CHAPTER

21

Review and problem-solving

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21A Review

Chapter 11: Circles, hyperbolas and simultaneous equations

1 Sketch the graph of:

a $x^2 + y^2 = 49$ **b** $x^2 + y^2 = 7$

c $(x - 2)^2 + y^2 = 4$ **d** $(x + 1)^2 + (y - 2)^2 = 16$

2 Write the equation of the circle with:

a centre (3, 0) and radius 4 **b** centre (–1, 2) and radius $\sqrt{3}$

3 Express each equation in the form $(x - h)^2 + (y - k)^2 = r^2$ and hence state the coordinates of the centre and the radius of the circle.

a $x^2 - 4x + y^2 + 6y + 9 = 0$ **b** $x^2 + 2x + y^2 + 8y + 1 = 0$

4 Sketch the graph of:

a $y = \frac{2}{x}$ **b** $y = 2 - \frac{1}{x}$ **c** $y = \frac{3}{x - 2}$ **d** $y = \frac{1}{x + 3} - 2$

5 Find the intersection points of:

a $y = x^2 + 2x - 3$, $y = 3x + 3$ **b** $y = 2x^2 + 3x - 3$, $y = 2x + 3$ **c** $y = 2x + 1$, $y = \frac{3}{x}$ **d** $y = 3x + 7$, $y = \frac{6}{x}$

6 Find the intersection points of:

a $x^2 + y^2 = 9$, $y = 2$ **b** $x^2 + y^2 = 4$, $x = 1$

c $x^2 + y^2 = 4$, $y = x + 2$ **d** $x^2 + y^2 = 16$, $y = 4 - \sqrt{2x}$

7 Find the coordinates of the points of intersection of $y + 2x = 1$ and $x^2 + y^2 = 13$.

8 Find the coordinates of the points of intersection of $4y = x^2 - 4$ and $2y - x = 10$.

9 Sketch each inequality.

a $y < 2x + 3$ **b** $x + 2y \le 6$ **c** $(x - 2)^2 + y^2 \le 1$

d $x^2 + (y - 2)^2 \le 4$ **e** $x^2 + y^2 > 9$ **f** $y > \frac{1}{x + 1}$

10 Sketch each region.

a $y \ge x$ and $x \ge 0$ and $x + y \le 6$

b $y \ge x$ and $y \le 2x$ and $y \le 6$

c $x \ge 0$ and $y \ge 0$ and $y \le 2x + 1$ and $x + y \le 8$

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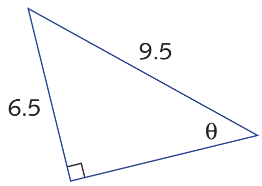
Chapter 12: Further trigonometry

Unless otherwise stated, values should be calculated to one decimal place.

1 Find the value of each pronumeral.

a



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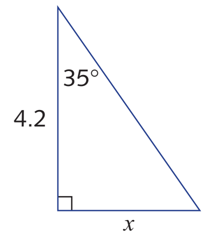

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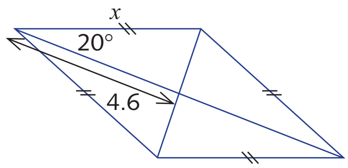


2 A 3 m ladder leans against a wall so that it makes an angle of 40° with the vertical.

a How far up the wall does it reach?

b How far is the foot of the ladder from the wall?

3 Find the angle of elevation of the sun when a tree 1.5 m high casts a shadow of 75 cm.

4 A hiker walks due south for 6 km then on a bearing of 270°T for 10 km and finally due north for 15 km.

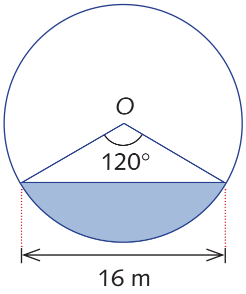
a Calculate the distance between the starting point and the finishing point.

b Calculate, to the nearest degree, the bearing of the starting point from the final position.

5 An aeroplane flies on a bearing of 060°T for 80 km and then on a bearing of 150°T for 70 km. What is the bearing of the starting point from the final position of the aeroplane?

6 An observer is 350 m from the shoreline, where a man is standing. Between the observer and the man is a sand dune 15 m high and 100 m from the sea. What is the minimum height above sea level that the observer's eye must be in order for him to see the man's feet?

7 The surface of the water in a horizontal pipe is 16 m wide and subtends an angle of 120° at the centre of the pipe, as shown. Find, correct to three decimal places:



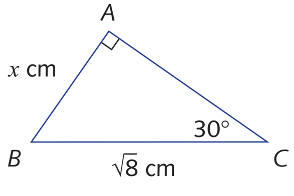
a the distance from the centre of the pipe to the water surface

b the diameter of the pipe

c the maximum depth of the water

8 Find the exact value of x.

a



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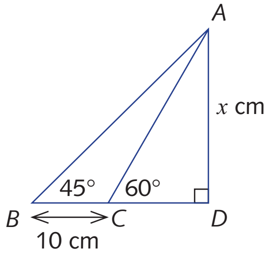

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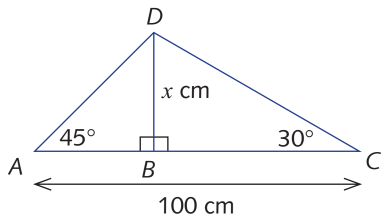
9 For the diagram shown, find the exact value of x.



