CHAPTER

16

Number and Algebra

Direct and inverse proportion

People working in science, economics and many other areas look for relationships between various quantities of interest. These relationships often turn out to be linear, quadratic or hyperbolic. That is, the graph relating these quantities is a straight line, a parabola or a rectangular hyperbola.

In Chapter 3, we revised the use of formulas. In this chapter we are mainly concerned with formulas for which the associated graphs are either straight lines or rectangular hyperbolas. In the first case we have direct proportion, and in the second we have inverse proportion. We have met direct proportion in Chapter 18 of *ICE-EM Mathematics Year 9.*

To take a very simple example, the formula $V = IR$ is called Ohm's law and relates voltage V, current I, and resistance R. The law is fundamental in the study of electricity. If R is a constant, V is directly proportional to I. If V is a constant, I is inversely proportional to R.

In part, because our examples are drawn from physical problems, in this chapter variables will take mostly positive values.

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16A Direct proportion

Andrew drives from his home at a constant speed of 100 km/h.

The formula for the distance, d km, travelled in t hours is:

$$d = 100t$$

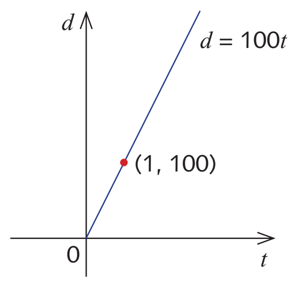
Andrew will go twice as far in twice the time, three times as far in three times the time and so on.

We say that d is **directly proportional** to t. The number 100 is called the **constant of proportionality**.

The statement 'd is directly proportional to t' is written as:

$$d \propto t$$

The graph of d against t is a straight line passing through the origin. The gradient of the line is 100.



By considering the gradient of the line, we see that for values t_1 and t_2 with corresponding values d_1 and d_2:

$$\frac{d_1}{t_1} = \frac{d_2}{t_2} = 100$$

That is, the constant of proportionality is the gradient of the straight line graph, $d = 100t$, which, in this example, is the speed of the car.

Quantities proportional to the square or cube

A metal ball is dropped from the top of a tall building and the distance it falls is recorded each second.

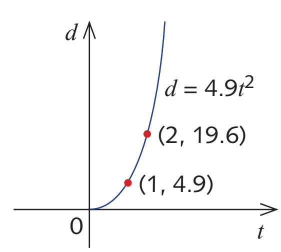
From physics, the formula for the distance, d metres, the ball has fallen in t seconds, is given by:

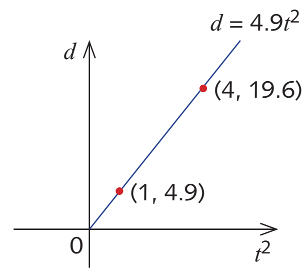
$$d = 4.9t^2$$

In this case, we say that d is **directly proportional** to the square of t.

The first diagram to the right is a graph of d against t. Since t is positive, the graph is half a parabola.

The second diagram to the right is a graph of d against t^2.





t	0	1	2	3
t^2	0	1	4	9
d	0	4.9	19.6	44.1

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This graph is now a straight line passing through the origin. The gradient of this line is 4.9.

The statement 'd is directly proportional to t^2' is written as:

$$d \propto t^2$$

This means that for any two values, t_1 and t_2, with corresponding values d_1 and d_2:

$$\frac{d_1}{t_1^2} = \frac{d_2}{t_2^2} = 4.9$$

So once again the gradient of the line is the constant of proportionality.

Finding the constant of proportionality

If we can relate two variables so that the graph is a straight line through the origin, then the constant of proportionality is the gradient of that line. Thus, to find the constant of proportionality, just one pair of non-zero values is needed.

