

Answers to exercises

Chapter 1 answers

Exercise 1A

- 1 a 0.56 b 0.082 c 0.12
d 0.0375 e 2.15 f 0.008
g 0.8825 h 0.008 75
- 2 a $\frac{9}{20}$ b $\frac{16}{25}$ c $\frac{27}{40}$
d $\frac{2}{3}$ e $\frac{33}{400}$ f $\frac{7}{125}$
g $\frac{6}{5}$ h $\frac{3}{2}$ i $\frac{29}{400}$
j $\frac{51}{400}$ k $\frac{1}{200}$ l $\frac{39}{500}$
- 3 a 80% b 87.5% c 43.75%
d 150% e 45% f $166\frac{2}{3}\%$
g 46% h 2.5% i 140%
j 112.5% k 0.075% l 225%
- 4 a $\frac{16}{25}$, 0.64 b 60%, 0.6 c $16\%, \frac{4}{25}$
d $\frac{41}{200}$, 0.205 e $140\%, 1\frac{2}{5}$ f 62.5%, 0.625
- 5 a 9 b 188.64 c 523.2
d 1011.6 e 2.77 f 1.1
- 6 a \$26 b \$254.10 c \$1749
d \$2.70 e \$165 f \$7.95
- 7 a 6% b 1.3%
c 4.7% d 160%
- 8 a 4.80% b 1.70% c 4.17%
d 3.13% e 3200% f 0.14%
- 9 45
- 10 a 1.508 g b 0.007 28 g
- 11 61 minutes and 6 seconds
- 12 a 16500 b 31500
- 13 a \$480 b 240 mins c 240 cm
d \$218 e 800 mm f $373\frac{1}{3}$ mm
- 14 \$1900
- 15 a \$575 b \$12 929
- 16 a 100% profit b 17.6% loss
- 17 a costs \$518 519, total sales \$546 519
b costs \$11 538 462, total sales \$10 788 462

- 18 a i \$0 ii \$450
iii \$4700 iv \$127 700
b i 0% ii 3%
iii 12.37% iv 31.93%
c i \$22 533.33 ii \$34 640.00
iii \$78 571.43 iv \$78 000.00

- 19 a \$1600 b \$8000
- 20 a \$720 b \$2025
- 21 a 7.5% p.a. b 6.8% p.a.
- 22 a 50 years b 38 years
- 23 a \$15 000 b \$24 790.12

Exercise 1B

- 1 a \$627 b \$9786
c \$483.36 d \$3369.60
- 2 a \$8100 b \$4332
c \$801.22 d \$9139.20
- 3 a 35 840 b 171 360 c 278
- 4 a 434 mm b 88 mm c 786 mm
- 5 a 6% increase b 88% increase
c 22% decrease d 44% decrease
- 6 a \$630 b \$5160
- 7 a 200 b 7299 c 68 142
- 8 a 4750 megalitres b 11 340 megalitres
c 1840 megalitres d 63 330 megalitres
- 9 a \$52 b \$31.20
c \$442 d \$1.04
- 10 a \$3111.11 b \$726.77 c \$54.44
- 11 a \$28 000 b \$5992.56 c \$13 806.10
d \$69 565.22 e \$31 890.04 f \$421 741.12
g 4% h 60.22% i -17.16%
- 12 a i \$3014 ii \$147 400 iii \$9.02
b i \$3570 ii \$388 070 iii \$2.90
c i \$4862 ii \$3 818 254 iii \$5.17
- 13 a i \$3102 ii \$24.97 iii \$373.27
b i \$4061.90 ii \$13.83 iii \$73.50
- 14 a 50% b 20% c 81.8% d 72.4%

- 15 a i 9.09% ii 15.25%
 iii 78.26% iv 3.94%
 b i 11.11% ii 21.95%
 iii 400% iv 4.28%

Exercise 1C

- 1 a \$12 474 b \$8042.64 c \$233 436.82
 2 \$3.83
 3 a \$7706.79 b \$96 119.13
 c \$128.24 d \$3947.50
 4 a 50.4%
 b i \$1503.63 ii \$25 965.05 iii \$324.84
 5 25.75%
 6 a decreased by 4% b decreased by 64%
 7 a \$2.52 b \$2.19 c \$1.99
 8 a i \$370.74 ii \$8897.76 iii \$4 199 930.76
 b 62.9%
 9 a 5158 b 51 589 c 61 895
 10 a i 49.8°C ii 24.8°C iii 3.1°C
 b i 20.2°C ii 34.2°C iii 68.6°C
 11 a 18.6% b 11.6%
 12 a 90.9% b 100.0% c 100.0%
 13 a decrease of 25%
 b decrease of 25%
 c As multiplication is commutative, if the percentage increases and decreases are the same, but in different orders, the overall effect is the same.

Exercise 1D

- 1 a \$107 000 b \$114 490 c \$140 255.17
 d 40.26% e \$40 255.17 f \$35 000
 2 a 59 750 b 76%
 3 a i \$1 630 910.87 ii 329%
 b i \$1 696 688.53 ii 346%
 4 a \$327 000 b \$548 411.74 c 82.80%
 d \$248 411.74 e \$189 000
 5 a \$83 521 265.73 b \$153 336 556.81 c \$12 000
 6 a \$33 542.00 b \$13 542.00
 7 a \$58 349.04 b 71%
 8 a \$154 880.98 b 61%

- 9 a \$43 297.57 b \$12 335.56
 10 a 74 488 b 69 356 c 55 985 d 39 178
 11 a \$16 282.30, \$13 282.30
 b \$5 512 011.72, \$2 512 011.72
 c \$552.75, \$2447.25
 d \$1 632 797.69, \$1 367 202.31
 12 a \$5400, \$5724, \$6067.44
 b \$5551.00, \$5893.38, \$6256.87
 13 \$1 202 680.28
 14 a 3.72% b 7.57%
 15 a 32.3% b 33.1% c 33.8%
 d 34.0% e 34.4% f 34.6%
 g The total percentage growth is approximately the same, but increases as the number of years increases.
 16 87%

Exercise 1E

- 1 a \$420 000 b \$294 000 c \$70 589.40
 d 88.2% e \$88 235.10
 2 a \$83 886.08, 73.8%
 b \$507 799.78
 3 Depreciated value is \$15 126. Price obtained is better by \$4874, to the nearest dollar.
 4 \$2 724 000, \$1 634 400, \$980 640, \$588 384;
 \$1 816 000, \$1 089 600, \$653 760, \$392 256
 5 Sandra \$116 731.38, Kevin \$20 971.52
 6 a taxis 98.4%, other cars 78.3%
 b taxis \$156 250, other cars \$2 166 757.03,
 difference = \$2 010 507.03
 7 a \$8387 b \$10 822 c \$83 157
 d 92.2% e \$7666
 8 a \$15 523.20 b \$33 348.49
 c \$83.2% d \$3964.07
 e i \$2480.80 ii \$5941.76
 9 0.4096%
 10 gain of 58.86%
 11 a i 27.75% ii 27.10% iii 26.61%
 iv 26.49% v 26.26% vi 26.14%
 b The total depreciation is approximately the same, but decreases as the number of years increases.
 12 92.09% 13 2.6%

Review exercise

- 1 a \$5400 b \$2700 c \$1500
- 2 a \$5000 b \$3333.33
c \$2083.33 d \$4000
- 3 a 8% b $12\frac{1}{2}\%$ c $11\frac{1}{9}\%$
- 4 16% 5 \$85.50
- 6 a 66 b 125 c 64
d 225 e 390 f 294.84
- 7 \$93.50 8 \$67.82
- 9 a i 99 ii 96
iii 99.84 iv 99.36
b No, they all end up close to 100.
- 10 a 2250 b \$2025 c \$1822.50 d 27.1%
- 11 a \$390 b 56%
- 12 7.58%
- 13 a \$4317.85 b \$10 235.50 c \$5320.45
- 14 a \$19 467.20 b \$16 477.04
c \$10 859.71 d $25\,000(0.92)^n$
- 15 a \$13 721.03 b \$7241.03 c 7 years
- 16 \$40 000

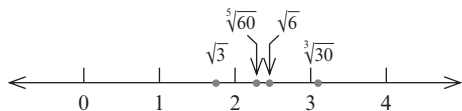
Challenge exercise

- 1 a \$37036 b 0.32% decrease
- 2 a Bank B b \$9653.19
- 3 a $\frac{50}{n}$ b \$50.51
c \$47.98 d \$53.03
- 4 12.36% 5 0.8% increase
- 6 25% 7 80.4%
- 8 \$369 814.11 9 $r \approx 4.88$

Chapter 2 answers

Exercise 2A

1



- 2 a $2\sqrt{2}$ b $2\sqrt{3}$ c $4\sqrt{2}$
d $5\sqrt{2}$ e $3\sqrt{6}$ f $6\sqrt{3}$

- g $7\sqrt{2}$ h $10\sqrt{2}$ i $12\sqrt{2}$
j $7\sqrt{3}$ k $4\sqrt{7}$ l $5\sqrt{7}$
m $7\sqrt{5}$ n $7\sqrt{6}$ o 15
p 30 q $15\sqrt{2}$ r $20\sqrt{2}$
s $10\sqrt{10}$ t $24\sqrt{3}$

- 3 a $10\sqrt{3}$ b $20\sqrt{5}$ c $18\sqrt{11}$
d $15\sqrt{6}$ e $24\sqrt{5}$ f $35\sqrt{5}$
g $24\sqrt{2}$ h $35\sqrt{2}$ i $66\sqrt{3}$
j 560 k $35\sqrt{3}$ l $12\sqrt{11}$
m $8\sqrt{13}$ n $25\sqrt{11}$ o $40\sqrt{3}$

- 4 a $\sqrt{8}$ b $\sqrt{45}$ c $\sqrt{147}$
d $\sqrt{216}$ e $\sqrt{300}$ f $\sqrt{160}$
g $\sqrt{605}$ h $\sqrt{2450}$ i $\sqrt{108}$
j $\sqrt{180}$

- 5 a $\sqrt{6}$ b $\sqrt{77}$ c $\sqrt{40} = 2\sqrt{10}$
d $\sqrt{39}$ e $\sqrt{48} = 4\sqrt{3}$

- 6 a $\sqrt{5}$ b $\sqrt{2}$ c $\sqrt{6}$
d $\sqrt{5}$ e $\sqrt{3}$ f $\sqrt{8} = 2\sqrt{2}$

- 7 a $\sqrt{14}$ b 4 c $3\sqrt{2}$
d $6\sqrt{2}$ e $\sqrt{6}$ f 0
g 25 h 3

- 8 a $\sqrt{5} \times \sqrt{6} = \sqrt{30}$ b $\sqrt{12} \times \sqrt{3} = \sqrt{36}$
c $\sqrt{15} \times \sqrt{3} = \sqrt{45}$ d $\frac{\sqrt{100}}{\sqrt{5}} = \sqrt{20}$
e $\frac{\sqrt{33}}{\sqrt{11}} = \sqrt{3}$ f $\sqrt{5}$

- 9 a $\sqrt{21} \text{ cm}^2$ b $\frac{1}{2}\sqrt{30} \text{ cm}^2$ c 17 cm^2
d $\sqrt{11} \text{ cm}$ e $3\sqrt{7} \text{ cm}$

- 10 a $3\sqrt{2}$ b $\sqrt{6}$ c $3\sqrt{5}$
d $2\sqrt{2}$ e $5\sqrt{2}$ f $3\sqrt{5}$

- 11 a 112 cm^2 b $4\sqrt{14} \text{ cm}$

- 12 a $7\sqrt{3} \text{ cm}^2$ b $\sqrt{52} = 2\sqrt{13} \text{ cm}$

Exercise 2B

- 1 a $13\sqrt{2}$ b $25\sqrt{3}$ c $3\sqrt{2}$
d $-27\sqrt{5}$ e $-2\sqrt{3}$ f $22\sqrt{5}$

- 2 a $12\sqrt{2} + 2\sqrt{3}$ b $10\sqrt{7} - 10\sqrt{5}$
c $13\sqrt{11} - 3\sqrt{10}$ d $10\sqrt{2} - 4\sqrt{3}$
e $6\sqrt{15} - 4\sqrt{7}$ f $11\sqrt{5} - 2\sqrt{2}$

- 3 a $6\sqrt{2}$ b $5\sqrt{3}$ c $5\sqrt{5}$
d $15\sqrt{2}$ e $-5\sqrt{3}$ f $-2\sqrt{5}$
g $3\sqrt{3} + 4\sqrt{5}$ h $\sqrt{11} + 8\sqrt{5}$ i $3\sqrt{3} - 8\sqrt{10}$
j $-4\sqrt{5} - 8\sqrt{2}$
- 4 a $5\sqrt{3}$ b $5\sqrt{2}$ c $2\sqrt{2}$
d $-3\sqrt{5}$ e $27\sqrt{2} - 12\sqrt{3}$ f $45\sqrt{3}$
g $18\sqrt{5}$ h $\sqrt{7}$ i $28\sqrt{11}$
j $-15\sqrt{3}$
- 5 a $\sqrt{2}$ b $6\sqrt{3}$ c $6\sqrt{2}$
d $\sqrt{3}$ e $9\sqrt{2}$ f $-7\sqrt{5}$
g $5\sqrt{6}$ h $4\sqrt{3}$ i $2\sqrt{5}$
j $11\sqrt{2}$ k $\frac{5\sqrt{5}}{2}$ l $15\sqrt{2} - 10\sqrt{5}$
- 6 a $10\sqrt{2} - 4\sqrt{3}$ b $24\sqrt{2} - \sqrt{5}$
c $6\sqrt{3} + 15\sqrt{10}$ d $24\sqrt{3} - 19\sqrt{2}$
- 7 a $x = 7$ b $x = 5$ c $x = 6$
- 8 a $2\sqrt{2} + 2\sqrt{3}, \sqrt{6}, \sqrt{5}$ b $2\sqrt{3} + 2\sqrt{5}, \sqrt{15}, 2\sqrt{2}$

Exercise 2C

- 1 a $\sqrt{15}$ b $\sqrt{55}$ c $\sqrt{112} = 4\sqrt{7}$
d $\sqrt{65}$ e $\sqrt{12} = 2\sqrt{3}$ f $\sqrt{54} = 3\sqrt{6}$
- 2 a $\sqrt{5}$ b $\sqrt{2}$ c $\sqrt{6}$
d $\sqrt{5}$ e $\sqrt{3}$ f $\sqrt{8} = 2\sqrt{2}$
- 3 a $12\sqrt{10}$ b $77\sqrt{15}$ c $54\sqrt{35}$
d $32\sqrt{6}$ e $-30\sqrt{6}$ f $-28\sqrt{22}$
g $12\sqrt{77}$ h $18\sqrt{14}$ i $-22\sqrt{42}$
- 4 a $2\sqrt{3}$ b $\frac{5\sqrt{5}}{2}$ c $\frac{2\sqrt{6}}{3}$
d $\frac{-5\sqrt{2}}{2}$ e $-2\sqrt{3}$ f $\frac{1}{3\sqrt{2}}$
- 5 a 2 b 11 c 9
d 40 e 30 f 90
g -56 h 24 i -126
- 6 a $\sqrt{3}$ b $\sqrt{2}$ c $2\sqrt{2}$
d $3\sqrt{3}$ e $3\sqrt{7}$ f $6\sqrt{5}$
g $6\sqrt{2}$ h $12\sqrt{3}$ i $10\sqrt{5}$
j $8\sqrt{2}$ k $32\sqrt{2}$ l $50\sqrt{5}$
- 7 a $-3\sqrt{3}$ b $-3\sqrt{5}$ c $3\sqrt{3}$
d $7\sqrt{2}$ e $25\sqrt{35}$ f $24\sqrt{42}$
g $3\sqrt{3}$ h $6\sqrt{2}$ i $3\sqrt{3}$
j $16\sqrt{15}$
- 8 a $16\sqrt{6}$ b $43\sqrt{15}$ c $6\sqrt{6}$
d $6\sqrt{10}$ e $-17\sqrt{30}$ f $-\sqrt{40} = -2\sqrt{10}$

- g $8\sqrt{2}$ h $-4\sqrt{5}$ i $2\sqrt{2}$
j $\sqrt{2}$
- 9 a 8 b 12 c 45
d 150 e 63 f 44
g 250 h a^2b
- 10 a $\sqrt{2}$ b 2 c $2\sqrt{2}$
d 4 e $4\sqrt{2}$ f 8
g $3\sqrt{3}$ h $5\sqrt{5}$ i $16\sqrt{2}$
j $54\sqrt{2}$ k $192\sqrt{3}$ l $40\sqrt{5}$
m $625\sqrt{5}$ n $56\sqrt{7}$ o 512
p $2187\sqrt{3}$
- 11 a $\sqrt{6} + \sqrt{10}$ b $\sqrt{35} + \sqrt{42}$ c $\sqrt{35} - \sqrt{10}$
d $6\sqrt{10} - 9\sqrt{6}$ e $20\sqrt{6} + 24\sqrt{15}$ f $4\sqrt{6} - 4\sqrt{3}$
g $6\sqrt{15} + 15$ h $18\sqrt{2} + 8\sqrt{3}$ i $60\sqrt{2} - 32\sqrt{5}$
j $24 - 6\sqrt{2}$ k $30 + 24\sqrt{5}$ l $36\sqrt{2} - 18$
m $80 - 24\sqrt{10}$ n $105\sqrt{5} - 42\sqrt{3}$ o $66\sqrt{2} - 132\sqrt{3}$
- 12 a $\sqrt{15} + \sqrt{21} + \sqrt{10} + \sqrt{14}$
b $\sqrt{55} + \sqrt{30} + \sqrt{77} + \sqrt{42}$
c $\sqrt{15} - \sqrt{35} + \sqrt{6} - \sqrt{14}$
d $\sqrt{15} - \sqrt{21} + \sqrt{30} - \sqrt{42}$
e $5\sqrt{6}$
f $45 - 17\sqrt{15}$
g $37 - 10\sqrt{5}$
h $10\sqrt{6} - 6\sqrt{2} + 30\sqrt{3} - 18$
i $-11 - 11\sqrt{14}$
j $2\sqrt{21} - 4\sqrt{35} + \sqrt{15} - 10$
k $34 - 9\sqrt{14}$
l $3\sqrt{21} - 9\sqrt{35} - 8\sqrt{3} + 24\sqrt{5}$

- 13 a $-18\sqrt{2}$ b $\frac{-3\sqrt{2}}{2}$ c 66
d $\frac{11}{108}$ e $24\sqrt{3}$ f $1296\sqrt{3}$
g $\frac{9}{4\sqrt{3}} = \frac{3\sqrt{3}}{4}$ h -42
- 14 a 4 b $6 - \sqrt{3}$ c $\sqrt{3} + 10$
d $2\sqrt{3}$ e $3\sqrt{3} - 2$ f 1
g $\sqrt{3}$ h 1
- 15 a $8\sqrt{6}, 4\sqrt{3} + 8\sqrt{2}$ b $24, 12\sqrt{3}$
c 29, 28 d $7 + 3\sqrt{5}, 6 + 4\sqrt{5}$
- 16 a $\sqrt{15}$ b $10 + 5\sqrt{3} + \sqrt{15}$ c $6\sqrt{5} + \sqrt{15}$
- 17 a $8 + 20\sqrt{3}$ b $79 + 20\sqrt{3}$

Exercise 2D

- 1 a $7 + 2\sqrt{10}$ b $10 + 2\sqrt{21}$ c $5 - 2\sqrt{6}$
d $13 - 2\sqrt{42}$ e $13 + 4\sqrt{3}$ f $22 - 12\sqrt{2}$
g $14 + 4\sqrt{6}$ h $59 - 24\sqrt{6}$ i $83 + 12\sqrt{35}$

- j $98 - 24\sqrt{10}$ k $34 - 24\sqrt{2}$ l $79 - 20\sqrt{3}$
 m $\frac{13}{4} - \sqrt{3}$ n $\frac{103}{4} - 5\sqrt{3}$
- 2 a 13 b 3 c 2
 d 1 e 11 f 2
 g 6 h 34 i 1
 j -3 k $-\frac{1}{2}$ l $\frac{33}{4}$
- 3 a $7 + 4\sqrt{3}$ b $7 - 4\sqrt{3}$
 c $37 + 20\sqrt{3}$ d $37 - 20\sqrt{3}$
- 4 a 1 b $2\sqrt{5}$ c 18 d $8\sqrt{5}$
- 5 a $\frac{75}{2}$ b $5\sqrt{6}$
- 6 a $7\sqrt{3} + 7$ b $3 - 3\sqrt{3}$ c $29\sqrt{3} + 40$
 d $40\sqrt{3} + 116$ e -42
- 7 a $\sqrt{122}$ b $14 + \sqrt{122}$ c $\frac{37}{2}$

Exercise 2E

- 1 a $\frac{5\sqrt{3}}{3}$ b $3\sqrt{2}$ c $\sqrt{7}$
 d $\frac{3\sqrt{5}}{5}$ e $\sqrt{3}$ f $\frac{\sqrt{10}}{2}$
 g $\sqrt{2}$ h $\frac{\sqrt{2}}{3}$ i $\frac{2\sqrt{6}}{9}$
 j $\frac{2\sqrt{15}}{15}$ k $\frac{\sqrt{15}}{4}$ l $\frac{1}{6}(8\sqrt{3} + 3)$
 m $\frac{1}{10}(3\sqrt{2} - 4)$ n $\frac{1}{3}(3 + 2\sqrt{6})$
 o $\frac{1}{8}(\sqrt{10} - 2\sqrt{6})$
- 2 a $\frac{1}{6}(9\sqrt{2} + 8\sqrt{3})$ b $\frac{1}{35}(15\sqrt{7} - 14\sqrt{5})$
 c $\frac{1}{20}(5\sqrt{6} + 2\sqrt{5})$ d $\frac{1}{28}(12\sqrt{14} - 21\sqrt{2})$
 e $\frac{20\sqrt{7} - 14\sqrt{5}}{35}$ f $\frac{11\sqrt{3} - 3\sqrt{11}}{33}$
 g $\frac{14\sqrt{3} - 15\sqrt{7}}{21}$ h $\frac{22\sqrt{3} - 12\sqrt{11}}{33}$
- 3 a $\sqrt{5} + 2$ b $\frac{7 - \sqrt{6}}{43}$ c $\frac{3(\sqrt{6} - 2)}{2}$
 d $6(2 + \sqrt{3})$ e $2(\sqrt{3} + \sqrt{2})$ f $\frac{4(\sqrt{5} + \sqrt{2})}{3}$
 g $\frac{1}{3}(\sqrt{15} - \sqrt{6})$ h $\sqrt{21} - \sqrt{14}$ i $\sqrt{5} + \sqrt{3}$
 j $\sqrt{7} + \sqrt{3}$ k $\frac{4}{5}(4 + \sqrt{6})$ l $\frac{2}{5}(2\sqrt{6} - 3)$
 m $\frac{3}{8}(3\sqrt{10} + 5\sqrt{2})$ n $\frac{2}{17}(3\sqrt{6} + \sqrt{3})$ o $\frac{4}{3}(2\sqrt{6} - 3\sqrt{2})$
 p $\frac{1}{15}(27 + 7\sqrt{6})$ q $-4 - \sqrt{15}$ r $\frac{1}{19}(21 + 4\sqrt{5})$
 s $\frac{1}{27}(29 + 7\sqrt{10})$ t $\frac{1}{13}(23 - 6\sqrt{10})$

- 4 a $\frac{1}{4}(5 + 5\sqrt{3} - 4\sqrt{2})$ b $\frac{1}{57}(37\sqrt{5} + 19\sqrt{2} - 9)$
 c $\frac{1}{17}(17\sqrt{10} + 4\sqrt{2} - 10)$ d $-\frac{2}{17}(3\sqrt{3} + 2\sqrt{5})$
 e $\frac{1}{10}(6\sqrt{3} - 5\sqrt{2})$ f -10
- 5 a $\frac{13\sqrt{3}}{3}$ b $-\frac{\sqrt{3}}{3}$ c 14
 d $\frac{6}{7}$ e $\frac{559\sqrt{3}}{9}$
- 6 $10(\sqrt{2} + 1)$
- 7 a $\frac{3 - \sqrt{3}}{2}$ b $\frac{16 - 9\sqrt{3}}{2}$
 c $6\sqrt{3} + 10$ d $15\sqrt{3} + 26$
- 8 a $\frac{4}{289}(19 + 6\sqrt{2})$ b $\frac{1}{2}(3\sqrt{2} - 1)$
 c $\frac{1}{4}(19 - 6\sqrt{2})$ d $\frac{1}{34}(63\sqrt{2} - 13)$
 e $\frac{1}{578}(915\sqrt{2} - 137)$ f $\frac{1}{1156}(5795 - 1638\sqrt{2})$
- 9 i a $\frac{1}{8}(15 - 5\sqrt{5})$ b $\frac{1}{5}(5 + \sqrt{5})$
 c $\frac{1}{5}(6 + 2\sqrt{5})$ d $\frac{1}{20}(45 - \sqrt{5})$
 e $\frac{1}{40}(115 - 17\sqrt{5})$ f $\frac{1}{40}(123 - 9\sqrt{5})$
 ii a $588 + 240\sqrt{6}$ b $\frac{1}{6}(5\sqrt{3} - 6\sqrt{2})$
 c $\frac{1}{36}(147 - 60\sqrt{6})$ d $\frac{1}{6}(65\sqrt{3} + 66\sqrt{2})$
 e $\frac{1}{6}(3528 + 1440\sqrt{6} + 5\sqrt{3} - 6\sqrt{2})$
 f $\frac{5}{12}(1421 + 572\sqrt{6})$

Review exercise

- 1 a $4\sqrt{2}$ b $\sqrt{6}$
 c $3\sqrt{3} - \sqrt{2}$ d $4\sqrt{2} - 3\sqrt{5}$
- 2 a $5\sqrt{2}$ b $\sqrt{2}$
 c $-4\sqrt{7}$ d $11\sqrt{3}$
- 3 a $30\sqrt{2}$ b $30\sqrt{2}$ c $12\sqrt{10}$ d $28\sqrt{42}$
- 4 a $6\sqrt{2}$ b $3\sqrt{5}$ c $2\sqrt{6}$
 d $3\sqrt{3}$ e $4\sqrt{5}$ f $2\sqrt{11}$
 g $6\sqrt{2}$ h $8\sqrt{3}$ i $45\sqrt{2}$
 j $18\sqrt{3}$ k $40\sqrt{2}$
- 5 a $\sqrt{75}$ b $\sqrt{112}$ c $\sqrt{242}$
 d $\sqrt{125}$ e $\sqrt{384}$ f $\sqrt{891}$
 g $\sqrt{208}$ h $\sqrt{176}$

- 6 a $9\sqrt{2}$ b $2\sqrt{5} + 5\sqrt{3}$ c $3\sqrt{3}$
d $19\sqrt{7}$ e $11\sqrt{5}$ f $55\sqrt{6}$
- 7 a $19\sqrt{2}$ b $4\sqrt{2}$
c $15\sqrt{2} + \sqrt{6}$ d $16\sqrt{7} - 27\sqrt{3}$
- 8 a $6\sqrt{3} + 6$ b $30 - 10\sqrt{2}$ c $24 - 16\sqrt{21}$
d $30\sqrt{5} - 50$ e $12\sqrt{7} - 21$ f $45 - 12\sqrt{6}$
- 9 a $10 - \sqrt{2}$ b $32 - 9\sqrt{3}$ c 2
d 17 e 50 f $8 + 2\sqrt{15}$
g $14 + 4\sqrt{6}$ h $14 - 4\sqrt{6}$ i 10
j $a - b$ k $11 - 4\sqrt{7}$ l $28 + 10\sqrt{3}$
- 10 a $\frac{5\sqrt{3}}{3}$ b $\frac{7\sqrt{3}}{6}$ c $\frac{2\sqrt{2}}{3}$
d $\frac{3\sqrt{2}}{7}$ e $\frac{5\sqrt{3}}{9}$ f $\frac{\sqrt{6}}{4}$
g $\frac{\sqrt{42}}{7}$ h $\frac{7\sqrt{42}}{12}$
- 11 a $-\frac{1}{2}(\sqrt{5} + \sqrt{7})$ b $\frac{1}{11}(10 + 3\sqrt{5})$
c $\frac{1}{2}(3 - \sqrt{5})$ d $\frac{1}{2}(3 + \sqrt{5})$
- 12 a $\sqrt{2} + 1$ b $2 - \sqrt{3}$
c $\sqrt{3} - \sqrt{2}$ d $\sqrt{3} + \sqrt{2}$
- 13 $p = 5, q = 2$
- 14 a $5\sqrt{5} + 2$ b $\frac{7}{3}$
- 15 a $2(\sqrt{3} + 2)$ b 0
c $4\sqrt{3} + 7$ d 1
- 16 a 6 b $2\sqrt{3} - 2$ c $26 - 4\sqrt{3}$
d $12\sqrt{3} - 12$ e $2 - \sqrt{3}$ f $\frac{30 - 12\sqrt{3}}{13}$
g $\frac{22 - 14\sqrt{3}}{13}$ h $2\sqrt{3} + 5$
- 17 a $4\sqrt{3} + 8$ b $4\sqrt{3} + 7$
- 18 $40 - 20\sqrt{3}$
- 19 a $\sqrt{14}$ b 6 c $\sqrt{118}$
d $\sqrt{42}$ e $\frac{30 - 10\sqrt{3}}{3}$ f $20\sqrt{2} - 10\sqrt{6}$
- 20 a i $\frac{3 + \sqrt{5}}{2}$ ii $\frac{\sqrt{5} - 1}{2}$ iii $\frac{1 + \sqrt{5}}{2}$
iv $2 + \sqrt{5}$ v 2
- 21 $20\sqrt{2}$
- 22 a $-4\sqrt{3}$ b -8 c -4 d $-7 - 4\sqrt{3}$
- 23 $14\sqrt{5}$
- 24 a 6 b -11 c $-\frac{6}{11}$

Challenge exercise

- 1 a $(\sqrt{x})^2 + 2\sqrt{x}\sqrt{y} + (\sqrt{y})^2 = x + y + 2\sqrt{xy}$
b i $\sqrt{11} + \sqrt{5}$ ii $\sqrt{11} - \sqrt{5}$ iii $\sqrt{6} + \sqrt{5}$
- 2 a $a + b - a^2$ b $4(1 + x^2) - 1 = 3 + 4x^2$
c $2x$ d $\sqrt{a} + \sqrt{b}$
e $2a + 2\sqrt{a^2 - b^2}$
- 3 a $x = \frac{9}{4}$ b $x = 9$ c $x = 2$
d $x = 9$ e $x = 9$ f $x = 8$
- 4 $x - 2\sqrt{x} - \frac{2}{\sqrt{x}} + \frac{1}{x} + 3$

5 $\frac{5^{\frac{2}{3}}}{5}$

6 $\frac{\sqrt{15} - \sqrt{5} - \sqrt{3} + 1}{8}$

7 $\frac{2\sqrt{3} + 3\sqrt{2} - \sqrt{30}}{12}$

8 $a + b - \sqrt{ab}$

9 a $\sqrt{\frac{9}{80}} = \sqrt{\frac{729}{80}} = 9\sqrt{\frac{9}{80}}$
 $\sqrt{4\frac{4}{15}} = \sqrt{\frac{64}{15}} = 4\sqrt{\frac{4}{15}}$

b $\sqrt{a + \frac{b}{c}} = a\sqrt{\frac{b}{c}}$
 $a + \frac{b}{c} = a^2 \frac{b}{c}$
 $a = \frac{b}{c}(a^2 - 1)$
 $\frac{a}{a^2 - 1} = \frac{b}{c}$

Mixed numbers of the form $a\frac{a}{a^2 - 1}$.

10 $(2\sqrt{6} + 5)$ cm

11 1 12 $\frac{2}{5}$

13 a 625 b 1799

14 a i $\sqrt{5}$ ii $\sqrt{5} - 1$
iii $\frac{\sqrt{5} - 1}{2}$ iv $\frac{\sqrt{5} - 1}{2}$

b $\left(\frac{\sqrt{5} - 1}{2}\right)^2 + \left(\frac{\sqrt{5} - 1}{2}\right) = 1$

Chapter 3 answers

Exercise 3A

- 1 a $8x$ b $2x$ c $-6x$
d $4a + 7b$ e $2t - s (= -s + 2t)$ f $-3x + 2y$
g $4a + b$ h $-4a - 4b$ i $3x - 4y$

- 2 a $3ab$ b $6x^2y$ c $12cd^3$
d $9st^2 + s^2t$ e $2mn + 4m^2n$ f $12x^2y$
- 3 a $3a + 6$ b $7b + 21$ c $4a - 10$
d $24p + 108$ e $-12b - 27$ f $-20m - 25$
g $2 - 6a$ h $42 - 24b$ i $35 - 10b$
j $x + 3$ k $4a + 2$ l $4t - 3$
m $3 - 5\ell$ n $4x + \frac{1}{2}$ o $\frac{1}{5} - \frac{x}{10}$
- 4 a $a^2 + 4a$ b $c^2 - 5c$ c $6g^2 - 10g$
d $20h^2 - 28h$ e $12j^2 + 21j$ f $4k - 5k^2$
g $\ell - 3\ell^2$ h $6ac + 3bc$ i $10d^2 - 20de$
j $10m^2 - 20mn$ k $10x^2y + 15x$ l $6p - 15p^2q$
- 5 a $4a + 4b$ b $5a + 5$
c $8p - 56$ d $15 - 3p$
e $a^2 + ab$ f $6m^2 + 10m$
g $30 - 6x$ h $12a^2 - 21a$
i $12a^2 + 20a$ j $6x^2 - 21xy$
- 6 a $8a + 23$ b $3d + 3$ c $2e + 5$
d $13g + 10$ e $3i - 30$ f $8a^2 + 13a$
g $10b^2 - 9b$ h $2b^2 - 15b$ i 10
j $\frac{5x}{6} + 2$ k $\frac{9}{5}$ l $\frac{3x}{4} - \frac{9}{4}$
- 7 a $5y + 14$ b $7a - 12$
c $16b - 9$ d $a + 1$
e -5 f $x^2 + x - 6$
g $2p^2 + 7p + 5$ h $10z$
i $4y^2 - 16y$ j $15z^2 - 10z$
- 8 a $x^2 + 10x + 21$ b $a^2 + 13a + 40$
c $a^2 + 12a + 27$ d $x^2 - 6x + 5$
e $x^2 - 5x + 6$ f $a^2 - 11a + 28$
g $x^2 - x - 42$ h $x^2 + 8x - 33$
i $x^2 - 5x - 24$ j $6x^2 + 17x + 12$
k $15x^2 + 22x + 8$ l $6x^2 + 7x - 5$
m $20x^2 - 11x - 3$ n $2a^2 - 11a + 15$
o $6p^2 + 11p - 10$ p $12x^2 + 13x - 14$
q $15x^2 - 46x + 16$ r $6x^2 - 17x + 5$
- 9 a $2a^2 + 7ab + 3b^2$ b $6m^2 - 7mn - 3n^2$
c $6p^2 + pq - 5q^2$ d $3x^2 + 13ax - 10a^2$
e $6x^2 + 13xy - 5y^2$ f $6c^2 + 35cd - 6d^2$
g $8p^2 + 2pq - 3q^2$ h $3b^2 + 5ab - 2a^2$
i $15q^2 - 4pq - 4p^2$
- 10 a $x^2 + 2x + 1$ b $9x^2 + 12x + 4$
c $9a^2 + 24ab + 16b^2$ d $4x^2 + 12xy + 9y^2$
e $x^2 - 14x + 49$ f $x^2 - 2xy + y^2$
g $4a^2 - 12ab + 9b^2$ h $9a^2 - 24ab + 16b^2$
i $4x^2 - 12xy + 9y^2$

- 11 a $x^2 - 9$ b $a^2 - 49$ c $4a^2 - 9$
d $25 - x^2$ e $49 - 9y^2$ f $4m^2 - p^2$
g $9x^2 - 4y^2$ h $64m^2 - 25n^2$ i $49r^2 - 4t^2$
- 12 a $x^2 - 64$ b $4a^2 + 4a + 1$
c $4a^2 - 1$ d $9 - 4x^2$
e $9 + 12a + 4a^2$ f $9 - 12x + 4x^2$
g $a^2 + 6ab + 9b^2$ h $a^2 - 9b^2$
i $a^2 - 6ab + 9b^2$

Exercise 3B

- 1 a $a = 2$ b $b = 3$ c $x = \frac{9}{2}$
d $y = -5$ e $x = 6$ f $y = \frac{22}{5}$
g $x = 4$ h $y = 6$ i $a = 3$
j $b = -1$
- 2 a $x = -3$ b $x = 2$ c $x = -3$
d $x = -2$ e $x = \frac{23}{5}$ f $a = \frac{26}{17}$
g $y = -2$ h $x = \frac{55}{28}$
- 3 a $x > 3$ b $x \leq 2$ c $x \leq -4$
d $x > -8$ e $x \geq 1$ f $x < 8$
g $x > \frac{1}{2}$ h $x \leq \frac{2}{3}$
- 4 a $x > -\frac{11}{2}$ b $x \leq 7$ c $x \geq 13$
d $x < 11$ e $x \leq -12$ f $x < \frac{9}{2}$
g $x < 4$ h $x \geq -\frac{29}{4}$ i $x \leq -20$
j $x < 5$

- 5 28 6 width 5 m, length 13 m
7 17 and 25 8 \$90
9 4 km/h 10 \$840 000

Exercise 3C

- 1 Giorgio 7 km/h, Fred 5 km/h
2 2 L
3 17 20-cent pieces, 10 \$2 coins
4 \$3600
5 $12\frac{1}{2}$ hours, 7 hours
6 1.5 L
7 A to B 305 km, B to C 335 km
8 a \$129 b \$107, \$137
9 26 hours

10 when sales exceed \$1700

11 a $y \geq \frac{150 - x}{2}$

b i $y \geq 57.5$

ii $y \geq 50.5$

Exercise 3D

1 127

2 43.2

3 44.7

4 $\frac{6}{5}$

5 2.45

6 2.92

7 a 339 cm^2

b 109 cm^2

c 3695 cm^2

8 a 465

b 1275

c 7260

d 5985

9 88.36

10 99 cm

11 20 mm

12 a $y = z - x$

b $r = \frac{A}{\pi s}$

c $y = \sqrt{\frac{ax}{t}}$

d $v = \sqrt{\frac{2w}{m}}$

e $x = \frac{m^2}{4}$

f $m = \frac{k^2}{4t}$

g $p = \frac{b^2}{a+b}$

h $t = \frac{x-1}{x+1}$

i $a = \sqrt{c^2 - b^2}$

j $r = \frac{n-q}{v^2}$

k $q = n - v^3 r$

l $L = g\left(\frac{t}{2\pi}\right)^2$

m $k = h - g\left(\frac{t}{2\pi}\right)^2$

n $a = \frac{mbc}{bc - mc - mb}$

o $A = \frac{B^2 + C^2}{2B}$

p $s = \frac{rc}{c - rv}$

Exercise 3E

1 a $(b+2)(b-2)$

b $(3+a)(3-a)$

c $(3x+4)(3x-4)$

d $(5a+1)(5a-1)$

e $16(y+2)(y-2)$

f $(a+2b)(a-2b)$

g $(3x+y)(3x-y)$

h $2(2x+1)(2x-1)$

i $3(2+y)(2-y)$

j $5(m+2n)(m-2n)$

k $3(3r+t)(3r-t)$

l $(1+3ab)(1-3ab)$

2 a $(x+\sqrt{3})(x-\sqrt{3})$

b $(x+\sqrt{7})(x-\sqrt{7})$

c $(x+\sqrt{13})(x-\sqrt{13})$

d $(x+\sqrt{6})(x-\sqrt{6})$

3 a $(x+2\sqrt{5})(x-2\sqrt{5})$

b $(x+3\sqrt{2})(x-3\sqrt{2})$

c $(x+3\sqrt{3})(x-3\sqrt{3})$

d $(x+2\sqrt{6})(x-2\sqrt{6})$

e $(x+2\sqrt{10})(x-2\sqrt{10})$

f $(x+2\sqrt{7})(x-2\sqrt{7})$

g $(x+5\sqrt{5})(x-5\sqrt{5})$

h $(x+10\sqrt{2})(x-10\sqrt{2})$

i $\left(x + \frac{\sqrt{7}}{5}\right)\left(x - \frac{\sqrt{7}}{5}\right)$

j $\left(x + \frac{\sqrt{3}}{2}\right)\left(x - \frac{\sqrt{3}}{2}\right)$

k $\left(x + \frac{\sqrt{5}}{3}\right)\left(x - \frac{\sqrt{5}}{3}\right)$

l $\left(x + \frac{\sqrt{11}}{4}\right)\left(x - \frac{\sqrt{11}}{4}\right)$

4 a $(x+2+\sqrt{2})(x+2-\sqrt{2})$

b $(x-1+\sqrt{3})(x-1-\sqrt{3})$

c $(x-5+\sqrt{5})(x-5-\sqrt{5})$

d $\left(x - 4 + \frac{\sqrt{2}}{2}\right)\left(x - 4 - \frac{\sqrt{2}}{2}\right)$

e $(x-1+2\sqrt{3})(x-1-2\sqrt{3})$

f $(x+3+2\sqrt{2})(x+3-2\sqrt{2})$

g $\left(x + 3 + \frac{\sqrt{5}}{3}\right)\left(x + 3 - \frac{\sqrt{5}}{3}\right)$

h $\left(x - 2 + \frac{\sqrt{7}}{3}\right)\left(x - 2 - \frac{\sqrt{7}}{3}\right)$

i $(2x+1+3\sqrt{3})(2x+1-3\sqrt{3})$

j $\left(2x - 3 + \frac{\sqrt{6}}{3}\right)\left(2x - 3 - \frac{\sqrt{6}}{3}\right)$

k $\left(4x - 1 + \frac{\sqrt{15}}{5}\right)\left(4x - 1 - \frac{\sqrt{15}}{5}\right)$

l $12z(3z+2)$

5 a $2(x+\sqrt{7})(x-\sqrt{7})$

b $3(x+\sqrt{2})(x-\sqrt{2})$

c $5(x+\sqrt{3})(x-\sqrt{3})$

d $4(x+\sqrt{7})(x-\sqrt{7})$

e $3(x+2\sqrt{3})(x-2\sqrt{3})$

f $2(x+3\sqrt{2})(x-3\sqrt{2})$

g $2(x+1+\sqrt{5})(x+1-\sqrt{5})$

h $3(x+2+\sqrt{2})(x+2-\sqrt{2})$

i $3(x-2+2\sqrt{2})(x-2-2\sqrt{2})$

j $2(x+1+2\sqrt{3})(x+1-2\sqrt{3})$

k $-2(x+3+\sqrt{2})(x+3-\sqrt{2})$

l $-4(x-2+\sqrt{5})(x-2-\sqrt{5})$

6 a $(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})$

b $\sqrt{a} + \sqrt{b}$

Exercise 3F

1 a $(x+1)(x+8)$

b $(x+2)(x+6)$

c $(x+2)(x+4)$

d $(x+6)^2$

e $(x+3)^2$

f $(x+3)(x+8)$

g $(x+3)(x-2)$

h $(x+6)(x-5)$

i $(x+8)(x-5)$

j $(x+10)(x-6)$

k $(x-3)(x-4)$

l $(x-5)^2$

m $(x-2)(x-16)$

n $(x+5)(x-7)$

o $(x+3)(x-7)$

p $3(x-1)(x-5)$

q $2(x+8)(x-3)$

r $4(x+4)(x-6)$

s $5(x+12)(x-2)$

t $3(x+1)(x-3)$

u $2(x+7)(x-4)$

2 a $(m+3)(x+2)$

b $(x-4)(x-3)$

c $(a-9)(b-7)$

d $(2x+5)(x-1)$

e $(3a+5)(b+2)$

f $(2x+y)(3x-2y)$

g $(2a-5)(b-6)$

h $(x+2)^2(x-2)$

i $(4x-5)(3-2y)$

j $(5x-2y)(2x+3y)$

- 3 a $(x-11)(x+7)$ b $(z-5)(z+5)$
 c $2x(x-3)$ d $5a^2(4-a)$
 e $(x+3)(y+2)$ f $(x+y)(x-y)$
 g $(x+10)(x+4)$ h $(x-7)^2$
 i $(x-\sqrt{11})(x+\sqrt{11})$ j $(x-6-3\sqrt{2})(x-6+3\sqrt{2})$
 k $(11-a)(7+a)$ l $(x-13)(x-2)$
 m $(z+3)^2$ n $-(x+5)(x-3)$
 o $2(x-3)(x-4)$ p $3(x-8)(x-1)$
 q $25(2-x)(2+x)$ r $(8p-9q)(8p+9q)$
 s $(x+y)(x+z)$ t $(3a-b)(2a-3)$

Exercise 3G

- 1 a $(2x+1)(x+1)$ b $(5x-2)(x-1)$
 c $(3x-2)(2x-1)$ d $(4x+3)(2x-1)$
 e $(5x-4)(4x-3)$ f $12(x-3)(x+2)$
- 2 a $(2x+3)(x+2)$ b $(2x+1)(x+5)$
 c $(2x+3)(x+3)$ d $(3x+2)(x+3)$
 e $(3x+1)(x+3)$ f $(3x+2)(x+4)$
 g $(2x+5)(x-2)$ h $(2x+1)(x-7)$
 i $(3x+5)(x-2)$ j $(5x-3)(x+2)$
 k $(3x-1)(x+9)$ l $(7x+10)(x-1)$
 m $(5x+6)(x-2)$ n $(3x-2)(x+2)$
 o $(4x-1)(x+3)$
- 3 a $(3x-2)(2x+1)$ b $(4x-3)(2x+1)$
 c $(3x-2)(2x+3)$ d $(5x-2)(3x-2)$
 e $(3x-2)^2$ f $(2x+3)^2$
 g $(5x+3)(2x-3)$ h $(4x+9)(3x-2)$
 i $(5x-2)(2x-3)$ j $(4x-3)(3x-2)$
 k $(4x-3)(3x-1)$ l $(5x-2)(3x-5)$
 m $2(5x-2)(2x-5)$ n $3(4x+3)(2x-1)$
 o $2(3x-2)(2x-3)$ p $2(3x+4)(2x+3)$
 q $5(4x-3)(2x+1)$ r $2(4x-3)(2x-7)$
- 4 a $(2x+1)(3x+2)$ b $(2x-3)(3x-1)$
 c $(2x+3)(3x+5)$ d $(3x-4)(3x-2)$
 e $(a-8)(a+7)$ f $-(2x-1)(3x+4)$
 g $-(x-9)(x+1)$ h $4(2x+1)(3x-2)$
 i $(x-8)(3x-7)$ j $5(2\sqrt{5}-x)(2\sqrt{5}+x)$
 k $(2x-1-\sqrt{15})(2x-1+\sqrt{15})$
 l $(3\sqrt{10}-\sqrt{2}(a-3))(3\sqrt{10}+\sqrt{2}(a-3))$
- 5 a $(x+y-3)(x-y)$ b $(x-y-5)(x+y)$
 c $(x-4)(x-1)$ d $(x+6)(x+1)$
 e $(a+2b)(a-2b-1)$ f $(3x+2y)(3x-2y-1)$
 g $(a+2d)(a-2d-1)$ h $(x+3)(x-9)$
 i $(2x+1)(2x+4)$ j $(x-11)(x-4)$

Exercise 3H

- 1 a $\frac{4x}{5}$ b $\frac{a}{4}$ c 3
 d $x-6$ e $2x+9$ f -1
 g $-\frac{x+4}{x-4}$ h $\frac{2x-5}{x-4}$ i $\frac{x+2}{x+4}$
- 2 a $\frac{9x}{20}$ b $\frac{x}{15}$ c $\frac{11x}{20}$
 d $\frac{23a}{20}$ e $\frac{9x+3}{20}$ f $\frac{5-x}{12}$
 g $\frac{1-2z}{35}$ h $\frac{13c+1}{15}$ i $\frac{4b+13}{12}$
 j $\frac{5k-3}{42}$ k $\frac{10-5x}{6}$ l $\frac{50x-3}{30}$
- 3 a $x=6$ b $x=8$ c $x=3$
 d $x=\frac{42}{13}$ e $x=2$ f $x=20$
 g $x=9$ h $x=3$ i $x=3$
 j $x=4$ k $x=3$ l $x=-2$

Exercise 3I

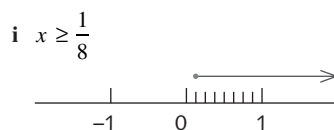
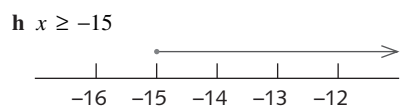
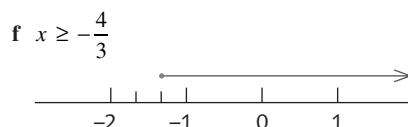
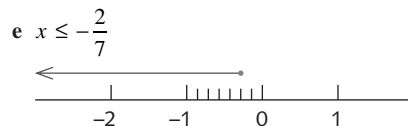
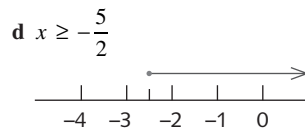
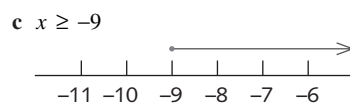
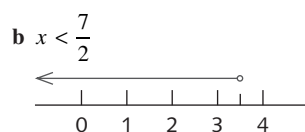
- 1 a $\frac{a+b}{b}$ b $\frac{x+y}{x-y}$
 c $\frac{a-b}{a+b}$ d $\frac{b+1}{b}$
 e $\frac{x+y}{x-y}$ f $3(x-y)$
 g $\frac{x+4}{x-2}$ h $\frac{a-2}{2}$
 i $\frac{x-3}{x+7}$ j $\frac{x+2}{x+1}$
 k $\frac{2x-1}{4(x-3)}$ l $\frac{3x-y}{4x-y}$
- 2 a $\frac{2y+x}{3(2x+y)}$ b $\frac{(a-3)(a+2)}{a^2}$
 c $\frac{x}{(x+3)}$ d $\frac{(x-2)(3x-1)}{6(x+2)}$
 e $\frac{a(a-1)}{a+1}$ f $2x$
 g $\frac{3x-2}{9}$ h $2x$
 i $\frac{x^2}{6y}$ j $\frac{x-y}{x}$
- 3 a $\frac{x^2-x+6}{(x-2)(x+2)}$ b $\frac{x}{6(x-3)}$
 c $\frac{4x}{x^2-9}$ d $\frac{2x+5}{(x+2)(x+3)}$
 e $\frac{2}{(x-2)(x+2)(x+4)}$ f $\frac{x^2+2}{(x-1)(x-3)(x+2)}$
 g $\frac{x+5}{(x-1)(x+3)}$ h $\frac{2}{2x+3}$
 i $\frac{a}{(a-4)(a+1)(a-2)}$

4 a $\frac{-2(x+4)}{x+3}$ b $\frac{5x+49}{49-x^2}$
 c $\frac{12}{a^2-1}$ d $\frac{a^2+3x^2}{2ax}$
 e $\frac{2}{(x-4)(x-6)}$ f $\frac{1}{(2x+1)(x-1)(3x-2)}$
 g $\frac{-4(4a-1)}{(5a+3)(2a-1)(a+4)}$ h $\frac{13a+2}{a^2-9}$

5 a $\frac{2x+3}{3x+5}$ b $\frac{x+7}{x+13}$ c $\frac{x}{x+1}$
 d $\frac{a-11}{a-2}$ e $\frac{4x+3a}{x+2}$ f $\frac{x+1}{x+5}$
 6 a $x=2$ b $x=\frac{13}{2}$ c $x=-17$
 d $x=4$ e $x=-\frac{1}{7}$ f $x=7$
 7 a $\frac{28}{5x}$ b $x=\frac{28}{5}$
 8 $x=-18$ 9 $x=2$
 10 a $x=8$ b $x=\frac{21}{5}$ c $x=\frac{54}{13}$
 d $x=6$ e $x=2$ f $x=\frac{2}{5}$

Review exercise

1 a $13x$ b $7p^3q$
 c $-5ab - ab^2$ d $20x^3y - 10x^2y$
 2 a $-2a - 8$ b $-3b - 18$ c $-12b + 20$
 d $-20b + 35$ e $8p + 4$ f $-4p + 5$
 3 a $d^2 - 9d$ b $3e^2 + e$ c $5f^2 + 6f$
 d $-10m^2 + 8m$ e $-15n^2 - 21n$ f $6pq - 10pr$
 g $-6x^2 - 15xy$ h $-6z^2 + 8zy$ i $6a + 8a^2b$
 4 a $2c + 5$ b $7h + 4$
 c $4 + 3q$ d $4ab$
 e $\frac{19x}{12} - \frac{10}{3}$ f $b - 22$
 g $12y^2 - 11y - 15$ h $6p^2 - 6p - 4$
 i $m^2 - m - 20$ j $6x^2 + 7x - 10$
 k $7x^2 - 36x + 5$ l $2m^2 + 7mn + 3n^2$
 5 a $x=3$ b $x=5$ c $x=2$
 d $x=15$ e $x=4$ f $x=4$
 g $x=\frac{24}{13}$ h $x=20$ i $x=2$



7 a $\frac{12}{7}$ b 3

8 a 52 b $a = \frac{p}{4} - b$ c -1

9 a $h = \frac{A - \pi r^2}{\pi r}$ b $h = \sqrt{\left(\frac{A}{\pi r}\right)^2 - r^2}$
 c $u = \sqrt{v^2 - \frac{2E}{m}}$ d $x = \frac{cb + by}{c - a} = \frac{b(c + y)}{c - a}$

10 a $(7+z)(7-z)$ b $(3+2a)(3-2a)$
 c $(5+8x)(5-8x)$ d $(ab+2cd)(ab-2cd)$
 e $(3x+1)(3x-1)$ f $(6p+7q)(6p-7q)$
 g $(1+10b)(1-10b)$ h $(11a+9b)(11a-9b)$

11 a $(x - \sqrt{11})(x + \sqrt{11})$ b $(x - \sqrt{15})(x + \sqrt{15})$
 c $(x - 2\sqrt{7})(x + 2\sqrt{7})$ d $(x - 4\sqrt{2})(x + 4\sqrt{2})$

- 12 a $(x+4-\sqrt{2})(x+4+\sqrt{2})$
 b $(x-3-\sqrt{5})(x-3+\sqrt{5})$
 c $(2x+1-2\sqrt{5})(2x+1+2\sqrt{5})$
 d $(3-2x-\sqrt{10})(3-2x+\sqrt{10})$
- 13 a $(a+b+c)(a+b-c)$
 b $(a-b+c)(a-b-c)$
 c $(x+y+2z)(x+y-2z)$
- 14 a $(a+3)(2x-1)$ b $(a+5)(b-2)$
 c $(2x+3)(y+5)$ d $(m-7)(3x+2)$
- 15 a $(x-1)(x-2)$ b $(c+1)(c+11)$
 c $(x+1)(x-5)$ d $(y-1)(y+10)$
 e $(z-8)(z+2)$ f $(c+10)(c-2)$
 g $(p-13)(p+2)$ h $(k-14)(k+3)$
 i $(x+10)(x-6)$
- 16 a $2(x-6)(x+11)$ b $3(x-7)(x+11)$
 c $3(x-7)(x+1)$ d $3(x-5)(x+2)$
 e $3(x-5)(x-6)$ f $6(x-1)(x+2)$
- 17 a $(2x+3)(x+4)$ b $(3s+2t)(2s-5t)$
 c $(4x+3)(3x-4)$ d $(3x+2)(x+2)$
 e $(x-1)(2x+3)$ f $(2x-1)(2x+3)$
 g $(3x-1)(x+3)$ h $(3x-2)(2x+5)$
 i $(3x-2)(x+1)$
- 18 a $x = 4$ b $x = 19$ c $x = 11$
 d $x = \frac{6}{7}$ e $x = 10$ f $x = \frac{68}{11}$
 g $x = -\frac{86}{37}$ h $x = 33$
- 19 a $\frac{3x-7}{(x-2)(x-1)(x-3)}$
 b $\frac{7}{(x-1)(x-2)}$
 c $\frac{13}{(x-1)(x-2)(x-3)}$
- 20 a 5 b 70 bottles of each

Challenge exercise

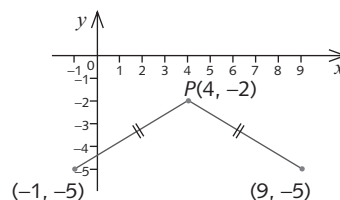
- 1 a $(x+10y)(5x+4y)$ b $(8x+y)(2x+3y)$
- 2 a $(a-x+2b)(a-x-2b)$
 b $(3a-x+c)(3a+x-c)$
 c $(b+d+a-c)(b+d-a+c)$
 d $(y+b+a+3x)(y+b-a-3x)$
- 3 a $a^2+b^2+c^2+2ab+2bc+2ac$
 b $343a^6-27x^3$
- 4 a $4x^2y(7x-5)(x+3)$
 b $(p+2q)(p-2q)(x+2\sqrt{2}y)(x-2\sqrt{2}y)$

- 5 $x = 6$
- 6 a length 18 m, width 12 m
 b length 15 m, width 12 m
- 7 $\frac{am}{bp}$ km
- 8 $\frac{cp}{p-q}$ km

Chapter 4 answers

Exercise 4A

- 1 a $\sqrt{10} \approx 3.16$ b 4 c 7
- 2 a $\sqrt{5} \approx 2.24$ b $\sqrt{29} \approx 5.39$ c $4\sqrt{5} \approx 8.94$
 d $\sqrt{89} \approx 9.43$ e $5\sqrt{2} \approx 7.07$ f 10
- 3 a (1, 4) b $(\frac{1}{2}, 5)$ c (4, 6)
 d (-3, -7) e $(\frac{1}{2}, 6)$ f (-5, 12)
- 4 a (6, 5) b i 5 ii 6
 c AC and BC are equal, so $\triangle ABC$ is isosceles.
- 5 a $u = 4$ b $v = -1$ or 9 c $w = -5$ or 11



- 6 a 5 b $M(1\frac{1}{2}, 2)$
 c i $2\frac{1}{2}$ ii $2\frac{1}{2}$ iii $2\frac{1}{2}$
- 7 a (-4, 1) b (22, -10)
- 8 PQ is perpendicular to PR.
- 9 XY = XZ
- 10 AB = CD = DA = CA
- 11 (-2, -3)
- 12 AB = CD = $\sqrt{20}$ and BC = DA = $\sqrt{50}$

Exercise 4B

- 1 a $\frac{3}{4}$ b 2 c -2
 d $-\frac{1}{3}$ e $\frac{5}{4}$ f -1
 g 1 h 0 i $-\frac{13}{9}$

2 both gradients = 1

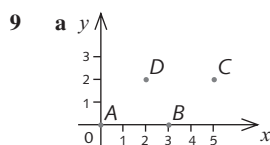
3 $a = 8$ 4 $b = -12$

5 a 4 b -4 c 8
d 7 e 1 f -2

6 gradient of $AB = 8$, gradient of $PQ' = -\frac{1}{8}$

7 a $-\frac{1}{6}$ b 2 c $-\frac{2}{3}$
d $\frac{5}{4}$ e 1

8 a 2 b 2 c $-\frac{1}{2}$



b gradient of $AB = \text{gradient of } DC = 0$;
gradient of $AD = \text{gradient of } BC = 1$

c $\left(\frac{5}{2}, 1\right)$; Because the diagonals of a parallelogram bisect each other

10 $M_{EF} = M_{GH} = \frac{1}{4}$ and $M_{EH} = M_{FG} = 12$

11 a $\sqrt{2}$ b 1 c 1

12 a gradient of $AB = \frac{1}{2}$, gradient of $BC = \frac{7}{5}$, points not collinear

b both gradients = 3, points are collinear

c both gradients = 3, points are collinear

d gradient of $AB = \frac{11}{5}$, gradient of $BC = \frac{13}{4}$, points not collinear

13 a 1 b 1 c $(6, -3)$ d $\left(5\frac{1}{2}, 3\frac{1}{2}\right)$

Exercise 4C

1 a gradient = 4, y-intercept is 2

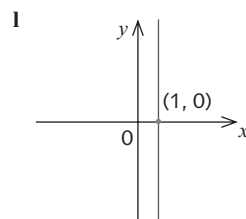
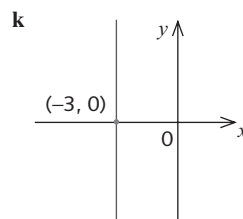
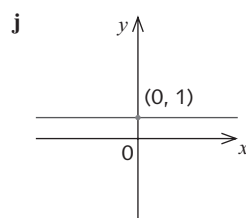
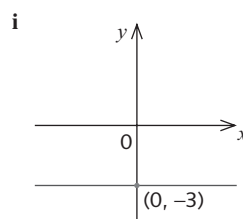
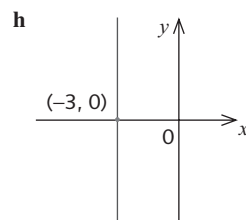
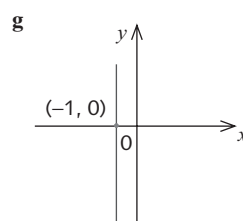
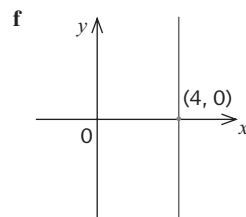
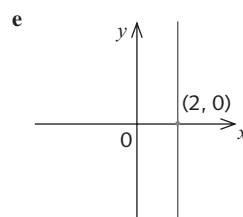
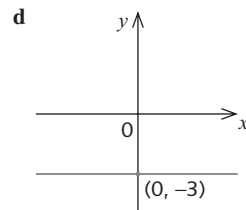
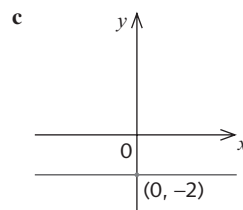
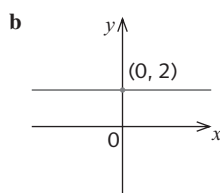
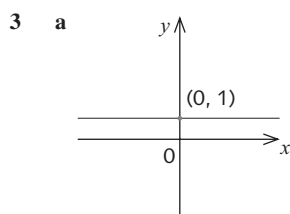
b gradient = $-\frac{2}{3}$, y-intercept is 5

c gradient = -7, y-intercept is 10

d gradient = $-\frac{4}{11}$, y-intercept is $\frac{2}{3}$

2 a $y = 8x + 3$ b $y = 11x + 5$

c $y = -6x - 7$ d $y = -\frac{3}{4}x + \frac{2}{5}$



a, b, c, d, i and j have gradient = 0.

e, f, g, h, k and l do not have a gradient.

