AUSTRALIAN MATHEMATICAL SCIENCES INSTITUTE

ICE-EM MATHEMATICS 8

THIRD EDITION

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INTERNATIONAL CENTRE OF EXCELLENCE FOR EDUCATION IN MATHEMATICS

CAMBRIDGE UNIVERSITY PRESS

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

4843/24, 2nd Floor, Ansari Road, Daryaganj, Delhi – 110002, India

79 Anson Road, #06–04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781108404297

First published 2017
20 19 18 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1

Cover designed by Loupe Studio
Typeset by diacriTech
Printed in China by C & C Offset Printing Co. Ltd.

A Catalogue-in-Publication entry is available from the catalogue of the National Library of Australia at www.nla.gov.au

ISBN 978-1-108-40429-7 Paperback

Additional resources for this publication at www.cambridge.edu.au/GO

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Preface

ICE-EM Mathematics Third Edition is a series of textbooks for students in years 5 to 10 throughout Australia who study the Australian Curriculum and its state variations.

The program and textbooks were developed in recognition of the importance of mathematics in modern society and the need to enhance the mathematical capabilities of Australian students. Students who use the series will have a strong foundation for work or further study.

Background

The International Centre of Excellence for Education in Mathematics (ICE-EM) was established in 2004 with the assistance of the Australian Government and is managed by the Australian Mathematical Sciences Institute (AMSI). The Centre originally published the series as part of a program to improve mathematics teaching and learning in Australia. In 2012, AMSI and Cambridge University Press collaborated to publish the Second Edition of the series to coincide with the introduction of the Australian Curriculum, and we now bring you the Third Edition.

The series

ICE-EM Mathematics Third Edition provides a progressive development from upper primary to middle secondary school. The writers of the series are some of Australia's most outstanding mathematics teachers and subject experts. The textbooks are clearly and carefully written, and contain background information, examples and worked problems.

For the Third Edition, the series has been carefully edited to present the content in a more streamlined way without compromising quality. There is now one book per year level and the flow of topics from chapter to chapter and from one year level to the next has been improved.

The year 10 textbook incorporates all material for the 10A course, and selected topics in earlier books carefully prepare students for this. *ICE-EM Mathematics Third Edition* provides excellent preparation for all of the Australian Curriculum's year 11 and 12 mathematics courses.

For the Third Edition, *ICE-EM Mathematics* now comes with an Interactive Textbook: a cutting-edge digital resource where all textbook material can be answered online (with students' working-out), additional quizzes and features are included at no extra cost. See 'The Interactive Textbook and Online Teaching Suite' on page xiii for more information.

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Author biographies

Peter Brown

Peter Brown studied Pure Mathematics and Ancient Greek at Newcastle University, and completed postgraduate degrees in each subject at the University of Sydney. He worked for nine years as a mathematics teacher in NSW State schools. Since 1990, he has taught Pure Mathematics at the School of Mathematics and Statistics at the University of New South Wales (UNSW). He was appointed Director of First Year Studies at UNSW from 2011 to 2015. He specialises in Number Theory and History of Mathematics and has published in both areas. Peter regularly speaks at teacher inservices, Talented Student days and Mathematics Olympiad Camps. In 2008 he received a UNSW Vice Chancellor's Teaching Award for educational leadership.

Michael Evans

Michael Evans has a PhD in Mathematics from Monash University and a Diploma of Education from La Trobe University. He currently holds the honorary position of Senior Consultant at the Australian Mathematical Sciences Institute at the University of Melbourne. He was Head of Mathematics at Scotch College, Melbourne and has also taught in public schools returning to classroom teaching in recent years. He has been very involved with curriculum development at both state and national levels. In 1999, Michael was awarded an honorary Doctor of Laws by Monash University for his contribution to mathematics education, and in 2001 he received the Bernhard Neumann Award for contributions to mathematics enrichment in Australia.

Garth Gaudry

Garth Gaudry was Head of Mathematics at Flinders University before moving to UNSW, where he became Head of School. He was the inaugural Director of AMSI before he became the Director of AMSI's International Centre of Excellence for Education in Mathematics. Previous positions include membership of the South Australian Mathematics Subject Committee and the Eltis Committee appointed by the NSW Government to enquire into Outcomes and Profiles. He was a life member of the Australian Mathematical Society and Emeritus Professor of Mathematics, UNSW.

David Hunt

David Hunt graduated from the University of Sydney in 1967 with an Honours degree in Mathematics and Physics, then obtained a master's degree and a doctorate from the University of Warwick. He was appointed to a lectureship in Pure Mathematics at UNSW in early 1971, where he is currently an honorary Associate Professor. David has taught courses in Pure Mathematics from first year to master's level and was Director of First Year Studies in Mathematics for five years. Many of David's activities outside UNSW have centred on the Australian Mathematics Trust. In 2016 David was awarded the Paul Erdos medal, in recognition of his contributions to education, as well as his work with the International Mathematical Olympiad movement.

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Janine McIntosh

Janine McIntosh works at The Australian Mathematical Sciences Institute, where she manages AMSI Schools. Janine leads a professional development and schools visit program for teachers across the country. Through clusters of schools supported by industry and government partners, Janine's aim is to encourage more Australians to enjoy and study mathematics. Janine has developed a suite of online and careers materials in her time at AMSI and was one of the writers for the Australian Curriculum: Mathematics F – 10. She is an experienced primary teacher, who has worked as a lecturer in mathematics education at the University of Melbourne and serves on the Maths Challenge and AMOC Committees of the Australian Mathematics Trust.

Bill Pender

Bill Pender has a PhD in Pure Mathematics from Sydney University and a BA (Hons) in Early English from Macquarie University. After a year at Bonn University, he taught at Sydney Grammar School from 1975 to 2008, where he was Subject Master for many years. He has been involved in the development of NSW Mathematics syllabuses since the early 1990s, and was a foundation member of the Education Advisory Committee of AMSI. He has also lectured and tutored at Sydney University and at UNSW, and given various inservice courses. Bill is the lead author of the NSW calculus series *Cambridge Mathematics*.

Jacqui Ramagge

Jacqui Ramagge is currently Head of the School of Mathematics and Statistics at the University of Sydney. After graduating in 1993 with a PhD in Mathematics from the University of Warwick (UK) she worked at the University of Newcastle (Australia) until 2007 and then at the University of Wollongong until 2015 when she moved to the University of Sydney. She has served on the Australian Research Council College of Experts, including as Chair of Australian Laureate Fellowships Selection Advisory Committee. She teaches mathematics at all levels from primary school to PhD courses and has won a teaching award. She contributed to the Vermont Mathematics Initiative (USA) and is a founding member of the Australian Mathematics Trust Primary Problems Committee. In 2013 she received a BH Neumann Award from the Australian Mathematics Trust for her significant contribution to the enrichment of mathematics learning in Australia.

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Acknowledgements

We are grateful to Professor Peter Taylor OA, former Director of the Australian Mathematical Trust, for his support and guidance as chairman of the Australian Mathematical Sciences Institute Education Advisory Committee, 2003–2011.

We gratefully acknowledge the major contributions made by those schools that participated in the Pilot Program during the development of the *ICE-EM Mathematics* program.

We also gratefully acknowledge the assistance of:

Sue Avery
Robin Bailey
Brian Dorofaeff
Andy Edwards
Claire Ho
Nikolas Sakellaropolous
James Wan

We also gratefully acknowledge the hard work and professionalism of David Treeby in the development and editing of this Third Edition.

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How to use this resource

The textbook

Each chapter in the textbook addresses a specific Australian Curriculum content strand and set of sub-strands. The exercises within chapters take an integrated approach to the concept of proficiency strands, rather than separating them out. Students are encouraged to develop and apply Understanding, Fluency, Problem-solving and Reasoning skills in every exercise.

The series places a strong emphasis on understanding basic ideas, along with mastering essential technical skills. Mental arithmetic and other mental processes are major focuses, as is the development of spatial intuition, logical reasoning and understanding of the concepts.

Problem-solving lies at the heart of mathematics, so *ICE-EM Mathematics* gives students a variety of different types of problems to work on, which help them develop their reasoning skills. Challenge exercises at the end of each chapter contain problems and investigations of varying difficulty that should catch the imagination and interest of students. Further, two 'Review and Problem-solving' chapters in each 7–10 textbook contain additional problems that cover new concepts for students who wish to explore the subject even further.

The Interactive Textbook and Online Teaching Suite

Included with the purchase of the textbook is the Interactive Textbook. This is the online version of the textbook and is accessed using the 16-character code on the inside cover of this book.

The Online Teaching Suite is the teacher version of the Interactive Textbook and contains all the support material for the series, including tests, worksheets, skillsheets, curriculum documentation and more.

For more information on the Interactive Textbook and Online Teaching Suite, see page xiii.

The Interactive Textbook and Online Teaching Suite are delivered on the *Cambridge HOTmaths* platform, providing access to a world-class Learning Management System for testing, task management and reporting. They do not provide access to the *Cambridge HOTmaths* stand-alone resource that you or your school may have used previously. For more information on this resource, contact Cambridge University Press.

AMSI's TIMES and SAM modules

The TIMES and SAM web resources were developed by the *ICE-EM Mathematics* author team at AMSI and are written around the structure of the Australian Curriculum. These resources have been mapped against your *ICE-EM Mathematics* book and are available to teachers and students via the AMSI icon on the Dashboard of the Interactive Textbook and Online Teaching Suite.

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The Interactive Textbook and Online Teaching Suite

Interactive Textbook

The Interactive Textbook is the online version of the print textbook and comes included with purchase of the print textbook. It is accessed by first activating the code on the inside cover. It is easy to navigate and is a valuable accompaniment to the print textbook.

Students can show their working

All textbook questions can be answered online within the Interactive Textbook. Students can show their working for each question using either the Draw tool for handwriting (if they are using a device with a touch-screen), the Type tool for using their keyboard in conjunction with the pop-up symbol palette, or by importing a file using the Import tool.

Once a student has completed an exercise they can save their work and submit it to the teacher, who can then view the student's working and give feedback to the student, as they see appropriate.

Auto-marked quizzes

The Interactive Textbook also contains material not included in the textbook, such as a short auto-marked quiz for each section. The quiz contains 10 questions which increase in difficulty from question 1 to 10 and cover all proficiency strands. There is also space for the student to do their working underneath each quiz question. The auto-marked quizzes are a great way for students to track their progress through the course.

Additional material for Year 5 and 6

For Years 5 and 6, the end-of-chapter Challenge activities as well as a set of Blackline Masters are now located in the Interactive Textbook. These can be found in the 'More resources' section, accessed via the Dashboard, and can then easily be downloaded and printed.

Online Teaching Suite

The Online Teaching Suite is the teacher's version of the Interactive Textbook. Much more than a 'Teacher Edition', the Online Teaching Suite features the following:

- The ability to view students' working and give feedback – When a student has submitted their work online for an exercise, the teacher can view the student's work and can give feedback on each question.
- For Years 5 and 6, access to Chapter tests, Blackline Masters, Challenge exercises, curriculum support material, and more.
- For Years 7 to 10, access to Pre-tests, Chapter tests, Skillsheets, Homework sheets, curriculum support material, and more.
- A Learning Management System that combines task-management tools, a powerful test generator, and comprehensive student and whole-class reporting tools.

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CHAPTER

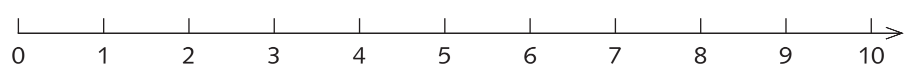
1

Number and Algebra

Whole numbers

This chapter is a brief review of whole numbers. Whole numbers, also known as the **counting numbers**, are the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, …

The whole numbers can be visualised as points equally spaced on the number line.



We can add, subtract, multiply, divide and take powers of numbers. Performing calculations using these operations can be done mentally using a variety of strategies or by using written algorithms.

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1A Addition and subtraction of whole numbers

We begin this chapter with a review of addition and subtraction of whole numbers.

Addition

Any two numbers can be added. When two or more whole numbers are added, the result is called the **sum**.

The order in which we add two or more whole numbers does not matter. Adding them in any order will give the same answer.

Subtraction

Subtraction gives the difference between two numbers. For example, the difference of 8 and 5 is 3, and this is written $8 - 5 = 3$.

We can think of subtraction as:

- taking away one number from another. For example: I had eight tennis balls, but I gave five away, so now I only have three tennis balls.
- finding what we have to add to one number to get the other. For example: I have eight tennis balls and she has five. I have to add 3 to 5 to get 8, so the difference is 3.

Example 1

Calculate these sums mentally by adding the numbers in a more convenient order.

a $29 + 60 + 21$ **b** $33 + 42 + 67$

Solution

a
$$\begin{aligned} 29 + 60 + 21 &= 29 + 21 + 60 \\ &= 50 + 60 \\ &= 110 \end{aligned}$$

b
$$\begin{aligned} 33 + 42 + 67 &= 33 + 67 + 42 \\ &= 100 + 42 \\ &= 142 \end{aligned}$$

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Example 2

Calculate these differences mentally

a $45 - 27$ **b** $136 - 77$

Solution

a $45 - 27 = 25 - 7$
$= 18$

or

$45 - 27 = 48 - 30$
$= 18$

or

$27 + 3 + 15 = 45$

Therefore $45 - 27 = 18$

Note: Other strategies are possible

b $136 - 77 = 139 - 80$
$= 59$

or

$77 + 23 + 36 = 136$

Therefore $136 - 77 = 23 + 36$
$= 59$

The standard algorithms

Example 3

The numbers of students and staff who attended the local swimming carnival from five schools were 1023, 896, 704, 199 and 1189. How many people attended in total?

Solution

$$\begin{array}{rrrrr} & 1 & 0 & 2 & 3 \\ & & 8 & 9 & 6 \\ & & 7 & 0 & 4 \\ & & 1 & 9 & 9 \\ + & 1_2 & 1_3 & 8_3 & 9 \\ \hline & 4 & 0 & 1 & 1 \\ \hline \end{array}$$

4011 people attended the swimming carnival.

There are two standard algorithms for subtraction. It is easier to check your calculations if you use Method 1 in Example 4.

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Example 4

Of the 4011 people who attended the swimming carnival, 2118 left by the end of lunchtime. How many were present for the start of the afternoon races?

Solution

Method 1

$$
\begin{array}{rcccc}
 & 4 & {}^{1}0 & {}^{1}1 & {}^{1}1 \\
- & 2_{1} & 1_{1} & 1_{1} & 8 \\
\hline
 & 1 & 8 & 9 & 3 \\
\hline
\end{array}
$$

Method 2

$$
\begin{array}{rcccc}
 & {}^{3}\cancel{4} & {}^{9}\cancel{{}^{1}0} & {}^{10}\cancel{{}^{1}1} & {}^{1}1 \\
- & 2 & 1 & 1 & 8 \\
\hline
 & 1 & 8 & 9 & 3 \\
\hline
\end{array}
$$

1893 people were at the swimming carnival for the start of the afternoon races. A subtraction can always be checked by addition. In this case, $1893 + 2118 = 4011$.

Addition and subtraction

- The **any-order property for addition** states that a list of numbers can be added together in any order to give the sum of the numbers.
- The any-order property summarises the commutative and associative laws for addition.
- Numbers in addition and subtraction calculations can often be rearranged to make mental calculations simpler.
- Written algorithms for addition and subtraction are useful when the numbers are large.
- Subtraction is the reverse of addition. For example, $8 + 5 = 13$ and $13 - 5 = 8$.

The commutative and associative laws of addition say, for example, that $3 + 4 = 4 + 3$ and $(3 + 4) + 5 = 3 + (4 + 5)$.

Exercise 1A

Example 1

1 Calculate these sums mentally by adding the numbers in a more convenient order.

a $19 + 48 + 11$ **b** $28 + 53 + 22$

c $15 + 57 + 85$ **d** $37 + 88 + 13$

e $22 + 24 + 26 + 28$ **f** $31 + 33 + 37 + 39$

2 Calculate these sums mentally by adding the numbers in stages.

a $25 + 36$ **b** $37 + 89$

c $54 + 27$ **d** $189 + 34$

e $12 + 16 + 15 + 22$ **f** $23 + 24 + 28 + 21$

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Example 2

3 Calculate these differences mentally, either by subtracting in stages or by adding in stages.

a $35-17$ **b** $84-36$

c $134-76$ **d** $53-27$

e $96-28$ **f** $252-177$

4 Calculate:

a $36+130+1644$ **b** $910+49+8371$

c $9221+839+65$ **d** $324+538+718$

e $229+1038+432$ **f** $251+489+12+37$

5 Carry out these subtractions.

a $762-387$ **b** $1921-428$

c $1405-386$ **d** $2764-1689$

e $918-579$ **f** $1297-399$

Example 3

6 The numbers of people living in five apartment blocks in the city are 1098, 956, 423, 156 and 42. How many people live in these apartment blocks in total?

Example 4

7 In 1975, the population of Hobart was 217 135. In 1985, the population was 231 135. What was the population increase from 1975 to 1985?

8 There are 3047 cats living in Brownville and 6857 in Wugtown. What is the combined cat population of the two towns?

9 A furniture store ordered 234 tables and 587 more chairs than tables. How many chairs were ordered? How many items were ordered in total?

10 The Mitchell Dam holds 496 709 000 litres of water. The Gaudry Weir holds 278 700 900 litres. If both water storage facilities are filled to capacity, how much water do they hold in total? What is the difference, in litres, between the capacities of the two facilities?

11 Doug is 1708 mm tall, Colin is 1639 mm tall and Peter is 1836 mm tall. How much taller is:

a Doug than Colin?

b Peter than Colin?

c Peter than Doug?

12 In 2004, there were 5938 students in state primary schools and 4133 students in state secondary schools in Toowoomba. What was the total number of students in state primary and secondary schools in that year?

13 In 1898, the population of the German Empire was 45 234 061. If 28 318 592 were Lutherans, 124 567 belonged to other protestant denominations, 561 612 were Jewish and the remainder Roman Catholic, how many Roman Catholics were there?

14 In 1891, the population of London was 4 211 743. The counties of Durham, Gloucestershire and Devonshire had populations of 1 024 369, 548 886 and 636 225, respectively. By how many did the population of London exceed the total population of these three counties?

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1B Multiplication and division of whole numbers

Multiplication and division are useful arithmetic operations. We use them all the time in everyday life. Repeated addition can be completed as a multiplication.

For example:

$$3 \times 5 = 5 + 5 + 5$$

Multiplication

Any two whole numbers can be multiplied together. The result is called the **product** of the numbers.

For example, the product of 15 and 13 is $15 \times 13 = 195$.

A list of whole numbers can be multiplied, two at a time, in any order, and the result will always be the same.

The following strategies are useful when doing multiplication mentally.

- Grouping numbers together in ways that will make the calculations easier.

Example 5

Find the product $8 \times 5 \times 7 \times 2$.

Solution

$$\begin{aligned} 8 \times 5 \times 7 \times 2 &= 8 \times 7 \times 10 \\ &= 560 \end{aligned}$$

(Multiply $5 \times 2 = 10$ first.)

- Using the distributive law. For example, $3 \times 23 = 3 \times (20 + 3) = 60 + 9 = 69$

Example 6

Calculate:

a 21×36

b 28×36

Solution

a $$\begin{aligned} 21 \times 36 &= 20 \times 36 + 1 \times 36 \\ &= 720 + 36 \\ &= 756 \end{aligned}$$

b $$\begin{aligned} 28 \times 36 &= 30 \times 36 - 2 \times 36 \\ &= 1080 - 72 \\ &= 1008 \end{aligned}$$

There are other possible methods.

The **multiplication algorithm** is used for larger numbers.

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Example 7

A brick wall is to have 37 rows, with 128 bricks in each row.

How many bricks are needed?

Solution

$$\begin{array}{r} 1\ 2\ 8 \\ \times \quad 3\ 7 \\ \hline 8\ 9\ 6 \\ 3\ 8\ 4\ 0 \\ \hline 4\ 7\ 3\ 6 \\ \hline \end{array}$$

(Multiply 128 by 7.)

(Multiply 128 by 30.)

4736 bricks are required.

Multiplication of whole numbers

- The **any-order property for multiplication** states that a list of numbers can be multiplied together, in any order, to give the product of the numbers.
- The any-order property summarises the commutative and associative laws for multiplication.
- Multiplication is **distributive** over addition and subtraction:

  $(20 + 1) \times 36 = 20 \times 36 + 1 \times 36 = 720 + 36 = 756$

  $(30 - 2) \times 36 = 30 \times 36 - 2 \times 36 = 1080 - 72 = 1008$
- When multiplying mentally, it is often effective to:
  - take the factors in a different order
  - use the distributive law.
- Larger numbers can be multiplied using the multiplication algorithm.
- It is important to know your multiplication tables.

The commutative and associative laws of multiplication say, for example, that $3 \times 4 = 4 \times 3$ and $(3 \times 4) \times 5 = 3 \times (4 \times 5)$.

Division

Any whole number can be divided by any non-zero whole number, called the **divisor**, to give a **quotient** and a **remainder**.

Division without remainder is the reverse process of multiplication. For example, $30 \div 5 = 6$ is the reverse of $30 = 6 \times 5$. Six is the quotient and the remainder is 0.

Often, we do not have exact division. For example, 5 does not divide exactly into 32. We write $32 \div 5 = 6$ remainder 2. This is another way of writing $32 = 6 \times 5 + 2$.

There are two quite different ways to interpret a division such as '$32 \div 5 = 6$ remainder 2'.

- 'Divide 32 people into groups of 5. There will be 6 groups, with 2 people left over.'
- 'Divide 32 people into 5 equal groups. Each group will have 6 people, and there will be 2 people left over.'

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Example 8

53 chocolates are put into boxes of 12. How many boxes are filled, and how many chocolates are left over?

Solution

$53 \div 12 = 4$ remainder 5

That is, $53 = 4 \times 12 + 5$

We can fill 4 boxes with 5 chocolates left over.

There are several useful strategies for mental division.

