = 0$

g $14x - 2x^2 = 0$ **h** $\frac{1}{2}x^2 - 6x = 0$ **i** $24x^2 = -6x$

3 Solve:

a $a^2 - a - 12 = 0$ **b** $t^2 + 8t + 15 = 0$ **c** $m^2 + 4m - 21 = 0$

d $n^2 - 3n - 4 = 0$ **e** $x^2 - 8x + 16 = 0$ **f** $b^2 - 6b = 27$

4 Solve:

a $2x^2 - 19x + 35 = 0$ **b** $9f^2 - 36f + 11 = 0$ **c** $-3x^2 - 23x + 8 = 0$

d $12y^2 + 21 = -32y$ **e** $3x^2 - 2x - 1 = 0$ **f** $12x^2 + 8x = 15$

g $-2x^2 - 5x + 12 = 0$ **h** $3x^2 = 18x - 27$

5 Solve:

a $b^2 - 6b + 9 = 0$ **b** $x^2 + 10x + 25 = 0$ **c** $2x^2 + 4x + 2 = 0$

d $3b^2 - 24b + 48 = 0$ **e** $4x^2 + 12x + 9 = 0$ **f** $3y^2 - 30y + 75 = 0$

6 Factorise, using surds:

a $x^2 - 5$

b $(x + 2)^2 - 8$

c $2(x - 3)^2 - 10$

ICE-EM Mathematics 10 3ed ISBN 978-1-108-40434-1

7 Solve each equation by completing the square.

a $y^2 + 2y - 4 = 0$ **b** $a^2 - 4a - 2 = 0$

c $x^2 - 2x = \frac{5}{2}$ **d** $x^2 - 7x + 2 = 0$

e $y^2 + \frac{1}{2}y = \frac{1}{16}$ **f** $2x^2 - x = 4$

g $n^2 = 5n + 4$ **h** $16x^2 + 8x = 1$

8 Solve:

a $5d^2 - 10 = 0$ **b** $\frac{2y^2}{3} - 5 = 0$ **c** $\frac{3(x - 10)^2}{5} - 12 = 0$

d $y^2 - 8y + 3 = 0$ **e** $1 = m^2 - m$ **f** $3n + 3 = n^2$

9 In a right-angled triangle, the hypotenuse is 8 cm longer than the shortest side, and the third side of the triangle is 7 cm longer than the short side. Let x cm be the shortest side length.

a Express the other two side lengths in terms of x.

b Hence, form an equation and solve it to find the side lengths of the triangle.

10 The height, h metres above sea level, to which a rocket has risen t seconds after launching from sea level is given by $h = ut - 4.9t^2$, where u metres per second is the launch velocity.

a Calculate the height above sea level 4 seconds after the launch of a rocket with a launch velocity of 115 m/s.

b If the launch velocity can be a maximum of 500 m/s, calculate the longest possible time of flight, to the nearest second.

(*Hint*: At the end of a flight, the height above sea level is 0 m.)

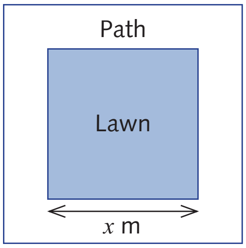
11 A sheet of cardboard 24 cm long and 17 cm wide has squares of side length x cm cut from each corner so that it can be folded to form an open box with base area of 228 cm^2.

a Express the length and width of the base in terms of x.

b Write an expression involving x and solve it for x.

c Find the dimensions of the box.

12 A square lawn is surrounded by a concrete path 2 m wide. If the lawn has sides of length x metres, find, in terms of x:



a the area of the lawn

b the area of the concrete path

The area of the concrete path is $1\frac{1}{4}$ times that of the lawn.

c Write an equation that can be used to find x.

d Solve this equation to find the dimensions of the lawn.

13 For the quadratic equation $x^2 + bx + 4 = 0$, find the values of b for which the equation has:

a one solution **b** two solutions **c** no solutions

14 For the quadratic equation $ax^2 - 4x + 3 = 0$, find the values of a for which the equation has:

a one solution **b** two solutions **c** no solutions

ICE-EM Mathematics 10 3ed ISBN 978-1-108-40434-1 Cambridge University Press
 Updated June 2022

Chapter 6: Surface area and volume

1 For a rectangular prism measuring 30 cm × 20 cm × 10 cm, calculate:

a the surface area

b the volume

2 A rectangular prism has a surface area of 550 cm^2. If its length is 15 cm and its width is 10 cm, calculate the height of the rectangular prism.

3 A rectangular prism has a volume of 660 cm^3. If its length is 12 cm and its width is 11 cm, calculate the height of the rectangular prism.

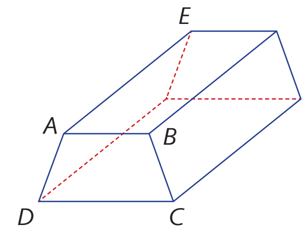
4 The cross-section *ABCD* of the prism shown is an isosceles trapezium with $AB = 8$ cm, $DC = 14$ cm, $AD = BC = 5$ cm and $AE = 20$ cm.

Calculate:

a the area of *ABCD*

b the surface area of the prism

c the volume of the prism



5 A cylindrical water tank stands on its circular base. It has a diameter of 2 m and a height of 1.5 m.

a Calculate the volume of the tank, to the nearest litre.

b Calculate the depth of water in the tank, to the nearest centimetre, when it contains 2000 litres of water.

6 Find answers to these questions in cm^2 and cm^3.

a A square-based pyramid has base side length 10 cm and perpendicular height 12 cm.

Calculate:

i the surface area **ii** the volume

b A cone has a radius of 6 cm and a slant height of 10 cm.

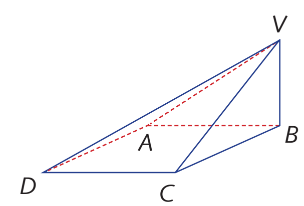
Calculate:

i the surface area **ii** the volume

7 In the pyramid *VABCD* shown, *VB* is perpendicular to rectangle *ABCD*, $AB = 12$ m, $BC = 8$ m and $VB = 5$ m.

a Calculate the surface area of the pyramid in m^2, correct to one decimal place.

b Calculate the volume of the pyramid.