Example 1

From physics, the kinetic energy, E mJ (mJ is the abbreviation for microjoules), of a body in motion is directly proportional to the square of its speed, v m/s. If a body travelling at a speed of 10 m/s has energy 400 mJ, find:

a the constant of proportionality

b the formula for E in terms of v

c the energy of the body when it travels at a speed of 15 m/s

d the speed if the moving body has energy 500 mJ

Solution

a Kinetic energy is directly proportional to the square of the speed.

$$E \propto v^2$$

so $E = kv^2$, for some constant k

We know that $E = 400$ when $v = 10$

so $400 = 100k$

$k = 4$

b From part **a**, $E = 4v^2$.

c When $v = 15$,

$$\begin{aligned} E &= 4 \times 15^2 \\ &= 900 \end{aligned}$$

Therefore, the body travelling at speed 15 m/s has energy of 900 mJ.

d When $E = 500$,

$$\begin{aligned} 500 &= 4 \times v^2 \\ v^2 &= 125 \\ v &= \sqrt{125} \quad (\text{since } v > 0) \\ &= 5\sqrt{5} \\ &\approx 11.18 \text{ m/s} \quad (\text{Correct to two decimal places.}) \end{aligned}$$

Therefore, the body has energy 500 mJ when travelling at $5\sqrt{5}$ m/s.

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The procedure for solving the previous example was as follows.

- Write down the statement of proportionality.
- Write this statement as an equation involving a constant, k.
- Substitute the given information to obtain the value of k.
- Rewrite the formula with the determined value of k.

Example 2

The mass, w grams, of a plastic material required to mould a solid ball is directly proportional to the cube of the radius, r cm, of the ball. If 40 grams of plastic is needed to make a ball of radius 2.5 cm, what size ball can be made from 200 grams of the same type of plastic?

Solution

$w \propto r^3$ so $w = kr^3$ for some constant k.

We know that $w = 40$ when $r = 2.5$

so, $40 = k \times (2.5)^3$
$k = 2.56$
Thus the formula is $w = 2.56r^3$

When $w = 200$, $200 = 2.56r^3$

$$r^3 = 78.125$$
$$r = \sqrt[3]{78.125}$$
$$r \approx 4.27$$

Thus, a ball with a radius of approximately 4.3 cm can be made from 200 grams of plastic.

Note: It is a fact that the mass of a ball of constant density is given by density × volume.
The volume is $\frac{4}{3}\pi r^3$ and so the mass of a ball is proportional to r^3.

Increase and decrease

If one quantity is proportional to another, we can investigate what happens to one of the quantities when the other is changed.

Suppose that $a \propto b$, then $a = kb$ for a positive constant, k.

If the value of b is doubled, then the value of a is doubled. For example, if $b = 1$, then $a = k$. So $b = 2$ gives $a = 2k$.

Similarly, if the value of b is tripled, then the value of a is tripled.

These ideas can be used in a variety of situations.

Example 3

Given that $y \propto \sqrt{x}$, what is the percentage change in:

a y when x is increased by 20%?

b x when y is decreased by 30%?

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Solution

Since $y \propto \sqrt{x}$, $y = k\sqrt{x}$

a When $x = 1$, $y = k$

If x is increased by 20%, then $x = 1.2$,

$$\text{so } y = k\sqrt{1.2} \approx 1.095k$$

and y is approximately 109.5% of its previous value.

Thus, y has increased by approximately 9.5%.

b Making x the subject in $y = k\sqrt{x}$:

$$y^2 = k^2 x$$

$$x = \frac{y^2}{k^2}$$

When $y = 1$, $x = \frac{1}{k^2}$

If y is decreased by 30%, then $y = 0.7$ and $x = \frac{0.49}{k^2}$, so x is 49% of its previous value.

Thus, x has decreased by 51%.

Direct proportion

- y is directly **proportional** to x^n if there is a positive constant k such that $y = kx^n$.
- The symbol $\propto$ is used for 'is proportional to'. We write $y \propto x^n$.
- The constant k is called the **constant of proportionality**.
- If y is directly proportional to x, then the graph of y against x^n is a straight line through the origin. The gradient of the line is the constant of proportionality.