Example 9

a Calculate $864 \div 8$ mentally.

b Calculate $752 \div 8$ mentally.

Solution

a Split 864 into $800 + 64$.

$$\begin{aligned} 864 \div 8 &= 800 \div 8 + 64 \div 8 \\ &= 100 + 8 \\ &= 108 \end{aligned}$$

or Divide successively by 2.

$$\begin{aligned} 864 \div 8 &= 432 \div 4 \\ &= 216 \div 2 \\ &= 108 \end{aligned}$$

b Split 752 into $800 - 48$.

$$\begin{aligned} 752 \div 8 &= 800 \div 8 - 48 \div 8 \\ &= 100 - 6 \\ &= 94 \end{aligned}$$

or

$$\begin{aligned} 752 \div 8 &= 376 \div 4 \\ &= 188 \div 2 \\ &= 94 \end{aligned}$$

The short division algorithm

The short division algorithm is effective when the divisor is small.

Example 10

The humbug-making machine produced 2592 humbugs in an afternoon, packaged into bags of 8. How many bags were produced?

Solution

In this case the divisor is 8, so the short division algorithm can be used.

$$\begin{array}{r} 3\;2\;4 \\ 8\overline{)2\;5\;{}^{1}9\;{}^{3}2} \end{array}$$

324 bags of humbugs were produced that afternoon.

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Fraction notation for division

We can also write a division as a fraction.

For example, $864 \div 8 = 108$ can be written as:

$$\frac{864}{8} = 108$$

Other examples using this notation are:

$$\frac{0}{3} = 0 \quad \text{and} \quad \frac{108}{12} = 9$$

When there is a remainder, we can write the answer using a mixed numeral:

$$\frac{27}{8} = 3\tfrac{3}{8}$$

In the next chapter we will study calculations such as:

$$8 \times 3\tfrac{3}{8} = 27$$

The long division algorithm

When the divisor is large, we use the **long division algorithm**.

Example 11

On Fridays, the humbugs produced by the humbug-making machine are packed into mega-bags of 36 humbugs. One Friday, 8318 humbugs were produced. How many mega-bags were produced?

Solution

$$
\begin{array}{r l}
2\;3\;1 & \quad \text{remainder 2} \\
36\overline{\big)\boxed{8\;3}\;1\;8} & \\
7\;2 & \\
\hline
1\;1\;1 & \quad \text{(Bring down 1.)} \\
1\;0\;8 & \\
\hline
3\;8 & \quad \text{(Bring down 8.)} \\
\underline{3\;6} & \\
\underline{2} &
\end{array}
$$

We use: $36 \times 1 = 36$
$36 \times 2 = 72$
$36 \times 3 = 108$

To start, consider $88 \div 36$:

$36 \times 2 = 72$ and $36 \times 3 = 108$

The 2 is placed above the line, and multiplied by 36 to give 72, which is subtracted from 83 to give the correct ‘carry’.

Thus $8318 \div 36 = 231$ remainder 2,

so 231 mega-bags were produced, with 2 humbugs left over.

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Example 12

Find $9617 \div 27$.

Solution

It is useful to begin by writing down at least some of the multiples of 27 to the right of the calculation.

$$
\begin{array}{r|rrrr}
 & & 3 & 5 & 6 \\ \cline{2-5}
27 & 9 & 6 & 1 & 7 \\
 & 8 & 1 & & \\ \cline{2-4}
 & 1 & 5 & 1 & \\
 & 1 & 3 & 5 & \\ \cline{2-5}
 & & 1 & 6 & 7 \\
 & & 1 & 6 & 2 \\ \cline{3-5}
 & & & & 5
\end{array}
$$

$27 \times 1 = 27$

$27 \times 2 = 54$

$27 \times 3 = 81$

$27 \times 4 = 108$

$27 \times 5 = 135$

$27 \times 6 = 162$

$27 \times 7 = 189$

$27 \times 8 = 216$

$27 \times 9 = 243$

$9617 \div 27 = 356$ remainder 5.

Division of whole numbers

- Any whole number can be divided by any non-zero whole number to give a quotient and remainder.
- A division such as $30 \div 5 = 6$ can be interpreted in two ways:
  - dividing 30 people into 5 equal groups
  - dividing 300 people into groups of 5
- Division is **distributive** over addition and subtraction. For example:

  $(800 + 64) \div 8 = 800 \div 8 + 64 \div 8 = 100 + 8 = 108$

  $(800 - 48) \div 8 = 800 \div 8 - 48 \div 8 = 100 - 6 = 94$
- A division statement such as $30 \div 7 = 4$ remainder 2 is equivalent to the multiplication and addition statement $30 = 7 \times 4 + 2$.
- Every multiplication statement is equivalent to a division statement.
- To perform divisions, you must know your multiplication tables.
- When performing division mentally, the distributive law is often useful. For example:

$$
\begin{aligned}
330 \div 15 &= (300 + 30) \div 15 \\
&= 20 + 2 \\
&= 22
\end{aligned}
$$

- Larger numbers can be divided using the long division algorithm.

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Exercise 1B

Example 5

1 Calculate these products mentally by multiplying the numbers in a more convenient order.

a $7\times5\times9\times2$ **b** $12\times25\times11\times4$

c $15\times9\times4$ **d** $35\times11\times2$

Example 6

2 Calculate these products mentally by using the fact that one factor is close to a multiple of 10.

a 21×33 **b** 18×28 **c** 42×16 **d** 49×34

Example 9

3 Calculate these divisions mentally.

a $749\div7$ **b** $352\div4$ **c** $1272\div12$ **d** $2964\div6$

Example 8

4 Calculate the quotient and remainder in each division.

a $165\div7$ **b** $441\div9$ **c** $800\div12$ **d** $1244\div11$

Example 7

5 Carry out each calculation using the multiplication algorithm.

a 353×14 **b** 572×29 **c** 79×186 **d** 612×49

e 432×88 **f** 895×35 **g** 135×36 **h** 936×24

Example 10

6 Use short division to calculate:

a $556\div2$ **b** $540\div4$ **c** $8624\div8$ **d** $498\div6$

e $4050\div6$ **f** $1496\div11$ **g** $78\,093\div9$ **h** $39\,240\div9$

Example 12

7 Use the long division algorithm to calculate:

a $828\div13$ **b** $1050\div16$ **c** $2344\div16$ **d** $8554\div17$

e $2806\div23$ **f** $4042\div19$ **g** $1498\div18$ **h** $4708\div19$

Example 7

8 A school hall has 30 rows of seats. Each row has 28 seats. How many seats are there?

9 An apartment block has 44 floors. Each floor has 18 apartments. How many apartments are there in the block?

10 A car park has 34 rows and each row has 42 parking spaces. How many cars can be parked?

11 Pete planted 80 rows of 30 tomato plants. If each plant produced 43 tomatoes, what was the total crop?

12 An office building has 15 floors, with 23 rooms and 4 corridors on each floor. If each room has 7 lights and each corridor has 15 lights, what is the total number of lights in the building?

13 Tara is comparing mobile phone plans. She uses her phone for SMS messages only. Plan A costs $33.00 per month and includes a maximum of 150 SMS messages. Plan B has no monthly charge but charges 24c for each SMS. Which plan would have been cheaper last month, when she sent 149 SMSs?

14 The average distance between Mercury and Venus is 51 078 090 km. What is the average distance between Mars and Jupiter if it is 11 times the average distance between Mercury and Venus?

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15 Tony is offered two options for buying a particular car. He can pay the marked price of \$52 800 in cash. Alternatively, he can pay a \$10 000 deposit and then pay instalments of \$1805 a month for 24 months. How much more would he pay for the car on the instalment plan?

Example 10

16 A greengrocer bought a sack of potatoes weighing 51 kg. He divided the potatoes into bags so that each bag held 3 kg of potatoes. How many bags of potatoes did he get from his sack?

Example 11

17 A humbug-making machine produced 12 429 humbugs in three days.

a How many bags of 8 is this? How many are left over?

b How many mega-bags of 36 is this? How many are left over?

18 **a** A calendar is to be invented for a newly settled planet, which has 2304 days in its year. The designer has to split the days up into weeks. How many weeks will there be if he uses weeks of:

i 6 days? **ii** 8 days? **iii** 12 days?

b If he makes each month have 144 days, how many weeks will there be in a month for each of part **a**?

19 A family were stuck in their house after a serious storm. During the storm, the clock was damaged so that only the hour hand worked. The family stayed inside the house for 636 hours after the storm. How many times did the hour hand go around the clock before they left?

Review exercise

1 Calculate these sums mentally.

a $38 + 22$	**b** $32 + 25$	**c** $35 + 27$	**d** $42 + 19$
e $13 + 17$	**f** $8 + 89$	**g** $14 + 76$	**h** $29 + 46$
i $84 + 6$	**j** $32 + 9$	**k** $11 + 9 + 33$	**l** $2 + 15 + 38$
m $61 + 24 + 9$	**n** $4 + 42 + 36$	**o** $27 + 6 + 3$	**p** $16 + 24 + 5$
q $16 + 55 + 24 + 45$	**r** $72 + 19 + 28 + 21$	**s** $22 + 17 + 18 + 23$	**t** $23 + 37 + 64 + 6$
u $15 + 64 + 26 + 45$			

2 Carry out these subtractions mentally.

a $31 - 9$	**b** $32 - 28$	**c** $94 - 66$	**d** $56 - 48$
e $521 - 428$	**f** $405 - 386$	**g** $764 - 689$	**h** $618 - 509$
i $813 - 619$	**j** $754 - 749$	**k** $328 - 68$	**l** $462 - 387$

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3 Carry out these multiplications mentally.

a $25 \times 4 \times 9$ **b** $50 \times 78 \times 2$ **c** $1 \times 35 \times 20$ **d** $3 \times 7 \times 4 \times 5$

e $5 \times 76 \times 2$ **f** $8 \times 5 \times 40$ **g** $6 \times 5 \times 10 \times 2$ **h** $5 \times 47 \times 20$

i $13 \times 6 \times 0$

4 Lake Superior, the largest freshwater lake in the world, has an area of 82 732 km^2. The second largest, Lake Victoria, has an area of 69 753 km^2. How much larger is Lake Superior?

5 Copy and complete the calculations by finding a digit for each *.

a
$$\begin{array}{r} 2\ 3\ 1 \\ 2\ 6\ \ast \\ +\ 1\ \ast\ 6 \\ \hline \ast\ 3\ 1 \\ \hline \end{array}$$

b
$$\begin{array}{r} \ast\ \ast\ \ast \\ 6\ 3\ 7 \\ +\ 5\ 6\ 7 \\ \hline \ast\ 0\ 4\ 7 \\ \hline \end{array}$$

c
$$\begin{array}{r} \ast\ 7\ \ast \\ \times\ \quad 9 \\ \hline 3\ 3\ \ast\ 4 \\ \hline \end{array}$$

6 There are 3257 dogs in Snugsville and 2673 in Cudstown. What is the combined dog population of the two towns?

7 A school has 2137 students. The number of boys at the school is 1586. How many girls are there at the school?

8 There are 576 chocolates to be divided among 24 people. How many chocolates will each person receive?

9 A youth club has 80 members. There are 10 more boys than girls. How many boys are there?

10 There are 4000 apples packed into boxes, each box holding 75 apples. How many boxes are required?

11 A club started the year with 82 members. During the year, 36 people left and 28 people joined. How many people belonged to the club at the end of the year?

12 A fruit grower sold 780 boxes of oranges. On average, the boxes weigh 21 kg each. What was the total weight (in kg) of the boxes of oranges?

13 An orchard of 60 hectares (ha) has 2100 trees. How many trees are there on each hectare (on average)?

Challenge exercise

1 The towns Thomas, Callaghan, Barker, Dixon and Evans are located on a straight highway in that order. The distance from Thomas to Evans is 20 km. The distance from Thomas to Dixon is 15 km. The distance from Callaghan to Evans is 10 km. Barker is halfway between Callaghan and Dixon. What is the distance from Callaghan to Barker?

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2 Using each of the digits 1 to 9 only once, write them in such a way that you produce two numbers that add up to 99 999.

3 A family had a collection of animals. When the council asked them how many birds and how many beasts they had, they answered, 'Well, we have 36 heads and 100 feet in total.' How many birds and how many beasts did the family have?

4 Take a number that does not contain zero and in which all the digits are different.

- Double the number and then add 4.
- Multiply by 5 and then add 12.
- Multiply by 10 and then subtract 320.
- Cross out the zeroes in your answer.

What number do you get? Explain your answer.

5 **a** What is the smallest number that must be subtracted from 5762, so that the result is exactly divisible by 19?

b What number divided by 367 will give 59 as the quotient and 126 as the remainder?

c Divide 931 into two parts such that one part is greater than the other by 127.

6 **a** The average of 7 numbers is 36. An eighth number is chosen so that the average is now 45. What is the eighth number?

b The average of 7 numbers is 17 and the average of four of them is 14. What is the average of the other three numbers?

c The average of 6 numbers is 29 and the average of four other numbers is 44. What is the average of the 10 numbers?

d A batsman had an average of 38 runs for 5 innings. If he scored 62 in his next innings, what was then his average?

Division by factors

7 **a** Divide 263 052 by 36 by first dividing by 9 and then by 4.

b Divide 56 133 by 231 by first dividing by 3, then by 7 and finally by 11 ($231 = 3 \times 7 \times 11$).

8 Consider the following division using factors: $69\,237 \div 45$:

$$
\begin{array}{r|l l}
5 & 6\,9\,2\,3\,7 & \\ \hline
9 & 1\,3\,8\,4\,7 & \text{remainder } 2 \\ \hline
 & 1\,5\,3\,8 & \text{remainder } 5
\end{array}
$$

Quotient $= 1538$

$$
\begin{aligned}
\text{Remainder} &= 5 \times 5 + 2 \\
&= 27
\end{aligned}
$$

Use this method to divide 69 237 by 231.

9 If the current time is 1 p.m., what will the time be in 1000 hours?

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CHAPTER

2

Number and Algebra

Fractions and decimals

In Chapter 1 we reviewed the whole numbers. This chapter is a review of fractions and decimals.

Fractions have been used since ancient times. For instance, dividing a harvest into equal parts and distributing different amounts to different families would often have involved fractions.

Fractions were used by the ancient Egyptians and Babylonians. Systematic use of decimal fractions did not appear until the sixteenth century. Decimals are now a central part of day-to-day calculations in almost every walk of life.

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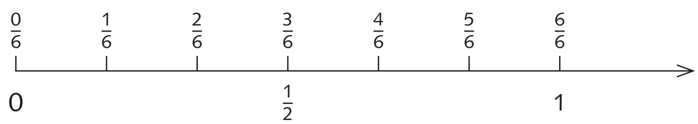
2A Equivalent fractions and simplest form

Equivalent fractions

Two fractions are **equivalent** if they are represented by the same point on the number line. We regard equivalent fractions as being equal.

The markers $\frac{3}{6}$ and $\frac{1}{2}$ are represented by the same point on the number line below, so they are equivalent.



This is simply the statement that, starting at 0, taking one step of length $\frac{1}{2}$ and taking 3 steps of length $\frac{1}{6}$ both get us to the same point.

$$\begin{aligned}\frac{1}{2} &= \frac{1 \times 3}{2 \times 3} \\ &= \frac{3}{6}\end{aligned}$$

To form equivalent fractions, start with a fraction and either:

- multiply its numerator and denominator by the same non-zero whole number.

  For example:

  $$\frac{3}{4} = \frac{3 \times 6}{4 \times 6} = \frac{18}{24}$$

- divide the numerator and denominator by the same common factor.

  For example:

  $$\frac{18}{24} = \frac{18 \div 6}{24 \div 6} = \frac{3}{4}$$

Simplest form and cancelling

A fraction is in **simplest form** or **lowest terms** if the numerator and denominator have no common factor other than 1.

For example, the fraction $\frac{5}{12}$ is in simplest form because the highest common factor of 5 and 12 is 1. However, $\frac{9}{12}$ is not in simplest form since 9 and 12 have 3 as a common factor.

Indeed $\frac{9}{12} = \frac{3}{4}$.

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Example 1

Simplify:

a $\dfrac{36}{45}$

b $\dfrac{98}{21}$

Solution

a $\dfrac{36}{45} = \dfrac{\cancel{36}^{4}}{\cancel{45}_{5}}$ (dividing each by 9)

$= \dfrac{4}{5}$

b $\dfrac{98}{21} = \dfrac{\cancel{98}^{14}}{\cancel{21}_{3}}$ (dividing each by 7)

$= \dfrac{14}{3}$

$= 4\frac{2}{3}$

Note: Two fractions are equivalent if they have the same simplest form.

Example 2

Write 'yes' or 'no' to indicate whether the fractions in each pair are equivalent.

a Is $\dfrac{3}{4}$ equivalent to $\dfrac{45}{60}$?

b Is $\dfrac{5}{9}$ equivalent to $\dfrac{70}{108}$?

c Is $\dfrac{99}{100}$ equivalent to $\dfrac{9}{10}$?

d Is $\dfrac{9}{27}$ equivalent to $\dfrac{33}{99}$?

Solution

a Yes. (Multiply numerator and denominator of $\dfrac{3}{4}$ by 15.)

b No. $\left(\dfrac{5}{9}\right.$ is in simplest form. The simplest form of $\dfrac{70}{108}$ is $\left.\dfrac{35}{54}.\right)$

c No. (Both fractions are in simplest form and they are different.)

d Yes. $\left(\text{The simplest form of both fractions is } \dfrac{1}{3}.\right)$

Equivalent fractions and simplest form

- Two fractions are said to be **equivalent** if they are represented by the same point on a number line. Equivalent fractions are equal.
- We say that a fraction is in **simplest form** if the numerator and denominator have no common factor other than 1.
- We can test when two fractions are **equivalent** by seeing if they have the same simplest form.
- Starting with a given fraction, the fractions obtained by multiplying its numerator and its denominator by the same non-zero whole number are equivalent to it.
- Starting with a given fraction, the fractions obtained by dividing its numerator and its denominator by the same common factor are equivalent to it.

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Comparison of fractions: which is larger?

If two or more fractions have the same denominator, then the one with the larger numerator is the larger fraction.

Equivalent fractions are used to compare fractions when the denominators of the fractions given are different. First find equivalent fractions with the same denominator for the given fractions, and then compare the numerators.

Example 3

Order the following fractions from smallest to largest:

$\frac{3}{10}, \frac{3}{4}, \frac{1}{2}, \frac{7}{20}$ and $\frac{3}{5}$.

Solution

The LCM of the denominators $2, 4, 5, 10$ and 20 is 20.

$$\frac{3}{10} = \frac{6}{20}, \frac{3}{4} = \frac{15}{20}, \frac{1}{2} = \frac{10}{20}, \frac{7}{20} = \frac{7}{20}, \frac{3}{5} = \frac{12}{20}$$

Writing these in order: $\frac{6}{20}, \frac{7}{20}, \frac{10}{20}, \frac{12}{20}, \frac{15}{20}$

The order is $\frac{3}{10} < \frac{7}{20} < \frac{1}{2} < \frac{3}{5} < \frac{3}{4}$.

Exercise 2A

Example 1

1 Write each fraction in simplest form.

a $\frac{10}{15}$ **b** $\frac{17}{34}$ **c** $\frac{18}{21}$ **d** $\frac{20}{24}$

e $\frac{45}{35}$ **f** $\frac{56}{14}$ **g** $\frac{105}{147}$ **h** $\frac{84}{224}$

i $\frac{20}{25}$ **j** $\frac{24}{36}$ **k** $\frac{14}{21}$ **l** $\frac{24}{32}$

m $\frac{45}{36}$ **n** $\frac{56}{42}$ **o** $\frac{105}{84}$ **p** $\frac{112}{48}$

Example 2

2 Test whether the fractions in each pair are equivalent.

a $\frac{2}{3}, \frac{10}{15}$ **b** $\frac{2}{3}, \frac{12}{21}$ **c** $\frac{2}{3}, \frac{22}{33}$

d $\frac{7}{8}, \frac{56}{64}$ **e** $\frac{7}{8}, \frac{49}{64}$ **f** $\frac{3}{5}, \frac{18}{30}$

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Example 3

3 Write the six fractions with a common denominator, and hence order them from smallest to largest.

a $\frac{11}{10}, \frac{2}{5}, \frac{5}{4}, \frac{9}{10}, \frac{3}{4}, \frac{4}{5}$

b $\frac{5}{6}, \frac{4}{5}, \frac{7}{10}, \frac{3}{4}, \frac{11}{15}, \frac{23}{30}$

4 Rewrite these fractions with denominator 63.

$\frac{1}{3}, \frac{2}{3}, \frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{1}{9}, \frac{2}{9}, \frac{5}{9}, \frac{8}{21}$

5 Arrange the fractions from smallest to largest.

a $\frac{7}{8}, \frac{2}{3}, \frac{5}{6}$

b $\frac{11}{16}, \frac{2}{3}, \frac{3}{4}$

c $\frac{11}{12}, \frac{31}{45}, \frac{2}{3}$

d $\frac{7}{12}, \frac{5}{8}, \frac{2}{3}$

e $\frac{25}{54}, \frac{1}{2}, \frac{5}{9}$

f $\frac{31}{48}, \frac{2}{3}, \frac{11}{16}$

2B Addition and subtraction of fractions

A **proper fraction** is greater than or equal to 0 and less than 1. For example, $\frac{2}{7}$ and $\frac{3}{8}$.

A **mixed numeral** is a whole number plus a proper fraction. For example, $1\frac{1}{3}$ and $2\frac{2}{5}$ are mixed numerals.

An **improper fraction** is a fraction whose numerator is greater than or equal to the denominator, so that it is greater than or equal to 1. For example, $\frac{9}{8}$ and $\frac{13}{7}$.

An improper fraction can be written as a mixed numeral or a whole number. In general, improper fractions should be written as mixed numerals with the fractional parts in simplest form.

For example:

$$\frac{20}{6} = 3\tfrac{1}{3}$$

To add and subtract fractions with the same denominator, simply add or subtract the numerators.