8 The curved surface area of a cone is 80π cm^2 and the area of the circular base is 16π cm^2.

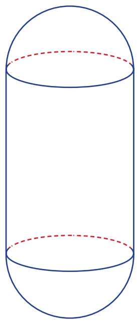
a Calculate the radius of the cone.

b Calculate the exact perpendicular height of the cone.

c Calculate the volume of the cone, correct to the nearest cm^3.

ICE-EM Mathematics 10 3ed ISBN 978-1-108-40434-1

9 A storage tank is constructed as a cylinder with a hemisphere at each end of the cylinder. The radius of the cylinder is 1.5 m and the overall length of the tank is 6 m.



Calculate:

a the surface area of the tank in m^2

b the volume of the tank

10 Fill in the missing entries in the table below.

	Length scale factor	Area scale factor	Volume scale factor
a	3		
b	1.5		
c		4	
d		36	
e			125
f			729

Chapter 7: The parabola

1 Find the x-intercepts of the graph for each equation.

a $y = x^2 + 4x + 3$

b $y = 2x^2 - 11x - 6$

c $y = (x + 4)^2 - 3$

d $y = 3(x - 2)^2 - 6$

2 Express each equation in the form $y = a(x - h)^2 + k$, and hence state the coordinates of the vertex of each graph.

a $y = x^2 + 6x + 3$

b $y = x^2 - 4x + 2$

c $y = 2x^2 + 6x + 1$

d $y = 3x^2 + 8x + 2$

3 **a** A parabola has x-intercepts -1 and 4, and y-intercept 8. Find the equation of the parabola.

b A parabola has x-intercepts 3 and 5, and passes through the point (1, 8). Find the equation of the parabola.

4 **a** A parabola has vertex (3, –2) and y-intercept 16. Find the equation of the parabola.

b A parabola has vertex (2, 5) and passes through the point (1, 2). Find the equation of the parabola.

5 Write the equation of the parabola obtained when the graph of $y = x^2$ is:

a stretched by a factor of 3 from the x-axis and translated 2 units to the right

b reflected in the x-axis and then translated 1 unit to the left and 3 units up

c translated 5 units to the right and 4 units down

ICE-EM Mathematics 10 3ed ISBN 978-1-108-40434-1

6 Sketch the graph, labelling the vertex, axes of symmetry and the intercepts.

a $y = x^2 - 6x + 5$ **b** $y = -x^2 - x + 6$ **c** $y = 4 - x^2$

d $y = (x + 3)^2$ **e** $y = (x - 1)^2 - 4$ **f** $y = x^2 - 2$

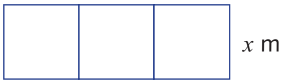
g $y = (x - 3)^2 + 2$ **h** $y = 2 - (x + 1)^2$ **i** $y = x^2 + 5x - 3$

j $y = 10 - 6x^2 - 11x$ **k** $y = 4x^2 + 7x + 6$ **l** $y = 2x^2 - x - 7$

7 For the graph with equation $y = 3x^2 - 2x - 1$, find the coordinates of the:

a vertex **b** x-intercepts

8 A gardener is planning to establish a vegetable garden. The garden will have a wooden border and two wooden dividers to form three partitions, as shown in the diagram. Twenty-four metres of timber is used for the border and the dividers. Let x m be the length of the dividers and two of the sides of the garden, as indicated in the diagram.



a Express the other side length of the garden in terms of x.

b Let A m^2 be the area of the garden. Write an equation for the area of the garden in terms of x.

c Find the length and width of the garden in order for the area to be a maximum.

9 **a** By expressing the quadratic equation $y = x^2 + 2x - 7$ in the form $y = a(x - h)^2 + k$, find the coordinates of the turning point.

b Find the points of intersection with the axes of the graph of $y = x^2 + 2x - 7$.

c Sketch the graph of $y = x^2 + 2x - 7$, marking on your sketch the points found in **a** and **b**.

d Solve $x^2 + 2x - 7 \le 0$ for x.

10 **a** Sketch the graph of $y = 4x^2 - 8x + 1$, labelling clearly the coordinates of the turning point and the points of intersection with the axes.

b Solve $4x^2 - 8x + 1 < 0$ for x.

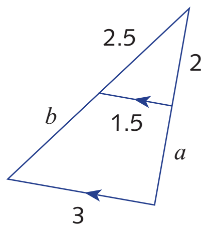
11 Solve for x:

a $x^2 + x < 30$ **b** $x^2 + 5x \ge -6$ **c** $-x^2 + 4x + 60 \le 0$

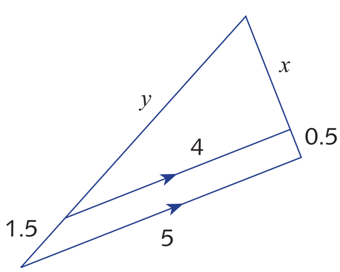
Chapter 8: Review of congruence and similarity

1 Find the value of the pronumerals.

a



b



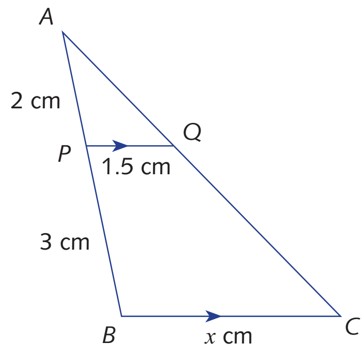
2 A vertical stick of length 30 cm casts a shadow of length 5 cm. Find the length of the shadow cast by a 1 metre ruler placed in the same position at the same time of day.