Exercise 16A

Throughout this exercise, all variables take only positive values.

Example 1

1 **a** Given that $a \propto b$, and that $b = 3$ when $a = 1$, find the formula for a in terms of b.

b Given that $m \propto n$, and that $m = 15$ when $n = 3$, find the formula for m in terms of n.

2 Consider the following table of values.

p	0	1	4	9	16
q	0	4	8	12	16
$\sqrt{p}$					

a Plot the graph of q against p.

b Complete the table of values and calculate $\frac{q}{\sqrt{p}}$ for each pair $(q, \sqrt{p})$.

c Assuming that there is a simple relationship between the two variables, find a formula for q in terms of p.

Example 2

3 **a** Given that $m \propto n^2$ and that $m = 12$ when $n = 2$, find the formula for m in terms of n and the exact value of:

i m when $n = 5$ **ii** n when $m = 27$

b Given that $a \propto \sqrt{b}$ and that $a = 30$ when $b = 9$, find the formula for a in terms of b and the exact value of:

i a when $b = 16$ **ii** b when $a = 25$

4 In each part, find the formula connecting the pronumerals.

a $R \propto s$ and $s = 7$ when $R = 28$

b $P \propto T$ and $P = 12$ when $T = 100$

c a is directly proportional to the square root of b and $a = 12$ when $b = 9$

d V is directly proportional to r^3 and $V = 216$ when $r = 3$

5 In each of the following tables, $y \propto x$. Find the constant of proportionality and complete the tables.

a

x	2	8	12	18
y	$\frac{1}{2}$			

b

x		3	6	15
y	16		48	

6 On a particular road map, a distance of 0.5 cm on the map represents an actual distance of 10 km. What actual distance would a distance of 6.5 cm on the map represent?

7 The estimated cost $\$C$ of building a brick veneer house on a concrete slab is directly proportional to the area A of floor space in square metres. If it costs \$90 000 for 150 m^2, how much floor space would you expect for \$126 300?

8 The power p kW needed to run a boat varies as the cube of its speed, s m/s. If 400 kW will run a boat at 3 m/s, what power, correct to the nearest kW, is needed to run the same boat at 5 m/s?

9 If air resistance is neglected, the distance d metres that an object falls from rest is directly proportional to the square of the time t seconds of the fall. An object falls to 9.6 metres in 1.4 seconds. How far will the object fall in 4.2 seconds?

Example 3

10 The surface area of a sphere, A cm^2, is directly proportional to the square of the radius, r cm. What is the effect on:

a the surface area when the radius is tripled?

b the radius when the surface area is tripled?

11 Given that $m \propto n^5$, what is the effect on:

a m when n is doubled? **b** m when n is halved?

c n when m is multiplied by 243? **d** n when m is divided by 1024?

12 Given that $a \propto \sqrt{b}$, what is the effect, correct to two decimal places, on a when b is:

a increased by 25%? **b** decreased by 8%?

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13 Given that $p \propto \sqrt[3]{q}$, what is the effect on:

a p when q is increased by 20%?

b p when q is decreased by 5%?

c q when p is increased by 10%?

d q when p is decreased by 10%?

16B Inverse proportion

We know that:

$$\text{distance} = \text{speed} \times \text{time} \quad (d = vt)$$

Rearranging gives:

$$\text{time} = \frac{\text{distance}}{\text{speed}} \quad \left(t = \frac{d}{v}\right)$$

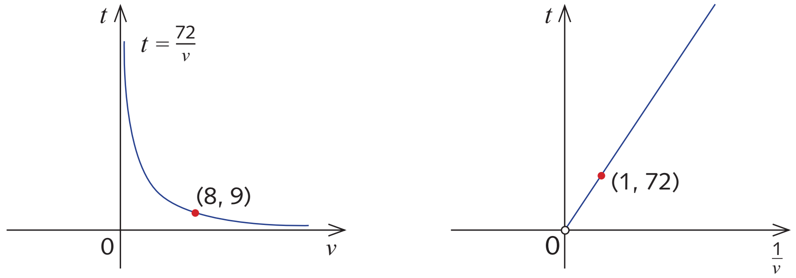
The distance between two towns is 72 km. The time t hours taken to cover this distance at v km/h is given by the formula:

$$t = \frac{72}{v}$$

As v increases, t decreases, and as v decreases, t increases.