10 A piece of wire 20 cm long is bent in the shape of a triangle with interior angles 30°, 60° and 90°. Find the length of the hypotenuse, giving your answer in surd form with a rational denominator.

11 A boat was sailing off the coast of Wilson's Promontory on a bearing of 350°T. At 1400 hours (2 p.m.), the bearing from the boat to South-East Point Lighthouse was 020°T and, at 1600 hours (4 p.m.), the bearing from the boat to the same lighthouse was 050°T. If the boat was travelling at 6 km/h, how far from the lighthouse was the boat at 1600 hours?

12 Find the missing side-lengths and angles for triangle ABC, given that:

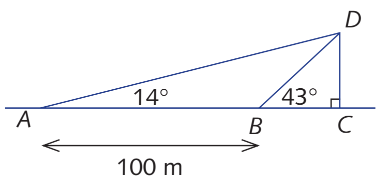
a $AB = 3$, $BC = 5$ and $\angle BAC = 50°$ **b** $AB = 6$, $AC = 4$ and $\angle ACB = 70°$

c $BC = 2$, $\angle BAC = 65°$ and $\angle ABC = 80°$

13 A hiker walks 5 km on a bearing of 143°T and then turns on a bearing of 121°T and walks a further 10 km. How far is the hiker from his starting position?

14 A scout measures the magnitudes of the angles of elevation to the top of a flagpole, CD, from two points (A and B) at ground level. A is 100 metres further away from the flagpole than B.

A, B, C and D are in the one vertical plane. If the angles are 43° and 14°, calculate the height of the flagpole, giving your answer correct to four significant figures.

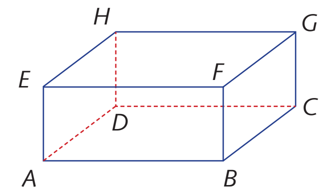


15 The bearing of a boat is taken from two points, A and B, which are on a jetty. The bearing of B from A is 090°T and $AB = 100$ m. The bearing of the boat from A is 045°T and from B is 030°. Find the distance of the boat from B, giving your answer as an exact value.

16 In the prism $ABCDEFGH$, $AB = 12$ cm, $BC = 5$ cm and $CG = 6$ cm. Find:

a the inclination of AG to the plane $ABCD$

b the inclination of HB to the plane $BCGF$



17 A right pyramid $VABCD$ stands on a square base $ABCD$ of side length 42 cm. If each sloping face makes an angle of 60° with the base, find:

a the height of the pyramid (correct to four significant figures)

b the angle a sloping edge makes with the base (correct to one decimal place)

c the length of a sloping edge (correct to four significant figures)

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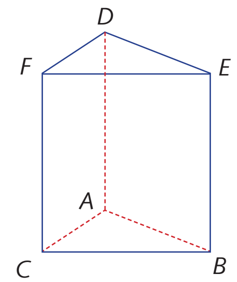
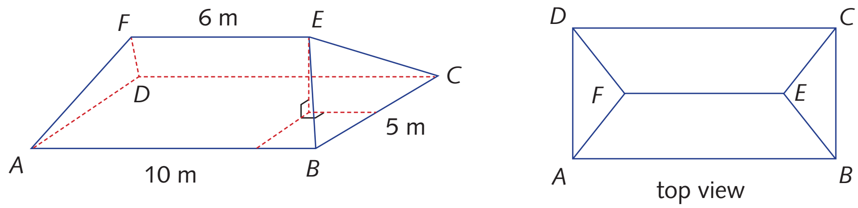
18 *ABCDEF* is a right prism where $\angle BAC$ is a right angle. Given that $AB = 8$ cm, $AC = 3$ cm and $AD = 15$ cm, find the inclination of the interval *CE* to the face *ADEB*.



19 In the gable roof shown below, the ceiling *ABCD* lies in a horizontal plane and the slope of the opposite faces is the same. The ridge beam *FE* is parallel to the ceiling plane and 2 m above it.



Calculate:

a the inclination of the face *EBC* to the ceiling

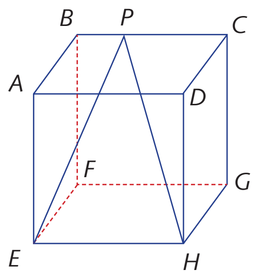
b the inclination of the rafter *EB* to the ceiling

20 *ABCDEFGH* is a cube with sides of length 5 cm.

P is a point on *BC*. Describe the location(s) of *P* so that $\angle EPH$ is:

a least **b** greatest

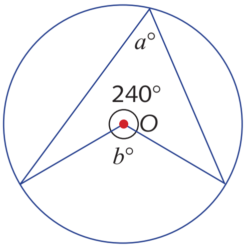
and state the size of $\angle EPH$ in each case, to two decimal places



Chapter 13: Circle geometry

1 Find the value of the pronumerals.

a



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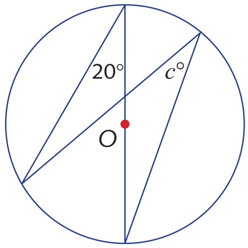