ICE-EM Mathematics 10 3ed ISBN 978-1-108-40434-1

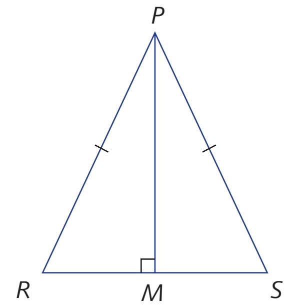
3 **a** In the figure shown, $PQ \parallel BC$.

i Prove that ΔAPQ is similar to ΔABC.

ii Find the value of x.

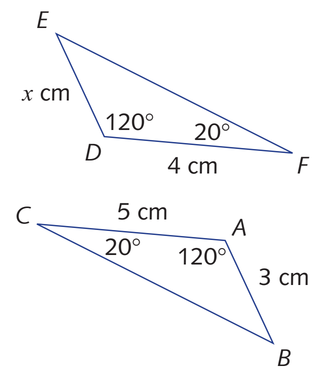


b In the figure shown, $PM \perp RS$ and $PR = PS$. Prove that $\Delta PMR \equiv \Delta PMS$.

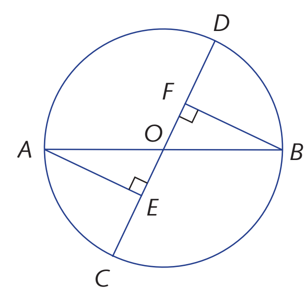


4 **a** **i** State, in abbreviated form, why ΔABC is similar to ΔDEF.

ii Calculate x.



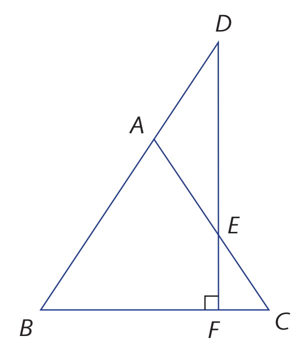
b In the diagram, AB and CD are diameters of the circle with centre O, and AE and BF are perpendicular to CD. State, in abbreviated form, why $\Delta AEO \equiv \Delta BFO$.



5 Complete the proof that, in the figure shown, ΔDAE is isosceles.

Given: In ΔABC, $AB = AC$, D is on the ray from B through A, $DF \perp BC$ and DF intersects AC at E.

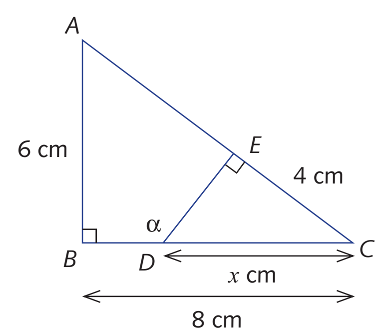
Prove: ΔDAE is isosceles.



6 **a** Prove that ΔDEC is similar to ΔABC.

b Calculate x.

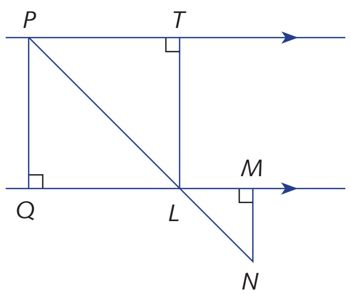
c Use trigonometry to calculate α, correct to two decimal places.



ICE-EM Mathematics 10 3ed ISBN 978-1-108-40434-1

Cambridge University Press
Updated June 2022

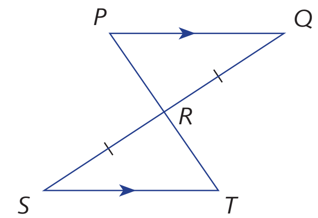
7 In order to calculate the distance across a straight canal, some scouts place markers Q, L, M and N in the positions shown. P is a pumping station and T is a large tree.



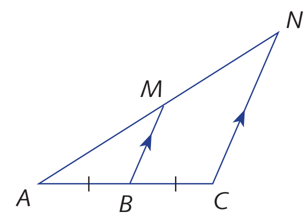
a Name all pairs of similar triangles in the diagram and give the abbreviated reason why they are similar.

b The scouts measure QL to be 60 m, LM to be 40 m and MN to be 50 m. Calculate the distance across the canal.

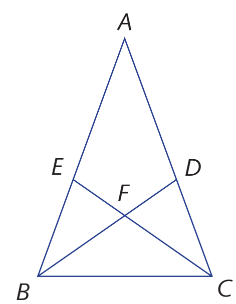
8 In the diagram, $PQ \parallel ST$ and $QR = SR$. Prove that triangles PQR and TSR are congruent.



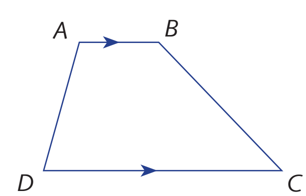
9 In the diagram, $AB = BC$ and $BM \parallel CN$. Prove that $CN = 2BM$.



10 In this diagram, ΔABC is isosceles. $AB = AC$ and $BE = CD$. Prove that $EC = DB$.



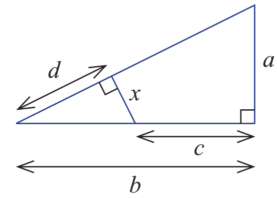
11 $ABCD$ is a trapezium with $AB \parallel DC$ and $AB = \frac{1}{3}DC$. The diagonals of this trapezium intersect at O.



a Prove that ΔABO is similar to ΔCDO.

b Hence, prove that $3AC = 4OC$.

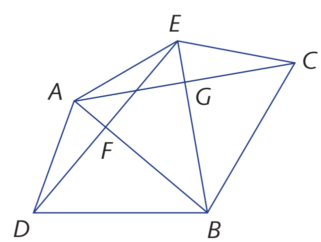
12 Find the formula, with x as its subject, that can be used to calculate the value of x if a, b, c and d are known.



13 In the diagram, $AB = BC$, $BE = BD$, BA intersects DE at right angles and BE intersects AC at right angles.

a Prove that $\Delta DFB \equiv \Delta EFB$.

b Prove that $\Delta ABD \equiv \Delta CBE$.