This is an example of **inverse proportion**. We write $t \propto \frac{1}{v}$ and say t is **inversely proportional** to v. The number 72 is **the constant of proportionality**.

The graph of t against v is a branch of the rectangular hyperbola $t = \frac{72}{v}$, and the graph of t against $\frac{1}{v}$ is a straight line with gradient 72.



Example 4

Suppose that two towns, A and B, are 144 km apart.

a Write down the formula for the time taken, t hours, to travel from A to B at a speed of v km/h.

b Draw a graph of t against v.

c If the car is driven at 24 km/h, how long does it take to complete the journey?

d If the trip takes 90 minutes, at what speed is the car driven?

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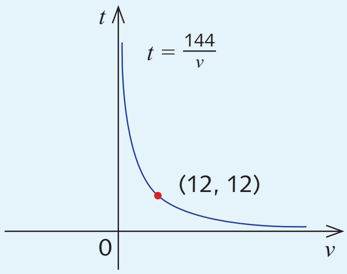
Solution

a $t = \dfrac{144}{v}$

b



c When $v = 24$,

$t = \dfrac{144}{24}$

$= 6$

It takes 6 hours.

d When $t = \dfrac{90}{60} = \dfrac{3}{2}$ h,

$\dfrac{3}{2} = \dfrac{144}{v}$

so $v = 144 \times \dfrac{2}{3} = 96$

The speed is 96 km/h.

Note: If $a \propto \dfrac{1}{b}$ then $ab = k$, where k is a positive constant. Conversely, if $ab = k$ for all values of a and b, then $a \propto \dfrac{1}{b}$.

Example 5

The volume, V cm^3, of a quantity of gas kept at a constant temperature is inversely proportional to the pressure, P kPa. If the volume is 500 cm^3 when the pressure is 80 kPa, find the volume when the pressure is 25 kPa.

Solution

V is inversely proportional to P $\left(V \propto \dfrac{1}{P}\right)$

so $V = \dfrac{k}{P}$ or $VP = k$ for some constant k.

We know that $V = 500$ when $P = 80$

$k = 500 \times 80$

$= 40\,000$

so $V = \dfrac{40\,000}{P}$

When $P = 25$, $V = \dfrac{40\,000}{25}$

$= 1600$

Thus, the volume of the gas at 25 kPa is 1600 cm^3.

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Example 6

If a is inversely proportional to the cube of b and $a = 2$ when $b = 3$, find the formula relating a and b. Then find:

a a when $b = 2$

b b when $a = \frac{27}{32}$

Solution

$a \propto \frac{1}{b^3}$

That is, $a = \frac{k}{b^3}$ or $ab^3 = k$ for some positive constant k.

We know that $a = 2$ when $b = 3$,

so $k = 54$

Hence, $a = \frac{54}{b^3}$

a When $b = 2$, $a = \frac{54}{8}$
$= 6.75$

b When $a = \frac{27}{32}$, $b^3 = 54 \div \frac{27}{32}$
$= 64$
Hence, $b = 4$

As with direct proportion, we are sometimes interested in the effect on one variable when the other one is changed. As before, we can take a particular value of one variable to work out the change in the other variable.

Example 7

Given that $y \propto \frac{1}{x^2}$, find, correct to the nearest 0.1%:

a the percentage change in y when x is decreased by 10%

b the percentage change in x when y is increased by 10%

Solution

$y \propto \frac{1}{x^2}$

that is, $y = \frac{k}{x^2}$, for some positive constant k.

a When $x = 1$, $y = k$.