4 a $2x + y - 6 = 0$ b $2x + 3y - 33 = 0$

c $9x + 15y + 10 = 0$ d $-24x + 42y - 7 = 0$

e $2x + 5y + 4 = 0$ f $-4x + 10y + 3 = 0$

g $3x - y - 12 = 0$ h $9x - 16y + 36 = 0$

5 a $y = \frac{3}{2}x - 3$ b $y = -\frac{5}{2}x - 5$ c $y = \frac{2}{3}x - 4$

d $y = \frac{1}{6}x - 3$ e $y = \frac{2}{15}x - \frac{6}{5}$ f $y = \frac{2}{3}x + 4$

g $y = -\frac{5}{4}x - \frac{5}{2}$ h $y = \frac{3}{2}x - 6$ i $y = \frac{3}{5}x + 3$

6 a gradient = $\frac{3}{2}$, y-intercept is 6

b gradient = -4, y-intercept is -24

c gradient = $-\frac{3}{8}$, y-intercept is -6

d gradient = $\frac{4}{7}$, y-intercept is 8

e gradient = $-\frac{11}{4}$, y-intercept is 11

f gradient = 2, y-intercept is 4

g gradient = $\frac{3}{7}$, y-intercept is 6

h gradient = $\frac{2}{7}$, y-intercept is -2

i gradient = -5, y-intercept is 20

7 a x-intercept is 5, y-intercept is -10

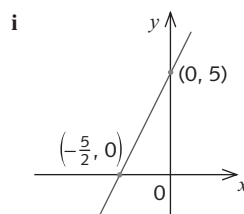
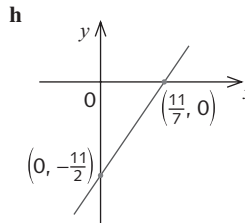
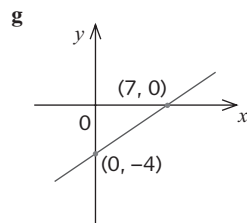
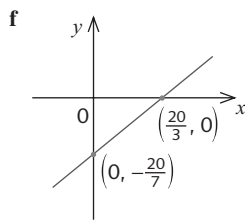
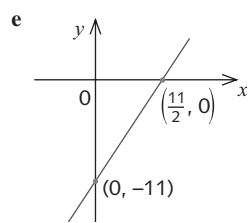
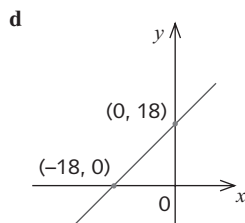
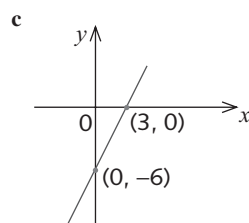
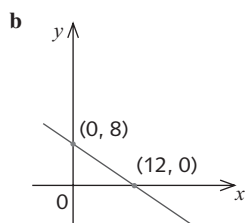
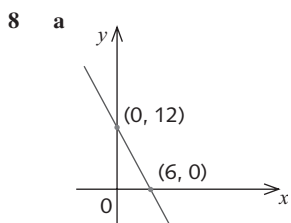
b x-intercept is $-\frac{11}{3}$, y-intercept is 11

c x-intercept is 16, y-intercept is 6

d x-intercept is -16, y-intercept is 20

e x-intercept is 14, y-intercept is -6

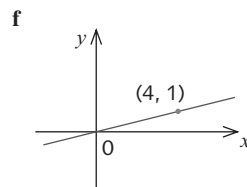
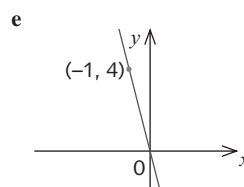
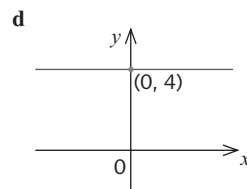
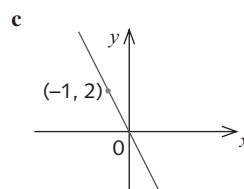
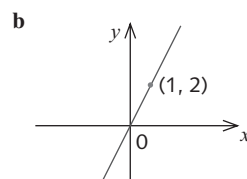
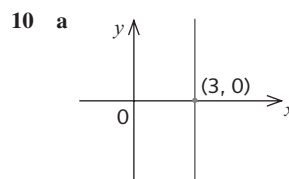
f x-intercept is $-\frac{11}{5}$, y-intercept is $\frac{11}{2}$



9 a $x = 1$

b $y = 5$

c $x = -4$



11 a $a = -\frac{5}{2}$

b $b = 10$

c $c = -\frac{9}{4}$

d $d = -14$

e $e = -\frac{3}{2}$

f $f = 50$

12 a $a = -4$

b $b = -8$

c $c = -\frac{9}{2}$

d $d = 4$

e $e = -6$

f $f = -28$

13 34

14 $-\frac{19}{5}$

15 12

16 -6

Exercise 4D

1 a $y = 6x - 24$

b $y = -4x + 14$

c $y = \frac{1}{2}x + 8\frac{1}{2}$

d $y = -3x + 11$

e $y = 6$

f $x = -7$

2 a $y = \frac{1}{3}x + 8\frac{2}{3}$

b $y = 2x$

c $y = -\frac{1}{2}x$

3 a $\left(\frac{5}{2}, \frac{5}{2}\right)$

b 7

c $y = 7x - 15$

d $y = -\frac{1}{7}x + 2\frac{6}{7}$

4 a $y = -4x - 6$

b $y = -4x + 20$

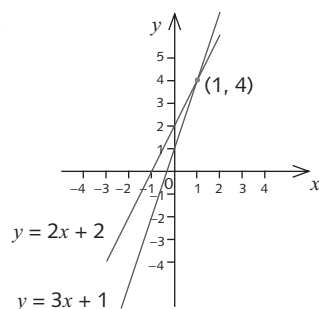
c $y = 5x + 28$

d $y = -2x - 12$

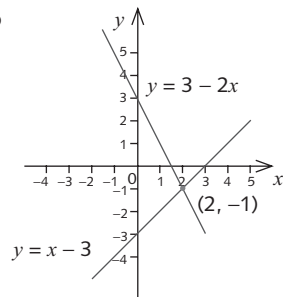
- 5 a $y = x - 4$ b $y = -3x - 9$ c $y = -x + 6$
d $y = 3$ e $x = 1$ f $y = \frac{9}{4}x - 3$
6 $y = 5x - 4$
7 a $y = \frac{1}{2}x + 5$ b $y = -x + 8$ c $y = \frac{1}{2}x + 2$
d -1 e $2\sqrt{2}$ f $2\sqrt{2}$
8 a i $(3\frac{1}{2}, 3\frac{1}{2})$ ii $(3\frac{1}{2}, 3\frac{1}{2})$
b $m_{AC} = 1, m_{BD} = -1$

Exercise 4E

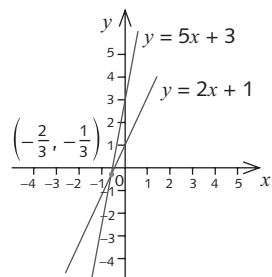
1 a



b



c



- 2 a $x = -\frac{9}{7}, y = -\frac{27}{7}$ b $x = \frac{3}{2}, y = \frac{3}{4}$
c $x = -\frac{7}{5}, y = -\frac{9}{5}$ d $x = \frac{13}{5}, y = -\frac{8}{15}$
e $x = -\frac{1}{7}, y = \frac{9}{7}$ f $x = \frac{5}{2}, y = \frac{3}{2}$
3 a $x = 4, y = 1$ b $x = 1, y = 2$
c $x = 3, y = -2$ d $x = -1, y = 2$

- e $x = 5, y = 2$ f $x = -3, y = 7$
g $x = 1, y = 2$ h $x = 1, y = -1$
i $x = -4, y = 3$ j $x = 2, y = -1$
k $x = \frac{60}{17}, y = \frac{10}{17}$ l $x = -\frac{5}{13}, y = -\frac{46}{13}$
m $x = 3, y = 5$ n $x = -\frac{6}{7}, y = \frac{24}{7}$
o $x = -\frac{30}{19}, y = -\frac{121}{19}$

- 4 a $x = -\frac{28}{33}, y = -\frac{173}{33}$ b $x = \frac{864}{101}, y = -\frac{35}{101}$
c $x = -\frac{97}{6}, y = \frac{38}{3}$ d $x = -\frac{63}{19}, y = \frac{30}{19}$
e $x = \frac{9}{13}, y = -\frac{51}{13}$ f $x = \frac{552}{59}, y = \frac{60}{59}$
g $x = \frac{385}{24}, y = -\frac{19}{12}$ h $x = \frac{10}{11}, y = \frac{8}{11}$
i $x = \frac{168}{25}, y = -\frac{24}{25}$ j $x = 0, y = -6$
k $x = 26, y = \frac{31}{3}$ l $x = -\frac{220}{17}, y = \frac{64}{17}$

5 $y = 3x - 6$

6 a $y = 0$ b $x = 2$

7 $A(-1, -1), B(3, 3), C(\frac{11}{5}, \frac{27}{5})$

8 $y = 2x + 2, x + y = 9, (\frac{7}{3}, \frac{20}{3})$

9 a i $y = \frac{1}{2}x + \frac{7}{2}$ ii $y = -2x + 16$
b $(5, 6)$

10 a i $y = x$ ii $y = -x$
b $(0, 0)$

11 $m = -\frac{27}{4}$

12 a $y = \frac{1}{2}x - 1$ b $y = -2x + 19$ c $(\frac{26}{5}, \frac{43}{5})$

d $\frac{14\sqrt{5}}{5} \approx 6.26$ e $5\sqrt{5} \approx 11.18$ f 70

13 $a = 4, b = -15$

14 The point is $(11, 5)$.

15 a $y = 3x$ b $y = x$

16 $a = 9, b = -6$

17 $(\frac{21}{5}, \frac{26}{5})$

18 a $BD : 6y - x = 6; AC : y + 6x = 21$

b $\left(\frac{120}{37}, \frac{57}{37}\right)$ c $\frac{9\sqrt{37}}{37}$ d 9

Exercise 4F

1 45 and 67 2 46 and 37

3 The daughter is 8 and the father is 36.

4 58 \$2 items, 43 \$5 items

5 Andrew is 36 and Brian is 14

6 10 metres and 14 metres

7 standard: 5 kg, jumbo: 7.5 kg

8 8000 \$60 tickets sold and 2000 \$80 tickets sold

9 1.875 L 10 5 hours 11 3:20 p.m.

12 50 tonnes of the 52% iron alloy, 150 tonnes of the 36% iron alloy

13 480 km/h and 640 km/h

14 model horse \$30.00; model cow \$10.00

15 $\frac{2}{15}$ 16 30 km

Review exercise

1 a $2\sqrt{17}$ b 6 c 12
d $\sqrt{26}$ e $\sqrt{34}$ f $4\sqrt{5}$

2 a $\left(\frac{3}{2}, 1\right)$ b $\left(-1, 4\frac{1}{2}\right)$ c $\left(-\frac{1}{2}, 0\right)$
d $\left(\frac{9}{2}, 5\right)$ e $\left(-\frac{3}{2}, \frac{13}{2}\right)$ f $\left(\frac{9}{2}, -3\right)$

3 a 4 b 3 c 3
d 1 e $\frac{2}{3}$ f $-\frac{5}{2}$
g -3 h 0 i 0

4 -4

5 a gradient = 2, y-intercept is 4
b gradient = 1, y-intercept is -4
c gradient = $\frac{1}{2}$, y-intercept is 1
d gradient = -2, y-intercept is 5
e gradient = -1, y-intercept is 6
f gradient = -1, y-intercept is 2
g gradient = -3, y-intercept is 4
h gradient = -3, y-intercept is 4
i gradient = $\frac{2}{3}$, y-intercept is -2

j gradient = $-\frac{3}{4}$, y-intercept is 3

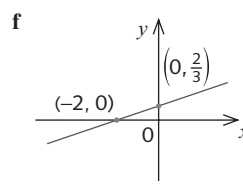
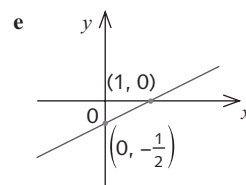
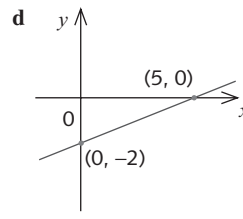
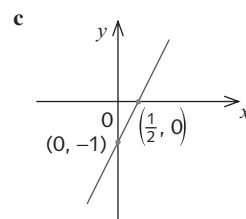
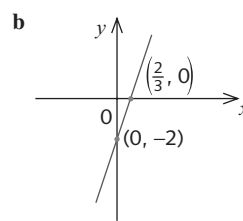
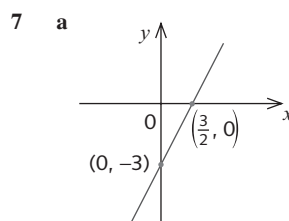
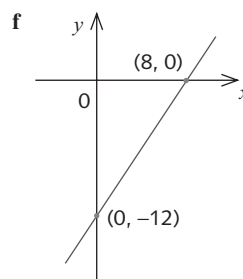
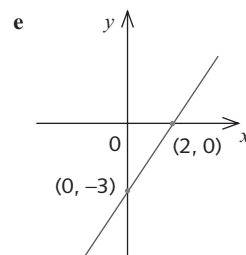
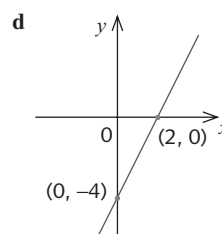
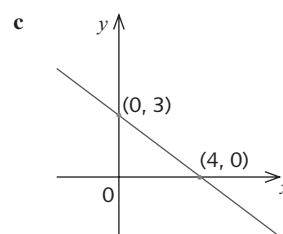
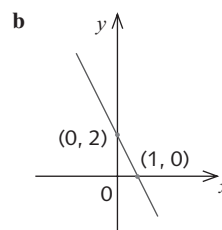
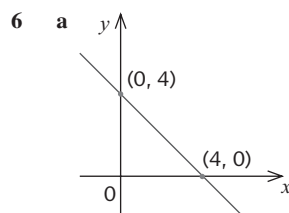
k gradient = 2, y-intercept is 0

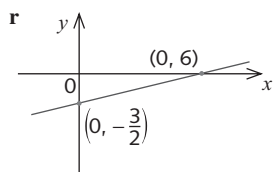
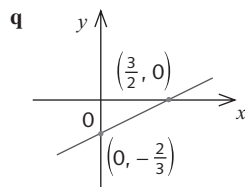
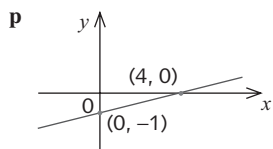
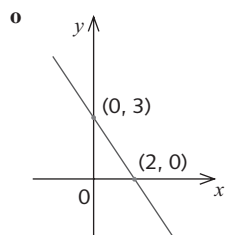
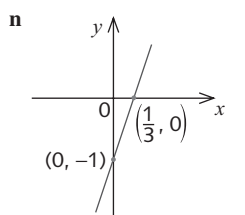
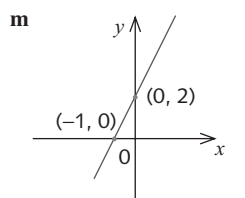
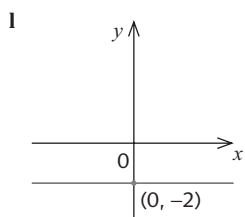
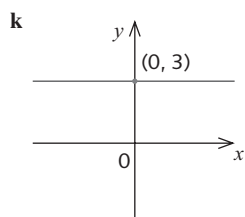
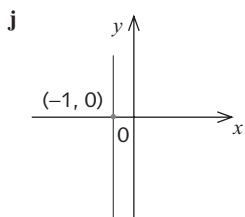
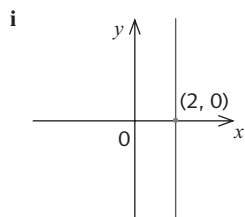
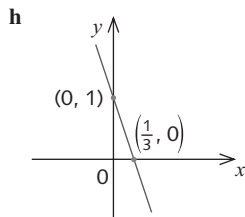
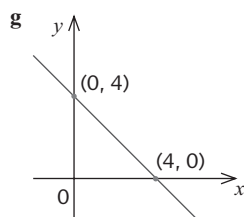
l gradient = 3, y-intercept is 0

m gradient = -4, y-intercept is 0

n gradient = $\frac{3}{2}$, y-intercept is 0

o gradient = $-\frac{3}{2}$, y-intercept is 3





8 a $6x + y = 11$

b $2x - 3y + 14 = 0$

9 a $4x + 9y = 74$

b $4x + 5y = 32$

c $4x - 3y + 26 = 0$

10 a $x = 4\frac{1}{8}, y = -1\frac{7}{8}$

b $x = 1\frac{7}{8}, y = 1\frac{7}{8}$

11 a $x = -3, y = -7$

b $x = -2, y = 8$

c $x = 5, y = 0$

d $x = 1, y = 1$

e $x = \frac{156}{11}, y = \frac{42}{11}$

f $x = \frac{138}{65}, y = -\frac{3}{13}$

12 a i $y = -\frac{5}{6}x + \frac{139}{12}$

ii $y = -\frac{3}{11}x + \frac{69}{11}$

b $\left(\frac{701}{74}, \frac{273}{74}\right)$

13 $\left(\frac{57}{13}, -\frac{14}{13}\right)$

14 $y = x + 3; y = x - 2; 3x + 2y = 21; 3x + 2y = 6$

15 The gradient of $AC = -1$ and the gradient of $AB = 1$.

16 b $12\sqrt{2}, 4\sqrt{2}$

17 113 and 25

18 stool \$30; chair \$100

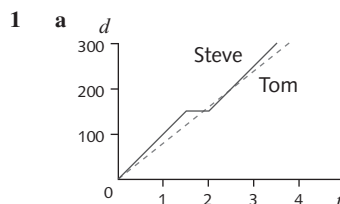
19 a $p = \frac{5}{2}$

b $p = -23$

20 $4x + 3y + 5 = 30$

21 The point is (4, 3).

Challenge exercise



b i After 1 hour 52½ minutes, 150 km from Mildura

ii After 2½ hours, 200 km from Mildura

iii 20 km

2 a $\left(\frac{1}{4}(x_1 + 3x_2), \frac{1}{4}(y_1 + 3y_2)\right)$

b $\left(\frac{1}{5}(2x_1 + 3x_2), \frac{1}{5}(2y_1 + 3y_2)\right)$

3 $\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$

4 a $\left(\frac{c}{2}, \frac{b}{2}\right)$

b i $\frac{\sqrt{c^2 + b^2}}{2}$

ii $\frac{\sqrt{b^2 + c^2}}{2}$

iii $\frac{\sqrt{c^2 + b^2}}{2}$

5 a i $y = \frac{b}{a+c}x$

ii $y = \frac{b}{a-c}(x-c)$

b i $\left(\frac{a+c}{2}, \frac{b}{2}\right)$

ii $\left(\frac{a+c}{2}, \frac{b}{2}\right)$

6 a i $\frac{b}{a+c}$

ii $\frac{b}{a-c}$

c $\sqrt{a^2 + b^2}$

$$10 \quad \text{a } \frac{p}{m}; -\frac{p}{\ell}$$

$$\text{c } y = \frac{1}{p}x + \frac{\ell m}{p}$$

$$\text{b } y = -\frac{m}{p}x + \frac{m\ell}{p}$$

Chapter 5 answers

Exercise 5A

- | | | |
|--------------------------------|--------------------------------|-------------------------------|
| 1 a 0, -3 | b 0, 7 | c 0, -5 |
| d 3, -6 | e -7, -9 | f 10, 7 |
| g 0, $-\frac{4}{5}$ | h $-\frac{3}{4}, \frac{2}{3}$ | i $-\frac{7}{2}, -4$ |
| j $\frac{3}{2}, -\frac{4}{3}$ | k $\frac{5}{2}, -4$ | l $\frac{2}{3}, \frac{4}{3}$ |
| 2 a 0, 5 | b 0, -7 | c 0, -8 |
| d 0, 25 | e 0, -18 | f $0, \frac{1}{2}$ |
| 3 a -1, -8 | b -2, -6 | c -3, -9 |
| d -6 | e 2, 4 | f 2, -3 |
| g 5, -6 | h 5, -8 | i 6, -10 |
| j 1, 6 | k 3, 4 | l 5 |
| m 2, 16 | n -3, 7 | o 10 |
| 4 a 0, 8 | b 1, 16 | c 1, 2 |
| d no solution | e 3, 5 | f no solution |
| g 0, -3 | h -5, 4 | i no solution |
| j -10, 1 | k 7, 1 | l -1, 1 |
| 5 a -1, -2 | b 3, 2 | c -7, 5 |
| 6 a $-\frac{3}{2}, -4$ | b $-\frac{1}{3}, -4$ | c $-\frac{3}{2}, -2$ |
| d $-\frac{1}{2}, 2$ | e $\frac{3}{2}, 3$ | f $\frac{4}{3}, 2$ |
| g $-\frac{3}{2}, -\frac{4}{5}$ | h $\frac{3}{2}, \frac{4}{3}$ | i $-\frac{1}{2}, \frac{5}{4}$ |
| j $-\frac{2}{3}, \frac{3}{4}$ | k $-\frac{1}{4}, \frac{2}{3}$ | l $\frac{2}{3}, -\frac{5}{2}$ |
| m $-\frac{2}{3}, 7$ | n $-\frac{2}{5}, -3$ | o $-\frac{15}{12}, 3$ |
| p $\frac{1}{15}, -15$ | q $\frac{9}{8}, \frac{8}{9}$ | r $-\frac{2}{3}, \frac{3}{2}$ |
| 7 a $\frac{1}{2}, \frac{4}{3}$ | b $-\frac{1}{6}, \frac{5}{4}$ | c 1, $\frac{3}{4}$ |
| d -2, $\frac{3}{2}$ | e $\frac{2}{3}, \frac{5}{2}$ | f $-\frac{1}{2}, \frac{4}{3}$ |
| g $\frac{5}{2}, \frac{2}{5}$ | h $-\frac{1}{4}, -\frac{3}{5}$ | i 2, $\frac{1}{2}$ |
| j 3, $-\frac{5}{4}$ | k $-\frac{2}{7}, \frac{1}{3}$ | l $\frac{5}{4}, -7$ |

Exercise 5B

- | | | |
|--------------------------------|---------------------------------|----------------------|
| 1 a 9, -2 | b 8, -4 | c $-\frac{3}{2}, 4$ |
| d -5, 5 | e no solution | f -3, -1 |
| g 0, -5 | h 5, 1 | i 8, -8 |
| j $-\frac{4}{3}, \frac{3}{2}$ | k $\frac{3}{2}, -\frac{7}{5}$ | l 4, 8 |
| 2 a -7, 2 | b 5, -3 | c -3, 2 |
| d 2, -1 | e 6, 1 | f 16, 2 |
| g -6, 5 | h -5, 4 | i $-\frac{1}{2}, -4$ |
| j 4 | k -13, 11 | l $-\frac{1}{2}, 3$ |
| m $-\frac{5}{4}, -\frac{7}{6}$ | n $-\frac{7}{3}, 3$ | |
| 3 a $\frac{50}{x}$ | b $\frac{50}{x} = x + 5, x = 5$ | |

Exercise 5C

- | | | | |
|------------------------|--------------------------------|------|-----|
| 1 11 sides | 2 12 cm | 3 16 | 4 5 |
| 5 9 and -7 or 7 and -9 | 6 8 cm | | |
| 7 4 cm, 32 cm, 42 cm | 8 5 and 7 | | |
| 9 6 km/h | | | |
| 10 a $20 - x$ cm | b 14 cm, 6 cm | | |
| 11 4 m | 12 8 cm, 12 cm, $4\sqrt{3}$ cm | | |
| 13 25 km/h | 14 20 cm and 25 cm | | |

Exercise 5D

- | | | |
|--------------------|-----------------------------|-------------------|
| 1 a -1 | b -2 | c -4 |
| d 5 | e $-\frac{1}{2}$ | f $\frac{3}{2}$ |
| g $\frac{2}{5}$ | h $-\frac{3}{4}$ | i $\frac{1}{3}$ |
| j $-\frac{1}{5}$ | k $\frac{2}{5}$ | l $-\frac{2}{7}$ |
| m $\frac{5}{7}$ | n $-\frac{5}{3}$ | |
| 2 a parts i, v, ix | b parts ii, iv, vii, x, xii | |
| 3 a 4 | b 16 | c 25 |
| d 36 | e 100 | f $\frac{9}{4}$ |
| g $\frac{1}{4}$ | h $\frac{49}{4}$ | i $\frac{121}{4}$ |

4 a $(x+3)^2 + 1$ b $(x+4)^2 - 21$
 c $(x+6)^2 - 46$ d $(x-5)^2 - 19$
 e $(x-3)^2 - 17$ f $(x-10)^2 - 95$
 g $\left(x + \frac{3}{2}\right)^2 - \frac{17}{4}$ h $\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$
 i $\left(x - \frac{5}{2}\right)^2 - \frac{1}{4}$ j $\left(x - \frac{1}{2}\right)^2 - \frac{41}{4}$
 k $\left(x + \frac{3}{2}\right)^2 + \frac{19}{4}$ l $\left(x - \frac{11}{2}\right)^2 - \frac{117}{4}$

5 a $3(x+1)^2 + 9$ b $5(x+3)^2 - 35$
 c $3(x-2)^2 + 3$ d $-(x+1)^2 + 5$
 e $-(x-4)^2 + 6$ f $-(x+3)^2 + 13$
 g $3(x-1)^2 - 4$ h $2(x-3)^2 + 15$
 i $4(x-6)^2 - 45$ j $2\left(x + \frac{3}{4}\right)^2 + \frac{7}{8}$
 k $4\left(x - \frac{1}{8}\right)^2 - \frac{65}{16}$ l $3\left(x - \frac{4}{3}\right)^2 + \frac{11}{3}$
 m $5\left(x - \frac{1}{10}\right)^2 + \frac{19}{20}$ n $2\left(x - \frac{5}{4}\right)^2 - \frac{81}{8}$
 o $-3\left(x + \frac{1}{6}\right)^2 + \frac{49}{12}$

Exercise 5E

1 a $\pm\sqrt{5}$ b $\pm\sqrt{11}$ c $\pm\sqrt{3}$
 d $\pm\sqrt{2}$ e $\pm\sqrt{10}$ f $\pm\sqrt{5}$

2 a $-1 \pm \sqrt{2}$ b $-2 \pm \sqrt{3}$ c $6 \pm \sqrt{13}$
 d $-3 \pm \sqrt{2}$ e $4 \pm \sqrt{17}$ f $-4 \pm 2\sqrt{5}$
 g $-5 \pm 2\sqrt{6}$ h $-6 \pm \sqrt{41}$ i $5 \pm 5\sqrt{3}$
 j $-10 \pm \sqrt{95}$ k $50 \pm 2\sqrt{645}$ l $25 \pm \sqrt{615}$

3 a $\frac{-1 \pm \sqrt{5}}{2}$ b $\frac{3 \pm \sqrt{5}}{2}$ c $\frac{5 \pm \sqrt{29}}{2}$
 d $\frac{-3 \pm \sqrt{17}}{2}$ e $\frac{-5 \pm \sqrt{41}}{2}$ f $\frac{3 \pm \sqrt{29}}{2}$
 g $\frac{7 \pm \sqrt{449}}{2}$ h $\frac{3 \pm \sqrt{33}}{2}$ i $\frac{9 \pm \sqrt{101}}{2}$
 j $\frac{1 \pm \sqrt{21}}{2}$ k $\frac{3 \pm \sqrt{5}}{2}$ l $\frac{5 \pm \sqrt{13}}{2}$

4 a $2 \pm \sqrt{3}$ b $-1 \pm \sqrt{5}$ c $-1 \pm \sqrt{5}$
 d $4 \pm \sqrt{6}$ e $-3 \pm \sqrt{21}$ f $-4 \pm 2\sqrt{5}$
 g $2, -\frac{1}{2}$ h $\frac{4 \pm \sqrt{34}}{3}$ i $\frac{1 \pm \sqrt{65}}{8}$
 j $\frac{-1 \pm \sqrt{21}}{10}$ k $3, -\frac{1}{2}$ l $\frac{-5 \pm \sqrt{70}}{3}$
 m $-1 \pm \sqrt{7}$ n $1 \pm \sqrt{5}$ o $\sqrt{2} \pm 2$

5 a $-3 \pm \sqrt{17}$ b $5, -2$ c $1, -7$
 d $2 \pm \sqrt{7}$ e $2, -3$ f $\frac{1 \pm \sqrt{13}}{2}$
 g $-2, -\frac{1}{2}$ h $1, -\frac{1}{3}$ i $-1 \pm \sqrt{6}$
 j $-3 \pm \sqrt{14}$ k no solution l no solution
 m $\pm\frac{5}{2}$ n $\pm\frac{1}{3}$ o $-7, 5$
 p $-3, 4$ q $\pm\frac{\sqrt{5}}{2}$ r no solution
 s $-\frac{3}{2}, \frac{4}{3}$ t $-\frac{5}{3}, -\frac{1}{4}$ u -3
 v $-1 \pm \frac{\sqrt{3}}{3}$ w $2 \pm \frac{\sqrt{6}}{2}$ x $-\frac{1 \pm \sqrt{26}}{5}$
 y $-\frac{2}{3}, \frac{1}{4}$ z no solution

6 a $-1 \pm \sqrt{6}$ b $1 \pm \sqrt{2}$ c $-2 \pm \sqrt{11}$
 d $-3 \pm \sqrt{5}$ e $\frac{1 \pm \sqrt{5}}{2}$ f $3, -\frac{3}{2}$

Exercise 5F

1 a $4 \pm \sqrt{15}$ b $-2, 4$
 c $\frac{3 \pm \sqrt{13}}{2}$ d $-2, 6$
 e $\frac{-5 \pm \sqrt{17}}{2}$ f $\frac{-9 \pm \sqrt{69}}{2}$
 g $4 \pm \sqrt{14}$ h $-1 \pm \sqrt{5}$
 i $-6 \pm \sqrt{33}$

2 a $\frac{-1 \pm \sqrt{22}}{3}$ b $\frac{-3 \pm \sqrt{29}}{10}$
 c $\frac{3 \pm \sqrt{5}}{4}$ d $1, \frac{2}{7}$
 e $-1, \frac{2}{5}$ f $\frac{1 \pm \sqrt{29}}{14}$
 g $\frac{-6 \pm \sqrt{38}}{2}$ h $\frac{10 \pm \sqrt{106}}{3}$
 i $\frac{2 \pm \sqrt{19}}{3}$

3 a $1.53, -0.13$ b $7.87, 0.13$
 c $-2, 5$ d $-0.20, -14.80$
 e $2, 3$ f $-0.41, 3.41$
 g $-0.35, 2.85$ h $-0.24, 0.84$
 i $0.15, 3.35$

4 a i no solutions ii one solution
 iii two solutions

Chapter 6 answers

Exercise 6A

- 1 a 318 cm² b 448 cm²
c $192\pi \text{ cm}^2 \approx 603.2 \text{ cm}^2$ d $54 + 36\pi \text{ cm}^2 \approx 167.1 \text{ cm}^2$
e 4140 cm² f $100 + 37.5\pi \text{ m}^2 \approx 217.8 \text{ m}^2$
g 1000 cm² h 596 cm²
i 194 cm² j $144 + 40\sqrt{2} \approx 200.6 \text{ cm}^2$
- 2 a $3\sqrt{3} \approx 5.1962 \text{ cm}$
b $216 + 18\sqrt{3} \approx 247.1769 \text{ cm}^2$
- 3 9 m 4 4.5 cm
- 5 a $1425\pi \text{ cm}^2 \approx 4476.8 \text{ cm}^2$
b $\frac{350}{3\pi} \text{ cm} \approx 37.1 \text{ cm}$
- 6 a 280 cm³ b 216 m³ c 144 m³
d 28 000 mm³ e $300\pi \text{ cm}^3 \approx 942.5 \text{ cm}^3$
f $1215\pi \text{ cm}^3 \approx 3817.0 \text{ cm}^3$ g 23 040 cm³
h $375\pi \text{ cm}^3 \approx 1178.1 \text{ cm}^3$ i 3240 cm³
j 1080 cm³ k 480 cm³ l 360 cm³
- 7 a 5.29 cm b 761.98 cm³
- 8 a 19.9 cm b 6.31 cm
- 9 a 4 cm b 7.4 cm³
- 10 16.25 m²
- 11 4 cm
- 12 $\frac{1500}{2 + 3\sqrt{2}} \text{ cm} \approx 240.28 \text{ cm}$
- 13 a 1099.56 cm³ b 1400 cm³ c 300.44 cm³

Exercise 6B

- 1 60 cm³
- 2 a 80 cm³ b 100 cm³ c 216 cm³ d 180 cm³
- 3 a 400 cm³ b $53\frac{1}{3} \text{ cm}^3$ c 80 cm³ d 96 cm³
- 4 2 547 758.6 m³
- 5 a 800 cm³ b 2333.33 cm³
- 6 a 5 cm b 13 cm c 65 cm² d 360 cm²
- 7 a $64 + 32\sqrt{13} \approx 179.378 \text{ cm}^2$
b $36 + 12\sqrt{34} \approx 105.971 \text{ cm}^2$
- 8 a $\sqrt{61} \text{ cm} \approx 7.810 \text{ cm}$ b $2\sqrt{13} \text{ cm} \approx 7.211 \text{ cm}$
c $(80 + 8\sqrt{61} + 20\sqrt{13}) \text{ cm} \approx 214.593 \text{ cm}^2$
- 9 a $\Delta VDA, \Delta VAB, \Delta VCB, \Delta VDC$: all are right-angled triangles.
b i 10 cm ii $2\sqrt{41} \text{ cm} \approx 12.806 \text{ cm}$
iii 10 cm
c 192 cm²

Exercise 6C

- 1 a 138.23 cm² b 235.62 cm² c 1099.56 cm²
- 2 a $24\pi \text{ cm}^2$ b $(16\pi + 16\pi\sqrt{10}) \text{ cm}^2$
c $70\pi \text{ cm}^2$
- 3 a 10 cm b 9.1652 cm
- 4 a 685.84 cm² b 21.83 cm c 19.41 cm
- 5 a 104.72 cm²
b i 10 cm ii 3.33 cm iii 9.43 cm
- 6 a i $5\sqrt{5} \text{ cm} \approx 11.18 \text{ cm}$ ii $25\pi\sqrt{5} \text{ cm}^2 \approx 175.62 \text{ cm}^2$
iii $\left(\frac{360}{\sqrt{5}}\right)^\circ \approx 161^\circ$
b $\left(\frac{360}{\sqrt{2}}\right)^\circ \approx 255^\circ$
- 7 a 100.53 cm³ b 335.10 cm³
c 376.99 cm³ d 339.29 cm³
- 8 127 mm
- 9 a $648\pi \text{ cm}^3$ b $270\pi \text{ cm}^2$
- 10 a $(1800 + 36\sqrt{2})\pi \text{ cm}^3$ b $726\pi \text{ cm}^3$
- 11 \$19 635

Exercise 6D

- 1 a 804.25 cm² b 2827.43 cm²
c 615.75 cm² d 1385.44 cm²
- 2 a $64\pi \text{ cm}^2 \approx 201.06 \text{ cm}^2$ b $128\pi \text{ cm}^2 \approx 402.12 \text{ cm}^2$
c $192\pi \text{ cm}^2 \approx 603.19 \text{ cm}^2$
- 3 6.3078 cm
- 4 a 2.11 m b 1.72 m
- 5 a $195\pi \text{ cm}^2$ b $115\pi \text{ cm}^2$ c $(440 + 16\pi) \text{ cm}^2$
- 6 a 2144.66 b 4188.79 c 904.78
d 452.39 cm³ e 3619.11 cm³ f 883.57 cm³
- 7 a 6.20 cm b 179 mm c 9.8 cm
- 8 a 368.61 cm³ b 575.96 cm³ c 329.87 cm³
- 9 8249 cm³
- 10 545 131.78 mm³

Exercise 6E

- 1 a i 9 ii 9
b i $\frac{36}{25}$ ii $\frac{36}{25}$
c i $\frac{9}{4}$ ii $\frac{9}{4}$

- 2 a i 4 ii 8
 b i $\frac{9}{4}$ ii $\frac{27}{8}$
 c i $\frac{36}{25}$ ii $\frac{216}{125}$
 d i $\frac{25}{4}$ ii $\frac{125}{8}$
- 3 a $42\,875\text{ cm}^3$ b 7350 cm^2
- 4 8 5 960 cm^2 6 $2\sqrt{2}$
- 7 a $172\,800\text{ cm}^2$ b 325.5 cm^3
- 8 a 192 m^3 b 1536 cm^3
 c $(96 + 16\sqrt{13})\text{ m}^2 \approx 153.7\text{ m}^2$ d 614.8 cm^2
- 9 a The area of the triangle increases by a factor of 7.
 b 36 c 9 d 81

Review exercise

- 1 a i 1000 cm^3 ii 600 cm^2
 b i 600 cm^3 ii 460 cm^2
 c i 480 cm^3 ii 528 cm^2
 d i $168\pi\text{ cm}^3$ ii $(36\pi + 12\sqrt{58}\pi)\text{ cm}^2$
 e i $90\pi\text{ m}^3$ ii $78\pi\text{ m}^2$
 f i $\frac{2048}{3}\pi\text{ m}^3$ ii $256\pi\text{ m}^2$
- 2 a i 550 m^3 ii $(375 + 25\sqrt{17})\text{ cm}^2 \approx 478.1\text{ cm}^2$
 b i $200\pi\text{ cm}^3 \approx 628.3\text{ cm}^3$
 ii $\left(\frac{340\pi}{3} + 120\right)\text{ cm}^2 \approx 476.0\text{ cm}^2$
- 3 2 cm
- 4 a $1\,500\,000\text{ m}^3$ b 80 cm^3
 c $\frac{250\pi}{3}\text{ cm}^3 \approx 261.8\text{ cm}^3$ d $156\pi\text{ cm}^3 \approx 490.1\text{ cm}^3$
- 5 8 cm
- 6 a i $\frac{592\pi}{3}\text{ cm}^3 \approx 619.94\text{ cm}^3$
 ii $168\pi\text{ cm}^2 \approx 527.8\text{ cm}^2$
 b i $\left(4 + \frac{2\pi}{3}\right)\text{ m}^3 \approx 6.09\text{ m}^3$
 ii $(16 + \pi)\text{ m}^2 \approx 19.1\text{ m}^2$
- 7 1.2 m
- 8 a 662.5 cm^2 b 937.5 cm^3
- 9 a 80 m^3 b $(63 + 8\sqrt{34} + 3\sqrt{89})\text{ m}^3 \approx 137.95\text{ m}^3$
- 10 a $V = \frac{250\,000}{3}\pi\text{ mm}^3$, $S = \left(20\,000 + \frac{12\,500}{3}\pi\right)\text{ mm}^2$
 b $V = 2250\text{ cm}^3$, $S = 675\pi\text{ m}^2$
 c $V = 4\pi\text{ m}^3$, $S = (6 + \sqrt{13})\pi\text{ m}^2$
 d $V = \frac{875}{3}\pi\text{ m}^3$, $S = 150\pi\text{ m}^2$

Challenge exercise

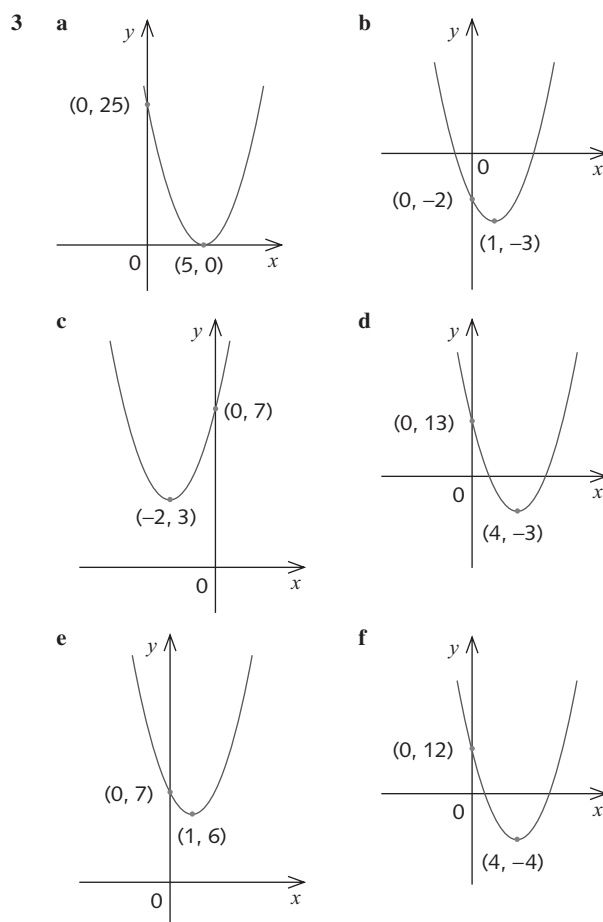
- 1 4 cm
 3 6 cm
 6 c Substitute the result of part b into that of part a.
 7 b 7 : 8
 9 a Use similar triangles.
 c Show $s^2 = h^2 + (R - r)^2$
 10 a Use Pythagoras' theorem
 c cross-section area = area of cylinder cross-section –
 area of cone cross-section
 d $\pi r^3 - \frac{1}{3}\pi r^3 = \frac{2}{3}\pi r^3$
 e Volume of a sphere is twice the result in part d.

Chapter 7 answers

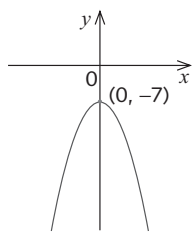
Exercise 7A

- 1 a $x = 4, (4, 0)$ b $x = 0, (0, -4)$ c $x = 2, (2, 6)$
 d $x = -3, (-3, 7)$ e $x = -2, (-2, 3)$ f $x = 0, (0, 9)$
 g $x = 3, (3, -4)$ h $x = -2, (-2, -3)$ i $x = 6, (6, 6)$
 j $x = -1, (-1, 0)$ k $x = 2, (2, 1)$ l $x = -3, (-3, 5)$

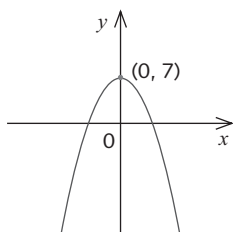
- 2 a -3 b 46 c 5
 d -9 e -8 f 2



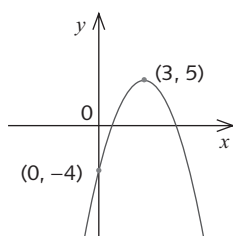
4 a



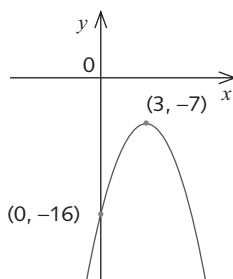
b



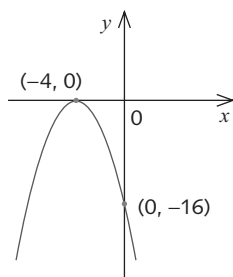
c



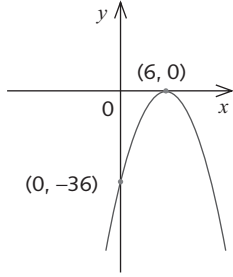
d



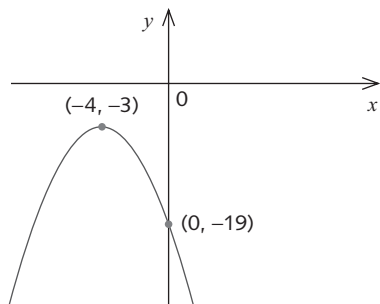
e



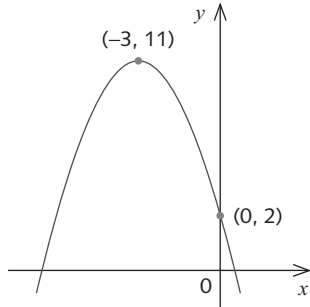
f



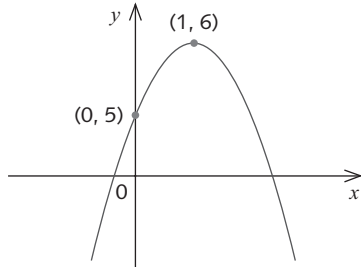
g



h



i



5

a $y = -(x - 3)^2$

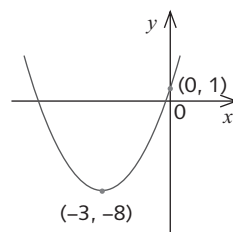
c $y = -x^2 - 6$

6

a $y = (x + 4)^2 + 3$

c $y = (x - a)^2 + b$

7



a $y = x^2$

c $y = (x + 3 - a)^2 - 8 + b$

8

a $a = 0$

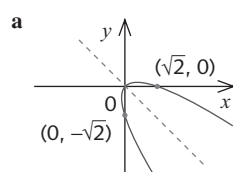
b $a = 2$

9

a $b = 5$

b $b = 7$

10



b $y = -(x + b)^2$

d $y = -x^2 + c$

b $y = (x - 6)^2 - 5$

d $y = (x + d)^2 - c$

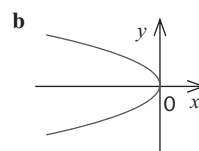
b $y = (x + 5)^2 - 11$

c $a = -1$

d $a = -8$

c $b = 0$

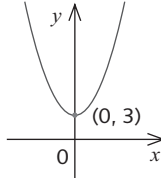
d $b = -3$



Exercise 7B

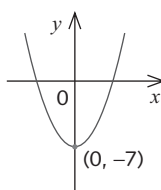
1 a i 3

iii



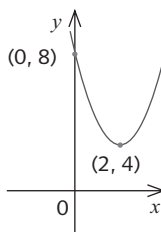
b i -7

iii



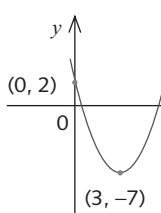
c i 8

iii



d i 2

iii



ii $x = 0, (0, 3)$

iv no x -intercepts

ii $x = 0, (0, -7)$

iv two x -intercepts

ii $x = 2, (2, 4)$

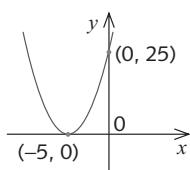
iv no x -intercepts

ii $x = 3, (3, -7)$

iv two x -intercepts

e i 25

iii

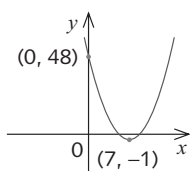


ii $x = -5, (-5, 0)$

iv one x -intercept

f i 48

iii

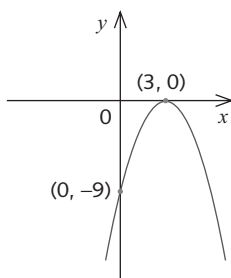


ii $x = 7, (7, -1)$

iv two x -intercepts

g i -9

iii

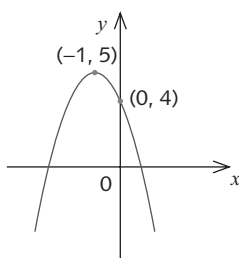


ii $x = 3, (3, 0)$

iv one x -intercept

h i 4

iii

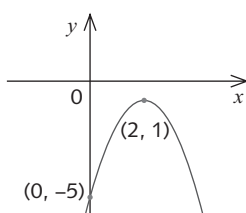


ii $x = -1, (-1, 5)$

iv two x -intercepts

i i -5

iii



ii $x = 2, (2, -1)$

iv no x -intercepts

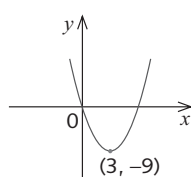
2 a i 0

iii $x = 3, (3, -9)$

v two x -intercepts

ii $y = (x - 3)^2 - 9$

iv



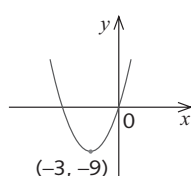
b i 0

iii $x = -3, (-3, -9)$

v two x -intercepts

ii $y = (x + 3)^2 - 9$

iv



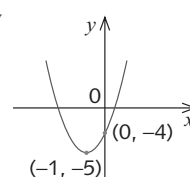
c i -4

iii $x = -1, (-1, -5)$

v two x -intercepts

ii $y = (x + 1)^2 - 5$

iv



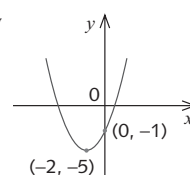
d i -1

iii $x = -2, (-2, -5)$

v two x -intercepts

ii $y = (x + 2)^2 - 5$

iv



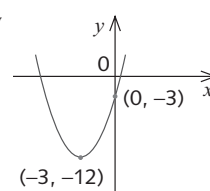
e i -3

iii $x = -3, (-3, -12)$

v two x -intercepts

ii $y = (x + 3)^2 - 12$

iv



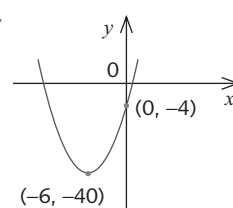
f i -4

iii $x = -6, (-6, -40)$

v two x -intercepts

ii $y = (x + 6)^2 - 40$

iv



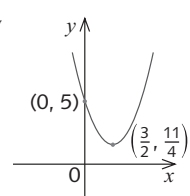
g i 5

iii $x = \frac{3}{2}, \left(\frac{3}{2}, \frac{11}{4}\right)$

v two x -intercepts

ii $y = \left(x - \frac{3}{2}\right)^2 + \frac{11}{4}$

iv



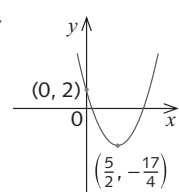
h i 2

iii $x = \frac{5}{2}, \left(\frac{5}{2}, -\frac{17}{4}\right)$

v two x -intercepts

ii $y = \left(x - \frac{5}{2}\right)^2 - \frac{17}{4}$

iv



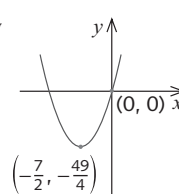
i i 0

iii $x = -\frac{7}{2}, \left(-\frac{7}{2}, -\frac{49}{4}\right)$

v two x -intercepts

ii $y = \left(x + \frac{7}{2}\right)^2 - \frac{49}{4}$

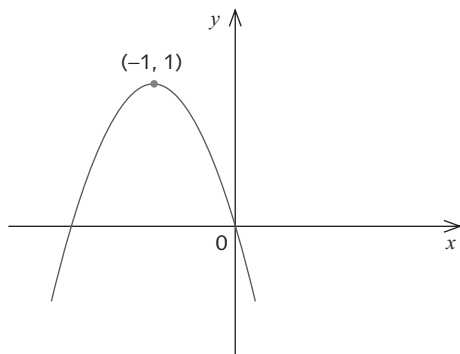
iv



j i 0

iii $x = -1, (-1, 1)$

iv

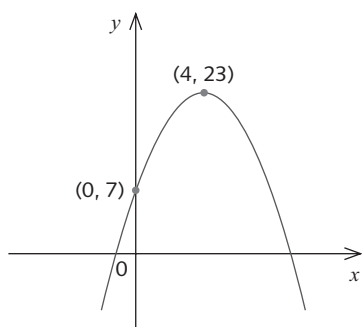


v two x -intercepts

k i 7

iii $x = 4, (4, 23)$

iv

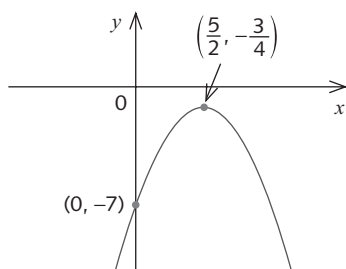


v two x -intercepts

l i -15

iii $x = \frac{5}{2}, \left(\frac{5}{2}, -\frac{3}{4}\right)$

iv



v no x -intercepts

3 a -3, 1

c -2, 8

e $-3 - \sqrt{11}, -3 + \sqrt{11}$

g $2 + 2\sqrt{2}, 2 - 2\sqrt{2}$

i $3 + 5\sqrt{2}, 3 - 5\sqrt{2}$

k $-2 + 4\sqrt{2}, -2 - 4\sqrt{2}$

4 a $-1 - \sqrt{5}, -1 + \sqrt{5}$

c $4 - \sqrt{3}, 4 + \sqrt{3}$

e -11, 1

g -1, 13

i $2 - 2\sqrt{3}, 2 + 2\sqrt{3}$

b -5, 1

d $4 - \sqrt{7}, 4 + \sqrt{7}$

f 1, -3

h $5 + 3\sqrt{2}, 5 - 3\sqrt{2}$

j $-3 + 2\sqrt{5}, -3 - 2\sqrt{5}$

l 3, -5

b $3 - \sqrt{2}, 3 + \sqrt{2}$

d $-2 - 2\sqrt{2}, -2 + 2\sqrt{2}$

f $10 - 5\sqrt{6}, 10 + 5\sqrt{6}$

h $3 + \sqrt{5}, 3 - \sqrt{5}$

ii $y = -(x+1)^2 + 1$

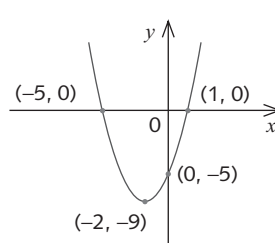
ii $y = -(x-4)^2 + 23$

ii $y = -\left(x - \frac{5}{2}\right)^2 - \frac{3}{4}$

5 a i -5

iii $x = -2, (-2, -9)$

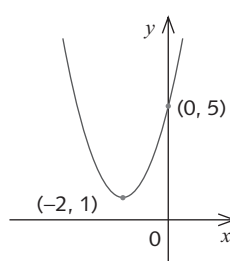
v



b i 5

iii $x = -2, (-2, 1)$

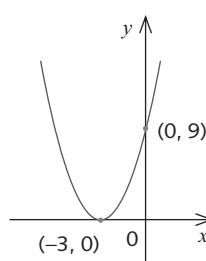
v



c i 9

iii $x = -3, (-3, 0)$

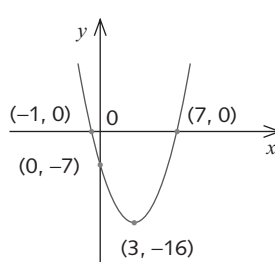
v



d i -7

iii $x = 3, (3, -16)$

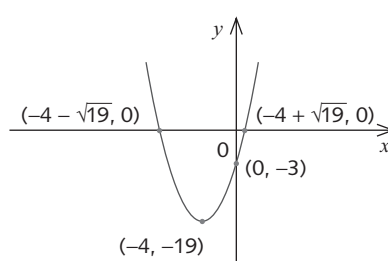
v



e i -3

iii $x = -4, (-4, -19)$

v



ii $y = (x+2)^2 - 9$

iv -5, 1

ii $y = (x+2)^2 + 1$

iv no x -intercepts

ii $y = (x+3)^2$

iv -3

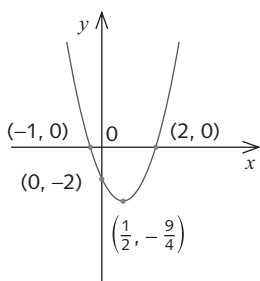
ii $y = (x-3)^2 - 16$

iv -1, 7

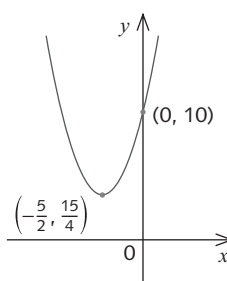
ii $y = (x+4)^2 - 19$

iv $-4 - \sqrt{19}, -4 + \sqrt{19}$

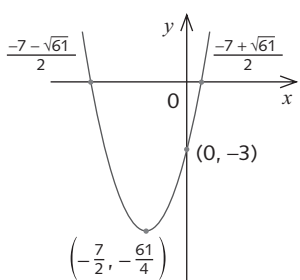
f i -2
 iii $x = \frac{1}{2}, \left(\frac{1}{2}, -\frac{9}{4}\right)$
 v
 ii $y = \left(x - \frac{1}{2}\right)^2 - \frac{9}{4}$
 iv -1, 2



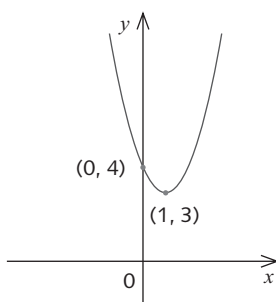
g i 10
 iii $x = -\frac{5}{2}, \left(-\frac{5}{2}, \frac{15}{4}\right)$
 v
 ii $y = \left(x + \frac{5}{2}\right)^2 + \frac{15}{4}$
 iv no x -intercepts



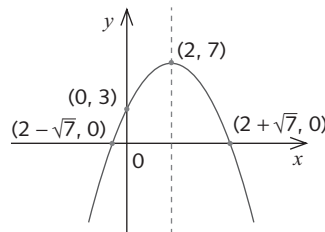
h i -3
 iii $x = -\frac{7}{2}, \left(-\frac{7}{2}, -\frac{61}{4}\right)$
 v
 ii $y = \left(x + \frac{7}{2}\right)^2 - \frac{61}{4}$
 iv $\frac{-7 + \sqrt{61}}{2}, \frac{-7 - \sqrt{61}}{2}$



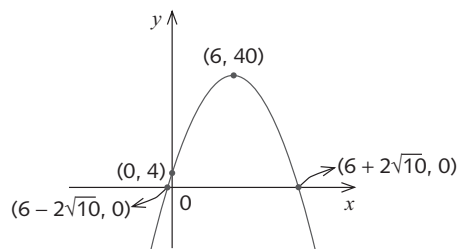
i i 4
 ii $y = (x - 1)^2 + 3$
 iii $x = 1, (1, 3)$
 iv no x -intercepts
 v



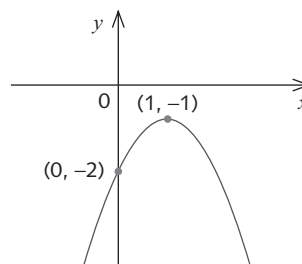
j i 3
 iii $x = 2, (2, 7)$
 v
 ii $y = -(x - 2)^2 + 7$
 iv $2 - \sqrt{7}, 2 + \sqrt{7}$



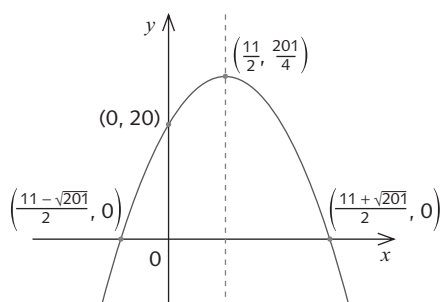
k i 6
 ii $y = -(x - 6)^2 + 40$
 iii $x = 6, (6, 40)$
 iv $6 - 2\sqrt{10}, 6 + 2\sqrt{10}$
 v



l i -2
 ii $y = -(x - 1)^2 - 1$
 iii $x = 1, (1, -1)$
 iv no x -intercepts
 v



m i 1
 ii $y = -\left(x - \frac{1}{2}\right)^2 + \frac{5}{4}$
 iii $x = \frac{1}{2}, \left(\frac{1}{2}, \frac{5}{4}\right)$
 iv $\frac{1 - \sqrt{5}}{2}, \frac{1 + \sqrt{5}}{2}$
 v