14 $ABCD$ is a parallelogram with the size of $\angle BAD < 90°$. E is on the ray CB such that ΔABE is isosceles with $AB = AE$. F is on the ray CD such that ΔADF is isosceles with $AD = AF$.

a Prove that ΔABE is similar to ΔADF.

b Prove that $DE = BF$.

15 ABC is a triangle, M is a point in the interval AB such that $AB = 3AM$, and N is a point in the interval AC such that $AC = 3AN$.

a Prove that $BC \parallel MN$.

b If BN and CM intersect at P, prove that $BP = 3PN$.

Chapter 9: Indices, exponentials and logarithms – part 1

1 Simplify each expression, writing your answers with positive powers.

a $(a^3)^2 \times a^{-2}$ **b** $(2x^2y)^3 \times 3xy^2$ **c** $\left(\frac{a}{b}\right)^2 \times b^3$

d $\frac{a^{-2}b^3}{a^3b^4} \times \frac{a^2b^3}{ab^2}$ **e** $\frac{8x^4y^2}{4x^3y}$ **f** $\frac{3x^2y^3}{9x^3y} \times \frac{6xy}{y^2}$

g $\frac{12xy^2}{x^2y} \div \frac{6x^3y}{y^3}$ **h** $\frac{ab^2}{a^3b^{-2}} \div \frac{a^2b^{-1}}{a^3b^3}$ **i** $\frac{x^2y^3}{x^{-1}y^2} \times \frac{x^{-3}y^2}{x^2y^{-1}}$

2 Express each number in scientific notation.

a 3200 **b** 576 000 **c** 0.000 267 **d** 0.025

3 Evaluate each expression, giving your answers to four significant figures and in scientific notation.

a $3.267 \times 10^6 \times 2.76 \times 10^{-2}$ **b** $\frac{5.567 \times 10^2 \times 2.78 \times 10^{-2}}{3.4 \times 10^4}$

c $\frac{2.34 \times 10^{-6} \times 1.76 \times 10^{-4}}{6.32 \times 10^{-5}}$ **d** $\frac{1.267 \times 10^{-10} \times 2.543 \times 10^{-12}}{1.27 \times 10^{-4} + 3.276 \times 10^{-3}}$

4 Evaluate:

a $\sqrt[3]{27}$ **b** $\sqrt[4]{81}$ **c** $\sqrt[4]{16}$ **d** $\sqrt[5]{32}$ **e** $\sqrt[5]{243}$ **f** $\sqrt[3]{64}$

5 Evaluate:

a $8^{\frac{2}{3}}$ **b** $16^{\frac{3}{4}}$ **c** $27^{\frac{2}{3}}$ **d** $4^{\frac{3}{2}}$ **e** $9^{-\frac{3}{2}}$ **f** $125^{-\frac{2}{3}}$

6 Simplify:

a $\left(b^{\frac{2}{3}}\right)^3 \times b^2$ **b** $\sqrt{\frac{a^4}{b^2}}$ **c** $\sqrt[3]{\frac{a^6}{b^3}}$ **d** $\frac{a^{\frac{1}{3}}b^{\frac{2}{3}}}{ab^2} \div \frac{ab^{\frac{4}{3}}}{a^2b^3}$

e $\sqrt[5]{\frac{a^5}{b^{10}}}$ **f** $\sqrt[3]{\frac{27a^2}{b^3}}$ **g** $\sqrt{\frac{32a^4b}{2ab^3}}$ **h** $\left(a^{\frac{1}{4}}\right)^2 \times \left(a^{\frac{1}{4}}\right)^3$

ICE-EM Mathematics 10 3ed ISBN 978-1-108-40434-1

7 Sketch the graph of each equation.

a $y = 2^x$ b $y = 3^x$ c $y = 5^{-x}$ d $y = -5^x$

8 Solve for x.

a $243^x = 3$ b $625^x = 25$ c $\left(\frac{1}{9}\right)^x = 81$

d $10\,000^x = 1000$ e $(0.0001)^x = 1000$ f $(0.001)^x = 0.000\,01$

9 Solve for x.

a $7^{x-3} = 49$ b $5^{5-x} = 625$ c $4^{2x-3} = 32$

d $16^{2x-1} = 32^{3-2x}$ e $5^{-5-7x} = 625^{3+2x}$ f $10^{4-3x} = 100^{5-2x}$

10 A biologist discovers that the number of organisms present in a Petri dish increases by 8% each minute. If there are initially 5000 organisms present in the dish, find the number of organisms in the dish:

a after 1 minute b after 2 minutes c after x minutes d after 20 minutes

11 The population of a town is initially 4200, and each year the population decreases by 2%.

a What is the population of the town after:

i 1 year? ii 2 years? iii x years?

b On a single set of axes, sketch the graphs of:

i $y = 4200 \times 0.98^x$ ii $y = 3200$

c Use your calculator and your answer to part **b** to find the minimum number of years it will take for the population of the town to drop below 3200.

12 Evaluate:

a $\log_2 16$ b $\log_7 49$ c $\log_{25} 5$ d $\log_{25} 125$

10B Miscellaneous questions

1 Two trains travel between towns A and B. They leave at the same time, with one train travelling from A to B and the other from B to A. They arrive at their destination one hour and four hours, respectively, after passing one another. The slower train travels at 35 km/h.

a How far does the slower train travel after they pass?

b If the faster train travels at x km/h, how far, in terms of x, does the faster train travel after they pass?

c Hence, find the number of hours, in terms of x, each train has travelled before they pass.

d Hence, find the speed of the faster train.