When x is decreased by 10%, the new value of $x = 0.9$.

Then, $y = \frac{k}{0.9^2}$ so new value of $y \approx 1.235k$.

Thus, y is approximately 123.5% of its previous value.

That is, y has increased by approximately 23.5%.

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b $y = \dfrac{k}{x^2}$ so when $y = 1$,

$$x^2 = k$$
$$x = \sqrt{k}$$

When y is increased by 10%,

the new value of $y = 1.1$

so the new value of x is given by:

$$1.1 = \frac{k}{x^2}$$
$$x^2 = \frac{k}{1.1}$$
$$x \approx 0.953\sqrt{k}$$

[Alternatively, make x as the subject of the relationship.

$$x^2 = \frac{k}{y}$$
$$x = \sqrt{\frac{k}{y}}$$

When $y = 1$, $x = \sqrt{k}$

When $y = 1.1$, $x = \sqrt{\dfrac{k}{1.1}} = \dfrac{\sqrt{k}}{\sqrt{1.1}}$]

That is, x is approximately 95.3% of its previous value.

Thus, x has decreased by approximately 4.7%.

Inverse proportion

- y is **inversely proportional** to x^n when y is directly proportional to $\dfrac{1}{x}$.
- We write $y \propto \dfrac{1}{x}$ when $y = \dfrac{k}{x}$ or $xy = k$, where k is a positive constant.
- If y is inversely proportional to x, then the graph of y against $\dfrac{1}{x^n}$ is a straight line and the gradient of the line is equal to the **constant of proportionality**.
- If $y \propto \dfrac{1}{x}$, then for any pair of values x_1 and y_1, $x_1 y_1 = k$.

Exercise 16B

1 Consider the following table of values.

a	1	2	3	4	5
b	15	7.5	5	3.75	3

a Plot the graph of b against $\dfrac{1}{a}$.

b Assuming that there is a simple relationship between the two variables, find a formula for b in terms of a.

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2 Consider the following table of values.

x	1	2	5	10
y	100	25	4	1
x^2y	100			

a Complete the table of values for x^2y.

b Assuming that there is a simple relationship between the two variables, find a formula for y in terms of x.

3 Write each statement in symbols.

a The speed v km/h of a car over a given distance is inversely proportional to the time t hours of travel.

b m is inversely proportional to the square root of n.

c s is inversely proportional to the cube of t.

Example 5 **4** y is inversely proportional to x. If $x = 2$ when $y = 3$, find a formula relating x and y, and calculate:

a y when $x = \frac{3}{2}$ **b** x when $y = \frac{2}{3}$

Example 6 **5** Given that a is inversely proportional to b^2 and that $a = 6$ when $b = 2$, find a formula for a in terms of b, and calculate:

a a when $b = 3$ **b** b when $a = 3$

6 Given that $p \propto \frac{1}{\sqrt{q}}$ and that $p = 5$ when $q = 4$, find a formula for p in terms of q, and calculate:

a p when $q = 9$ **b** q when $p = 4$

7 For the data below, we assume that $y \propto \frac{1}{x}$. Find the constant of proportionality and complete the table.

x	1	2		4	
y	12		4		24

8 For the data below, we assume that $y \propto \frac{1}{x^2}$. Find the constant of proportionality and complete the table.

x	2		8		16
y	8	2		0.32	

Example 4 **9** If a car travels at an average speed of 60 km/h, it takes 77 minutes to complete a certain trip. To complete the same trip in 84 minutes, what average speed is required?

10 Timber dowelling comes in fixed lengths. If 48 pieces, each 3.6 cm long, can be cut from a fixed length, how many pieces 3.2 cm long can be cut from the same fixed length?