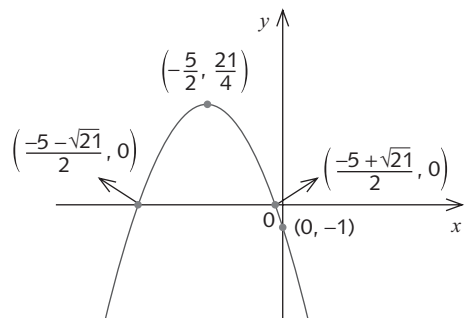
n i -1

ii $-\left(x + \frac{5}{2}\right)^2 + \frac{21}{4}$

iii $x = \frac{-5}{2}, \left(\frac{-5}{2}, \frac{21}{4}\right)$

iv $\frac{-5 - \sqrt{21}}{2}, \frac{-5 + \sqrt{21}}{2}$

v



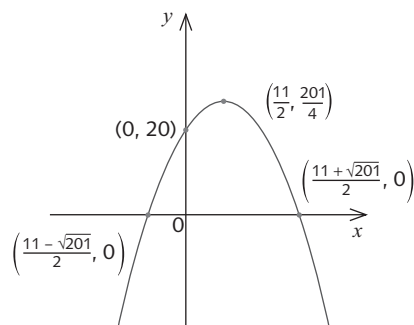
o i 20

ii $y = -\left(x - \frac{11}{2}\right)^2 + \frac{201}{4}$

iii $x = \frac{11}{2}, \left(\frac{11}{2}, \frac{201}{4}\right)$

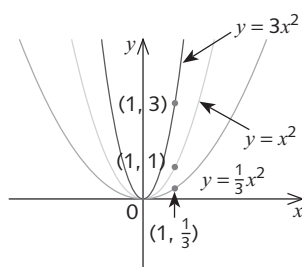
iv $\frac{11 - \sqrt{201}}{2}, \frac{11 + \sqrt{201}}{2}$

v

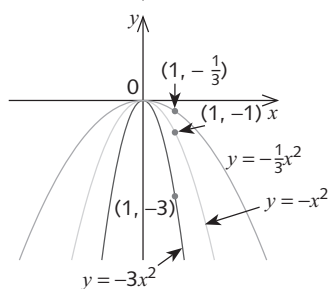


Exercise 7C

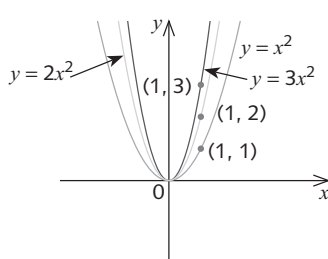
1 a



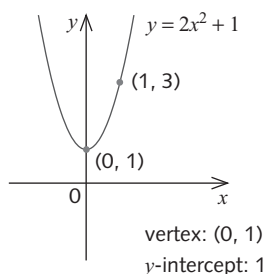
b



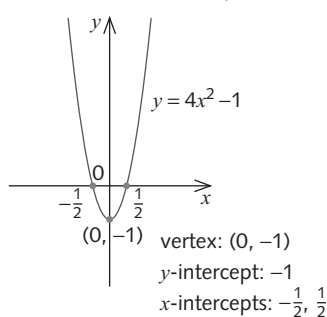
c



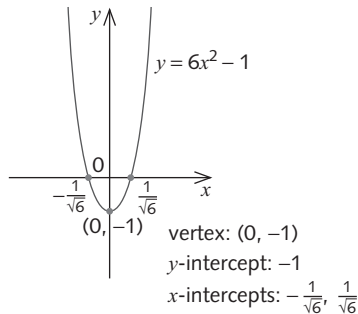
2 a



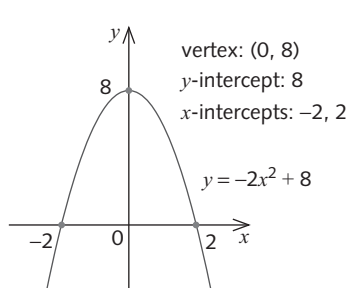
b



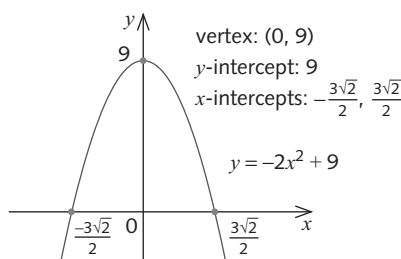
c



d



e



3 a, d, e, g are congruent; b, f, i are congruent; c, h are congruent; j and k are congruent

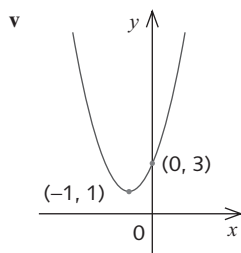
4 a $y = 3(x + 1)^2$ b $y = 3\left(x - \frac{3}{2}\right)^2 + \frac{41}{4}$

c $y = 2\left(x + \frac{5}{2}\right)^2 - \frac{51}{2}$ d $y = 2\left(x + \frac{7}{4}\right)^2 - \frac{25}{8}$

e $y = 3\left(x + \frac{5}{6}\right)^2 + \frac{59}{12}$ f $y = -5(x - 2)^2 + 57$

5 a i 3

iii $x = -1, (-1, 1)$

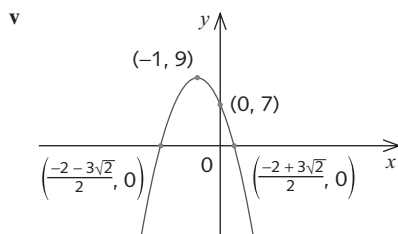


ii $y = 2(x+1)^2 + 1$

iv no x -intercepts

b i 7

iii $x = -1, (-1, 9)$



ii $y = -2(x+1)^2 + 9$

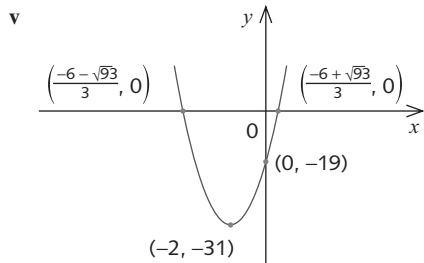
iv $\frac{-2+3\sqrt{2}}{2}, \frac{-2-3\sqrt{2}}{2}$

c i -19

ii $y = 3(x+2)^2 - 31$

iii $x = -2, (-2, -31)$

iv $\frac{-6+\sqrt{93}}{3}, \frac{-6-\sqrt{93}}{3}$

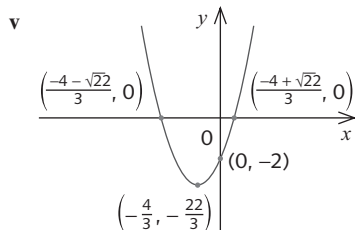


d i -2

ii $y = 3\left(x + \frac{4}{3}\right)^2 - \frac{22}{3}$

iii $x = -\frac{4}{3}, \left(-\frac{4}{3}, -\frac{22}{3}\right)$

iv $\frac{-4+\sqrt{22}}{3}, \frac{-4-\sqrt{22}}{3}$

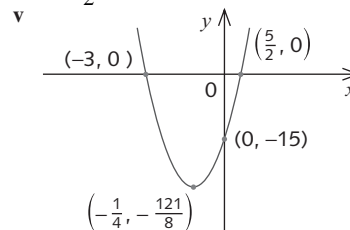


e i -15

ii $y = 2\left(x + \frac{1}{4}\right)^2 - \frac{121}{8}$

iii $x = -\frac{1}{4}, \left(-\frac{1}{4}, -\frac{121}{8}\right)$

iv $-3, \frac{5}{2}$

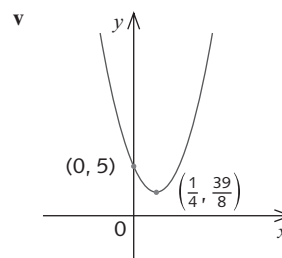


f i 5

ii $y = 2\left(x - \frac{1}{4}\right)^2 + \frac{39}{8}$

iii $x = \frac{1}{4}, \left(\frac{1}{4}, \frac{39}{8}\right)$

iv no x -intercepts



6 a $x = \frac{1}{2}, \left(\frac{1}{2}, \frac{11}{4}\right)$, no x -intercepts

b $x = -\frac{5}{6}, \left(-\frac{5}{6}, -\frac{181}{12}\right)$, has x -intercepts

c $x = -\frac{1}{4}, \left(-\frac{1}{4}, \frac{7}{8}\right)$, no x -intercepts

d $x = -\frac{3}{10}, \left(-\frac{3}{10}, \frac{131}{20}\right)$, no x -intercepts

e $x = \frac{5}{6}, \left(\frac{5}{6}, -\frac{59}{12}\right)$, no x -intercepts

f $x = \frac{3}{2}, \left(\frac{3}{2}, \frac{17}{4}\right)$, has x -intercepts

7 a 6

b 8

c -8

d -2

8 a 3

b 1

c $\frac{1}{2}$

d -3

9 a ± 1

b ± 3

c $\pm \frac{\sqrt{2}}{2}$

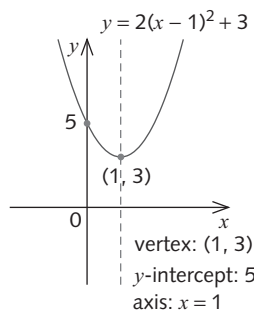
d $\pm \sqrt{3}$

10 $y = (x-1)^2 - 2$ or $y = x^2 - 2x - 1$

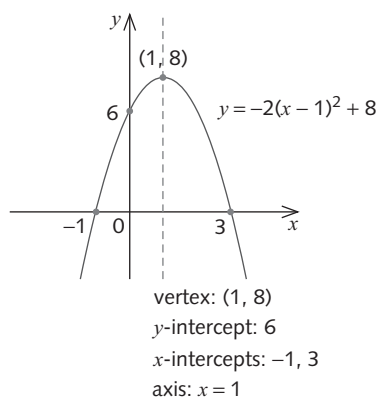
11 $y = 3(x+2)^2 - 1$ or $y = 3x^2 + 12x + 11$

12 $y = -2(x-1)^2 + 6$ or $y = -2x^2 + 4x + 4$

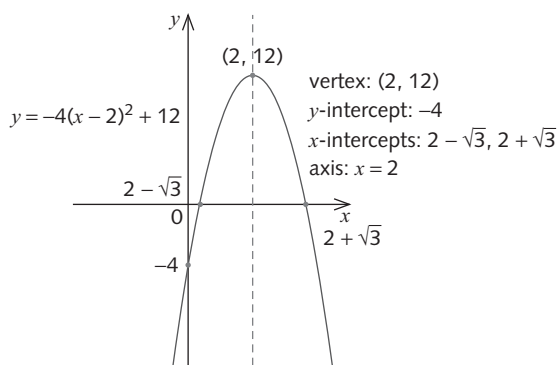
13 a



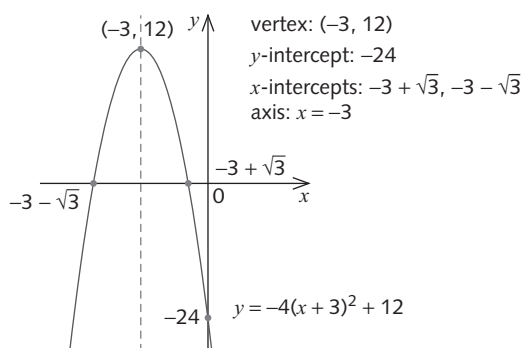
b



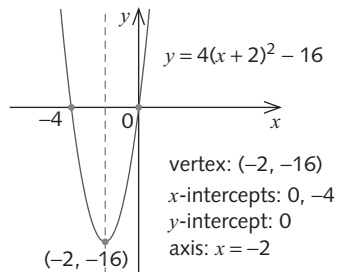
c



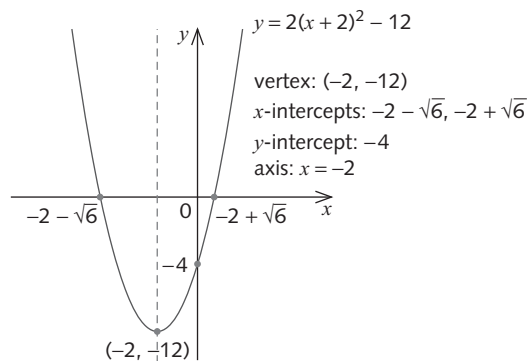
d



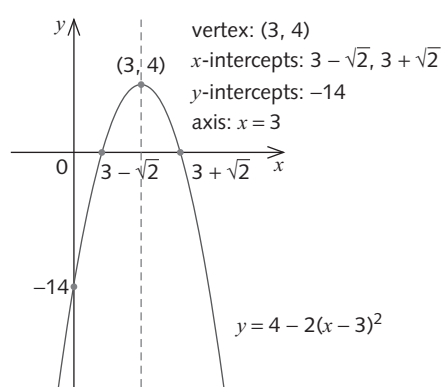
e



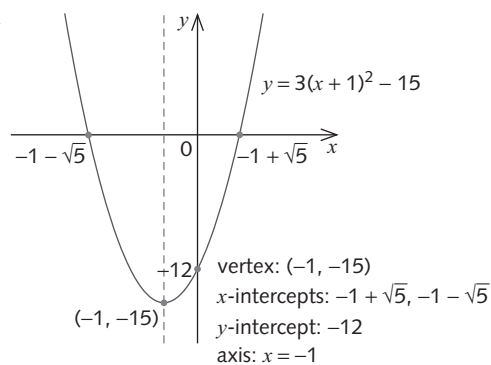
f



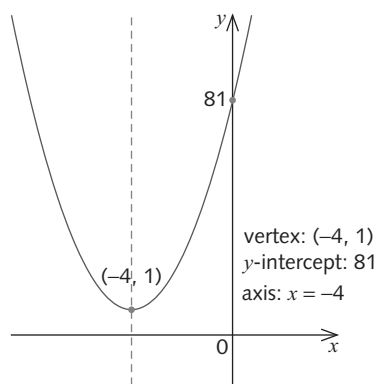
g



h



i

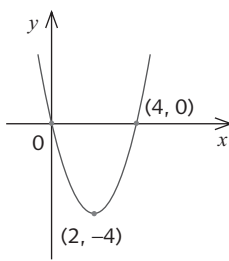


Exercise 7D

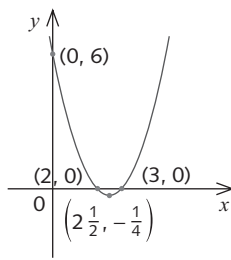
1 4

2 1

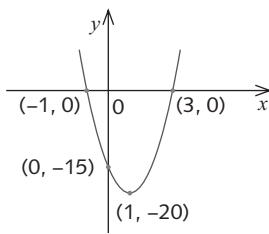
3 a



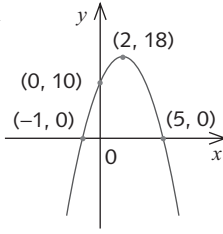
b



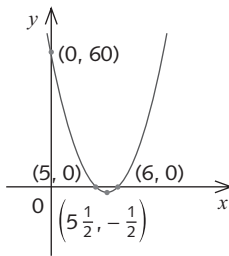
c



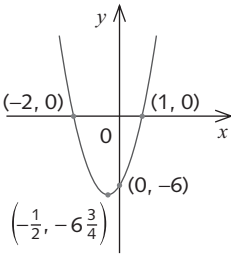
d



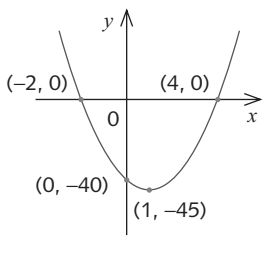
e



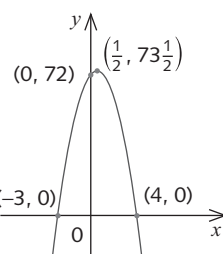
f



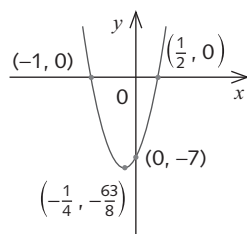
g



h



i



4

a -5, -1

b -4, -3

c -3, 6

d -5, 3

e -1/2, 10

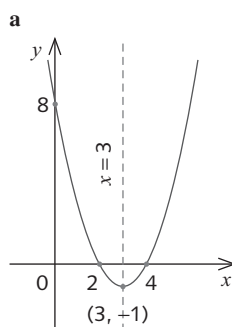
f -4, 4

g -1/2, 1/2

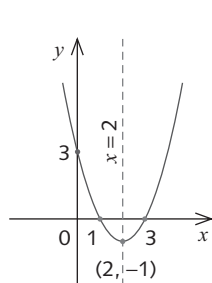
h 0, 3

i -4, 0

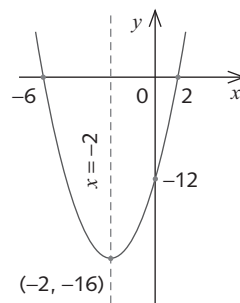
5



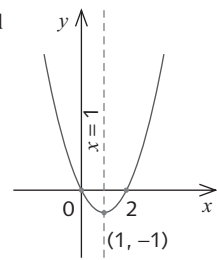
b



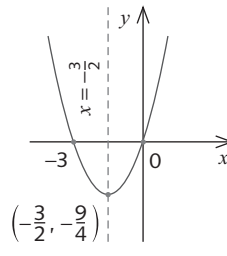
c



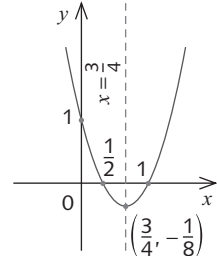
d



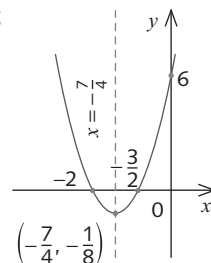
e



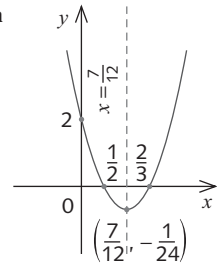
f



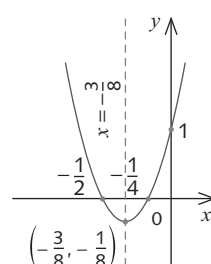
g



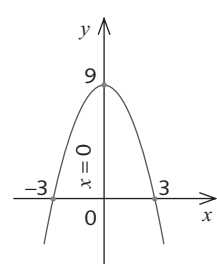
h



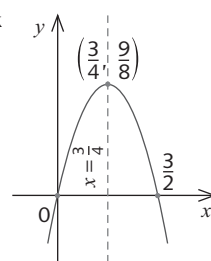
i



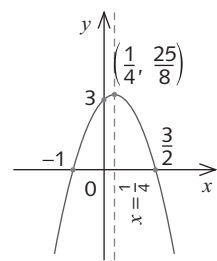
j



k



l



6

a $y = x^2 - 4x$

b $y = 9 - x^2$

c $y = -3x^2 + 6x + 9$

d $y = -\frac{3x^2}{5} - \frac{9x}{5} + 6$

7

$y = x^2 - x - 2$

8

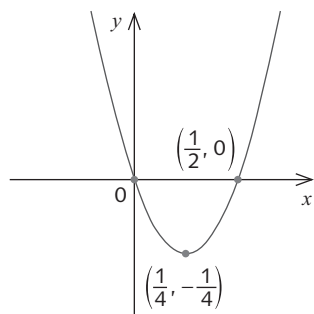
$y = \frac{1}{2}x^2 - \frac{x}{2} - 3$

9

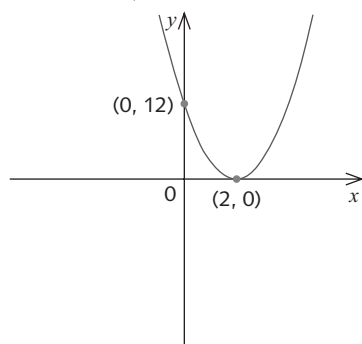
$y = 6x^2 - 15x + 6$

Exercise 7E

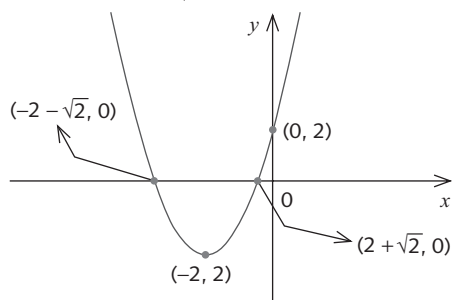
a



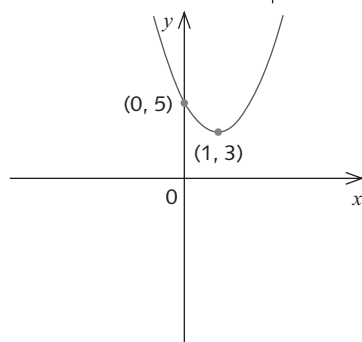
b



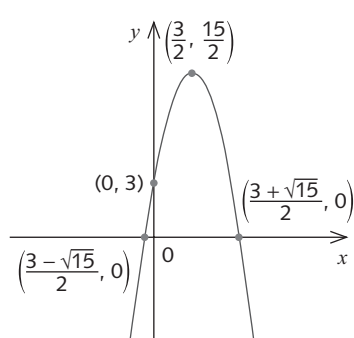
c



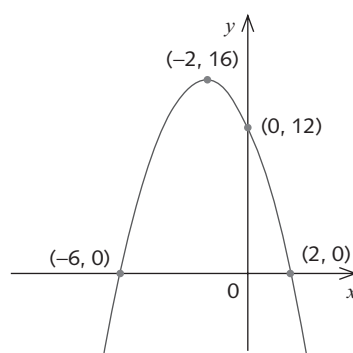
d



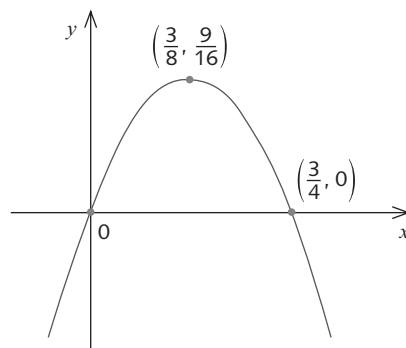
e



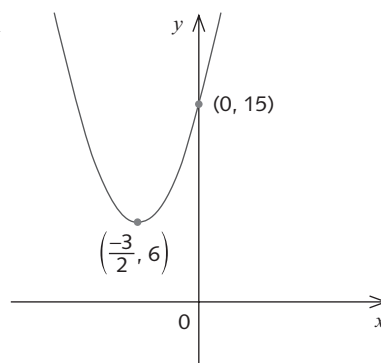
f



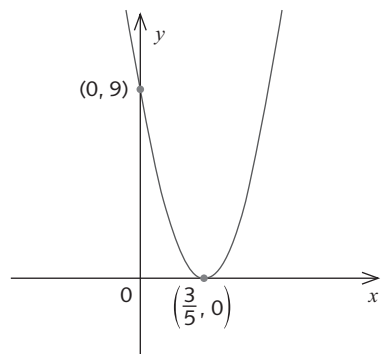
g



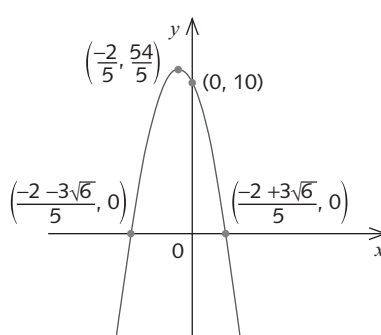
h

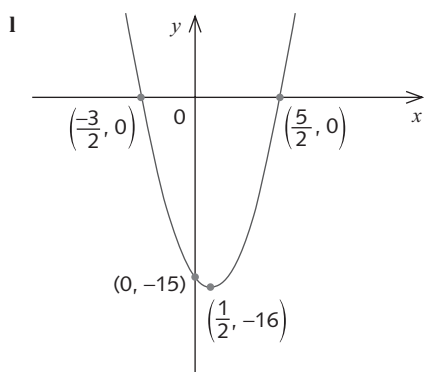
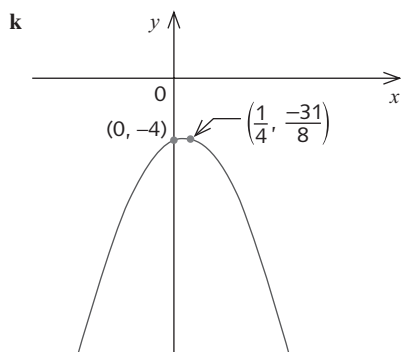


i



j





Exercise 7F

- | | | |
|--|------------------|-----------------|
| 1 80 m, 130 m | 2 60 cm | 3 25, 26 |
| 4 14, 16 | 5 13, 15 | 6 3 |
| 7 8 years | 8 \$6 | |
| 9 8 cm, 15 cm, 17 cm | | |
| 10 20 cm, 21 cm, 29 cm | | |
| 11 5 cm, 30 cm, 40 cm | | |
| 12 44 sides | 13 16 | 14 1.84 seconds |
| 15 -7 | 16 $\frac{5}{4}$ | |
| 17 a max = 4, min = $-\frac{73}{12}$ | | |
| b max = 46, min = -2 | | |
| 18 25 cm by 25 cm, 17 cm | | |
| 19 75 m by 150 m | | |
| 20 31.888 m, 5.102 seconds | | |
| 21 62.5 m by 80 m, or 160 m by 31.25 m | | |
| 22 $3\frac{3}{8}$ square units | | |

Exercise 7G

- | | |
|---------------------|---|
| 1 a $-3 < x < 5$ | b $x < -4$ or $x > 1$ |
| c $x = -2$ | d $x \leq -1$ or $x \geq 2$ |
| e all values of x | f $x < -\frac{3}{2}$ or $x > \frac{3}{2}$ |

- | | |
|------------------------------|-----------------------|
| 2 a $x < -2$ or $x > 3$ | b $-4 \leq x \leq -1$ |
| c $x \leq 2$ or $x \geq 5$ | d $-3 < x < 0$ |
| 3 a $x < -10$ or $x > 7$ | b $-3 < x < 8$ |
| c $x \leq -5$ or $x \geq -4$ | d $3 \leq x \leq 4$ |
| 4 a $-8 < x < 5$ | b $-3 \leq x \leq 8$ |
| c $x \leq 5$ or $x \geq 7$ | d $x < 0$ or $x > 11$ |

Review exercise

- | | | |
|--|--------------------------------|-------------------------------|
| 1 a 2 | b 24 | c 0 |
| d 0 | e 4 | f -16 |
| 2 a 0 | b ± 4 | |
| c ± 3 | d ± 2 | |
| 3 a -4, 1 | b $-6, -\frac{1}{2}$ | c $-\frac{3}{4}, \frac{3}{2}$ |
| d $-\frac{1}{2}, \frac{5}{2}$ | e -7, 7 | f 0, 5 |
| 4 a $-2 - \sqrt{5}, -2 + \sqrt{5}$ | b $3 - \sqrt{2}, 3 + \sqrt{2}$ | |
| c $-1 - \sqrt{5}, -1 + \sqrt{5}$ | d $2 - \sqrt{5}, 2 + \sqrt{5}$ | |
| e $3 + \frac{\sqrt{35}}{5}, 3 - \frac{\sqrt{35}}{5}$ | f $2 + \sqrt{2}, 2 - \sqrt{2}$ | |
| 5 a $-2 - \sqrt{6}, -2 + \sqrt{6}$ | | |
| b $3 - 2\sqrt{2}, 3 + 2\sqrt{2}$ | | |
| c $-\frac{5}{2} - \frac{\sqrt{19}}{2}, -\frac{5}{2} + \frac{\sqrt{19}}{2}$ | | |
| d $-2 - \frac{\sqrt{26}}{2}, -2 + \frac{\sqrt{26}}{2}$ | | |
| 6 a minimum | b maximum | |
| c maximum | d minimum | |
| 7 $y = x^2$ is congruent to $y = x^2 - x$, | | |
| $y = -2x^2$ is congruent to $y = 3 - 2x^2$, | | |
| $y = 3x^2$ is congruent to $y = 3x^2 + 1$, | | |
| $y = 2 + 3x - 4x^2$ is congruent to $y = 1 + 4x^2$ | | |
| 8 a translate 1 unit down | | |
| b translate 2 units up | | |
| c reflect in the x -axis then translate 4 units up | | |
| d reflect in the x -axis then translate 1 unit up | | |
| (There are other correct answers to this question.) | | |
| 9 a translate 2 units left | | |
| b translate 1 unit right | | |
| c reflect in the x -axis then translate 1 unit left | | |
| d translate 1 unit left and 3 units down | | |
| e translate 2 units right and 3 units down | | |
| f reflect in the x -axis then translate 3 units right and 1 unit up | | |
| (There are other correct answers to this question.) | | |
| 10 a $y = (x + 2)^2$ | b $y = (x - 3)^2 + 1$ | |
| c $y = (x - 5)^2 - 2$ | d $y = (x + 3)^2 - 8$ | |

11 a $y = 3(x+3)^2 + 2$

b $y = 3(x-3)^2 - 2$

12 a $y = -(x-1)^2$

b $y = -(x+2)^2$

c $y = -(x+1)^2 - 2$

13 a (1, 2)

c (-4, -2)

e (-2, -1)

b (-2, 3)

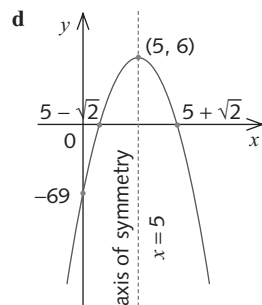
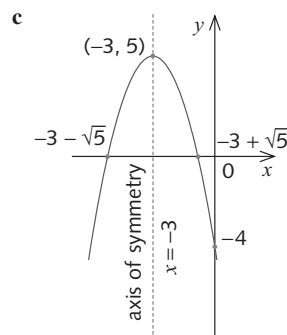
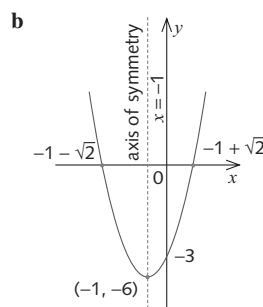
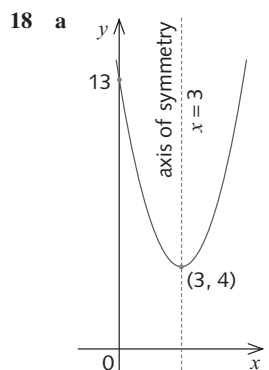
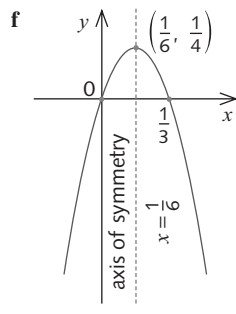
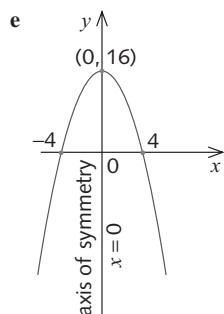
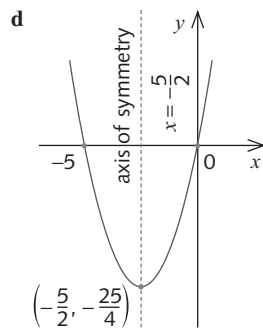
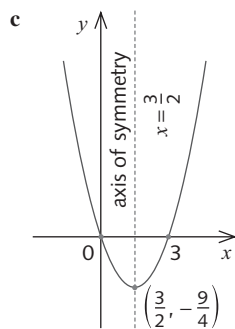
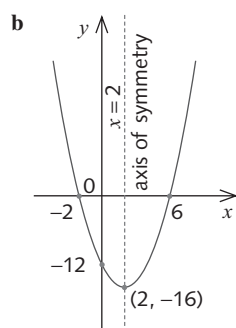
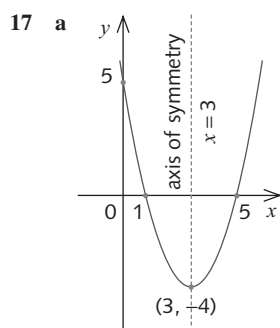
d (5, 11)

f (3, 4)

14 $y = x^2 - 2x - 1$

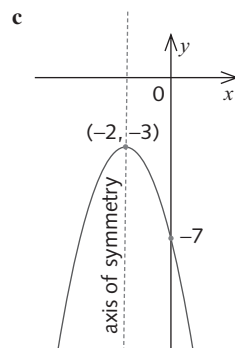
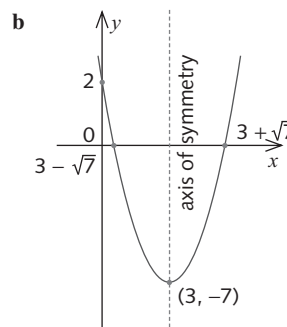
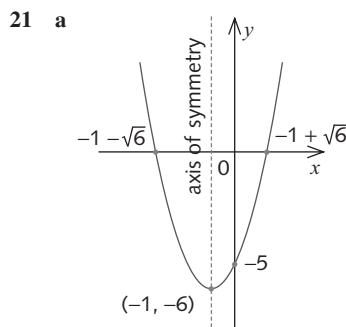
15 $y = x^2 + 2x - 15$

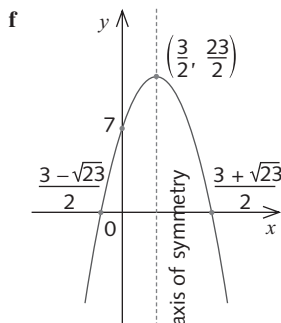
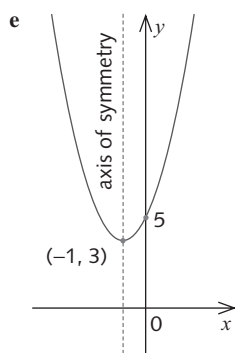
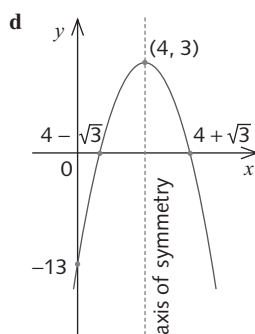
16 $y = -x^2 - 6x - 8$



19 $y = 11(x-2)^2 - 4$

20 $h = 1$ or $h = -1$





22 The side lengths are 5 cm, 12 cm and 13 cm.

23 52 cm, 42 cm and 4 cm

24 125 m by 250 m

25 a $-3 < x < 5$

b $-5 \leq x \leq -2$

c $x < 0$ or $x > 2$

Challenge exercise

1 a -5

b -5

2 a -21

b 4

c $c > 4$

3 d $x = 80, t = \frac{3}{2}; x = 90, t = \frac{4}{3}$

4 40 km/h

5 45 cattle

6 32 km/h, 40 km/h

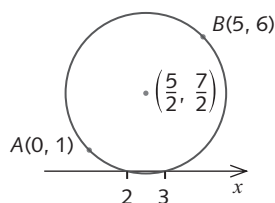
7 75 runs

8 4.844 m/s, 5.844 m/s

9 40 km/h, 60 km/h

10 $a = \frac{24}{5}$

11 a $\left(x - \frac{5}{2}\right)^2 + \left(y - \frac{7}{2}\right)^2 = \frac{25}{2}$



b Let $A(0, 1)$ and $B(g, h)$ be the given points and $P(x, y)$ be any point. P is on the circle with

diameter AB if $\angle APB = 90^\circ$ (angle in semicircle).

If gradient of $AP \times$ gradient of $BP = -1$, then

$$\frac{y-1}{x} \times \frac{y-h}{x-g} = 1.$$

So $(y-1)(y-h) + x(x-g) = 0$ is the equation of the circle. This meets the x -axis when $y = 0$. That is when $x^2 - gx + h = 0$.

12 a $\frac{x^2}{4\pi} + \frac{(100-x)^2}{16}$

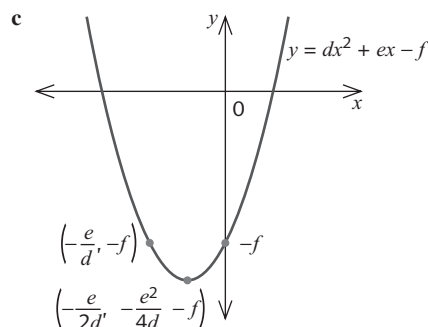
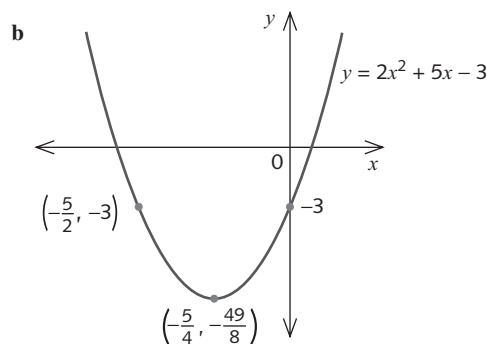
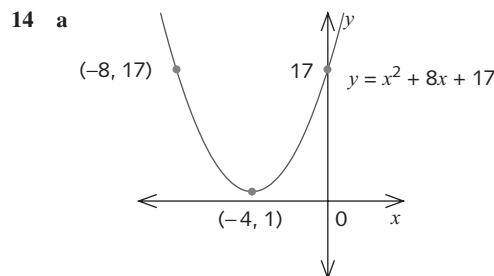
b The minimum area is $\frac{2500}{\pi+4} \text{ cm}^2$ when $x = \frac{100\pi}{\pi+4}$

13 d If (x, y) goes to $(x, 5y)$, then (a, a^2) goes to $(a, 5a^2)$. The graph of $y = x^2$ goes to the graph of $y = 5x^2$. $(0, 0)$ goes to $(0, 0)$, $(1, 0)$ goes to $(1, 0)$, and $(0, 1)$ goes to $(0, 5)$. It is not a similarity transformation since some distances are unchanged and others multiplied by 5. Note that 5 can be replaced by any $a > 0$.

e The enlargement with centre at $(0, 0)$ and enlargement factor $\frac{1}{5}$ maps $A(a, a^2)$ to $B\left(\frac{a}{5}, \frac{a^2}{5}\right)$. Since $\frac{a^2}{5} = 5\left(\frac{a}{5}\right)^2$,

B lies on $y = 5x^2$. That is $y = x^2$ is similar to $y = 5x^2$.

f By **c** $y = \pm ax^2 + bx + c$ is congruent to $y = ax^2$, $a > 0$. By **d** $y = ax^2$ is similar to $y = x^2$. That is, all parabolas are similar.



Chapter 8 answers

Exercise 8A

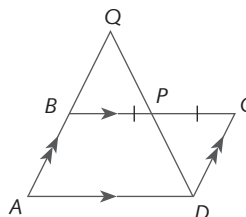
- 1 a $y = 6$, $\beta = 45^\circ$ b $x = 5$, $\alpha = 30^\circ$
c $\alpha = \gamma = 60^\circ$, $\beta = 120^\circ$, $x = 6$ d $\alpha = 90^\circ$, $\beta = 60^\circ$
e $x = 4$, $\alpha = 45^\circ$ f $\alpha = 60^\circ$, $x = 5$
- 2 a $\alpha = \beta = \gamma = 60^\circ$ (equilateral $\triangle RST$)
b $\theta = 20^\circ$ (angle sum of $\triangle WXY$),
 $x = 12$ cm (isosceles $\triangle WXY$)
c $\angle RQP = 60^\circ$ (base angle of isosceles $\triangle PQR$),
 $\theta = 60^\circ$ (angle sum of $\triangle PQR$),
 $y = 7$ cm (equilateral $\triangle RPQ$)
d $y = 5$, $\alpha = 60^\circ$ (equilateral $\triangle LNQ$),
 $\alpha = 2\beta$ (exterior angle of isosceles $\triangle LQM$), $\beta = 30^\circ$
e $\angle ABD = 55^\circ$ (straight angle at B),
 $\angle BAD = 55^\circ$ (base angle of isosceles $\triangle ABD$),
 $\angle ADB = 70^\circ$ (angle sum of $\triangle ABD$),
 $\theta = 70^\circ$ (vertically opposite angles at D),
 $\angle AGE = 55^\circ$ (alternate angles, $AC \parallel HE$),
 $\alpha = 125^\circ$ (straight angle at G)
f $\angle DCG = 55^\circ$ (vertically opposite angles at C),
 $\angle CDG = 55^\circ$ (base angle of isosceles $\triangle CDG$),
 $\alpha = 70^\circ$ (angle sum of $\triangle CDG$),
 $\angle AGE = 55^\circ$ (corresponding angles, $BD \parallel HE$),
 $\angle EGD = 55^\circ + 70^\circ = 125^\circ$,
 $\beta = 125^\circ$ (vertically opposite angles at G)
g $\angle DAB = 50^\circ$ (complementary angles),
 $\alpha = 50^\circ$ (alternate angles, $AB \parallel CE$),
 $\angle AFD = 40^\circ$ (base angle of isosceles $\triangle ADF$),
 $\alpha + \angle FDC = 100^\circ$ (angle sum of $\triangle ADF$),
 $\angle FDC = 50^\circ$, $\theta = 130^\circ$ (straight angle at D)
h $y = 2$ m (radii of circle),
 $\beta = 25^\circ$ (base angle of isosceles $\triangle ABO$),
 $\alpha = 60^\circ$ (base angle of isosceles $\triangle BCO$), angle sum of $\triangle BCO$
i $3\alpha + \beta = 180^\circ$ (co-interior angles, $AD \parallel BC$),
 $\alpha + 2\beta = 180^\circ$ (angle sum of isosceles triangle),
 $\alpha = 36^\circ$, $\beta = 72^\circ$
j $\angle CBO = 2\alpha$ ($\triangle BOC$ isosceles),
 $\angle OBA = 90 - \frac{3\alpha}{2}$ (angle sum of isosceles triangle),
 $\angle OBC + \angle ABO = 95^\circ$ (alternate angles, $AB \parallel CD$),
 $\alpha = 10^\circ$
k $\alpha = 130^\circ$ (angle sum of quadrilateral)
l $\alpha = 108^\circ$ (angle sum of regular pentagon)
m $\alpha = 120^\circ$ (co-interior angles, $AB \parallel DC$),
 $\beta = 60^\circ$ (co-interior angles, $BC \parallel AD$)
- 3 6 sides 4 36°

Exercise 8B

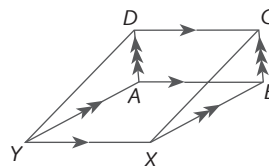
- 1 a $\triangle BAC \equiv \triangle YXZ$ (SAS) b $\triangle ABC \equiv \triangle XZY$ (ASA)
c $\triangle ABC \equiv \triangle PQR$ (SSS) d $\triangle ABC \equiv \triangle XYZ$ (ASA)
e $\triangle PQR \equiv \triangle XYZ$ (ASA)
- 2 a $\angle BCA = 40^\circ$, $\angle FED = 100^\circ$, $\angle FDE = 40^\circ$,
 $AB = BC = 3$ cm, $FD = 4.6$ cm
b $\angle ABC = 10^\circ$, $\angle EFD = 130^\circ$, $CB = 4.2$ cm,
 $AC = 1.1$ cm, $ED = 5$ cm
c $\angle FED = 32^\circ$, $\angle CAB = 111^\circ$, $\angle BCA = 37^\circ$,
 $AB = 9.1$ cm, $AC = 8$ cm, $FE = 14.1$ cm

- d $\angle FED = 67^\circ$, $\angle CAB = 67^\circ$, $\angle ABC = 67^\circ$,
 $\angle BCA = 46^\circ$, $CB = 38.3$ mm, $AC = 38.3$ mm,
 $FE = 38.3$ mm, $ED = 30$ mm

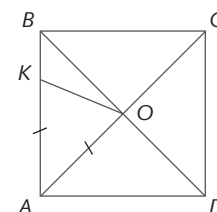
- 3 a $AD = BC$ (equal side lengths of square), $DE = CE$ (given),
 $\angle ADE = \angle BCE$ (given),
 $\triangle ADE \equiv \triangle BCE$ (SAS)
b $AE = BE$ (matching sides in congruent triangles)
- 4 a $PR = RS$ (equal side lengths of square), $PX = ST$
(half side lengths of square),
 $\angle RPX = \angle RST = 90^\circ$,
 $\triangle RPX \equiv \triangle RST$ (SAS),
 $RX = RT$ (matching sides in congruent triangles),
b $RX = RT$ (from a),
 $VR = VR$ (common side),
 $VT = VX$ (half side lengths of square)
 $\triangle RXV \equiv \triangle RTV$ (SSS),
 $\angle TRV = \angle XRV$ (matching angles of congruent triangles)
- 5 a Diagonals of a rectangle are equal and bisect each other.
b i $\angle AOD = 56^\circ$ ii $\angle AOB = 124^\circ$
iii $\angle OBC = 62^\circ$ iv $\angle ABO = 28^\circ$
- 6 $\angle YDO = \angle OBX$ (alternate angles, $AB \parallel CD$),
 $\angle BOX = \angle DOY$ (vertically opposite),
 $OB = OD$ (diagonals of a parallelogram bisect each other),
 $\triangle BOX \equiv \triangle DOY$ (AAS),
 $OX = OY$ (matching sides of congruent triangles)
- 7 $BP = PC$ (P is the midpoint of BC),
 $\angle QPB = \angle CPD$ (vertically opposite),
 $\angle PQB = \angle CDP$ (alternate angles, $AQ \parallel DC$),
 $\triangle BQP \equiv \triangle CDP$ (AAS),
 $BQ = CD$ (matching sides of congruent triangles),
 $CD = AB$ (opposite sides of parallelogram), $AQ = 2AB$



- 8 $DC = AB = YX$ (opposite sides of parallelogram),
 $DC \parallel AB \parallel YX$ (opposite sides of parallelogram)
Therefore, $DCXY$ is a parallelogram (opposite sides equal and parallel).



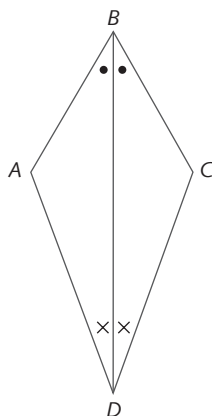
- 9 $\angle AKO = \angle AOK$ (isosceles triangle AKO),
 $\angle KAO = 45^\circ$ (diagonals of a square bisect the vertex angles),
 $\angle AKO = \angle AOK = 67\frac{1}{2}^\circ$ (angle sum of triangle),
 $\angle BOA = 90^\circ$ (diagonals of a square intersect at right angles),
 $\angle KBO = 45^\circ$ (diagonals of a square bisect the vertex angles),
 $\angle BOK = 22\frac{1}{2}^\circ$, $\angle AOK = 3\angle BOK$



- 10 a $\triangle DBC \equiv \triangle DBA$ (AAS)

$AB = CB$ (matching sides of congruent triangles)

$DA = DC$ (matching sides of congruent triangles)

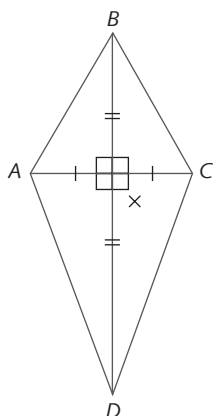


- b $\triangle AXB \equiv \triangle CXB$ (SAS)

So $AB = CB$ (matching sides of congruent triangles)

$\triangle CDX \equiv \triangle ADX$ (SAS)

$AD = CD$ (matching sides of congruent triangles)



- 11 $\angle DAC = \angle BCA$ (alternate angles, $AD \parallel BC$)

$\angle ACD = \angle CAB$ (alternate angles, $AB \parallel CD$)

$\triangle ABC \equiv \triangle CDA$ (SAS)

$CD = BA$ (matching sides of congruent triangles)

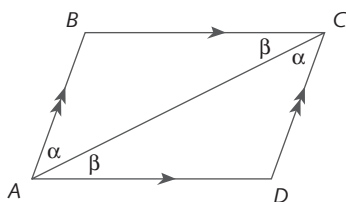
$AD = BC$ (matching sides of congruent triangles)

Thus, opposite sides are equal.

$\angle ABC = \angle CDA$ (matching angles of congruent triangles)

$\angle BAD = \alpha + \beta = \angle OCB$

Thus, opposite angles are equal.



- 12 Let X be the point of intersection of diagonals BD and AC .

We use the results of Question 11.

$AD = CB$

$\angle DAC = \angle BCA$ (alternate angles, $BC \parallel AD$)

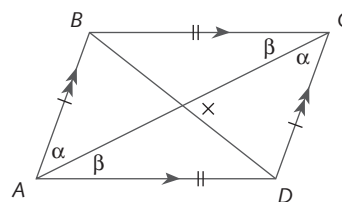
$\angle BXC = \angle DXA$ (vertically opposite)

$\angle BXC \equiv \angle DAX$ (AAS)

$CX = AX$ (matching sides of congruent triangles)

$DX = BX$ (matching sides of congruent triangles)

Thus, the diagonals of a parallelogram bisect each other.



- 13 A rhombus is a parallelogram.

$BC = CD$ (all sides of a rhombus are equal)

$BX = DX$ (diagonals of a parallelogram bisect each other)

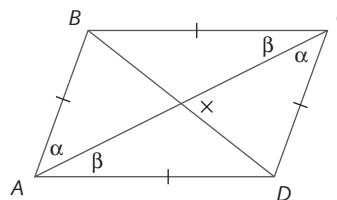
$\triangle BXC \equiv \triangle DXC$ (SSS)

$\angle BXC = \angle DXC$ (matching angles of congruent triangles)

$\angle BXC + \angle DXC = 180^\circ$

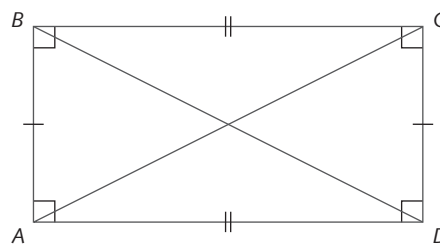
So $\angle BXC = \angle DXC = 90^\circ$

Thus, diagonals of a rhombus are perpendicular.



- 14 $\triangle BAD \equiv \triangle DCB$ (SAS)

$BD = AC$ (matching sides of congruent triangles)



- 15 Let $\angle ABK = \angle DBK = \alpha$

Then $\angle CBD = 2\alpha$

(diagonals of rhombus bisect the vertex angles)

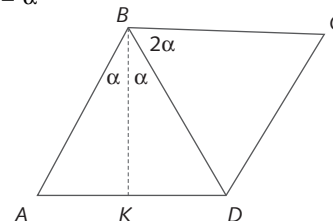
$\angle BAK = 180^\circ - 4\alpha$

(co-interior angles $AD \parallel BC$)

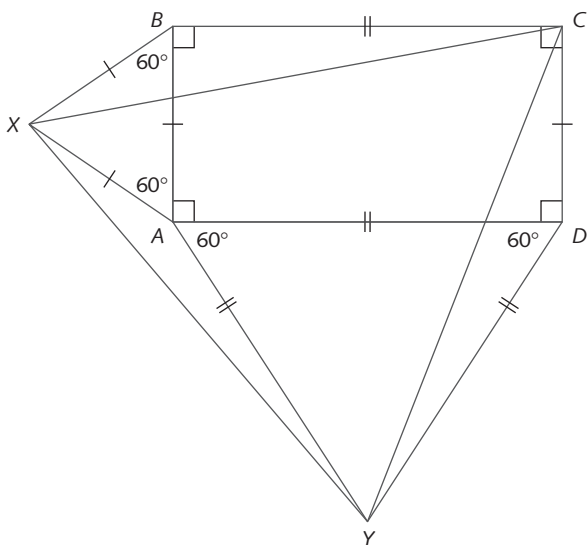
$\angle BKA = 180^\circ - (180^\circ - 4\alpha) - \alpha$ (angle sum of triangle)

$= 3\alpha$

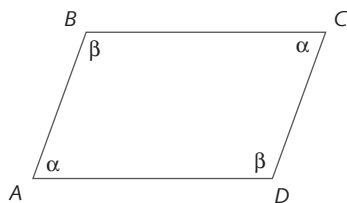
$= 3\angle ABK$



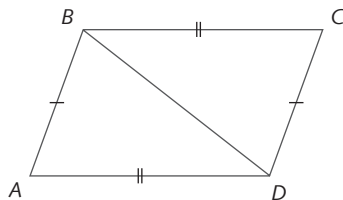
- 16 $\angle CBX = \angle CDY = 90^\circ + 60^\circ = 150^\circ$
 $\angle XAY = 360^\circ - (90^\circ + 60^\circ + 60^\circ) = 150^\circ$
 $\triangle XBC \equiv \triangle CDY \equiv \triangle XAY$ (SAS congruence test)
Hence, $XC = CY = YX$ (matching sides of congruent triangles)



- 17 $2\alpha + 2\beta = 360^\circ$ (angle sum of quadrilateral $ABCD$)
 $\alpha + \beta = 180^\circ$
so $\angle ABC + \angle BAD = 180^\circ$
so $\angle BCD + \angle ADC = 180^\circ$
Thus, $AB \parallel DC$ and $BC \parallel AD$ (co-interior angles are supplementary).



- 18 $BC = AD$ and $AB = DC$
 $\triangle BCD \equiv \triangle DAB$ (SSS)
Hence,
 $\angle ABD = \angle CDB$ (matching angles of congruent triangles)
 $\angle ADB = \angle CBD$ (matching angles of congruent triangles)
Hence, $AB \parallel CD$ (alternate angles are equal)
and $BC \parallel AD$ (alternate angles are equal)



Exercise 8C

- $\triangle ABC$ is similar to $\triangle RQP$ (SSS)
 - $\triangle LMN$ is similar to $\triangle QRP$ (AAA)
 - $\triangle STU$ is similar to $\triangle XZY$ (AAA)
 - $\triangle LMN$ is similar to $\triangle TSU$ (SAS)
 - $\triangle BCA$ is similar to $\triangle ECD$ (AAA)
 - $\triangle GHF$ is similar to $\triangle JHI$ (SAS)
 - $\triangle OMK$ is similar to $\triangle LMN$ (AAA)
 - $\triangle STR$ is similar to $\triangle SUQ$ (AAA)
 - $\triangle EDC$ is similar to $\triangle ADB$ (SAS)
 - $\triangle IFG$ is similar to $\triangle IHF$ is similar to $\triangle FHG$ (AAA)
- AAA
 - $x = 3.5$
- SAS
 - $y = 14.4$
- $\triangle JKL$ is similar to $\triangle IGH$ (SSS)
 - $\alpha = 49^\circ$, $a = \beta$, $b = 131 - \beta$
- $\triangle DEC$ is similar to $\triangle DBA$ (SAS)
 - $\frac{EC}{BA} = \frac{ED}{DB} = \frac{DC}{DA}$
 - $x = 2$
- $\angle ACB = \angle DCE$, $\angle CAB = \angle CED$, $\angle ABC = \angle EDC$
 - 3.75 cm
 - 7.2 cm
- $\triangle ADE$ is similar to $\triangle ACB$ (AAA)
 - $8\frac{4}{7}$ cm
- AAA
 - $LM = 16\frac{2}{3}$ cm
- 18.75 m
- 130 m
- 33 m
- $\angle DOC = \angle AOB$ (vertically opposite),
 $\angle ABO = \angle CDO$ (alternate angles, $AB \parallel CD$),
 $\triangle AOB$ is similar to $\triangle COD$ (AAA)
 $\frac{OB}{OD} = \frac{OA}{OC} = 2$, $OB = 2OD$
- $\angle BAD = \angle BAC$ (common angle),
 $\angle ADB = \angle ABC = 90^\circ$,
 $\triangle ABD$ is similar to $\triangle ACB$ (AAA)
 - $\angle BCD = \angle BCA$ (common angle),
 $\angle ABC = \angle BDC = 90^\circ$, $\triangle BCD$ is similar to $\triangle ACB$ (AAA)
- $\angle AEP = \angle CBP$ (alternate angles, $CB \parallel AD$),
 $\angle EPA = \angle BPC$ (vertically opposite),
 $\triangle APE$ is similar to $\triangle CPB$ (AAA)
 - $\frac{AP}{PC} = \frac{AE}{AD} = \frac{AE}{CB}$ (ratios of matching sides of similar triangles) and $AD = CB$ (opposite sides of a parallelogram)
 $\frac{AD}{PC} = \frac{AE}{CD} = \frac{AE}{CB} = \frac{1}{2}$,
 $AP = 2PC$ and $AC = 3AP$
- $\angle EDB = 180^\circ - \angle ACB$ (given),
 $\angle ADE = \angle ACB$ (supplementary),
 $\angle BAC = \angle DAE$ (common),
 $\triangle ADE$ is similar to $\triangle ACB$ (AAA)
 - $\frac{AE}{AB} = \frac{AD}{AC}$ (equal ratios of matching sides of similar triangles)

- 16 a $\angle PDB = 90^\circ$ (given), $\angle AFB = 90^\circ$ (given)
 $\angle DBP = \angle ABF$ (common)
 $\triangle PBD$ is similar to $\triangle ABF$ (AAA)
 $\triangle ABF \cong \triangle ACF$ (RHS)
 So $\triangle PDB$ is similar to $\triangle ACF$
- b $\frac{FC}{DB} = \frac{AC}{PB}$ (matching sides of similar triangles)
- 17 $LM = \frac{1}{2} AC$ and $NM = \frac{1}{2} AB$. But $AC = AB$. Therefore,
 $LM = NM$.
- 18 a $\angle ABC = \angle ADB = 90^\circ$, $\angle BAC = \angle BAD$ (common),
 $\triangle ABC$ is similar to $\triangle ADB$ (AAA)
- b $\frac{b}{x} = \frac{a}{c} = \frac{c}{a-y}$. Therefore, $a(a-y) = c^2$ (1)
- c $\angle ABC = \angle BDC = 90^\circ$, $\angle BCA = \angle BCD$ (common),
 $\triangle ABC$ is similar to $\triangle BDC$ (AAA)
- d $\frac{c}{x} = \frac{b}{y} = \frac{a}{b}$, so $ay = b^2$ (2)
- e From (1) $a(a-y) = c^2$ and $a^2 = c^2 + ay$
 From (2) $ay = b^2$
 $a^2 = b^2 + c^2$

Exercise 8D

- 1 Let M be the midpoint of AB .

Let N be the midpoint of BC .

$$MB = \frac{1}{2} AB \text{ and } NB = \frac{1}{2} CB$$

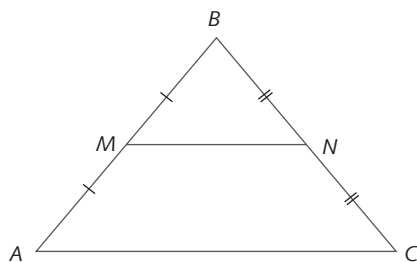
$$\text{Hence, } \frac{MB}{AB} = \frac{1}{2} \text{ and } \frac{NB}{CB} = \frac{1}{2}$$

$\triangle MBN$ is similar to $\triangle ABC$ (SAS)

Hence, $\angle BAC = \angle BMN$ (matching angles of similar triangles)

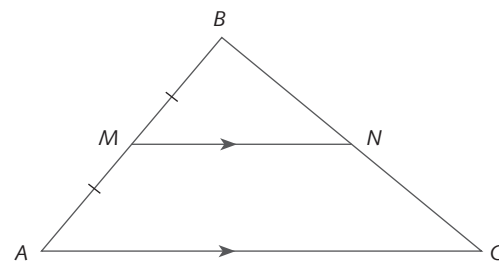
$MN \parallel AC$ (corresponding angles equal)

Also, $MN = \frac{1}{2} AC$ (similarity factor of $\frac{1}{2}$)



- 2 Let M be the midpoint of AB and $MN \parallel AC$.
 $\angle BMN = \angle BAC$ (corresponding angles, $MN \parallel AC$)
 $\triangle BMN$ is similar to $\triangle BAC$ (AAA)
 $BM = \frac{1}{2} BA$ (matching sides of similar angles)
 $\frac{BM}{BA} = \frac{BN}{BC} = \frac{1}{2}$

Thus, N is the midpoint of BC .



- 3 M is the midpoint of AB .

N is the midpoint of BC .

P is the midpoint of AC .

From question 1,

$$MN = \frac{1}{2} AC, MP = \frac{1}{2} BC, PN = \frac{1}{2} AB$$

Therefore,

$$NP = MB = AM$$

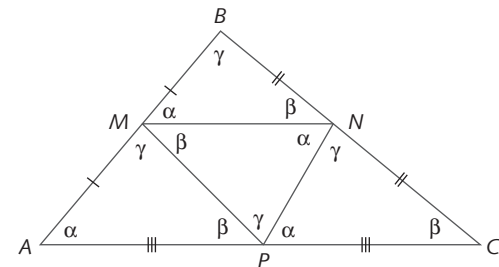
$$MN = AP = PC$$

$$MP = BN = NC$$

Hence, all the triangles are congruent by the SSS test.

All triangles have angles α , β and γ .

Therefore, each of the small triangles is similar to the large triangle.



- 4 Let M, N, Q and P be the midpoints of AB, BC, CD and DA respectively.

Join vertices B and D .

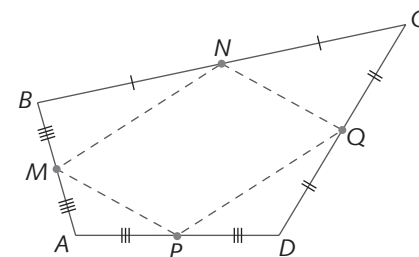
In $\triangle CBD$, $NQ \parallel BD$ (Question 1)

In $\triangle ABD$, $MP \parallel BD$ (Question 1)

Therefore, $NQ \parallel MP$.

In a similar way, $MN \parallel PQ$.

Thus, $MNPQ$ is a parallelogram.