ICE-EM Mathematics 10 3ed ISBN 978-1-108-40434-1

2 **a** A car left town A and travelled at a constant speed towards town B, 150 km away. Half an hour later, an express train left A travelling at a constant speed towards B, and overtook the car 90 km from A. The speed of the car was 80 km/h. Find:

i the time for which the car had been travelling before it was overtaken by the train

ii the time for which the train had been travelling before it overtook the car

iii the speed of the train

b A car left town A and travelled at a constant speed to town B, which is d km away. At a time n hours later, an express train left A travelling at a constant speed to B and overtook the car m km from B. The speed of the car was v km/h. Find formulas for:

i the distance from town A to the point where the train passes the car

ii the time, T hours, for which the car was travelling before it was overtaken by the train in terms of d, m and v

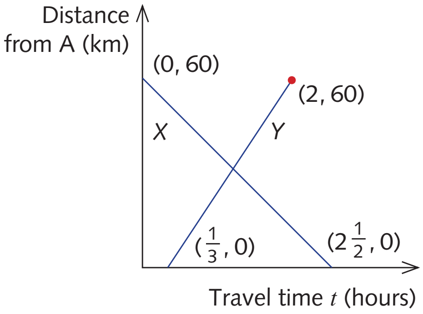
iii the time, t hours, for which the train was travelling before it overtook the car in terms of d, m, v and n

iv the speed of the train, w km/h, in terms of d, m, v and n

v the speed of the car, v km/h, in terms of d, m, n and w

c Given that $d = 150$, $m = 90$, $n = 0.5$ and $w = 108$, find the speed of the car.

3 Two cyclists are riding on the same road between two points, A and B, which are 60 km apart. Cyclist X starts first and is riding from B to A. Cyclist Y starts 20 minutes later and is riding from A to B. The distance–time graph opposite shows all the information.



Find:

a how long it takes each cyclist to ride between A and B

b the average speed of each cyclist on the ride

c how far from A they pass each other

4 In a triathlon event, two competitors, Alan and Shen, are keen rivals. The event consists of an 800 m swim, a 50 km bicycle ride and a 20 km run. Alan can swim at 2 km/h, cycle at 35 km/h and run at 10 km/h (all average speeds). Shen can swim at 2.4 km/h, cycle at 30 km/h and run at 12 km/h (all average speeds). Assume no time is lost when transitioning between legs.

a Find the distance between Shen and Alan when Alan has completed the swim.

b Find which of the two competitors finishes first, and the difference between their times, to the nearest minute.

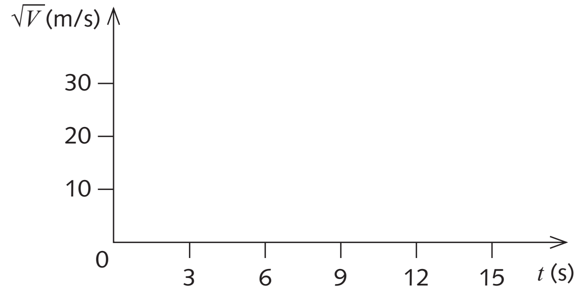
ICE-EM Mathematics 10 3ed ISBN 978-1-108-40434-1

5 Lindy is speeding in her car along a straight road at a constant speed of 20 m/s (72 km/h). She passes a stationary police motorcyclist, John. Three seconds later, John starts in pursuit. He accelerates for 6 seconds until he reaches his maximum speed, which he maintains until he overtakes Lindy. Let t seconds be the time elapsed since Lindy passed John.

John's speed, v m/s, at any time until he reaches his maximum speed at $t = 9$, is given by $v = 5(t - 3)$ for $3 \le t \le 9$.

a Find John's maximum speed.

b Copy this set of axes.



i Sketch the speed–time graph for Lindy.

ii On the same set of axes, sketch John's speed–time graph for $3 \le t \le 9$.

iii On the same set of axes, sketch John's speed–time graph for $t \ge 9$.

c Find the value of t when John and Lindy are travelling at equal speeds.

d What is John's acceleration (rate of change of speed) for $3 \le t \le 9$?

e Given that the distance travelled by an object is equal to the area under its speed–time graph (above the t-axis), find:

i the distance travelled by Lindy in the first 9 seconds

ii an expression for the distance travelled by Lindy after t seconds

f Find:

i the distance travelled by John by the time he reaches his maximum speed

ii the total distance travelled by John when $t = 12$

iii an expression for the total distance travelled by John, t seconds after Lindy passed him, for $t \ge 9$

g i Use your answers to parts **e** and **f** to find the value of t when John draws level with Lindy.

ii How far has John travelled by the time he draws level with Lindy?

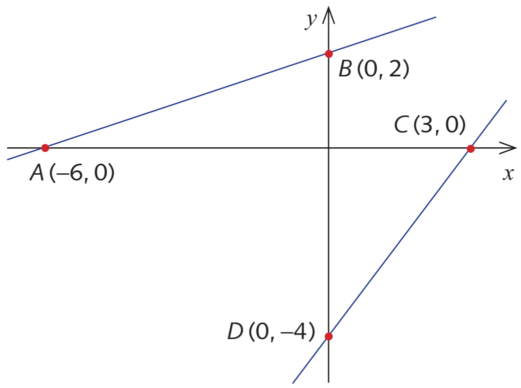
6 Consider the lines shown in the diagram.

a Find the gradient of:

i AB **ii** CD

b Find the equation of:

i AB **ii** CD



c Find the coordinates of E, the point of intersection of the line AB and the line CD.

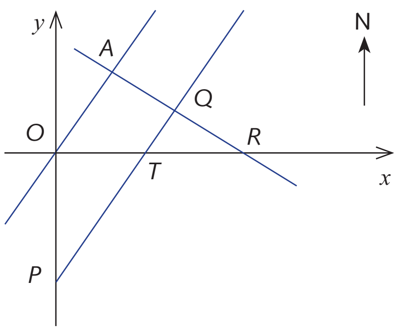
d Find the area of quadrilateral $ABCD$.

e Find the area of ΔDBE.

f If A and D remain fixed but $B = (0, 2b)$ and $C = (3b, 0)$, find the coordinates of E, the point of intersection of the line AB and the line CD, commenting on the special cases when $b = 0$, $b = 2$ and $b = -2$.

7 A surveyor has drawn lines on a map to represent straight roads between towns positioned at O, A, P, Q, T and R, as shown. Cartesian axes have been drawn so that equations can be assigned to roads. Distances are measured in kilometres.