11 The illumination from a light is inversely proportional to the square of the distance from the light source. If the illumination is 3 units when seen from 4 metres away, find:

a the illumination when seen from 6 metres

b the distance from the light source when the illumination is 12 units

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12 Given that $m \propto \dfrac{1}{n^2}$, what is the effect on:

a m when n is doubled?

b m when n is halved?

c n when m is multiplied by 16?

d n when m is divided by 9?

Example 7

13 Given that $a \propto \dfrac{1}{b}$, what is the effect, correct to the nearest 0.1%, on a when b is:

a increased by 15%?

b decreased by 12%?

14 Given that $p \propto \dfrac{1}{q^3}$, what is the effect, correct to two decimal places, on:

a p when q is increased by 10%?

b p when q is decreased by 10%?

c q when p is increased by 20%?

d q when p is decreased by 20%?

15 We know that a cone of height h and radius r has volume $V = \dfrac{1}{3}\pi r^2 h$.

For cones of the same volume, height is inversely proportional to the radius squared.

a What is the effect on:

i the height when the radius is doubled?

ii the radius when the height is multiplied by 9?

b For cones of volume 12π cm^3, state the constant of proportionality in the relationship $h \propto \dfrac{1}{r^2}$.

16C Proportionality in several variables

Often a particular physical quantity is dependent on several other variables. For example, the distance d a motorist travels depends on both the speed v at which he travels and the time t taken for the trip. These variables are related by the formula $d = vt$. We say that d is directly proportional to v and t.

If $y = kxz$ for a positive constant k, we say that y is directly proportional to x and z. Similarly, if $a = \dfrac{kb^3}{c^2}$, where k is a positive constant, we say that a is directly proportional to b^3 and inversely proportional to c^2.

Example 8

Suppose that a is directly proportional to b and to the square of c.

If $a = 36$ when $b = 3$ and $c = 2$, find:

a the formula connecting a, b and c

b the value of a when $b = 4$ and $c = 1$

c the value of b when $a = 48$ and $c = 3$

d the value of c when $a = 64$ and $b = 6$

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Solution

a $a \propto bc^2$, so $a = kbc^2$ for some constant k.

Substitute $a = 36$, $b = 3$ and $c = 2$ to find k.

$36 = k \times 3 \times 2^2$; hence, $k = 3$.

Thus, $a = 3bc^2$.

b When $b = 4$ and $c = 1$:

$$a = 3 \times 4 \times 1^2 = 12$$

c When $a = 48$ and $c = 3$:

$$48 = 3 \times b \times 3^2$$
$$b = \frac{16}{9}$$

d When $a = 64$ and $b = 6$:

$$64 = 3 \times 6 \times c^2$$
$$c^2 = \frac{32}{9}$$
$$c = \frac{4\sqrt{2}}{3}$$

Example 9

Suppose that y is directly proportional to x and inversely proportional to z.

If $y = \frac{1}{5}$ when $x = \frac{2}{5}$ and $z = \frac{3}{5}$, find:

a the formula for y in terms of x and z

b the value of y when $x = 1$ and $z = \frac{3}{8}$

c the value of z when $x = 2$ and $y = \frac{1}{6}$

Solution

a $y = \frac{kx}{z}$, for some positive constant k.

We know that $y = \frac{1}{5}$ when $x = \frac{2}{5}$ and $z = \frac{3}{5}$

$$\text{so} \quad \frac{1}{5} = k \times \frac{2}{5} \div \frac{3}{5}$$
$$\text{and} \quad k = \frac{1}{5} \times \frac{5}{2} \times \frac{3}{5} = \frac{3}{10}$$

Hence, $k = \frac{3}{10}$ and $y = \frac{3x}{10z}$.

b $y = \frac{3x}{10z}$ so when $x = 1$ and $z = \frac{3}{8}$

$$\text{so} \quad y = \frac{3}{10} \div \frac{3}{8} = \frac{4}{5}$$

c $y = \frac{3x}{10z}$ so when $x = 2$ and $y = \frac{1}{6}$

$$\frac{1}{6} = \frac{3 \times 2}{10z}$$
$$\text{so} \quad 10z = 36$$
$$z = 3.6$$