- 5 In $\triangle PAQ$ and $\triangle BAC$

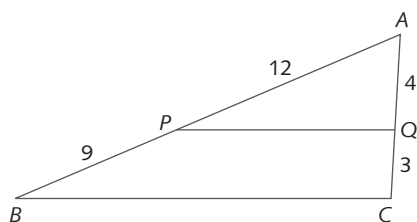
$\angle PAQ = \angle BAC$ (common)

$$\frac{BA}{PA} = \frac{AC}{AQ} = \frac{7}{4}$$

$\triangle PAQ$ is similar to $\triangle BAC$ (SAS)

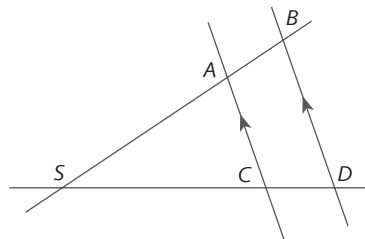
$\angle APQ = \angle ABC$ (matching angles of similar triangles)

$PQ \parallel BC$ (corresponding angles are equal)



- 6 a $\triangle SAC$ is similar to $\triangle SBD$

$$\begin{aligned}\frac{SA}{SA+AB} &= \frac{SC}{SC+CD} \\ SA \times (SC+CD) &= SC \times (SA+AB) \\ SA \times SC + SA \times CD &= SC \times SA + SC \times AB \\ SA \times CD &= SC \times AB \\ \frac{SA}{AB} &= \frac{SC}{CD} \\ SA:AB &= SC:CD\end{aligned}$$



- b $\triangle SAC$ is similar to $\triangle SBD$

$$\begin{aligned}\frac{SB}{SB-AB} &= \frac{SD}{SD-CD} \\ SB \times (SD-CD) &= SD \times (SB-AB) \\ SB \times SD - SB \times CD &= SD \times SB - SD \times AB \\ SB \times CD &= SD \times AB\end{aligned}$$

$$\frac{SB}{AB} = \frac{SD}{CD}$$

That is, $SB:AB = SD:CD$.

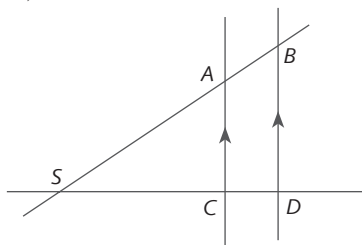
- c $\frac{SA}{SB} = \frac{SC}{SD}$ or $\triangle SAC$ is similar to $\triangle SBD$.
 $SA:SB = SC:SD$

7

$$\begin{aligned}\frac{SA}{AB} &= \frac{SC}{CD} \\ SA \times CD &= AB \times SC \\ SA \times SC + SA \times CD &= AB \times SC + SC \times SA \\ SA \times (SC+CD) &= SC \times (AB+SA) \\ SA \times SD &= SC \times SB \\ \frac{SA}{SB} &= \frac{SC}{SD}\end{aligned}$$

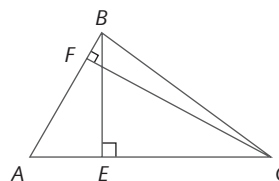
Therefore, $\triangle SAC$ is similar to $\triangle SBD$ (SAS).

Hence, $AC \parallel BD$.



Review exercise

- $\alpha = 37^\circ, \beta = 80^\circ, \gamma = 20^\circ$
- a $\triangle PCQ$ is similar to $\triangle ACB$
b $x = 5$
- a In $\triangle BEC$ and $\triangle FED$, $\angle CBE = \angle DFE$ (alternate angles, $BC \parallel AD$)
 $\angle CEB = \angle FED$ (vertically opposite),
 $\triangle BEC$ is similar to $\triangle FED$ (AAA)
b i 2 ii 4
- a $\alpha = 112^\circ$
b $\alpha = 78^\circ, \beta = 58^\circ$
c $\alpha = 20^\circ$
d $\beta = 104^\circ, \alpha = 76^\circ, \gamma = 52^\circ$
e $\alpha = 36^\circ, \beta = 144^\circ$
f $\alpha = 130^\circ, \beta = 50^\circ, \gamma = 130^\circ$
- a $\angle BAC = 70^\circ, \angle ACB = 55^\circ$
b $\angle FIC = 55^\circ$ (corresponding angles, $FI \parallel AB$).
 $\angle FCI = 55^\circ$. So $\triangle FIC$ is isosceles.
c $\angle DIB = 55^\circ$ (corresponding angles, $AC \parallel DI$). $\triangle DBI$ is isosceles (equal base angles), $DB = DI$
d $\angle GIF = 53^\circ, \angle EFG = 17^\circ$
- a $\triangle ONM \equiv \triangle ZYX$ (AAS)
b $\triangle ABC \equiv \triangle RTS$ (RHS)
- $\triangle AOB$ is similar to $\triangle COD$ (AAA).
Ratio of matching sides gives: $\frac{BO}{OD} = \frac{AO}{OC}$
- In $\triangle CFA$ and $\triangle BEA$,
 $\angle CFA = \angle BEA = 90^\circ$ (altitudes),
 $\angle FAC = \angle EAB$ (common),
 $\triangle CFA$ is similar to $\triangle BEA$ (AAA)
 $\frac{BE}{CF} = \frac{AB}{AC}$ (ratios of matching sides)



- a $x = \frac{102}{11}$ b $x = \frac{21}{4}$
- a $x = 13$ b $EF = \frac{119}{13}$
- 20 cm

Challenge exercise

Only outlines of proofs are given in the challenge exercise

- a 272 cm b $50\sqrt{55}$ cm
c $16\sqrt{55}$ cm d approximately 67 cm
- Produce AY to Z . Let $\angle BAY = \alpha$, $\angle AZC = \alpha$ (alternate angles, $BA \parallel ZC$)
Then $\angle YAC = \alpha$ (angle bisector), and $\triangle ZCA$ is isosceles
So $ZC = CA = 2OA$
Next, $\triangle AOX$ is similar to $\triangle ZCY$ (AAA)

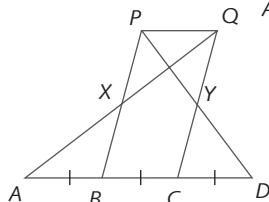
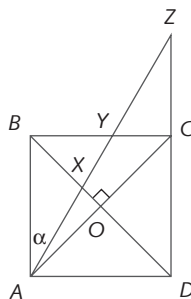
$$\frac{CY}{OX} = \frac{ZC}{OA} = 2 \text{ or } CY = 2OX$$

- 3 $PQ = BC = AB = CD = x$ and
 $BP = CQ = 2x$

$\triangle AXB \equiv \triangle QXP$ (AAS)

$\therefore PX = XB = x = QY = YC$

$PQYX$ is a rhombus and XQ and PY are diagonals of a rhombus. Hence,
 $AQ \perp DP$.

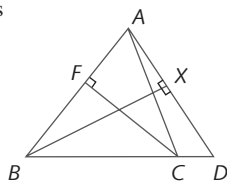


- 4 Show that $\angle BAD = \angle BDA = \angle ACF = 54^\circ$.

Hence, $\triangle ABD$ is isosceles with $AB = BD$.

Draw the perpendicular from B to AD . Let X be the point where the perpendicular meets AD . X is the midpoint of AD .

$\triangle AFC \equiv \triangle BXD$ (AAS). Hence,
 $FC = XD$ and $AD = 2FC$.



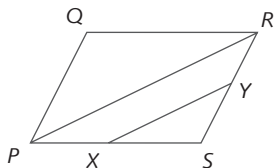
- 5 $\triangle SXY$ is similar to $\triangle SPR$.

$$SX = \frac{1}{2} PS, SY = \frac{1}{2} SR$$

Therefore, area of $\triangle SXY = \frac{1}{4}$ area of $\triangle SPR$

Therefore, area of $\triangle SPR = \frac{1}{2}$ area of parallelogram $PQRS$

Therefore, area of $\triangle SXY = \frac{1}{8}$ area of parallelogram $PQRS$



- 6 a $\angle BAD = \angle BEC$ (corresponding angles, $AD \parallel EC$)
 $\angle DAC = \angle ACE$ (alternate angles, $AD \parallel EC$)
 $\triangle BAD$ is similar to $\triangle BEC$ (AAA)
 $\triangle AEC$ is isosceles; $AC = AE$ (base angles of $\triangle AEC$)

$$\frac{BD}{BD + DC} = \frac{BA}{BA + AE}$$

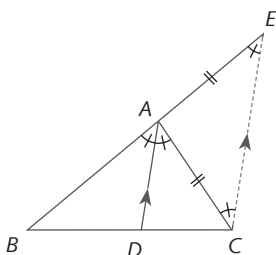
$$\frac{BD}{BD + DC} = \frac{BA}{BA + AC} \quad (AC = AE)$$

$$BD(BA + AC) = BA(BD + DC)$$

$$BD \times BA + BD \times AC = BA \times BD + BA \times DC$$

$$BD \times AC = BA \times DC$$

$$\frac{BD}{DC} = \frac{BA}{AC}$$



- b From a, $\frac{BC}{CD} = \frac{BE}{ED}$ and $\frac{BA}{AD} = \frac{BE}{ED}$.

$$\text{Therefore, } \frac{BC}{CD} = \frac{BA}{AD}$$

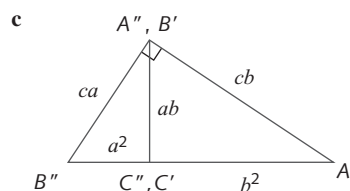
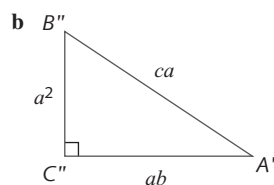
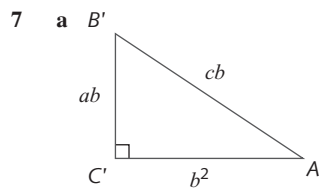
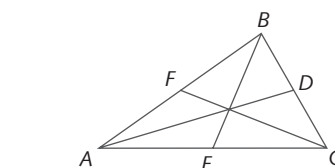
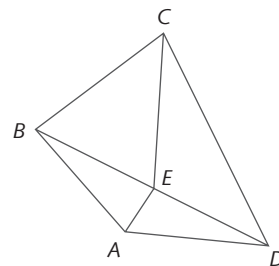
$$\text{Therefore, } \frac{AD}{AB} = \frac{CD}{CB}$$

- c From a, $\frac{AF}{FB} = \frac{AC}{BC}$

$$\frac{BD}{DC} = \frac{BA}{CA}$$

$$\frac{AE}{EC} = \frac{AB}{BC}$$

$$\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = \frac{BA}{CA} \times \frac{BC}{AB} \times \frac{AC}{BC} = 1$$

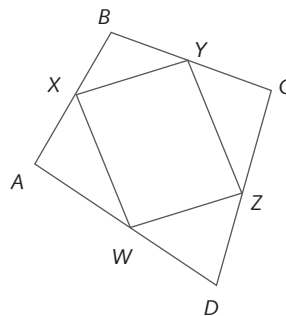


- d $\angle B''A''A'$ is a right angle. The new triangle is similar to triangle ABC .

e c f $a^2 + b^2 = c^2$

- 8 $XY = \frac{1}{2} AC = WZ$ and $YZ = \frac{1}{2} BD = XW$.

Therefore, $XYZW$ is a parallelogram. Diagonals bisect each other.



Chapter 9 answers

Exercise 9A

- | | | |
|--------------------------|-----------------------------------|----------------------------|
| 1 a 16 | b 125 | c 64 |
| d 27 | e 10 000 | f 216 |
| 2 a 2^3 | b 2^6 | c 3^4 |
| d 2^5 | e 5^4 | f 3^5 |
| 3 a 13^{-1} | b 7^{-2} | c 13^{-3} |
| d 2^{-10} | e 3^{-4} | f 11^{-10} |
| 4 a a^{15} | b a^{11} | c m^{15} |
| d p^{11} | e a^6b^7 | f m^9n^6 |
| g $8a^5b^5$ | h $15x^8y^4$ | i $15x^8y^9$ |
| 5 a $\frac{x}{y}$ | b $\frac{3xy^2}{2}$ | c a^4m^3 |
| d $\frac{5xy}{4}$ | e $\frac{2c^2}{3b^2}$ | f $\frac{4x^3}{y}$ |
| 6 a a^6b^{12} | b $x^{21}y^{35}$ | c $a^5b^{10}c^{15}d^{20}$ |
| d $4a^6b^2$ | e $9a^4b^8$ | f $64a^9b^6$ |
| 7 a $18m^{12}$ | b p^4 | c a^6b^3 |
| d m^5n | e a^5b^2 | f $p^{12}q$ |
| 8 a $\frac{1}{2}$ | b $\frac{1}{4}$ | c $\frac{1}{3}$ |
| d $\frac{1}{9}$ | e $\frac{1}{1000}$ | f $\frac{8}{7}$ |
| g $\frac{14}{15}$ | h $\frac{25}{9}$ | i $\frac{27}{8}$ |
| j $\frac{1331}{125}$ | k $\frac{1}{81}$ | l 6 |
| m 4 | n 1 | o 1 |
| 9 a $\frac{1}{2x^2y}$ | b $\frac{1}{9x^4y^4}$ | c $\frac{y^3}{64x^3}$ |
| d $\frac{y^6}{8x^6}$ | e $\frac{x^6y^6}{27}$ | f $4x^{10}y^{10}$ |
| 10 a $\frac{1}{m}$ | b $\frac{8}{a^4b^3}$ | c $\frac{15p^3}{q^5}$ |
| d $\frac{3p^{11}}{2q^6}$ | e $\frac{y}{2x^6}$ | f $\frac{x^4}{16}$ |
| g $\frac{1}{a^4b^9}$ | h m^7n^6 | i $\frac{p^8n^2}{m^4}$ |
| j $\frac{a^2}{b^8c^2}$ | k $\frac{4ab^5}{3}$ | l $\frac{1}{m^9n^{17}}$ |
| m $\frac{x^7}{y^7}$ | n a^3b^{11} | o $\frac{2a^{15}}{b^8c^3}$ |
| p mn^3p^9 | q $\frac{a^8}{b^7}$ | r $\frac{8a^{14}}{b^6}$ |
| 11 a $\frac{1}{27}$ | b $\frac{1}{64}$ | c $-\frac{1}{64}$ |
| 12 $\frac{7}{208}$ | 13 $\frac{13}{3}$ | |
| 14 a $-\frac{x}{y}$ | b $\frac{1}{x^2} - \frac{1}{y^2}$ | |
| c $\frac{3x^2y^2}{x+y}$ | d $\frac{x^2y+xy^2}{x^2+y^2}$ | |

$$\text{e } \frac{x^4}{1+2x^2y+x^4y^2} \quad \text{f } \frac{x^2y}{x^2+y}$$

Exercise 9B

- | | | |
|---|------------------------------------|----------------------------------|
| 1 a 6.3×10^1 | b 4×10^{-1} | c 6.2×10^{-1} |
| d 7.4×10^3 | e 2.1×10^7 | f 2.6×10^{-4} |
| g -8.6×10^{-2} | h 2×10^{12} | i 9.1345×10^{-5} |
| j 5.732×10^4 | k 3.012×10^{-3} | l $1.000\,51 \times 10^{-1}$ |
| 2 a 2.25×10^7 | b $6.700 \times 10^{-7} \text{ m}$ | c $1.5 \times 10^{11} \text{ m}$ |
| 3 a 6400 | b 92 000 | c 0.048 |
| d 0.0087 | e 7 412 000 | f -402 |
| g -0.004 657 | h 47.26 | |
| 4 a 8×10^5 | b 1×10^6 | c 1.26×10^8 |
| d 2.04×10^6 | e 2×10^{-4} | f 4×10^{-6} |
| g 1.21×10^{-16} | h 2×10^{-21} | i 2.5×10^1 |
| j 3×10^{-2} | k 5×10^6 | l 5×10^0 |
| 5 a 1.026×10^9 | b 5.83×10^1 | c 6×10^{-7} |
| d 4.34×10^9 | e 1.2×10^2 | f 1.6×10^2 |
| 6 $5.8473 \times 10^9 \text{ kg} = 5.8473 \times 10^6 \text{ tonnes}$ | | |
| 7 a $2.18 \times 10^{18} \text{ km}$ | b $1.23 \times 10^{23} \text{ km}$ | |
| c approximately 8 minutes 18 seconds | | |
| 8 1840 electrons | | |
| 9 a 5.766×10^2 | b 4.73×10^2 | c 4.7×10^2 |
| d 5×10^2 | e 5.124×10^{-2} | f 5.12×10^{-2} |
| g 5.1×10^{-2} | h 5×10^{-2} | i 1.603×10^3 |
| j 1.60×10^3 | k 1.6×10^3 | l 2×10^3 |
| m 2.994×10^{27} | n 2.99×10^{27} | o 3.0×10^{27} |
| p 3×10^{27} | q 5.73×10^5 | r 7×10^{-3} |
| 10 a 5.60 | b 538 | c 9670 |
| d 732 000 | e 0.003 51 | f 0.0142 |
| g 372 | h 478 000 | |
| 11 $2.93 \times 10^{-8} \text{ m}^3$ | | |
| 12 a 4.14×10^4 | b 2.97×10^9 | |
| 13 a between 14.5 cm and 15.5 cm | | |
| b between $1.995 \times 10^3 \text{ kg}$ and $2.005 \times 10^3 \text{ kg}$ | | |
| c between 18.665 m and 18.675 m | | |
| d between $4.874\,45 \times 10^7 \text{ mL}$ and $4.874\,55 \times 10^7 \text{ mL}$ | | |

Exercise 9C

- | | | |
|-------------------|------------------|-----------------|
| 1 a 2 | b 7 | c 3 |
| d 2 | e 10 | f 5 |
| 2 a $\frac{1}{5}$ | b $\frac{1}{8}$ | c $\frac{1}{2}$ |
| d $\frac{1}{100}$ | e $\frac{1}{10}$ | |

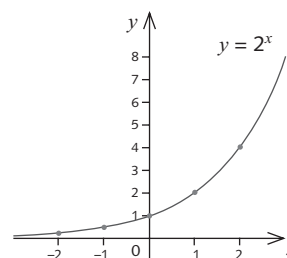
- 3 a 9 b 16 c 27
d 8 e 243 f 100 000
g $\frac{1}{4}$ h $\frac{16}{81}$ i 1331
j 49 k $\frac{8}{125}$ l $\frac{125}{216}$
m $\frac{1}{1000}$ n $\frac{7}{10}$ o $\frac{16}{81}$
- 4 a $\sqrt[3]{2}$ b $\sqrt[4]{3}$ c $\sqrt[5]{20}$
d $\sqrt[4]{10}$ e $\sqrt[5]{9}$
- 5 a $4^{\frac{1}{3}}$ b $13^{\frac{1}{7}}$ c $5^{\frac{1}{4}}$
d $11^{\frac{2}{7}}$ e $5^{\frac{2}{3}}$ f $7^{\frac{3}{2}}$
g $11^{\frac{3}{5}}$ h $10^{\frac{2}{7}}$
- 6 a $5^{\frac{3}{2}}$ b $5^{\frac{4}{3}}$ c $6^{\frac{5}{4}}$
d $7^{\frac{6}{5}}$ e $5^{\frac{7}{3}}$ f $11^{\frac{5}{2}}$
- 7 a $\frac{1}{3}$ b $\frac{1}{2}$ c $\frac{1}{11}$
d $\frac{1}{10}$ e $\frac{1}{1000}$ f $\frac{1}{11}$
- 8 a $\frac{1}{8}$ b $\frac{1}{1000}$ c 4
d $\frac{1}{625}$ e $\frac{1}{100}$ f $\frac{1}{8}$
- 9 a 2 b $3^{\frac{17}{15}}$ c $7^{\frac{3}{5}}$
d $3^{\frac{7}{12}}$ e $10^{\frac{3}{2}}$ f $10^{\frac{11}{12}}$
g $3^{\frac{7}{15}}$ h $\frac{1}{3^{\frac{1}{20}}}$ i $\frac{1}{5^{\frac{10}{3}}}$
- 10 a $2^{\frac{2}{5}}$ b 7 c $2^{\frac{9}{28}}$
d $7^{\frac{1}{6}}$ e 2 f $\frac{1}{2^{\frac{1}{7}}}$
- 11 a 3.9811 b 8.3203 c 12.748
d 3.7477 e 9.3152 f 2.4150
g 2202.7 h 2.4721
- 12 a $m^{\frac{11}{12}}$ b $a^{\frac{11}{15}}$ c $x^{\frac{3}{4}}y^{\frac{11}{15}}$
d $a^{\frac{10}{11}}b^{\frac{5}{6}}$ e $a^{\frac{1}{2}}$ f $\frac{1}{m^{\frac{1}{6}}}$
g $b^{\frac{5}{21}}$ h $4m^{\frac{8}{5}}$ i $\frac{1}{27m^{\frac{3}{2}}}$
j $\frac{a^2}{625}$ k $\frac{80}{m^{12}}$ l $m^{\frac{13}{6}}$
m $4m^{\frac{5}{2}}$ n $\frac{3}{8m^3}$

- 13 a 8.586 b 458.2 c 1.130
d 0.9547 e 0.010 46 f 0.001 406
- 14 a $a^{4.8}$ b m^6 c $p^{3.6}$
d $b^{1.25}$ e $4p^{2.6}$ f $64p^{6.3}$
g $\frac{b^{1.6}}{2a^{0.7}}$ h $\frac{2n^8}{3m^{4.2}}$ i $\frac{b^{5.1}}{a^{0.8}}$
j $\frac{1}{m^{2.1}n^{5.8}}$

Exercise 9D

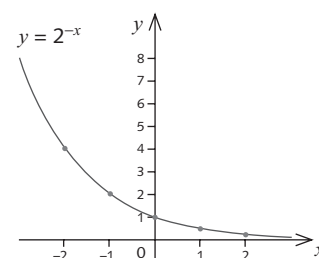
1 a

x	-2	-1	0	1	2
2^x	0.25	0.5	1	2	4



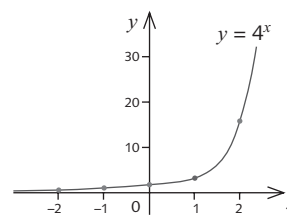
b

x	-2	-1	0	1	2
2^{-x}	4	2	1	0.5	0.25



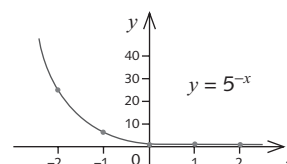
c

x	-2	-1	0	1	2
4^x	0.0625	0.25	1	4	16

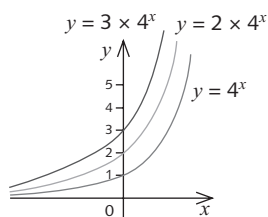


d

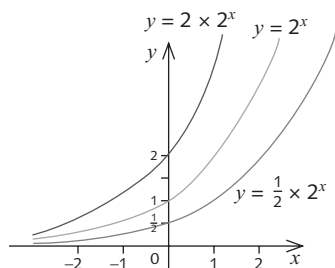
x	-2	-1	0	1	2
5^{-x}	25	5	1	0.2	0.04



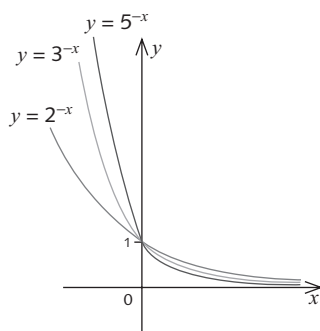
2



3



4



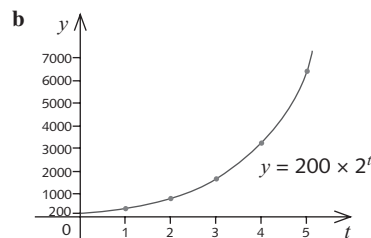
Exercise 9E

- 1 a 3 b 9 c 5 d 2
e 3 f 2 g 3 h 5
i 3 j 6 k 4 l 5
- 2 a -4 b -4 c 0 d -3
e -5 f -3 g -5 h -10
- 3 a $\frac{1}{2}$ b $\frac{3}{2}$ c $\frac{3}{2}$ d $\frac{2}{3}$
e $\frac{3}{2}$ f $\frac{2}{3}$ g $\frac{2}{3}$ h $\frac{3}{4}$
- 4 a $\frac{5}{3}$ b $\frac{7}{2}$ c $\frac{5}{7}$ d $\frac{3}{4}$
e $\frac{1}{3}$ f $-\frac{2}{3}$ g $-\frac{1}{3}$ h $-\frac{3}{2}$
- 5 a 5 b -2 c $\frac{4}{3}$ d $\frac{2}{15}$
e $\frac{3}{2}$ f 6 g $-\frac{5}{4}$ h $\frac{3}{2}$
- 6 a 4 and 5 b 2 and 3 c 5 and 6
d 2 and 3 e 4 and 5 f 2 and 3
g 1 and 2 h 2 and 3 i 3 and 4
j -1 and 0 k 0 and 1 l 1 and 2

Exercise 9F

1 a

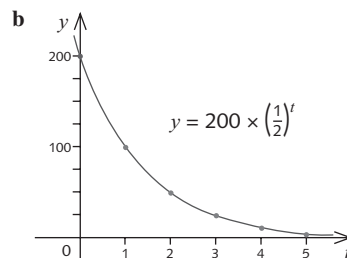
t	0	1	2	3	4	5
y	200	400	800	1600	3200	6400



c i 303.14 ii 918.96 iii 2262.74

2 a

t	0	1	2	3	4	5
y	200	100	50	25	12.5	6.25



c i 131.95 ii 21.76 iii 8.25

- 3 a i 60 ii 3840 iii 10 861
b i 1000 ii 100 iv 0.1
iii 1

4 a 8.2007×10^9 b 1.360×10^{11}

5 a 63.2° b 42.0°

6 a 0.8% b 80 000 c 86 635

7 a 1000
b i 250 ii 125 iii 31

Exercise 9G

- 1 a $\log_2 8 = 3$ b $\log_{10} 100 = 2$ c $\log_7 49 = 2$
d $3^4 = 81$ is equivalent to $\log_3 81 = 4$
e $5^3 = 125$ is equivalent to $\log_5 125 = 3$
f $7^3 = 343$ is equivalent to $\log_7 343 = 3$
g $2^5 = 32$ is equivalent to $\log_2 32 = 5$
h $10^4 = 10\,000$ is equivalent to $\log_{10} 10\,000 = 4$
i $10^{-3} = 0.001$ is equivalent to $\log_{10} 0.001 = -3$
j $2^{-1} = 0.5$ is equivalent to $\log_2 0.5 = -1$
- 2 a 2 b 6 c 7 d 12
e 0 f 8 g 3 h 2
- 3 a 3 b 4 c 3 d 2
e 3 f 0 g 4 h 3

- 4 a -2 b -1 c -2 d -2
e -3 f -5 g -4 h -3
- 5 a 1 b 0 c 3 d 5
e 100 f -2 g -7 h -13
- 6 1
- 7 0
- 8 a 2.8971 b -3.5229 c 7.8573
d -2.2680 e 56.8028 f -3.9085
- 9 a 6 b 5 c -4
d -3 e 27 f 25
g $\frac{1}{8}$ h $\frac{1}{5}$ i 2
j 4 k 5 l 2

Review exercise

- 1 a a^{17} b $1296m^{16}$ c a^6b^2
- 2 a $\frac{1}{16}$ b 6
c $\frac{1}{10\,000}$ d $\frac{81}{16}$
- 3 a $\frac{1}{a^8}$ b $\frac{14}{a^3}$
c $\frac{4}{a^2}$ d $\frac{1}{4a^6}$
- 4 a $\frac{1}{b^3}$ b $\frac{2}{x^4}$ c $\frac{5}{x^3}$
d $\frac{1}{2x^3}$ e $\frac{1}{5a^4}$ f $2x^2$
g $4x^3$ h $\frac{4b^3}{a^2}$ i $\frac{5m^3}{6}$
- 5 a $\frac{1}{36}$ b $\frac{1}{64}$ c $\frac{1}{16}$
d $\frac{1}{5}$ e $\frac{1}{100}$
- 6 a 1 b 5 c 1
d 7 e 1 f 5
g 1 h $\frac{1}{3}$ i 4
- 7 a $8ab$ b ab^5 c $2b$
d $\frac{1}{32a}$ e $\frac{9}{a^2}$ f $\frac{a^4}{2}$
- 8 2^{2n-4}
- 9 6^{3x}
- 10 a $2^{-\frac{1}{6}}$ b $a^{\frac{11}{20}}$
c $2^{\frac{7}{3}}$ d $2^{\frac{3}{5}}$
- 11 a 4.2×10^3 b 6.2×10^{-3}
c 7.4×10^8 d 2×10^{-7}

- 12 a 5400 b 112 000
c 0.068 d 0.0097
e 0.18 f 0.000 064
g 7 410 000 h 402
- 13 a 20 b 500 c 420
d 34 000 e 0.0068 f 0.0492
g 480 h 600 i 0.007

- 14 a 3 b 4 c -2
d 0 e -2 f -3
g -4 h -3

- 15 a 1 b 5 c 15
d -1 e -2 f -6

- 16 a $-\frac{5}{3}$ b $-\frac{7}{3}$ c $-\frac{1}{5}$
d -2 e -5 f 6

- 17 a 12 b $\frac{1}{2}$ c 4
d $\frac{1}{128}$ e $\frac{1}{2}$ f $7^{\frac{1}{2}}$

- 18 a $\frac{3^7b^2}{3a^4}$ b $\frac{1}{x^{10}}$
c $\frac{3^7a^8}{2^7b^9}$ d ab^4

- 19 a 32 b $3^7 = 2187$
c 1 d $7^2 = 49$
e $\frac{1}{10}$ f $\frac{\sqrt{5}}{5}$
g 5 h 3 i 10

- 20 a 8400 b 9261 c $8000 \times (1.05)^n$

Challenge exercise

- 1 a $120x$ b 128 c $\frac{x}{16}$
- 2 -8 3 $\frac{1}{6}$ 4 -1
- 5 a 2306, 2308, 2312, 2320, 2336, 2368, 2432, 2560
b $n = 12$
- 7 a $x = 3, y = 2$ b $x = \frac{1}{2}, y = 2$
c $x = 5, y = 3$ d $x = 3, y = 7$
- 8 a $\frac{1}{a^3} + \frac{1}{b^3} + \frac{1}{c^3}$ b $x + x^{-1} + 3^{\frac{1}{2}}$
- 9 a $3a - 2a^{-\frac{1}{3}}b^{\frac{1}{2}} - a^{\frac{1}{3}}b^{-\frac{1}{2}} - 6a^{\frac{2}{3}}b + 4a^{-\frac{2}{3}}b^{\frac{3}{2}} + 2b^{\frac{1}{2}}$
b $a - b^2$
- 10 $\frac{1}{3^3} > 2^{\frac{1}{2}} > 5^{\frac{1}{5}}$ 11 xy
- 12 $\frac{1}{5}$ 13 18 063

Chapter 10 answers

10A Review

Chapter 1: Consumer arithmetic

- 1 \$4500 2 3 years 3 8.7% p.a.
- 4 a \$239.40 b \$72
- 5 20%
- 6 a \$24 b \$120
- 7 a 108 b 103.5
c 36 d 20
e 25% decrease f 150% increase
g 253.75
- 8 a 4% decrease b 15.5% increase
c 4.32% decrease d 19% decrease
- 9 11.384% increase
- 10 a 20% increase b 5% increase
c 20% decrease d 15% decrease
- 11 \$56 314.12
- 12 a \$19 042.49 b \$19 301.25 c \$19 362.03

Chapter 2: Review of surds

- 1 a $5\sqrt{2} + 2\sqrt{3}$ b $7\sqrt{5} + 4$
- 2 a $3\sqrt{2}$ b $8\sqrt{2}$
c $24\sqrt{2}$ d $9\sqrt{3}$
- 3 a $-\sqrt{2}$ b $26\sqrt{3}$
c $16\sqrt{3}$ d $8\sqrt{7}$
e $7\sqrt{5} + 18\sqrt{2}$ f $2\sqrt{a}$
- 4 a $4\sqrt{3} - 4$ b $2\sqrt{2} - 13$
c 13 d $5 - 2\sqrt{6}$
- 5 a 8 b $a - b$
c $a + 2\sqrt{ab} + b$
- 6 a $\sqrt{6}$ b $\sqrt{2} + \sqrt{3}$
c $\frac{3 + 5\sqrt{3}}{3}$ d $\frac{15 - \sqrt{5}}{5}$
e $2\sqrt{15} + 2\sqrt{2}$ f $\frac{2\sqrt{2} - 1}{3}$
- 7 a $\frac{7 + 2\sqrt{10}}{3}$ b $\frac{13 + 5\sqrt{6}}{19}$
c $\frac{4\sqrt{3} + 4\sqrt{2} + \sqrt{6} + 2}{2}$ d $\frac{14\sqrt{2} - \sqrt{5}}{9}$

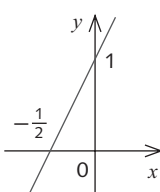
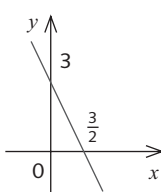
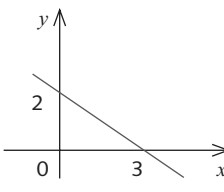
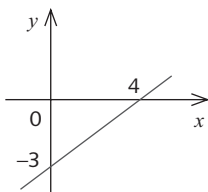
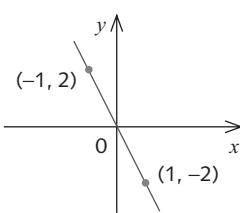
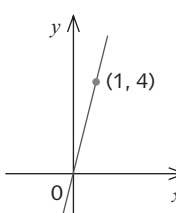
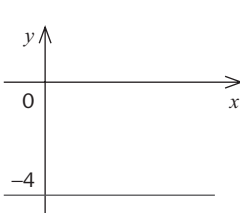
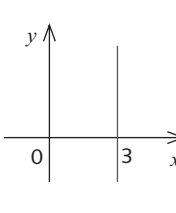
Chapter 3: Algebra review

- 1 a $9mn^2 - 3m$ b 0
c $7ab^2 + a^2$ d $10p^2q^2 - 5p^3$
- 2 a $5a + 4$ b $2b + 21$
c $6x^2 - 2x$ d $17y - 5y^2$
e $2x^2 + 11x + 5$ f $6a^2 + 17a - 14$
g $y^2 + 5y + 9$ h $b^2 + 23b - 1$
- 3 a $6x^2 + 33x + 45$ b $12a^2 - 10a - 8$
c $10y^2 + 7y - 14$ d $7b^2 + 37b + 1$
- 4 a $\frac{5}{4}a + 1$ b $\frac{7}{6}b + \frac{7}{3}$
c $\frac{11x^2}{12} + \frac{31x^2}{12}$ d $-\frac{7y^2}{10} - \frac{y}{10}$
e $\frac{2x^2}{3} + \frac{7x}{6} + \frac{1}{4}$ f $-\frac{4y^2}{9} + \frac{15y}{4} + \frac{73}{12}$
- 5 a $x = \frac{17}{2}$ b $x = -\frac{10}{3}$ c $x = -\frac{11}{3}$
d $y = -4$ e $x = \frac{38}{3}$ f $y = -\frac{10}{3}$
g $x = 16$ h $x = \frac{79}{7}$ i $y = -9$
j $x = \frac{65}{2}$
- 6 a $(x - 5)$ metres
b length 17.5 m, width 12.5 m
- 7 24 km/h
- 8 4 litres
- 9 a $x > 4$ b $x \geq \frac{11}{6}$ c $x < 37$
d $x \leq -\frac{17}{7}$ e $x \leq -5$ f $x > 3$
- 10 9.61×10^6 watts
- 11 a $s = 42.225$ b $a = \frac{2(s - ut)}{t^2}$
- 12 a $g = \frac{4\pi^2 p}{T^2}$ b $g = 9.8$
- 13 a $x = \frac{c - b}{a}$ b $x = \frac{c - ab}{a}$ c $x = \frac{cd - b}{a}$
d $x = \frac{c - b}{r - t}$ e $x = ya^2$ f $x = \frac{yc}{y - c}$
g $x = \frac{n - p}{m^2}$ h $x = \frac{-(a + b)}{a}$ i $x = \frac{3y - 1}{2}$
- 14 a $x^2 - 25$ b $x^2 - 4$ c $9x^2 - 1$
d $25x^2 - 4y^2$ e $\frac{1}{4}a^2 - 1$ f $\frac{1}{16}x^2 - \frac{4}{9}y^2$
- 15 a $(x - 6)(x + 6)$ b $(a - 8)(a + 8)$
c $(9b - 1)(9b + 1)$ d $(3x - 2y)(3x + 2y)$

- 16 a $x(x-18)$
 c $2(3b-5)(3b+5)$
 e $7(2x-3y)(2x+3y)$
 g $\left(\frac{1}{2}x-y\right)\left(\frac{1}{2}x+y\right)$
- 17 a $(x+2)(x+3)$
 c $(x-1)(x-2)$
 e $(x-3)(x-6)$
 g $(x-5)(x+2)$
 i $(x-7)(x+3)$
- 18 a $(x+2)(2x+3)$
 c $(5x+4)(x+3)$
 e $(3x-10)(x-1)$
 g $(3x+2)(x-3)$
 i $(2x+3)(x-7)$
- 19 a $(2x+1)(2x+3)$
 c $(4x+3)(x+4)$
 e $(2x-5)(3x-2)$
 g $(2x-5)(2x-3)$
 i $(4x+5)(2x-3)$
- 20 a $2(x+3)(x-7)$
 c $4(x+1)(x-4)$
 e $3(2x+5)(x-1)$
 g $5(2x+1)(x-6)$
 i $-2(3x+4)(x+1)$
- 21 a $\frac{34x+13}{35}$
 c $\frac{-3x-11}{(x+2)(x-3)}$
 e $\frac{x-4}{x(x-1)(x+1)}$
- 22 a $\frac{x+2}{x+4}$
 c $\frac{1}{x}$
- b $3x(x-6)$
 d $3(2b-3)(2b+3)$
 f $6(3a-2b)(3a+2b)$
 h $3\left(\frac{1}{2}x-\frac{2}{5}y\right)\left(\frac{1}{2}x+\frac{2}{5}y\right)$
- b $(x+2)(x+6)$
 d $(x-1)(x-5)$
 f $(x-6)(x+1)$
 h $(x-4)(x+2)$
- b $(3x+1)(x+6)$
 d $(2x-1)(x-2)$
 f $(7x-9)(x-2)$
 h $(5x+4)(x-2)$
- b $(3x+2)(2x+3)$
 d $(2x-5)(2x-3)$
 f $(5x-6)(2x-3)$
 h $(2x-5)(3x+2)$
- b $5(x+2)(x+3)$
 d $3(x-1)(x+5)$
 f $4(2x+1)(x+2)$
 h $(4-x)(x-6)$
- b $\frac{5x+7}{(x+2)(x+1)}$
 d $\frac{4x+5}{x(x+2)(2x+3)}$
 f $\frac{5x+9}{(x-3)(x+3)(x+1)}$
- b $\frac{(x+1)(x+6)}{(x-1)(x+1)}$
 d $\frac{-x(x+3)}{3(x+1)}$

Chapter 4: Lines and linear equations

- 1 a $2\sqrt{13}$
 c $\sqrt{34}$
- 2 a $(3, 2)$
 c $\left(-\frac{1}{2}, \frac{7}{2}\right)$
- 3 a $\frac{2}{3}$
 c $\frac{3}{5}$
- b $2\sqrt{2}$
 d $\sqrt{34}$
- b $(4, 3)$
 d $\left(-\frac{5}{2}, -\frac{1}{2}\right)$
- b 1
 d $\frac{5}{3}$

- 4 a $m = 2, c = -1$
 c $m = -1, c = 7$
 e $m = -\frac{2}{3}, c = \frac{1}{3}$
- 5 a i 3
 c i $\frac{1}{2}$
- ii $-\frac{1}{3}$
 ii -2
- 6 a 
- b 
- c 
- d 
- e 
- f 
- g 
- h 
- 7 a $y = 3x + 2$
 c $y = -x - 3$
- 8 a $3x + 2y = 6$
 c $y = 3$
 e $y = 2x - 1$
- 9 a $x = 2, y = 3$
 c $x = 6, y = -5$
 e $x = 6, y = -5$
- 10 a $3, b = 2$
- 11 a 3
 b i $a - 2$
 c i $\frac{2}{a}$
- b $m = 1, c = 3$
 d $m = -1, c = 4$
 f $m = \frac{3}{4}, c = -\frac{1}{2}$
- b i -2
 d i $-\frac{2}{3}$
- ii $\frac{1}{2}$
 ii $\frac{3}{2}$
- b $y = 2x + 1$
 d $x + 2y = 8$
- b $2y - x = 2$
 d $x = 5$
 f $y = 1 - x$
- b $x = 1, y = 1$
 d $x = \frac{1}{2}, y = \frac{2}{3}$
 f $x = \frac{1}{13}, y = \frac{44}{13}$

$$\text{d i } \frac{2}{5} \quad \text{ii } y = \frac{2}{5}x + 1 \quad \text{iii } \left(\frac{5}{2}, 2\right)$$

$$\text{e i } \sqrt{a^2 + 4} \quad \text{ii } 3\sqrt{5}$$

- 12 a 31 000 L b 40 L/h
c $V = 31\,000 - 40t$ d after 775 hours

- 13 a i A started in Melbourne, B started 100 km from Melbourne.
ii A finished 160 km from Melbourne, B finished in Melbourne.
iii A took 2 hours 20 minutes, B took 1 hour 36 minutes.
iv A averaged $68\frac{4}{7}$ km/h, B averaged 62.5 km/h.

b A: $d = \frac{480}{7}t$, B: $d = -62.5t + 100$

c They passed after 46 minutes of travelling.

Chapter 5: Quadratic equations

- 1 a -4, 4 b -2, 2
c -7, 7 d $-\frac{5}{2}, \frac{5}{2}$
e $\frac{1}{2}, -\frac{1}{2}$ f $\frac{5}{2}, -\frac{5}{2}$

- 2 a 0, $\frac{3}{2}$ b 0, $-\frac{1}{3}$
c 0, $\frac{3}{5}$ d 0, $\frac{1}{2}$
e 0, $\frac{2}{7}$ f 0, -5
g 0, 7 h 0, 12
i 0, $-\frac{1}{4}$

- 3 a $a = 4$ or -3 b $t = -3$ or -5
c $m = -7$ or 3 d $m = 4$ or -1
e $x = 4$ f $b = 9$ or -3

- 4 a $\frac{5}{2}, 7$ b $\frac{1}{3}, \frac{11}{3}$
c $\frac{1}{3}, -8$ d $-\frac{7}{6}, -\frac{3}{2}$
e $-\frac{1}{3}, 1$ f $-\frac{3}{2}, \frac{5}{6}$
g $\frac{3}{2}, -4$ h 3

- 5 a 3 b -5 c -1
d 4 e $-\frac{3}{2}$ f 5

- 6 a $(x + \sqrt{5})(x - \sqrt{5})$
b $(x + 2 - 2\sqrt{2})(x + 2 + 2\sqrt{2})$
c $2(x - 3 - \sqrt{5})(x - 3 + \sqrt{5})$

- 7 a $y = -1 \pm \sqrt{5}$ b $a = 2 \pm \sqrt{6}$
c $x = 1 \pm \frac{\sqrt{41}}{2}$ d $x = \frac{7 \pm \sqrt{41}}{2}$
e $y = -\frac{1}{4} \pm \frac{\sqrt{2}}{4}$ f $x = \frac{1 \pm \sqrt{33}}{4}$

$$\text{g } n = \frac{5 \pm \sqrt{41}}{2} \quad \text{h } x = \frac{-1 \pm \sqrt{2}}{4}$$

- 8 a $d = \pm\sqrt{2}$ b $y = \pm\frac{\sqrt{30}}{2}$
c $x = 10 \pm 2\sqrt{5}$ d $y = 4 \pm \sqrt{13}$

$$\text{e } m = \frac{1}{2} \pm \frac{\sqrt{5}}{2} \quad \text{f } n = \frac{3 \pm \sqrt{21}}{2}$$

- 9 a $(x + 7)$ metres, $(x + 8)$ metres
b 5 cm, 12 cm, 13 cm

- 10 a 381.6 m b 102 seconds

- 11 a length = $24 - 2x$ cm, width = $17 - 2x$ cm
b $x = 2.5$
c $19 \text{ cm} \times 12 \text{ cm} \times 2.5 \text{ cm}$

- 12 a $x^2 \text{ m}^2$ b $(8x + 16) \text{ m}^2$
c $8x + 16 = \frac{5x^2}{4}$ d $8 \text{ m} \times 8 \text{ m}$

- 13 a $b = \pm 4$ b $b < -4$ or $b > 4$
c $-4 < b < 4$

- 14 a $a = \frac{4}{3}$ b $a < \frac{4}{3}$ c $a > \frac{4}{3}$

Chapter 6: Surface area and volume

- 1 a 2200 cm^2 b 6000 cm^3
2 5 cm 3 5 cm
4 a 44 cm^2 b 728 cm^2 c 880 cm^3
5 a 4712 litres b 64 cm

- 6 a i 360 cm^2 ii 400 cm^3
b i $96\pi \text{ cm}^2$ ii $96\pi \text{ cm}^3$

- 7 a 254.6 m^2 b 160 m^3

- 8 a 4 cm b $8\sqrt{6} \text{ cm}$ c 328 cm^3

- 9 a $18\pi \text{ m}^2$ b $\frac{45}{4}\pi \text{ m}^3$

- 10 a 9, 27 b 2.25, 3.375 c 2, 8
d 6, 216 e 5, 25 f 9, 81

Chapter 7: The parabola

- 1 a -1, -3 b $-\frac{1}{2}, 6$
c $-4 + \sqrt{3}, -4 - \sqrt{3}$ d $2 + \sqrt{2}, 2 - \sqrt{2}$

- 2 a $y = (x + 3)^2 - 6, (-3, -6)$
b $y = (x - 2)^2 - 2, (2, -2)$
c $y = 2\left(x + \frac{3}{2}\right)^2 - \frac{7}{2}, \left(-\frac{3}{2}, -\frac{7}{2}\right)$
d $y = 3\left(x + \frac{4}{3}\right)^2 - \frac{10}{3}, \left(-\frac{4}{3}, -\frac{10}{3}\right)$

3 a $y = -2x^2 + 6x + 8$

b $y = x^2 - 8x + 15$

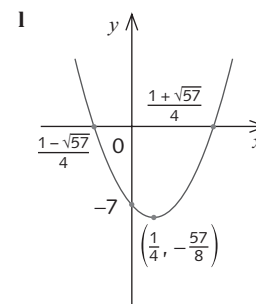
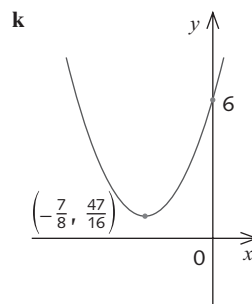
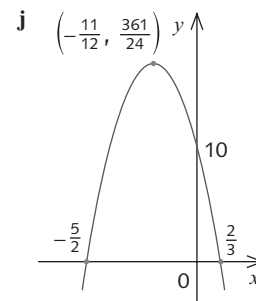
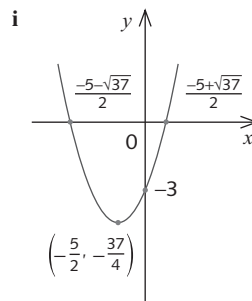
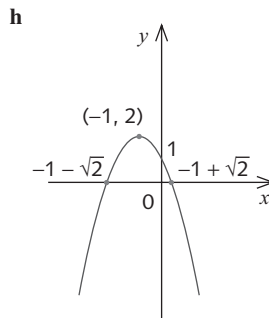
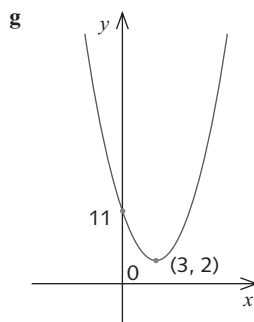
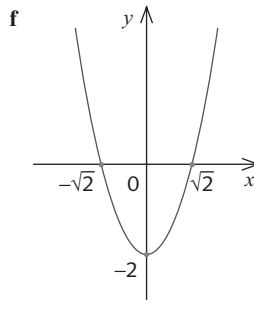
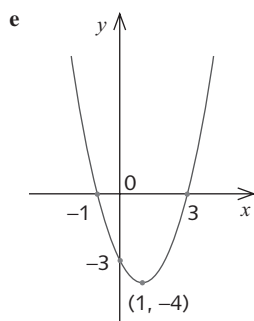
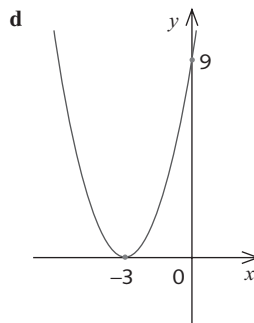
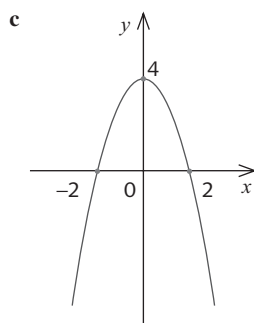
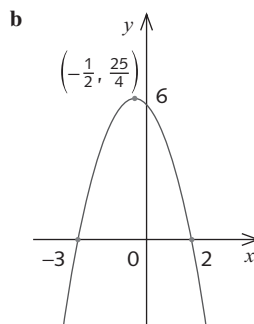
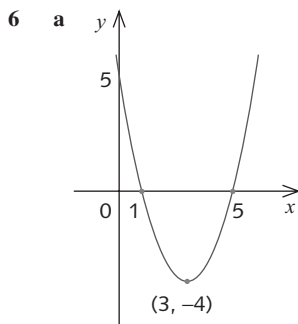
4 a $y = 2x^2 - 12x + 16$

b $y = -3x^2 + 12x - 7$

5 a $y = 3x^2 - 12x + 12$

b $y = -x^2 - 2x + 2$

c $y = x^2 - 10x + 21$



7 a $\left(\frac{1}{3}, -\frac{4}{3}\right)$

b $\left(-\frac{1}{3}, 0\right), (1, 0)$

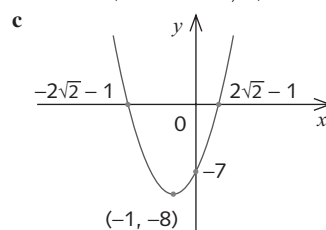
8 a $12 - 2x$

b $A = x(12 - 2x)$

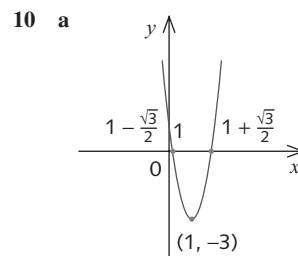
c 3 m by 6 m

9 a $(-1, -8)$

b $(0, -7), (-1 - 2\sqrt{2}, 0), (-1 + 2\sqrt{2}, 0)$



d $-2\sqrt{2} - 1 \leq x \leq 2\sqrt{2} - 1$



b $1 - \frac{\sqrt{3}}{2} < x < 1 + \frac{\sqrt{3}}{2}$

11 a $-6 < x < 5$

b $x \leq -3$ or $x \geq -2$

c $x \leq -6$ or $x \geq 10$

Chapter 8: Review of congruence and similarity

Note: Proofs are not given in answers for this section.

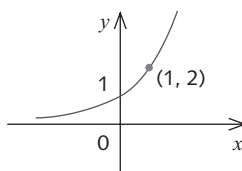
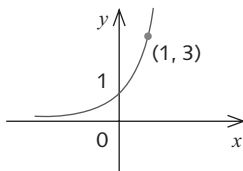
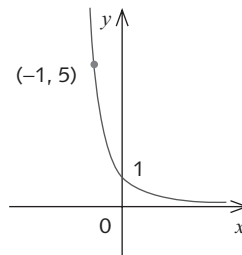
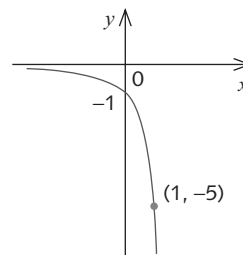
1 a $a = 2, b = 2.5$

b $x = 2, y = 6$

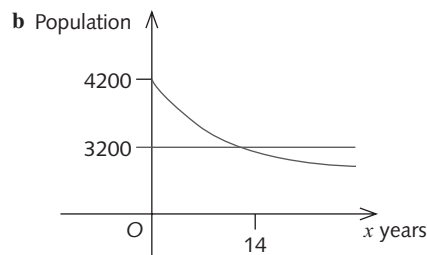
2 $16\frac{2}{3}$ cm

- 3 a i AAA ii $x = 3.75$
b RHS
- 4 a i AAA ii $x = \frac{12}{5}$
b ASA
- 6 a $\angle DEC = \angle ABC$ (given both 90°), $\angle ECD = \angle BCA$ (common), $\triangle DEC$ is similar to $\triangle ABC$ (AAA)
b $x = 5$ c $\alpha = 126.87^\circ$
- 7 a $\triangle PQL$ is similar to $\triangle LTP$ is similar to $\triangle NML$ (AAA)
b $LT = 75$ m
- 12 $x = \frac{ad}{b}$

Chapter 9: Indices, exponentials and logarithms – part 1

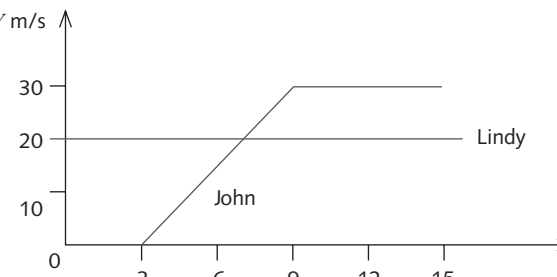
- 1 a a^4 b $24x^7y^5$ c a^2b
d $\frac{1}{a^4}$ e $2xy$ f $2y$
g $\frac{2y^3}{x^4}$ h $\frac{b^8}{a}$ i $\frac{y^4}{x^2}$
- 2 a 3.2×10^3 b 5.76×10^5
c 2.67×10^{-4} d 2.5×10^{-2}
- 3 a 9.017×10^4 b 4.552×10^{-4}
c 6.516×10^{-6} d 9.468×10^{-20}
- 4 a 3 b 3 c 2 d 2 e 3 f 4
- 5 a 4 b 8 c 9 d 8 e $\frac{1}{27}$ f $\frac{1}{25}$
- 6 a b^4 b $\frac{a^2}{b}$ c $\frac{a^2}{b}$ d $a^{\frac{1}{3}}b^{\frac{1}{3}}$
e $\frac{a}{b^2}$ f $\frac{3a^{\frac{2}{3}}}{b}$ g $\frac{4a^{\frac{3}{2}}}{b}$ h a^2
- 7 a  b 
c  d 
- 8 a $x = \frac{1}{5}$ b $x = \frac{1}{2}$ c $x = -2$
d $x = \frac{3}{4}$ e $x = -\frac{3}{4}$ f $x = \frac{5}{3}$
- 9 a $x = 5$ b $x = 1$ c $x = \frac{11}{4}$
d $x = \frac{19}{18}$ e $x = -\frac{17}{15}$ f $x = 6$

- 10 a 5400 b 5832
c 5000×1.08^x d 23 305
- 11 a i 4116 ii 4037 iii 4200×0.98^x



- c 14 years
- 12 a 4 b 2 c $\frac{1}{2}$ d $\frac{3}{2}$

10B Miscellaneous questions

- 1 a 140 km b x km
c $\frac{x}{35}$ hours, $\frac{140}{x}$ hours d 70 km/h
- 2 a i $1\frac{1}{8}$ hours ii $\frac{5}{8}$ hours iii 144 km/h
b i $(d - m)$ km ii $T = \frac{d - m}{v}$
iii $t = \frac{d - m}{v} - n$ iv $w = \frac{v(d - m)}{d - m - nv}$
v $v = \frac{w(d - m)}{wn + d - m}$
- c $56\frac{16}{19}$ km/h
- 3 a X takes $2\frac{1}{2}$ hours, Y takes $1\frac{2}{3}$ hours
b X averages 24 km/h, Y averages 36 km/h
c They pass 31.2 km from A.
- 4 a 2 km
b Shen finishes first by approximately 10 minutes.
- 5 a 30 m/s
b 
- c 7 seconds d 5 m/s^2
e i 180 m ii $20t$ metres
f i 90 m ii 180 m
iii $(30t - 180)$ metres
g i 18 seconds ii 360 m
- 6 a i $\frac{1}{3}$ ii $\frac{4}{3}$
b i $y = \frac{1}{3}x + 2$ ii $y = \frac{4}{3}x - 4$

- c (6, 4) d 27 e 18

f $\left(\frac{6b^2 - 12b}{4 - b^2}, \frac{8b + 4b^2}{4 - b^2}\right)$; when $b = 0$, $E = (0, 0)$; when $b = 2$ the lines are parallel so E does not exist; when $b = -2$ the lines are the same.

- 7 a 20 km
b i $10\sqrt{2}$ km ii $25\sqrt{2}$ km
c $(15\sqrt{2}, 10)$
d i $P = (0, -20)$, $R = (25\sqrt{2}, 0)$
ii $\frac{2\sqrt{5}}{5}$
iii $y = \frac{2\sqrt{2}}{5}x - 20$

- 8 a i $h = 3r$ ii $V = \pi r^3$ iii $A = \sqrt{10}\pi r^2$
b i 7.55 cm ii 62.9 cm^3

- 9 a $\angle AGB = \angle CGF$ (vertically opposite angles at G),
 $\angle GAB = \angle GCF$ (or $\angle ABG = \angle CFG$) (alternate angles in parallel lines), so $\triangle AGB$ is similar to $\triangle CGF$ (AAA)

b $\triangle EBA, \triangle BFC$

- c i 3 : 2 ii 2 : 3

- 10 a i AAA ii $x = 5\frac{1}{7}$ iii $\frac{24}{49}$

- b i $x = \frac{3y}{2}$ ii $\frac{4}{9}$

- 11 a i 3 ii $3\sqrt{5}$
iii $\sqrt{6}$ iv $1 + \sqrt{3}$

b $y = \frac{1}{x}$

- 12 a $\angle ABE = \angle ACD$ (given), $\angle BAE = \angle CAD$ (common), so $\triangle ABE$ is similar to $\triangle ACD$ (AAA)

- b i $(x + 10)$ metres ii $\frac{x}{x + 10} = \frac{2}{3}$

c 20 metres

- d i $\frac{1}{2}x^2 \text{ m}^2$ ii $\left(450 - \frac{1}{2}x^2\right) \text{ m}^2$

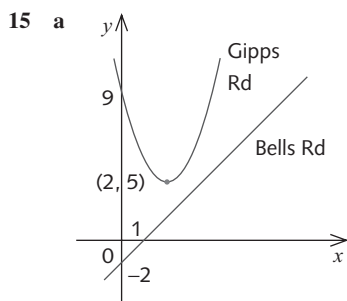
- e i $\frac{1}{2}x^2 = 450 - \frac{1}{2}x^2$ ii $15\sqrt{2}$ metres

- 13 a $A = (15 - x)^2 - (15 - 2x)^2$
b 75 m^2

- 14 a 10 km b 0.5 km

c $A = (1.13, 0.2)$, $B = (8.87, 0.2)$

d 7.75 km



b i $y = (x - 2)^2 + 5$

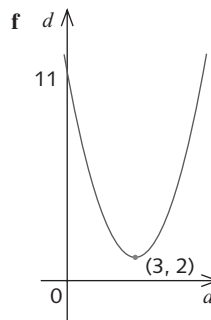
ii 9

iii see graph

c 11 km

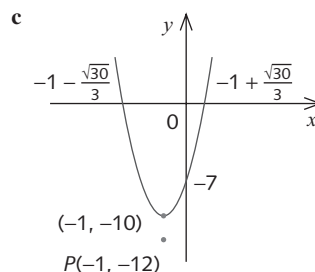
d i $y = 2a - 2$

ii $y = a^2 - 4a + 9$



g 2 km, 3 km east of origin

- 16 b i $(-1, -10)$ ii $\left(-1 + \frac{\sqrt{30}}{3}, 0\right)$, $\left(-1 - \frac{\sqrt{30}}{3}, 0\right)$



d 200 km

e $3x^2 + 6x - 7 = 5x + 3$
 $3x^2 + x - 10 = 0$

f $\left(\frac{5}{3}, \frac{34}{3}\right)$, $(-2, -7)$

- 17 a i $h = r \tan \theta$ ii $s = \frac{r}{\cos \theta}$

b i Area (A) = $\frac{1}{2}\pi r^2$

ii Area (B) = $r^2 \tan \theta$

d 57.5°

e $4\pi \approx 12.57 \text{ mm}^2$

10C Problem-solving

1 $a = \frac{2r}{5}$, $b = \frac{6r}{5} - 3$

- 2 $x = 12$ and $y = 2$, or $x = 12$ and $y = -2$
or $x = -12$ and $y = 2$, or $x = -12$ and $y = -2$
or $x = 36$ and $y = 34$, or $x = 36$ and $y = -34$,
or $x = -36$ and $y = 34$, or $x = -36$ and $y = -34$

- 3 a i $2\sqrt{259}$ cm ii $2\sqrt{257}$ cm

b 5 cm

4 $(4 - 2\sqrt{3})$ m

5 a $16\pi \text{ cm}^3$

b $\frac{16\pi}{3} \text{ cm}^3$

c $\frac{32\pi}{3} \text{ cm}^3$

d $8\pi \text{ cm}^3$

6 Midpoint of $AB = X\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

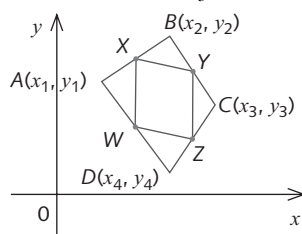
Midpoint of $BC = Y\left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$

Midpoint of $CD = Z\left(\frac{x_3 + x_4}{2}, \frac{y_3 + y_4}{2}\right)$

Midpoint of $DA = W\left(\frac{x_4 + x_1}{2}, \frac{y_4 + y_1}{2}\right)$

Gradient of $XY = \frac{y_3 - y_1}{x_3 - x_1}$

Gradient of $WZ = \frac{y_3 - y_1}{x_3 - x_1}$



Therefore, $XY \parallel WZ$

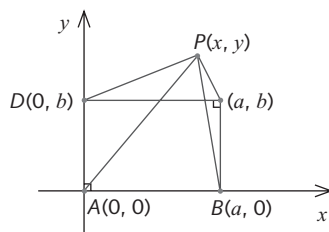
$$XY = \frac{1}{2} \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$$

$$WZ = \frac{1}{2} \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$$

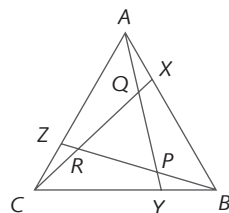
so $XY = WZ$

$XYZW$ is a parallelogram.