The road through towns O and A has equation $y = \sqrt{2}x$, while the road through towns P and Q has equation $y = \sqrt{2}x - 20$. The road through towns A, Q and R has equation $y = \dfrac{-1}{\sqrt{2}}x + 25$. The direction due north is shown on the diagram.



Express all answers in parts **a** to **d** as exact values in surd form.

a Find the distance from town O to town P (OP).

b Find the distances:

i OT **ii** OR

c Find the coordinates of town Q at the intersection of the road from A to R with the road from P to Q.

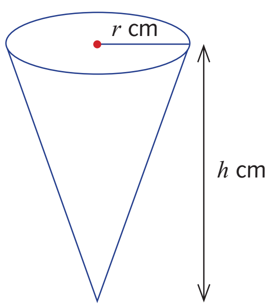
d A new road is to be built through towns positioned at P and R.

i Find the coordinates of P and R.

ii Find the gradient of the line from P to R.

iii Find the equation of the line that runs through P and R.

8 The cone shown in this diagram has an open circular top of radius r cm and depth h cm. The radius of the cone is equal to one-third of the height; that is, $r = \dfrac{h}{3}$.



a Express, in terms of r:

i h **ii** V **iii** A

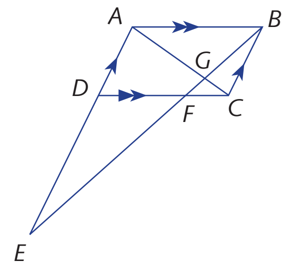
ICE-EM Mathematics 10 3ed ISBN 978-1-108-40434-1

b If the cone holds $50\ \text{cm}^3$ of water, find:

i the depth of water in the cone, correct to three significant figures

ii the curved surface area of the cone covered by water, correct to three significant figures

9 Using the diagram shown:



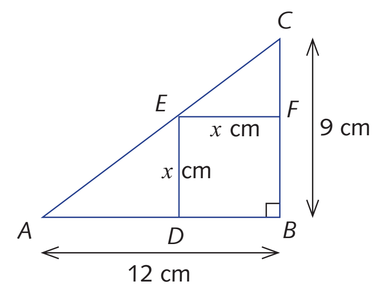
a prove that ΔAGB is similar to ΔCGF

b name two triangles similar to ΔEFD

c given that $DF : FC = 2 : 1$, and using your answers to parts **a** and **b**, find:

i $AB : DF$ **ii** $EF : EB$

10 a In the right-angled triangle ABC, there is a square $BDEF$, as shown.

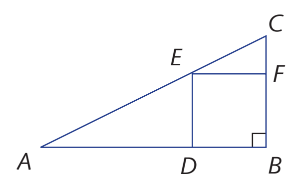


i What is the abbreviated reason for ΔEFC to be similar to ΔABC?

ii Hence, find x.

iii Hence, find the area of the square $BDEF$ as a fraction of the area of ΔABC.

b In this diagram, square $BDEF$ is inside ΔABC, as shown. If $BC = x$ cm, $EF = y$ cm and $AB = 2BC$:

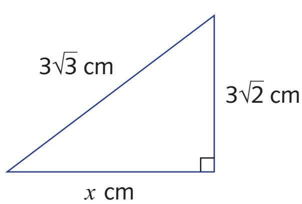


i find the relationship between x and y

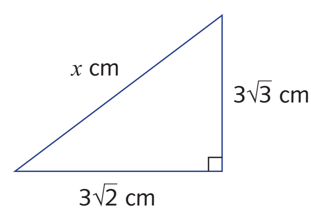
ii hence, find the area of the square $BDEF$ as a fraction of the area of ΔABC

11 a Find the exact value of x in the following diagrams.

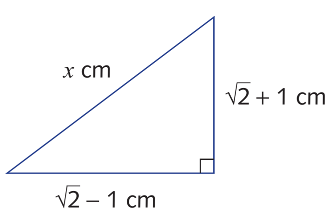
i



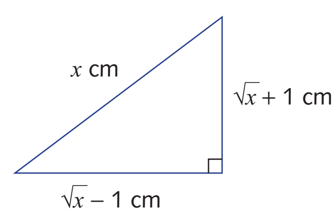
ii



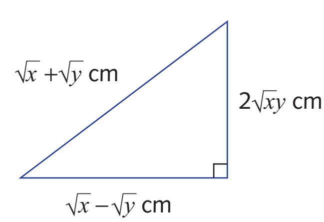
iii



iv

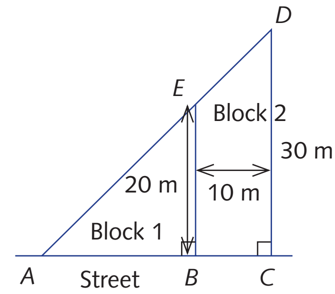


b Find the relationship between x and y, with y as the subject of the formula.



ICE-EM Mathematics 10 3ed ISBN 978-1-108-40434-1

12 A triangular region of land (ΔACD) is divided up into two areas, Block 1 and Block 2, to make way for residential development. Details regarding the plan are provided in the given diagram.



a Prove that ΔABE is similar to ΔACD.

b By letting $AB = x$ metres, write, in terms of x:

i the length AC

ii an equation linking x with the lengths EB, AC and CD

c Solve the equation in part **b ii** to find the length AB.