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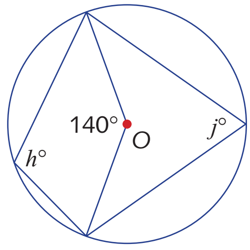
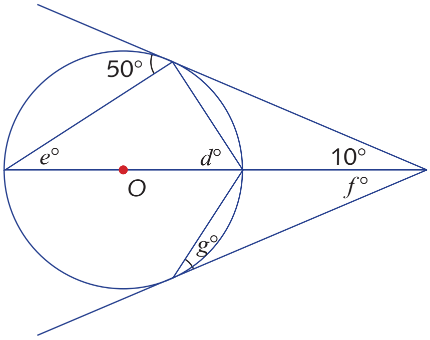
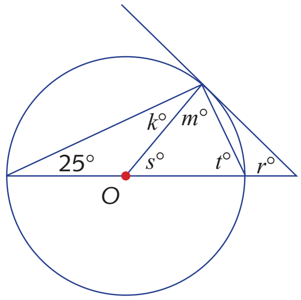


d



e



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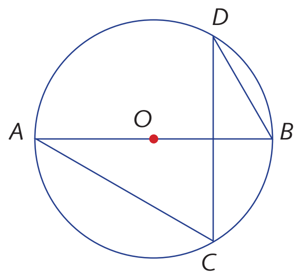
2 In the circle with centre O, AB is a diameter and $BC = OB$.

a Find the size of:

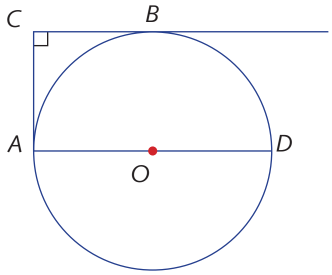
i $\angle ACB$ **ii** $\angle BOC$

iii $\angle CAB$ **iv** $\angle CDB$

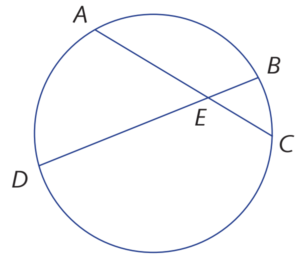
b If the radius of the circle is 6 cm, find AC.



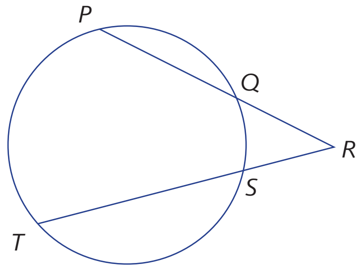
3 AD is the diameter of a circle ADB, with centre O. BC is the tangent to the circle at B, $AC \perp BC$ and AC is tangent to the circle at A. Prove that BA bisects $\angle CAD$.



4 AC and BD are two chords of a circle intersecting internally at E. Given that $AE = 6$ cm, $EC = 3$ cm and $DE = 9$ cm, find the length of BE.



5 PQ and TS are two secants of a circle intersecting externally at R. Given that $PQ = 5$ cm, $QR = 7$ cm and $SR = 4$ cm, find the length of TS.



6 $ABCD$ is a cyclic quadrilateral with BA and CD extended to meet at E. If $AD = 2$ cm, $BC = 5$ cm, $EA = 4$ cm and $AB = 11$ cm, find EC and ED.

7 P is a point inside triangle ABC. BP is extended to cut AC at Q and CP is extended to cut AB at R. If $BP \times PQ = CP \times PR$, prove that $AR \times AB = AQ \times AC$.

8 PT is a tangent to a circle where T is the point of tangency, and PXY is a secant.

a If $PT = 6$ cm and $PX = 4$ cm, find XY and PY.

b If $XY = 24$ cm and $PX = 3$ cm, find PT.

c If $XY = 21$ cm and $PT = 10$ cm, find PX.

9 AB is a chord of a circle ABC with centre O and TC is a tangent at C. If $\angle BCT = 75°$, find the size of $\angle BOC$.

10 AB is a chord of a circle and XAY is the tangent at A. AK and AL are chords bisecting $\angle XAB$ and $\angle YAB$, respectively. Prove that:

a $AL = BL$ **b** KL is the diameter of the circle

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Chapter 14: Indices, exponentials and logarithms – part 2

1 Calculate each logarithm.

a $\log_2 16$ **b** $\log_3 81$ **c** $\log_2 1024$

d $\log_7 1$ **e** $\log_{10} 100\,000$

2 Calculate each logarithm.

a $\log_2 \frac{1}{32}$ **b** $\log_3 \frac{1}{243}$ **c** $\log_{10} \frac{1}{10\,000}$ **d** $\log_{10} 0.01$

e $\log_5 \frac{1}{625}$ **f** $\log_6 \frac{1}{216}$ **g** $\log_2 \frac{1}{2048}$ **h** $\log_{10} 0.000\,01$

3 Simplify:

a $\log_2 15 + \log_2 5$ **b** $\log_2 7 + \log_2 9$ **c** $\log_2 11 + \log_2 3$

d $\log_3 1000 - \log_3 10$ **e** $\log_7 200 - \log_7 5$ **f** $\log_7 42 - \log_7 6$

g $\log_3 15 - \log_3 45$ **h** $\log_5 1000 - \log_5 200$ **i** $\log_5 30 - \log_5 6$