7 $(PA)^2 + (PC)^2 = x^2 + y^2 + (x - a)^2 + (y - b)^2$
 $(PB)^2 + (PD)^2 = (x - a)^2 + y^2 + x^2 + (y - b)^2$
 so $(PA)^2 + (PC)^2 = (PB)^2 + (PD)^2$



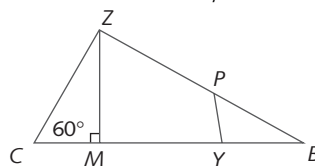
8 $\triangle AQX \cong \triangle CRZ \cong \triangle BPY$
 Quad $AQRZ \cong$ Quad $BPQX \cong$ Quad $CRPY$
 $\triangle PQR$ is equilateral.
 $\triangle BPY$ is similar to $\triangle BCZ$ (AAA).
 Assume $AB = BC = CA = 1$.
 Then $CZ = AX = YB = \frac{1}{3}$.



Apply Pythagoras' theorem twice to find $ZB = \frac{\sqrt{7}}{3}$.

The enlargement factor from $\triangle BPY$ to $\triangle BCZ$ is $\sqrt{7}$.

$\therefore$ area of $\triangle BPY = \frac{1}{7}$ area of $\triangle BCZ$
 $= \frac{1}{21}$ area of $\triangle ABC$
 area of $\triangle PQR = 3 \times$ area of $\triangle BPY$
 $= \frac{1}{7} \times$ area of $\triangle ABC$



9 $\triangle ARD$ is similar to $\triangle PRB$

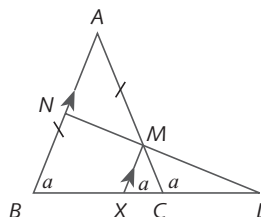
Therefore $\frac{AR}{RP} = \frac{RD}{RB}$

$\triangle BRA$ is similar to $\triangle DRQ$

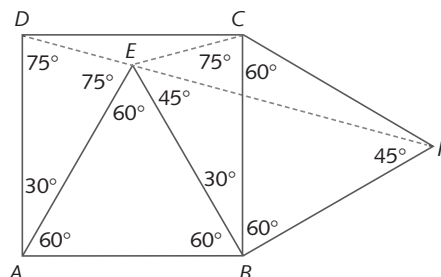
$\therefore \frac{RD}{RB} = \frac{RQ}{AR}$
 $\therefore \frac{AR}{RP} = \frac{RQ}{AR}$
 $\therefore AR^2 = RP \times RQ$

10 Draw $MX \parallel AB$.

$\angle ABC = \angle MXC$ (corresponding angles $AB \parallel MX$)
 $\angle ABC = \angle MCX$ ($\triangle ABC$ isosceles)
 $\therefore \angle MXC = \angle MCX$ ($\triangle MXC$ isosceles)
 $\therefore MX = MC$
 $\triangle LNB$ is similar to $\triangle LMX$
 $NM = ML$ (given)
 $\therefore LN = 2LM$ and $NB = 2MX = 2MC$



11 $\angle AEB = 60^\circ$ ($\triangle AEB$ is isosceles)
 $\angle DEA = 30^\circ$ (complementary to $\angle EAB$)
 $\angle CBE = 30^\circ$ (complementary to $\angle EBC$)
 $\triangle BEC = \triangle AED$ (SAS)
 $\angle ADE = \angle AED = \angle BEC = \angle ECB = 75^\circ$
 ($\triangle BEC$ and $\triangle AED$ are isosceles)
 $\angle EBF = 30^\circ + 60^\circ$ (square and equilateral triangle)
 $\triangle BEF$ is isosceles
 $\angle BEF = 45^\circ$ (angle sum of $\triangle BEF$)
 Therefore,
 $\angle AED + \angle AEB + \angle AEB + \angle BEF = 75^\circ + 60^\circ + 45^\circ = 180^\circ$
 Points A, E and F are collinear.



12 2 cm

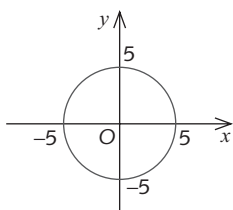
13 2 : 1

14 $120\sqrt{2}$

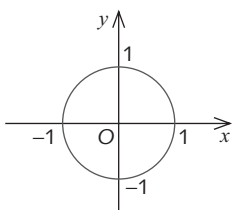
Chapter 11 answers

Exercise 11A

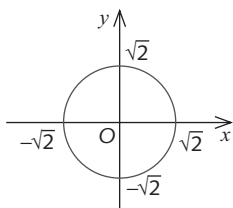
1 a



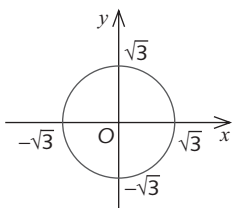
b



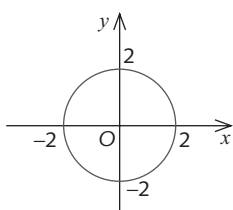
c



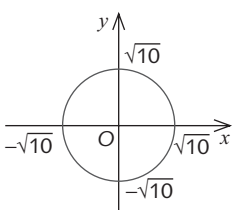
d



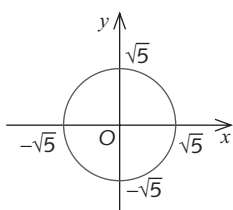
2 a



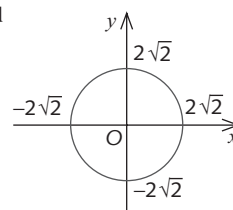
b



c



d



3 a yes

b no

c no

d yes

e yes

f yes

4 a yes

b no

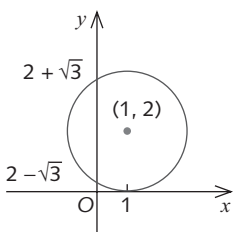
c yes

d yes

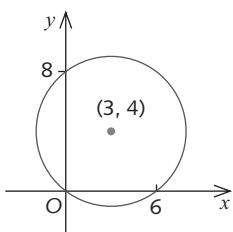
e no

f yes

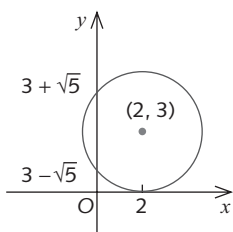
5 a



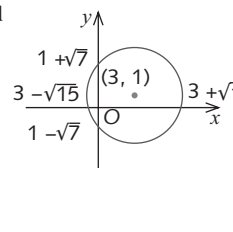
b



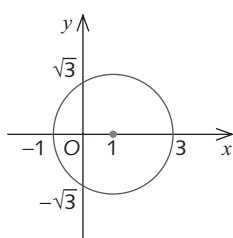
c



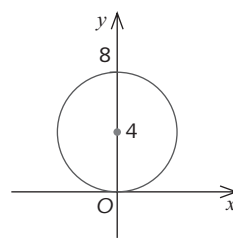
d



e



f



- 6 a centre $(-2, -3)$, radius 2
 b centre $(1, -4)$, radius $\sqrt{13}$
 c centre $(3, 4)$, radius 8
 d centre $(7, 4)$, radius 5
 e centre $(4, 3)$, radius $\sqrt{10}$
 f centre $(4, 2)$, radius $\sqrt{10}$

7 a $(x-1)^2 + (y-3)^2 = 9$ b $(x+2)^2 + (y-1)^2 = 16$

c $(x-4)^2 + (y+1)^2 = 1$ d $(x-2)^2 + y^2 = 4$

8 $\sqrt{(17-5)^2 + (17-12)^2} = 13$, so the point $(17, 17)$ is 13 units from the centre at $(5, 12)$. The equation of the circle is $(x-5)^2 + (y-12)^2 = 169$.

9 $(x-3)^2 + (y+4)^2 = 25$

10 a $(x-6)^2 + (y-7)^2 = 36$ b $(x-6)^2 + (y-7)^2 = 49$

11 a 6 b $(5, 6)$

c $(x-5)^2 + (y-6)^2 = 9$

12 a $10\sqrt{2}$ b $(2, -1)$

c $(x-2)^2 + (y+1)^2 = 50$

Exercise 11B

1 a $y = \frac{1}{2}$ b $y = -\frac{1}{2}$ c $y = 2$

d $y = \frac{2}{3}$ e $y = -\frac{3}{2}$

2 a $y = 4$ b $y = 3$ c $y = -24$

d $y = -8$ e $y = 16$

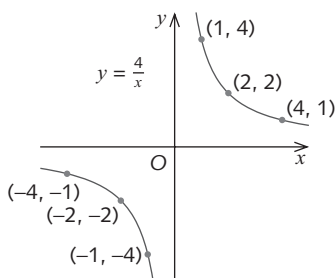
3 a $y = 1$ b $y = \frac{1}{2}$ c $y = 2$

d $y = -\frac{2}{3}$ e $y = \frac{2}{3}$

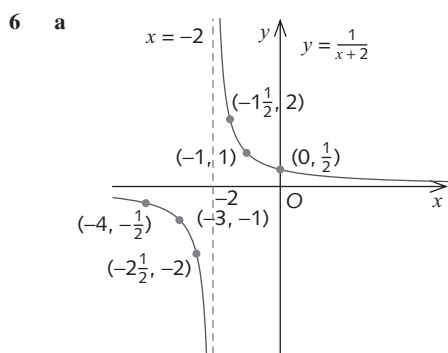
4 a $y = 1$ b $y = -1$ c $y = \frac{1}{2}$

d $y = 2$ e $y = -4$

5 a



b $-1, -2, -4, 4, 2, 1$



b $-\frac{1}{2}, -1, -2, 2, 1, \frac{1}{2}$

7 a -12 000

b -60

c 4

d $\frac{1}{2}$

e $\frac{1}{12}$

8 a -2

b -3

c -600

d 600

e $\frac{6}{997}$

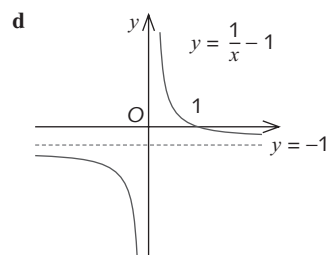
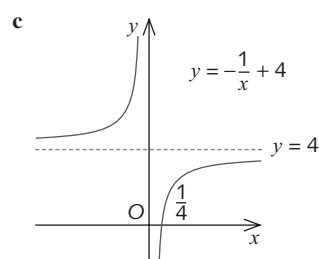
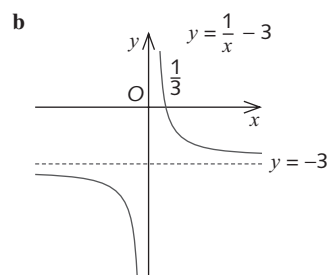
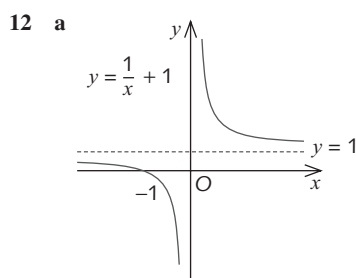
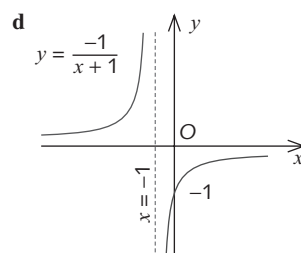
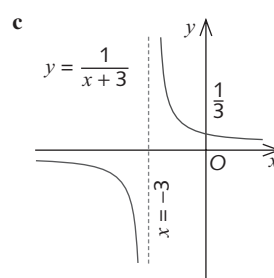
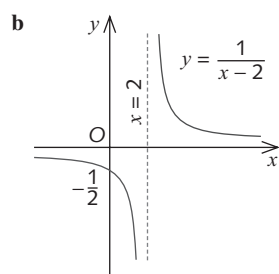
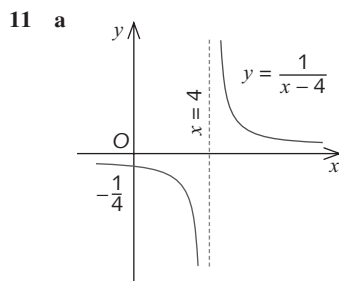
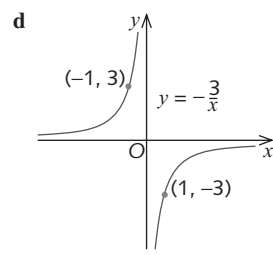
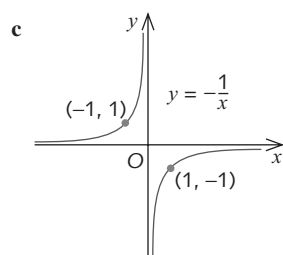
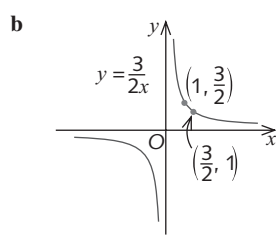
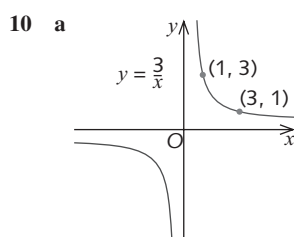
9 a 4

b 120

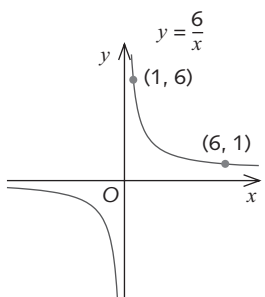
c 1200

d -1200

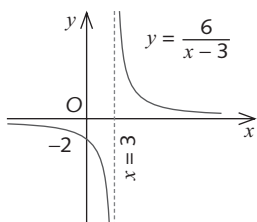
e -12 000



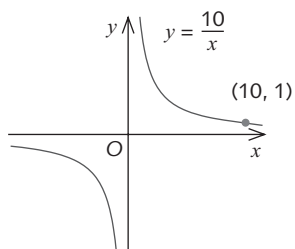
13 a i



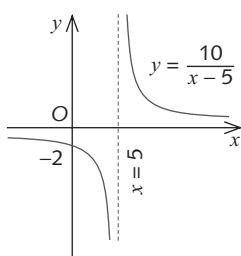
ii



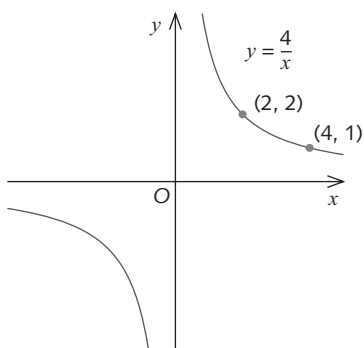
b i



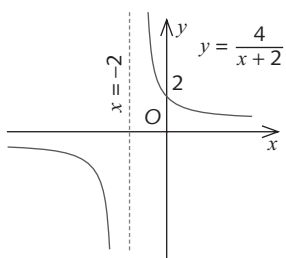
ii



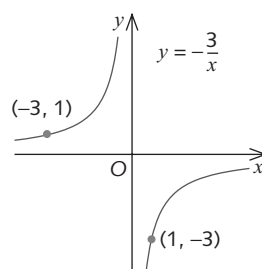
c i



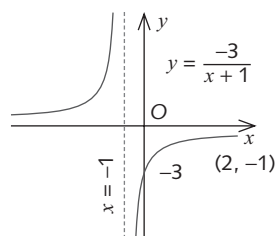
ii



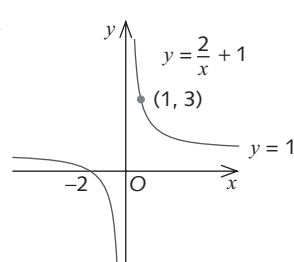
d i



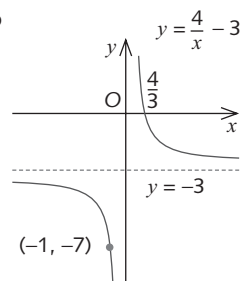
ii



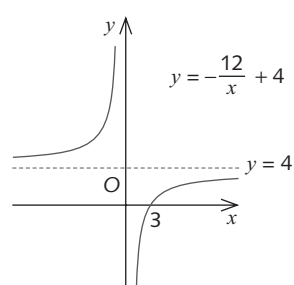
14 a



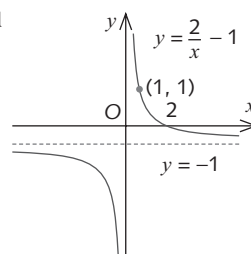
b



c

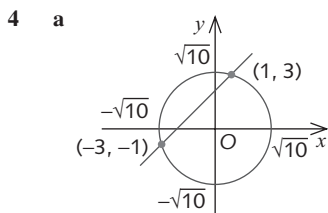


d

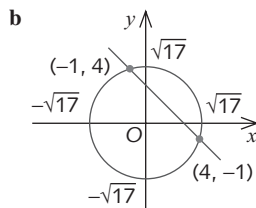


Exercise 11C

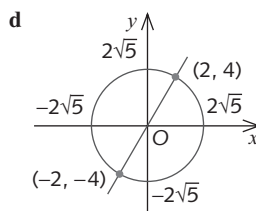
- 1 a (2, 4), (-2, 4) b (1, 1), (-1, 1)
 c (2, 1) d (3, 9), (4, 16)
 2 a (-1, 1) b (-5, 2), (1, 8) c (-2, 4), (1, 7)
 d $(-\frac{1}{2}, 0)$, (0, 1) e $(-\frac{1}{3}, 2)$, (1, 6)
 f $(-\frac{2}{3}, \frac{5}{3})$, $(-\frac{1}{2}, 2)$
 3 a (2, 0) b (-3, 0), (3, 0)
 c (4, 4), (-4, -4) d $(-3, -6\sqrt{2})$, $(3, 6\sqrt{2})$



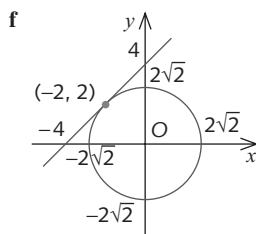
intersection points:
 (1, 3), (-3, -1)



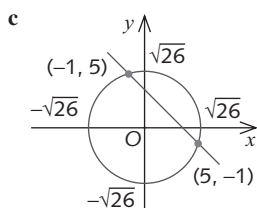
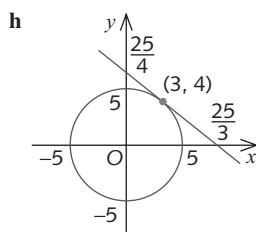
intersection points:
 (4, -1), (-1, 4)



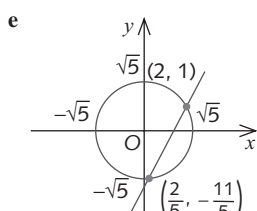
intersection points:
 (-2, -4), (2, 4)



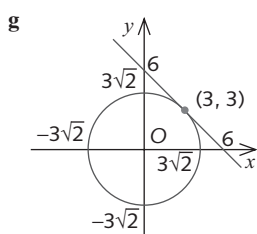
intersection point:
 (-2, 2)



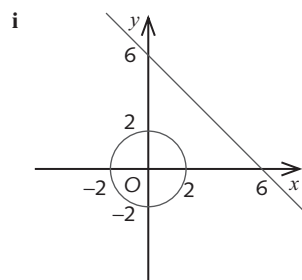
intersection points:
 (-1, 5), (5, -1)



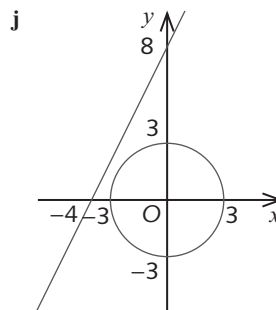
intersection points:
 $(\frac{2}{5}, -\frac{11}{5})$, (2, 1)



intersection point:
 (3, 3)
 intersection point:
 (3, 4)

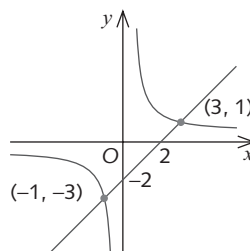


no intersection point

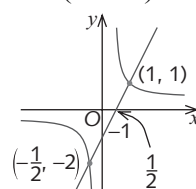


no intersection point

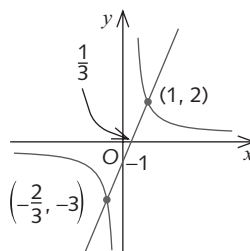
- 5 a (3, 1), (-1, -3)



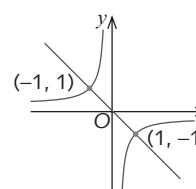
- b $(1, 1), (-\frac{1}{2}, -2)$



- c $(1, 2), (-\frac{2}{3}, -3)$



- d (-1, 1), (1, -1)



- 6 a $A = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$, $B = (1, 1)$

b area of $\triangle OAX$ is $\frac{1}{4}$, area of $\triangle OBY$ is $\frac{1}{2}$

- 7 (2, 3)

- 8 $(x-5)^2 + 4x^2 = 4$, $(x-1)^2 = -\frac{16}{5}$, so this quadratic equation has no solution and the line does not meet the circle.

- 9 a $a = 3\sqrt{2}$ or $-3\sqrt{2}$

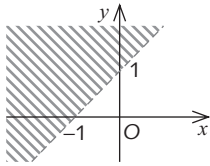
- b $-3\sqrt{2} < a < 3\sqrt{2}$

- c $a < -3\sqrt{2}$ or $a > 3\sqrt{2}$

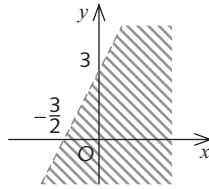
- 10 $(1, 2\sqrt{2})$, $(1, -2\sqrt{2})$

Exercise 11D

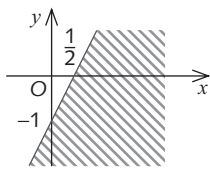
1 a



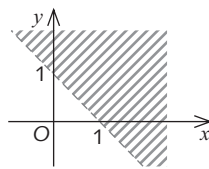
b



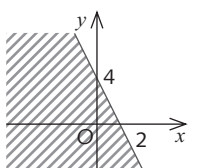
c



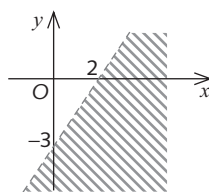
d



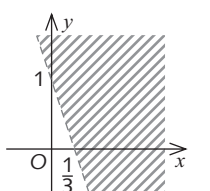
e



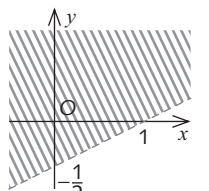
f



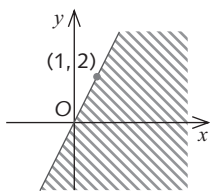
g



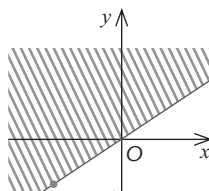
h



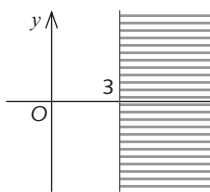
i



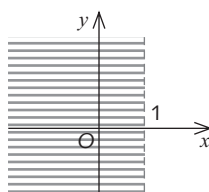
j



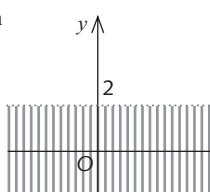
k



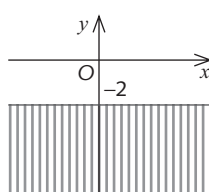
l



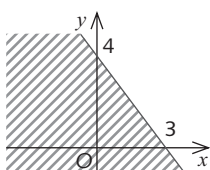
m



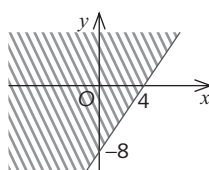
n



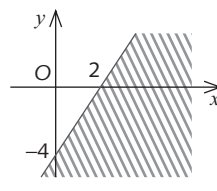
o



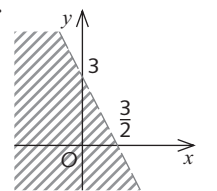
p



q



r



2

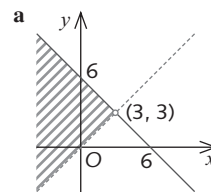
$$a \ y \geq 2x + 4$$

$$b \ y < -x + 2$$

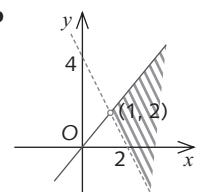
$$c \ x + y < 3$$

$$d \ y \geq 2x + 1$$

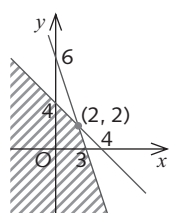
3



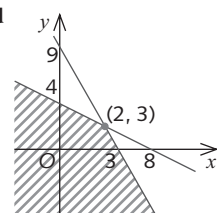
b



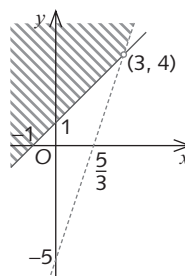
c



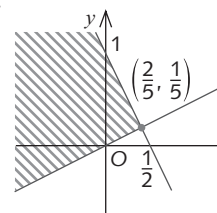
d



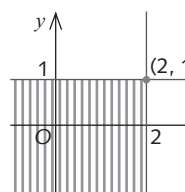
e



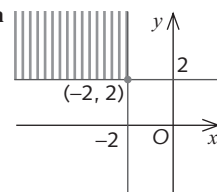
f



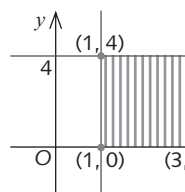
g



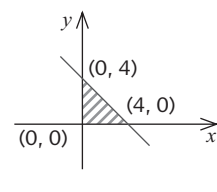
h



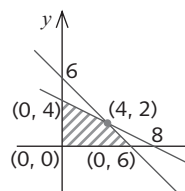
i



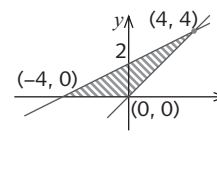
j



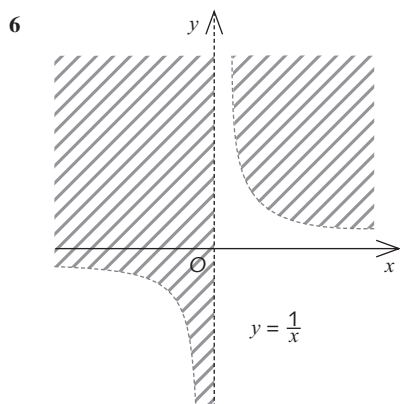
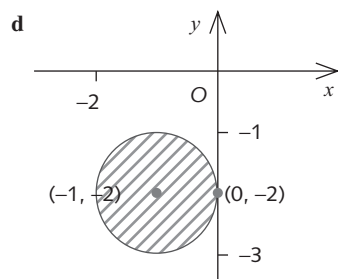
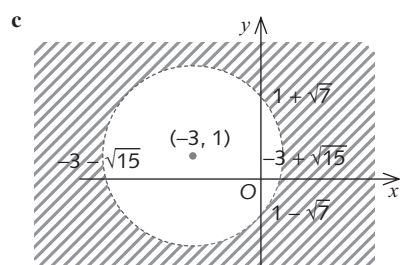
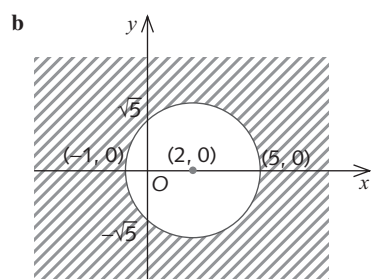
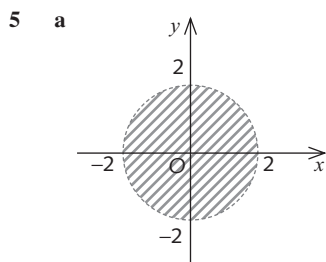
k



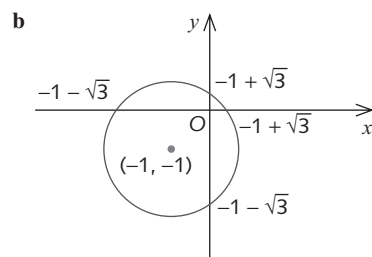
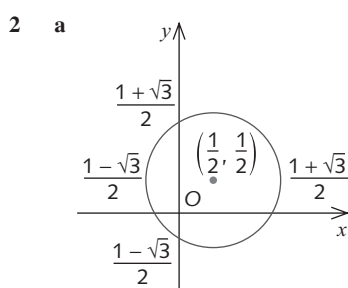
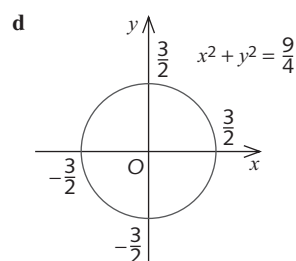
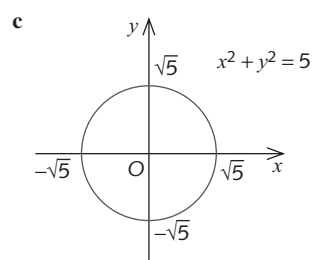
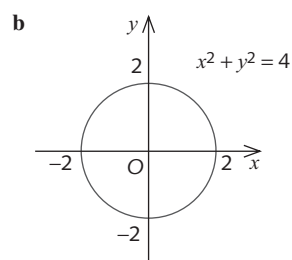
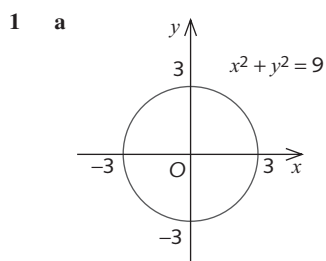
l

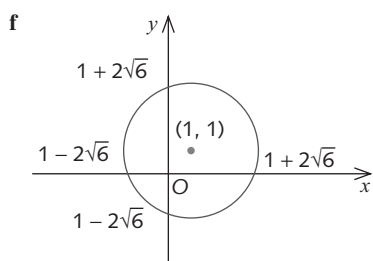
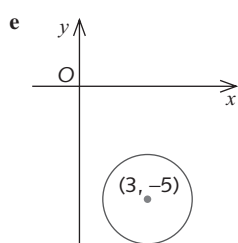
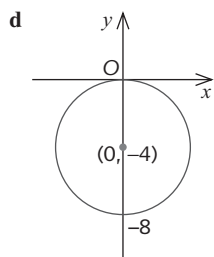
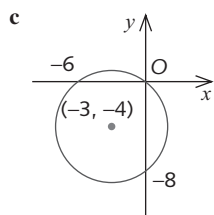


- 4 a $x + y \leq 2, y - x \leq 2$ b $y \geq x - 1, y \leq 3x + 3$
 c $y \geq 2, x < 2$ d $y \geq x, y < 2$
 e $y \leq x, x \leq 2, y \geq 0$
 f $x \geq 0, y \geq 0, x \leq 3, x + y \leq 6$



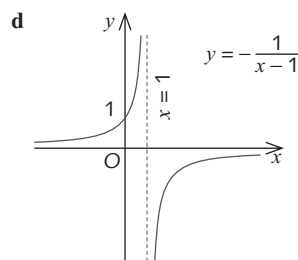
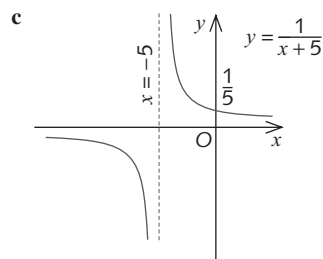
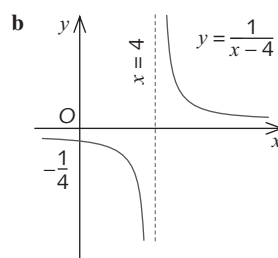
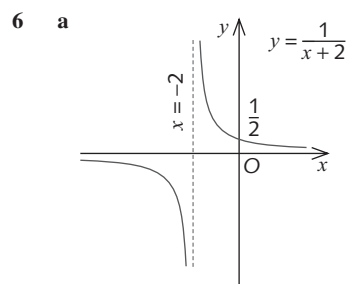
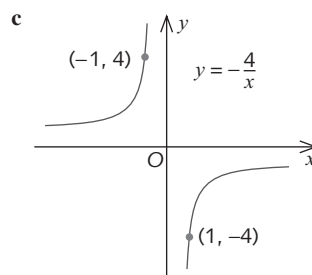
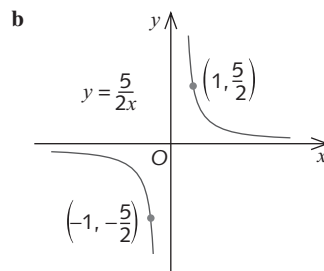
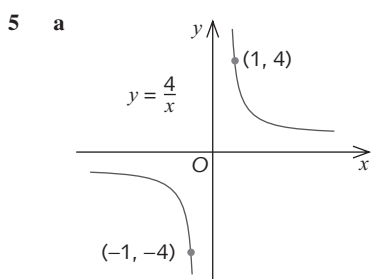
Review exercise





- 3 a centre $(-2, -4)$, radius $2\sqrt{5}$
 b centre $(-2, -1)$, radius $\sqrt{10}$
 c centre $\left(2, -\frac{5}{4}\right)$, radius $\frac{\sqrt{65}}{4}$
 d centre $(2, -3)$, radius $5\sqrt{2}$

- 4 a $x^2 + y^2 = 9$
 b $(x + 1)^2 + (y - 4)^2 = 36$
 c $(x - 2)^2 + (y - 5)^2 = 1$
 d $(x + 2)^2 + (y + 6)^2 = 16$



7 a $(-2, 18), (2, 6)$

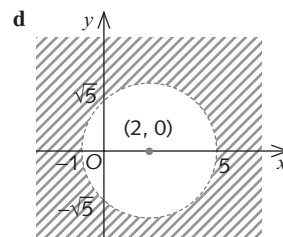
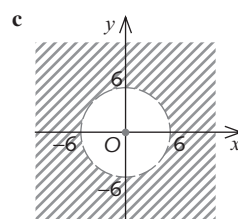
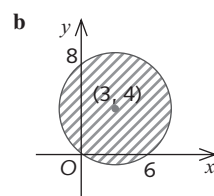
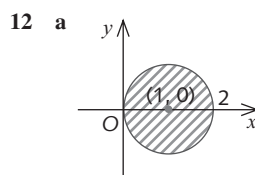
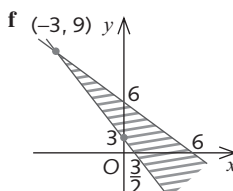
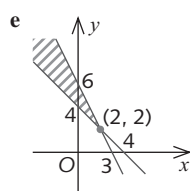
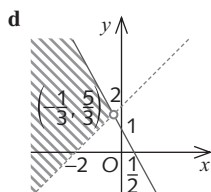
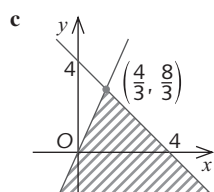
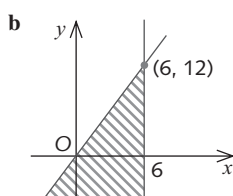
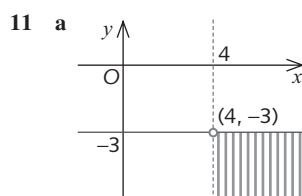
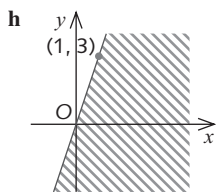
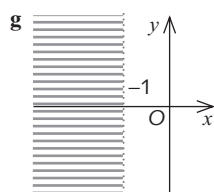
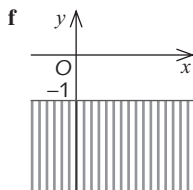
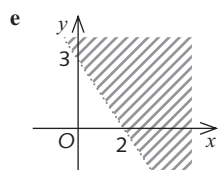
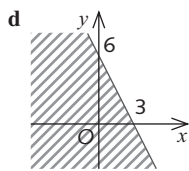
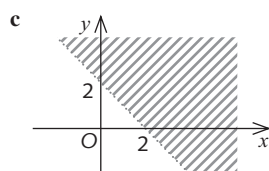
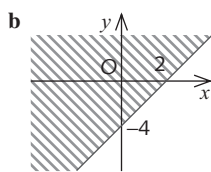
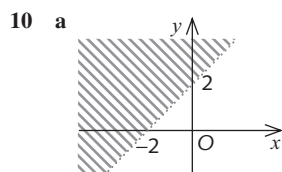
8 a $(3, 0)$
c $(-2, -2\sqrt{3}), (2, 2\sqrt{3})$

9 a $(4, 3)$

b $(0, 2), (2, -12)$

b $(-4, 0), (4, 0)$

b $\left(-\frac{7}{5}, -\frac{26}{5}\right), (2, 5)$



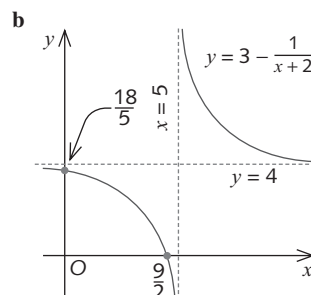
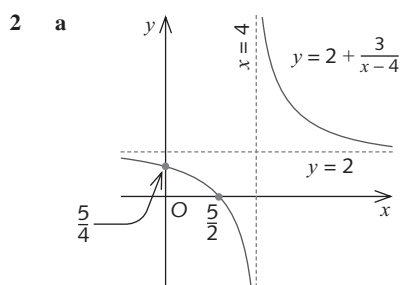
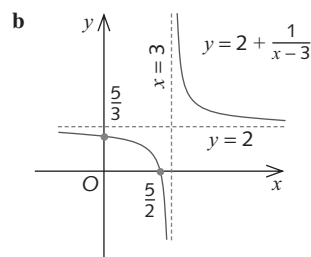
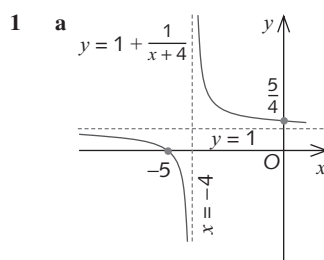
13 a $(4, 3), (-3, -4)$

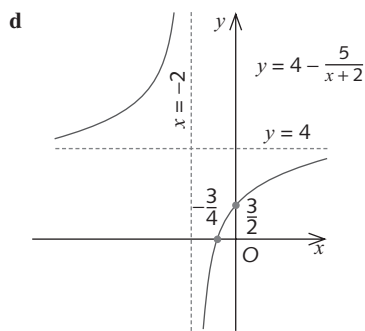
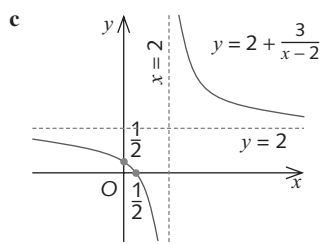
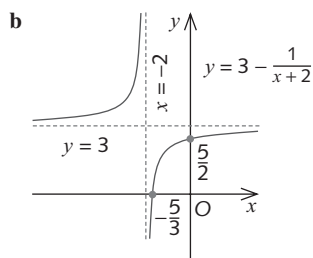
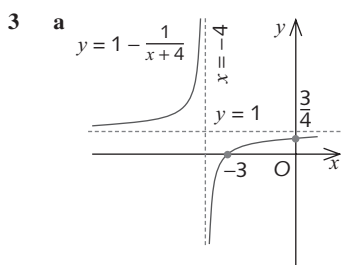
c $(3, 2)$

d no points of intersection

b $\left(\frac{1}{2}, -6\right), (3, -1)$

Challenge exercise





4 a i $y = -\frac{1}{\sqrt{3}}x + \frac{a}{3}$ ii $y = \frac{1}{\sqrt{3}}x + \frac{a}{3}$
 c $\frac{2}{3}a$ d $x^2 + \left(y - \frac{a}{3}\right)^2 = \frac{4a^2}{9}$

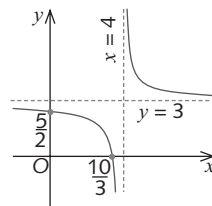
6 Centre $\left(\frac{a}{2}, \frac{a}{2}\right)$, radius $\frac{\sqrt{2}}{2}a$, expand
 $\left(x - \frac{a}{2}\right)^2 + \left(y - \frac{a}{2}\right)^2 = \left(\frac{\sqrt{2}}{2}a\right)^2$

7 a i $x = \frac{a}{2}$ ii $y = \frac{b}{2}$ b $\left(\frac{a}{2}, \frac{b}{2}\right)$
 d $\left(x - \frac{a}{2}\right)^2 + \left(y - \frac{b}{2}\right)^2 = \frac{a^2 + b^2}{4}$

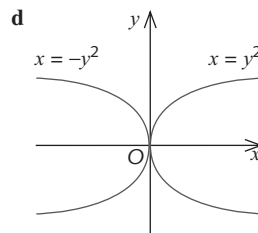
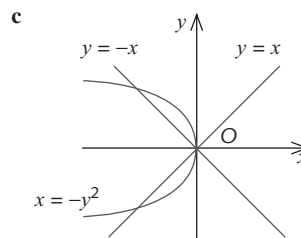
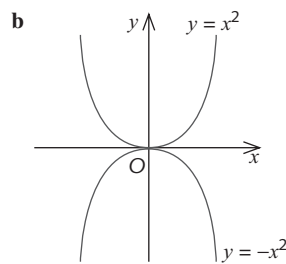
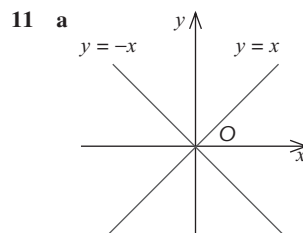
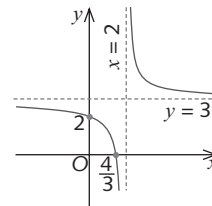
8 $\left(x - \frac{a^2 + b^2}{2b}\right)^2 + y^2 = \frac{(a-b)^4 + 4b^4}{4b^2}$

9 $\left(x - \frac{5}{2}\right)^2 + y^2 = \frac{5}{4}$

10 a $y - 3 = \frac{2}{x-4}$
 $y = 3 + \frac{2}{x-4}$



b $y - 3 = \frac{2}{x-2}$
 $y = 3 + \frac{2}{x-2}$



12 (0, 3) and (2, 0) 13 $\left(-\frac{9}{5}, \frac{12}{5}\right)$ and (-1, 3)

14 $x^2 + y^2 - x - 5y + 4 = 0$

15 $ax + b = \frac{1}{x}$, eliminating y .

$ax^2 + bx = 1$

$ax^2 + bx - 1 = 0$

$x = \frac{-b \pm \sqrt{b^2 + 4a}}{2a}$ and $b^2 + 4a > 0$, since $a > 0$

So there are always two points of intersection between the line and the hyperbola.

Chapter 12 answers

Exercise 12A

- a** $a = 7.42$ **b** $b = 3.71$ **c** $c = 6.17$
d $d = 2.29$ **e** $i = 3.51$ **f** $j = 13.59$
- a** $a = 14.97$ **b** $b = 9.01$ **c** $e = 11.75$
d $h = 38.70$
- a** 25.4° **b** 56.3° **c** 60.3°
d 70.9° **e** 39.6° **f** 70.8°
- a** $x = 7.10$ **b** $\theta = 44.9^\circ$ **c** $a = 8.08$
d $a = 2.26$ **e** $\theta = 61.9^\circ$ **f** $\theta = 52.1^\circ$
g $x = 10.15$ **h** $y = 10.63$ **i** $a = 9.39$
- a** $3.92 \text{ cm}, 2.52 \text{ cm}, 50^\circ$ **b** $6.97 \text{ cm}, 40.7^\circ, 49.3^\circ$
c $9.78 \text{ cm}, 30.8^\circ, 59.2^\circ$

Exercise 12B

- a** 5 **b** $6\sqrt{2}$ **c** $6\sqrt{3}$
d $4\sqrt{3}$ **e** $4\sqrt{3}$
- a** 1 **b** -1 **c** $2 - \sqrt{3}$
d $\frac{\sqrt{2} + \sqrt{6}}{4}$ **e** $\frac{\sqrt{3}}{2}$ **f** 0
- 10 cm and $10\sqrt{3}$ cm
- a** $16\sqrt{3}$ cm **b** $12\sqrt{3}$ cm **c** $8\sqrt{3}$ cm
d $4\sqrt{3}$ cm **e** 12 cm
- a** $a = \frac{40\sqrt{3}}{3}$ and $x = \frac{40}{3}$ **b** $50(\sqrt{3} + 1)$
- $5\sqrt{3}$ cm

Exercise 12C

- a** $2\sqrt{41}$ cm **b** 38.7° **c** $2\sqrt{77}$ cm
d 27.1° **e** $2\sqrt{34}$ cm **f** 34.4°
- a** $12\sqrt{3}$ cm **b** 35.3° **c** 90°
d 35.3°
- a** $10\sqrt{2}$ cm **b** $5\sqrt{2}$ cm **c** $\sqrt{194}$ cm
d 59.5° **e** 5 cm **f** 67.4°
g 69.0°
- a** 47.5 cm **b** 40 cm **c** 45.7°
- a** 31.2 m
b i $50\sqrt{5} \text{ m} \approx 111.8 \text{ m}$
 ii 15.6°
 iii 296.6°
- a** 130.6 m **b** 96.2 m **c** 162.2 m
d 216.4°T
- 81 m
- a** 73 m **b** 260 m **c** 16°
- 42 m

Exercise 12D

- a** 10.46 **b** 6.88 **c** 8.69
- a** 8.63 **b** 10.89 **c** 9.34

- a** 7.33 **b** 10.63
- a** 54° **b** 34° **c** 66°
- a** 47.34° **b** 61.66° **c** 16.76 cm
- $\angle ACD \approx 35.07^\circ, 26.01^\circ$
- a** 708 m **b** 547 m
- a** 27° **b** 132.56 m **c** 119 m
- 16 m **10** 7.11 m

Exercise 12E

- a** 65° **b** 57° **c** 42° **d** 85°

- a** $\frac{1}{\sqrt{2}}$ **b** $-\frac{1}{\sqrt{2}}$

- a** $30^\circ, 150^\circ$ **b** $35^\circ, 145^\circ$ **c** 151°

- | θ | 30° | 120° | 150° | 90° | 135° |
|---------------|----------------------|----------------------|-----------------------|------------|-----------------------|
| $\sin \theta$ | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ | 1 | $\frac{1}{\sqrt{2}}$ |
| $\cos \theta$ | $\frac{\sqrt{3}}{2}$ | $-\frac{1}{2}$ | $-\frac{\sqrt{3}}{2}$ | 0 | $-\frac{1}{\sqrt{2}}$ |

- a** 44.42 **b** 6.47
- a** 140° **b** 155°
- a** The two angles are supplementary, and the sines of two supplementary angles are the same.
b $h = b \sin A$ **c** $h = a \sin B$
- a** 59° **b** 25.1 m
- a** 234°T **b** 1285 m
- a** 341°T **b** 1282 m

Exercise 12F

- a** 8.65 cm **b** 3.82 cm
c 16.34 cm **d** 6.93 cm
- 17.91 cm **3** 6.2 m
- 410 km **5** 103 km
- a** 14.38 m **b** 7.01 m **c** 50.43 m^2
- From $\triangle BXA$

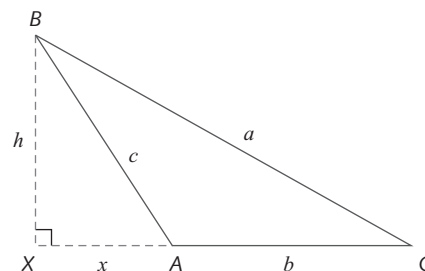
$$c^2 = h^2 + x^2 \text{ and } x = c \cos(180 - A) = -\cos A$$

From $\triangle BXC$,

$$a^2 = (b + x)^2 + h^2$$

Hence,

$$\begin{aligned} a^2 &= b^2 + 2bx + x^2 + c^2 - x^2 \\ &= b^2 + c^2 - 2bc \cos A \end{aligned}$$

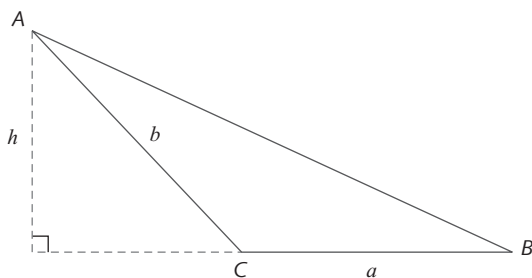


Exercise 12G

- 1 a $x^2 = y^2 + z^2 - 2yz \cos R$
b $b^2 = a^2 + c^2 - 2ac \cos B$
c $p^2 = q^2 + r^2 - 2qr \cos B$
- 2 a 52.6° b 54.8° c 78.5°
- 3 a $A = 39.8^\circ, B = 45.4^\circ, C = 94.8^\circ$
b $A = B = 29.0^\circ, C = 122.1^\circ$
c $A = 21.0^\circ, B = 18.6^\circ, C = 140.4^\circ$
- 4 19.4°
- 5 108.4°
- 6 $16^\circ, 34^\circ, 130^\circ$
- 7 92.1°
- 8 a i 21.8° ii 4.4 cm b $9.5 \text{ cm}, 11.8 \text{ cm}$

Exercise 12H

- 1 a 39.5 cm^2 b 29.8 cm^2 c 48.5 cm^2
- 2 a 10.08 cm^2 b 46.98 cm^2 c 43.30 cm^2
- 3 a 73.2° b 56.8° c 134 cm^2
- 4 a 36.3° b 21 cm^2
- 5 a 57 cm^2 b 30 cm^2 c 199 cm^2
- 6 a 8.5 cm b 9.6 cm c 36.7°
- 7 22°
- 8 a 152.6 m b 159.7° c 6213.6 m^2
- 9 a 108° b 47.55 cm^2 c 16.18 cm
d 36° e 72° f 36°
g 76.94 cm^2 h 172.05 cm^2
- 10 98 cm^2
- 11 Area $= \frac{1}{2} \times (ka) \times (kb) \sin C = \frac{1}{2} k^2 \times ab \sin C$
 $= k^2 \times \text{area of triangle } ABC$
- 12 Area $= \frac{1}{2} \times a \times h$
 $= \frac{1}{2} ab \sin (180 - C)$
 $= \frac{1}{2} ab \sin C$



- 13 a 31.29° b 42.85 cm^2 c 34 cm
d 42.85 cm^2

Review exercise

- 1 a 31.0° b 33.6° c 10.58
d 7.20

- 2 a 10 b $18\sqrt{2}$ c $25\sqrt{3}$
- 3 $A = 26.4^\circ, B = 117.3^\circ, C = 36.3^\circ$
- 4 a 92° b 1211 m^2
- 5 48 6 7 7 24.47 cm^2
- 8 a 49.0° b 71.0° c 20.3 cm
d 142.5 cm^2
- 9 141 m
- 10 a $\angle C = 91^\circ, BC \approx 5.44 \text{ cm}, AC \approx 4.31 \text{ cm}$
b $BC \approx 4.46 \text{ cm}, \angle C \approx 90.19^\circ, \angle B \approx 41.81^\circ$
c $AB \approx 11.17 \text{ cm}, \angle B \approx 61.88^\circ, \angle A \approx 38.12^\circ$
d $AC \approx 19.08 \text{ cm}, \angle A \approx 33.00^\circ, \angle C \approx 27.00^\circ$
e $\angle B \approx 105.05^\circ, \angle C \approx 43.95^\circ, AB \approx 10.78 \text{ cm}$
f $\angle C \approx 149.00^\circ, \angle A \approx 13.00^\circ, BC \approx 10.92 \text{ cm}$

Challenge exercise

- 1 $\frac{1}{2} ab \sin C = \frac{1}{2} ac \sin B, \frac{b}{\sin B} = \frac{c}{\sin C}$
- 2 a i $\frac{h}{\tan 12^\circ}$ ii $\frac{h}{\tan 9^\circ}$
b 47.5 m c 318°T
- 4 b $b = a \cos C + c \cos A, c = a \cos B + b \cos A$
- 7 If given angle A and sides b and a :
0 triangles if $a < b \sin A$ or A is obtuse and $a < b$
1 triangle if A is obtuse and $a > b$ or A is acute and $a = b \sin A$
2 triangles if A is acute and $b \sin A < a < b$

Chapter 13 answers

Exercise 13A

- 1 a ii 90°
b ii The angle at the centre is twice any angle at the circumference standing on the same minor arc.
iii The reflex angle at the centre is twice any angle at the circumference standing on the same major arc.
- 2 a $\alpha = 90^\circ$ (Thales' theorem),
 $\beta = 75^\circ$ (sum of angles in a triangle is 180°)
b $\theta = 90^\circ$ (Thales' theorem)
c $\theta = 10^\circ$ ($\angle JLK = 90^\circ$, Thales' theorem, sum of angles in a triangle is 180°)
d $\gamma = 70^\circ$ ($OS = OT$ radii of circle and base angles of an isosceles triangle are equal),
 $\alpha = 40^\circ$ (sum of angles in a triangle is 180°),
 $\theta = 140^\circ$ (straight angle),
 $\beta = 20^\circ$ (base angles of an isosceles triangle)
e $\theta = 55^\circ$ ($OZ = OY$ radii of circle, base angles of an isosceles triangle are equal and the sum of angles in a triangle is 180°),
 $\beta = 35^\circ$ (external angle of isosceles triangle is 70°)
f $\theta = 80^\circ$ ($\angle AOC = 20^\circ$ and $\triangle AOC$ is isosceles)

- 3 a $\theta = 110^\circ$ (angle at the centre is twice angle at the circumference standing on the same arc)
 b $\gamma = 44^\circ$ (angle at the circumference is half angle at the centre standing on the same arc)
 c $\alpha = 190^\circ$ (angle at the centre is twice angle at the circumference standing on the same arc)
 d $\gamma = 100^\circ$ (angle at the circumference is half angle at the centre standing on the same arc)
 e $\theta = 129^\circ$ (angle at the circumference is half angle at the centre standing on the same arc)
 f $\alpha = 40^\circ$ (angle at the circumference is half angle at the centre standing on the same arc)
 g $\beta = 30^\circ$ (angle at the circumference is half angle at the centre standing on the same arc)
 h $\gamma = 100^\circ$ (angle at the centre is twice angle at the circumference standing on the same arc)
 i $\theta = 24^\circ$ (angle at the centre is twice angle at the circumference standing on the same arc)
- 4 a $\alpha = 120^\circ$ (sum of angles about a point is 360°),
 $\beta = 60^\circ$ (angle at the circumference is half angle at the centre standing on the same arc)
 b $\theta = 30^\circ$ ($\angle AOB = 60^\circ$, sum of angles about a point is 360° and angle at the circumference is half angle at the centre standing on the same arc so θ is half of $\angle AOB$)
 c $\theta = 220^\circ$ ($\angle SOR = 140^\circ$, angle at the centre is twice angle at the circumference standing on the same arc and $\theta + \angle SOR = 360^\circ$, sum of angles about a point is 360°)
 d $\alpha = \beta = 40^\circ$ (any angle at the circumference is half angle at the centre standing on the same arc)
 e $\theta = 320^\circ$ ($\angle SOR = 40^\circ$, angle at the centre is twice angle at the circumference standing on the same arc and $\theta + \angle SOR = 360^\circ$, sum of angles about a point is 360°)
 $\alpha = 20^\circ$ (angle at the circumference is half angle at the centre standing on the same arc, $\angle SOR = 40^\circ$)
 f $\alpha = 100^\circ$ ($OR = OQ$ radii so base angles of an isosceles triangle are both 40° , sum of angles in a triangle is 180°),
 $\beta = 140^\circ$ (sum of angles about a point is 360°),
 $\gamma = 20^\circ$ ($OR = OP$ radii, γ is a base angle of an isosceles triangle)
 g $\alpha = 80^\circ$ (angle at the circumference is half angle at the centre standing on the same arc),
 $\beta = 200^\circ$ (sum of angles about a point is 360°),
 $\gamma = 100^\circ$ (angles at the circumference is half angle at the centre standing on the same arc)
 h $\alpha = 60^\circ$ ($\angle DAB = 90^\circ$, Thales' theorem, so $\alpha + 30^\circ = 90^\circ$),
 $\beta = 60^\circ$ ($OA = OB$ radii, so $\alpha = \beta$, base angles of isosceles triangle),
 $\gamma = 30^\circ$ ($\angle ABC = 90^\circ$, Thales' theorem, so $\gamma + \beta = 90^\circ$)
 i $\alpha = \beta = 45^\circ$ (both are base angles in isosceles triangles with the third angle 90°)
- 5 a $\alpha = 90^\circ$ (Thales' theorem),
 $\beta = 10^\circ$ (alternate angles, $AB \parallel FG$)
 b $\alpha = 60^\circ$ ($OP = OA = AP$ so the triangle is equilateral),
 $\beta = 30^\circ$ (angle at the circumference is half angle at the centre standing on the same arc)
 c $\alpha = 20^\circ$ (alternate angles, $PO \parallel QR$),
 $\gamma = 40^\circ$ (angle at the centre is twice angle at the circumference standing on the same arc),
 $\beta = 40^\circ$ (alternate angles, $PO \parallel QR$)

- d $\alpha = 220^\circ$ (sum of angles about a point),
 $\beta = 110^\circ$ (angle at the circumference is half angle at the centre standing on the same arc),
 $\gamma = 60^\circ$ (sum of angles in a quadrilateral is 360°)
- e $\alpha = 200^\circ$ (sum of angles about a point),
 $\beta = 100^\circ$ (angle at the circumference is half angle at the centre standing on the same arc),
 $\gamma = 80^\circ$ (co-interior angles, $PQ \parallel OR$)
- f $\alpha = 100^\circ$ (angle at the circumference is half angle at the centre standing on the same arc),
 $\beta = 60^\circ$ (construct OA , as $OB = AB = OA$ the triangle is equilateral),
 $\gamma = 40^\circ$ (sum of angles in a quadrilateral is 360°)
- 6 a i $OA = OP$ so $\triangle APO$ is isosceles with
 $\angle PAO = \angle APO = \alpha$,
 $OB = OP$ so $\triangle PBO$ is isosceles with
 $\angle PBO = \angle BPO = \beta$,
 so $\angle APB = \alpha + \beta$; $\angle XPB = \alpha + \beta$, external angle of $\triangle APB$
 ii $\angle XPB + \angle APB = 180^\circ$, $\alpha + \beta + \alpha + \beta = 180^\circ$,
 $2(\alpha + \beta) = 180^\circ$, $\alpha + \beta = 90^\circ$
- b i $OA = OP$ so $\triangle APO$ is isosceles with
 $\angle PAO = \angle APO = \alpha$, $OB = OP$ so $\triangle PBO$ is isosceles with
 $\angle PBO = \angle BPO = \beta$, $\angle AOM = 2\alpha$
 and $\angle BOM = 2\beta$, external angles of $\triangle APO$
 and $\triangle PBO$
 ii $\angle AOM + \angle BOM = 180^\circ$ so $2\alpha + 2\beta = 180^\circ$
 iii Thus $\alpha + \beta = 90^\circ$
- 7 a i $OB = OP$ so $\triangle PBO$ is isosceles with
 $\angle PBO = \angle BPO = \beta$, so $\angle APB = \beta$
 ii $\angle AOB = 2\beta$, (exterior angle of triangle)
- b i $OA = OP$ so $\triangle APO$ is isosceles with
 $\angle PAO = \angle APO = \alpha$,
 $OB = OP$ so $\triangle PBO$ is isosceles with
 $\angle PBO = \angle BPO = \beta$,
 so $\angle APB = \angle BPO - \angle APO = \beta - \alpha$
 ii $\angle XO B = 2\beta$ (exterior angle of triangle), $\angle XO A = 2\alpha$
 (exterior angle of triangle),
 $\angle AOB = 2\beta - 2\alpha = 2(\beta - \alpha) = 2\angle APB$
- 8 a The diagonals bisect each other.
 b A parallelogram with a right angle is a rectangle.
 c The diagonals of a rectangle are equal and bisect each other, so $OA = OB = OP$. A circle with diameter AB has centre O and passes through P .
- 9 The angle at the centre is twice the angle at the circumference. As the horse moves from position 1 to position 2, angles are subtended both at the binoculars and the centre.