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Example 4

Evaluate:

a $\frac{18}{19}+\frac{13}{19}$ **b** $1\frac{1}{6}+2\frac{5}{6}$ **c** $\frac{5}{16}+\frac{11}{16}+\frac{7}{16}$

Solution

a $\frac{18}{19}+\frac{13}{19}=\frac{31}{19}$
$=1\frac{12}{19}$

b $1\frac{1}{6}+2\frac{5}{6}=1+2+\frac{1}{6}+\frac{5}{6}$
$=3+\frac{6}{6}$
$=4$

c $\frac{5}{16}+\frac{11}{16}+\frac{7}{16}=\frac{23}{16}$
$=1\frac{7}{16}$

Example 5

Evaluate:

a $\frac{12}{13}-\frac{5}{13}$ **b** $1\frac{8}{19}-\frac{9}{19}$

Solution

a $\frac{12}{13}-\frac{5}{13}=\frac{7}{13}$

b $1\frac{8}{19}-\frac{9}{19}=\frac{27}{19}-\frac{9}{19}$
$=\frac{18}{19}$

If the denominators are different, use the lowest common multiple of the denominators to find equivalent fractions.

Example 6

Evaluate:

a $\frac{2}{3}+\frac{5}{8}$ **b** $\frac{5}{9}-\frac{1}{6}$

Solution

a The LCM of 3 and 8 is 24.

$\frac{2}{3}+\frac{5}{8}=\frac{16}{24}+\frac{15}{24}$
$=\frac{31}{24}$
$=1\frac{7}{24}$

b The LCM of 9 and 6 is 18.

$\frac{5}{9}-\frac{1}{6}=\frac{10}{18}-\frac{3}{18}$
$=\frac{7}{18}$

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Example 7

Evaluate:

a $2\frac{3}{8}+1\frac{5}{6}$

b $21\frac{1}{6}-19\frac{5}{8}$

Solution

a
$$\begin{aligned}2\tfrac{3}{8}+1\tfrac{5}{6} &= 2\tfrac{9}{24}+1\tfrac{20}{24}\\ &= 3+\frac{9}{24}+\frac{20}{24}\\ &= 3+\frac{29}{24}\\ &= 3+1+\frac{5}{24}\\ &= 4\tfrac{5}{24}\end{aligned}$$

b Notice that $\frac{1}{6}<\frac{5}{8}$.

$$\begin{aligned}21\tfrac{1}{6}-19\tfrac{5}{8} &= 21\tfrac{4}{24}-19\tfrac{15}{24}\\ &= 20+1\tfrac{4}{24}-\left(19+\frac{15}{24}\right)\\ &= (20-19)+\left(\frac{28}{24}-\frac{15}{24}\right)\\ &= 1\tfrac{13}{24}\end{aligned}$$

or

$$\begin{aligned}21\tfrac{1}{6}-19\tfrac{5}{8} &= 21+\frac{1}{6}-19-\frac{5}{8}\\ &= 21-19+\frac{4}{24}-\frac{15}{24}\\ &= 2-\frac{11}{24}\\ &= 1\tfrac{13}{24}\end{aligned}$$

Many applications involve addition and subtraction of fractions.

Example 8

Five identical trucks, carrying $\frac{5}{6}$ load, $\frac{3}{8}$ load, $\frac{3}{4}$ load, $\frac{1}{2}$ load and $\frac{2}{3}$ load of potatoes, arrived at the chip factory. How many truckloads of potatoes were delivered in total?

Solution

$$\begin{aligned}\frac{5}{6}+\frac{3}{8}+\frac{3}{4}+\frac{1}{2}+\frac{2}{3} &= \frac{20}{24}+\frac{9}{24}+\frac{18}{24}+\frac{12}{24}+\frac{16}{24}\\ &= \frac{75}{24}\\ &= \frac{25}{8}\\ &= 3\tfrac{1}{8}\end{aligned}$$

$3\frac{1}{8}$ truckloads of potatoes were delivered.

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Example 9

Brad has a roll of electrical cable with $3\frac{3}{10}$ m left on it. How much is left if he uses $2\frac{7}{8}$ m?

Solution

$$
\begin{aligned}
3\tfrac{3}{10} - 2\tfrac{7}{8} &= \frac{33}{10} - \frac{23}{8} \\
&= \frac{132}{40} - \frac{115}{40} \quad \text{(LCM is 40.)} \\
&= \frac{17}{40}
\end{aligned}
$$

There is $\frac{17}{40}$ m of cable left.

Exercise 2B

Example 4

1 Find the value of:

a $\frac{3}{4}+\frac{5}{4}$ **b** $\frac{1}{5}+\frac{3}{5}+\frac{4}{5}$ **c** $\frac{3}{10}+\frac{2}{10}$

d $\frac{3}{5}+\frac{2}{5}+\frac{7}{5}$ **e** $\frac{2}{3}+\frac{5}{3}$ **f** $\frac{2}{7}+\frac{8}{7}$

Example 5

2 Evaluate:

a $\frac{3}{4}-\frac{1}{4}$ **b** $\frac{7}{12}-\frac{5}{12}$ **c** $2\frac{2}{5}-1\frac{1}{5}$

d $1\frac{7}{19}-\frac{9}{19}$ **e** $2\frac{2}{5}-1\frac{4}{5}$ **f** $3\frac{5}{12}-2\frac{7}{12}$

Example 6a

3 Evaluate:

a $\frac{2}{3}+\frac{3}{4}$ **b** $\frac{9}{10}+\frac{10}{11}$ **c** $\frac{5}{6}+\frac{3}{10}$

d $\frac{1}{9}+\frac{1}{10}$ **e** $\frac{8}{9}+\frac{7}{8}$ **f** $\frac{5}{7}+\frac{3}{5}$

g $\frac{1}{3}+\frac{5}{8}$ **h** $\frac{7}{8}+\frac{3}{7}$ **i** $\frac{5}{10}+\frac{3}{5}$

Example 6b

4 Evaluate:

a $\frac{5}{6}-\frac{2}{3}$ **b** $\frac{1}{3}-\frac{1}{4}$ **c** $\frac{20}{33}-\frac{35}{77}$

d $\frac{5}{7}-\frac{3}{5}$ **e** $\frac{7}{8}-\frac{5}{6}$ **f** $\frac{8}{9}-\frac{7}{8}$

g $\frac{1}{9}-\frac{1}{10}$ **h** $\frac{2}{7}-\frac{1}{5}$ **i** $\frac{11}{12}-\frac{5}{6}$

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Example 7a

5 Evaluate:

a $2\frac{1}{5}+3\frac{2}{3}$ **b** $1\frac{2}{3}+1\frac{1}{4}$ **c** $2\frac{4}{5}+1\frac{1}{4}$

d $3\frac{1}{2}+2\frac{2}{3}$ **e** $1\frac{1}{5}+2\frac{2}{3}$ **f** $5\frac{1}{10}+3\frac{7}{8}$

g $1\frac{1}{11}+3\frac{4}{5}$ **h** $7\frac{3}{8}+2\frac{1}{5}$ **i** $6\frac{4}{5}+7\frac{5}{8}$

Example 7b

6 Evaluate:

a $3\frac{3}{4}-1\frac{1}{2}$ **b** $13\frac{3}{4}-2\frac{7}{8}$ **c** $27\frac{5}{8}-14\frac{1}{3}$

d $52\frac{3}{4}-26\frac{7}{8}$ **e** $22\frac{1}{4}-11\frac{1}{3}$ **f** $52\frac{7}{10}-3\frac{5}{11}$

Example 8

7 A man walks $2\frac{7}{8}$ km and then runs for $1\frac{3}{4}$ km. How far has he travelled in total?

Example 9

8 The distance from Davidson to Clare is $5\frac{3}{10}$ km. A boy rides $2\frac{3}{4}$ km from Davidson along the road to Clare. How much further does he have to ride to Clare?

9 Jacinta has $\frac{2}{3}$ of a litre of water in a jug and then pours in $\frac{1}{4}$ of a litre. How much water is in the jug now?

10 A box of chocolates has 24 chocolates in it. Marie ate $\frac{1}{2}$ of the box of chocolates and then ate $\frac{3}{8}$ of the same box of chocolates the next day. What fraction of the box of chocolates did she eat in total?

11 I have a length of wire and cut off $\frac{3}{7}$ of it. What fraction of the wire do I have left?

12 Dimitri devotes $\frac{1}{3}$ of the day to schoolwork and he spends $\frac{1}{10}$ of the day watching television. As a fraction of the day, how much more time is spent on schoolwork than on television?

13 A family travelling to Albury cover one-third of the journey before 1 p.m. and a further one-quarter of the journey between 1 p.m. and 2 p.m. What fraction of the journey have they travelled by 2 p.m.?

14 In a packet of jelly beans, $\frac{1}{3}$ of the jelly beans are purple, $\frac{1}{8}$ are black and $\frac{1}{4}$ are red.

a What fraction of the jelly beans are either purple or black?

b What fraction of the jelly beans are either purple or red?

c What fraction of the jelly beans are purple or red or black?

d What fraction of the jelly beans are not purple or red or black?

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2C Multiplication and division of fractions

Multiplication of fractions

When multiplying fractions, we first multiply the two numerators, then multiply the two denominators, and then simplify if we can.

Always look out for cancelling. In most cases, cancel common factors first, then multiply.

Example 10

Evaluate $\frac{10}{21} \times \frac{9}{16}$.

Solution

$$\frac{10}{21} \times \frac{9}{16} = \frac{\cancel{10}^{5}}{\cancel{21}^{7}} \times \frac{\cancel{9}^{3}}{\cancel{16}^{8}}$$

(Divide 10 and 16 by 2; divide 9 and 21 by 3.)

$$= \frac{5}{7} \times \frac{3}{8}$$

$$= \frac{15}{56}$$

When multiplying mixed numerals, first convert to improper fractions, cancel common factors if possible and then multiply.

Example 11

Evaluate $5\frac{1}{4} \times 2\frac{1}{3}$.

Solution

$$5\tfrac{1}{4} \times 2\tfrac{1}{3} = \frac{21}{4} \times \frac{7}{3}$$

(Convert to improper fractions.)

$$= \frac{\cancel{21}^{7}}{4} \times \frac{7}{\cancel{3}^{1}}$$

(Divide 21 and 3 by 3.)

$$= \frac{7}{4} \times \frac{7}{1}$$

$$= \frac{49}{4}$$

$$= 12\tfrac{1}{4}$$

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Remember that multiplication of fractions is required when the word 'of' is used.

An everyday use of multiplication of fractions arises when we take part of a quantity or measurement. The key word here is 'of'.

Example 12

Approval has been given for $\frac{4}{5}$ of a class of 30 students to be immunised. How many injections will the nurse give to students in that class?

Solution

$$\begin{aligned}\frac{4}{5} \text{ of } 30 &= \frac{4}{5} \times 30 \\ &= \frac{4}{5} \times \frac{30}{1} \\ &= \frac{4}{\cancel{5}^{1}} \times \frac{\cancel{30}^{6}}{1} \\ &= \frac{4}{1} \times \frac{6}{1} \\ &= 24\end{aligned}$$

Hence, 24 injections will be given.

Example 13

There is $\frac{5}{8}$ of a packet of cereal at the start of the week, and Jack eats $\frac{2}{3}$ of that amount by the end of the week.

a What fraction of a whole packet of cereal has Jack eaten by the end of the week?

b Assuming Jack will eat the same amount of cereal each week, how much does he eat in 5 weeks?

Solution

a We need to work out $\frac{2}{3}$ of $\frac{5}{8}$.

$$\begin{aligned}\frac{2}{3} \times \frac{5}{8} &= \frac{2 \times 5}{3 \times 8} \\ &= \frac{10}{24} \\ &= \frac{5}{12}\end{aligned}$$

b Jack has eaten $\frac{5}{12}$ of a packet of cereal over the week.

For 5 weeks, Jack needs

$$\begin{aligned}\frac{5}{12} \times 5 &= \frac{5}{12} \times \frac{5}{1} \\ &= \frac{25}{12} \\ &= 2\tfrac{1}{12} \text{ packets.}\end{aligned}$$

Jack will eat $2\frac{1}{12}$ packets of cereal over 5 weeks.

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Division of fractions

The **reciprocal** of a non-zero fraction is obtained by interchanging the numerator and the denominator. For example, the reciprocal of $\frac{3}{7}$ is $\frac{7}{3}$ and the reciprocal of 5 is $\frac{1}{5}$.

The product of a fraction and its reciprocal is always 1.

$$\frac{3}{7} \times \frac{7}{3} = 1 \quad \text{and} \quad 5 \times \frac{1}{5} = 1$$

Division of fractions is the reverse process of multiplication, so to divide by a non-zero fraction we multiply it by its reciprocal.

Example 14

Evaluate:

a $\frac{2}{3} \div \frac{4}{27}$

b $16 \div \frac{2}{3}$

c $\frac{2}{3} \div 16$

Solution

a The reciprocal of $\frac{4}{27}$ is $\frac{27}{4}$.

$$\begin{aligned}\frac{2}{3} \div \frac{4}{27} &= \frac{2}{3} \times \frac{27}{4}\\ &= \frac{\cancel{2}^{1}}{\cancel{3}_{1}} \times \frac{\cancel{27}^{9}}{\cancel{4}^{2}}\\ &= \frac{9}{2}\\ &= 4\tfrac{1}{2}\end{aligned}$$

b The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

$$\begin{aligned}16 \div \frac{2}{3} &= \frac{16}{1} \times \frac{3}{2}\\ &= \frac{\cancel{16}^{8}}{1} \times \frac{3}{\cancel{2}^{1}}\\ &= \frac{24}{1}\\ &= 24\end{aligned}$$

c The reciprocal of 16 is $\frac{1}{16}$.

$$\begin{aligned}\frac{2}{3} \div 16 &= \frac{2}{3} \times \frac{1}{16}\\ &= \frac{\cancel{2}^{1}}{3} \times \frac{1}{\cancel{16}^{8}}\\ &= \frac{1}{24}\end{aligned}$$

As with multiplication, mixed numerals must first be converted to improper fractions before dividing.

Example 15

Robin has $6\frac{3}{4}$ m of timber and wants to cut from it pieces that are $2\frac{1}{4}$ m long.

How many such lengths can he cut?

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Solution

$$\begin{aligned} 6\tfrac{3}{4} \div 2\tfrac{1}{4} &= \frac{27}{4} \div \frac{9}{4} && \text{(Convert each mixed numeral to an improper fraction.)} \\ &= \frac{27}{4} \times \frac{4}{9} && \left(\text{Multiply by the reciprocal of } \frac{9}{4}, \text{ that is } \frac{4}{9}.\right) \\ &= \frac{\cancel{27}^{3}}{\cancel{4}_{1}} \times \frac{\cancel{4}^{1}}{\cancel{9}_{1}} && \text{(Cancel and then multiply.)} \\ &= 3 \end{aligned}$$

Thus cutting $6\frac{3}{4}$ m of timber into pieces of length $2\frac{1}{4}$ m gives exactly 3 pieces.

Multiplying and dividing fractions

- To multiply fractions:
  - first cancel factors common to a numerator and a denominator
  - then multiply the numerators and multiply the denominators.
- The **reciprocal** of a non-zero fraction is formed by interchanging the numerator and the denominator.
- To divide by a non-zero fraction, multiply by its reciprocal.
- To multiply or divide mixed numerals, first change them to improper fractions.

Exercise 2C

1 Evaluate:

a $\frac{2}{3}$ of $\frac{5}{16}$ **b** $\frac{3}{4}$ of $\frac{11}{16}$ **c** $\frac{5}{11}$ of 22

d $\frac{3}{5}$ of $\frac{11}{20}$ **e** $\frac{5}{12}$ of 32 **f** $\frac{6}{11}$ of 25

Example 10

2 Evaluate:

a $\frac{1}{2}\times\frac{1}{3}$ **b** $\frac{1}{7}\times\frac{1}{11}$ **c** $\frac{3}{4}\times\frac{1}{3}$ **d** $\frac{7}{8}\times\frac{4}{5}$ **e** $\frac{3}{28}\times\frac{2}{3}$

f $\frac{5}{18}\times\frac{3}{5}$ **g** $\frac{16}{21}\times\frac{7}{8}$ **h** $\frac{14}{15}\times\frac{7}{8}$ **i** $\frac{14}{3}\times\frac{3}{7}$ **j** $\frac{7}{11}\times\frac{22}{35}$

k $\frac{7}{5}\times\frac{5}{13}$ **l** $\frac{3}{4}\times\frac{8}{15}$ **m** $\frac{5}{8}\times\frac{7}{12}$ **n** $\frac{15}{22}\times\frac{11}{30}$ **o** $\frac{7}{4}\times\frac{12}{35}$

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Example 11

3 Evaluate:

a $1\frac{1}{5}\times 2\frac{1}{3}$ **b** $2\frac{2}{3}\times 3\frac{1}{5}$ **c** $4\frac{1}{10}\times 5\frac{2}{5}$

d $2\frac{4}{11}\times 1\frac{1}{3}$ **e** $1\frac{3}{5}\times 1\frac{2}{3}$ **f** $2\frac{1}{4}\times 1\frac{1}{5}$

g $2\frac{1}{5}\times \frac{3}{11}$ **h** $2\frac{1}{4}\times 1\frac{1}{8}$ **i** $3\frac{1}{3}\times 1\frac{2}{3}$

Example 12

4 $\frac{7}{8}$ of a group of 968 students have brown eyes. How many students is this?

5 Find the value of:

a $\frac{3}{5}$ of 20 L **b** $\frac{5}{8}$ of 10 km **c** $\frac{5}{8}$ of 10 000 m **d** $\frac{3}{4}$ of 50 m

e $\frac{4}{5}$ of 60 kg **f** $\frac{3}{5}$ of 800 mm **g** $\frac{3}{7}$ of 140 m **h** $\frac{7}{8}$ of 100 km

Example 14

6 Evaluate:

a $\frac{5}{8}\div\frac{4}{11}$ **b** $\frac{7}{15}\div\frac{2}{11}$ **c** $\frac{2}{3}\div\frac{4}{7}$ **d** $15\div\frac{18}{5}$

e $\frac{3}{4}\div 10$ **f** $\frac{5}{11}\div 7$ **g** $\frac{5}{8}\div\frac{11}{3}$ **h** $\frac{5}{12}\div 15$

i $\frac{2}{3}\div\frac{5}{8}$ **j** $16\div\frac{2}{3}$ **k** $10\div\frac{2}{5}$ **l** $\frac{15}{8}\div\frac{3}{4}$

7 Evaluate:

a $6\frac{3}{4}\div 1\frac{1}{5}$ **b** $2\frac{3}{5}\div 1\frac{1}{2}$ **c** $5\frac{1}{4}\div 1\frac{1}{2}$ **d** $10\div 1\frac{1}{5}$

e $5\frac{1}{3}\div 10$ **f** $2\frac{1}{5}\div 1\frac{1}{3}$ **g** $7\frac{1}{5}\div 9$ **h** $2\frac{1}{5}\div 3\frac{3}{4}$

i $5\frac{1}{3}\div 6\frac{1}{4}$

Example 13

8 It takes a cabinet maker $2\frac{1}{4}$ days to make a polished table top. How many can he make in 9 weeks?

Example 15

9 How many seconds does it take for a man running at $9\frac{1}{2}$ m/s to run 100 m?

10 **a** What number multiplied by $2\frac{1}{4}$ gives $1\frac{1}{8}$?

b What number divided by $1\frac{1}{3}$ gives $2\frac{1}{4}$?

11 At Castlefield College, $\frac{4}{7}$ of the students are boys.

a What fraction of students are girls?

b If there are 360 boys, how many girls are there?

12 Four-fifths of the jelly beans in a jar are black. There are 288 black jelly beans in the jar. How many jelly beans are there in total?

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13 Three-quarters of the trees in a forest are acacias. It is known that there are 6000 trees in the forest.

a How many acacias are there in the forest?

b What fraction of the trees are not acacias?

14 Three-sevenths of a sum of money is \$36. How much is $\frac{4}{7}$ of the sum of money?

15 Five-twelfths of a farm covers 325 hectares (ha). What is the area of the whole farm?

16 A tank that is $\frac{2}{3}$ full contains 1350 L of water. How many litres does it hold when it is full?

17 After cycling $\frac{1}{3}$ of a journey, a cyclist has 28 km further to go. What is the length of the journey?

18 A rope $10\frac{7}{8}$ m is cut into five equal length pieces of rope. How long is each piece?

2D The unitary method

The idea of the **unitary method** is to base calculations on one part of the whole. Often, the ‘part’ is a fraction of the whole. Here we take another look at ‘of’ and multiplication from this point of view.

Fraction of a quantity: the unitary method

In the **unitary method** of solving problems, the most important step is the very first line, where the ‘parts’ and the ‘whole’ are identified.

Example 16

The class was told that each pupil must read $\frac{3}{8}$ of a 496-page book before the first day of term. How many pages must each pupil read?

Solution

The ‘whole’ is the 496-page book.

The ‘part’ is $\frac{1}{8}$ of the pages in the book,

so 8 parts is 496 pages. (This line is the key step.)

$\boxed{\div 8}$ 1 part is 62 pages. (The operation is in the box to the left.)

$\boxed{\times 3}$ 3 parts is 186 pages. (We have now got to the ‘3’ in $\frac{3}{8}$.)

Hence, each pupil must read 186 pages.

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Going from a fraction to the whole

The unitary method can be used to solve problems where we are given a fraction of the whole.

Example 17

There are 51 people at Sacha's barbecue. This is $\frac{3}{5}$ of those invited. How many people were invited?

Solution

The 'part' to use is $\frac{1}{5}$ of those invited,

so 3 parts is 51 people. (This line is the key step.)

$\boxed{\div 3}$ 1 part is 17 people.

$\boxed{\times 5}$ 5 parts is 85 people. (This is the 'whole'.)

Hence, 85 people were invited to the party.