4 Simplify:

a $\log_2 7 - \log_2 11 + \log_2 22$ **b** $\log_3 1000 - \log_3 10 - \log_3 5$

c $\log_5 7 + \log_5 49 - 2\log_5 343$ **d** $\log_{11} 25 + \log_{11} 3 - \log_{11} 125$

5 Solve each logarithmic equation for x.

a $\log_5 x = 3$ **b** $\log_2 x = 8$ **c** $\log_5(x + 5) = 4$

d $\log_2(6x - 3) = 10$ **e** $\log_2(5 - x) = 6$ **f** $\log_{10}(2x - 1) = 4$

6 Solve each logarithmic equation for x.

a $\log_x 27 = 3$ **b** $\log_x 16 = 6$ **c** $\log_x 2048 = 6$ **d** $\log_x 1000 = 3$

7 Sketch each graph.

a $y = \log_5 x, \quad x > 0$ **b** $y = \log_3(x - 2), \quad x > 2$

c $y = \log_3(x + 5), \quad x > -5$ **d** $y = 3\log_2 x, \quad x > 0$

e $y = \log_3(x) - 2, \quad x > 0$

Chapter 15: Probability

1 A fair die is rolled once. Find the probability that the number showing on the die is:

a divisible by 3 **b** an even number

2 A card is drawn at random from a standard deck of playing cards. Find the probability that the card is:

a a Heart **b** a Jack **c** the Ace of Hearts

d a court card (i.e. a Jack, King or Queen)

3 Two thousand tickets are sold in a raffle. If you buy 10 tickets, what is the probability that you will win first prize?

4 A fair coin is tossed 5 times. What is the probability of getting 3 heads from the 5 tosses?

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5 Two dice are rolled and the sum of the values on the uppermost faces is noted. Find the probability that the sum is:

a 10

b 12

c less than 9

6 From a box containing 6 red and 4 blue spheres, 2 spheres are taken at random:

i with replacement

ii without replacement

In each case, find the probability that:

a both spheres are blue

b one is red and one is blue

7 A group of 1000 people, eligible to vote, were asked their age and their preferred candidate in an upcoming election, with the following results.

	18–25 years	26–40 years	Over 40 years	Total
Candidate *A*	200	100	85	385
Candidate *B*	250	230	50	530
Candidate *C*	50	20	15	85
Total	500	350	150	1000

What is the probability that a person chosen at random from this group:

a is between 18 and 25 years old?

b prefers Candidate *A*?

c is between 18 and 25 years old, given that they prefer Candidate *A*?

d prefers Candidate *A*, given that they are between 18 and 25 years old?

8 $P(A) = p$, $P(B) = \dfrac{3p}{2}$ and $P(A \cup B) = \dfrac{2}{3}$. Find p if:

a A and B are mutually exclusive

b A and B are independent

9 Of the patients reporting to a clinic, 35% have a headache, 50% have a fever, and 10% have both.

a What is the probability that a patient selected at random has either a headache, a fever or both?

b Are the events 'headache' and 'fever' independent? Explain your answer.

10 Records indicate that 60% of secondary students participate in sport, and 50% of secondary students regularly read books for leisure. They also show that 20% of students participate in sport and also read books for leisure. Use this information to find:

a the probability that a person selected at random does not read books for leisure

b the probability that a person selected at random does not read books for leisure, given that they do not participate in sport

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Chapter 16: Direct and inverse proportion

1 In each of the following:

i find the constant of proportion and the formula for y in terms of x

ii find the missing numbers in the tables

a

x	1	4	8	
y		2	4	10

$y \propto x$

b

x	1	3	5	
y	2	18		14

$y \propto x^2$

c

x	2	5	7	11
y	$\frac{5}{2}$	1		

$y \propto \frac{1}{x}$

d

x	2	3		7
y	$\frac{1}{4}$		8	$\frac{1}{49}$

$y \propto \frac{1}{x^2}$

2 Given that $y \propto \sqrt{x}$, and if $y = 27$ when $x = 9$, find the formula for y in terms of x, and find:

a y when $x = 4$

b x when $y = 75$

3 The surface area of a sphere is directly proportional to the square of the radius. If the surface area of a spherical ball of radius 7 cm is 616 cm^2, find the surface area of a sphere of radius 3.5 cm.

4 Given that y is inversely proportional to x^2 and $y = 10$ when $x = 2$, find the formula for y in terms of x, and find:

a y when $x = 9$

b x when $y = 9$

5 Given that $c \propto ab^2$, find:

a the constant of proportionality and the formula for c in terms of a and b

b the missing numbers in the table

a	5		6	
b	1	2		3
c	10	24	48	54

6 a is proportional to x and inversely proportional to y. If $a = 8$ when $x = 7$ and $y = 14$, find a when $x = 14$ and $y = 7$.

7 z is proportional to the square of x and proportional to the square root of y. If $z = 72$ when $x = 2$ and $y = 4$, find z when $x = 3$ and $y = 9$.

8 The energy of a moving body is proportional to its mass and the square of its velocity. A mass of 3 kg has a velocity of 10 m/sec and its kinetic energy is 150 joule.

a Find the kinetic energy of a mass of 5 kg, moving with a velocity of 30 m/sec.

b What is the effect on the kinetic energy of doubling the mass and doubling the velocity?