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Example 10

Suppose that a is directly proportional to the square of b and inversely proportional to c. Find the effect on a when:

a b is halved and c is doubled

b b is increased by 10% and c is increased by 20%

Solution

$a = \dfrac{kb^2}{c}$ for some positive constant k.

a When $b = 1$ and $c = 1$, $a = k$

When $b = \dfrac{1}{2}$ and $c = 2$, $a = \dfrac{1}{4}k \div 2$

$$= \frac{k}{8}$$

Thus, the value of a is divided by 8.

b When $b = 1$ and $c = 1$, $a = k$

When b is increased by 10% and c is increased by 20%.

So, $b = 1.1$ and $c = 1.2$

$$a = \frac{1.1^2}{1.2}k$$

$$= \frac{121}{120}k$$

Thus, a is increased by approximately 0.83%.

Exercise 16C

Example 8

1 If $a \propto bc$, write down the formula relating the variables and complete the following table.

a	12	24		48	72
b	1		2		2
c	1	2	1	2	

2 If $r \propto \dfrac{s}{t}$, write down the formula relating the variables and complete the following table.

r	24	12		48	4
s	1		2		2
t	1	2	2	1	

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Example 9

3 Suppose that y is directly proportional to x and inversely proportional to w. If $y = 2$ when $x = 7$ and $w = 14$, find y when $x = 10$ and $w = 8$.

4 Assume that y is directly proportional to the square of x and inversely proportional to the square root of z.

a Write a formula for y in terms of x and z.

b If $y = 6$ when $x = 2$ and $z = 4$, find y when $x = 3$ and $z = 16$.

5 Suppose that a is directly proportional to b and the cube of c.

a Write a formula for a in terms of b and c.

b If $a = 96$ when $b = 3$ and $c = 2$, find b when $a = 16$ and $c = \frac{1}{2}$.

6 The amount of heat, H units, produced by an electric heater element is directly proportional to the square of the current, i amperes, flowing through the element, to the electrical resistance, R ohms, and to the time, t seconds, for which the current has been flowing.

a Write down the formula for H in terms of i, R and t.

b If 256 units of heat are produced by a current of 2 amp through a resistance of 40 ohms for 10 seconds, how much heat is produced by a current of 4.5 amp through a resistance of 60 ohms for 15 seconds?

7 A model aeroplane attached to one end of a string moves in a horizontal circle. The tension, T N (or newtons), in the string is directly proportional to the square of the speed, v m/s, and inversely proportional to the radius, r m, of the circle. If the radius is 10 m and the speed is 20 m/s, the tension is 60 N. Find the tension if the radius is 15 m and the speed is 30 m/s.

8 The frequency n (the number of vibrations per second) of a piano string varies directly as the square root of the tension, T N, in the string and inversely as the length, ℓ cm, of the string. A string 30 cm long under a tension 25 N has a frequency of 256 vibrations per second (this is the pitch called 'middle C'). If the tension is changed to 30 N, to what must the length be changed, correct to two decimal places, for the string to emit the same note?

9 The quantity t is directly proportional to m and n, and is inversely proportional to the square of r. If $t = \frac{45}{4}$ when $m = 3$, $n = 5$ and $r = 4$, find:

a r when $t = 6$, $m = 9$ and $n = 8$

b n when $t = 8$, $r = 12$ and $m = 4$

Example 10

10 If y is directly proportional to the cube of x and inversely proportional to the square of z, what is the effect on y if:

a both x and z are doubled?

b x is increased in the ratio $3 : 2$ and z is decreased in the ratio $1 : 2$?

11 If y is directly proportional to the square of x and inversely proportional to the square root of z, what is the effect on y if:

a x and z are increased by 10%?

b x is increased by 20% and z is decreased by 15%?