Exercise 13B

- 1 b They are equal.
 c They are equal.
- 2 a $\alpha = 50^\circ$ (angles at the circumference standing on the same arc)
 b $\alpha = \beta = 20^\circ$ (angles at the circumference standing on the same arc)

- c** $\alpha = 20^\circ$ (angles at the circumference standing on the same arc)
 $\beta = 40^\circ$ (angles at the circumference standing on the same arc)
- d** $\alpha = 40^\circ$ (angles at the circumference standing on the same arc)
 $\beta = 30^\circ$ (angles at the circumference standing on the same arc)
- e** $\theta = 90^\circ$ (angles at the circumference standing on the same arc)
 $\alpha = 40^\circ$ ($\alpha + \theta = 130^\circ$, exterior angle of triangle)
- f** $\alpha = 20^\circ$ (angles at the circumference standing on the same arc)
 $\theta = 100^\circ$ ($\angle QPK = 60^\circ$, angles at the circumference standing on the same arc, sum of angles in triangle is 180°)
- 3**
- a** $\theta = 80^\circ$ (opposite angles in a cyclic quadrilateral are supplementary)
- b** $\alpha = 100^\circ$ (opposite angles in a cyclic quadrilateral are supplementary)
 $\beta = 95^\circ$ (opposite angles in a cyclic quadrilateral are supplementary)
- c** $\alpha = 40^\circ$ (angles at the circumference standing on the same arc)
 $\beta = 45^\circ$ (angles at the circumference standing on the same arc)
 $\theta = 35^\circ$ ($80 = \beta + \theta$ exterior angle),
 $\gamma = 60^\circ$ (sum of angles in triangle ACD is 180°)
- d** $\alpha = 50^\circ$ (opposite angles in a cyclic quadrilateral are supplementary)
 $\beta = 90^\circ$ (sum of angles in triangle is 180°),
 $\gamma = 30^\circ$ (sum of angles in triangle is 180°)
- e** $\alpha = 110^\circ$ (straight angle),
 $\gamma = 70^\circ$ (opposite angles in a cyclic quadrilateral are supplementary)
 $\beta = 80^\circ$ (straight angle),
 $\theta = 100^\circ$ (opposite angles in a cyclic quadrilateral are supplementary)
- f** $\alpha = 20^\circ$ (angles at the circumference standing on the same arc),
 $\beta = 90^\circ$ (Thales' theorem),
 $\gamma = 90^\circ$ (Thales' theorem),
 $\theta = 70^\circ$ (opposite angles in a cyclic quadrilateral are supplementary)
- 4**
- a** $\alpha = 70^\circ$ (co-interior angles, $DC \parallel AB$),
 $\gamma = 110^\circ$ (opposite angles in a cyclic quadrilateral are supplementary),
 $\beta = 70^\circ$ (opposite angles in a cyclic quadrilateral are supplementary)
- b** $\alpha = 65^\circ$ (angles at the circumference standing on the same arc),
 $\gamma = 65^\circ$ (alternate, $TU \parallel SR$)
 $\beta = 65^\circ$ (alternate, $TU \parallel SR$)
- c** $\alpha = 30^\circ$ ($\angle BQM = 30^\circ$, alternate, $BQ \parallel AP$),
 $\beta = 30^\circ$ (angles at the circumference standing on the same arc),
 $\gamma = 30^\circ$ (angles at the circumference standing on the same arc)
- d** $\beta = 70^\circ$ ($\angle JML = 90^\circ$, Thales' theorem, sum of angles in triangle is 180°)
 $\alpha = 20^\circ$ ($\angle MJK = 90^\circ$, Thales' theorem, $\alpha + \beta = 90^\circ$)
- e** $\alpha = 70^\circ$ (alternate, $AD \parallel BC$),
 $\beta = 40^\circ$ ($\triangle AOB$ is isosceles with base angles α , so $\angle AOB = 40^\circ$, alternate, $AD \parallel BC$),
 $\gamma = 40^\circ$ ($\triangle BOC$ is isosceles with base angles γ and β),
 $\theta = 70^\circ$ ($\angle DOC = 40^\circ$, alternate, $AD \parallel BC$ and $\triangle DOC$ is isosceles with base angles θ)
- f** $\alpha = 60^\circ$ (construct SO , $\triangle RSO$ is equilateral),
 $\beta = 120^\circ$ (opposite angles in a cyclic quadrilateral are supplementary),
 $\gamma = 60^\circ$ (co-interior, $ST \parallel RU$)
- 5**
- a**
- i** $\angle Q = \angle T = \theta$ (angles at the circumference standing on the same arc),
 $\angle P = \angle T = \theta$ (alternate, $PQ \parallel ST$), $\angle P = \angle S = \theta$ (angles at the circumference standing on the same arc),
so $\angle P = \angle Q = \angle S = \angle T = \theta$
- ii** $\angle P = \angle Q = \angle S = \angle T = \theta$, $\triangle SMT$ and $\triangle PMQ$ are isosceles, so $SM = TM$ and $PM = QM$, so $SM + MQ = TM + MP$ giving $SQ = TP$
- b**
- i** $\alpha = 60^\circ$ ($\triangle PQR$ is equilateral),
 $\beta = 120^\circ$ (opposite angles in a cyclic quadrilateral are supplementary),
 $\gamma = 30^\circ$ (base angle of isosceles triangle PGQ)
- ii** $\angle GRP = 30^\circ$, $\angle GRQ = 30^\circ$ (angles standing on the same arc),
 $\angle PMR = 90^\circ$ so $PQ \perp GR$
- 6**
- a**
- i** $\alpha = 110^\circ$ (opposite angles in a cyclic quadrilateral are supplementary),
 $\beta = 70^\circ$ (straight angle),
 $\gamma = 110^\circ$ (opposite angles in a cyclic quadrilateral are supplementary)
- ii** As $\angle TQP$ and $\angle SPQ$ are co-interior and supplementary then $PS \parallel QT$.
- b**
- i** $\alpha = 130^\circ$ (opposite angles in a cyclic quadrilateral are supplementary),
 $\beta = 50^\circ$ ($\triangle ABM$ is isosceles, straight angle at A),
 $\gamma = 50^\circ$ ($\angle QBA = 130^\circ$, straight angle at B , opposite angles in a cyclic quadrilateral are supplementary)
- ii** $\angle QPM$ and $\angle BAP$ are co-interior and supplementary, so $PQ \parallel AB$
- iii** $\angle QPM = \angle PQM$, QMP is isosceles with $QM = PM$, also $BM = AM$, so $QB = PA$
- 7**
- a**
- i** $\angle ABS = 90^\circ$ and $\angle ABT = 90^\circ$ (Thales' theorem)
- ii** $\angle TBS = \angle ABS + \angle ABT = 180^\circ$, so $\angle TBS$ is a straight angle $\therefore T, B$ and S are collinear
- b**
- i** $\angle ABC = 90^\circ$; (opposite angles in a cyclic quadrilateral are supplementary), $\angle ABD = 90^\circ$ (Thales' theorem)
- ii** $\angle CBD = \angle ABC + \angle ABD = 180^\circ$, so $\angle CBD$ is a straight angle $\therefore C, B$ and D are collinear
- iii** $\angle ANC = 90^\circ$; by the converse of Thales' theorem $\angle ANC$ is in a semicircle with AC the diameter

- 8 a $\angle DBC = \angle DAC = \alpha$ (angles at the circumference standing on the same arc)
 $\angle BDC = \angle BAC = \beta$ (angles at the circumference standing on the same arc)
 b $\angle DBC + \angle BDC + \angle BCD = 180^\circ$ (sum of angles in a triangle),
 $\alpha + \beta + \angle BCD = 180^\circ$, so $\angle BCD = 180^\circ - \alpha - \beta$
 c $\angle BAD = \alpha + \beta$; thus, $\angle BAD + \angle BCD = 180^\circ$
- 9 a i $\angle C = 180^\circ - \theta$ (opposite angles in a cyclic quadrilateral are supplementary),
 $\angle C = \theta$ (opposite angles in a parallelogram are equal)
 ii From a $180^\circ - \theta = \theta$, so $\theta = 90^\circ$, so
 $\angle C = \angle A = 90^\circ$, similarly $\angle B = \angle D = 90^\circ$, so $ABCD$ is a rectangle.
 b A rhombus is a parallelogram with all sides equal and so if a cyclic parallelogram is a rectangle, a cyclic rhombus must be a square.
- 10 a $\angle MDC = 180^\circ - \theta$ (co-interior, $AB \parallel DC$) and $\angle BCD = 180^\circ - \theta$ (opposite angles in a cyclic quadrilateral are supplementary) $\therefore \triangle MDC$ is isosceles
 $\angle MBA = 180^\circ - \theta$ (corresponding angles, $AB \parallel DC$) and $\angle MAB = 180^\circ - \theta$ (straight angle at A) $\therefore \triangle MAB$ is isosceles
 b As $\triangle MDC$ is isosceles, $MD = MC$ and as $\triangle MAB$ is isosceles, $MA = MB$, so $AD = BC$

Exercise 13C

- 2 a $x = 3, \theta = 53.1^\circ$
 b $x = 2\sqrt{13}, \theta = 33.7^\circ$
 c $x = 4\sqrt{3}, \theta = 30^\circ$
- 3 a i 17.0 cm ii 8.5 cm
 b i 24 cm ii 73.7°
 c i 6 cm ii 106.3°
- 4 a $\theta = 50^\circ$ and $\angle UOT = 50^\circ$ (chords of equal length subtend equal angles at the centre of a circle)
 $\alpha = 65^\circ$ since $\triangle UOT$ is isosceles
 b $\alpha = 60^\circ$ ($\triangle ABO$ is equilateral),
 $\beta = 240^\circ$ ($\triangle ABO$ and $\triangle BCO$ are equilateral, sum of angles at a point)
 c $\theta = 30^\circ$ ($\triangle RQO$ is equilateral, so $\angle QRO = 60^\circ$ and $\angle RQP = 90^\circ$, Thales' theorem)
 d $\alpha = 45^\circ$ (base angle of isosceles triangle with third angle 90°),
 $\beta = 25^\circ$ (base angle of isosceles triangle with third angle 130°),
 $\gamma = 20^\circ$ (base angle of isosceles triangle with third angle 140°)
 e $\theta = 50^\circ$ ($\angle HOF = 130^\circ$ as $\triangle HOF$ is isosceles, $\angle GOF$ is a straight angle)
 f $\alpha = 36^\circ$ (10 congruent triangles in a circle),
 $\beta = 72^\circ$ (base angle of isosceles triangle with third angle 36°)
- 5 a $\angle FPO = 60^\circ$ ($\triangle FPO$ is equilateral), $\angle FGO = 30^\circ$ (angle at the centre is twice angle at the circumference standing on the same arc) $\angle FMO = 90^\circ$ (angle sum of triangle)
 b $FG = \sqrt{3}$
- 6 a i $AO = BO$ (radii of the circle), $AM = BM$ (as M is midpoint of AB),
 OM is common $\therefore \triangle AOM \equiv \triangle BOM$ (SSS)
 ii $\angle AMO = \angle BMO$ (matching angles, $\triangle AOM \equiv \triangle BOM$) and
 $\angle AMO + \angle BMO = 180^\circ$ (straight angle at M), so
 $\angle AMO = \angle BMO = 90^\circ$
 $\angle AOM = \angle BOM$ (matching angles, $\triangle AOM \equiv \triangle BOM$)
- b i $\angle AMO = \angle BMO = 90^\circ$ (given), $AO = BO$ (radii of the circle),
 OM is common $\therefore \triangle AOM \equiv \triangle BOM$ (RHS)
 ii $\angle AOM = \angle BOM$ (matching angles, $\triangle AOM \equiv \triangle BOM$) and
 $AM = BM$ (matching sides, $\triangle AOM \equiv \triangle BOM$)
 so OM bisects $\angle AOB$
- c i $\angle AOM = \angle BOM = \alpha$ (given), $AO = BO$ (radii of the circle),
 OM is common $\therefore \triangle AOM \equiv \triangle BOM$ (SAS)
 ii $\angle AMO = \angle BMO$ (matching angles, $\triangle AOM \equiv \triangle BOM$) and
 $\angle AMO + \angle BMO = 180^\circ$ (straight angle at M), so
 $\angle AMO = \angle BMO = 90^\circ$ $AM = BM$ (matching sides, $\triangle AOM \equiv \triangle BOM$)
- 7 a i Join OA and OP . $OA = OP = OQ$ (radii of the circle) so $\triangle AOP$ and $\triangle AOQ$ are isosceles with base angles θ making the third angle in both triangles $180^\circ - 2\theta$, so $\triangle AOP \equiv \triangle AOQ$ (SAS)
 ii $AP = AQ$ (matching sides, $\triangle AOP \equiv \triangle AOQ$)
 b i Join OS . $\angle TFS = \angle SFO = \theta$ (given) and $\triangle FSO$ is isosceles ($OF = OS$, radii),
 so $\angle SFO = \angle FSO \therefore \angle TFS = \angle FSO$
 ii Join OF . $\angle TFS$ and $\angle FSO$ are alternate and equal, so $FT \parallel OS$
 c i $SQ = SP$ (given), $QT = PT$ (given), ST is common $\therefore \triangle SQT \equiv \triangle SPT$ (SSS)
 ii $\angle P = \angle Q$ (matching angles, $\triangle SQT \equiv \triangle SPT$)
 iii $\angle P + \angle Q = 180^\circ$ (opposite angles in a cyclic quadrilateral are supplementary)
 iv As $\angle P = \angle Q$ from ii and $\angle P + \angle Q = 180^\circ$ from iii then $\angle P = \angle Q = 90^\circ$, so ST is a diameter (converse of Thales' theorem)
 d i Join OR, OS, OT and OU . $\triangle OST$ is isosceles with $OT = OS$ (radii of circle centre O through T), angles opposite equal sides are equal so $\angle OST = \angle OTS$
 ii $\triangle OUR$ is isosceles with $OR = OU$ (radii of circle centre O through U), angles opposite equal sides are equal so $\angle OUR = \angle ORU$
 iii $\angle OST = \angle OTS$ (from i), $\angle OUT = \angle ORS$ (from ii) and $OR = OU$ (radii), so $\triangle ORT \equiv \triangle OUS$ (AAS)
 iv $RT = SU$ (matching sides, $\triangle ORT \equiv \triangle OUS$)
- 8 a $GO = FO$ (radii of the circle centre O), $PG = PF$ (radii of the circle centre P), OP is common $\therefore \triangle GOP \equiv \triangle FOP$ (SSS)
 b $\angle FOM = \angle GOM$ (matching angles, $\triangle GOP \equiv \triangle FOP$)
 $GO = FO$ (radii of the circle centre O),
 $\angle FOM = \angle GOM$ (from b), OM is common $\therefore \triangle GOM \equiv \triangle FOM$ (SAS)

- c $\angle GMO = \angle FMO$ (matching angles, $\triangle FOM \equiv \triangle GOM$) and $\angle GMO + \angle FMO = 180^\circ$ (straight angle at M), so $\angle GMO = \angle FMO = 90^\circ$ and $OP \perp FG$, $FM = GM$ (matching sides, $\triangle GOM \equiv \triangle FOM$)
- 9 a $\triangle PAM \equiv \triangle PBM$ (SSS), so $\angle AMP = \angle BMP = 90^\circ$
 b $\triangle PAM \equiv \triangle PBM$ (SAS), so $AP = BP$
 c Take three points A, B and C on the circle. Construct the perpendicular bisectors of AB and BC . By parts a and b, the centre lies on the perpendicular bisectors of AB and BC , so the intersection is the only centre of the circle.
 d i Imagine the vertical line through the centre of the circle, and let P be any point on this line. Then P is equidistant from all the points on the circle.
 ii A sphere
 iii An infinite cylinder
 iv A cylinder with hemispherical ends
 v A plane perpendicular to the interval through the midpoint of the interval
 vi One method is to take three chords, not in a plane. Take the plane perpendicular to each chord through its midpoint, then the intersection of the three planes is the centre of the sphere.

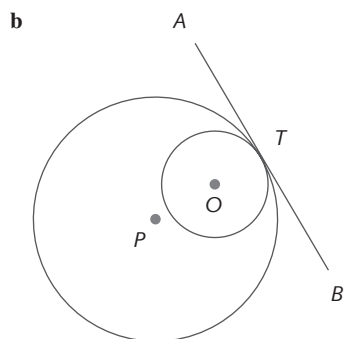
Exercise 13D

- 1 a $\alpha = 90^\circ$ (tangent perpendicular to radius at common point on circle),
 $\beta = 40^\circ$ (sum of angles in a triangle)
 b $\alpha = 75^\circ$ ($\angle OFP = 90^\circ$, sum of angles in a triangle),
 $\beta = 37.5^\circ$ (base angle of isosceles triangle with external angle 75°)
 c $\alpha = 70^\circ$ (angle at the centre is twice angle at the circumference standing on the same arc),
 $\beta = 20^\circ$ ($\angle OTU = 90^\circ$, sum of angles in a triangle)
 d $\alpha = 62^\circ$ ($\angle MTO = 90^\circ$, sum of angles in a triangle),
 $\beta = 31^\circ$ (angles at the circumference is half angle at the centre standing on the same arc)
 e $\theta = 50^\circ$ ($\angle UOT = 40^\circ$, angle at the centre is twice angle at the circumference standing on the same arc and $\angle OTU = 90^\circ$, sum of angles in a triangle)
 f $\alpha = 90^\circ$ ($\angle OTU = 90^\circ$, alternate angles, $TU \parallel BA$),
 $\beta = 45^\circ$ (base angle of isosceles triangle with third angle 90°)
 g $\beta = 70^\circ$ (alternate angle, $DT \parallel AC$),
 $\alpha = 20^\circ$ ($\alpha + \beta = 90^\circ = \angle DTO$)
 h $\alpha = 25^\circ$ (alternate angles, $TR \parallel SD$), $\beta = 90^\circ$ (Thales' theorem),
 $\gamma = 65^\circ$ (sum of angles in a triangle)
 i $\alpha = 35^\circ$ ($\angle BTS = 90^\circ$),
 $\beta = 55^\circ$ ($\angle SQT = 90^\circ$, Thales' theorem, sum of angles in triangle)
 $\theta = 70^\circ$ ($\angle SPT = 90^\circ$, Thales' theorem, sum of angles in triangle)
 j $\alpha = 30^\circ$ ($\angle DTO = 15^\circ$, angle at circumference is half angle at centre standing on same arc),
 $\beta = 60^\circ$ ($\angle OTB = 90^\circ$, sum of angles in a triangle)
 k $\theta = 30^\circ$ ($\triangle TXO$ is equilateral, $\angle XTO = 60^\circ$, $\angle LTO = 90^\circ$),
 $\beta = 120^\circ$ ($\angle YZO = 60^\circ$, sum of opposite angles in a cyclic quadrilateral)
- 1 $\alpha = 30^\circ$ ($\triangle TQO$ is equilateral, $\angle TOQ = 60^\circ$, angle at the circumference is half angle at the centre standing on the same arc),
 $\theta = 30^\circ$ (complementary with $\angle QTO = 60^\circ$)
- 2 a $x = 5$ (tangents to a circle from an external point have equal length),
 $\alpha = 70^\circ$ (base angle of an isosceles triangle),
 $\beta = 40^\circ$ (sum of angles in a triangle),
 $\gamma = 20^\circ$ ($\angle OSP = 90^\circ$)
 b $x = 8$ (tangents to a circle from an external point have equal length),
 $\alpha = 70^\circ$ (base angle of an isosceles triangle),
 $\theta = 140^\circ$ ($\angle T = \angle S = 90^\circ$, sum of angles in a quadrilateral)
 c $x = 2$ (tangents to a circle from an external point have equal length),
 $y = 3, z = 3$ (tangents to a circle from an external point have equal length)
 d $x = 7$ ($SQ = 4$, tangents to a circle from an external point have equal length, so $RS = 7$)
 e $x = 9$ ($SB = 4$ (equal tangents), $SP = 14$ and $TP = 14$ (equal tangents), $TA = 5$ (equal tangents), $x = 14 - 5$)
 f $\alpha = 100^\circ$ (reflex $\angle SOT = 260^\circ$, angle at the centre is twice angle at the circumference standing on the same arc, sum of angles at point O),
 $\beta = 70^\circ$ (sum of angles in a quadrilateral),
 $\theta = 20^\circ$ ($\angle PSO = 90^\circ$)
- 3 a $x = 5$ (radius), $y = 8$ ($OB = 13$ by Pythagoras' theorem), $\theta \approx 22.62^\circ$
 b $x = \sqrt{133} \approx 11.53$
 c $x \approx 19.23, y \approx 13.47$
- 4 $\angle PAB = 90^\circ$ (tangent perpendicular to radius) and $\angle VBA = 90^\circ$ (tangent perpendicular to radius), $\angle PAB$ and $\angle VBA$ are alternate and equal therefore $PQ \parallel UV$
- 5 $AB + CD = AP + PB + CR + DR$
 $= AS + BQ + QC + SD$ (tangents to a circle from an external point have equal length)
 $= AD + BC$
- 6 b Join OS and OT . $\angle OSP$ and $\angle OTP$ are angles in a semi-circle, centre M , so by Thales' theorem $\angle OSP = \angle OTP = 90^\circ$. As OS and OT are radii of circle, centre O , $PS \perp OS$ and $PT \perp OT$, so PS and PT are tangents to circle centre O .
 c $PS = PT$ (tangents to a circle from an external point have equal length)
- 7 a i $\angle PSO = \angle PTO$ (tangent perpendicular to radius), $SO = TO$ (radii of circle), PO is common. Thus $\triangle PSO \equiv \triangle PTO$ (RHS)
 ii $PS = PT$ (matching sides, $\triangle PSO \equiv \triangle PTO$), $\angle SPO = \angle TPO$ (matching angles, $\triangle PSO \equiv \triangle PTO$), $\angle SOP = \angle TOP$ (matching angles, $\triangle PSO \equiv \triangle PTO$)
 b i $\angle SPO = \angle TPO$ (from aii), $SP = TP$ (from aii), PM is common
 Thus $\triangle PSM \equiv \triangle PTM$ (SAS)
 ii $SM = TM$ (matching sides, $\triangle PSM \equiv \triangle PTM$), $\angle SMP = \angle TMP$ (matching angles, $\triangle PSM \equiv \triangle PTM$) and
 $\angle SMP + \angle TMP = 180^\circ$ (straight angle at M), so $\angle SMP = \angle TMP = 90^\circ$.
 Thus OP is the perpendicular bisector of ST

- 8 $RO \perp AB, QO \perp AC, PO \perp BC$ (tangent perpendicular to radius)

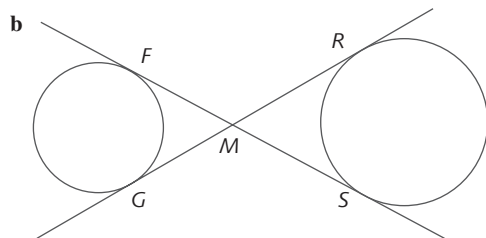
$$\begin{aligned}\text{Area of } \triangle ABC &= \text{area of } \triangle AOC + \text{area of } \triangle BOC + \text{area of } \triangle AOB \\ &= \frac{1}{2} \times AC \times OQ + \frac{1}{2} \times BC \times OP + \frac{1}{2} \times AB \times OR \\ &= \frac{1}{2} \times \text{radius} \times (AC + BC + AB) \\ &= \frac{1}{2} \times (\text{radius of circle}) \times (\text{perimeter of triangle})\end{aligned}$$

- 9 a i $\angle BTA = \alpha$ (base angle of isosceles triangle),
 $\angle BTO = \beta$ (base angle of isosceles triangle),
 $\alpha + \beta = 90^\circ$ (tangent perpendicular to radius),
 $\beta = 2\alpha$ (exterior angle of $\triangle ABT$).
 So $3\alpha = 90^\circ, \alpha = 30^\circ$.
 ii $\beta = 60^\circ$
 b i $OT = OB$ (radii) and $OB = BA$ (given), so
 $OA = 2OT, \angle OTA = 90^\circ, \sin \alpha = \frac{OT}{OA} = \frac{OT}{2OT} = \frac{1}{2}$
 ii $\sin \alpha = \frac{1}{2}, \alpha = 30^\circ$,
 so $\angle TOA = 60^\circ$ (sum of angles in a triangle), $\triangle OBT$ is equilateral and $\beta = 60^\circ$
 10 a i $\angle ATO = 90^\circ$ (tangent perpendicular to radius),
 $\angle ATP = 90^\circ$ (tangent perpendicular to radius)
 ii $\angle OTP = \angle ATO + \angle ATP = 90^\circ + 90^\circ = 180^\circ$, so
 $\angle OTP$ is a straight angle so O, T and P are collinear



$\angle ATO = 90^\circ$ (tangent perpendicular to radius),
 $\angle ATP = 90^\circ$ (tangent perpendicular to radius),
 $\angle ATO = \angle ATP = 90^\circ$ so O, T and P are collinear

- 11 a i $MF = MG$ (tangents to a circle from an external point have equal length)
 $MR = MS$ (tangents to a circle from an external point have equal length)
 ii $FR = MF - MR = MG - MS = GS$



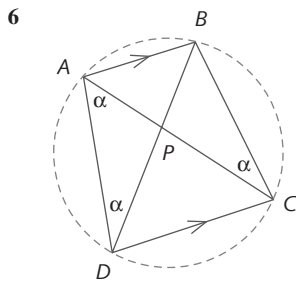
$MF = MG$ (tangents to a circle from an external point have equal length)
 $MR = MS$ (tangents to a circle from an external point have equal length)
 $FS = FM + MS = GM + MR = GR$

- 12 a $MA = MT$ (tangents to a circle from an external point have equal length)
 $MB = MT$ (tangents to a circle from an external point have equal length)
 $\therefore MA = MT = MB$
 b Circle centre M and passing through A, B and T has a diameter AB , so $\angle ATB$ is in a semicircle, hence by Thales' theorem $\angle ATB = 90^\circ \therefore AT \perp BT$

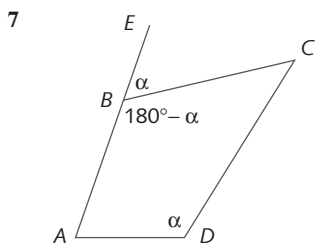
Exercise 13E

- 2 a $\alpha = 35^\circ$ (alternate segment theorem)
 b $\theta = 40^\circ$ (alternate segment theorem)
 c $\beta = 110^\circ$ (alternate segment theorem)
 d $\gamma = 150^\circ$ (alternate segment theorem)
 e $\alpha = 70^\circ$ (alternate segment theorem),
 $\beta = 110^\circ$ (opposite angles in a cyclic quadrilateral are supplementary)
 f $\beta = 125^\circ$ (alternate segment theorem),
 $\alpha = 55^\circ$ (opposite angles in a cyclic quadrilateral are supplementary)
 3 a $\alpha = 50^\circ$ (alternate segment theorem),
 $\beta = 50^\circ$ (alternate angles, $FG \parallel LM$ alternate segment theorem),
 $\gamma = 80^\circ$ (straight angle)
 b $\beta = 70^\circ$ (base angles of isosceles triangle are equal),
 $\alpha = 70^\circ$ (alternate segment theorem),
 $\gamma = 40^\circ$ (angles in a triangle)
 c $\beta = 80^\circ$ (alternate segment theorem)
 d $\beta = 35^\circ$ ($\triangle TSF$ is isosceles),
 $\alpha = 35^\circ$ (alternate segment theorem)
 e $\alpha = 130^\circ$ (alternate segment theorem),
 $\beta = 80^\circ$ ($\angle BTS = 50^\circ$, straight angle at T and $\triangle BTS$ is isosceles, tangents to a circle from an external point have equal length)
 f $\alpha = 50^\circ$ (alternate segment theorem),
 $\beta = 80^\circ$ ($\triangle UAT$ is isosceles, tangents to a circle from an external point have equal length),
 $\gamma = 55^\circ$ ($\triangle BUS$ is isosceles, tangents to a circle from an external point have equal length)
 4 a $\angle BAO = 90^\circ - \theta$ ($\angle OAT = 90^\circ$, tangent perpendicular to radius)
 b $\angle AOB = 180^\circ - 2(90^\circ - \theta)$, ($\triangle AOB$ is isosceles with base angles $\angle BAO$ and $\angle OBA$, sum of angles in a triangle), so $\angle AOB = 2\theta$
 c $\angle APB = \theta$ (angle at the circumference is half angle at the centre standing on the same arc)
 5 a $\angle ANB = 180 - \alpha$ (opposite angles in a cyclic quadrilateral are supplementary)
 b $\angle ABN = 90^\circ$ (Thales' theorem)
 $\angle NAB = 180 - (\angle ABN + \angle ANB)$
 $= 180 - (180 - \alpha + 90)$
 $= \alpha - 90$
 c $\angle SAB = 90 + (\alpha - 90)$
 $= \alpha$

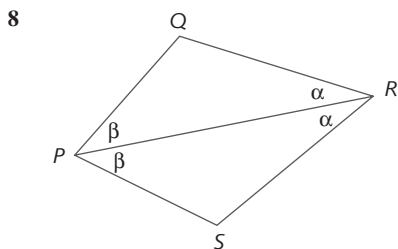
$\angle BDA = \angle BCA = 90^\circ$ (Thales' theorem)
 $AB = BA$ (common)
 $\triangle ABC \equiv \triangle BAD$ (RHS)
 Thus, $AD = BC$ (matching sides of congruent triangles)
 Alternatively, use Pythagoras' theorem



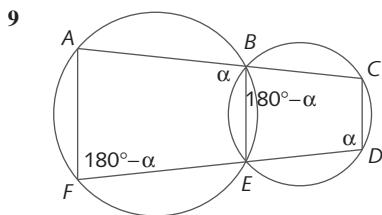
Let $\angle BCA = \alpha$
 $\angle BDA = \alpha$ (angles subtended by the same arc)
 $\angle DAC = \alpha$ (alternate angles $BC \parallel AD$)
 $\angle BPA = 2\angle ACB$ (exterior angle of $\triangle APD$)



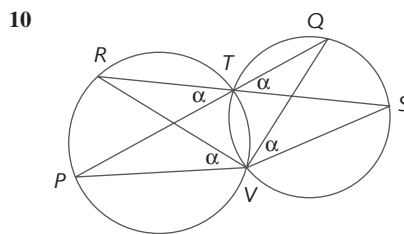
$\angle ABC = 180 - \angle CDA$ (cyclic quadrilateral)
 $\angle EBC = 180 - \angle ABC$ (straight line)
 $\angle EBC = \angle CDA$



$\triangle PQR \equiv \triangle PSR$ (AAS)
 Let $\angle QPR = \angle RPS = \beta$ and $\angle QRP = \angle SRP = \alpha$
 $2\alpha + 2\beta = 180^\circ$
 $\alpha + \beta = 90^\circ$
 Thus $\angle PQR = \angle RSP = 90^\circ$



Join B to E . Let $\angle EDC = \alpha$.
 $\angle CBE = 180^\circ - \alpha$ (opposite angles of cyclic quadrilateral)
 $\angle ABE = \alpha$ (supplementary angles)
 $\angle AFE = 180^\circ - \alpha$ (opposite angles of a cyclic quadrilateral)
 $\angle AFE$ and $\angle CDE$ are co-interior angles and
 $\angle CDE + \angle AFD = 180^\circ$
 Thus, $AF \parallel CD$

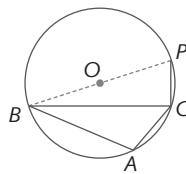


Let $\angle RVP = \alpha$
 $\angle RTP = \angle RTP = \alpha$ (angles subtended by the same arc)
 $\angle STQ = \alpha$ (vertically opposite)
 $\angle QTS = \angle SVQ$ (angles subtended by the same arc)
 Thus, $\angle QVS = \angle PVR$

- 11 a $\angle ADC = \frac{1}{2}\angle AQC = \angle AQX$ (angle subtended at the centre is twice the angle subtended at the circumference in the small circle)
 $\angle ABC = \frac{1}{2}\angle ADC$ (angle subtended at the centre is twice the angle subtended at the circumference in the large circle)
 $\therefore \angle AQX = 2\angle ABC$
- b $\angle ACB = 90^\circ$ (Thales' theorem)
 $AB^2 = BC^2 + AC^2$ (Pythagoras' theorem in $\triangle ABC$)
 $= BC^2 + 4(AQ^2 - XQ^2)$ (Pythagoras' theorem in $\triangle AQX$)

Challenge exercise

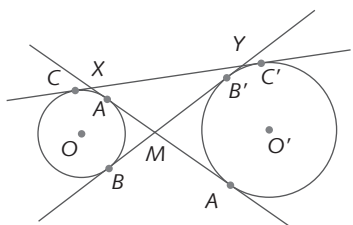
- 1 a $\angle BCP = 90^\circ$ (Thales' theorem)
 $\angle P = \angle A$ (angles on the circumference standing on the same arc)
 In $\triangle BCP$, $\sin P = \frac{a}{BP}$, so $BP = \frac{a}{\sin A}$
 So $2R = \frac{a}{\sin A}$
- b $\angle P = 180^\circ - \angle A$ (opposite angles in a cyclic quadrilateral are supplementary)
 In $\triangle BCP$, $\sin(180^\circ - A) = \frac{a}{BP}$, so $2R = \frac{a}{\sin A}$,
 since $\sin(180^\circ - \alpha) = \sin \alpha$



- 2 a $\triangle BRI \equiv \triangle BPI$ (AAS) as $\angle RBI = \angle PBI$ ($\angle B$ is bisected), $\angle BRI = \angle BPI$ (right angles), and BI is common $\therefore IR = IP$ (corresponding sides of congruent triangles)
 $\triangle CPI \equiv \triangle CQI$ (AAS) as $\angle PCI = \angle QCI$ ($\angle C$ is bisected), $\angle CPI = \angle CQI$ (right angles), and CI is common $\therefore IP = IQ$ (corresponding sides of congruent triangles)
- b From a $IP = IQ = IR$, $\angle ARI = \angle AQI$ (right angles), and AI is common
 $\therefore \triangle ARI \equiv \triangle AQI$ (RHS). So $\angle RAI = \angle QAI$ (corresponding angles of congruent triangles)
 $\therefore IA$ bisects $\angle A$
- c $IP = IQ = IR = \text{radius}$, $IR \perp AB$, $IP \perp BC$, $IQ \perp AC$, so AB , BC and CA are tangents to the circle

- 3 a $\angle CLH = 90^\circ$ so CH is a diameter of a circle that passes through L (converse of Thales' theorem) and $\angle CKH = 90^\circ$ so CH is a diameter of a circle that passes through K (converse of Thales' theorem), so C, K, H and L are concyclic.
- b C, K, H and L are concyclic, so $\angle HCL = \angle HKL$ (angles on the circumference standing on the same arc) and $\angle AKL = \angle HKL = \theta$
- c $\angle AKB = 90^\circ$ so AB is a diameter of a circle that passes through K (converse of Thales' theorem) and $\angle BLA = 90^\circ$ so AB is a diameter of a circle that passes through L (converse of Thales' theorem), so B, K, L and A are concyclic. As B, K, L and A are concyclic $\angle ABL = \angle AKL$ (angles on the circumference standing on the same arc) and $\angle ABL = \angle AKL = \theta$
- d AMC is similar to ALB (AA) as $\angle ABL = \angle ACM$ (both θ), $\angle BAL = \angle CAM$ (common)
 $\therefore \angle BLA = \angle CMA = 90^\circ$ (corresponding angles in similar triangles) meaning CM is an altitude of $\triangle ABC$
- 4 a $MG : GO = 2 : 1$ (given), $AG : GF = 2 : 1$ (centroid property),
 $\angle AGM = \angle FGO$ (vertically opposite) $\therefore GMA$ is similar to $\triangle GOF$ (SAS)
- b $\angle OFG = \angle MAG$ (corresponding angles in similar triangles), so $MA \parallel OF$ (alternate angles equal)
 $\therefore$ line $AM \perp CB$ since $FO \perp CB$, so M lies on the altitude from A
- c By the same argument, $BM \perp AC$ and $CM \perp AB$.
Hence, $M = H$ from question 3.
- 5 a $\angle ABC + \angle APC = 180^\circ$ (as $APCB$ is a cyclic quadrilateral), $\angle ABC + \angle ADC = 180^\circ$ (given), so $\angle APC = \angle ADC$
- b D is on AP and $\angle APC = \angle ADC$, so D and P coincide.
- 6 a $PM \times CM = AM \times BM$ (The product of the intervals on intersecting chords are equal)
- b $DM \times CM = AM \times BM$ (given) and $PM \times CM = AM \times DM$ (from a), then $DM = PM$. D is on MP and $DM = PM$, so P and D coincide.

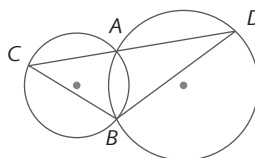
7



- a Let the intersection of AA' and BB' be M .
 $AM = BM$ and $A'M = B'M$ (tangents to a circle from an external point have equal length)
As $AA' = AM + A'M$ and $BB' = BM + B'M$, then $AA' = BB'$
- b $YC' = YB'$, $XC = XA$, $XA' = XC'$, $YC = YB$, (tangents to a circle from an external point have equal length),
 $XY = XA + AA' - YC' = \text{eqn 1}$ and
 $XY = YB' + BB' - CX = \text{eqn 2}$
eqn 1 + eqn 2 gives
 $2XY = XA + AA' - YC' + YB' + BB' - CX$
 $2XY = XA - CX + AA' + BB' - YC' + YB'$
 $2XY = AA' + BB'$
 $2XY = 2AA'$ so $XY = AA'$

- c The indirect common tangents (AA' and BB') become the same common tangent at the point of contact and $AA' = BB' = XY$.

8 a

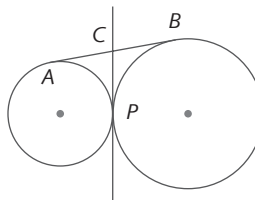


$\angle D$ is constant as angles on the circumference standing on the same arc are equal and $\angle C$ is constant as angles on the circumference standing on the same arc are equal.
So any triangles drawn as described are similar (AAA).

b Area = $\frac{1}{2} BC \times BD \sin B$

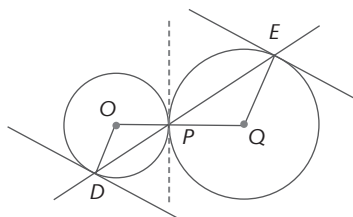
B is a constant. $\therefore$ largest area when BC and BD are diameters. (If BC is a diameter, $\angle CAB = 90^\circ$. Therefore $\angle BAD = 90^\circ$. Thus BD is a diameter by the converse of Thales' theorem.)

9 a



$CA = CP$ and $CB = CP$ (tangents to a circle from an external point have equal length), so $CA = CP = CB$
 $\therefore C$ is the midpoint of AB .

b



Let the centres of the circles be O and Q .
As the circles touch at P , the radii OP and QP are $\perp$ to the common tangent at P , so OPQ is a line. $\triangle DOP$ is isosceles as $OD = OP$ (radii), so $\angle ODP = \angle OPD$ and $\triangle EQP$ is isosceles as $QE = QP$ (radii), so $\angle QPE = \angle QEP$.
 $\angle OPD = \angle QPE$ (vertically opposite)
 $\therefore \angle ODP = \angle QEP$, so $OD \parallel QE$ (alternate and equal angles). Finally the tangent at D is perpendicular to OD and the tangent at E is perpendicular to EQ , so the tangents must also be parallel.

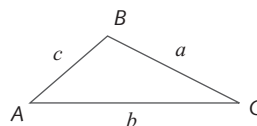
10 Area of $\triangle ABC = \frac{1}{2} ab \sin C$

$$= \frac{1}{2} bc \sin A$$

$$= \frac{1}{2} ac \sin B$$

$$\therefore \frac{2 \times \text{Area of } \triangle ABC}{abc} = \frac{ab \sin C}{abc} = \frac{bc \sin A}{abc} = \frac{ac \sin B}{abc}$$

$$= \frac{\sin C}{c} = \frac{\sin A}{a} = \frac{\sin B}{b}$$



Chapter 14 answers

Exercise 14A

- 1 a 3 d 0 g 4 b 3 e 4 h 6 c 11 f 3
- 2 a -4 d -2 g -10 b -3 e -3 h -4 c -1 f -2
- 3 a $\frac{3}{2}$ d $\frac{5}{2}$ g $\frac{7}{3}$ b $\frac{5}{2}$ e $\frac{7}{2}$ h $\frac{1}{6}$ c $\frac{5}{2}$ f $-\frac{5}{2}$
- 4 a 32 d $\frac{1}{1000}$ g 5 b 729 e $\frac{1}{10\,000}$ h 60 i 13 c 1000 f 625
- 5 a 9 d 2 b $\sqrt{2}$ e 3 c 4 f 10
- 6 a $\log_{\sqrt{2}} 2 = 2$ d $\log_{32} 1024 = 2$ g $\log_5 1 = 0$ b $\log_{10} 0.001 = -3$ e $\log_{10} N = x$ h $\log_{13} 13 = 1$ c $\log_{\frac{1}{2}} 2 = -1$ f $\log_5 5\sqrt{2} = \frac{3}{2}$
- 7 a $2^5 = 32$ d $3^{\frac{7}{2}} = 27\sqrt{3}$ b $3^4 = 81$ e $b^x = y$ c $10^{-3} = 0.001$ f $a^x = N$
- 8 a $\log_3 35$ d 2 b $\log_2 15$ e 2 c $\log_2 63$ f 0
- 9 a $\log_3 10$ d -1 b $\log_7 2$ e $\log_5 10$ c 1 f 1
- 10 a $\log_2 105$ d 0 b $\log_3 5$ c 0
- 11 a $2\alpha + \beta$ d $7\alpha + \beta + 6\gamma$ g $a\alpha + b\beta + c\gamma + d\delta$ h 1 b $\beta + 2\gamma$ e $\beta + 4\gamma$ c $\alpha + \beta + \gamma + \delta$ f $\alpha + \beta + 2\gamma + \delta$
- 12 a $xy = x + y$ d $\frac{1+y}{1-y} = 7^x$ b $x^2 = \frac{1}{10}y^3$ c $y = 125x^2$
- 13 $\log_2 V = \log_2 \frac{4\pi}{3} + 3\log_2 r$,
or $\log_2 V = 2 + \log_2 \pi - \log_2 3 + 3\log_2 r$

14 $x = \frac{1}{b} \log_{10} \frac{y}{a}$

15 $A = P \times 10^{bt}$

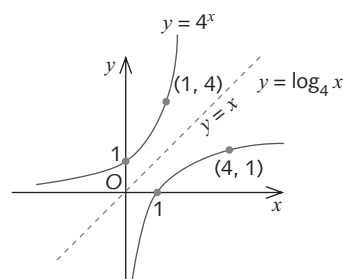
Exercise 14B

- 1 a $\frac{5}{2}$ b $\frac{5}{3}$

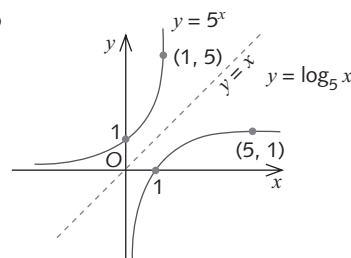
- 2 a 1.1292 d 2.3347 b 0.6826 e 0.9622 c 1.4650 f -0.7124
- 3 a 2.3219 d 1.7604 b 2.6309 e -2.8074 c 0.4307 f -1.4650
- 4 a -0.4225 d 1 b 0.1587 c -5.3847
- 5 a 1 b 1
- 6 a $\frac{3}{2}$ d $\frac{3}{11}$ b $\frac{7}{2}$ e 12 c $\frac{5}{3}$ f $\frac{133}{60}$

Exercise 14C

1 a

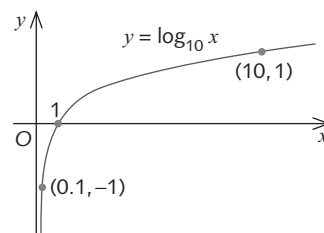


b



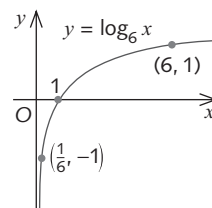
2 a

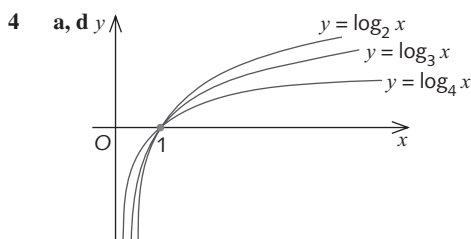
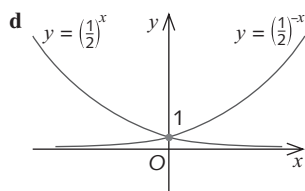
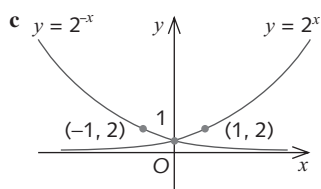
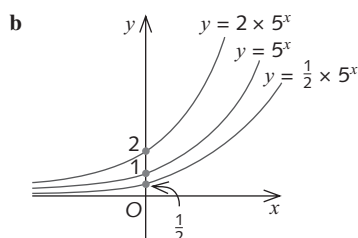
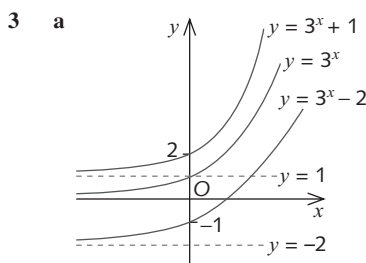
x	0.01	0.1	1	10	100
$\log_{10} x$	-2	-1	0	1	2



b

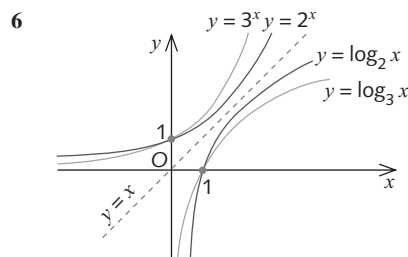
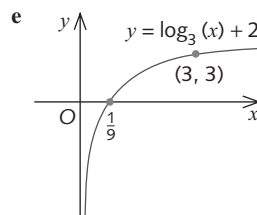
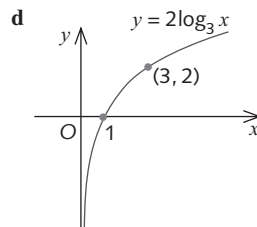
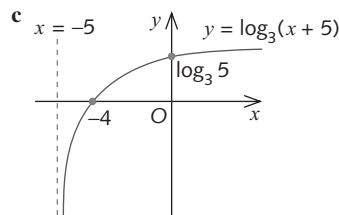
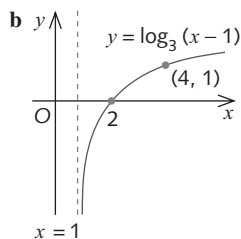
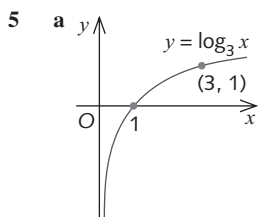
x	$\frac{1}{36}$	$\frac{1}{6}$	1	6	36
$\log_6 x$	-2	-1	0	1	2





b They have the same x -intercept, no y -intercept and the same asymptote, the y -axis.

c $\log_2 x$ grows faster than $\log_3 x$ as x grows



Exercise 14D

1 4 hours 19 minutes

2 a $M = 0.72 \times 5^t$ g

b 18 g

c 10 hours 20 minutes

d 39.94 days

e This is unrealistic, as there will not be enough food to keep the bacteria alive, so they will not multiply at the same rate for long.

3 a $4 \times 1.02^{t-1976}$ billion

b 28.98 billion

c 2022

4 a $750 \times 1.04^{t-1970}$ million

b 1995

c 2014

d 2050 (this is clearly impossible, because if the population of China increases at the same rate, the world population will increase at a rate getting closer to 4%, not the assumed 2%)

5 a $M_0 = 10$, $k = 8.864 \times 10^{-4}$

b 339.59 years

6 a 5

b 14

7 6

8 a \$62 985.60

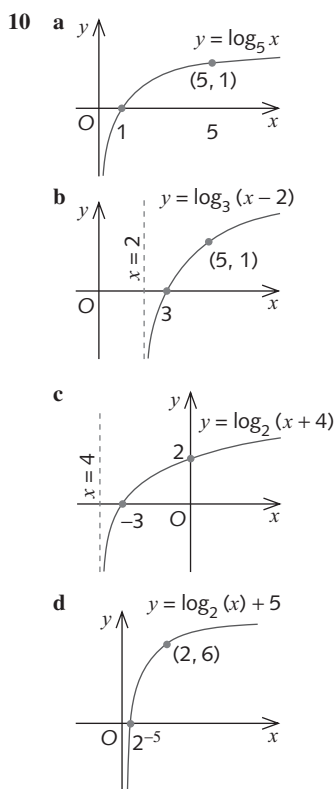
b \$65 816.20

c 55.5

d 38.3

Review exercise

- 1 a 4 b 3 c 9
d 0 e -3 f -6
g 4 h -3
- 2 a 4 b 2 c 2
d 8 e 5 f 5
- 3 a $\log_2 1024 = 10$ b $\log_{10} a = x$
c $\log_6 1 = 0$ d $\log_{11} 11 = 1$
e $\log_3 b = x$ f $\log_5 625 = 4$
- 4 a $3^4 = 81$ b $2^6 = 64$ c $10^{-2} = 0.01$
d $b^a = c$ e $a^c = b$
- 5 a $\log_2 55$ b $\log_2 35$ c $\log_6 77$
d $-\log_3 4$ e 1 f 1
- 6 a $\log_2 140$ b 0 c -2
d -2
- 7 a 1.2323 b 1.2091 c 2.8928
d 3.2362 e 0.75 f -1.2619
- 8 a 2.8074 b 3.9656 c 2.0704
d 3.0339 e -2.3219 f -2.4650
- 9 a $\frac{19}{2}$ b 27 c -1
d 10 000 e $-\frac{59}{2}$ f 10



- 11 a $y = 10x$ b $y + 1 = 100x$
- 12 $\log_2 3 - 1$
- 13 1.1

- 14 a $\log_2 40$ b $\log_2 \left(\frac{32}{5}\right)$
- 15 a 4 years b 7 years

Challenge exercise

- 1 From Pythagoras' theorem, $a^2 = c^2 - b^2 = (c - b)(c + b)$; now take log of both sides
- 2 1
- 4 a $\frac{9}{7}$ b 10 or $10^{\frac{-5}{2}}$
c 4 or $2^{\frac{5}{4}}$ d 9 or 27
- 5 a $x = 3, y = 5$ b $x = 5, y = 2$
c $x = -\frac{63}{8}, y = -\frac{9}{4}$ d $x = 2, y = 3$
- 6 Change base: $\frac{\log_a x \log_a x}{\log_a b} = \log_a b$, so $\log_a x = \pm \log_a b$,
 $x = b$ or $\frac{1}{b}$
- 7 $a = \frac{2b}{3}$
- 8 b Suppose $\log_{10} n = \frac{p}{q}$, then $10^{\frac{p}{q}} = n$, $10^p = n^q$, then
for n to be an integer, it must contain an equal power
of 2 and 5, and no other prime factors; that is, it is a
power of 10.
- 10 Let $\alpha = \frac{\log_a x}{y - z} = \frac{\log_a y}{z - x} = \frac{\log_a z}{x - y}$
 $\log_a x = \alpha y - \alpha z$
 $\log_a y = \alpha z - \alpha x$
 $\log_a z = \alpha x - \alpha y$
Add together
 $\log_a x + \log_a y + \log_a z = 0$
 $xyz = 1$
Now, $x = a^{\alpha y - \alpha z}$, $y = a^{\alpha z - \alpha x}$ and $z = a^{\alpha x - \alpha y}$
 $x^x y^y z^z = a^{\alpha y x - \alpha z x + \alpha z y - \alpha x y + \alpha x z - \alpha y z}$
 $= a^0$
 $= 1$
- 11 $x = 2a$ or $x = 5a$

Chapter 15 answers

Exercise 15A

- 1 $\frac{3}{13}$ 2 $\frac{7}{12}$ 3 $\frac{3}{5}$
- 4 $\frac{4}{5}$ 5 $\frac{8}{11}$
- 6 a $\xi = \{5, 6, 7, 8, 9\}$
b $P(5) = \frac{1}{10}, P(6) = \frac{1}{10}, P(7) = \frac{1}{4}, P(8) = \frac{7}{20}, P(9) = \frac{1}{5}$
c $\frac{4}{5}$

7 a $\xi = \{(1, 2), (1, 4), (1, 7), (2, 2), (2, 4), (2, 7), (3, 2), (3, 4), (3, 7)\}$

b equally likely, $\frac{1}{9}$ c $\frac{2}{9}$

8 a $\xi = \{2, 3, 4, 5, 6, 7\}$

b $P(2) = \frac{1}{12}, P(3) = \frac{1}{6}, P(4) = \frac{1}{4}, P(5) = \frac{1}{4},$

$P(6) = \frac{1}{6}, P(7) = \frac{1}{12}$

c $P(\text{sum is less than } 5) = \frac{1}{2}$

9 a $\frac{1}{4}$

b $\frac{5}{18}$

c $\frac{1}{6}$

10 a $\xi = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$

b $\frac{5}{12}$

c $\frac{1}{6}$

d $\frac{5}{36}$

e $\frac{1}{3}$

11 a $\xi = \{(R_1, R_1), (R_1, R_2), (R_1, R_3), (R_1, W_1), (R_1, W_2), (R_1, Y), (R_2, R_1), (R_2, R_2), (R_2, R_3), (R_2, W_1), (R_2, W_2), (R_2, Y), (R_3, R_1), (R_3, R_2), (R_3, R_3), (R_3, W_1), (R_3, W_2), (R_3, Y), (W_1, R_1), (W_1, R_2), (W_1, R_3), (W_1, W_1), (W_1, W_2), (W_1, Y), (W_2, R_1), (W_2, R_2), (W_2, R_3), (W_2, W_1), (W_2, W_2), (W_2, Y), (Y, R_1), (Y, R_2), (Y, R_3), (Y, W_1), (Y, W_2), (Y, Y)\}$

b i $\frac{7}{18}$

ii $\frac{11}{18}$

12 a $\frac{1}{8}$

b $\frac{33}{400}$

c $\frac{29}{200}$

Exercise 15B

1 $\frac{3}{5}$

2 $\frac{12}{13}$

3 $\frac{13}{15}$

4 $\frac{19}{20}$

5 a $\frac{1}{10}$

b $\frac{1}{5}$

c $\frac{2}{5}$

d $\frac{4}{5}$

6 a $\frac{1}{4}$

b $\frac{4}{13}$

c $\frac{1}{13}$

d $\frac{25}{52}$

e 0

f $\frac{8}{13}$

7 a $\frac{1}{6}$

b $\frac{1}{2}$

c 0

d $\frac{2}{3}$

e $\frac{1}{3}$

f $\frac{2}{3}$

8 a $\frac{29}{100}$

b $\frac{23}{100}$

c $\frac{49}{100}$

d $\frac{13}{20}$

e $\frac{11}{100}$

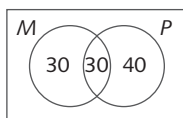
f $\frac{13}{20}$

g $\frac{77}{100}$

h $\frac{1}{8}$

i $\frac{7}{8}$

9 a ξ

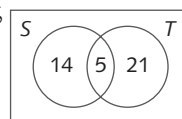


b i $\frac{3}{10}$

ii $\frac{2}{5}$

iii 0

10 ξ



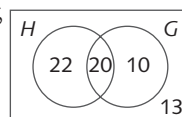
a $\frac{1}{8}$

b $\frac{21}{40}$

c $\frac{7}{8}$

d $\frac{7}{20}$

11 ξ



a $\frac{4}{5}$

b $\frac{1}{5}$

c $\frac{22}{65}$

d $\frac{32}{65}$

12 a $\frac{1}{5}$

b $\frac{111}{1000}$

c $\frac{9}{100}$

d $\frac{11}{500}$

e $\frac{9}{500}$

f $\frac{1}{100}$

g $\frac{1}{500}$

13 a $\frac{6}{85}$

b $\frac{61}{85}$

c $\frac{16}{85}$

14 a $\frac{7}{26}$

b $\frac{4}{13}$

c $\frac{1}{2}$

15 $\frac{7}{15}$

Exercise 15C

1 a $\frac{17}{48}$

b $\frac{11}{24}$

c $\frac{13}{48}$

d $\frac{13}{31}$

e $\frac{4}{13}$

f $\frac{9}{22}$

2 a $\frac{1}{3}$

b $\frac{1}{2}$

3 $\frac{1}{3}$

4 a $\frac{4}{13}$

b $\frac{1}{13}$

5 a $\frac{4}{11}$

b $\frac{7}{11}$

c $\frac{2}{5}$

d $\frac{1}{3}$

6 a $\frac{7}{20}$

b $\frac{53}{100}$

c $\frac{20}{77}$

d $\frac{1}{2}$

7 a $\frac{211}{228}$

b $\frac{77}{114}$

c $\frac{49}{57}$

d $\frac{9}{14}$

e $\frac{186}{211}$

8 a

	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

b

Outcome	0	1	2	3	4	5
Probability	$\frac{1}{6}$	$\frac{5}{18}$	$\frac{2}{9}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{18}$

c $\frac{2}{3}$

d $\frac{1}{3}$

e $\frac{4}{5}$

f $\frac{1}{5}$

- 9 a $\frac{1}{13}$ b $\frac{9}{19}$ c $\frac{3}{4}$ d $\frac{9}{19}$ e $\frac{2}{5}$
- 10 a $\frac{3}{14}$ b $\frac{9}{28}$ c $\frac{1}{3}$
- 11 a $\frac{1}{6}$ b $\frac{1}{2}$
- 12 0.3

Exercise 15D

- 1 No. Example: $P(M \cap \text{Yes}) \neq P(M) \times P(\text{Yes})$ [$0.6 \times 0.55 = 0.33 \neq 0.25$]
- 2 No. Example: $P(< 30 \cap \text{Yes}) \neq P(< 30) \times P(\text{Yes})$ [$0.5625 \times 0.6875 \approx 0.387 \neq 0.4375$]
- 3 a 0.27 b $\frac{21}{38} \approx 0.55$
c $P(M) \neq P(M | > 10 \text{ km/h})$
d $P(\text{Minor} \cap \text{Not speeding}) \neq P(\text{Minor}) \times P(\text{Not speeding})$
 $\left[\frac{41}{200} \neq \frac{146}{200} \times \frac{43}{200} \right]$
- 4 a $\frac{3}{35}$ b $\frac{18}{35}$ c $\frac{12}{35}$ d $\frac{23}{35}$
- 5 $P(A) = P(A|B) = \frac{3}{8}$ or $P(A \cap B) = P(A) \times P(B) = \frac{3}{20}$
or $P(B) = P(B|A) = \frac{2}{5}$

6 a

		Die 2					
		1	2	3	4	5	6
Die 1	1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
	6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

- b $P(A) = \frac{6}{36} = \frac{1}{6}$, $P(B) = \frac{6}{36} = \frac{1}{6}$, $P(A \cap B) = \frac{1}{36}$.
Events independent since $P(A \cap B) = P(A) \times P(B)$.
- c $P(A \cap B) \neq 0$. Therefore, events A and B are not mutually exclusive.

Exercise 15E

- 1 a $\frac{1}{16}$ b $\frac{9}{16}$ c $\frac{3}{8}$
d $\frac{3}{16}$ e $\frac{3}{16}$ f $\frac{7}{16}$
- 2 a $\frac{1}{17}$ b $\frac{19}{34}$ c $\frac{13}{34}$
d $\frac{13}{68}$ e $\frac{13}{68}$ f $\frac{15}{34}$
- 3 a $\frac{10}{39}$ b $\frac{20}{39}$ c $\frac{14}{39}$ d $\frac{5}{39}$
- 4 a $\frac{10}{91}$ b $\frac{15}{91}$ c $\frac{30}{91}$ d $\frac{3}{91}$
- 5 a $\frac{5}{36}$ b $\frac{1}{18}$ c $\frac{5}{18}$ d $\frac{5}{18}$ e $\frac{17}{18}$

- 6 a $\frac{351}{1891}$ b $\frac{595}{1891}$ c $\frac{945}{1891}$
- 7 a $\frac{6}{25}$ b $\frac{12}{25}$ c $\frac{4}{25}$ d $\frac{13}{25}$
- 8 a $\frac{1}{16}$ b $\frac{1}{16}$ c $\frac{1}{16}$ d $\frac{1}{16}$ e $\frac{15}{16}$
- 9 a $\frac{1}{216}$ b $\frac{125}{216}$ c $\frac{1}{8}$
d $\frac{1}{8}$ e $\frac{5}{216}$ f $\frac{5}{216}$
- 10 $P(G) = P(G|C) = \frac{4}{9}$

Review exercise

- 1 a i $\frac{9}{25}$ ii $\frac{4}{25}$ iii $\frac{12}{25}$ iv $\frac{16}{25}$
b i $\frac{1}{10}$ ii $\frac{3}{5}$ iii $\frac{2}{5}$
- 2 a $\frac{1}{10}$ b $\frac{1}{2}$ c $\frac{1}{20}$

3 a

	1	2	3	4	5	6
2	3	4	5	6	7	8
4	5	6	7	8	9	10
6	7	8	9	10	11	12
8	9	10	11	12	13	14

- b i $P(A) = \frac{1}{2}$ ii $P(B) = \frac{1}{8}$
- c $A \cap C = \{9, 10, 11, 12\}$; $P(A \cap C) = \frac{5}{12}$
- d No. $P(A) \times P(C) = \frac{1}{2} \times \frac{11}{12} = \frac{11}{24} \neq \frac{10}{24} = P(A \cap C)$
- 4 $\frac{1}{3}$ 5 $\frac{2}{5}$
- 6 a $\frac{13}{20}$ b $\frac{13}{20}$ c $\frac{2}{5}$
- 7 $\frac{1}{2}$
- 8 a $\frac{33}{100}$ b $\frac{7}{50}$ c $\frac{1}{25}$ d $\frac{29}{100}$ e $\frac{1}{10}$

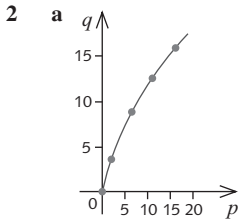
Challenge exercise

- 1 a i 330 ii 150 iii 215 iv 215
b 2
c i 0.28 ii 0.288 iii 0.49
- 2 a $\frac{22}{119}$ b $\frac{11}{30}$ c $\frac{6}{11}$
- 3 a i 1296 ii 360 iii 810
b $\frac{671}{1296}$
- 4 $\frac{2}{3}$ 5 $\frac{10(n-3)}{n(n-1)}$ for $n > 4$
- 7 a 210 b $\frac{5}{21}$

Chapter 16 answers

Exercise 16A

1 a $a = \frac{b}{3}$ b $m = 5n$



b

$\sqrt{p}$	0	1	2	3	4
q	0	4	8	12	16

$\frac{q}{\sqrt{p}} = 4$, for each pair (p, q)

c $q = 4\sqrt{p}$

3 a $m = 3n^2$
i 75 ii 3

b $a = 10\sqrt{b}$
i 40 ii 6.25

4 a $R = 4s$ b $P = 0.12T$
c $a = 4\sqrt{b}$ d $V = 8r^3$

5 a $y = \frac{x}{4}$

x	2	8	12	18
y	$\frac{1}{2}$	2	3	$\frac{9}{2}$

b $y = 8x$

x	2	3	6	15
y	16	24	48	120

6 130 km 7 210.5 m^2

8 1852 kW 9 86.4 m

10 a The surface area is multiplied by 9.

b The radius is multiplied by $\sqrt{3}$.