The subdivider now considers moving the position of the fence BE with the given information: $AC = 30$ m, $BE = AB$ and $BE \perp AC$. (The length DC remains at 30 m and $AC \perp CD$.)

d Given that $AB = x$ metres, find, in terms of x:

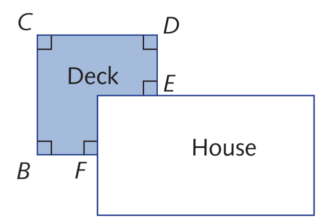
i the area of Block 1

ii the area of Block 2

e If the subdivider requires that the area of Block 1 is to be the same as the area of Block 2:

i write an equation in x to represent this situation

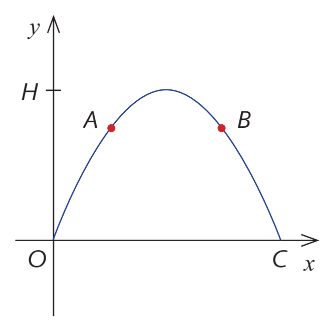
ii find the length AB (x metres) as a surd in simplest form



13 A builder has been contracted to construct a deck for a family on the corner of their house, as shown. The contract requirements are that $BF = DE$ and $BC = CD$, the total length of railing is $BF + BC + CD + DE = 30$ metres and that the deck has the maximum possible area. If $BF = x$ m and A m^2 = area of the deck:

a construct a formula relating A and x with A the subject

b hence, find the maximum possible area of the deck



14 A weather rocket is fired so that it follows a parabolic path, just over weather balloons A and B, as shown. It has been fired to follow the path with equation $y = \frac{1}{5}x - \frac{1}{50}x^2$, where x and y are measured in kilometres.

a Find how far the rocket travels horizontally from O to point C.

b Find H kilometres, the maximum height reached by the rocket.

Given that A and B are both at a height of 200 metres:

c find the coordinates of A and B, expressing your answer correct to two decimal places

d hence, find the horizontal distance, AB, between the balloons, correct to two decimal places

ICE-EM Mathematics 10 3ed ISBN 978-1-108-40434-1

15 Gipps Road and Bells Road are two non-intersecting roads in the country. The government wishes to build a road running North–South that connects these two roads. They employ you to work out where to build this connecting road in order to minimise its length. The path of Gipps Road is given by the equation $y = x^2 - 4x + 9$, while the path of Bells Road is given by the equation $y = 2x - 2$. In this model the positive direction of the x-axis runs due East while the positive direction of the y-axis runs due North. All lengths are in kilometres.

a Sketch a graph of Bells Road. Clearly mark in the x- and y-intercepts.

b **i** By completing the square, write the equation for Gipps Road in the form $y = (x - h)^2 + k$.

ii What will be the y-value for Gipps Road when $x = 0$?

iii Hence, on the same set of axes used to sketch Bells Road, sketch the graph for the path of Gipps Road. Clearly label the turning point and y-intercept.

c Find the distance between the two points on the roads where $x = 0$.

d When $x = a$, find, in terms of a, the y-value of:

i Bells Road

ii Gipps Road

e Hence, show that the North–South distance, d km, between the two roads when $x = a$ is given by $d = a^2 - 6a + 11$.

f On a new set of axes, sketch a graph of d against a. Clearly label the turning point.

g Hence, report back to the government on how long and how far East of the origin the North–South connection road should be built in order to minimise its length.

16 **a** Show, by completing the square, that $y = 3x^2 + 6x - 7$ can be written in the form $y = 3(x + 1)^2 - 10$.

A two-dimensional *Space Invaders*-type game involves a coordinate system whereby the x- and y-axes are centrally located on the screen and a space station is located at $P(-1, -12)$. An enemy spacecraft approaches and attacks the space station while flying on the path described by $y = 3x^2 + 6x - 7$.

b Use the result from part **a** to complete the following.

i Write the coordinates of the turning point of the path of the spacecraft.

ii Find the exact coordinates of where the path of the spacecraft cuts the x-axis, leaving your answer in surd form.

c Sketch a graph showing the path of the spacecraft and the position of the space station, P. Label the turning point and the x- and y-intercepts for the path of the spacecraft.

d If one unit represents 100 km, find the distance between the spacecraft and the space station when they are closest to each other.

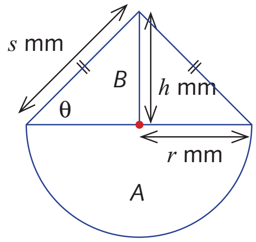
A second spacecraft flies on the path $y = 5x + 3$.

e Show that the x-coordinates of the intersection points of the paths of the two spacecraft can be found by solving $3x^2 + x - 10 = 0$.

f Solve the equation in part **e** and hence state the coordinates of the intersection points of the paths of the two spacecraft.

ICE-EM Mathematics 10 3ed ISBN 978-1-108-40434-1

17 The cross-section through the centre of a diamond cut at The Perfect Diamond Company is of the shape shown in the diagram. Region A is semicircular and region B is an isosceles triangle. The semicircle has radius r mm and the isosceles triangle has height h mm, slant height s mm and slant angle θ, as shown.



a Use trigonometric ratios to find a formula for:

i h in terms of r and θ **ii** s in terms of r and θ

b Find a formula for the area of:

i region A in terms of r and π **ii** region B in terms of r and θ

The Perfect Diamond Company's secret is to make sure that the cross-sectional areas of regions A and B are equal.

c Show that this leads to an equation that can be simplified to $\tan\theta = \frac{\pi}{2}$.

d Solve the equation in part **c** to find the value of θ for diamonds cut at The Perfect Diamond Company. Round off your answer to the nearest tenth of a degree.

e Find, to two decimal places, the total area of the cross-section through the centre of a diamond if the radius, r, is 2 mm.

10C Problem-solving

1 A line with gradient –2 passes through the point $(r, -3)$. A second line, perpendicular to the first, meets this line at the point (a, b). The second line passes through the point $(6, r)$. Find a and b in terms of r.

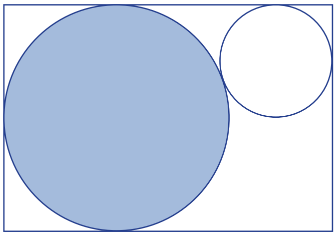
2 Find all the ordered pairs of integers such that $x^2 - y^2 = 140$.

3 **a** The sum of the lengths of the shorter sides of a right-angled triangle is 34. Find the length of the hypotenuse of the triangle if the area is:

i 30 cm^2 **ii** 32 cm^2

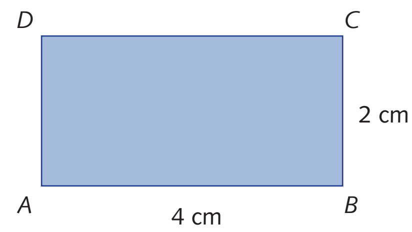
b The area of a rectangle is 12 cm^2 and its perimeter is 14 cm. What is the length of the diagonal of the rectangle?