Exercise 2D

Example 16

1 Use the unitary method to solve these problems.

a $\frac{2}{3}$ of 540 **b** $\frac{5}{8}$ of 968 **c** $\frac{5}{12}$ of 1440

2 Use the unitary method to solve these problems.

a Find $\frac{4}{5}$ of \$160.

b Find $\frac{3}{7}$ of 210 kg.

c Amina's father agrees to pay $\frac{7}{12}$ of the price of a new saxophone. If the saxophone costs \$3780, how much will Amina's father pay?

d Binh's farm has produced 4600 cubic metres of hay. If he keeps $\frac{3}{8}$ of this to feed his stock, how much will he have left to sell at the market?

Example 17

3 **a** $\frac{3}{8}$ of an amount of money is \$543. What is the amount of money?

b $\frac{3}{4}$ of the students in a school are boys. There are 726 boys in the school. How many students are there in the school?

4 Twenty-one students have signed up for the school music camp. This is $\frac{3}{7}$ of the maximum number of people allowed to attend the camp. How many more students can still sign up?

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5 Use the unitary method in reverse to solve these problems.

a If $\frac{3}{5}$ of a container used for storing milk is 150 litres, what is the capacity of the container?

b Massima has saved \$495, which is $\frac{11}{12}$ of the cost of a stereo. What is the price of the stereo?

c Jehan saves \$105 every week, which is $\frac{3}{5}$ of his weekly wage. What is his weekly wage?

d Conrad is recovering from an operation and has been told to walk 10 000 steps per day. In 45 minutes, he has walked 3000 steps. If he walks at the same rate, how long will it take him to walk the required number of steps?

6 In the basketball grand final, Simon scored 16 goals. If Simon scored $\frac{1}{6}$ of his team's goals, what was their final number of goals?

7 One goat needs two-fifths of a hectare if it is to produce high-quality milk. How many hectares are needed for a herd of 72 goats?

8 Sam wants to make five banana pancakes. He has a recipe for 20 pancakes, and the recipe requires two-thirds of a cup of milk. How much milk is needed for 5 pancakes?

2E Decimal notation

Decimals are an extension of the base-ten number system. The whole-number place-value notation is extended to include tenths, hundredths, thousandths, and so on. The **decimal point** separates the whole-number part from the fractional part.

Here is a reminder of what a decimal means. The decimal:

$$123.456 = 100 + 20 + 3 + \frac{4}{10} + \frac{5}{100} + \frac{6}{1000}$$

This process is called writing 123.456 in **expanded form**. We can also write $123.456 = 123\frac{456}{1000}$.

Using expanded form, we can write a decimal as a fraction or mixed numeral.

Example 18

Write 2.1255 as a mixed numeral with the fractional part in simplest form.

Solution

$$\begin{aligned} 2.1255 &= 2\tfrac{1255}{10\,000} \\ &= 2\tfrac{251}{2000} \end{aligned}$$

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Converting fractions to decimals

Converting fractions with denominators that are powers of 10 to decimals is straightforward.

Example 19

Convert $8\frac{123}{1000}$ to a decimal.

Solution

$8\frac{123}{1000} = 8.123$

Some fractions do not have denominators that are powers of 10 but they can also be converted into decimals.

Example 20

Write each fraction as a decimal.

a $\dfrac{3}{25}$ **b** $\dfrac{7}{40}$ **c** $\dfrac{19}{125}$

Solution

a $\dfrac{3}{25} = \dfrac{3 \times 4}{25 \times 4} = \dfrac{12}{100} = 0.12$

b $\dfrac{7}{40} = \dfrac{7 \times 25}{40 \times 25} = \dfrac{175}{1000} = 0.175$

c $\dfrac{19}{125} = \dfrac{19 \times 8}{125 \times 8} = \dfrac{152}{1000} = 0.152$

Comparing and ordering decimals

Selecting the larger of two decimals is done by comparing the leftmost digit and then comparing from left to right until the digits being compared are different. You can then decide which is larger.

Example 21

Five children measure their arm spans. Their results are 1.64 m, 1.5 m, 1.595 m, 1.328 m and 1.593 m.

Put these measurements in order from smallest to largest.

Solution

First compare the units. They are all the same, so compare the tenths digit. The largest tenths digit is the 6 in 1.64 and the smallest is the 3 in 1.328. Now compare the hundredths digits and finally the thousandths to see that the order is:

$1.328 < 1.5 < 1.593 < 1.595 < 1.64$

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Example 22

Order these fractions from smallest to largest by first converting them to decimals:

$\frac{3}{4}$, $7\frac{5}{8}$, $\frac{6265}{700}$ and $\frac{228}{400}$.

Solution

$$\begin{aligned}\frac{3}{4} &= \frac{75}{100}\\ &= 0.75\end{aligned}$$

$$\begin{aligned}7\tfrac{5}{8} &= 7\tfrac{625}{1000}\\ &= 7.625\end{aligned}$$ (Multiply numerator and denominator by 125.)

$$\begin{aligned}\frac{6265}{700} &= \frac{895}{100}\\ &= 8.95\end{aligned}$$ (Divide numerator and denominator by 7.)

$$\begin{aligned}\frac{228}{400} &= \frac{57}{100}\\ &= 0.57\end{aligned}$$ (Divide numerator and denominator by 4.)

So the order is $\frac{280}{400} < \frac{3}{4} < 7\frac{5}{8} < \frac{6265}{700}$ or $0.57 < 0.75 < 7.625 < 8.95$.

Exercise 2E

Example 18

1 Convert these numbers to a proper fraction or mixed numeral.

a 2.1 **b** 5.023 **c** 6.71 **d** 2.006 **e** 0.076

f 5.68 **g** 0.0085 **h** 2.008 **i** 16.875 **j** 23.625

Example 19, 20

2 Convert these fractions and mixed numerals to decimals.

a $51\frac{3}{4}$ **b** $\frac{7}{8}$ **c** $36\frac{3}{25}$ **d** $112\frac{17}{50}$ **e** $87\frac{39}{200}$

Example 21

3 Arrange these numbers in order from smallest to largest.

a 2.5834, 2.35, 2.83, 2.435, 2.5

b 18.009 9573, 18.1, 18.02, 18.1002, 18.21

c 6.6, 6.66, 66.06, 60.66, 60.006

d 55.2, 47.682, 55.24, 55.16, 56.001

Example 22

4 Convert these mixed numerals to decimals, then put them in increasing order.

$4\frac{3}{4}, 4\frac{3}{5}, 4\frac{31}{100}, 4\frac{5}{8}$

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2F Operations on decimals

Addition and subtraction

Addition and subtraction of decimals follow the same ideas as for whole numbers and use the standard algorithms. It is important that we only add like to like: we can only add tens to tens, ones to ones, tenths to tenths, and so on. For this reason, we must line up the decimal points of the numbers when we use the algorithms for addition and subtraction.

Example 23

The wiring plan for an electronic robot calls for 0.82 m, 1.5 m and 13.285 m of cable. How much cable is needed in total?

Solution

$$\begin{array}{rcccc} & 0 & . & 8 & 2 & 0 \\ & 1 & . & 5 & 0 & 0 \\ + & 13_1 & . & 2_1 & 8 & 5 \\ \hline & 15 & . & 6 & 0 & 5 \\ \hline \end{array}$$

15.605 m of cable is needed in total.

Example 24

Coffee comes in bags that weigh 4.2 kg. In one week, 1.83 kg of coffee was used. How much was left?

Solution

$$\begin{array}{rcccc} & 4 & . & {}^{1}2 & {}^{1}0 \\ - & 1_1 & . & 8_1 & 3 \\ \hline & 2 & . & 3 & 7 \\ \hline \end{array}$$

2.37 kg of coffee was left.

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Multiplication and division by powers of 10

Multiplying and dividing decimals by 10, 100, 1000, 10 000 and so on is easy.

Multiplication and division of decimals by powers of 10

- When any number is multiplied by 10, each digit is multiplied by 10. This corresponds to moving the decimal point one place to the right and inserting a zero if necessary.
- Multiplying by $100 = 10^2$ corresponds to moving the decimal point 2 places to the right and inserting zeros if necessary.

  Multiplying by $1000 = 10^3$ corresponds to moving the decimal point 3 places to the right and inserting zeros if necessary.
- When any number is divided by 10, each digit is divided by 10. This corresponds to moving the decimal point one place to the left and inserting a zero if necessary.
- Dividing by $100 = 10^2$ corresponds to moving the decimal point 2 places to the left and inserting zeros if necessary.

  Dividing by $1000 = 10^3$ corresponds to moving the decimal point 3 places to the left and inserting zeros if necessary.

Multiplication of decimals

Example 25

Evaluate 2.451×100.

Solution

$2.451 \times 100 = 245.1$ (The decimal point is moved two places to the right.)

Example 26

Evaluate $2.451 \div 1000$.

Solution

$2.451 \div 1000 = 0.002\,451$ (The decimal point is moved 3 places to the left and the 2 zeros are inserted.)

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Multiplying a decimal by a whole number

The following example presents a suitable setting out.

Example 27

Six children collected lengths of timber for recycling. They collected 3.87 m of timber each. How much timber did they collect in total?

Solution

Multiply the length collected by one child by 6.

$$\begin{array}{r} 3.87 \\ \times \quad 6 \\ \hline 23.22 \\ \hline \end{array}$$

The children collected 23.22 m of timber in total.

Multiplying one decimal by another

We can multiply decimals by converting each decimal to a fraction, multiplying the fractions (without cancelling) and then converting the result back to a decimal.

Example 28

Calculate 0.03×0.18.

Solution

$$\begin{aligned} 0.03 \times 0.18 &= \frac{3}{100} \times \frac{18}{100} \\ &= \frac{54}{10\,000} \\ &= 0.0054 \end{aligned}$$

or

$0.03 \times 0.18 = 0.0054$.
Multiply 18 by 3 and place the decimal point so that the total number of places after the decimal point is the same on both sides of the equation.

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Division of decimals

Dividing a decimal by a whole number

We can use the division algorithm to do this.

$$4\overline{)3.6}\quad 0.9$$

The procedure is the same as for whole numbers. The decimal point in the quotient is aligned directly above the decimal point in the dividend.

Example 29

Divide 3.6 by 5.

Solution

$$5\overline{)3.6^{1}0}\quad 0.72$$

Dividing one decimal by another

There are two methods for dividing one decimal by another.

Example 30

Evaluate $3.6 \div 0.05$.

Solution

Method 1

Multiply numerator and denominator by the same factor to obtain a whole-number divisor.

$$\begin{aligned}3.6 \div 0.05 &= \frac{3.6 \times 100}{0.05 \times 100}\\ &= \frac{360}{5}\\ &= 72\end{aligned}$$

Method 2

Convert each number to a fraction.

$$\begin{aligned}3.6 \div 0.05 &= \frac{36}{10} \div \frac{5}{100}\\ &= \frac{36}{10} \times \frac{100}{5}\\ &= \frac{36}{\cancel{10}^{1}} \times \frac{\cancel{100}^{10}}{5}\\ &= \frac{360}{5}\\ &= 72\end{aligned}$$

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Exercise 2F

Example 23, 24

1 Calculate these sums and differences using the standard algorithms.

a $9.77 + 37.8$ b $78.9 + 0.89 + 45 + 4.664$
c $562.6 - 43.18$ d $307.05 - 89.77$

Example 25, 26

2 Evaluate:

a 2.3×10 b 0.003×100 c $2.6 \div 10$
d $260 \div 1000$ e $260 \div 10\,000$ f 0.0075×100
g $56.1 \div 100$ h $2.63 \times 10\,000$ i 2.5×100

Example 27

3 Evaluate:

a 5.63×7 b 22.867×5 c 426.8×9
d 456.23×7 e 56.23×24 f 69.56×32

Example 28

4 Calculate:

a 0.3×0.7 b 0.8×0.06 c 0.9×0.07 d 0.006×0.7
e 0.06×0.004 f 0.05×0.08 g 31.504×1.2 h 0.061×0.002

Example 29

5 Evaluate:

a $2.376 \div 2$ b $9.2 \div 5$ c $5.48 \div 4$
d $42.7 \div 5$ e $9.7 \div 2$ f $43.2 \div 5$

Example 30

6 Calculate:

a $36 \div 0.9$ b $2800 \div 0.0007$ c $0.378 \div 0.03$
d $63.147 \div 0.07$ e $84 \div 2.4$ f $560 \div 0.008$
g $7200 \div 0.09$ h $0.144 \div 0.012$ i $450.56 \div 0.08$

7 The population of Sydney was 3.9 million in 1996 and increased to 4.2 million in 2001. By what number of people did the population increase over the 5 years? What was the average annual increase?

8 What is the perimeter of a table with side lengths 1.8 m, 0.87 m, 0.87 m and 1.43 m?

9 Between June 1996 and June 2001, the population of Brisbane increased by 133 400 people to reach 1.7 million. What was the population in June 1996?

10 What is the mass, in kilograms, of each of the following amounts of olive oil if one litre of olive oil has a mass of 0.878 kg?

a 2 L b 3.5 L c 0.8 L
d 1.7 L e 423 mL f 400 mL

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11 On average, Melburnians ate 18.34 kg of cheese each in 2001. If the population of Melbourne in 2001 was 3.5 million, what was the total amount of cheese eaten in Melbourne in that year?

12 If the 321 700 people living in Darwin produce an average of 1.87 kg of garbage per person per day, how much garbage is produced by Darwin residents in one non-leap year?

13 A rectangular block of land measures 15.78 m by 27.93 m. Calculate the perimeter and area of the block.

14 Find, as a decimal, the missing number in each statement.

a $10 \times \square = 0.4$

b $109.9 = \square \div 0.7$

c $\square \times 2.3 = 9.2$

d $\square = 5.3 \times 96.4$

e $\square \div 1.098 = 25.87$

f $4.52 \div 0.8 = \square$

15 Gold is measured in troy ounces. One troy ounce is equal to 31.103 g. What is the weight, in grams, of each of the following gold nuggets?

a The Welcome Nugget, found at Ballarat in 1858, weighing 2217 ounces

b The Welcome Stranger Nugget, found at Moliagul in 1869, weighing 2284 ounces

c The Beyers and Holterman specimen, which was reported to have contained 3000 ounces of gold

2G Fractions, decimals and rounding

We have already seen how to convert certain fractions to decimals.

For example:

$$\frac{3}{10} = 0.3, \frac{3}{4} = 0.75$$

The method used was to multiply the denominator by some number to produce a power of 10 and multiply the numerator by that same number. Thus,

$$\frac{3}{8} = \frac{3 \times 125}{8 \times 125} = \frac{375}{1000} = 0.375$$

This method will only work if the only prime factors of the denominator are 2 or 5, and will produce a decimal with a finite number of digits. Such decimals are called **terminating decimals**. We will see shortly that a fraction whose denominator has a prime factor other than 2 or 5 does not produce a terminating decimal.

A better method for converting fractions to decimals is to use the division algorithm.

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Example 31

Evaluate $\frac{3}{8}$ as a decimal by using the division algorithm.

Solution

$$\begin{array}{r}0.3\ 7\ 5\\ 8\overline{\big)3.0^{6}0^{4}0}\end{array}$$

So $\frac{3}{8} = 0.375$.

Recurring decimals

If we apply the division algorithm to a fraction whose denominator has a prime factor other than 2 or 5, we see that the process does not terminate.

Example 32

Use the division algorithm to write each fraction as a repeating decimal.

a $\frac{4}{11}$

b $\frac{5}{12}$

Solution

a

$$\begin{array}{r}0.\ 3\ 6\ 3\ 6\ 3\ 6\ 3\ldots\\ 11\overline{\big)4.^{4}0^{7}0^{4}0^{7}0^{4}0^{7}0^{4}0\ldots}\end{array}$$

Hence, $\frac{4}{11} = 0.363636363636\ldots$

We write $\frac{4}{11} = 0.\dot{3}\dot{6}$

b

$$\begin{array}{r}0.\ 4\ 1\ 6\ 6\ 6\ 6\ 6\ldots\\ 12\overline{\big)5.^{5}0^{2}0^{8}0^{8}0^{8}0^{8}0^{8}0\ldots}\end{array}$$

Hence, $\frac{5}{12} = 0.416666666666\ldots$

We write $\frac{5}{12} = 0.41\dot{6}$

We write 0.363636… as $0.\dot{3}\dot{6}$ to indicate that the digits 36 repeat indefinitely. This is an example of a recurring decimal. We place dots on the first and last digit of the recurring cycle.

We write $0.4166666666\ldots$ as $0.41\dot{6}$ to show that only the digit 6 repeats indefinitely. This is an example of an eventually recurring decimal.

Note the conventions for writing the repeating cycles in the final answers:

- In $0.41\dot{6} = 0.416666666666\ldots$ the dot goes over the repeating digit 6.
- In $0.\dot{3}\dot{6} = 0.363636363636\ldots$, the dots go over the first and last digits of the repeating cycle.

Other fractions give decimals with longer repeating strings of digits.

For example, $\frac{2}{7} = 0.285714285714\ldots$ This can be written as $0.\dot{2}85\,71\dot{4}$.

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The infinite string of digits in the statement $\frac{5}{12} = 0.416666666666\ldots$ means that the sequence 0.4, 0.41, 0.416, 0.4166, 0.41666, 0.416666, 0.4166666, 0.41666666 . . . gets closer and closer to $\frac{5}{12}$. No term can ever quite get to $\frac{5}{12}$, but we can find terms 'as close as we like' to $\frac{5}{12}$ by taking more and more digits in the string.

Similarly, the statement $\frac{4}{11} = 0.363636363636\ldots$ means that the sequence 0.3, 0.36, 0.363, 0.3636, 0.36363, 0.363636, 0.3636363, 0.36363636 gets 'as close as we like' to $\frac{4}{11}$.

Rounding decimals

The last few decimal places of a decimal such as 3.141 5927 may have no practical value. For example, if 3.141 5927 represents the number of kilometres between two farmyard gates, then the final digit 7 represents $\frac{7}{10}$ of a millimetre, which is completely irrelevant. When we round numbers, we write them correct to a certain number of decimal places.

Writing a decimal correct to a number of decimal places

The rules for rounding are as follows.

Suppose that we want to round 10.125 89 correct to two decimal places.

- Identify the rounding digit in the second decimal place – in this case it is 2.
- Look at the next digit to the right of the rounding digit – in this case it is 5.
  - If this next digit is 0, 1, 2, 3 or 4, leave the rounding digit alone.
  - If this next digit is 5, 6, 7, 8 or 9, increase the rounding digit by 1.

  In this case, the next digit is 5, so the rounding digit increases from 2 to 3.
- Now discard all the digits after the rounding digit.

Hence 10.125 89 is approximately 10.13, correct to two decimal places. We write this as 10.125 89 $\approx$ 10.13 correct to two decimal places.

Example 33

Write 3.164 93 correct to:

a 2 decimal places **b** 3 decimal places **c** 4 decimal places

Solution

a 3.164 93 $\approx$ 3.16 to 2 decimal places: the rounding digit is 6, and the digit to the right is smaller than 5.

b 3.164 93 $\approx$ 3.165 to 3 decimal places: the rounding digit is 4, and the digit to the right is larger than 5.

c 3.164 93 $\approx$ 3.1649 to 4 decimal places: the rounding digit is 9, and the digit to the right is smaller than 5.

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The rounding procedure can also be used with repeating decimals.