11 a m is multiplied by 32

b m is multiplied by $\frac{1}{32}$

c n is tripled

d n is divided by 4

12 a a is increased by 11.8%

b a is decreased by approximately 4.08%

13 a p is increased by 6.27%

b p is decreased by 1.70%

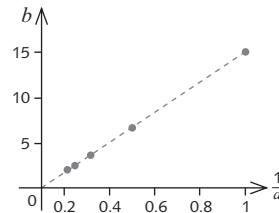
c q is increased by 33.1%

d q is decreased by 27.1%

Exercise 16B

1 a

$\frac{1}{a}$	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$
b	15	7.5	5	3.75	3



b $b = \frac{15}{a}$

2 a

x	1	2	5	10
y	100	25	4	1
x^2y	100	100	100	100

b $y = \frac{100}{x^2}$

3 a $v \propto \frac{1}{t}$ b $m \propto \frac{1}{\sqrt{n}}$ c $s \propto \frac{1}{t^3}$

4 $xy = 6$ or $y = \frac{6}{x}$

a 4 b 9

5 $ab^2 = 24$

a $\frac{8}{3}$ b $2\sqrt{2}$

6 $p^2q = 100$

a $\frac{10}{3}$ b $\frac{25}{4}$

7 $k = 12$ so $y = \frac{12}{x}$

x	1	2	3	4	$\frac{1}{2}$
y	12	6	4	3	24

8 $k = 32$ so $y = \frac{32}{x^2}$

x	2	4	8	10	16
y	8	2	0.5	0.32	0.125

9 55 km/h

10 54

11 a $1\frac{1}{3}$ units

b 2 m

12 a m is divided by 4

b m is multiplied by 4

c n is divided by 4

d n is tripled

13 a a is decreased by approximately 13.04%

b a is increased by approximately 13.64%

14 a p is decreased by approximately 24.87%

b p is increased by approximately 37.17%

c q is decreased by approximately 5.90%

d q is increased by approximately 7.72%

- 15 a i The height is divided by 4.
 ii The radius is divided by 3.
 b $k_1 = 36$

Exercise 16C

1 $a = 12bc$

a	12	24	24	48	72
b	1	1	2	2	2
c	1	2	1	2	3

2 $r = \frac{24s}{t}$

r	24	12	24	48	4
s	1	1	2	2	2
t	1	2	2	1	12

3 $y = 5$

4 a $y = \frac{kx^2}{\sqrt{z}}$ b $y = \frac{27}{4}$

5 a $a = kbc^3$ b $b = 32$

6 a $H = ki^2Rt$ b 2916 units

7 90 N

8 $6\sqrt{30} \text{ cm} \approx 32.86 \text{ cm}$

9 a $r = 12$ b $n = 24$

10 a y is doubled b y is multiplied by 13.5

11 a y is multiplied by approximately 1.154
 b y is multiplied by approximately 1.562

12 a F is divided by 4 b F is multiplied by 12

13 1.65 m/s^2

Review exercise

- 1 a x is directly proportional to y
 b p is directly proportional to the square of n
 c a is directly proportional to the square root of b
 d p is directly proportional to the cube of q

2 a i $p = 48$ ii $q = \frac{81}{8}$

b $a = \frac{5b^2}{4}$

i $a = \frac{125}{4}$ ii $a = 180$

3 a $k = 12, y = 12x$

x	0	1	2	3
y	0	12	24	36

b $k = 1.5, y = 1.5x$

x	2	8	12	18
y	3	12	18	27

- 4 a y is multiplied by 8
 b y is multiplied by 27
 c y is divided by 64

- 5 a m is multiplied by $\sqrt{2}$ (increase by 41% since $\sqrt{2} \approx 1.41$)
 b m is halved

- 6 a a is increased by 10.25%
 b a is decreased by 15.36%

7 a $y = \frac{80}{3}$

b $x = 60$

8 a $a = 2$

b $b = \frac{4\sqrt{2}}{3}$

9 $z = 27\sqrt{2}$

10 $x = \frac{128}{3}$

Challenge exercise

1 a $R = \frac{0.002 \ 24 \ L}{D^2}$

b $5.736 \approx 5.4 \text{ ohm}$

c resistance is halved

d resistance is increased by 21.9%

2 a $a = kc$ and $b = Kc$

$a + b = (k + K)c$ so $(a + b) \propto c$

$a - b = (k - K)c$ so $(a - b) \propto c$

$\sqrt{ab} = \sqrt{kc \times Kc} = \sqrt{kK}c$ so $\sqrt{ab} \propto c$

3 $y = \frac{150x}{19} + \frac{1080}{19x^2}$ 4 $y = \frac{12}{5}$ 5 $\frac{2abC}{(a+b)^2}$

6 $a + b = k(a - b)$

$a^2 + 2ab + b^2 = k^2(a^2 - 2ab + b^2)$

$2abk^2 + 2ab = k^2a^2 - a^2 + k^2b^2 - b^2$

$2ab(k^2 + 1) = (k^2 - 1)(a^2 + b^2)$

$a^2 + b^2 = \frac{k^2 + 1}{k^2 - 1} \times 2ab$

$a^2 + b^2 \propto ab$

Chapter 17 answers

Exercise 17A

- 1 a yes b no c yes
 d yes e yes f yes
 g yes h yes i no
 j yes k yes l no
- 2 a 3, 1, -6 b 4, 5, 0 c 1, -4, 7
 d 0, 15, 15 e 3, 7, 5 f 2, 3, 8
 g 3, -14, 0 h $5, \frac{1}{5}, 0$ i $6, -3\frac{1}{3}$
- 3 a monic b non-monic c monic
 d monic e non-monic f monic
- 4 a -6 b -6 c 0
 d -12 e -6 f -30
 g $a^3 - a - 6$ h $8a^3 - 2a - 6$ i $-a^3 + a - 6$

- 5 a 0, 60, -90 900, 0 b 39, 0, 11 712, 32
c -1, -55, 295 001, 1 d -27, 0, 0, -20
- 6 a $x^2 - 10x + 25$, x^2 , 2, 25
b $x^2 + 5x - 50$, x^2 , 2, -50
c $-3x^7 + x^3$, $-3x^7$, 7, 0
d $x^3 + 12x^2 + 36x$, x^3 , 3, 0
e $3x^7 + 6x^5 + 3x^3$, $3x^7$, 7, 0
f $-6x$, $-6x$, 1, 0
g $x^3 + 9x^2 + 26x + 24$, x^3 , 3, 24
h $3x^2 + 12x + 14$, $3x^2$, 2, 14
- 7 a $a = 6$
b $b = -1$
c $a = -15$, $d = 8$
- 8 a $P(x) = x^2 - 5x + 1$ b $Q(x) = 2x^4 + 10x$
c $R(x) = 7x^4 + 7x^3 + 7x^2 + 7x + 7$

Exercise 17B

- 1 a $3x^3 - 3x^2 - x + 5$, $-x^3 + 3x^2 + 7x + 5$
b $x + 3$, $4x^3 - 6x^2 - 9x + 7$
c $8x^2 - 6x$, 12
d $x^4 + x^3 - 2x^2 + 2x - 2$, $x^4 - x^3$
e 0, $10x^3 + 4x^2 - 2x - 10$
- 2 a $12x^5 + 4x^3 - 12x^2 + 20$
b $-16x^5 + 10x^3 - 7x^2 + 27$
c $-5x^5 + 5x^3 - 5x^2 + 15$
d 0
- 3 a $5x^6 - 2x^5 + 7x^4$ b $x^6 - 1$
c $x^6 - 1$ d $x^5 + 3x^4 - x^3 - 3x^2$
e $x^4 + x^2 + 1$
- 4 a equals the sum of the degrees of $P(x)$ and $Q(x)$
b the product of the constant terms of $P(x)$ and $Q(x)$
- 5 a $x^2 - 14x + 49$ b $x^4 - 6x^2 + 9$
c $x^6 - 14x^4 + 49x^2$ d $9x^{10} + 30x^8 + 25x^6$
e $x^4 + 2x^3 + 3x^2 + 2x + 1$
f $x^8 + 2x^6 + 3x^4 + 2x^2 + 1$
- 6 a is twice the degree of $P(x)$
b equal to the square of the constant term of $P(x)$
c the leading coefficient of $P(x)$ is 1 or -1
- 7 a $-5x^4 + 5x^3 - 7x - 3$
b $6x^4 - x^3 - 15x^2 - 2x + 3$
c $-4x^4 - 3x^3 + 26x^2 + 28x + 2$
- 8 a $x^4 - 5x^2 + 4$
b $x^4 + 10x^3 + 35x^2 + 50x + 24$

Exercise 17C

- 1 a $68 = 11 \times 6 + 2$ b $1454 = 12 \times 121 + 2$
c $2765 = 21 \times 131 + 14$

- 2 a $x^2 + 6x + 1 = (x + 2)(x + 4) - 7$
b $x^3 - 5x^2 - 12x + 30 = (x + 5)(x^2 - 10x + 38) - 160$
c $5x^3 - 7x^2 - 6 = (x - 3)(5x^2 + 8x + 24) + 66$
d $x^4 + 3x^2 - 3x = (x + 2)(x^3 - 2x^2 + 7x - 17) + 34$
e $4x^3 - 4x^2 + 1 = (2x + 1)\left(2x^2 - 3x + \frac{3}{2}\right) - \frac{1}{2}$
f $x^4 + 3x^3 - 3x^2 - 4x + 1 = (x + 1)(x^3 + 2x^2 - 5x + 1)$
- 3 a $R(x) = -5x + 1$, $Q(x) = x^2 - x + 3$
b $Q(x) = x^2 - 2x + 5$, $R(x) = 15 - 8x$
- 4 a $x^3 + 5x^2 - x + 2 = (x^2 + x + 1)(x + 4) - 6x - 2$
b $x^3 - 4x^2 - 3x + 7 = (x^2 - 2x + 3)(x - 2) - 10x + 13$
c $x^4 + 5x^2 + 3 = (x^2 - 3x - 3)(x^2 + 3x + 17) + 60x + 54$
d $x^5 - 3x^4 - 9x^2 + 9 = (x^3 - x^2 + x - 1)(x^2 - 2x - 3) - 9x^2 + x + 6$
- 5 a 0 b 0 or 1
c 3 or higher d 4
- 6 a $P(x) = (x + 5)(x + 3)(x - 7)$
b $P(x) = (x + 2)^2(x + 3)^2$
- 7 a i $x^4 - 3x^3 - 5x^2 + x - 7$
 $= (x + 5)(x^3 - 8x^2 + 35x - 174) + 863$
ii -870
b i $x^4 - 3x^3 - 5x^2 + x - 7$
 $= (x^2 + 5)(x^2 - 3x - 10) + 16x + 43$
ii $a = -15$, $b = -50$

Exercise 17D

- 1 a 38, not a factor b 0, is a factor
c -274, not a factor d 0, is a factor
- 2 a 0, is a factor b -6, not a factor
c 38, not a factor d 20, not a factor
- 3 a 0, is a factor b -14, not a factor
c 1, not a factor d -4, not a factor
e 6, not a factor f 0, is a factor
- 4 a $x + 1$, $x + 2$, $x - 2$ b $x + 1$, $x - 1$, $x + 4$
- 5 a $k = 4$ b $m = 4$
- 6 a $p = 28$ b $b = -23$
- 7 a $a = -14$, $b = -22$ b $a = 9$, $b = 5$
c $a = 17$, $b = 21$, $c = 0$

Exercise 17E

- 1 a $P(x) = (x - 12)(x + 9)$ b $x^2 - 3x - 108$
2 $P(x) = (x - 1)(x + 1)(x - 2)(x + 2)$

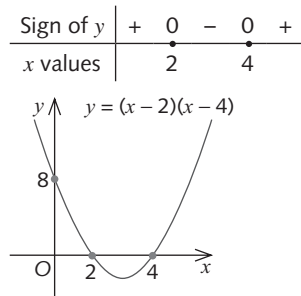
- 3 **b** $P(x) = (x-1)(x^2 - 5x + 6)$
c $P(x) = (x-1)(x-2)(x-3)$
- 4 **a** $(x+1)(x+2)(x+3)$ **b** $(x+1)(x-1)(x-7)$
c $(x+1)(x+5)(x-3)$ **d** $(x-5)(x+3)^2$
e $(x-1)^2(x+3)$ **f** $(x-1)(x+2)^2$
- 5 **a** $(x+1)(x-1)(x-2)(x-3)$ **b** $(x+1)^2(x+5)^2$
- 6 **a** $3(x-2)(x-1)(x+5)$ **b** $5(x-2)(x-1)(x+2)$
c $x(x-2)(x+1)(x+2)$ **d** $x(x-1)^2(x+3)^2$
- 7 **a** $(x-1)(x^2 + 3x + 5)$ **b** $(x+3)(x^2 + x + 1)$
c $x^2(x-2)(x^2 + 6x - 3)$ **d** $(x-2)(x+3)(x^2 + 3x + 1)$
- 8 If $P(\alpha) = 0$, then, $a_n\alpha^n + a_{n-1}\alpha^{n-1} + a_{n-2}\alpha^{n-2} + \dots + a_1\alpha + a_0 = 0$
Therefore, $a_0 = -\alpha(a_n\alpha^{n-1} + a_{n-1}\alpha^{n-2} + a_{n-2}\alpha^{n-3} + \dots + a_1)$
and α divides a_0

Exercise 17F

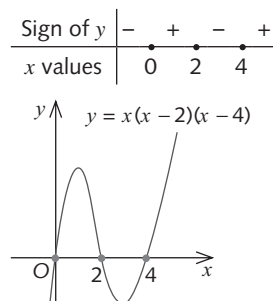
- 1 **a** -7, 5, -6 **b** 3, -1
c 2, 4, 6, 8 **d** 0, 7, -8
- 2 **a** $3, -3 + \sqrt{17}, -3 - \sqrt{17}$
b $-5, \frac{1}{3}(1 + \sqrt{7}), \frac{1}{3}(1 - \sqrt{7})$
c 0, 7, -6
d 2, 5, $\sqrt{10}, -\sqrt{10}$
- 3 **a** -2, -1, 5 **b** -2, 2, 3
c -7, 0, 1, 3 **d** -2, -1, 1, 7
- 4 **a** 1, 5 **b** -3, 2 **c** 1, 5
d -3, 2 **e** -6, -1, 0 **f** -1, 1, 2
- 5 **a** $(x-3)(x^2 - 4x - 1); 3, 2 - \sqrt{5}, 2 + \sqrt{5}$
b $(x+1)(x^2 + 3x + 7); -1$
c $x(x-2)(x-3)(x^2 + 3x - 1); 0, 2, 3,$
 $\frac{1}{2}(-3 - \sqrt{13}), \frac{1}{2}(-3 + \sqrt{13})$
d $(x+1)(x-1)(x-2)(x^2 + 2x + 2); -1, 1, 2$

Exercise 17G

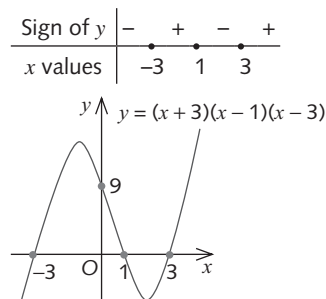
- 1 **a** $x = 2, x = 4$



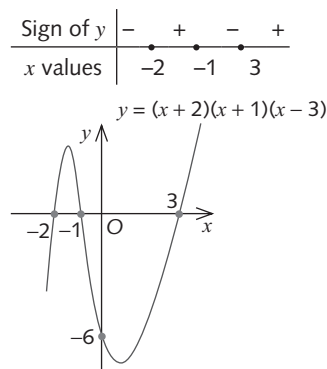
- b** $x = 0, x = 2, x = 4$



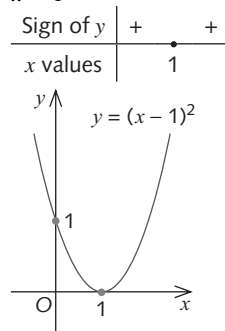
- c** $x = -3, x = 1, x = 3$



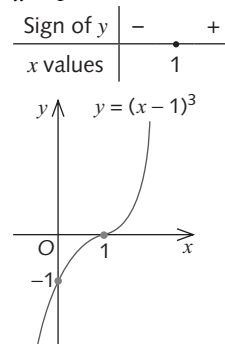
- d** $x = -2, x = -1, x = 3$



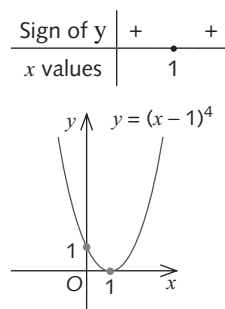
- 2 **a** $x = 1$

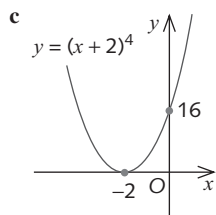
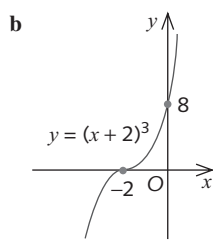
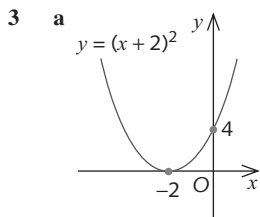


- b** $x = 1$

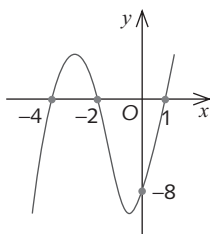


- c** $x = 1$





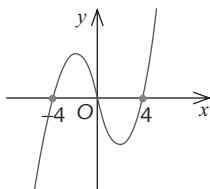
4 a $y = (x+2)(x-1)(x+4)$



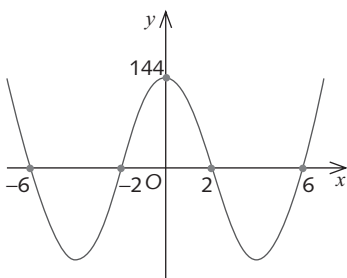
b $x > 1, -4 < x < -2$

c $x < -4, -2 < x < 1$

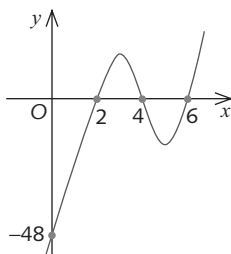
5 a $y = 3x(x-4)(x+4)$



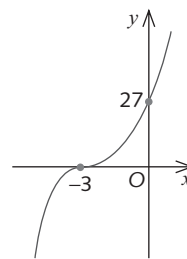
b $y = (x-6)(x+6)(x-2)(x+2)$



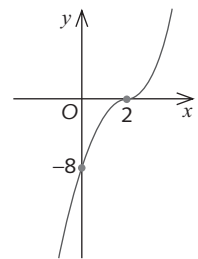
6 a $P(x) = (x-2)(x-4)(x-6)$



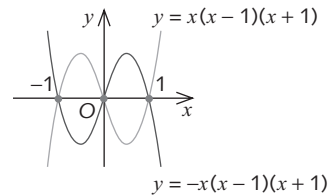
b $P(x) = (x+3)^3$



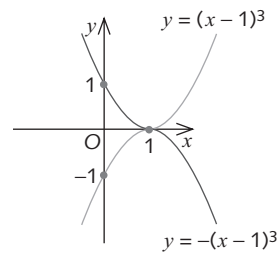
c $P(x) = (x-2)^3$



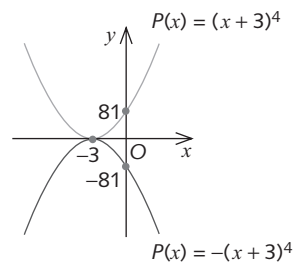
7 a - b



8 a - b



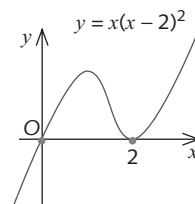
9 a - b



Exercise 17H

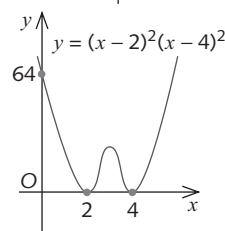
1 a $x = 0, x = 2$

Sign of y	-	0	+	0	+
x values		0		2	



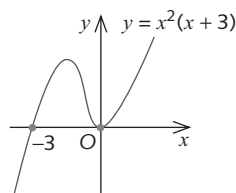
b $x = 2, x = 4$

Sign of y	+	0	+	0	+
x values		2		4	



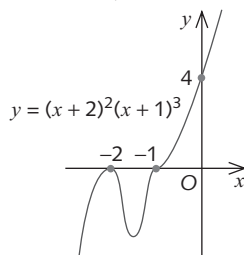
c $x = 0, x = -3$

Sign of y	-	0	+	0	+
x values		-3		0	



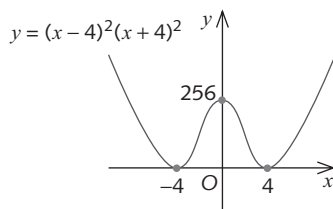
d $x = -2, x = -1$

Sign of y	-	0	-	0	+
x values		-2		-1	



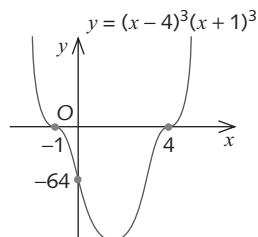
2 a $x = 4, x = -4$

Sign of y	+	0	+	0	+
x values		-4		4	



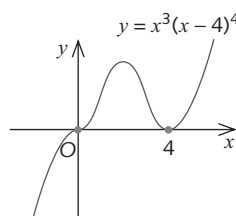
b $x = -1, x = 4$

Sign of y	+	0	-	0	+
x values		-1		4	



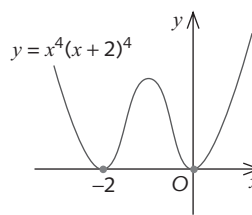
c $x = 0, x = 4$

Sign of y	-	+	0	+	+
x values		0		4	

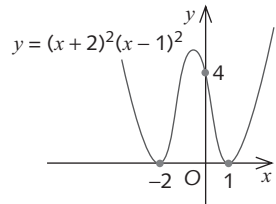


d $x = 0, x = -2$

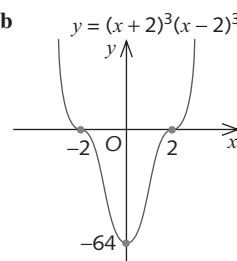
Sign of y	+	+	+	+	+
x values		-2		0	



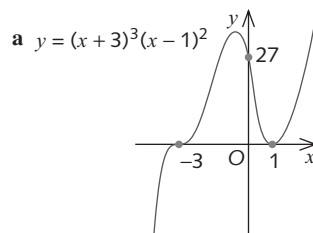
3 a



b



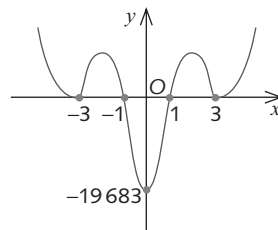
4 a



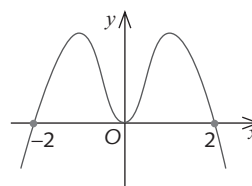
b $x > 1, -3 < x < 1$

c $x < -3$

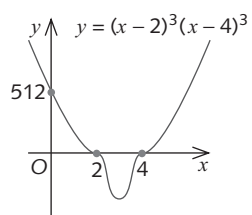
5 a $y = 3(x-1)(x+1)(x+3)^4(x-3)^4$



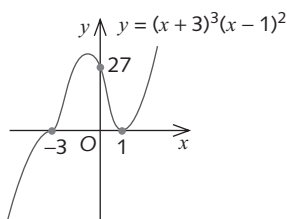
b $y = 5x^2(4-x^2) = -5x^2(x-2)(x+2)$



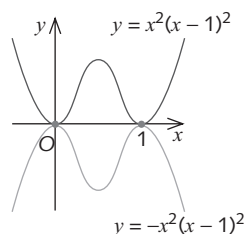
6 a $P(x) = (x-2)^3(x-4)^3$



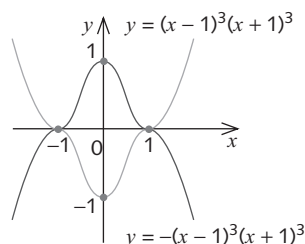
b $P(x) = (x+3)^3(x-1)^2$



7 a - b



8 a - b



Review exercise

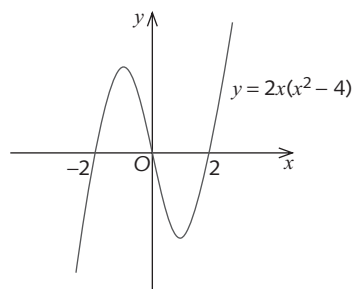
- 1 a polynomial b polynomial
c polynomial d not a polynomial
e not a polynomial f polynomial
- 2 a 2 b 3 c 4 d 2 e 3
- 3 a 2 b -4 c 11
d -13 e $a^3 + 2a - 1$ f $8a^3 + 4a - 1$
- 4 $a = -3$ 5 $a = -1$
- 6 a $P(x) + Q(x) = x^2 + 3x + 6$; $P(x) - Q(x) = -x^2 - x$; $P(x)Q(x) = x^3 + 5x^2 + 9x + 9$
b $P(x) + Q(x) = 2x^2 + 4$; $P(x) - Q(x) = -2$; $P(x)Q(x) = x^4 + 4x^2 + 3$
c $P(x) + Q(x) = x^2 + 2$; $P(x) - Q(x) = 4x - x^2$; $P(x)Q(x) = 2x^3 - 3x^2 + 1$
- 7 a $6x^2 + 13x - 2$, remainder 2
b $2x^2 - 5x + 10$, remainder -7
c $x^2 - 4x - 6$, remainder -17
d $x^2 + 3$, remainder 7
- 8 a $(x-3)(x-1)(x+2)$ b $(x+4)(x+1)(2x-3)$
c $(x-1)(x+4)(2x-3)$ d $(2x+3)(x-2)(3x-1)$
- 9 $a = 4, b = -1$ 10 $d = 4$
- 11 $a = -2, b = -19$

12 a $2w^3$

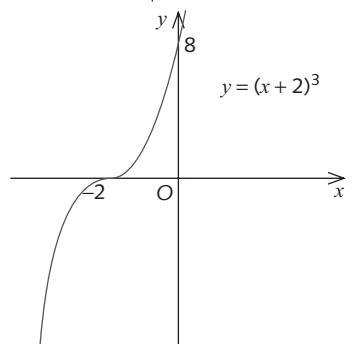
b $a = -2, b = 4$

- 13 a x-intercepts: -1, 0, 2 y-intercept: 0
b x-intercepts: -2, 1, 3 y-intercept: 6
c x-intercepts: -2, -1, 1 y-intercept: -2
d x-intercepts: $-1, -\frac{2}{3}, 3$ y-intercept: -6
e x-intercepts: $-4, -\frac{2}{5}, 2$ y-intercept: -16
f x-intercepts: $-\frac{1}{2}, \frac{1}{3}, 1$ y-intercept: 1

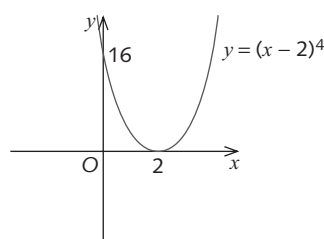
14 a



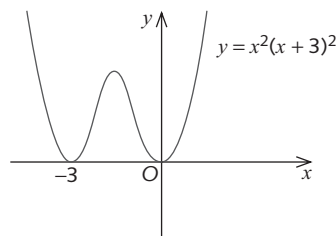
b



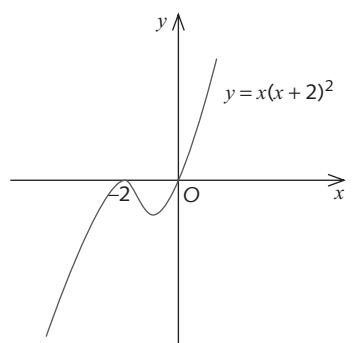
c

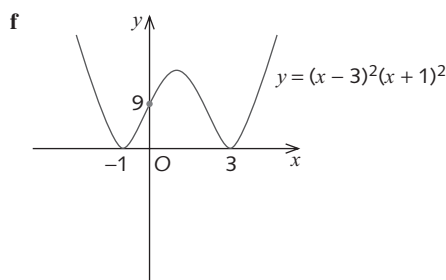


d



e





Challenge exercise

- $a = -3$
- $(x^2 - 2x + 2)(x^2 + 2x + 2)$
- $b = 14, a = -11$
- $b \ x^3 - 6x^2 + 11x - 6$
- $P(x) = 6x^5 - 15x^4 + 10x^3$
- $a = 1$ and $b = 2$
- $4x$
- $c \ (x^4 + 2x^2 + 2x)^2 + (x^3 - 2x^2 - 4)^2$
- $\frac{(A-B)}{a-b}x - \frac{(Ba-Ab)}{a-b}$
- $x = \frac{1+\sqrt{5}}{2}, y = \frac{1-\sqrt{5}}{2}; x = \frac{1-\sqrt{5}}{2}, y = \frac{1+\sqrt{5}}{2};$
 $x = 1, y = 1; x = -2, y = -2$

Chapter 18 answers

Exercise 18A

- range = 17, IQR = 9
 - range = 9, IQR = 3
 - range = 9, IQR = 2.5
 - range = 17, IQR = 9
- lower quartile = 27, median = 32, upper quartile = 38.5, IQR = 11.5
 - lower quartile = 64, median = 76, upper quartile = 82, IQR = 18
- mean = 5.7, mode = 10, median = 5.5, interquartile range = 6
- | | number of data items | lower quartile position | median position | upper quartile position |
|---|----------------------|-------------------------|-----------------|-------------------------|
| a | 100 | 25.5 | 50.5 | 75.5 |
| b | 101 | 25.5 | 51 | 76.5 |
- 38 cm
 - 161.5 cm
 - 22.5 cm
- IQR = 1.4 cm
- median = \$149, lower quartile = \$143, upper quartile = \$153.50
 - IQR = \$10.50
- 3 5 7 10 12 12 13 and 3 5 6 10 11 12 13 (others are possible)

- No. 111 223 18, mean = 4, lower quartile = 1 and upper quartile = 3;
113 131 41 416, mean = 12, lower quartile = 13 and upper quartile = 14
- 35
 - 13.7

Exercise 18B

- \$49 and \$10.50, respectively
- \$40 000
 - \$120 000
 - \$100 000
 - \$60 000
- 40
 - 65
 - 55
 - 20
-
- median = 75, upper quartile = 78.5, lower quartile = 69.5, IQR = 9
-
- 50%
 - 25%
 - 50%
 - 25%
- no
- It depends on the spread of the values between the lower quartile and the median compared with those between the median and the upper quartile.

- B
 - B
 - B
 - B
- A
 - B
 - B
 - B

e Class A. Lowest mark is higher, and lower quartile, median and upper quartile are all higher. Only the maximum mark is lower.

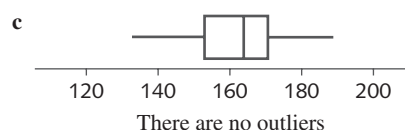
11 a

channel	minimum	lower quartile	median	upper quartile	maximum
A	7	10.5	15	16.3	23
B	7.8	11	14.6	16.7	25
C	8	10	13	14.5	16

- channel A
- channel B, channel A, channel C
- channel A, channel B, channel C
- If the criterion is the highest rating for the lowest rating show, then Channel C is the winner.

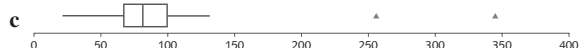
Exercise 18C

- $Q_1 = 151.5, Q_3 = 171.5$, median = 164
 - IQR = 20

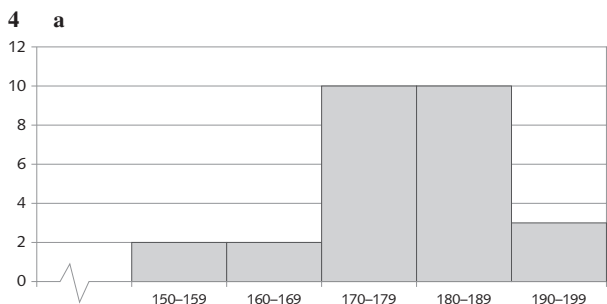


- 2 a $Q_1 = 67\,000$, $Q_3 = 100\,000$, median = 81500

b IQR = 33000



- 3 a iii, symmetric b i, negative skew
c ii, positive skew

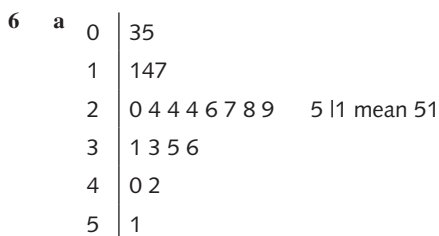


b $Q_1 = 173$, median = 179, $Q_3 = 187$, IQR = 14

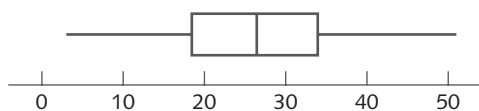


d symmetric

- 5 a and b are outliers; c is not an outlier

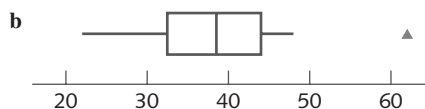


b median = 26.5, $Q_1 = 18.5$, $Q_3 = 34$, IQR = 15.5

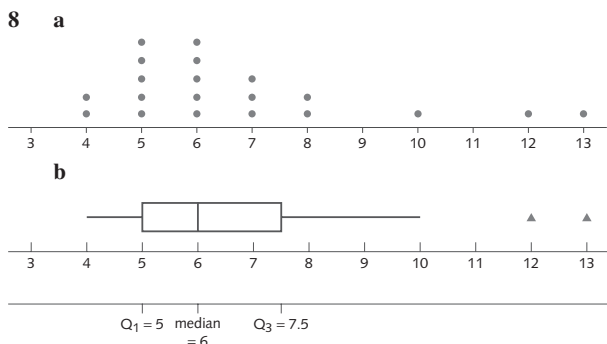


c The distribution is symmetric. There are no outliers.

- 7 a median = 38.5, $Q_1 = 32.5$, $Q_3 = 44$, IQR = 11.5



c 62 is an outlier



c The distribution has a slight positive skew.
minimum = 4, $Q_1 = 5$, median = 6, $Q_3 = 7.5$,
maximum = 13, 12 kg and 13 kg are outliers

Exercise 18D

- 1 15.54
2 City A : 30.43, City B : 31.86; City B has the greater mean daily maximum temperature.
3 60 kg 4 41.2
5 a i $\bar{x} = 5$, $\sigma = 3.16$ ii $\bar{x} = 5$, $\sigma = 1.34$
b Both data sets have the same mean but data set i is more spread out.

x_i	f_i	$f_i x_i$	$(x_i - \bar{x})$	$f_i(x_i - \bar{x})^2$
1	2	2	-2	8
2	7	14	-1	7
3	6	18	0	0
4	1	4	1	1
5	2	10	2	8
6	2	12	3	18
Total = 60		Total = 60		Total = 60

$$\sigma = 1.45, \bar{x} = 3$$

- 7 a $\bar{x} = 9.13$, $\sigma = 3.61$ b $\bar{x} = 14$, $\sigma = 3.46$

- 8 a $\bar{x} = 30.55$ b 10 c 10.22

- 9 a 6.15 b 10 c 2.46

10 a $\bar{x} = \frac{a+b+c}{3}$ and the
sum of the deviations = $a - \bar{x} + b - \bar{x} + c - \bar{x}$
= $(a+b+c) - 3\bar{x}$
= $(a+b+c) - 3 \times \frac{a+b+c}{3}$
= 0

b $\bar{x} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$ and
the sum of the deviations = $(x_1 - \bar{x}) + (x_2 - \bar{x}) + \dots + (x_n - \bar{x})$
= $(x_1 + x_2 + x_3 + \dots + x_n) - n\bar{x}$
= $(x_1 + x_2 + x_3 + \dots + x_n) - n \times \frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$
= 0

Exercise 18E

- 1 a $\bar{x} = 9$, $\sigma = 5.45$ b $\bar{x} = 9$, $\sigma = 1.31$

c $\bar{x} = 9$, $\sigma = 5.45$

All three data sets have the same mean. Data set b is less spread out than the other two, with a standard deviation of 1.31. Data sets a and c have the same mean and standard deviation. (The size of σ for a would be much smaller if the outlier 22 is omitted. It drops from 5.45 to 1.34. This does not happen when any one value is omitted from c.)

- 2 a i between 32.5 and 37.5

ii between 30 and 40

b i between 35 and 45

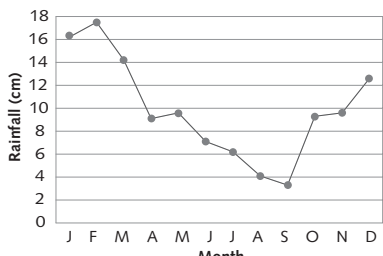
ii between 30 and 50

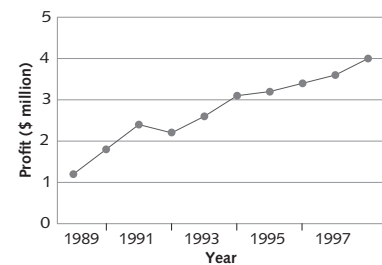
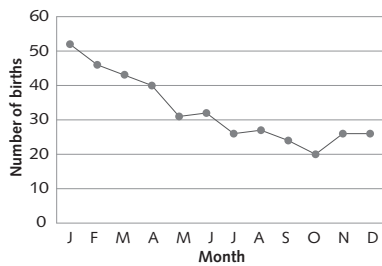
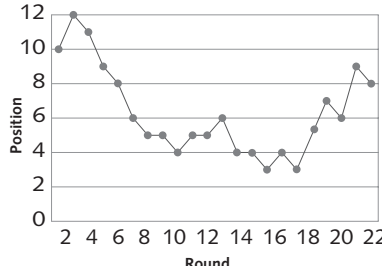
c i between 27 and 43

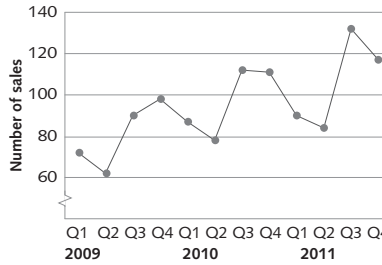
ii between 19 and 51

- 3 a Mathematics : $\bar{x} = 16.13$ $\sigma = 2.63$;
English : $\bar{x} = 15.87$ $\sigma = 2.83$
- b The mathematics result is about 0.05 standard deviations below the mean, whereas the English result is 0.66 standard deviations below the mean. The mathematics mark is better.
- 4 a David's mark for English is one standard deviation below the mean.
David's mathematics mark is more than one standard deviation below the mean.
His English mark is better.
- b Akira's mark for English is two standard deviations above the mean.
Akira's mathematics mark is less than two standard deviations above the mean.
Her English mark is better.
- c Katherine's mark for English is one standard deviation below the mean.
Katherine's mathematics mark is half a standard deviation below the mean.
Her mathematics mark is better.
- d Daniel's mark for English is more than one standard deviation above the mean.
Daniel's mathematics mark is one standard deviation above the mean.
His English mark is better.
- 5 a i $\bar{x} = 6.5$, $\sigma = 1.71$ ii $\bar{x} = 6.5$, $\sigma = 1.38$
iii $\bar{x} = 6.5$, $\sigma = 1.98$
- b i $\bar{x} = 11.5$, $\sigma = 1.71$ ii $\bar{x} = 11.5$, $\sigma = 1.38$
iii $\bar{x} = 11.5$, $\sigma = 1.98$
- c i $\bar{x} = 13$, $\sigma = 3.42$ ii $\bar{x} = 13$, $\sigma = 2.77$
iii $\bar{x} = 13.0$, $\sigma = 3.96$
- 6 a 1111155555 b 1111199999
c 1111559999
- 7 a 1111111111 b 1111111111
c 5555555555
- 8 a average annual salary increases by about \$5200
b the standard deviation remains unchanged
c an increase of \$2900

Exercise 18F

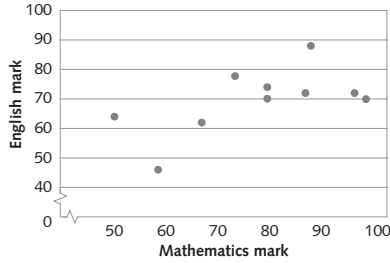
- 1 a 
- b The rainfall is high in January and February and decreases to reach a minimum in September. The rainfall from September to December increases significantly.

- 2 
- 3 a 
- b The number of births fell over the first half of the year. From July until the end of the year, the birth rate was roughly constant. The maximum occurred in January and the minimum in October.
- 4 a 
- b The team improved its position quite markedly during the first half of the season and maintained a position in the top five teams between rounds 7 and 18 (except for round 12, when it was sixth). However, during the last five rounds, the team's position deteriorated again to eighth at the end of the season.
- 5 a fourth quarter b third quarter
c yes
1st quarter sales: 45, 51, 55
2nd quarter sales: 63, 69, 71
3rd quarter sales: 67, 75, 79
4th quarter sales: 43, 39, 49

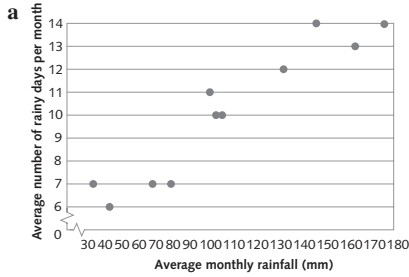
- 6 a 
- b Car sales per quarter have shown a general upward trend, with major fluctuations.
- c It seems that the car dealer is able to sell more cars in the third and fourth quarters each year than in the first and second quarters.

Exercise 18G

1

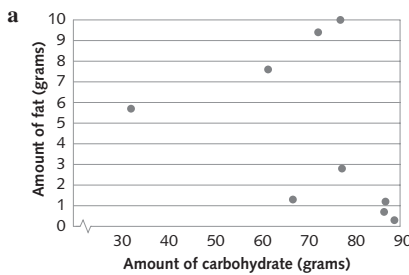


2



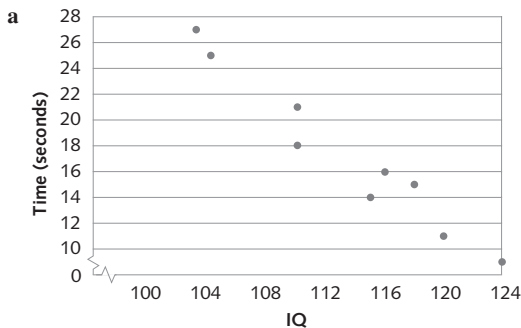
b The number of rainy days per month generally increases as the average monthly rainfall increases.

3



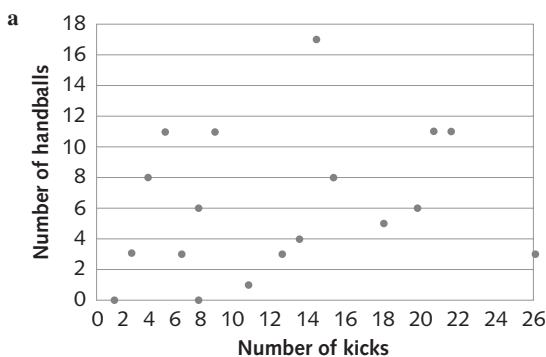
b There is no apparent relationship between carbohydrate content and fat content.

4



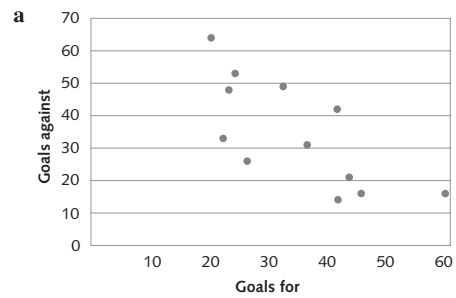
b The time taken to complete a puzzle tends to decrease as IQ increases.

5



b The scatter plot does not support the claim 'the greater the kicks, the greater the handballs'. This would only be supported by an upwards trend from left to right.

6



b i The best team is F, with easily the greatest 'goals for' and very nearly the least 'goals against'.
 ii The worst team is E, with the least 'goals for' and the greatest 'goals against'.
 iii J is better than H – while the 'goals for' are about the same, the 'goals against' clearly favour J.

7

a $B = v$ b $C = ii$ c $D = iv$
 d $E = viii$ e $F = vi$ f $G = iii$
 g $H = i$ h $I = vii$

8

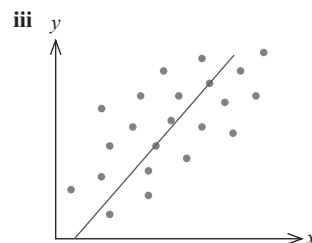
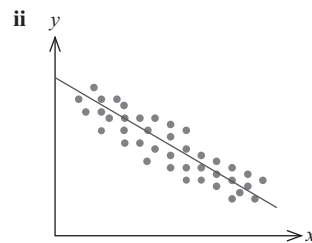
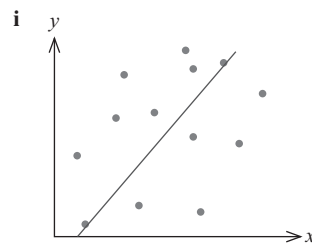
a $A = iii$ b $B = iv$ c $C = viii$
 d $D = vii$ e $E = vi$ f $F = v$
 g $G = i$ h $H = ii$

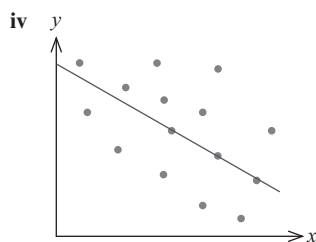
9

a i 54 ii 55 iii 36
 iv 84 v 67 or 68
 b i 52 ii 53 iii 32
 iv 84 v 66

Exercise 18H

1



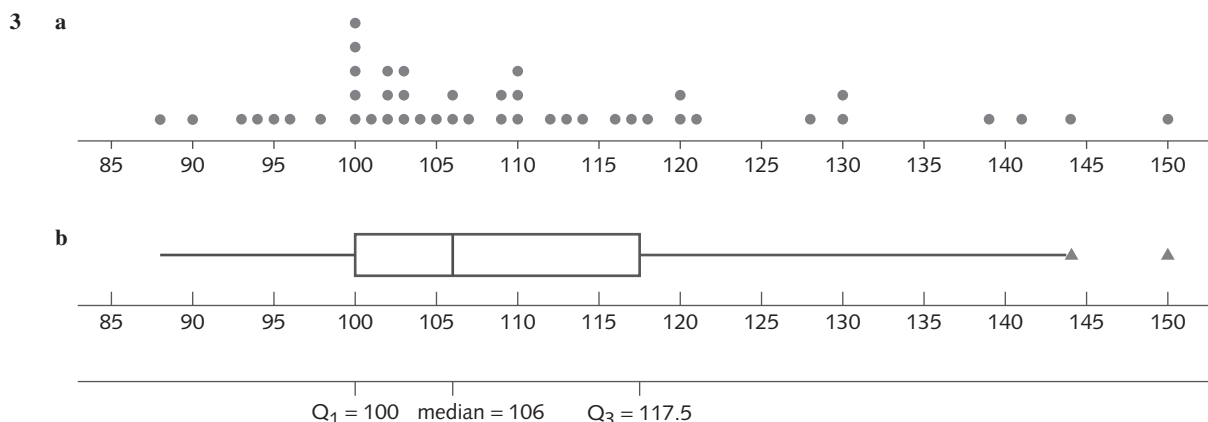


- 2 a i increasing ii decreasing
iii increasing iv decreasing
b i weak ii strong
iii moderate iv weak
- 3 a 161 cm, interpolation
b 185 cm, interpolation
c 195 cm, extrapolation
- 4 a $\text{number of seeds} = 40 \times (\text{weight kg}) + 260$
b 468 seeds
c 8.5 kg
- 5 a Using points (4, 69) and (8, 59) we get the equation,
 $\text{time} = -\frac{5}{2} \times (\text{fitness level}) + 79$
b 71.5 seconds
c 7 (rounded up from 6.8)
d Interpolation. A fitness level of 3 and time of 62 seconds is within the data range already obtained.
- 6 a Too few points to judge if there is a relationship between variables. If there is, it appears weak at best, so using the line of best fit to make predictions would be unreliable.
b The form of the scatterplot is not linear. Hence, using a linear equation to make predictions would be inappropriate.
- 7 \$4 300 300, extrapolation

Review exercise

- 1 a 6.7 seconds b 14.3 seconds
c 2.4 seconds d no

- 2 a Grade A; 29 hours, Grade B; 26 hours
b Grade A; 4 hours, Grade B; 8 hours
c Grade A; 2 hours, Grade B; 4 hours
d Grade A; 20(50% of 40), Grade B; 10(25% of 40)
e Grade A; symmetrical, Grade B; positively skewed
f In samples of 40 batteries, a Grade B battery lasted longer than any other (32 hours)
- 3 a See below on this page
b See below on this page
- 4 a 2.6 b 1.93 c 1 to 4
d The mean would increase by one (from 2.6 to 3.6), but the standard deviation would remain the same (1.93).
- 5 Kathryn performed better in history, relative to the rest of the class, since she achieved a score that was a full standard deviation above the mean, whereas in mathematics her score was $\frac{2}{3}$ of a standard deviation above the mean.
- 6 a
-
- b In the first quarter (Jan–March)
- c Sales figures are seasonally changing, with greatest sales in and around the summer period, and least sales in the winter to early spring period (July–September). Nevertheless, it can be observed that there is a general upwards trend in sales.
- 7 a $\text{height (cm)} = 2.5 \times \text{tibia (cm)} + 72.5$ (Determined using points (45, 185) and (31, 150).)
b 182.5 cm
c 29 cm
d The answer to (b) is interpolation since a tibia length of 44 cm is within the data range already collected. The answer to (c) is extrapolation since a height of 145 cm is outside the data range already collected.



Chapter 19 answers

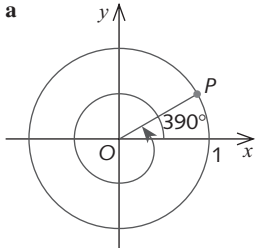
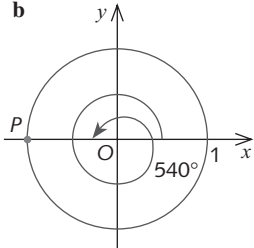
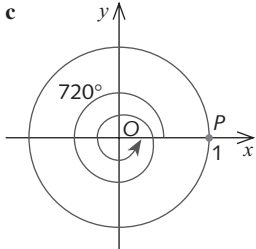
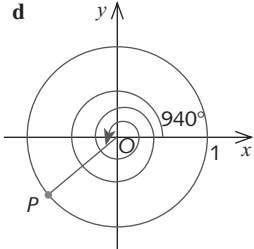
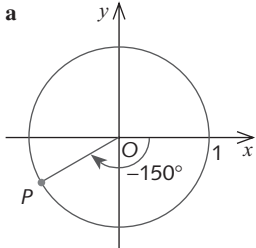
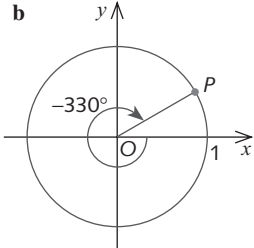
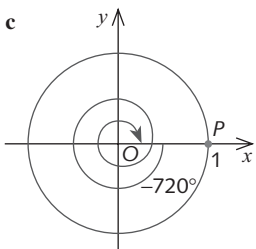
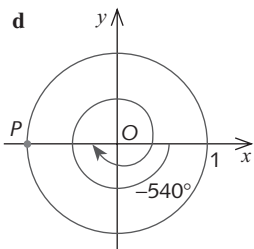
Exercise 19A

- 1 a second b third c third d fourth
e second f third g second h fourth
- 2 a $\sin 10^\circ$ b $-\cos 10^\circ$ c $-\tan 10^\circ$
d $-\sin 10^\circ$ e $-\cos 10^\circ$ f $\tan 10^\circ$
g $-\sin 10^\circ$ h $\cos 10^\circ$ i $-\tan 10^\circ$
- 3 a $\frac{1}{\sqrt{2}}$ b $-\frac{1}{\sqrt{2}}$ c $-\sqrt{3}$
d -1 e $-\frac{\sqrt{3}}{2}$ f $\frac{\sqrt{3}}{2}$
g $-\sqrt{3}$ h $\frac{1}{2}$ i $-\frac{1}{\sqrt{2}}$
- 4 a fourth b second c third
d third e second f third
- 5 a 1, 0, 0 b 0, 1
c $-1, 0, 0$ d 0, -1
e 1, 0, 0 f $\frac{1}{0}$ is not defined
- 6 a $\frac{1}{\sqrt{2}}$ b $-\frac{1}{2}$ c 0
d -1 e $-\frac{1}{\sqrt{2}}$ f $-\frac{3\sqrt{3}}{2} - 1$
- 7 a $\frac{1}{4}$ b $\frac{3}{4}$ c $\frac{1}{3}$ d $\frac{3}{4}$
e 3 f $\frac{3}{4}$ g 1 h 1
- 8 a $\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = \frac{a^2 + b^2}{c^2} = 1$, by Pythagoras' theorem
b $\cos^2 \theta = \left(\frac{-b}{c}\right)^2 = \frac{b^2}{c^2}$, the rest remains the same
c it is still true
- 9 a $\frac{2}{\sqrt{3}}$ b 1 c $\frac{2}{\sqrt{3}}$ d -2
e -2 f $\frac{1}{\sqrt{3}}$ g $-\frac{1}{\sqrt{3}}$ h $\frac{2}{\sqrt{3}}$
- 10 a From question 8, $\sin^2 \theta + \cos^2 \theta = 1$, now divide both sides of the equation by $\cos^2 \theta$.
b The identity holds for all θ between 0° and 360° except when $\cos \theta = 0$. That is, when $\theta = 90^\circ$ and 270° .