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Chapter 17: Polynomials

1 Let $P(x) = x^3 - 2x + 4$. Find:

a $P(1)$ **b** $P(-1)$ **c** $P(2)$

d $P(-2)$ **e** $P(0)$ **f** $P(a)$

2 **a** Find a, if $P(x) = x^4 - 3x^2 - 5x + a$ and $P(2) = 1$.

b Find b, if $Q(x) = x^3 - 3x^2 + bx + 6$ and $Q(-1) = 0$.

3 Find the sum $P(x) + Q(x)$ and the difference $P(x) - Q(x)$, given that:

a $P(x) = x^3 + 4x + 7$ and $Q(x) = -2x^3 + 3x^2 - 4x$

b $P(x) = -3x^5 - 3x + 7$ and $Q(x) = 3x^5 + x^2 - 7$

c $P(x) = 4x^3 - 5x^2 - 6x + 6$ and $Q(x) = -4x^3 + 5x^2 + 5x - 4$

4 Use the division algorithm to divide $P(x)$ by $D(x)$. Express each result in the form $P(x) = D(x)Q(x) + R(x)$, where either $R(x) = 0$ or the degree of $R(x)$ is less than the degree of $D(x)$.

a $P(x) = x^2 + 8x + 6$, $D(x) = x + 2$

b $P(x) = x^3 - 6x^2 - 12x + 30$, $D(x) = x + 6$

c $P(x) = 5x^3 - 7x^2 - 1$, $D(x) = x - 1$

5 Use the remainder theorem to find the remainder when the polynomial $P(x) = x^3 + 2x^2 - x + 3$ is divided by:

a $x - 3$ **b** $x - \frac{1}{2}$ **c** $x + \frac{1}{2}$

6 Find the value of a in the polynomial $ax^3 + 2x^2 + 3$ if the remainder is 3 when the polynomial is divided by $x - 2$.

7 Factorise each polynomial.

a $2x^3 + 5x^2 - x - 6$ **b** $2x^3 + x^2 - 7x - 6$ **c** $2x^4 - x^3 - 8x^2 + x + 6$

8 Solve each equation for x.

a $2x^3 + 5x^2 - x - 6 = 0$ **b** $2x^4 - x^3 - 8x^2 + x + 6 = 0$

9 Let $P(x) = x^3 - kx^2 + 2kx - k - 1$.

a Show that $P(x)$ is divisible by $x - 1$ for all k.

b If $P(x)$ is divisible by $x - 2$, find the value of k.

c Assuming that $x - 2$ divides $P(x)$, solve the equation $P(x) = 0$.

10 **a** Write $\frac{2x + 3}{x - 1}$ in the form $a + \frac{b}{x - 1}$.

b Write $\frac{4x^2 + 3x + 2}{x^2 + 2x}$ in the form $a + \frac{bx + c}{x^2 + 2x}$.

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Chapter 18: Statistics

1 Calculate, correct to two decimal places, the mean and standard deviation for each data set.

a 3, 5, 6, 10, 12, 14, 11, 12, 11, 15, 5

b 7, 9, 11, 13, 15, 16, 18, 12, 11, 10, 14, 16, 18, 19

2 The body mass and heart mass of 14 ten-month old male mice are given in the table below.

Body mass (grams)	27	30	37	38	32	36	32	32	38	42	36	44	33	38
Heart mass (milligrams)	118	136	156	150	140	155	157	114	144	149	159	149	131	160

a Draw a scatter plot of the heart mass against the body mass.

b Draw a line of best fit and describe the main features of the scatter plot.

3 The following table represents the results of two different tests for a group of students.

Student	Test 1	Test 2
1	214	216
2	281	270
3	212	221
4	324	326
5	340	330
6	205	207
7	208	213
8	304	312
9	303	311

Draw the scatter plot of Test 2 against Test 1 and comment on the result.

4 A woman keeps a record of how long it takes her to get to work each day for a month. The times in minutes are as follows.

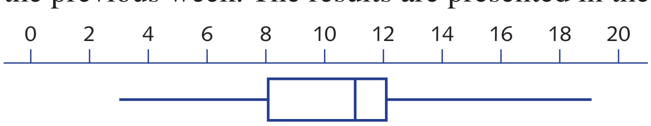
42	31	38	29	47	41	46	28	32	37	38
46	41	27	35	38	42	48	27	29	32	

a Find the median.

b Find the interquartile range.

c Use the information to construct a boxplot.

5 In a market survey, 200 people were asked how many hours of television they watched in the previous week. The results are presented in the boxplot below.



a What is the maximum number of hours anyone watched television?

b How many people watched more than 8 hours of television?

c What is the interquartile range?

d How many people watched between 8 hours and 11 hours of television?

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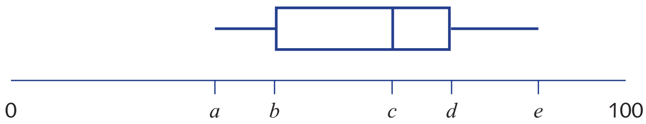
6 **a** The boxplot shows the distribution of test scores in a class (Class A) of 20 students.



The lowest score in the class was 38, the range of the scores was 50 and the median was 61.

i Write down the values of a, c and e.

ii When all the test scores were added up the total was 1240.

What was the mean of the test scores?

b The stem-and-leaf plot shows the distribution of test scores in Class B for the same test.