Example 34

Write $0.\dot{3}\dot{6}$ correct to:

a 2 decimal places **b** 3 decimal places **c** 4 decimal places

Solution

$0.\dot{3}\dot{6} = 0.363636\ldots$

a $0.\dot{3}\dot{6} = 0.36$ (correct to 2 decimal places)
b $0.\dot{3}\dot{6} = 0.364$ (correct to 3 decimal places)
c $0.\dot{3}\dot{6} = 0.3636$ (correct to 4 decimal places)

Exercise 2G

Example 31

1 Convert these fractions and mixed numerals to decimals.

a $\frac{2}{5}$ **b** $\frac{4}{5}$ **c** $\frac{3}{25}$ **d** $\frac{7}{8}$ **e** $4\frac{3}{20}$
f $7\frac{3}{8}$ **g** $45\frac{3}{4}$ **h** $7\frac{17}{20}$ **i** $\frac{17}{5}$ **j** $\frac{28}{25}$

Example 32

2 Convert these fractions to decimals.

a $\frac{2}{7}$ **b** $\frac{3}{7}$ **c** $\frac{2}{3}$ **d** $\frac{2}{9}$ **e** $\frac{3}{11}$
f $\frac{4}{9}$ **g** $\frac{7}{11}$ **h** $\frac{8}{9}$ **i** $\frac{2}{13}$

3 Convert these fractions to decimals.

a $\frac{1}{6}$ **b** $\frac{5}{6}$ **c** $\frac{5}{12}$ **d** $\frac{7}{12}$ **e** $\frac{11}{12}$ **f** $\frac{4}{9}$

4 Convert these fractions and mixed numerals to decimals. (They are all recurring decimals.)

a $64\frac{5}{9}$ **b** $78\frac{5}{6}$ **c** $45\frac{2}{11}$ **d** $\frac{11}{12}$ **e** $3\frac{2}{7}$

Example 33

5 Round these decimals correct to the given numbers of decimal places.

a 463.1529 (2 places) **b** 7.2811 (1 place) **c** 79.497 (2 places)
d 0.0649 (3places) **e** 7.99 (1 place) **f** 85.6 (nearest whole number)

6 Express each fraction as a decimal correct to 2 decimal places.

a $\frac{2}{7}$ **b** $\frac{5}{6}$ **c** $\frac{4}{7}$ **d** $\frac{7}{12}$ **e** $\frac{4}{9}$

Example 34

7 Write $0.\dot{7}4\dot{6}$ correct to:

a 2 decimal places **b** 4 decimal places **c** 8 decimal places

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Review exercise

1 From this list of fractions: $\frac{35}{42}, \frac{22}{33}, \frac{10}{15}, \frac{17}{34}, \frac{112}{224}, \frac{20}{24}, \frac{18}{21}, \frac{6}{7}$ write two fractions that are equivalent to each of the following.

a $\frac{12}{18}$ **b** $\frac{5}{6}$ **c** $\frac{42}{49}$ **d** $\frac{1}{2}$

2 Order each set of fractions from largest to smallest.

a $\frac{3}{2}, \frac{2}{3}, \frac{10}{12}, \frac{8}{6}, \frac{5}{4}, \frac{2}{4}$ **b** $2\tfrac{3}{7}, 3\tfrac{1}{8}, 2\tfrac{2}{9}, 2\tfrac{5}{9}, 3\tfrac{1}{9}, 2\tfrac{6}{7}$

3 Evaluate:

a $\frac{5}{12} \div \frac{1}{6}$ **b** $\frac{7}{15} \div \frac{14}{25}$ **c** $\frac{19}{13} \times \frac{52}{81} \div \frac{16}{45}$ **d** $\frac{5}{16} \times \frac{15}{16} \div \frac{125}{32}$

4 Convert mixed numerals to fractions before carrying out these calculations.

a $5\tfrac{1}{4} \times 2\tfrac{1}{7}$ **b** $3\tfrac{1}{5} \div 1\tfrac{1}{25}$ **c** $5\tfrac{1}{2} \times 6\tfrac{1}{2} \div 3\tfrac{1}{4}$

d $7\tfrac{6}{7} \times 3\tfrac{15}{16} \div 4\tfrac{2}{5}$ **e** $1\tfrac{1}{3} \times 2\tfrac{1}{4}$ **f** $\frac{5}{12} \times \frac{3}{14}$

g $\frac{5}{12} \div \frac{3}{4}$ **h** $3\tfrac{7}{8} \div \frac{1}{4}$ **i** $1\tfrac{1}{4} \times 2\tfrac{1}{8}$

5 Calculate:

a $\frac{5}{12} + \frac{1}{3}$ **b** $\frac{2}{3} + \frac{5}{12}$ **c** $\frac{5}{6} - \frac{3}{4}$ **d** $1\tfrac{1}{3} - 2\tfrac{1}{5}$

e $3\tfrac{1}{4} + 2\tfrac{3}{5}$ **f** $3\tfrac{3}{4} + 2\tfrac{1}{5}$ **g** $2\tfrac{1}{2} - \frac{3}{8}$ **h** $1\tfrac{1}{2} - \frac{1}{5}$

6 Evaluate:

a $1\tfrac{3}{4} - \frac{5}{6} + 2\tfrac{1}{2}$ **b** $5 - 1\tfrac{1}{2} + \frac{5}{8}$ **c** $3\tfrac{1}{4} - 2\tfrac{3}{4} + 2\tfrac{1}{2}$

d $6\tfrac{1}{2} + 2\tfrac{3}{5} - 1\tfrac{4}{5}$ **e** $2\tfrac{3}{4} + 9\tfrac{1}{2} + 1\tfrac{3}{4}$ **f** $2\tfrac{1}{3} - \frac{3}{4} + 1\tfrac{2}{3}$

7 Nina is training hard to become a better swimmer. She spends 17 hours per week doing freestyle, 9 hours per week doing backstroke, and 4 hours per week in the gym. What fraction of her training time is spent out of the pool?

8 Alejandro allocates $\frac{1}{4}$ of his pocket money to transport costs, $\frac{1}{10}$ to telephone calls and $\frac{3}{5}$ to clothes, and he saves the rest. If he receives \$18 per week, what are his expenses for transport, telephone calls and clothes? What fraction of his pocket money does Alejandro save each week?

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9 In a garden, $\frac{1}{3}$ of the area is used for vegetables and $\frac{2}{5}$ for flowers and plants. If the rest of the garden consists of lawns, what fraction of the garden is occupied by lawns?

10 There are 1236 boys at a school. This is $\frac{3}{5}$ of the total number of students at the school. How many students are there at the school?

11 A football squad consists of 24 players. Only $\frac{1}{8}$ of the squad are injured. How many players are fit to play?

12 Round these decimals correct to the given number of decimal places.

a 0.089 734	4 dec. places	**b** 654.0786	3 dec. places
c 3.141 592 65	5 dec. places	**d** 3.141 592 65	3 dec. places
e 2.718 281 828 46	10 dec. places	**f** 2.718 281 828 46	4 dec. places

13 Calculate:

a 4.56×10 **b** 0.0028×100 **c** $1.030\,52 \times 10\,000$

d $0.000\,043 \times 1000$ **e** $24.87 \div 10$ **f** $971.2 \div 100$

g $0.003 \div 1000$ **h** $340 \div 10\,000$ **i** $1000 \times 1.008\,36$

14 Calculate:

a $3.045 + 34.98$ **b** $34.7605 + 0.089$ **c** $452.906 + 8.006$

d $45 - 39.06$ **e** $56.089 - 7.308$ **f** $451.23 - 356.8$

g 67.9×3 **h** 0.41×0.6 **i** 0.345×0.002

j $4 \div 0.02$ **k** $67 \div 0.04$ **l** $0.08 \div 0.6$

15 Calculate the cost of:

a 3.75 kilograms of butter at \$1.50 a kilogram

b $3\frac{1}{2}$ dozen eggs at \$2.80 per dozen

c 4.35 kilograms of cheese at \$7.50 a kilogram

16 Inouk wanted to edge her garden beds with timber. She measured the lengths of the edges as 1.48 metres, 9.2 metres, 0.93 metres, 4.02 metres and 5.45 metres. How many 4.2-metre lengths of timber garden edging did she have to buy? If the timber garden edging costs \$3.44 per metre, how much did Inouk's garden edging project cost her?

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17 Calculate how much you save on each item when you buy in bulk.

a 3 cans of beetroot for \$2.85 or 98 cents each

b 6 jars of tomato paste for \$8.40 or \$1.50 each

c 4 bottles of soft drink for \$3.70 or 93 cents each

18 One-quarter of a class arrived at 7 p.m. for the school concert, half of them arrived at 7.15 p.m., three children told the teacher that they were not able to attend, and the rest are running late. If there are 60 children in the class:

a how many children arrived at 7 p.m.?

b what fraction of the class are running late? How many children is this?

c what fraction told the teacher they were not able to attend the concert?

19 Belgian cheese costs \$1.70 for $\frac{1}{2}$ a kilogram. How much will $2\frac{1}{2}$ kilograms of Belgian cheese cost?

20 Linda's doctor tells her she should get 10 hours of sleep each night. She knows that she sleeps $\frac{3}{8}$ of a day already. How much more sleep should she be getting each day?

21 Catherine is $\frac{5}{6}$ of her father's height and her brother Peter is $\frac{7}{9}$ of their father's height. If Peter is 140 cm tall, how tall is Catherine?

22 Zubin's large dog eats $3\frac{1}{3}$ tins of dog food every day. Each tin contains $1\frac{1}{7}$ kg of food.

a How much dog food does the dog eat in a seven-day week?

b How much dog food does the dog eat in a 52-week year?

23 Evaluate:

a $\dfrac{\frac{3}{5}-\frac{1}{2}}{\frac{4}{5}}$ **b** $\dfrac{\frac{6}{11}-\frac{1}{3}}{\frac{7}{9}}$ **c** $\dfrac{\frac{5}{8}}{3\frac{1}{2}-1\frac{1}{4}}$ **d** $\dfrac{\frac{7}{11}}{2\frac{2}{3}-1\frac{1}{2}}$

e $\dfrac{\frac{5}{8}-\frac{1}{3}}{\frac{5}{8}+\frac{1}{3}}$ **f** $\dfrac{5\frac{4}{9}}{2\frac{1}{3}+5\frac{1}{3}}$ **g** $\dfrac{\frac{1}{2}+\frac{3}{4}+\frac{5}{6}}{\frac{2}{3}+\frac{1}{4}+\frac{7}{6}}$ **h** $\dfrac{3\frac{1}{4}-2\frac{3}{5}}{3\frac{1}{4}-2\frac{3}{5}}$

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Challenge exercise

1 Rebecca has a collection of 45 books. The collection consists of novels and textbooks written in either Chinese or English. $\frac{4}{5}$ of the novels are in English and $\frac{3}{4}$ of the textbooks are in English. The total number of books written in English is 35. How many of her books are textbooks written in English?

2 Ava thought she had bought $3\frac{1}{2}$ m of cloth from the market. When she got home, she found that the stall holder had used a ruler that was 4 cm short of 1 m. What was the real length, in metres, of Ava's cloth?

3 Angel spent $\frac{7}{24}$ of her weekly salary on Monday, $\frac{1}{4}$ on Tuesday and $\frac{1}{3}$ of it on Wednesday. What fraction of her salary was left on Thursday?

4 The fractions $\frac{1}{2}$ and $\frac{1}{3}$ are said to be **consecutive unit fractions**.

a Complete each of these subtractions.

i $\frac{1}{2} - \frac{1}{3}$

ii $\frac{1}{4} - \frac{1}{5}$

iii $\frac{1}{7} - \frac{1}{8}$

iv $\frac{1}{9} - \frac{1}{10}$

b State a rule for subtracting consecutive unit fractions.

c Write $\frac{1}{12}$ as the difference of two consecutive unit fractions.

5 Which of the fractions $\frac{1}{4}, \frac{1}{2}, \frac{5}{8}$ and $1\frac{4}{5}$ is the closest to each of the fractions below?

a $\frac{4}{6}$ **b** $\frac{4}{12}$ **c** $\frac{4}{9}$ **d** $\frac{9}{17}$

e $\frac{47}{96}$ **f** $\frac{31}{120}$ **g** $\frac{199}{100}$ **h** $\frac{299}{402}$

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6 **a** Show that $27 \times 37 = 999$. Then use the fact that $0.\dot{9} = 1$ to show that $0.\dot{0}2\dot{7} \times 37 = 1$.

b Hence, show that $\frac{1}{37} = 0.\dot{0}2\dot{7}$.

c By multiplying $\frac{1}{37} = 0.\dot{0}2\dot{7}$ by 10 and 100, show that $\frac{10}{37} = 0.\dot{2}7\dot{0}$ and $\frac{100}{37} = 2.\dot{7}0\dot{2}$, and hence show that $\frac{26}{37} = 0.\dot{7}0\dot{2}$.

d By multiplying $\frac{1}{37} = 0.\dot{0}2\dot{7}$ by 2, 3, 4 and 26, show that

$\frac{2}{37} = 0.\dot{0}5\dot{4}$, $\frac{3}{37} = 0.\dot{0}8\dot{1}$, $\frac{4}{37} = 0.\dot{1}0\dot{8}$ and $\frac{26}{37} = 0.\dot{7}0\dot{2}$.

e Using similar methods, find $\frac{1}{27}, \frac{10}{27}, \frac{19}{27}$, and then $\frac{2}{27}, \frac{3}{27}$ and $\frac{4}{27}$ as recurring decimals.

7 **a** Show that $101 \times 99 = 9999$.

b Hence, find $\frac{1}{101}$ as a recurring decimal.

c Use the methods of the previous question to find $\frac{1}{101}, \frac{100}{101}$ and $\frac{1000}{101}$ as recurring decimals, and hence find $\frac{91}{101}$ as a recurring decimal.

d Find $\frac{2}{101}, \frac{3}{101}$ and $\frac{4}{101}$ as recurring decimals.

8 Four bells commence tolling together and ring every $1, 1\frac{1}{4}, 1\frac{1}{5}$ and $1\frac{1}{6}$ seconds, respectively. After what time will they first ring together again?

9 A unit fraction is of the form $\frac{1}{n}$ where n is a whole number. Remarkably, every fraction can be written as the sum of different unit fractions. For example, $\frac{2}{3} = \frac{1}{2} + \frac{1}{6}$

a Write the number $\frac{3}{10}$ as the sum of two different unit fractions.

b Write $\frac{19}{20}$ as the sum of three different unit fractions.

c Write $\frac{2}{5}$ as the sum of four different unit fractions.

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CHAPTER

3

Number and Algebra

Review of factors and indices

This chapter is about the whole numbers 0, 1, 2, 3, ... and multiplication. The whole number 6 divides exactly into the whole number 24. The number 1001 can be written as $7 \times 13 \times 11$, which is a product of prime numbers. Each whole number can be factorised into prime numbers. In this chapter, we will learn how to factor numbers. The numbers 72 and 81 share some common factors. The number 9 is called the **highest common factor** of 72 and 81. Factoring numbers into products of primes gives a simple way to find their highest common factor. It will also assist us in finding the square roots and cube roots of numbers. Primes and factoring have important applications in telecommunications technology.

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3A Factors, prime and composite numbers, multiples

All numbers in this chapter are whole numbers.

Factors

The numbers $1, 2, 3, 6, 9$ and 18 are the **factors** of 18, because each of them divides into 18 without remainder.

Common factors and the HCF

Below, we list the factors of 18 and 24 and compare the lists. The factors that are common to both lists have been boxed.

Factors of 18: $\boxed{1}, \boxed{2}, \boxed{3}, \boxed{6}, 9, 18$

Factors of 24: $\boxed{1}, \boxed{2}, \boxed{3}, 4, \boxed{6}, 8, 12, 24$

The **common factors** of 18 and 24 are $1, 2, 3$ and 6.

The largest of these common factors is 6. This is called the **HCF** – the **highest common factor** – of 18 and 24.

Example 1

Find the highest common factor of 18, 72 and 54.

Solution

The factors of 18 are 1, 2, 3, 6, 9 and 18.

The factors of 72 are 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36 and 72.

The factors of 54 are 1, 2, 3, 6, 9, 18, 27 and 54.

So the highest factor common to 18, 72 and 54 is 18.

Multiples

To write down the non-zero multiples of 4, count in 4s until you have as many as you want:

4, 8, 12, 16, 20, 24, 28, 32, …

Common multiples and the LCM

Look at the lists of the first few non-zero multiples of 18 and 24 below and compare them. The multiples that are common to both lists have been boxed.

Multiples of 18: $18, 36, 54, \boxed{72}, 90, 108, 126, \boxed{144}, 162, 180, 198, \boxed{216}, \ldots$

Multiples of 24: $24, 48, \boxed{72}, 96, 120, \boxed{144}, 168, 192, \boxed{216}, \ldots$

So the **common multiples** of 18 and 24 are 72, 144, 216, . . . The least of these common multiples is 72. This is called the **LCM** – the **lowest common multiple** – of 18 and 24.

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Example 2

Find the lowest common multiple of 6, 8 and 12.

Solution

Non-zero multiples of 6 are 6, 12, 18, 24, 30, 36, ...

Non-zero multiples of 8 are 8, 16, 24, 32, 40, ...

Non-zero multiples of 12 are 12, 24, 36, ...

So the lowest common multiple of 6, 8 and 12 is 24.

Factors and the HCF

- A **factor** of a number is a number that divides into it without remainder. For example, 6 is a factor of 24 because $24 = 6 \times 4$ or, equivalently, $24 \div 6 = 4$.
- There is natural pairing of the factors of any number. For example, the factors of 24 are 1 and 24, 2 and 12, 3 and 8, 4 and 6.
- The **highest common factor**, or **HCF**, of two or more numbers is the largest number that is a factor of all of them.

Multiples and the LCM

- The non-zero **multiples** of a number are found by counting in that number. For example, the non-zero multiples of 4 are 4, 8, 12, 16, 20, 24, 28, 32, ...
- The **lowest common multiple**, or **LCM**, of two or more numbers is the smallest number that is a multiple of all of them.

Prime numbers and composite numbers

A **prime number** is a number greater than 1 whose only factors are 1 and itself.

The list of prime numbers begins 2, 3, 5, 7, 11, ... There are infinitely many prime numbers.

The whole numbers greater than 1 that are not prime are called **composite numbers**. Such numbers have at least three factors, including 1 and itself. The list of composite numbers begins

4, 6, 8, 9, 10, ...

The numbers 0 and 1 are special, because they are neither prime nor composite.

Prime numbers and composite numbers

- A **prime number** is a number greater than 1 whose only factors are 1 and itself.
- A **composite number** is a whole number greater than 1 that is not prime. It will therefore have at least 3 factors.
- The numbers **0** and **1** are neither prime nor composite.

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Exercise 3A

Example 1

1 List all the factors of both numbers in each pair, then write down their HCF.

a 8 and 12 **b** 6 and 20 **c** 14 and 9
d 20 and 22 **e** 1 and 8 **f** 5 and 15

2 Find the HCF of:

a 30 and 24 **b** 15 and 21 **c** 12 and 72
d 25 and 16 **e** 36 and 16 **f** 26 and 65
g 12, 18 and 30 **h** 15, 6 and 14 **i** 60, 20 and 10

Example 2

3 List the first few non-zero multiples of both numbers in each pair, then write down their LCM.

a 4 and 6 **b** 10 and 12 **c** 3 and 7
d 5 and 10 **e** 1 and 5 **f** 6 and 15

4 Find the LCM of:

a 8 and 12 **b** 8 and 9 **c** 17 and 1
d 12 and 15 **e** 7 and 49 **f** 20 and 90
g 3, 4 and 6 **h** 8, 9 and 12 **i** 12, 8, 10 and 30

5 **a** **i** Count to 30 in 2s.
ii Count to 30 in 3s.
iii Count to 30 in 5s.

b Write down all the numbers from 2 to 30 that you did not list in part **a**. This list, together with 2, 3 and 5, is a list of all the 10 prime numbers less than 30.

c Write down all the composite numbers between 30 and 50.

6 Find all the prime divisors of:

a 8 **b** 12 **c** 15 **d** 30
e 44 **f** 38 **g** 125

7 **a** Write 10 and 14 as products of two prime numbers.

b Write 30 and 42 as products of three prime numbers.

c Write each of 8 and 9 as a power of a prime number.

8 Write each of these prime numbers as the sum of two squares.

a 5 **b** 13
c 17 **d** 29
e 37 **f** 41

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9 The Guavalo Palace has three long corridors, whose lengths are 24 m, 48 m and 60 m, respectively. The decorators are designing a standard piece of carpeting that will fit an exact number of times into each corridor. What is the longest piece of carpet that they can choose?

10 The baker's delivery truck visits the caravan park every 4 days. The fishmonger visits every 5 days. If they both visit today, how many days will pass before they visit on the same day again?

11 Three different bugs living in a puddle take 4, 8 and 3 minutes, respectively, to swim around the edge of the puddle. If they start at the same point at 12 p.m., what time will it be when they are all together again at that point?

12 1001 is the product of which three prime numbers?

13 **Goldbach's conjecture** states that every even number greater than 2 can be expressed as the sum of two prime numbers. Show that this is true for the even numbers between 140 and 160.

14 The prime factors of 210 are 2, 3, 5 and 7. Find six other factors of 210 without using division.

15 Explain why a square number has an odd number of factors, and a non-square number has an even number of factors. (*Hint*: Think about the pairing procedure that you used to find all the factors of a number.)

16 Using Question **15**, find the smallest and largest three-digit numbers that have exactly three factors.

3B Indices and the index laws

In this section we discuss some properties of products of repeated factors. We begin with a review of powers.

Powers

Powers provide a useful way of writing a product of repeated factors. For example, 16 can be written as the product of four 2s:

$$\begin{aligned} 16 &= 2 \times 2 \times 2 \times 2 \\ &= 2^4 \qquad \text{(We read this as ‘2 to the power 4’.)} \end{aligned}$$

- The small number 4 at the top right is called the **index** or **exponent**. (*Note*: The plural of *index* is *indices*.)
- The whole expression 2^4 is called a **power**. It is the fourth power of 2. We say ‘two to the fourth’.
- The number 2 in this expression is called the **base**.

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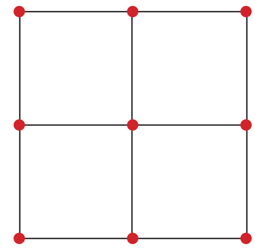
Arrays provide a useful way to visualise powers. Here are some examples with powers of 3.

$3^1 = 3$

Read 3^1 as 'three to the power of one'.

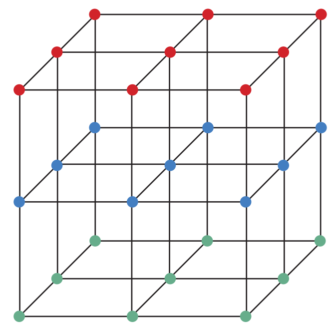
$3^2 = 3 \times 3 = 9$

Read 3^2 as 'three squared' or 'three to the power of two'.



$3^3 = 3 \times 3 \times 3 = 27$

Read 3^3 as 'three cubed' or 'three to the power of three'.



$3^4 = 3 \times 3 \times 3 \times 3 = 81$

Read 3^4 as 'three to the power of four'.

$3^n = 3 \times 3 \times 3 \times \ldots \times 3 \times 3$, with n factors of 3 in the product.

$a^m = a \times a \times a \times \ldots \times a \times a$, with m factors of a in the product.

Index notation

- A **power** is the product of a certain number of factors, all of which are the same. For example:
  $2^4 = 2 \times 2 \times 2 \times 2$
- The number 2 in the example on the previous page is called the **base** of the power.
- The number 4 in the example on the previous page is called the **index** or **exponent**.
- For any number a, $a^1 = a$.

Index laws

There are rules, called the **index laws**, for handling indices with the same base.

Index law 1: A rule for multiplying powers of the same base

There is a very simple way to multiply two or more powers of the same base. Here is an example that explains the rule.

$$\begin{aligned} 7^2 \times 7^3 &= (7 \times 7) \times (7 \times 7 \times 7) && \text{(Write out each power.)} \\ &= 7^5 && \text{(There are five 7s multiplied together.)} \end{aligned}$$

Notice that we add the two indices to get the result:

$$\begin{aligned} 7^2 \times 7^3 &= 7^{2+3} \\ &= 7^5 \end{aligned}$$

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Index law 1

When multiplying numbers written using powers, if the base number is the same, add the indices.

$a^m \times a^n = a^{m+n}$

Example 3

Write $5^2 \times 5^7$ as a single power of 5.