4 A circle (shown shaded) just fits inside a 2 m × 3 m rectangle. What is the radius, in metres, of the largest circle that will also fit inside the rectangle but will not intersect with the shaded circle?

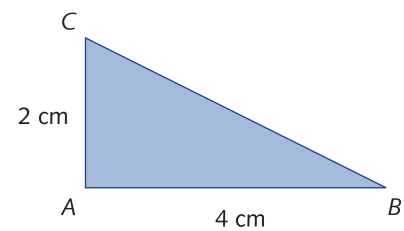


ICE-EM Mathematics 10 3ed ISBN 978-1-108-40434-1

5 **a** The rectangle $ABCD$ is rotated about the side AB. Find the volume of the solid defined by this rotation.

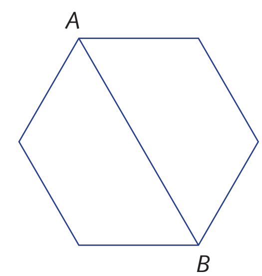


b Triangle ABC is rotated about the side AB. Find the volume of the solid defined by this rotation.



c A circle of radius 2 cm is rotated about a diameter. Find the volume of the solid defined by this rotation.

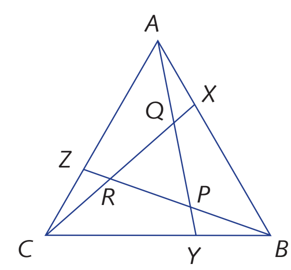
d A regular hexagon with side length 2 cm is rotated about the diagonal AB. Find the volume of the solid produced.



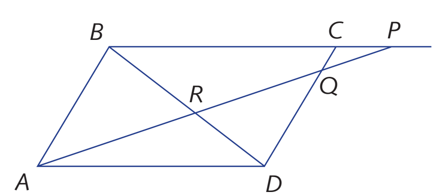
6 Prove, using coordinates, that the line intervals joining the midpoints of successive sides of any quadrilateral form a parallelogram.

7 If P is any point in the plane of a rectangle $ABCD$, prove that $(PA)^2 + (PC)^2 = (PB)^2 + (PD)^2$.

8 ΔABC is equilateral, X is on AB and $AX : XB = 1 : 2$. Y is on BC and $BY : YC = 1 : 2$. Z is on CA and $CZ : ZA = 1 : 2$. AY, BZ and CX intersect at P, Q and R. Prove that the area of ΔPQR is one-seventh of the area of ΔABC.

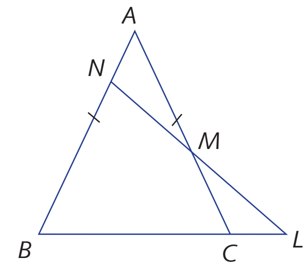


9 $ABCD$ is a parallelogram and P is any point on BC produced. Prove that: $AR^2 = RQ \times RP$.

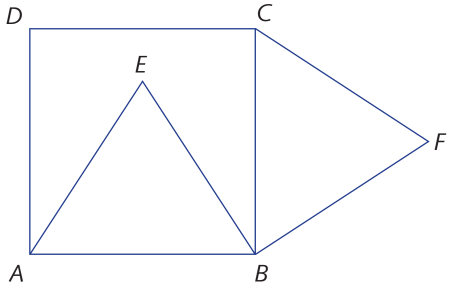


ICE-EM Mathematics 10 3ed ISBN 978-1-108-40434-1

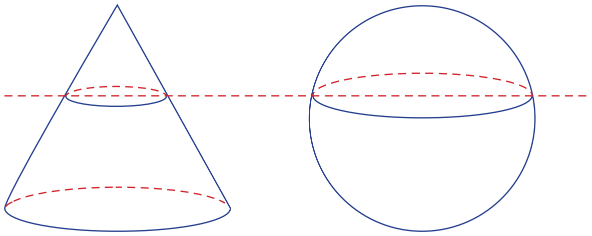
10 ΔABC is an isosceles triangle. L is a point on BC produced so that there are points N and M on AB and AC, respectively, so that $NM = ML$. Find the ratio $BN : CM$.



11 On square $ABCD$, an equilateral triangle ABE is constructed internally and an equilateral triangle BCF is constructed externally. Prove that the points D, E and F are collinear.



12 A sphere has radius 5 cm. A cone has height 10 cm and its base has radius 5 cm.

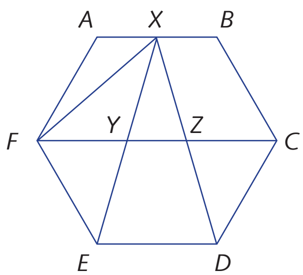


The sphere and the cone sit on a horizontal surface. Find the height of the horizontal plane above the surface that gives circular cross-sections of the sphere and the cone of equal area.

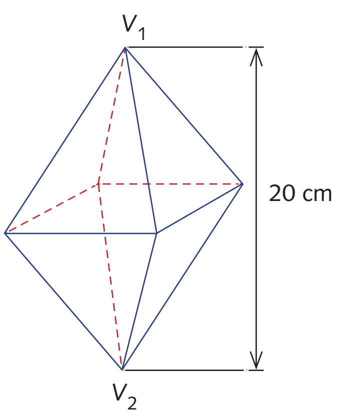
13 $ABCDEF$ is a regular hexagon. X is the midpoint of AB. XE and XD are drawn to meet FC at Y and Z, respectively.

Find the ratio:

Area of quadrilateral $YZDE$: Area of ΔFYX



14 The solid shown is a regular octahedron. The distance between the vertices V_1 and V_2 is 20 cm. Find the sum of the lengths of the edges of the octahedron.



ICE-EM Mathematics 10 3ed ISBN 978-1-108-40434-1