Stem	Leaf
4	4 7
5	2 3 3 6 9
6	2 3 7 8
7	1 5 6
8	3 6
9	0

4 | 7 is 47

i Assuming all students sat for the test, write down the number of students in Class B.

ii Find the median of the scores for Class B.

7 A community group is claiming that traffic volume on a suburban street has risen to 500 vehicles for the hour between 8 and 9 a.m. on weekdays. George lives on this street and decides to conduct his own test. The following data represents George's count of vehicles between 8 and 9 a.m. on Monday to Friday for 2 weeks.

	Monday	Tuesday	Wednesday	Thursday	Friday
Week 1	383	295	378	317	346
Week 2	15	339	311	341	357

a How might you explain the value of the outlier; that is, the value obtained for Monday of week 2?

For the remaining parts, ignore this outlier.

b Find the:

i mean, correct to one decimal place

ii median

iii interquartile range

c Represent the data as a boxplot.

d Give reasons which might explain the discrepancy between the community group's claim and the data gathered by George.

Chapter 19: Trigonometric functions

1 State which quadrant each angle is in.

a 160° **b** 245° **c** 240° **d** 300°

e 135° **f** 272° **g** 192° **h** 337°

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2 Without evaluating, express each number as the trigonometric function of an acute angle.

a $\sin 175°$ **b** $\cos 150°$ **c** $\tan 160°$ **d** $\sin 200°$

e $\cos 200°$ **f** $\tan 185°$ **g** $\sin 355°$ **h** $\cos 350°$

3 Find the exact value of:

a $\cos 135°$ **b** $\sin 225°$ **c** $\sin 120°$ **d** $\tan 120°$

e $\sin 330°$ **f** $\cos 315°$ **g** $\tan 315°$ **h** $\sin 240°$

4 Without using a calculator, find the exact value of:

a $\sin 90° \times \sin 225° \times \cos 135°$ **b** $\sin 330° \times \cos 240°$

c $\sin 360° \times \cos 275°$ **d** $2 \times \sin 120° \times \cos 120°$

5 Using exact values, find the angles θ between 0° and 360° inclusive, with the given trigonometric function.

a $\cos\theta = \frac{1}{2}$ **b** $\tan\theta = -\sqrt{3}$ **c** $\sin\theta = \frac{1}{\sqrt{2}}$

d $\sin\theta = -\frac{1}{2}$ **e** $\cos\theta = -\frac{\sqrt{3}}{2}$ **f** $\tan\theta = -1$

6 Using a calculator, find, correct to two decimal places, the angles θ between 0° and 360° inclusive, such that:

a $\sin\theta = 0.2745$ **b** $\cos\theta = -0.9165$ **c** $\tan\theta = 2.2465$

d $\sin\theta = -0.8976$ **e** $\cos\theta = 0.7010$ **f** $\tan\theta = -2.5884$

7 Find, in surd form, each of the following.

a $\cos(-60°)$ **b** $\sin(-225°)$ **c** $\tan(-135°)$

d $\cos(-210°)$ **e** $\cos(-330°)$ **f** $\sin(-405°)$

8 Solve each equation for $0° \le \theta < 360°$.

a $2\cos\theta = 1$ **b** $2\cos\theta = -\sqrt{3}$ **c** $2\sin\theta + \sqrt{3} = 0$

d $6\cos\theta + 3 = 0$ **e** $8\tan\theta = 8$ **f** $\sqrt{3}\tan\theta = 1$

Chapter 20: Functions and inverse functions

1 Given that $f(x) = 2x - 1$, find:

a $f(0)$ **b** $f(4)$ **c** $f(-1)$ **d** $f(-5)$

2 The function f is defined by $f(x) = \frac{4}{x}$, $x \neq 0$. Find:

a $f\left(\frac{1}{2}\right)$ **b** $f(2)$ **c** $f(8)$ **d** $f(-2)$

3 If $f(x) = 3 - x$, find:

a $f(1)$ **b** $f(-1)$ **c** $f(5)$ **d** $f(-3)$

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4 Find the value of a if:

a $f(x) = 5x - 4$ and $f(a) = 2$

b $f(x) = \frac{1}{x}$ $(x \neq 0)$ and $f(a) = 5$

5 Write down the domain for each function.

a $f(x) = \frac{1}{x+2}$

b $f(x) = \frac{1}{3x-6}$

c $f(x) = \sqrt{5-x}$

d $g(x) = \sqrt{2x-4}$

e $g(x) = \frac{1}{x^2-9}$

f $f(x) = \log_2(x+7)$

g $f(x) = 2^x + 6$

h $h(x) = \log_2(2x-1)$

i $h(x) = \log_2(6-x)$

6 Sketch each function and write down its domain and its range.

a $f(x) = x^2 - 3$

b $g(x) = 6 - x^2$

c $f(x) = \log_2(x+3)$

d $g(x) = 3^x + 6$

e $h(x) = 6 - 2^x$

f $f(x) = \sqrt{16 - x^2}$

7 Let $f(x) = x^3$. Sketch the graph of $y = f(x)$, $y = f(-x)$ and $y = 2f(x)$ on the one set of axes.

8 Suppose that $f(x) = x^2$ and $g(x) = 2x - 3$. Calculate:

a $f(g(1))$

b $g(f(1))$

c $g(f(x))$

d $f(g(x))$

9 For each function $f(x)$, find the inverse function $g(x)$ and state its domain.

a $f(x) = 2x - 3$

b $f(x) = \frac{x-1}{2}$

c $f(x) = 2^x - 3$

d $f(x) = \log_3(x+1)$

e $f(x) = 8 - x^3$

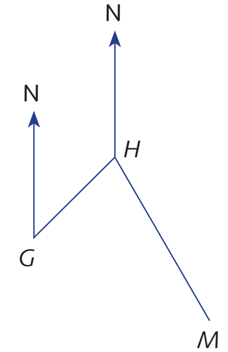
f $f(x) = x^3 - 8$

21B Problem-solving

1 A man starts from a point G and walks for 6 km on a bearing of 045°to a point H, then he walks 10 km on a bearing of 150° to a point M. From his position at M:

a how far is he from G, correct to one decimal place?

b what is the bearing of G from M, correct to one decimal place?



2 a A ladder $5\sqrt{3}$ m long leaning against a vertical wall makes an angle of $x°$ with the ground. If the foot of the ladder is a distance of $3\sqrt{3}$ m from the wall, then:

i find how far the ladder reaches up the wall

ii find x, correct to the nearest degree

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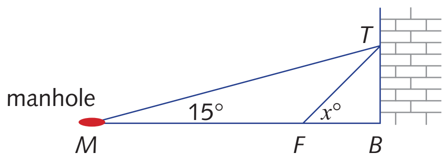
b A manhole is at a point (M) where the angle of elevation to the top of the ladder (T) is 15°, as shown in the diagram.

Find the exact distance from the manhole to the foot of the ladder, given that $\tan 15° = 2 - \sqrt{3}$.



3 A ship is sailing on a bearing of 350°T. At 2 p.m., the bearing from the ship to North Cape Light is 080°T and the bearing from the ship to South Light is 105°T. It is clear from a map that the bearing from South Light to North Cape Light is 355°T, and they are 1.5 km apart.

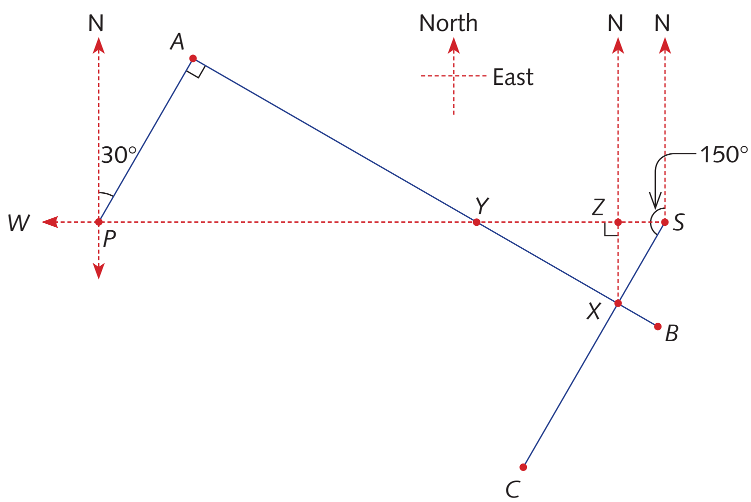
a Draw a diagram using A for the point that the bearings were taken from the ship, N for North Cape Light and S for South Light. Clearly label all bearings and true north directions.

b Draw ΔANS, indicating the angles and side lengths that are known.

c Find the distance from the 2 p.m. position of the ship to the North Cape Light, to the nearest metre.

d If the ship has maintained a constant course, find, to the nearest metre, the closest it came to South Light.

4 Pedro and Sam are both camping in the bush. Sam's campsite is 15 km due east of Pedro's campsite. At 9 a.m., they both walk out from their campsites. Initially Pedro walks 5 km to checkpoint A. From there, he turns right 90° and walks 15 km to checkpoint B. Sam just walks 10 km to checkpoint C. The paths Pedro and Sam follow from their campsites are indicated on the diagram below. The angles are given from due north. Let P and S represent Pedro and Sam's campsites, respectively.



Let X be the point where their paths cross and Y be the point of intersection of the lines PS and AX.

a Find each angle.

i $\angle APS$ **ii** $\angle AYP$ **iii** $\angle SYX$ **iv** $\angle SXY$

b i Find the distance PY.
ii Hence, calculate the distance YS.

c Prove that ΔPAY and ΔSXY are similar.

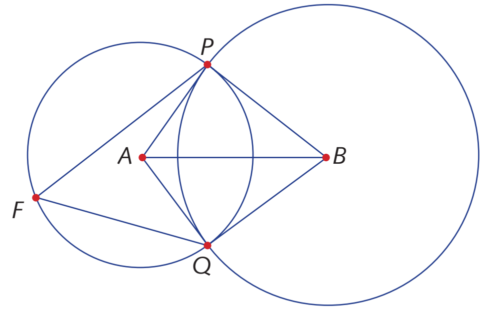
d Hence, find the distance SX.

e Find the exact values of:

i AX **ii** XB

f Hence, find how far apart Pedro and Sam finish up. Give your answer, correct to the nearest metre.