Solution

$$
\begin{aligned}
5^2 \times 5^7 &= 5^{2+7} \\
&= 5^9
\end{aligned}
$$

Index law 2: A rule for dividing powers of the same base

There is also a simple rule for dividing powers of the same base. Again, an example shows what the rule must be.

$$
\begin{aligned}
5^7 \div 5^3 &= \frac{5^7}{5^3} \\
&= \frac{\cancel{5}^1 \times \cancel{5}^1 \times \cancel{5}^1 \times 5 \times 5 \times 5 \times 5}{\cancel{5}^1 \times \cancel{5}^1 \times \cancel{5}^1} && \text{(Three factors cancel out.)} \\
&= 5 \times 5 \times 5 \times 5 && \text{(This leaves } 7 - 3 = 4 \text{ factors behind.)} \\
&= 5^{7-3} && \text{(Notice that we subtract the indices to get the result.)} \\
&= 5^4
\end{aligned}
$$

Index law 2

When dividing a number a^m by another number a^n, subtract the indices.

$a^m \div a^n = a^{m-n}$ or $\dfrac{a^m}{a^n} = a^{m-n}$

Example 4

Write: **a** $3^6 \div 3^2$ as a single power of 3 **b** $3^7 \div 3^6$ as a single power of 3

Solution

a
$$
\begin{aligned}
3^6 \div 3^2 &= \frac{3^6}{3^2} \\
&= 3^4
\end{aligned}
$$

b
$$
\begin{aligned}
3^7 \div 3^6 &= \frac{3^7}{3^6} \\
&= 3
\end{aligned}
$$

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Index law 3: A rule for raising a power to a power

Once again, an example makes the rule for raising a power to a power clear.

$$\begin{aligned}(8^3)^4 &= 8^3 \times 8^3 \times 8^3 \times 8^3 && \text{(This is what '}8^3\text{ to the power of 4' means.)}\\ &= 8^{3+3+3+3} && \text{(index law 1)}\\ &= 8^{3\times 4}\\ &= 8^{12}\end{aligned}$$

Index law 3

When a power is raised to another power, multiply the indices.

$(a^m)^n = a^{m \times n}$

Example 5

Write $(5^3)^2$ as a single power of 5.

Solution

$$\begin{aligned}(5^3)^2 &= 5^{3\times 2}\\ &= 5^6\end{aligned}$$

Index law 4: Powers of products

It is often useful to expand a power of a product.
For example, we can expand:

$$\begin{aligned}(3\times 4)^5 &= (3\times 4)\times(3\times 4)\times(3\times 4)\times(3\times 4)\times(3\times 4)\\ &= (3\times 3\times 3\times 3\times 3)\times(4\times 4\times 4\times 4\times 4)\\ &= 3^5 \times 4^5\end{aligned}$$

Index law 4

A power of a product is the product of the powers.

$(a\times b)^m = a^m \times b^m$

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Example 6

a Expand the brackets in $(5\times4)^2$ and write the answer as a product of powers. Then calculate the answer.

b Use the above index law to evaluate $5^4\times2^4$.

Solution

a $$\begin{aligned}(5\times4)^2 &= 5^2\times4^2\\ &= 25\times16\\ &= 400\end{aligned}$$

b $$\begin{aligned}5^4\times2^4 &= (5\times2)^4\\ &= 10^4\\ &= 10\,000\end{aligned}$$

Index law 5: Powers of quotients

The brackets in a power of a quotient can also be expanded.

$$\begin{aligned}\left(\frac{2}{3}\right)^5 &= \frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}\\ &= \frac{2\times2\times2\times2\times2}{3\times3\times3\times3\times3}\\ &= \frac{2^5}{3^5}\end{aligned}$$

Index law 5

A power of a quotient is the quotient of the powers.

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

Example 7

Evaluate $\dfrac{20^3}{5^3}$.

Solution

$$\begin{aligned}\frac{20^3}{5^3} &= \left(\frac{20}{5}\right)^3\\ &= 4^3\\ &= 64\end{aligned}$$

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Indices

Index law 1

To multiply powers of the same base, add the indices.

$a^m \times a^n = a^{m+n}$

Index law 2

To divide powers of the same base, subtract the indices.

$a^m \div a^n = a^{m-n}$

Index law 3

To raise a power to a power, multiply the indices.

$(a^m)^n = a^{m \times n}$

Index law 4

A power of a product is the product of the powers.

$(a \times b)^m = a^m \times b^m$

Index law 5

A power of a quotient is the quotient of the powers.

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

Zero index

Clearly $\frac{4^3}{4^3} = 1$. If the index laws are to apply then $4^{3-3} = 4^0 = 1$. Hence, we define $a^0 = 1$, for all non-zero a.

For example:

$2^0 = 1, 5^0 = 1, 2 \times 6^0 = 2s$ and $(2 \times 6)^0 = 1$.

Exercise 3B

1 Copy and complete this sentence.

'The expression 5^3 is a ________, in which 5 is the ________ and 3 is the ________.'

2 Evaluate:

a 4^2 **b** 9^2

c 12^2 **d** 100^2

e 2000^2 **f** 25^2

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3 **a** Write down three powers of base 7.

b Write down three powers with index 7.

c Write 49, 144 and 8 as powers with indices greater than 1.

d Write 64 as a power with:

i index 1

ii index 2

iii index 3

iv index 6

4 **a** Evaluate all the powers of 2 up to 2^{10}.

b Evaluate all the powers of 5 up to 5^4.

5 Simplify each of the following, using the index law given in brackets.

a $2^2 \times 2^3$ (index law 1)

b $3^2 \times 4^2 \times 3^2$ (index law 1)

c $3^3 \div 3^2$ (index law 2)

Example 3

6 Write each expression as a single power.

a $7^3 \times 7^2$ **b** $11^2 \times 11^4$ **c** $11^5 \times 11$

d $3^2 \times 3^7$ **e** $13^2 \times 13^5$ **f** $10^2 \times 10^3$

Example 4

7 Write each expression as a single power.

a $7^3 \div 7^2$ **b** $11^5 \div 11^2$ **c** $5^5 \div 5^2$

d $\dfrac{5^7}{5^2}$ **e** $\dfrac{7^{11}}{7^2}$ **f** $\dfrac{5^{12}}{5^7}$

Example 5

8 Write each expression as a single power.

a $(2^3)^2$ **b** $(3^5)^2$ **c** $(2^5)^2$

d $(11^2)^3$ **e** $(5^2)^3$ **f** $(7^2)^4$

9 Write each expression as a single power.

a $3^6 \times 3^7$ **b** $8^5 \times 8^2$ **c** $5^3 \times 5 \times 5^4$

d $10^{12} \times 10^{13} \times 10^{20}$ **e** $6^7 \div 6^3$ **f** $2^{12} \div 2^4$

g $7^{10} \div 7$ **h** $4^6 \times 4^5 \div 4^9$ **i** $(12^4)^3$

j $(11^7)^9$ **k** $(4^5)^2 \times 4^6$ **l** $(10^4)^6 \div 10^{20}$

Example 6

10 Evaluate:

a $2^4 \times 5^4$ **b** $4^3 \times 5^3$ **c** $2^5 \times 5^3$

11 Evaluate:

a 5^0 **b** 7^0 **c** 2×6^0

d $(2 \times 6)^0$ **e** 7×11^0 **f** $6^2 \times 6^0$

g $\dfrac{7^{11}}{7^{10} \times 7^0}$ **h** $(6^7)^0$ **i** $(5^0)^6$

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3C Order of operations

Brackets are used to control the order in which operations are done. For example:

a $(3+4)\times 5 = 7\times 5$
$= 35$

b $(5\times 4)^3 = 20^3$
$= 8000$

c $20-(8+3) = 20-11$
$= 9$

However, when there are no brackets, we need a set of conventions, known as the **order of operations**, to control the order in which operations are done.

Order of operations

- Evaluate expressions inside brackets first.
- In the absence of brackets, and within brackets, carry out operations in the following order:
  - indices
  - multiplication and division from left to right
  - addition and subtraction from left to right.

In the following examples, the brackets have been removed from the expressions **a** to **c** above.

The order of operations conventions mean that the operations are done in different order, and the answers are quite different. We can now see the effect of removing the brackets from **a**, **b** and **c**.

a $3+4\times 5 = 3+20$
$= 23$ (Do multiplication before addition.)

b $5\times 4^3 = 5\times 64$
$= 320$ (Calculate the power first.)

c $20-8+3 = 12+3$
$= 15$ (Do addition and subtraction from left to right.)

Example 8

Evaluate:

a $3\times(6+5)$ **b** $17+3\times 2$ **c** 7×3^2 **d** $(50+10)^2$

Solution

a $3\times(6+5) = 3\times 11$
$= 33$

b $17+3\times 2 = 17+6$
$= 23$

c $7\times 3^2 = 7\times 9$
$= 63$

d $(50+10)^2 = 60^2$
$= 3600$

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Example 9

Evaluate:

a $3^3 \times (2^2 + 12) \div 6 - 4^2$

b $3^3 \times 2^2 + 12 \div (6-4)^2$

Solution

a
$$\begin{aligned} 3^3 \times (2^2 + 12) \div 6 - 4^2 &= 27 \times (4 + 12) \div 6 - 16 \\ &= 27 \times 16 \div 6 - 16 \\ &= 432 \div 6 - 16 \\ &= 72 - 16 \\ &= 56 \end{aligned}$$

b
$$\begin{aligned} 3^3 \times 2^2 + 12 \div (6-4)^2 &= 27 \times 4 + 12 \div 4 \\ &= 108 + 3 \\ &= 111 \end{aligned}$$

Exercise 3C

Example 8

1 Use the order of operations conventions to evaluate these expressions.

a $3 \times (5 + 11)$ **b** $3 \times 5 + 11$ **c** $(20 - 8) \times 2$

d $20 - 8 \times 2$ **e** $(9 \times 2)^3$ **f** 9×2^3

g $(20 \div 2)^2$ **h** $20 \div 2^2$ **i** $(11 - 1)^4$

j $11 - 1^4$ **k** $(90 + 10)^2$ **l** $90 + 10^2$

2 Evaluate these expressions. You will need to use the 'work from left to right' convention.

a $40 - (17 - 12)$ **b** $40 - 17 - 12$ **c** $52 - (16 + 12)$

d $52 - 16 + 12$ **e** $(44 - 20) - (12 - 8)$ **f** $44 - 20 - 12 - 8$

g $(60 + 25) - (20 + 15)$ **h** $60 + 25 - 20 + 15$ **i** $48 \div (12 \times 2)$

j $48 \div 12 \times 2$ **k** $90 \div (6 \div 3)$ **l** $90 \div 6 \div 3$

Example 9

3 Evaluate:

a $6^2 + 20 \div 2^2 + 15$ **b** $4^3 + 6 \times 8 + 5^2$ **c** $2^3 + (2^2 - 2) \times 2^4$

d $2^2 \times 2^2 + 2 \div 2 - 2^2$ **e** $2^3 \times 2^3 + 2 \div 2 - 2^3$ **f** $2^3 \times (2^3 + 2) \div 2 - 2^3$

g $18 \div 6 - 5 + 3^2 \times 4^3$ **h** $7^3 \times 2^2 + 12$ **i** $\dfrac{3^3 \times 4^2}{2 \times 3^2}$

4 Copy these statements, inserting brackets if necessary to make them true.

a $4 \times 3 + 7 = 40$ **b** $70 - 20 \div 5 = 10$ **c** $2^2 + 6 \times 4 \div 2^2 = 7$

d $4 + 3^2 - 2 = 11$ **e** $3 + 2^2 \times 2^3 - 7^2 = 7$ **f** $4 + 4^4 \div 2^3 + 2 = 26$

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5 Copy these statements, inserting brackets if necessary to make them true.

a $4 \times 2 + 7 = 36$
b $4 \times 2 + 7 = 15$
c $20 - 2 + 2 - 2 + 2 = 12$
d $20 - 2 + 2 - 2 + 2 = 16$
e $70 - 20 \div 5 = 10$
f $70 - 20 \div 5 = 66$
g $2^2 + 2 \times 8 \div 2^2 = 12$
h $2^2 + 2 \times 8 \div 2^2 = 8$
i $2^2 + 2 \times 8 \div 2^2 = 144$
j $11 - 2 + 3^2 = 144$
k $11 - 2 + 3^2 = 18$
l $11 - 2 + 3^2 = 0$

3D Divisibility tests

We have already seen that a number is divisible by another number if the second number divides it exactly, with no remainder.

Sometimes it is easy to see that one number divides another exactly. For example, 2 divides 12 because $12 = 2 \times 6$.

But what about situations where it is not so easy to see if one number is divisible by another? Sometimes it is helpful to know about divisibility without having to use the division algorithm.

Summary of the divisibility tests

Number	Divisibility test	Examples	
2	Last digit is divisible by 2	12 876 yes	12 877 no
3	Sum of digits is divisible by 3	381 yes	382 no
4	The number consisting of last two digits is divisible by 4	3124 yes	3126 no
5	Last digit is 0 or 5	1245 yes	1246 no
6	Divisible by 2 and 3	3336 yes	3338 no
8	The number consisting of last three digits is divisible by 8	3016 yes	3014 no
9	Sum of digits is divisible by 9	3339 yes	9993 no
10	Last digit is 0	9990 yes	9909 no

We will now look in greater detail why the tests work.

Divisibility tests for 2, 10 and 5

A number that ends in a zero is divisible by 10. For example, 20, 450 and 23 890 are all divisible by 10. This can be seen in the counting pattern:

$$10, 20, 30, 40, \ldots, 670, 680, 690, 700, \ldots$$

Numbers that do not end in 0 are not divisible by 10.

The numbers that are divisible by 2 – the even numbers – can be listed by counting by 2s:

$$0, 2, 4, 6, 8, 10, \ldots, 452, 454, 456, \ldots$$

It is easy to see why this test for divisibility by 2 works.

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For example, we write 352 as a multiple of 10 plus its last digit: $35 \times 10 + 2$. Since we know that any multiple of 10 is divisible by 2, then we only need to look at the last digit. If this last digit is divisible by 2, then the entire number is divisible by 2. If the last digit of a number is not even, then the number is not divisible by 2.

If the last digit of a number is 5, then it is divisible by 5. For example, 15, 225 and 439 785 are all divisible by 5. Also, it follows that a number divisible by 10 is also divisible by 5.

So any number that ends in 0 or 5 is also divisible by 5. If the last digit of a number is not 0 or 5, then the number is not divisible by 5. The counting pattern shows the numbers that are divisible by 5:

$$5, 10, 15, 20, 25, \ldots, 985, 990, 995, 1000, \ldots$$

Divisibility tests for 3 and 9

A number is divisible by 3 if the sum of its digits is divisible by 3. Any number whose digit sum is not divisible by 3 is not itself divisible by 3. For example, 39 is divisible by 3 because the sum of its digits is equal to $3 + 9 = 12$, and 12 is divisible by 3.

A number is divisible by 9 if the sum of its digits is divisible by 9. Any number whose digit sum is not divisible by 9 is not itself divisible by 9. For example, 13 968 is divisible by 9 because the sum of its digits is equal to $1 + 3 + 9 + 6 + 8 = 27$ and 27 is divisible by 9.

The proof of why the test for divisibility by 9 works is illustrated for 684 in the example below.

In expanded form, 684 can be written as:

$$684 = 100 \times 6 + 10 \times 8 + 4$$

We know that $100 = 99 + 1$ and $10 = 9 + 1$, so we can rewrite the above expression as:

$$\begin{aligned} 684 &= (99 \times 6 + 1 \times 6) + (9 \times 8 + 1 \times 8) + 4 \\ &= 99 \times 6 + 6 + 9 \times 8 + 8 + 4 \\ &= (99 \times 6 + 9 \times 8) + 6 + 8 + 4 \end{aligned}$$

Since 9 divides 99, 9 divides $(99 \times 6 + 9 \times 8)$. We are left with $6 + 8 + 4$ (the sum of the digits), which is equal to $6 + 8 + 4 = 18$. This is divisible by 9, so the entire number is divisible by 9.

Note also that any number that is divisible by 9 is also divisible by 3, so 684 is divisible by both 9 and 3. The proof of the test for divisibility by 3 is essentially the same as that for divisibility by 9.

Divisibility test for 6

A number is divisible by 6 if it is both even and divisible by 3. This is because we know that $2 \times 3 = 6$. It follows that if a number is divisible by 3 and it is even, then it is divisible by 6.

Divisibility tests for 4 and 8

A number is divisible by 4 if the number formed by the last two digits is divisible by 4. Conversely, if the number formed by the last two digits of a number is not divisible by 4, then the number itself is not divisible by 4.

For example, 124 is divisible by 4 because the number formed by its last two digits is 24, which is divisible by 4.

The reason why the test for divisibility by 4 works is illustrated in the following example.

$$\begin{aligned} 324 &= 300 + 24 \\ &= (3 \times 100) + 24 \end{aligned}$$

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Since we know that 100 is divisible by 4, we are left with 24. Since 24 is divisible by 4, then 324 is divisible by 4.

A number is divisible by 8 if the number formed by the last three digits is divisible by 8. Conversely, if the number formed by the last 3 digits of a number is not divisible by 8, then the number itself is not divisible by 8.

For example, 5472 is divisible by 8 because 472 is divisible by 8. Since 8 divides 1000, 8 divides any multiple of 1000, so we only need to deal with the last three digits.

Divisibility test for 11

To test a number for divisibility by 11, we follow this procedure. Add up the digits in the odd positions, then add up the digits in the even positions. If the difference of these two numbers is divisible by 11, then the given number is divisible by 11.

For example, 82 786 is divisible by 11 because:

$$\begin{aligned}(8+7+6)-(2+8) &= 21-10\\ &= 11\end{aligned}$$

Example 10

Is 1848 divisible by 2, 3, 4, 5, 9 and 11?

Solution

1848 is divisible by 2 as it ends in 8.

The sum of the digits is $1+8+4+8=21$. Since 21 is divisible by 3, 1848 is divisible by 3.

Since the number formed from the last two digits (48) is divisible by 4, the given number is divisible by 4.

1848 does not end in 5 so it is not divisible by 5.

The sum of the digits is $1+8+4+8=21$. Since 21 is not divisible by 9, 1848 is not divisible by 9.

The sum of the odd-position digits minus the sum of the even-position digits is $(8+8)-(1+4)=16-5=11$. This is divisible by 11, so 1848 is also divisible by 11.

Exercise 3D

Example 10

1 Test each number for divisibility by 2, 4 and 8.

a 2896 **b** 56 374 **c** 1 858 732 **d** 280 082

2 Test each number for divisibility by 3, 9 and 11.

a 5679 **b** 7425 **c** 71 643 **d** 1727

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3 Test each number for divisibility by 6 and 12.

(A number is divisible by 12 if it is divisible by both 4 and 3, because $12 = 4 \times 3$.)

a 3798 b 5772 c 9909 d 48 882

4 Test each number for divisibility by 2, 3, 4, 5, 6, 8, 9, 10, 11 and 12.

a 672 b 49 395

c 136 290 d 242 010 000 437 000 361

5 Fill in the boxes to make a six-digit number 3__1__3__ divisible by:

a 3 b 5 c 3 and 5

d 2, 4 and 9 e 3, but not 9 f 11

g 4 and 11 h 6 and 8, but not 9 i 3, but not 6

6 List the other numbers by which a number is divisible if it is divisible by:

a 9 b 2 and 3 c 2 and 5 d 8

7 Test each number for divisibility by 12, 15 and 18.
(A number is divisible by 15 if it is divisible by both 3 and 5, because $15 = 3 \times 5$. A number is divisible by 12 if it is divisible by both 4 and 3, because $12 = 4 \times 3$. A number is divisible by 18 if it is divisible by both 2 and 9, because $18 = 2 \times 9$.)

a 36 450 b 21 942 c 2 041 200 d 2 007 000 000

8 Using only digits 3 and 2 write down the smallest number divisible by 6.

9 Using only digits 1 and 2, write down the smallest number divisible by 9.

3E Prime factorisation and its applications

Any whole number can be broken down, by factorisation, into a product of prime numbers. This means that prime numbers are the building blocks from which all whole numbers greater than 1 are built up by multiplication. For example:

$$\begin{aligned} 12 &= 3 \times 4 \\ &= 3 \times 2 \times 2 \\ &= 2^2 \times 3 \end{aligned} \qquad \text{or} \qquad \begin{aligned} 12 &= 6 \times 2 \\ &= 2 \times 3 \times 2 \\ &= 2^2 \times 3 \end{aligned}$$

We have just found the **prime factorisation** of 12 in two different ways. The prime factorisation of 12 is $2^2 \times 3$.

Factorisation into primes always gives the same answer, apart from the order of the factors. This fact is called the **fundamental theorem of arithmetic**. The final answer is usually written using index notation, with the primes placed in increasing order.

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Here are two of the many ways to carry out the factorisation of 420 into primes.

$$\begin{aligned} 420 &= 2\times 210 \\ &= 2\times 21\times 10 \\ &= 2\times 3\times 7\times 2\times 5 \\ &= 2^2\times 3\times 5\times 7 \end{aligned} \qquad \text{or} \qquad \begin{aligned} 420 &= 60\times 7 \\ &= 6\times 10\times 7 \\ &= 2\times 3\times 2\times 5\times 7 \\ &= 2^2\times 3\times 5\times 7 \end{aligned}$$

The example below shows another standard way to set out these calculations.

Example 11

Express 1260 as the product of its prime factors.

Solution

$$\begin{array}{r|r} 2 & 1260 \\ \hline 2 & 630 \\ \hline 3 & 315 \\ \hline 3 & 105 \\ \hline 5 & 35 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$$\begin{aligned} \text{Hence, } 1260 &= 2\times 3\times 5\times 7\times 3\times 2 \\ &= 2\times 2\times 3\times 3\times 5\times 7 \\ &= 2^2\times 3^2\times 5\times 7 \end{aligned}$$

Using prime factorisation to find the LCM

At the beginning of this chapter, we found the lowest common multiple (LCM) of two numbers by writing down lists of the multiples of each number. For larger numbers this is not very practical – prime factorisation provides a better method.