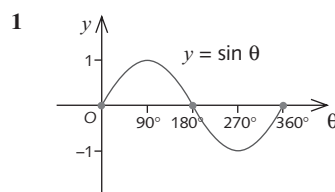
Exercise 19B

- 1 a $30^\circ, 150^\circ$ b $60^\circ, 240^\circ$ c $45^\circ, 315^\circ$
d $120^\circ, 240^\circ$ e $240^\circ, 300^\circ$ f $45^\circ, 225^\circ$
- 2 a $225^\circ, 315^\circ$ b $150^\circ, 330^\circ$
c $150^\circ, 210^\circ$ d 90°
e $90^\circ, 270^\circ$ f $0^\circ, 180^\circ, 360^\circ$
- 3 a $10^\circ, 170^\circ$ b $155^\circ, 205^\circ$ c $65^\circ, 245^\circ$
d $233^\circ, 307^\circ$ e $36^\circ, 324^\circ$ f $106^\circ, 286^\circ$
g $161^\circ, 199^\circ$ h $23^\circ, 203^\circ$ i $258^\circ, 282^\circ$

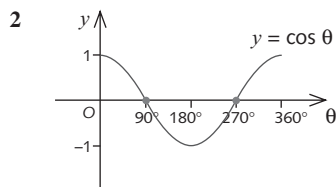
Exercise 19C

- 1 a  b 
- c  d 
- 2 a  b 
- c  d 
- 3 a 30° b 0° c 0° d 40°
- 4 a 30° b 30° c 0° d 0°
- 5 a 0 b -1 c 0 d $\frac{1}{2}$
e $-\frac{1}{2}$ f $-\sqrt{3}$ g $\frac{1}{2}$ h $\frac{1}{\sqrt{2}}$
- 6 a $-\frac{\sqrt{3}}{2}$ b $-\frac{1}{\sqrt{2}}$ c -1
d $-\frac{1}{2}$ e $\frac{1}{2}$ f $-\frac{1}{2}$
- 7 a 0 b 1 c 0 d 0
e 1 f 0 g 0

Exercise 19D



θ	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
$\sin \theta$	0.00	0.17	0.34	0.50	0.64	0.77	0.87	0.94	0.98	1.00

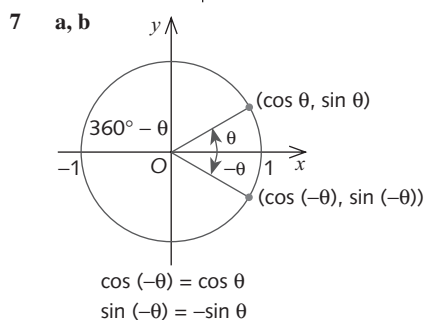
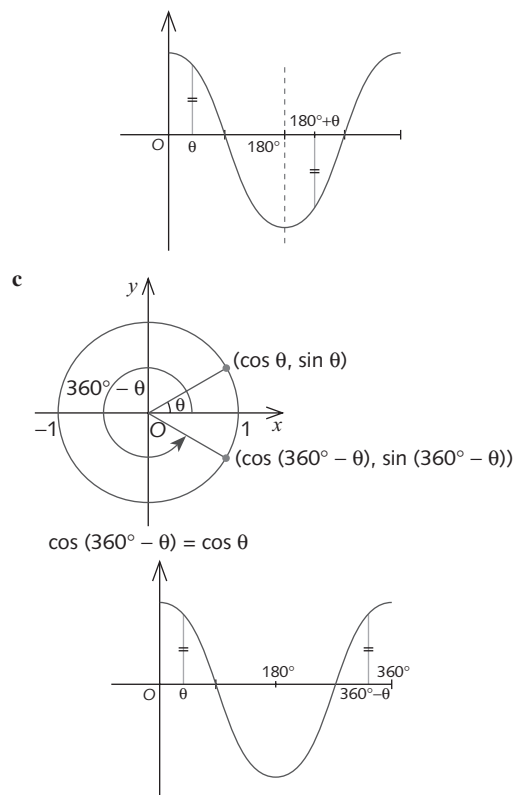
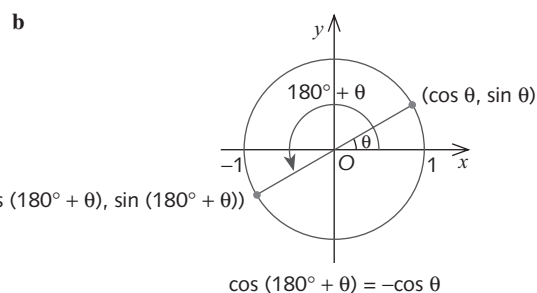
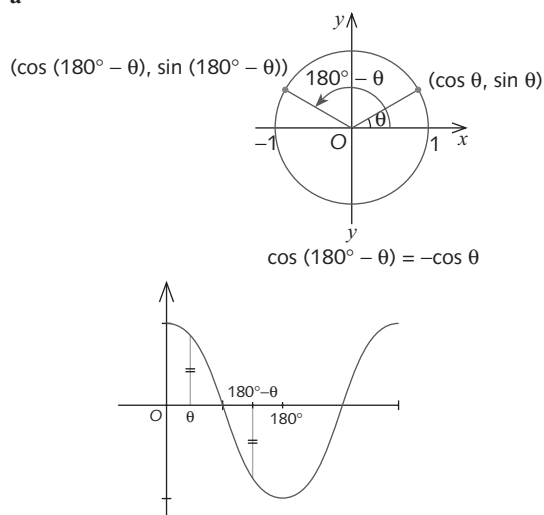


θ	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
$\cos \theta$	1.00	0.98	0.94	0.87	0.77	0.64	0.50	0.34	0.17	0.00

- 3 a i 0.5 ii -0.9 iii 0.9 iv 0.95
v -0.85 vi 0.40 vii -0.8 viii 0.55
b i $30^\circ, 150^\circ$ ii $120^\circ, 240^\circ$ iii $65^\circ, 115^\circ$
iv $55^\circ, 305^\circ$ v $55^\circ, 125^\circ$ vi $145^\circ, 215^\circ$
vii $205^\circ, 335^\circ$ viii $105^\circ, 255^\circ$

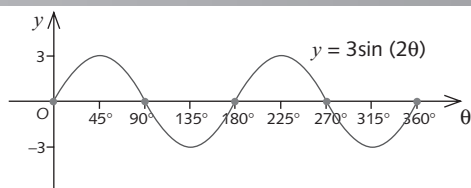
c 45° and 225°

- 4 a 1, -1
b maximum: 90° (plus multiples of 360°); minimum: 270° (plus multiples of 360°)
5 a 1, -1
b maximum: 0° (plus multiples of 360°); minimum: 180° (plus multiples of 360°)
6 a



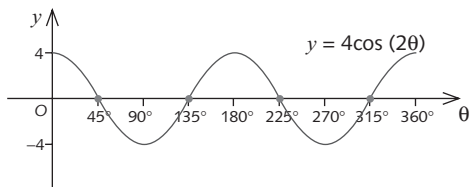
8

θ	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
$3\sin(2\theta)$	0	1.5	2.6	3	2.6	1.5	0	-1.5	-2.6	-3	-2.6	-1.5	0
θ		195°	210°	225°	240°	255°	270°	285°	300°	315°	330°	345°	360°
$3\sin(2\theta)$		1.5	2.6	3	2.6	1.5	0	-1.5	-2.6	-3	-2.6	-1.5	0



9

θ	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
$4\cos(2\theta)$	4	3.5	2	0	-2	-3.5	-4	-3.5	-2	0	2	3.5	4
θ		195°	210°	225°	240°	255°	270°	285°	300°	315°	330°	345°	360°
$4\cos(2\theta)$		3.5	2	0	-2	-3.5	-4	-3.5	-2	0	2	3.5	4

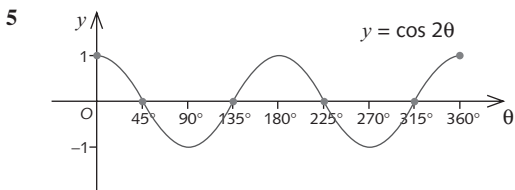
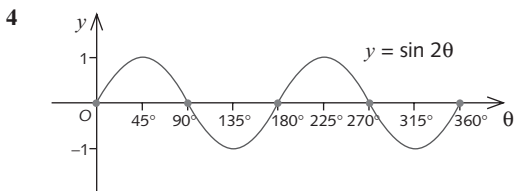


Exercise 19E

- 1 a $30^\circ, 150^\circ$ b $60^\circ, 120^\circ$ c $240^\circ, 300^\circ$
d $60^\circ, 300^\circ$ e $45^\circ, 225^\circ$ f $150^\circ, 330^\circ$
g $150^\circ, 210^\circ$ h $60^\circ, 240^\circ$
- 2 a $36^\circ, 144^\circ$ b $56^\circ, 304^\circ$ c $63^\circ, 117^\circ$
d $228^\circ, 312^\circ$ e $16^\circ, 344^\circ$ f $256^\circ, 284^\circ$
- 3 a $60^\circ, 120^\circ, 240^\circ, 300^\circ$ b $45^\circ, 135^\circ, 225^\circ, 315^\circ$
c $60^\circ, 120^\circ, 240^\circ, 300^\circ$ d $45^\circ, 135^\circ, 225^\circ, 315^\circ$
e $30^\circ, 150^\circ, 210^\circ, 330^\circ$ f $30^\circ, 150^\circ, 210^\circ, 330^\circ$
- 4 a $45^\circ, 225^\circ$ b $60^\circ, 240^\circ$

Review exercise

- 1 a 35° b 30° c 50° d 20°
e 70° f 60° g 60° h 60°
- 2 a $\frac{1}{2}, -\frac{\sqrt{3}}{2}, -\frac{1}{\sqrt{3}}$ b $\frac{\sqrt{3}}{2}, -\frac{1}{2}, -\sqrt{3}$
c $\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, -1$ d $-\frac{\sqrt{3}}{2}, \frac{1}{2}, -\sqrt{3}$
e $-\frac{1}{2}, -\frac{\sqrt{3}}{2}, \frac{1}{\sqrt{3}}$ f $-\frac{1}{2}, \frac{\sqrt{3}}{2}, -\frac{1}{\sqrt{3}}$
g $-\frac{\sqrt{3}}{2}, -\frac{1}{2}, \sqrt{3}$ h $-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, -1$
- 3 a $\frac{\sqrt{3}}{2}$ b 1 c $-\frac{1}{2}$
d 1 e $-\frac{1}{2}$ f -1



- 6 a $120^\circ, 240^\circ$ b $45^\circ, 135^\circ$ c $45^\circ, 225^\circ$
d $120^\circ, 300^\circ$ e $60^\circ, 120^\circ$ f $30^\circ, 150^\circ, 210^\circ, 330^\circ$

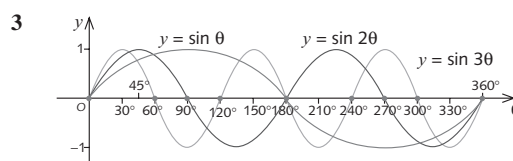
- 7 a $-\frac{1}{\sqrt{2}}$ b $-\frac{1}{2}$ c $-\frac{1}{\sqrt{3}}$
d $\frac{1}{2}$ e $-\frac{1}{2}$ f $-\frac{1}{\sqrt{3}}$

- 8 a $35^\circ, 145^\circ$ b $126^\circ, 234^\circ$
c $65^\circ, 245^\circ$ d $235^\circ, 305^\circ$

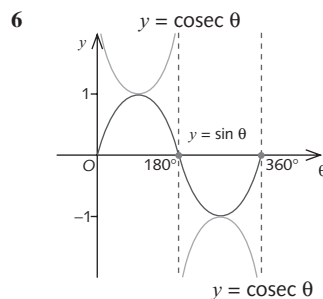
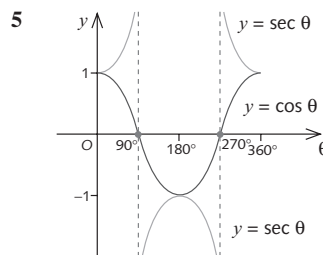
- 9 a $60^\circ, 300^\circ$ b $120^\circ, 240^\circ$ c $60^\circ, 120^\circ$
d $240^\circ, 300^\circ$ e 270° f $0^\circ, 360^\circ$

Challenge exercise

1 $\frac{3}{4}$



- 4 $0^\circ, 180^\circ, 360^\circ$



- 7 a $60^\circ, 300^\circ$ b $30^\circ, 150^\circ$ c $30^\circ, 150^\circ, 270^\circ$

8 b Area = $\frac{1}{2}ab \sin(\alpha + \beta) = \frac{1}{2}ya \sin \alpha + \frac{1}{2}yb \sin \beta$
 $= \frac{1}{2}(ab \sin \alpha \cos \beta + ab \sin \beta \cos \alpha)$

Thus $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \sin \beta \cos \alpha$

c Use $\alpha = 45^\circ$ and $\beta = 30^\circ$: $\sin 75^\circ = \frac{\sqrt{2} + \sqrt{6}}{4}$

- 9 a $2 \sin \theta \cos \theta$ b $\angle CGD = \theta + \theta = 2\theta$
c $2 \sin \theta \cos \theta = \text{area } ABC = \text{area } BCD = \frac{1}{2}BC \times DE = \sin 2\theta$

Chapter 20 answers

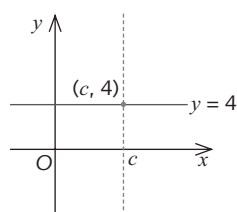
Exercise 20A

- 1 a all real numbers
c $x \neq 0$
e $x \neq -4$
g $x \neq 2$ and $x \neq -2$
- b all real numbers
d $x \neq 2$
f $x \neq 2$
h $x \neq 3$ and $x \neq -3$

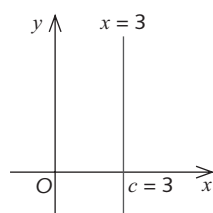
- 2 a $x \geq 0$
d $x \geq \frac{1}{7}$
- b $x \geq -7$
e $x > 0$
- c $x \leq 7$
f $x > 7$

- 3 a all real numbers
c $x > 0$
e $x < 0$
- b all real numbers
d $x > 2$
f all real numbers

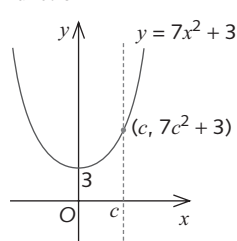
- 4 a function



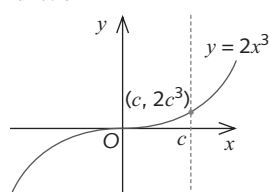
- b not a function



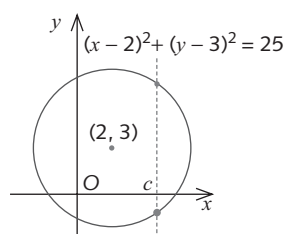
- c function



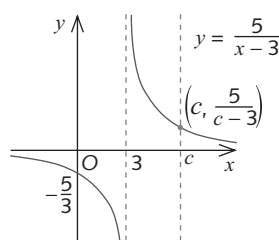
- d function



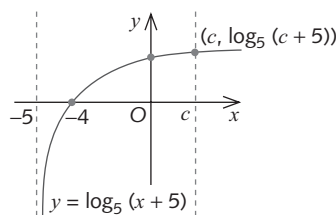
- e not a function



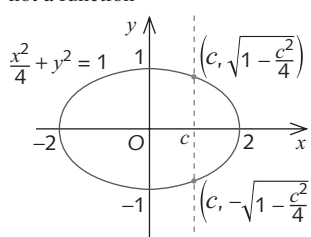
- f function



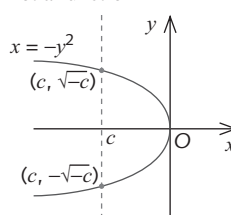
- g function



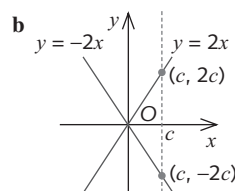
- h not a function



- i not a function



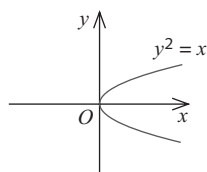
- 5 a $y = 2x$ or $y = -2x$



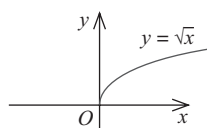
- c no

- d not a function

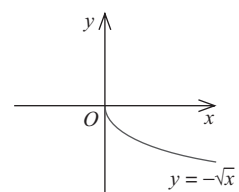
- 6 a



- b



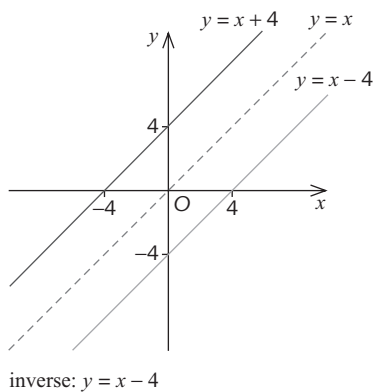
domain: $x \geq 0$



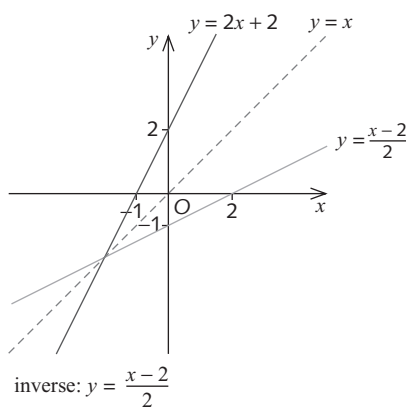
domain: $x \geq 0$

Exercise 20B

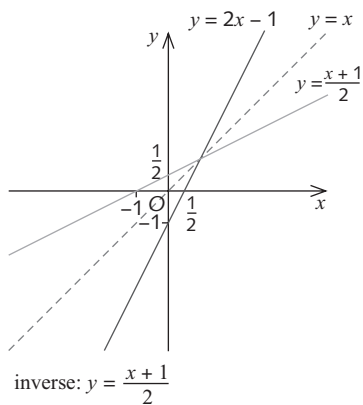
1 a



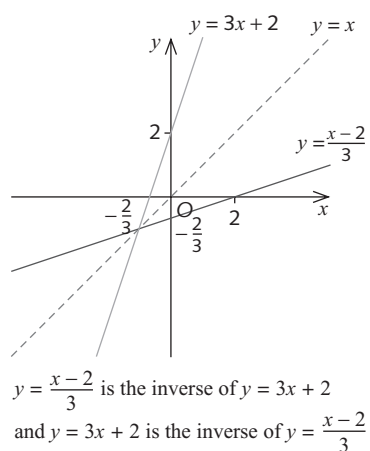
b



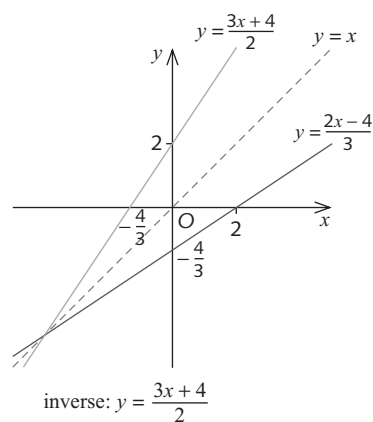
c



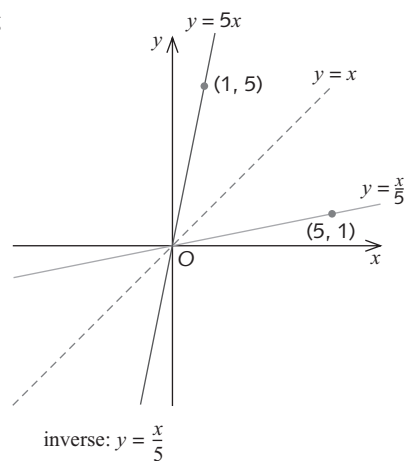
d, e



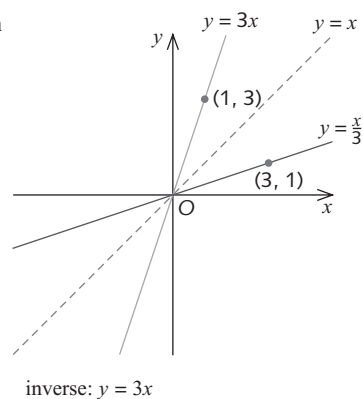
f



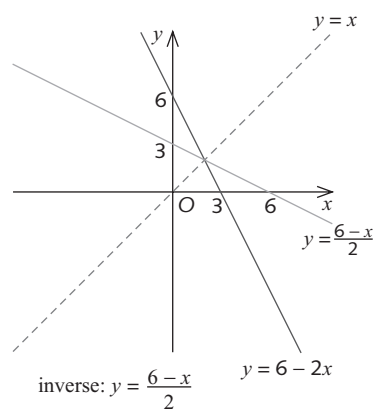
g

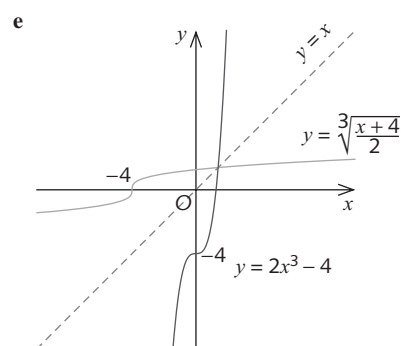
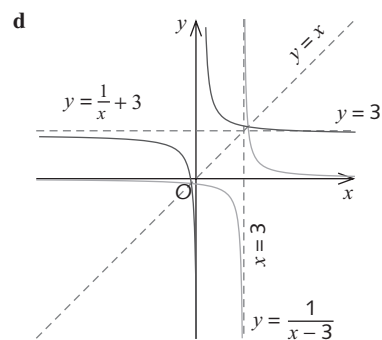
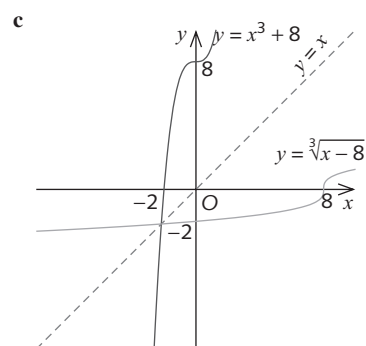
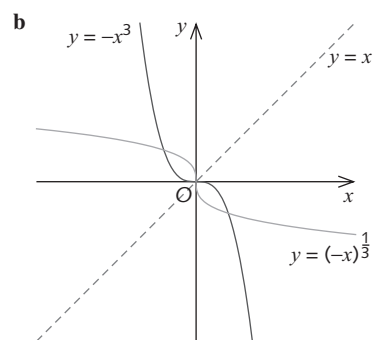
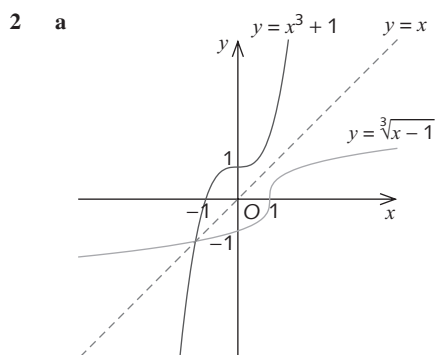
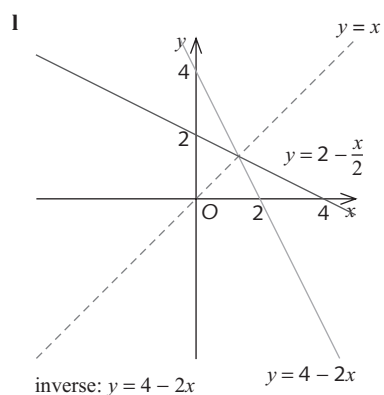
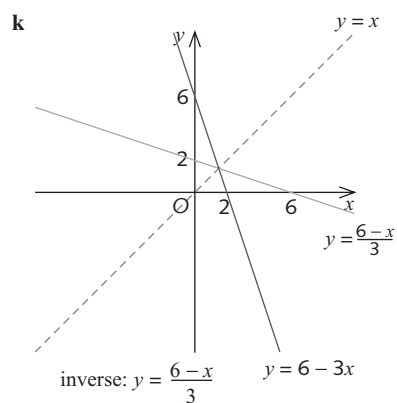
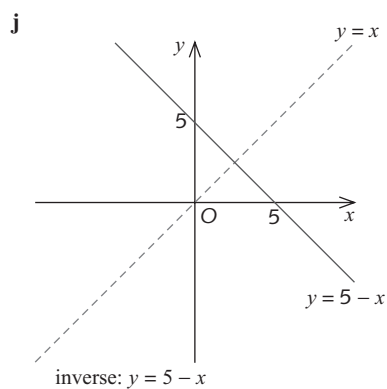


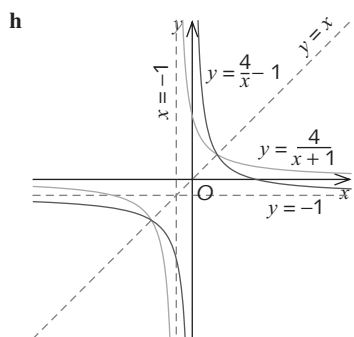
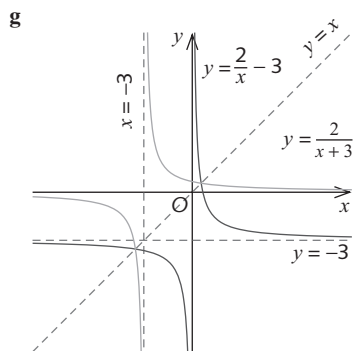
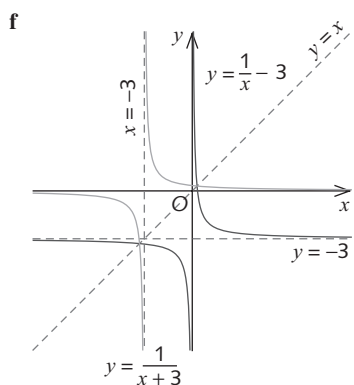
h



i







Exercise 20C

- 1 a 3 b -17 c 0
d -9 e 204 f -53
- 2 a 6 b 2 c 11
d $\frac{9}{4}$ e 4 f 504
- 3 a 1 b 0 c -6
d $\frac{3}{2}$ e 3 f $\frac{1}{3}$
- 4 a $\frac{1}{6}$ b $\frac{2}{5}$ c -2
- 5 a $x = 0$ or $x = 4$ b $x = 2$
c $x = -1$ or $x = 5$ d no values of x
e $x = 2 - \sqrt{5}$ or $x = 2 + \sqrt{5}$ f $x = 1$ or $x = 3$

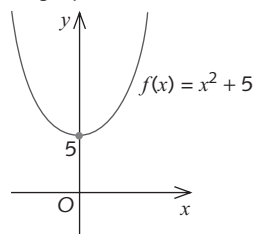
- 6 a $a^2 - 4$ b $y^2 + 4y$ c $4b^2 - 4$
d $9c^2 + 6c - 3$ e $x^4 - 4$ f $x^6 - 4$
- 7 a true b false c false
d true e true f true
- 8 a false b false c true
d false e false f true
- 9 a all real numbers, $y \leq 3$
b $x \neq 0, y \neq 0$
c all real numbers, $y > 4$
d $-3 \leq x \leq 3, 0 \leq y \leq 3$
e all real numbers, $y \leq 6$
f all real numbers, $y \geq 4$
g all real numbers, $y > -3$
h all real numbers, $y > 7$
i $-5 \leq x \leq 5, -5 \leq y \leq 0$
j all real numbers, all real numbers
k $x \neq 0, y \neq 0$
l $x < 7$, all real numbers
m all real numbers, $-1 \leq y \leq 1$
n $x \neq \dots, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \dots$, all real numbers

Exercise 20D

- 1 a b
c d
- 2 a b
c d
Indeed in this case, $-f(-x) = f(x)$

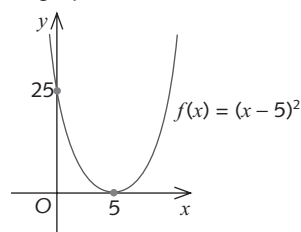
- 3 a domain: all real numbers

range: $y \geq 5$



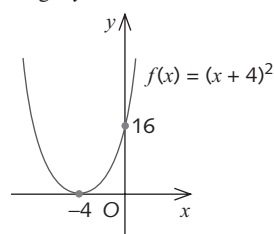
- b domain: all real numbers

range: $y \geq 0$



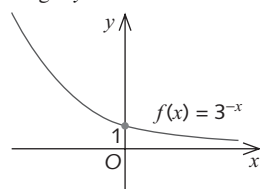
- c domain: all real numbers

range: $y \geq 0$



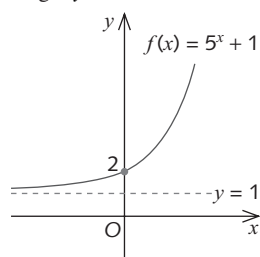
- d domain: all real numbers

range: $y > 0$



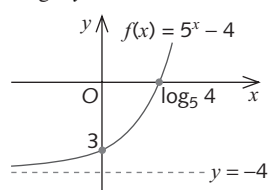
- e domain: all real numbers

range: $y > 1$



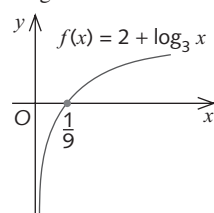
- f domain: all real numbers

range: $y > -4$



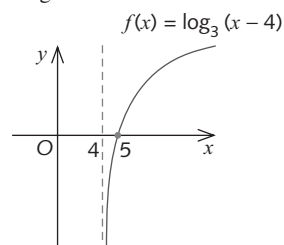
- g domain: $x > 0$

range: all real numbers



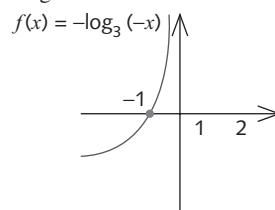
- h domain: $x > 4$

range: all real numbers



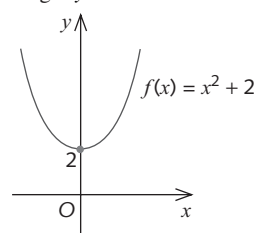
- i domain: $x < 0$

range: all real numbers



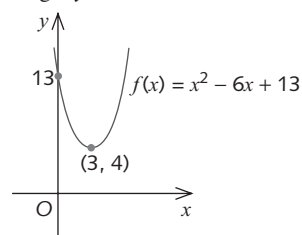
- 4 a domain: all real numbers

range: $y \geq 2$



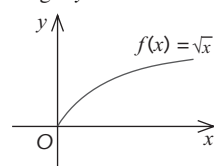
- b domain: all real numbers

range: $y \geq 4$



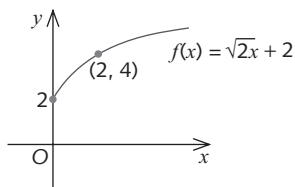
- c domain: $x \geq 0$

range: $y \geq 0$



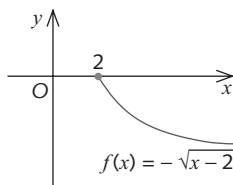
d domain: $x \geq 0$

range: $y \geq 2$



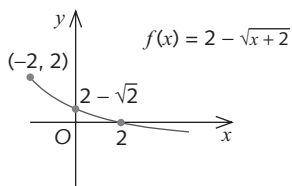
e domain: $x \geq 2$

range: $y \leq 0$

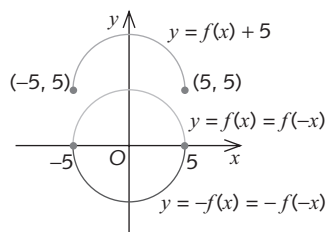


f domain: $x \geq -2$

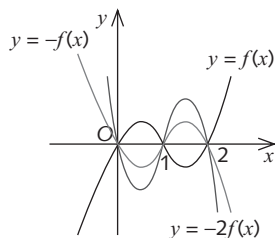
range: $y \leq 2$



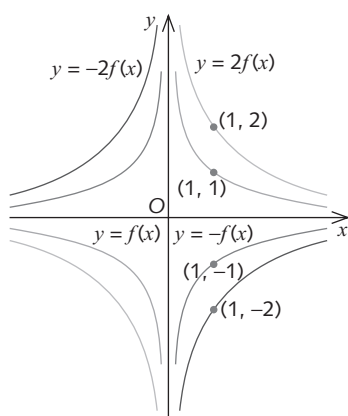
5



6



7



Exercise 20E

1 a 3 b 5 c 10

d $a + 3$ e $g(f(x)) = x + 3$

The result is a translation of 3 units to the right.

2 a 0 b -6 c -4

d -2 e 3 f -4

g $x^2 - 6$ h $x^2 - 4x$ i $x - 4$

j $x^4 - 8x^2 + 12$ k $f(g(x)) \neq g(f(x))$

3 a 2 b 2 c 4

d 4 e x f x

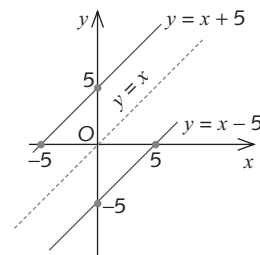
g $f(g(x)) = x$ and $g(f(x)) = x$, f and g are inverses of each other.

4 a 2 b 2 c 4

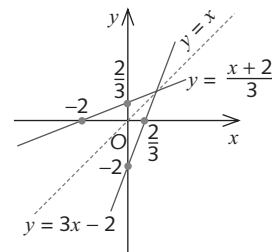
d 4 e x f x

g $f(g(x)) = x$ and $g(f(x)) = x$, f and g are inverses of each other.

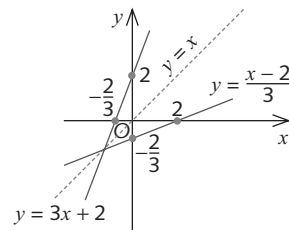
5 a $g(x) = x - 5$



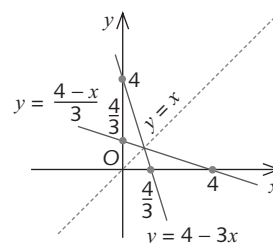
b $g(x) = \frac{x+2}{3}$



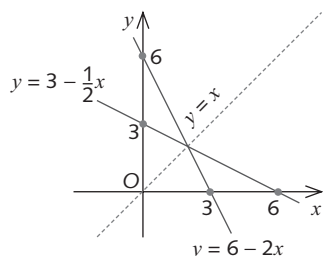
c $g(x) = \frac{x-2}{3}$



d $g(x) = \frac{4-x}{3}$



e $g(x) = 6 - 2x$



6 a $g(x) = \sqrt[3]{x+2}$

b $g(x) = \sqrt[3]{2-x}$

c $g(x) = \sqrt[5]{\frac{x}{32}}$

7 a domain: $x \neq 0$, $g(x) = \frac{1}{x-1}$, $x \neq 1$

b domain: $x \neq -1$, $g(x) = \frac{1}{x} - 1$, $x \neq 0$

c domain: $x \neq 2$, $g(x) = \frac{2(x+1)}{x-1}$, $x \neq 1$

d domain: $x \neq -2$, $g(x) = \frac{2x}{3-x}$, $x \neq 3$

8 a $f(f(x)) = f(5-x) = 5 - (5-x) = x$

b $f(f(x)) = f(-x) = x$

c $f(f(x)) = f\left(-\frac{1}{x}\right) = x$

d $f(f(x)) = f\left(\frac{6}{x}\right) = x$

e $f(f(x)) = f\left(\frac{2x-2}{x-2}\right) = x$

f $f(f(x)) = f\left(\frac{-3x-5}{x+3}\right) = x$

9 a domain: all real numbers; $g(x) = \frac{x}{3}$; domain of $g(x)$: all real numbers

b domain: all real numbers; $g(x) = \frac{1}{3}\log_2 x$; domain of $g(x)$: $x > 0$

c domain: all real numbers; $g(x) = \log_7 \frac{x}{5}$; domain of $g(x)$: $x > 0$

d domain: $x > 0$; $g(x) = 5^x$; domain of $g(x)$: all real numbers

e domain: $x > 0$; $g(x) = \frac{1}{3} \times 4^{\frac{x}{2}}$; domain of $g(x)$: all real numbers

f domain: $x > 3$; $g(x) = 2^x + 3$; domain of $g(x)$: all real numbers

g domain: all real numbers; $g(x) = \log_5(x) + 1$; domain of $g(x)$: $x > 0$

h domain: $x > 0$; $g(x) = 4^{x-4}$; domain of $g(x)$: all real numbers

i domain: all real numbers; $g(x) = \frac{1}{3}\log_5(x-5)$; domain of $g(x)$: $x > 5$

10 $y = \sqrt{49-x^2}$, $-7 \leq x \leq 0$; $y = \sqrt{49-x^2}$, $0 \leq x \leq 7$;

$y = -\sqrt{49-x^2}$, $-7 \leq x \leq 0$; $y = -\sqrt{49-x^2}$, $0 \leq x \leq 7$

Review exercise

1 a all real numbers

b $x \neq 0$

c $x \neq 5$

d $x \neq -8$

e $x \geq 2$

f all real numbers

g $x \neq -5$

h $x \geq -6$

2 a -4

b -3

c -3

d 12

e $a^2 - 4$

f $a^2 - 4$

g $4(a^2 - 1)$

h $a^2 - 4a$

3 a 3

b 1

c 5

d 11

e $3 - 2a$

f $3 + 2a$

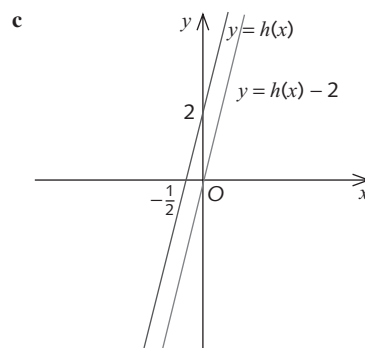
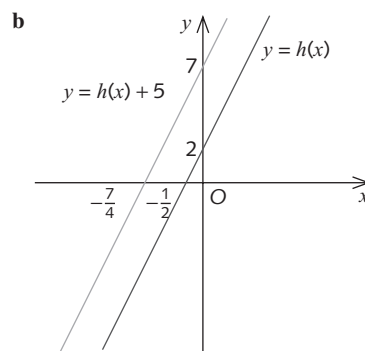
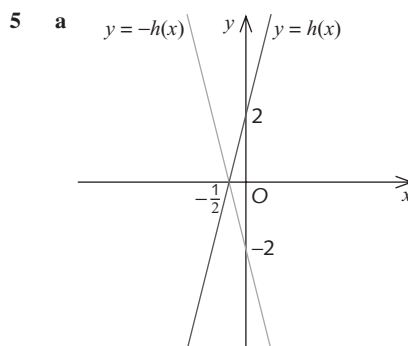
g $3 - 4a$

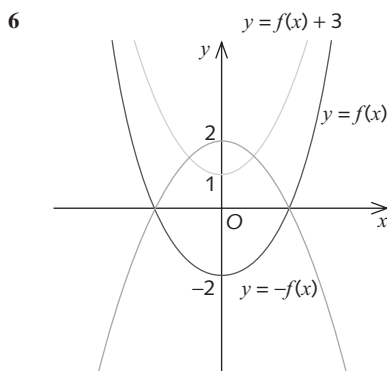
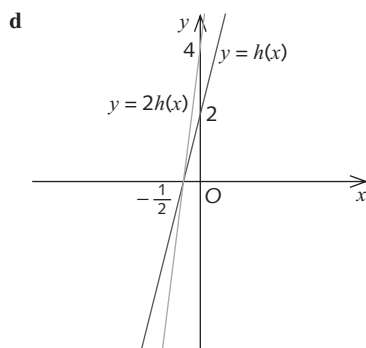
h $7 - 2a$

4 a domain: all real numbers; range: all real numbers

b domain: all real numbers; range: $y \leq 4$

c domain: $x \neq -6$; range: $y \neq 0$





- 7 a 4 b 11 c -20
 d 3 e 31 f 4
 g $11 - 2x^2$ h $-4x^2 - 4x + 4$ i $4x + 3$
 j $-x^4 + 10x^2 - 20$ k $f(g(x)) \neq g(f(x))$
- 8 a $y = \frac{x+4}{3}$ b $y = \frac{2-x}{3}$
 c $y = \sqrt[3]{x-2}$ d $y = \frac{1}{x} - 2$

Challenge exercise

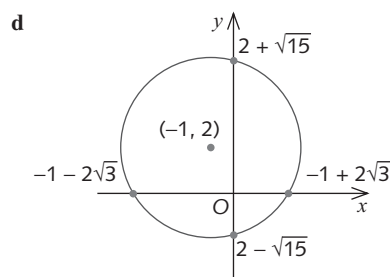
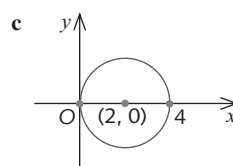
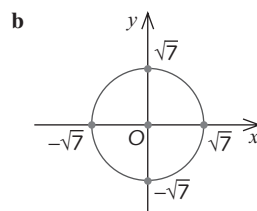
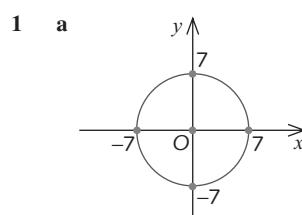
- 1 a $f(a+b) = 2(a+b) = 2a+2b = f(a)+f(b)$
 $f(ka) = 2ka = kf(a)$
 b Assume $f(a+b) = f(a)+f(b)$
 Then $a+b+2 = a+2+b+2 = a+b+4$, which is a contradiction.
 Assume $f(ka) = kf(a)$.
 Then, $ka+2 = ka+2k$.
 Thus, $k = 1$.
- 2 a $f(x+y) = 2^{x+y} = 2^x \times 2^y = f(x)f(y)$
 b $x+y = xy$. $x = 2$ and $y = 2$, and $x = y = 0$
- 3 a $f(x) = x^2$ is even and $f(x) = x^3$ is odd.
 b Let $f(x)$ and $g(x)$ be even functions. The sum function
 $(f+g)(x) = f(x)+g(x)$
 $(f+g)(-x) = f(-x)+g(-x) = f(x)+g(x) = (f+g)(x)$.
 c Let $f(x)$ and $g(x)$ be even functions. The product
 function $(fg)(x) = f(x)g(x)$
 $(fg)(-x) = f(-x)g(-x) = f(x)g(x) = (fg)(x)$

- d Let $f(x)$ and $g(x)$ be odd functions. The product
 function $(fg)(x) = f(x)g(x)$
 $(fg)(-x) = f(-x)g(-x) = -f(x) \times (-g(x))$
 $= f(x)g(x) = (fg)(x)$
 e Let $f(x)$ and $g(x)$ be odd functions. Then
 $f(g(-x)) = f(-g(x)) = -f(g(x))$

Chapter 21 answers

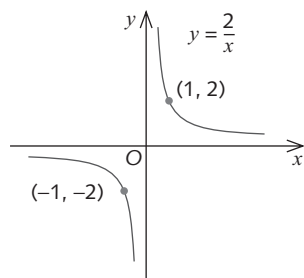
21A Review

Chapter 11: Circles, hyperbolas and simultaneous equations

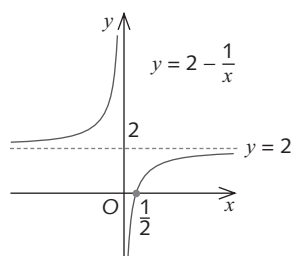


- 2 a $(x-3)^2 + y^2 = 16$
 b $(x+1)^2 + (y-2)^2 = 3$
- 3 a $(x-2)^2 + (y+3)^2 = 4$, centre $(2, -3)$, radius 2
 b $(x+1)^2 + (y+4)^2 = 16$, centre $(-1, -4)$, radius 4

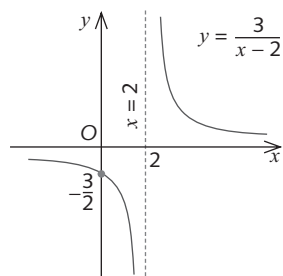
- 4 a asymptotes: $x = 0$ and $y = 0$



- b asymptotes: $x = 0$, $y = 2$; x -intercept $x = \frac{1}{2}$

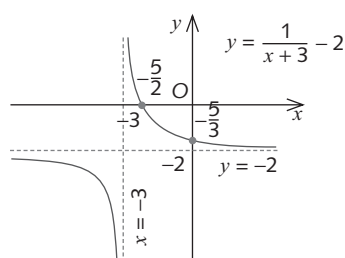


- c asymptotes: $y = 0$, $x = 2$, $x = 2$; y -intercept $= -\frac{3}{2}$



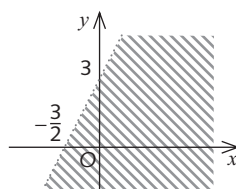
- d asymptotes: $x = -3$ and $y = -2$

x -intercept $= -\frac{5}{2}$, y -intercept $= -\frac{5}{3}$

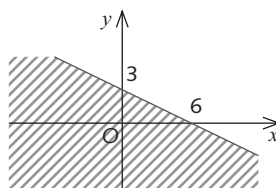


- 5 a $(3, 12), (-2, -3)$ b $(-2, -1), (\frac{3}{2}, 6)$
 c $(-\frac{3}{2}, -2), (1, 3)$ d $(-3, -2), (\frac{2}{3}, 9)$
 6 a $(\sqrt{5}, 2), (-\sqrt{5}, 2)$ b $(1, \sqrt{3}), (1, -\sqrt{3})$
 c $(0, 2), (-2, 0)$ d $(0, 4), (\frac{8\sqrt{2}}{3}, -\frac{4}{3})$
 7 $(-\frac{6}{5}, \frac{17}{5}), (2, -3)$
 8 $(-4, 3), (6, 8)$

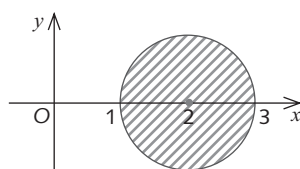
- 9 a



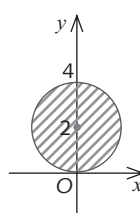
- b



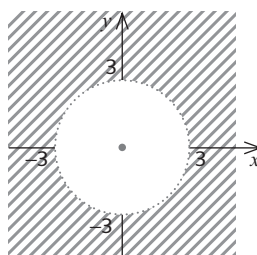
- c



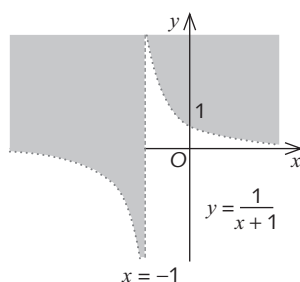
- d



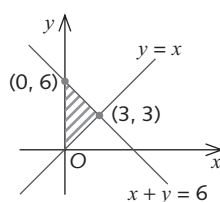
- e

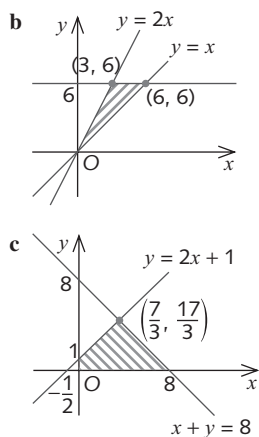


- f



- 10 a





Chapter 12: Further trigonometry

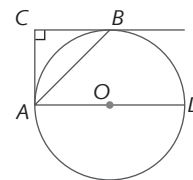
- 1 a $\theta \approx 43.2^\circ$ b $x \approx 2.9$ c $x \approx 4.9$
- 2 a 2.3 m b 1.9 m
- 3 63.4°
- 4 a 13.5 km b 132.0°T
- 5 281.2°T 6 52.5 m
- 7 a 4.619 m b 18.475 m c 4.619 m
- 8 a $x = \sqrt{2}$ b $x = 5(3 + \sqrt{3})$
- 9 $x = 50(\sqrt{3} - 1)$
- 10 $\frac{20(3 - \sqrt{3})}{3}$ cm
- 11 12 km
- 12 a $B \approx 102.6^\circ, C \approx 27.4^\circ, AC \approx 6.4$
b $A \approx 71.2^\circ, B \approx 38.8^\circ, BC \approx 6.0$
c $C = 35^\circ, AB \approx 1.3, AC \approx 2.17$
- 13 14.8 km 14 34.03 m
- 15 $100(\sqrt{3} + 1)$ m
- 16 a 24.8° b 56.9°
- 17 a 36.37 cm b 50.8° c 46.96 cm
- 18 10.0°
- 19 a 45° b 32.0°
- 20 a P at B or C, $\angle EPB \approx 35.26^\circ$
b P is midpoint of BC, $\angle EPB \approx 38.94^\circ$

Chapter 13: Circle geometry

- 1 a $a = 60, b = 120$
b $c = 20$
c $h = 110, j = 70$
d $d = 50, e = 40, f = 10, g = 40$
e $k = 25, m = 65, r = 40, s = 50, t = 65$

- 2 a i 90° ii 60° iii 30° iv 30°
b $6\sqrt{3}$ cm

- 3 $CB = CA$ (equal tangents). Hence, $\angle CAB = \angle CBA = 45^\circ$ (isosceles triangle), $\angle CAD = 90^\circ$ (tangent to circle at A). Therefore, BA bisects $\angle CAD$.



- 4 $BE = 2$ cm 5 $TS = 17$ cm
- 6 $EC = 10, ED = 6$

- 7 If $BP \times PQ = CP \times PR$

$$\text{then } \frac{BP}{CP} = \frac{PR}{PQ}$$

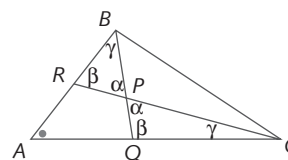
$\angle BPR = \angle CPQ$
(vertically opposite)

$\triangle RPB$ is similar to $\triangle QPC$ (SAS)

$\triangle ABQ$ is similar to $\triangle ACR$ (AAA)

$$\text{so } \frac{AR}{AC} = \frac{AQ}{AB}$$

$$AR \times AB = AC \times AQ$$



- 8 a $PY = 9$ cm, $XY = 5$ cm b $PT = 9$ cm
c $PX = 4$ cm

- 9 $\angle BOC = 150^\circ$

- 10 a Let $\angle YAL = \beta$

Then $\angle LAB = \beta$ (given)

$\angle ABL = \beta$ (alternate segment theorem)

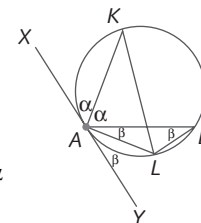
So $\triangle ALB$ is isosceles, $AL = BL$

- b Let $\angle XAK = \alpha$. Then $\angle KAB = \alpha$ (given)

$$\text{Therefore, } 2\alpha + 2\beta = 180^\circ$$

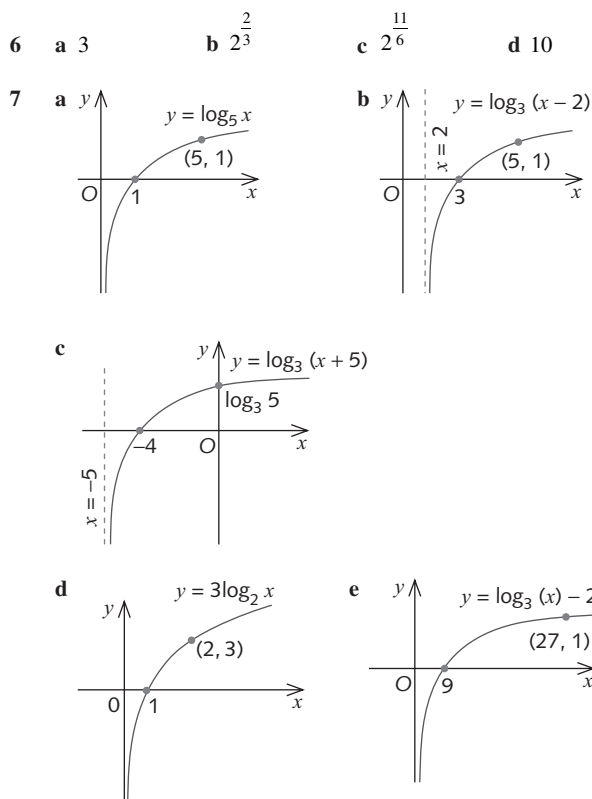
$$\alpha + \beta = 90^\circ$$

$\angle KAL$ is a right angle. Therefore, KL is a diameter (converse of Thale's theorem)



Chapter 14: Indices, exponentials and logarithms – part 2

- 1 a 4 b 4 c 10
d 0 e 5
- 2 a -5 b -5 c -4
d -2 e -4 f -3
g -11 h -5
- 3 a $\log_2 75$ b $\log_2 63$ c $\log_2 33$
d $\log_3 100$ e $\log_7 40$ f 1
g -1 h 1 i 1
- 4 a $\log_2 14$ b $\log_3 20$
c $-\log_5 343 = -3\log_5 7$ d $\log_{11} \frac{3}{5}$
- 5 a 125 b 256 c 620
d $\frac{1027}{6}$ e -59 f $\frac{10001}{2}$



Chapter 15: Probability

- 1 a $\frac{1}{3}$ b $\frac{1}{2}$
- 2 a $\frac{1}{4}$ b $\frac{1}{13}$ c $\frac{1}{52}$ d $\frac{3}{13}$
- 3 $\frac{1}{200}$ 4 $\frac{5}{16}$
- 5 a $\frac{1}{12}$ b $\frac{1}{36}$ c $\frac{13}{18}$
- 6 a i $\frac{4}{25}$ ii $\frac{2}{15}$
b i $\frac{12}{25}$ ii $\frac{8}{15}$
- 7 a $\frac{1}{2}$ b $\frac{77}{200}$ c $\frac{40}{77}$ d $\frac{2}{5}$
- 8 a $p = \frac{4}{15}$ b $p = \frac{1}{3}$
- 9 a 0.75
b They are not independent as $P(H \cap F) \neq P(H)P(F)$
- 10 a 0.5 b 0.25

Chapter 16: Direct and inverse proportion

- 1 a i $\frac{1}{2}, y = \frac{1}{2}x$ ii

x	1	4	8	20
y	$\frac{1}{2}$	2	4	10

- b i $2, y = 2x^2$ ii

x	1	3	5	$\sqrt{7}$
y	2	18	50	14

- c i $5, y = \frac{5}{x}$ ii

x	2	5	7	11
y	$\frac{1}{2}$	1	$\frac{5}{7}$	$\frac{5}{11}$

- d i $1, y = \frac{1}{x^2}$ ii

x	2	3	$\frac{1}{2\sqrt{2}}$	7
y	$\frac{1}{4}$	$\frac{1}{9}$	8	$\frac{1}{49}$

2 $y = 9\sqrt{x}$ a $y = 18$ b $x = \frac{625}{9}$

3 154 cm^2

4 $y = \frac{40}{x^2}$ a $y = \frac{40}{81}$ b $x = \frac{2\sqrt{10}}{3}$

- 5 a $2, c = 2ab^2$ b

a	5	3	6	3
b	1	2	2	3
c	10	24	48	54

6 $a = 32$ 7 $z = 243$

- 8 a 2250 joules
b 8 times the original energy

Chapter 17: Polynomials

- 1 a 3 b 5 c 8
d 0 e 4 f $a^3 - 2a + 4$
- 2 a $a = 7$ b $b = 2$
- 3 a $P(x) + Q(x) = -x^3 + 3x^2 + 7$,
 $P(x) - Q(x) = 3x^3 - 3x^2 + 8x + 7$
b $P(x) + Q(x) = x^2 - 3x$,
 $P(x) - Q(x) = -6x^5 - x^2 - 3x + 14$
c $P(x) + Q(x) = -x + 2$,
 $P(x) - Q(x) = 8x^3 - 10x^2 - 11x + 10$
- 4 a $P(x) = (x + 2)(x + 6) - 6$
b $P(x) = (x + 6)(x^2 - 12x + 60) - 330$
c $P(x) = (x - 1)(5x^2 - 2x - 2) - 3$
- 5 a 45 b $\frac{25}{8}$ c $\frac{31}{8}$
- 6 $a = -1$
- 7 a $(x - 1)(x + 2)(2x + 3)$
b $(x - 2)(x + 1)(2x + 3)$
c $(x - 2)(x - 1)(x + 1)(2x + 3)$
- 8 a $x = 1$ or $x = -2$ or $x = -\frac{3}{2}$
b $x = 2$ or $x = 1$ or $x = -1$ or $x = -\frac{3}{2}$
- 9 a $P(1) = 0$ for all x . b $k = 7$
c $x = 1$ or $x = 2$ or $x = 4$

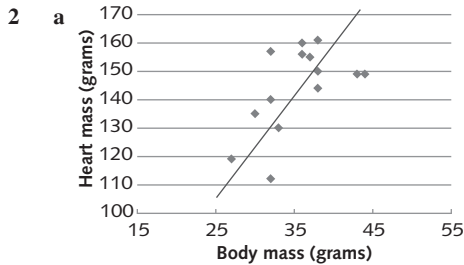
10 a $2 + \frac{5}{x-1}$

b $4 + \frac{2-5x}{x^2+2x}$

Chapter 18: Statistics

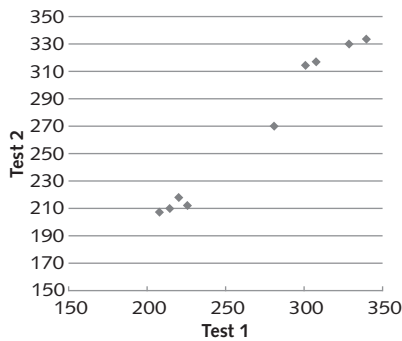
1 a $\bar{x} = 9.45, \sigma = 3.85$

b $\bar{x} = 13.50, \sigma = 3.54$



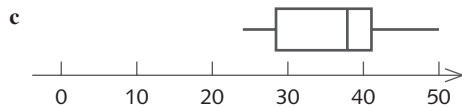
b There is some tendency for the heart mass to increase as the body mass increases. The relationship could be described as linear and of moderate strength.

3 As the scores on Test 1 increase, the scores on Test 2 increase.



4 a 38

b 11.5



5 a 19

b approximately 150

c 4 hours

d approximately 50

6 a i $a = 38, c = 61, e = 88$

ii 62

b i 17

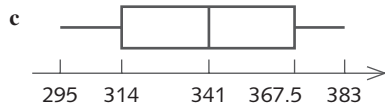
ii 63

7 a public holiday, roadworks, accident

b i 340.8

ii 341

iii 53.5



d George may have sampled over school holiday period. Community group may have sampled for a small number of days and obtained high counts, or may be exaggerating.

Chapter 19: Trigonometric functions

1 a second

b third

c third

d fourth

e second

f fourth

g third

h fourth

2 a $\sin 5^\circ$

b $-\cos 30^\circ$

c $-\tan 20^\circ$

d $-\sin 20^\circ$

e $-\cos 20^\circ$

f $\tan 5^\circ$

g $-\sin 5^\circ$

h $\cos 10^\circ$

3 a $-\frac{\sqrt{2}}{2}$

b $-\frac{\sqrt{2}}{2}$

c $\frac{\sqrt{3}}{2}$

d $-\sqrt{3}$

e $-\frac{1}{2}$

f $\frac{\sqrt{2}}{2}$

g -1

h $-\frac{\sqrt{3}}{2}$

4 a $\frac{1}{2}$

b $\frac{1}{4}$

c 0

d $-\frac{\sqrt{3}}{2}$

5 a $60^\circ, 300^\circ$

b $120^\circ, 300^\circ$

c $45^\circ, 135^\circ$

d $210^\circ, 330^\circ$

e $150^\circ, 210^\circ$

f $135^\circ, 315^\circ$

6 a $15.93^\circ, 164.07^\circ$

b $156.42^\circ, 203.58^\circ$

c $66.00^\circ, 246.00^\circ$

d $243.84^\circ, 296.16^\circ$

e $45.49^\circ, 314.51^\circ$

f $111.12^\circ, 291.12^\circ$

7 a $\frac{1}{2}$

b $\frac{\sqrt{2}}{2}$

c 1

d $-\frac{\sqrt{3}}{2}$

e $\frac{\sqrt{3}}{2}$

f $-\frac{\sqrt{2}}{2}$

8 a $60^\circ, 300^\circ$

b $150^\circ, 210^\circ$

c $240^\circ, 300^\circ$

d $120^\circ, 240^\circ$

e $45^\circ, 225^\circ$

f $30^\circ, 210^\circ$

Chapter 20: Functions and inverse functions

1 a -1

b 7

c -3

d -11

2 a 8

b 2

c $\frac{1}{2}$

d -2

3 a 2

b 4

c -2

d 6

4 a $\frac{6}{5}$

b $\frac{1}{5}$

5 a $x \neq -2$

b $x \neq 2$

c $x \leq 5$

d $x \geq 2$

e $x \neq 3$ and $x \neq -3$

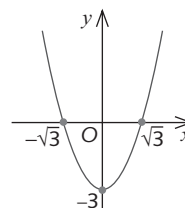
f $x > -7$

g all real numbers

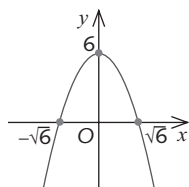
h $x > \frac{1}{2}$

i $x < 6$

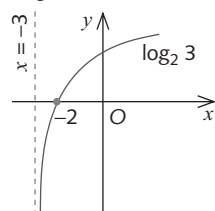
6 a domain: all real numbers, space and range: $y \geq -3$



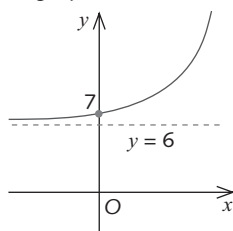
b domain: all real numbers, space and range: $y \leq 6$



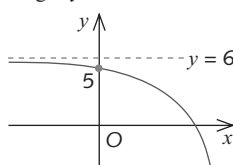
c domain: $x > -3$
range: all real numbers



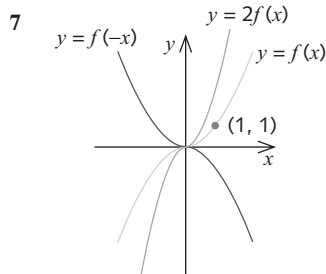
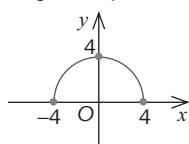
d domain: all real numbers
range: $y > 6$



e domain: all real numbers
range: $y < 6$



f domain: $-4 \leq x \leq 4$
range: $0 \leq y \leq 4$



8 a 1

c $g(f(x)) = 2x^2 - 3$

b -1

d $f(g(x)) = (2x - 3)^2$

9 The domain for **a**, **b**, **d**, **e** and **f** is all real numbers. The domain for **c** is the real numbers > -3 .

a $g(x) = \frac{x+3}{2}$

b $g(x) = 2x + 1$

c $g(x) = \log_2(x + 3)$

d $g(x) = 3^x - 1$

e $g(x) = \sqrt[3]{8-x}$

f $g(x) = \sqrt[3]{8+x}$

21B Problem-solving

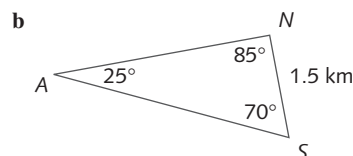
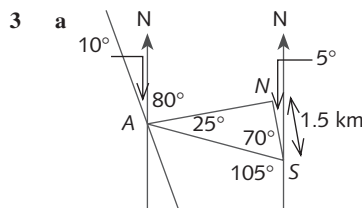
1 a 10.2 km

b 295.5°

2 a i $4\sqrt{3}$ m

ii 53°

b $(5\sqrt{3} + 12)$ m



c 3.335 km

d 3.205 km

4 a i 60° **ii** 30° **iii** 30° **iv** 90°

b i 10 km

ii 5 km

c $\angle PAY = \angle SXY = 90^\circ$, $\angle APY = \angle XSY = 60^\circ$,
 $\angle AYP = \angle SYX = 30^\circ$, so $\triangle PAY$ is similar to
 $\triangle SXY$ (AAA).

d 2.5 km

e i $\frac{15\sqrt{3}}{2}$ km

ii $\frac{30 - 15\sqrt{3}}{2}$ km

f 7.765 km

5 a 3, 4, 5 triangle

b $\angle APB$

c $AP = AQ$ (radii of smaller circle), $BP = BQ$
(radii of larger circle), $AB = AB$ (common), so
 $\triangle APE = \triangle AQB$ (SSS).

d i 53°

ii 106°

e 53° ($\angle PFQ$ is angle on circumference standing on the
same arc as $\angle PAQ$, the angle at the centre of circle)

6 a $\angle CEP = \angle DEQ$ (common), $\angle ECP = \angle EDQ = 90^\circ$
(tangent $\perp$ radius), so $\triangle CPE$ is similar to $\triangle DQE$ (AAA).

b $\angle GFP = \angle GHQ = 90^\circ$ (tangent $\perp$ radius),
 $\angle FGP = \angle HGQ$ (vertically opposite angles at G), so
 $\triangle GFP$ is similar to $\triangle GHQ$ (AAA).

c From $\triangle CPE$ and $\triangle DQE$, $\frac{CE}{DE} = \frac{CP}{DQ}$

From $\triangle GFP$ and $\triangle GHQ$, $\frac{FG}{GH} = \frac{PF}{HQ} = \frac{CP}{DQ}$ (radii), so

$$\frac{CE}{DE} = \frac{FG}{GH} \text{ so } \frac{CE}{FG} = \frac{DE}{GH}$$

7 a deviation of 4.30°

b $\sqrt{(275 - 220 \cos 10^\circ)^2 + (220 \sin 10^\circ + 75\sqrt{3})^2}$
 $\approx 177.94 \text{ m}$

8 a $x^2 + y^2 = 16$

b $y = 2x - 4$

c $P = \left(\frac{16}{5}, \frac{12}{5}\right)$, peg is 3.2 m east and 2.4 m north of the centre of the garden

9 a i $h = r \tan \theta$

ii $s = \frac{r}{\cos \theta}$

b i Area (A) = $\frac{1}{2}\pi r^2$

ii Area (B) = $r^2 \tan \theta$

c $\frac{1}{2}\pi r^2 = r^2 \tan \theta$, $\frac{\pi}{2} = \tan \theta$

d 57.5°

e $4\pi \approx 12.57 \text{ mm}^2$

10 a i 60°

ii 30°

b $a + b = 90$

c i $CE = (10 - x) \text{ cm}$

ii $DE = (10 - y) \text{ cm}$

d $x(10 - x) = y(10 - y) \Rightarrow 0 = 10(y - x) - (y^2 - x^2)$
 $\Rightarrow 0 = 10(y - x) - (y - x)(y + x)$
 $0 = (y - x)(10 - y - x)$

e i $\frac{xy}{2} \text{ cm}^2$ ii $\frac{1}{2}(10 - x)(10 - y) \text{ cm}^2$

f $\frac{xy}{2} = \frac{1}{2}(10 - x)(10 - y)$

$xy = 100 - 10y - 10x + xy$

$10(x + y) = 100$

$x + y = 10$

g $\frac{x(10 - x)}{2} \text{ cm}^2$

h $x = 6$ or 4