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12 The force of attraction F between two particles of masses m_1 and m_2 that are distance d apart varies directly as the product of the masses, and inversely as the square of the distance between them.

a What is the effect on F if the distance between the two masses is doubled?

b What is the effect on the force if the distance between the two particles is halved and the mass of one particle is trebled?

13 The value of g, the acceleration due to gravity on the surface of a planet or moon, varies directly as the planet or moon's mass and inversely as the square of the radius of the planet. The mass of the Moon is $\frac{1}{80}$ of the mass of the Earth, and the radius of the moon is $\frac{3}{11}$ the radius of the earth. Given that the value of g on the surface of the earth is 9.8 m/s^2, find the value of g, correct to two decimal places, on the surface of the moon.

Review exercise

1 Write each of the following in words.

a $x \propto y$ **b** $p \propto n^2$ **c** $a \propto \sqrt{b}$ **d** $p \propto q^3$

2 **a** Given that $p \propto q$ and $p = 12$ when $q = 1.5$, find the exact value of:

i p when $q = 6$ **ii** q when $p = 81$

b Given that $a \propto b^2$ and $a = 20$ when $b = 4$, find the formula for a in terms of b and:

i a when $b = 5$ **ii** a when $b = 12$

3 In each of the following tables, $y \propto x$. Find the constant of proportionality in each case and complete the tables.

a

x	0	1	2	3
y	0	12		

b

x	2	8	12	18
y	3			

4 Given that $y \propto x^3$, what is the effect on y when x is:

a doubled? **b** multiplied by 3? **c** divided by 4?

5 Given that $m \propto \sqrt{n}$, what is the effect on:

a m when n is doubled? **b** m when n is divided by 4?

6 Given that $a \propto b^2$, what is the effect on a when b is:

a increased by 5%? **b** decreased by 8%?

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7 y is inversely proportional to x. If $x = 5$ when $y = 8$, find:

a y when $x = \frac{3}{2}$

b x when $y = \frac{2}{3}$

8 a is inversely proportional to b^2. If $a = 8$ when $b = 2$, find:

a a when $b = 4$

b b when $a = 9$

9 z is directly proportional to the square of x and inversely proportional to the square root of y. If $z = 12$ when $x = 2$ and $y = 4$, find z when $x = 6$ and $y = 32$.

10 The quantity y is directly proportional to x and the cube of z. If $y = 108$ when $x = 3$ and $z = 2$, find x when $y = 24$ and $z = \frac{1}{2}$.

Challenge exercise

1 The electrical resistance, R ohms, in a wire is directly proportional to its length, L m, and inversely proportional to the square of its diameter, D mm. A certain wire 100 m long with a diameter 0.4 mm has a resistance 1.4 ohms.

a Find the equation connecting R, L and D.

b Find the resistance (correct to one decimal place) of a wire of the same material if it is 150 m in length and has a diameter of 0.25 mm.

c If the length and diameter are doubled, what is the effect on the resistance?

d If the length is increased by 10% and the diameter is decreased by 5%, what is the percentage change on the resistance? (Give your answer correct to one decimal place.)

2 If $a \propto c$ and $b \propto c$, prove that $a + b$, $a - b$ and $\sqrt{ab}$ are directly proportional to c.

3 It is known that $a \propto x$, $b \propto \frac{1}{x^2}$ and $y = a + b$. If $y = 30$ when $x = 2$ or $x = 3$, find the expression for y in terms of x.

4 If $x^2 + y^2$ is directly proportional to $x + y$ and $y = 2$ when $x = 2$, find the value of y when $x = \frac{4}{5}$.

5 For stones of the same quality, the value of a diamond is proportional to the square of its weight. Find the loss incurred by cutting a diamond worth $\$C$ into two pieces whose weights are in the ratio $a : b$.

6 If $a + b \propto a - b$, prove that $a^2 + b^2 \propto ab$.

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