To find the LCM of 24 and 18, we first factorise each number as a product of primes:

$$\begin{aligned} 24 &= 2\times 2\times 2\times 3 \\ &= 2^3\times 3 \end{aligned} \qquad \begin{aligned} 18 &= 2\times 3\times 3 \\ &= 2\times 3^2 \end{aligned}$$

Both numbers have powers of 2 as a factor. The larger, 2^3, must be taken as a factor of the LCM, otherwise the LCM could not be a multiple of 24. Similarly, the larger power of 3 is 3^2. It must be taken as a factor of the LCM because the LCM must be a multiple of 18. So the LCM of 24 and 18 is:

$$\begin{aligned} 2^3\times 3^2 &= 8\times 9 \\ &= 72 \end{aligned}$$

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Example 12

What is the LCM of 180 and 144?

Solution

Factorise each number into a product of primes.

$$180 = 2 \times 2 \times 3 \times 3 \times 5 \qquad 144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$$
$$= 2^2 \times 3^2 \times 5 \qquad = 2^4 \times 3^2$$

Take the greatest power of each prime.

The LCM of 180 and 144 is:

$$2^4 \times 3^2 \times 5^1 = 16 \times 9 \times 5$$
$$= 720$$

Using prime factorisation to find the HCF

Prime factorisation provides an effective method for finding the highest common factor (HCF) of two or more numbers.

To find the HCF of 24 and 18, we again factorise each number into primes:

$$24 = 2 \times 2 \times 2 \times 3 \qquad 18 = 2 \times 3 \times 3$$
$$= 2^3 \times 3^1 \qquad = 2^1 \times 3^2$$

Each number contains a power of 2 and a power of 3 in its prime factorisation.

To get the highest common factor of 24 and 18, we take the lower power of 2, which is $2 = 2^1$, and multiply it by the lower power of 3, which is $3 = 3^1$. No higher power than 3^1 can divide the HCF. No higher power than 2^1 can divide the HCF. The HCF of 24 and 18 is therefore $2 \times 3 = 6$.

The reason this works is that we cannot take any higher power of 2 and still have a factor of 18, nor can we take a higher power of 3 and still have a factor of 24.

Example 13

What is the HCF of 180 and 144?

Solution

Factorise both numbers into primes.

$$180 = 2 \times 2 \times 3 \times 3 \times 5 \qquad 144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$$
$$= 2^2 \times 3^2 \times 5^1 \qquad = 2^4 \times 3^2$$

Take the smallest index of each prime common factor.

The HCF of 180 and 144 is $2^2 \times 3^2 = 36$.

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Factors and divisibility

- The **fundamental theorem of arithmetic** states that every whole number greater than 1 can be expressed as a product of prime numbers. This product for each number is unique apart from the order of the prime factors.
- Prime factorisation can be used to find the lowest common multiple. Take the *greatest* power for each prime.
- Prime factorisation can be used to find the highest common factor. Take the *smallest* power for each prime.

Squares and square roots

The **square** of a number is the product of that number with itself. For example, 8 squared is the number $8 \times 8 = 64$. The number 64 is called a **perfect square**, because it is the square of a whole number.

The **square root** of a number is the number that when multiplied by itself gives the first number. For example, $8^2 = 64$, so 8 is the square root of 64.

We write $\sqrt{64} = 8$.

Cubes and cube roots

The **cube** of a number is defined similarly. So the cube of 4 is $4 \times 4 \times 4 = 4^3 = 64$.

The **cube root** of 64 is 4 because $4^3 = 64$, and we write $\sqrt[3]{64} = 4$.

The cube root of 125 is 5, because the cube of 5 is 125. We write $\sqrt[3]{125} = 5$, because $5^3 = 125$.

We can see that taking the cube root is the reverse operation to cubing.

Note that most numbers do not have a square or cube root that is a whole number. For example:

- Neither $\sqrt{2}$ nor $\sqrt[3]{2}$ is a whole number.
- $\sqrt{9} = 3$ because $3^2 = 9$, but $\sqrt[3]{9}$ is not a whole number.
- $\sqrt{8}$ is not a whole number, but $\sqrt[3]{8} = 2$ because $2^3 = 8$.

Example 14

What is the cube root of 1000?

Solution

$$\begin{aligned} 1000 &= 10 \times 10 \times 10 \\ &= 10^3 \end{aligned}$$

$$\sqrt[3]{1000} = 10$$

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Finding square roots and cube roots by prime factorisation

Prime factorisation can help us find the square root of a perfect square. For larger numbers, prime factorisation is the best way of finding their square roots and cube roots, if these are whole numbers.

For example, here is the prime factorisation of 324.

$$\begin{aligned}324 &= 4\times 81\\ &= 2\times 2\times 9\times 9\\ &= 2\times 2\times 3\times 3\times 3\times 3\\ &= 2^2\times 3^4\end{aligned}$$

Since both indices are even, we can use the index laws to write 324 as the square of a whole number.

$$\begin{aligned}324 &= 2^2\times 3^4\\ &= (2^1\times 3^2)^2 \quad \text{(index laws 3 and 4)}\end{aligned}$$

Since the square of $2^1\times 3^2$ is 324, $\sqrt{324} = 2\times 3^2 = 18$.

However, the indices are not multiples of 3, so $\sqrt[3]{324}$ is not a whole number.

Example 15

Find the square root of 7056, using the prime factorisation of 7056.

Solution

$$\begin{array}{r|r}7 & 7056\\ \hline 7 & 1008\\ \hline 3 & 144\\ \hline 3 & 48\\ \hline 2 & 16\\ \hline 2 & 8\\ \hline 2 & 4\\ \hline 2 & 2\\ \hline & 1\\ \hline\end{array}$$

$$\begin{aligned}\text{Hence, } 7056 &= 2^4\times 3^2\times 7^2\\ &= (2^2\times 3\times 7)^2\end{aligned}$$

Therefore, 7056 is a square number and $\sqrt{7056} = 2^2\times 3\times 7 = 84$.

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It is not necessary to factor one prime at a time, as the following example shows.

Example 16

Find $\sqrt[3]{1728}$.

Solution

The prime factorisation of 1728 is:

$$\begin{aligned}1728 &= 4 \times 432\\ &= 4 \times 4 \times 108\\ &= 4 \times 4 \times 4 \times 27\\ &= (2^2)^3 \times 3^3\\ &= (2^2 \times 3)^3\end{aligned}$$

so $\sqrt[3]{1728} = 2^2 \times 3 = 12$

Note that we could not calculate $\sqrt{1728}$ in this way because not all the indices are even, so $\sqrt{1728}$ is not a whole number.

Perfect squares and square roots

- A number multiplied by itself gives the **square** of the original number.
- The **square root** of a square number is the number that when multiplied by itself gives the original number.
- The square of a whole number is called a **perfect square**.
- We can use prime factorisation to determine whether a whole number is a square and to find the square root of a number.

Cubes and cube roots

- The **cube** of a number a is the product $a \times a \times a = a^3$.
- Taking the **cube root** is the inverse operation of cubing.
- We can use prime factorisation to determine whether a whole number is a cube and to find the cube root of a number.

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Exercise 3E

Example 11

1 Find the prime factorisation of:

a 8 b 24 c 45

d 891 e 735 f 2904

Example 12, 13

2 Given the prime factorisations $144 = 2^4 \times 3^2$, $108 = 2^2 \times 3^3$ and $405 = 3^4 \times 5$. Find the LCM and HCF of:

a 144 and 108 b 108 and 405 c 108, 144 and 405

3 Use prime factorisation to find:

a the HCF of 36 and 45

b the LCM of 36 and 45

c the HCF of 112 and 21

d the LCM of 112 and 21

e the LCM of 12, 15 and 40

f the LCM of 24, 72 and 108

g the HCF of 72 and 126

h the HCF of 24, 60 and 112

i the HCF of 70, 105 and 280

Example 14, 15

4 Given the prime factorisation, find the square root and the cube root of each number below, if they are whole numbers.

a $3375 = 3^3 \times 5^3$ b $729 = 3^6$ c $4356 = 2^2 \times 3^2 \times 11^2$

Example 15, 16

5 Find the prime factorisation of each number below. Then find its square root and cube root, if they are whole numbers.

a 225 b 324

c 512 d 1728

e 360 f 1936

g 216 h 10 648

6 Mr and Mrs Cinco have quintuplets, three boys and two girls. When buying toys, to avoid squabbles, they always make sure that the toy collection can be shared equally whether the five children play together, the girls play alone or the boys play alone.

At the beginning of the year, the number of toys in the collection is 180. What is the smallest number of toys they can buy for the children's birthday so that the family can continue sharing the toys in the same way as before?

7 Find the prime factorisation of 360, and use it to write down all 24 factors of 360.

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Review exercise

1 The product of the factors of 12 is:

A 12 **B** 28 **C** 1728 **D** 144 **E** 3556

2 The sum of the factors of 18 is:

A 9 **B** 36 **C** 5832 **D** 39 **E** 2916

3 Write each of these numbers as the sum of two multiples of 8.

a 96 **b** 40 **c** 184 **d** 480 **e** 296

4 Find the LCMs of the following sets of numbers.

a 8 and 6 **b** 10 and 18 **c** 16 and 24

d 60 and 72 **e** 6, 10 and 15 **f** 10, 12 and 18

g 36, 45 and 27 **h** 12, 60 and 102 **i** 30, 40, 50 and 75

5 Find the HCFs of the following sets of numbers.

a 6 and 9 **b** 45 and 75 **c** 12 and 35

d 60 and 70 **e** 21 and 35 **f** 132 and 680

g 12, 18 and 30 **h** 18, 30 and 90 **i** 42, 189 and 231

6 Evaluate:

a 6^2 **b** 11^2 **c** 50^2

d 31^2 **e** 87^2 **f** 143^2

7 Write each expression as a single power.

a $8^2 \times 8^4$ **b** $2^{13} \div 2^9$ **c** $3^5 \times 3^5 \times 3^5$

d $4^9 \times 4^{19}$ **e** $10^3 \times 10^2 \times 10^3 \times 10^4$ **f** $7^{11} \div 7^2 \times 7^8$

g $(5^2)^3$ **h** $(8^2)^3 \div 8^2$ **i** $12^4 \times (12^5)^2$

8 **a** The LCM of $2^2 \times 3^1 \times 5^4$ and $2^3 \times 3^2 \times 7^2$ is:

A $2 \times 5^4 \times 7^2$ **B** $2^3 \times 3^2 \times 5^4 \times 7^2$ **C** $2^2 \times 3^1 \times 5^1 \times 7^1$

b The HCF of $2^2 \times 3^1 \times 5^4$ and $2^3 \times 3^2 \times 7^2$ is:

A 75 **B** 3528 **C** 12 **D** 210 **E** $2^3 \times 3^2$

9 Standing on the seashore, Stephanie can see two ships' lights flashing. One light flashes at intervals of 12 seconds and the other at intervals of 18 seconds.

a If they flash together once, how long is it before they flash together again?

b If they flash together at the start of a 5-minute interval, how many times will they flash together in those 5 minutes?

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10 Show that each of the whole numbers between 28 and 37 can be written as the product of prime numbers.

11 Express each of these numbers as a product of its prime factors and so find its square root.

a 256 **b** 441 **c** 1156

d 7569 **e** 18 225 **f** 20 736

12 Write each prime number as the sum of two square numbers.

a 73 **b** 137 **c** 229

13 Test each of the following numbers for divisibility by 3, by 4 and by 12.

a 64 **b** 1336 **c** 3972

14 Write a five-digit number that has one of its digits repeated three times and is:

a divisible by 2 **b** divisible by 5 and 2

c divisible by 3 and 4 **d** divisible by 11

e divisible by 8 and 3 **f** divisible by 2 and 11 but not 4

15 Evaluate:

a $3\times 5^2+6^2-2^3$ **b** $(3\times 5^2+6)^2-2^3$ **c** $(3\times 5)^2+(6^2-2)^3$

d $[(3\times 5)^2+6]^2-2^3$ **e** $3\times(5^2+6^2-2)^3$ **f** $(3\times 5^2-2\times 6^2-2)^3$

16 A yellow bulb flashes every 4 minutes and a blue bulb flashes every 7 minutes. If both bulbs flash together at 9 a.m., what is the first time after 10 a.m. that both bulbs will flash together?

17 How many factors do each of the following have?

a 2^3 **b** 5^3 **c** 7^3 **d** 11^3

18 How many factors do each of the following have?

a 2^5 **b** 3^5 **c** 7^5 **d** 11^5

19 Write down the factors of:

a 111 **b** 1001 **c** 1111

20 Find the highest common factor of each of the following pairs.

a $2^3\times 3^4$, $2^2\times 3^6\times 5$ **b** $2^4\times 3\times 5$, $2^3\times 3^2\times 5^2$

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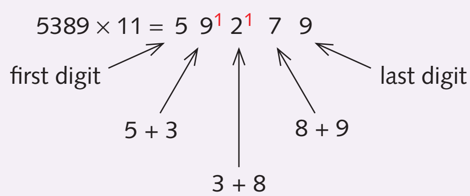
Challenge exercise

1 A number is greater than 48 squared and less than 49 squared. If 6 squared is one of its factors, and it is a multiple of 13, what is the number?

2 Given a six-digit number 328 xyz, find the values of x, y and z such that the number is divisible by 3, 4 and 5, and is the smallest six-digit number starting with 328 that has this property. How many other numbers have this property?

3 When multiplying a number by 11, we can use the following technique. First, write down the last digit of the number to be multiplied. Then, *working from right to left*, write down the sum of every pair of adjacent numbers. Remember to carry correctly. Finally, write down the first digit of the original number. For example:



a Write down, using the above method:

i $17\,089 \times 11$ **ii** 2223×11

iii $1\,654\,998 \times 11$ **iv** $1\,654\,998 \times 111$

b Briefly explain why this method works.

4 What three-digit number leaves a remainder of 1 when divided by 2, 3, 4, 5 or 6, and no remainder when divided by 7?

5 The local takeaway chicken shop has all of its employees working the same number of hours each day. The total number of hours for Thursday is 133. How many employees work on Thursdays?

6 Hoc knew that before he closed his novel, the product of the page numbers was 1190. What were the two open pages?

7 It is 7 p.m. now. What will the time be 23 999 999 996 hours later?

8 What is the smallest whole number that is divisible by 11 and leaves a remainder of 1 when divided by any of the numbers from 2 to 10?

9 What is the largest prime factor of 93 093?

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10 The product of two whole numbers is 10 000 000. Neither number is a multiple of 10. What are the two numbers?

11 A teacher multiplied the ages of all of the students (all teenagers) in her class and came up with the number 15 231 236 267 520.

a What is the prime factorisation of this number?

b Use the prime factorisation to help you work out the number of students in the class, their ages and the number of students of each age.

12 Twelve is the smallest whole number that has exactly 6 different factors. What is the smallest whole number that has exactly 24 different factors?

13 **a** What is the smallest whole number that satisfies all of the following conditions?

- Dividing by 7 gives a remainder of 4.
- Dividing by 8 gives a remainder of 5.
- Dividing by 9 gives a remainder of 6.

b What is the smallest whole number that satisfies the conditions in part **a** and also the following ones?

- Dividing by 6 gives a remainder of 3.
- Dividing by 11 gives a remainder of 8.

14 **a** If the four-digit number $8mn9$ is a perfect square, find the values of m and n.

b The digits 1, 2, 3, 4 and 5 are each used once in the five-digit number *abcde* such that the three-digit number *abc* is divisible by 4, *bcd* is divisible by 5, and *cde* is divisible by 3. Find the values of a, b, c, d and e.

c The digits 1, 2, 3, 4, 5 and 6 are each used in the six-digit number $abcdef$. The three-digit number *abc* is divisible by 4, *bcd* is divisible by 5, *cde* is divisible by 3, and *def* is divisible by 11. Find the values of a, b, c, d, e and f.

15 Helena has a basket of chocolates. There are fewer than 500 chocolates in the basket. Helena knows that she can divide all of the chocolates into bags holding 2 each, 3 each or 6 each. She also knows that it is not possible to put them into bags of 4, 5, 9 or 10. If she tries to put 7 in each bag, there are 3 left over. How many chocolates are in Helena's basket?

16 What is the next number after 64 whose square roots and cube roots are both whole numbers?

17 Find the smallest positive whole number m such that $60m$ is a perfect square.

18 Find the smallest positive whole number whose digits are all 0s and 1s that is divisible by 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

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CHAPTER

4

Number and Algebra

Negative numbers

You have probably come across examples of negative numbers already. They are the numbers that are less than zero. Negative numbers are used, for example, in the measurement of temperature.

The temperature 0°C is the temperature at which water freezes, known as *freezing point*. The temperature that is 5 degrees colder than freezing point is written as –5°C.

In some Australian cities, the temperature drops below zero. Canberra has a lowest recorded temperature of –10°C. The lowest recorded temperature in Australia was –23°C, recorded at Charlotte's Pass in NSW. Here are some other lowest temperatures:

Alice Springs	–7°C	Paris	–24°C	London	–16°C

Negative numbers are also used to record heights below sea level.

For example, the surface of the Dead Sea in Israel is at 423 metres below sea level. This can also be written as –423 metres above sea level. This is the lowest point on land anywhere on Earth. The lowest point on land in Australia is at Lake Eyre, which is 15 metres below sea level. This can also be written as –15 metres above sea level.

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4A Negative integers

The whole numbers together with the negative whole numbers are called the **integers**.

…, –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, …

The numbers 1, 2, 3, 4, 5, … are called the **positive integers**.

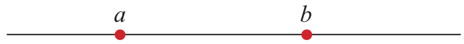
The numbers … , –5, –4, –3, –2, –1 are called the **negative integers**.

The number 0 is neither positive nor negative.

The number line

The integers can be represented by points on a number line. The line is infinite in both directions, with the positive integers to the right of zero and the negative integers to the left of zero. The numbers are equally spaced.

An integer a is **less than** another integer b if a lies to the left of b on the number line. The symbol $<$ is used for **less than**. For example, $-3 < -1$.



An integer b is **greater than** another integer a if b lies to the right of a on the number line. The symbol $>$ is used for **greater than**. For example, $1 > -5$.

A practical illustration of this is that a temperature of −10°C is colder than a temperature of −5°C, and $-10 < -5$. Similarly, $-5 < 0$.

Example 1

a List all the integers less than 5 and greater than −2.
b List all the integers less than 1 and greater than –4.

Solution

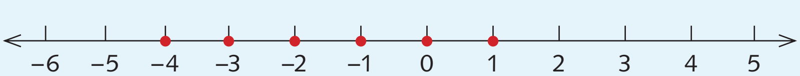
a −1, 0, 1, 2, 3, 4

b –3, –2, –1, 0

Example 2

Draw a number line and indicate on it with dots all the integers greater than −5 and less than 2.

Solution



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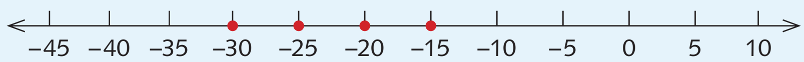
Example 3

a The sequence 10, 5, 0, –5, –10, ... is 'going down by fives'.
Write down the next four terms.

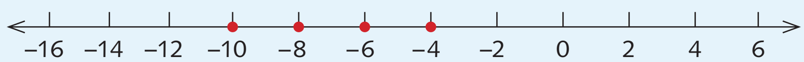
b The sequence –16, –14, –12, ... is 'going up by twos'.
Write down the next four terms.

Solution

a The next four terms are –15, –20, –25, –30.



b The next four terms are –10, –8, –6, –4.



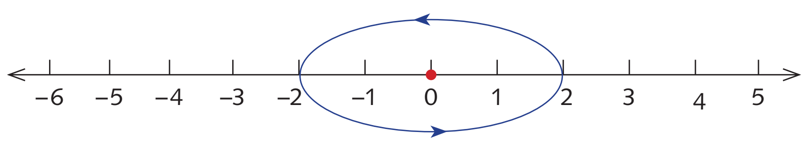
The opposite of an integer

We call –2 the number **opposite** to 2.

The integer –2 has opposite –(–2) = 2.

The operation of forming the opposite can be visualised by putting a pin in the number line at 0 and rotating the number line by 180°.

The opposite of 2 is –2.



The opposite of –2 is 2, so –(–2) = 2.

The number 0 is different from all of the other numbers. The opposite of 0 is 0.

The operation of taking the opposite can be visualised by swapping the number on one side of zero with the matching number the same distance from zero on the other side.

Exercise 4A

Example 1

1 a List the integers less than –16 and greater than –22.

b List the integers greater than –114 and less than –105.

c List the integers less than 3 and greater than –8.

Example 2

2 a Draw a number line and mark the numbers –12, –2, 2 and –8 on it.

b Draw a number line and mark the numbers –8, –3, –5 and 4 on it.

c Draw a number line and mark the integers less than 5 and greater than –6 on it.