5 Two sprinklers, A and B, are set up to spray the circular areas shown in the diagram. Sprinkler A has a spray radius of 3 m and sprinkler B has a spray radius of 4 m. Points P and Q show the intersection of the circles. The sprinklers are 5 m apart.



a Explain why ΔPAB is a right-angled triangle.

b Which angle in ΔPAB is the right angle?

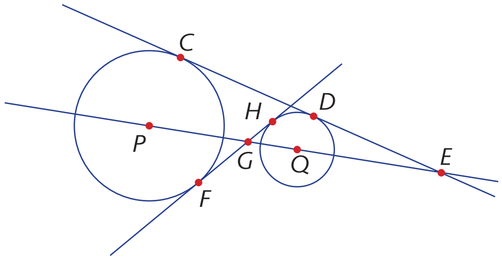
c Prove that $\Delta APB \equiv \Delta AQB$.

d Find, to the nearest degree, the size of:

i $\angle PAB$ **ii** $\angle QAP$

e F is a point on the circle with centre A.
Find $\angle PFQ$ and give a reason for your answer.

6 In the diagram to the right, the line CE and the line FH are tangents to both circles with centres P and Q. The points of tangency for CE are C and D, and the points of tangency for FH are F and H.

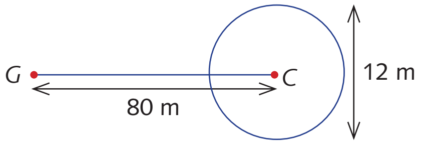


a Prove that ΔCPE is similar to ΔDQE.

b Prove that ΔGFP is similar to ΔGHQ.

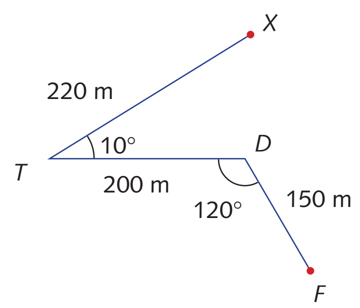
c Prove that $\dfrac{CE}{FG} = \dfrac{DE}{GH}$.

7 a This diagram represents a golfer at G, 80 m from the centre of a green, C, which can be represented by a circle of diameter 12 metres.

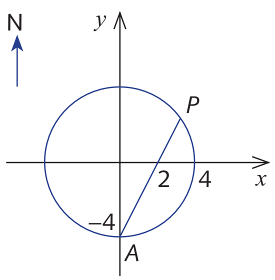


Calculate, to two decimal places, the greatest angle that the golfer can deviate either side of the direct line GC so that the golfer's ball can land on the green.

b This diagram below represents a 350 m golf hole. TD and DF represent the centre line of the fairway, with $\angle TDF = 120°$, $TD = 200$ m and $DF = 150$ m. The golfer hits 220 m, 10° left of the line TD, to a point, X. Find the distance, correct to two decimal places, from X to F.



8 Bob the gardener is planning a circular garden, as shown, that is divided into two sections by the string line AP. Dimensions are in metres. The direction north is indicated.

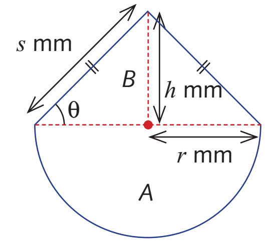


a Write down the equation of the circle.

b Find the equation for the straight line AP.

c A peg is placed at point P. By using your answers to parts **a** and **b**, find the coordinates of P, and hence state where the peg is relative to the centre of the garden.

9 The cross-section through the centre of a diamond cut at the Perfect Diamond Company is of the shape shown in the diagram. Region A is semicircular and region B is an isosceles triangle. The semicircle has radius r mm and the isosceles triangle has height h mm, slant height s mm and slant angle θ, as shown.



a Find a formula for:

i h in terms of r and θ

ii s in terms of r and θ

b Find a formula for the area of:

i region A in terms of r and π

ii region B in terms of r and θ

c The Perfect Diamond Company's secret is to make sure that the cross-sectional areas of regions A and B are equal. Show that this leads to an equation that can be simplified to $\tan\theta = \dfrac{\pi}{2}$.

d Solve the equation in part **c** to find the value of θ, correct to one decimal place, for diamonds cut at the Perfect Diamond Company.

e Find, to two decimal places, the total area of the cross-section through the centre of a diamond if the radius r is 2 mm.

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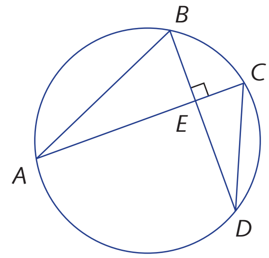
10 Suppose that the points A, B, C and D lie on a circle, with AC meeting BD at right-angles at E.



a If $\angle BAE = 30°$, find the size of:

i $\angle ABE$

ii $\angle CDE$

b If $\angle BAE = a°$ and $\angle ECD = b°$, find an equation relating a and b.

Next, suppose that $AC = 10$ cm, $BD = 10$ cm, $AE = x$ cm and $BE = y$ cm.

c Find:

i CE in terms of x

ii DE in terms of y

d Show that $x = y$ or $x + y = 10$.

e Find, in terms of x and y:

i area ΔABE

ii area ΔCED

Next, suppose that area (ΔABE) = area (ΔCED).

f Show that $x + y = 10$.

g Find the area of ΔABE in terms of x.

h find x if that the area of ΔABE is 12 cm^2.

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