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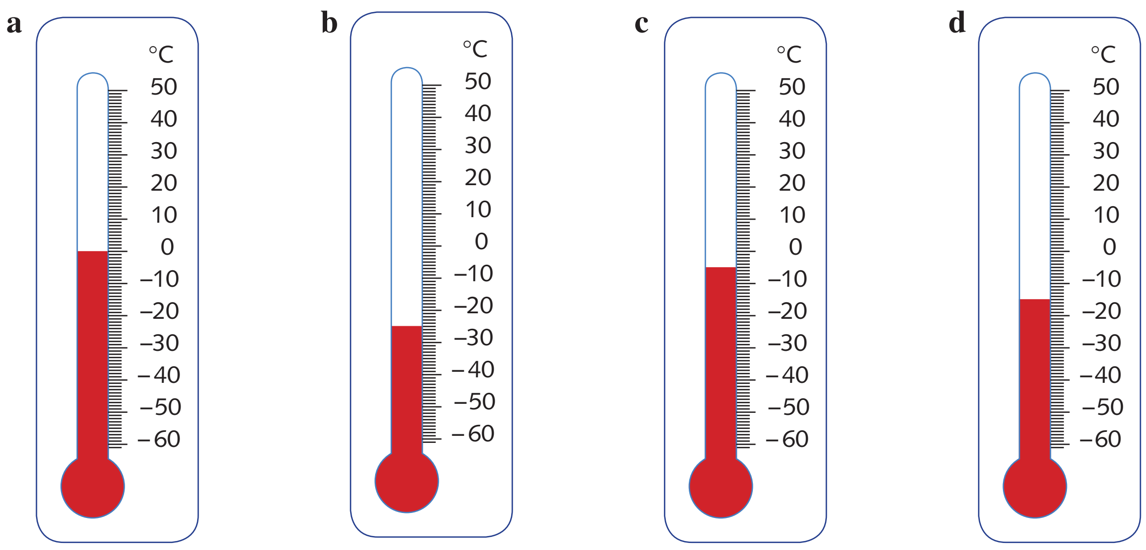
Example 3

3 The sequence –16, –13, –10, ... is 'going up by threes'. Find the next three terms.

4 The sequence 15, 11, 7, ... is 'going down by fours'. Find the next three terms.

5 The sequence –50, –43, –36, ... is 'going up by sevens'. Find the next four terms.

6 Determine the readings for each of the thermometers shown below.



7 Determine the opposites of these integers.

a 6	**b** –3	**c** 34	**d** –5
e –72	**f** 67	**g** –456	**h** 10 000

4B Addition and subtraction of a positive integer

If a submarine is at a depth of –250 m and then rises by 20 m, its final position is –230 m. This can be written as $-250 + 20 = -230$.

Joseph has \$3000 and he spends \$5000. He now has a debt of \$2000, so it is natural to interpret this as $3000 - 5000 = -2000$.

The number line and addition

The number line provides a useful picture for addition and subtraction of integers.

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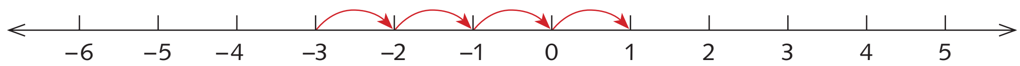
Addition of a positive integer

When you add a positive integer, move to the right along the number line.



For example, to calculate $-3+4$, start at -3 and move 4 steps to the right. We see that $-3+4=1$.

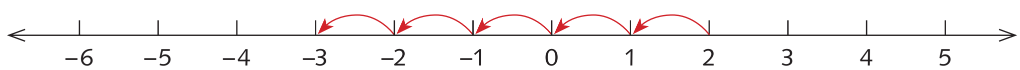
A practical example of this is: ‘I started with a debt of \$3 and I earned \$4. I now have \$1’.

Subtraction of a positive integer

We can think of subtraction as **taking away**.

When you subtract a positive integer, move to the left along the number line.

For example, to calculate $2-5$, start at 2 and move 5 steps to the left. We see that $2-5=-3$.



The same question arises in a practical way: ‘I had \$2 and I spent \$5. I now have a debt of \$3’.

Example 4

Calculate:

a $-6+9$ **b** $-1+7$ **c** $-2+9$ **d** $-9+2$

Solution

a $-6+9=3$ (Start at -6 on the number line and move 9 steps to the right.)
b $-1+7=6$ (Start at -1 on the number line and move 7 steps to the right.)
c $-2+9=7$ (Start at -2 on the number line and move 9 steps to the right.)
d $-9+2=-7$ (Start at -9 on the number line and move 2 steps to the right.)

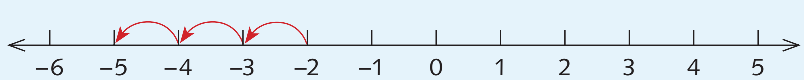
Example 5

Calculate:

a $-2-3$ **b** $2-8$

Solution

a



Start at -2 and move 3 steps to the left. We see that $-2-3=-5$.

b $2-8=-6$ (Start at 2 on the number line and move 8 steps to the left.)

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Exercise 4B

Example 4

1 Write the answers to these additions.

a $-5+4$ **b** $-7+11$ **c** $-6+2$
d $-11+15$ **e** $-4+9$ **f** $-7+11$
g $-1+10$ **h** $-3+10$ **i** $-15+7$
j $-11+11$ **k** $-17+10$ **l** $-13+4$
m $-16+16$ **n** $-70+33$ **o** $-60+99$
p $-21+15$ **q** $-75+12$ **r** $-96+6$
s $-345+600$ **t** $-23+89$

Example 5

2 Write the answers to these subtractions.

a $-5-6$ **b** $6-8$ **c** $1-9$
d $3-11$ **e** $7-12$ **f** $-2-8$
g $4-10$ **h** $2-5$ **i** $-3-4$
j $4-6$ **k** $-10-6$ **l** $-26-12$
m $-17-10$ **n** $-13-6$ **o** $14-20$
p $3-96$ **q** $54-59$ **r** $-66-30$
s $28-64$ **t** $12-21$ **u** $-100-28$
v $-136-144$

3 Write the answers to these additions and subtractions.

a $-450+50$ **b** $-500+80$
c $-100-70$ **d** $-315+430$
e $-420-45$ **f** $-300+500$
g $-500-700$ **h** $-1500+500$
i $-4000-600$ **j** $8000-10\,000$
k $4500-5000$ **l** $-5000-2500$

4 Work from left to right to calculate:

a $2-4+8$ **b** $4-10+20$
c $4-12+8$ **d** $23-30+17$
e $2+3-5$ **f** $3-7+11$
g $-3-7+12$ **h** $4-11-21$

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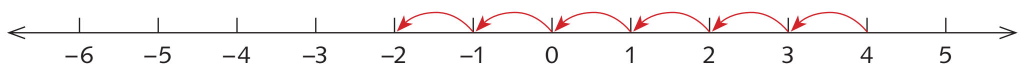
4C Addition and subtraction of a negative integer

In the previous section, we considered addition and subtraction of a positive integer. In this section, we will add and subtract negative integers.

Addition of a negative integer

Adding a negative integer to another integer means that you take a certain number of steps to the *left* on a number line.

The result of the addition $4+(-6)$ is the number you get by moving 6 steps to the left, starting at 4.



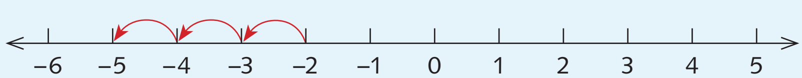
$4+(-6)=-2$

Note that $4+(-6)$ is the same as $4-6$.

Example 6

Calculate $-2+(-3)$.

Solution



$-2+(-3)$ is the number you get by moving 3 steps to the left, starting at -2. That is, -5.

Notice that $-2-3$ is also equal to -5.

All additions of this form can be completed in a similar way. For example:

$4+(-7)=-3$ and $4-7=-3$

$-11+(-3)=-14$ and $-11-3=-14$

This suggests the following rule.

To add a negative integer, subtract its opposite

For example:

$$4+(-10)=4-10 \\ =-6$$

$$-7+(-12)=-7-12 \\ =-19$$

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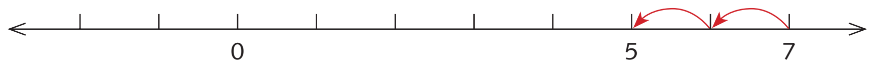
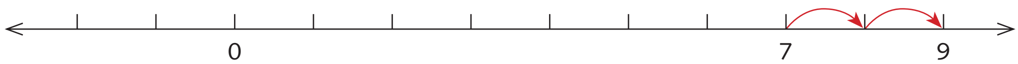
Subtraction of a negative integer

We have already seen that adding -2 means taking 2 steps to the left. For example:



$7 + (-2) = 5$

We want subtracting -2 to be the reverse process of adding -2, so to subtract -2, we will take 2 steps to the *right*. For example:



$7 - (-2) = 9$

This is a very simple way to state this rule:

To subtract a negative number, add its opposite

For example:

$$\begin{aligned} 7 - (-2) &= 7 + 2 \\ &= 9 \end{aligned}$$

Example 7

Evaluate:

a $20 + (-3)$ **b** $-3 + (-8)$ **c** $6 - (-20)$ **d** $-13 - (-8)$ **e** $-20 - (-3)$

Solution

a $\begin{aligned} 20 + (-3) &= 20 - 3 \\ &= 17 \end{aligned}$

b $\begin{aligned} -3 + (-8) &= -3 - 8 \\ &= -11 \end{aligned}$

c $\begin{aligned} 6 - (-20) &= 6 + 20 \\ &= 26 \end{aligned}$

d $\begin{aligned} -13 - (-8) &= -13 + 8 \\ &= -5 \end{aligned}$

e $\begin{aligned} -20 - (-3) &= -20 + 3 \\ &= -17 \end{aligned}$

Example 8

Calculate $6 - (-17)$.

Solution

$$\begin{aligned} 6 - (-17) &= 6 + 17 \\ &= 23 \end{aligned}$$

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Example 9

Calculate $-17 + (-6) - (-22)$.

Solution

$$\begin{aligned} -17 + (-6) - (-22) &= -17 - 6 + 22 \\ &= -23 + 22 \\ &= -1 \end{aligned}$$

Example 10

The minimum temperature on Saturday is −12°C and the maximum temperature is −1°C. Calculate the difference (maximum temperature − minimum temperature).

Solution

$$\begin{aligned} \text{maximum temperature} - \text{minimum temperature} &= -1 - (-12) \\ &= -1 + 12 \\ &= 11^\circ\text{C} \end{aligned}$$

Example 11

Evaluate:

a $347 + (-625)$

b $456 - (-356)$

c $-234 + 568$

d $-120 - (-105)$

Solution

a $$\begin{aligned} 347 + (-625) &= 347 - 625 \\ &= -(625 - 347) \\ &= -278 \end{aligned}$$

b $$\begin{aligned} 456 - (-356) &= 456 + 356 \\ &= 812 \end{aligned}$$

c $$\begin{aligned} -234 + 568 &= 568 + (-234) \\ &= 568 - 234 \\ &= 334 \end{aligned}$$

d $$\begin{aligned} -120 - (-105) &= -120 + 105 \\ &= -(120 - 105) \\ &= -15 \end{aligned}$$

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Exercise 4C

Example 6, 7a, b

1 Write the answers to these additions.

a $7 + (-2)$ **b** $-7 + 2$ **c** $-5 + 12$ **d** $-15 + (-3)$

e $-13 + 17$ **f** $-7 + (-3)$ **g** $-7 + (-2)$ **h** $-6 + (-11)$

i $12 + (-5)$ **j** $-13 + 5$ **k** $-13 + (-4)$ **l** $-24 + 8$

Example 7c, d, e, 8

2 Write the answers to these subtractions.

a $3 - 6$ **b** $5 - 11$ **c** $-4 - 7$ **d** $13 - (-3)$

e $-12 - 8$ **f** $-5 - (-3)$ **g** $-7 - (-5)$ **h** $6 - (-11)$

i $-11 - 5$ **j** $-14 - (-5)$ **k** $-23 - (-15)$ **l** $31 - (-10)$

3 Write the answer to these additions and subtractions.

a $2 + (-3)$ **b** $4 + (-6)$ **c** $5 + (-8)$ **d** $4 + (-7)$

e $6 + (-11)$ **f** $6 - (-3)$ **g** $-4 - (-1)$ **h** $-6 - (-2)$

i $-6 - 3$

Example 9

4 Evaluate:

a $13 - 27 + (-26)$ **b** $-12 - 14 + 8$ **c** $-41 + 56 - 2$

d $32 - 42 - (-8)$ **e** $7 + 13 - 16$ **f** $-32 - (-42) - (-3)$

g $-37 - 17 + 25$ **h** $6 - (-27) + 45$ **i** $16 + (-3) - (-7)$

j $-100 + 48 - (-80)$ **k** $-50 - 70 - (-100)$ **l** $20 - (-10) - (-30)$

Example 11

5 Evaluate:

a $256 - (-100)$ **b** $-850 - (-246)$ **c** $-658 - (-790)$ **d** $-9860 - (-3755)$

Example 10

6 The temperature in Warsaw on a winter's day went from a minimum of −17°C to a maximum of −1°C. By how much did the temperature rise?

7 The temperature in Daylesford on a very cold winter's day went from −2°C to 9°C. What was the change in temperature?

8 The following table shows a list of minimum and maximum temperatures for a number of cities. Complete the table, showing the increase in each case.

Minimum	Maximum	Increase
−11°C	6°C	
−16°C	7°C	
−35°C	−4°C	
−25°C	−15°C	
−7°C	−2°C	
−13°C	2°C	
−11°C	5°C	

9 The temperature in Canberra on a very cold day went from 8°C to −2°C. What was the change in temperature?

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4D Multiplication and division with negative integers

Multiplication with negative integers

$5 \times (-3)$ means 5 lots of -3 added together. That is:

$$\begin{aligned} 5 \times (-3) &= (-3) + (-3) + (-3) + (-3) + (-3) \\ &= -15 \end{aligned}$$

Just as $8 \times 6 = 6 \times 8$, we will take -3×5 to be the same as $5 \times (-3)$.

All products such as $5 \times (-3)$ and -3×5 are treated in the same way.

For example:

$$\begin{aligned} -6 \times 3 &= 3 \times (-6) \\ &= -18 \end{aligned} \qquad \begin{aligned} -15 \times 4 &= 4 \times (-15) \\ &= -60 \end{aligned}$$

The question remains as to what we might mean by multiplying two negative integers together. We first investigate this by looking at a multiplication table.

In the left-hand column below, we are taking multiples of 5. The products go down by 5 each time.

In the right-hand column, we are taking multiples of -5. The products go up by 5 each time.

$3 \times 5 = 15$	$3 \times (-5) = -15$
$2 \times 5 = 10$	$2 \times (-5) = -10$
$1 \times 5 = 5$	$1 \times (-5) = -5$
$0 \times 5 = 0$	$0 \times (-5) = 0$
$-1 \times 5 = -5$	$-1 \times (-5) = ?$
$-2 \times 5 = -10$	$-2 \times (-5) = ?$

The pattern suggests that it would be natural to take $-1 \times (-5)$ equal to 5 and $-2 \times (-5)$ equal to 10 so that the pattern continues in a natural way.

All products such as $-5 \times (-2)$ and $-5 \times (-1)$ are treated in the same way.
For example:

$$-6 \times (-2) = 12$$

$$-3 \times (-8) = 24$$

$$-20 \times (-5) = 100$$

We have the following rules.

The sign of the product of two integers

- The product of a negative number and a positive number is a negative number. For example: $-4 \times 7 = -28$.
- The product of two negative numbers is a positive number. For example: $-4 \times (-7) = 28$.

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Example 12

Evaluate each product.

a $4 \times (-20)$

b -7×10

c $-25 \times (-40)$

Solution

a $4 \times (-20) = -80$

b $-7 \times 10 = -70$

c $-25 \times (-40) = 25 \times 40$
$\quad = 1000$

Division with negative integers

Every multiplication statement, for non-zero numbers, has an equivalent division statement. For example, $7 \times 3 = 21$ is equivalent to $21 \div 3 = 7$ and to $21 \div 7 = 3$. We will use this fact to establish the rules for division involving the integers.

Here are some more examples:

$7 \times 6 = 42$ is equivalent to $42 \div 6 = 7$.

$$\begin{tikzcd} \boxed{7} \arrow[rr, bend left, "\times 6"] & & \boxed{42} \arrow[ll, bend left, "\div 6"] \end{tikzcd}$$

$7 \times (-6) = -42$ is equivalent to $-42 \div (-6) = 7$.

$$\begin{tikzcd} \boxed{7} \arrow[rr, bend left, "\times (-6)"] & & \boxed{-42} \arrow[ll, bend left, "\div (-6)"] \end{tikzcd}$$

$-7 \times 6 = -42$ is equivalent to $-42 \div 6 = -7$.

$$\begin{tikzcd} \boxed{-7} \arrow[rr, bend left, "\times 6"] & & \boxed{-42} \arrow[ll, bend left, "\div 6"] \end{tikzcd}$$

$-7 \times (-6) = 42$ is equivalent to $42 \div (-6) = -7$.

$$\begin{tikzcd} \boxed{-7} \arrow[rr, bend left, "\times (-6)"] & & \boxed{42} \arrow[ll, bend left, "\div (-6)"] \end{tikzcd}$$

The sign of the quotient of two integers

- The quotient of a positive number and a negative number is a negative number. For example: $28 \div (-7) = -4$.
- The quotient of a negative number and a positive number is a negative number. For example: $-28 \div 7 = -4$.
- The quotient of two negative numbers is a positive number. For example: $-28 \div (-7) = 4$.

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Example 13

Evaluate:

a $-54 \div 9$ **b** $-28 \div (-4)$ **c** $72 \div (-9)$

Solution

a $-54 \div 9 = -6$ **b** $-28 \div (-4) = 7$ **c** $72 \div (-9) = -8$

Example 14

Evaluate:

a $\dfrac{-63}{7}$ **b** $\dfrac{-72}{-4}$ **c** $\dfrac{108}{-12}$

Solution

a $\dfrac{-63}{7} = -9$ **b** $\dfrac{-72}{-4} = 18$ **c** $\dfrac{108}{-12} = -9$

Exercise 4D

Example 12

1 Carry out these multiplications.

a $6 \times (-2)$ **b** $7 \times (-2)$
c $7 \times (-10)$ **d** $12 \times (-4)$
e $12 \times (-18)$ **f** -5×3
g -6×4 **h** -5×12
i -11×7 **j** -12×5
k $-20 \times (-7)$ **l** $16 \times (-4)$
m $-7 \times (-20)$ **n** $-14 \times (-14)$
o -19×9 **p** $25 \times (-4)$

Example 13

2 Carry out these divisions.

a $-18 \div 3$ **b** $-28 \div 2$ **c** $-35 \div 5$ **d** $-27 \div 3$
e $-120 \div 4$ **f** $18 \div (-3)$ **g** $36 \div (-2)$ **h** $45 \div (-9)$
i $21 \div (-3)$ **j** $120 \div (-3)$ **k** $-52 \div (-4)$ **l** $-72 \div (-8)$
m $-52 \div (-13)$ **n** $-600 \div (-10)$ **o** $-168 \div 6$ **p** $-385 \div 11$

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Example 14

3 Evaluate:

a $\frac{4}{-1}$ **b** $\frac{-6}{-1}$ **c** $\frac{8}{-2}$ **d** $\frac{12}{-4}$

4 Evaluate:

a $\frac{-60}{-12}$ **b** $\frac{-65}{13}$ **c** $\frac{-72}{12}$ **d** $\frac{-120}{-8}$

e $\frac{136}{-4}$ **f** $\frac{-610}{5}$ **g** $\frac{-240}{15}$ **h** $\frac{396}{-4}$

i $\frac{-70}{10}$ **j** $\frac{720}{-24}$ **k** $\frac{-360}{-90}$ **l** $\frac{-4323}{3}$

5 Evaluate:

a $3 \times (-4) \times (-7)$ **b** $5 \times (-7) \times (-8)$

c $60 \times (-3) \times (-12)$ **d** $-45 \times (-8) \times 10$

e $48 \times (-10) \div 3$ **f** $8 \times (-10) \div 5$

6 Copy and complete these statements.

a $2 \times \ldots = -50$ **b** $5 \times \ldots = -75$

c $-7 \times \ldots = 63$ **d** $\ldots \times (-8) = 72$

e $80 \div \ldots = -10$ **f** $-45 \div \ldots = 5$

g $-321 \div \ldots = 3$

h $5664 \div \ldots = -708$

4E Indices and order of operations

You need to be particularly careful with the order of operations conventions when working with indices.

For example, $-4^2 = -16$ and $(-4)^2 = 16$. In the first case, 4 is first squared and then the opposite is taken. In the second case, -4 is squared. Notice how different the two answers are.

Also:

$$\begin{aligned} -3 \times 5 + 2 &= -15 + 2 \\ &= -13 \end{aligned} \quad \text{and} \quad \begin{aligned} -3 \times (5 + 2) &= -3 \times 7 \\ &= -21 \end{aligned}$$

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Order of operations

- Evaluate expressions inside brackets first.
- In the absence of brackets, carry out operations in the following order:
  - powers
  - multiplication and division from left to right
  - addition and subtraction from left to right.

Example 15

Evaluate:

a $(-5)^2$ **b** -5^2 **c** $-6-5+4$ **d** $6\times(-3)+8$

Solution

a $(-5)^2 = -5\times(-5)$
$= 25$

b $-5^2 = -(5\times 5)$
$= -25$

c $-6-5+4 = -11+4$
$= -7$

d $6\times(-3)+8 = -18+8$
$= -10$

Example 16

Evaluate:

a $7\times(-6+8)$

b $-3+8\times(7-12)$

c $6-(5+3)$

d $4\times(-6)+3\times 8$

e $3\times(-6)^2+3\times 21$

Solution

a $7\times(-6+8) = 7\times 2$
$= 14$

b $-3+8\times(7-12) = -3+8\times(-5)$
$= -3+(-40)$
$= -43$

c $6-(5+3) = 6-8$
$= -2$

d $4\times(-6)+3\times 8 = -24+24$
$= 0$

e $3\times(-6)^2+3\times 21 = 3\times 36+63$
$= 108+63$
$= 171$

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Exercise 4E

Example 15 a, b

1 Evaluate:

a $(-9)^2$ **b** $-(7)^2$ **c** $3 \times (-5)^2$ **d** $-9 \times (-4)^2$

e $(-20)^2 \times -(4)^2$ **f** $(-13)^2$ **g** $(-6)^3$ **h** $(-3)^4$

i $(-5)^5$ **j** $(-7)^6$ **k** $(-1)^7$

Example 15c

2 Evaluate:

a $-6 + 30 - 15$ **b** $-5 - (-10) + 30$ **c** $-8 + 12 - 18$

d $-6 + 12 - (-15)$ **e** $-15 + 7 - 10$ **f** $75 - (-40) + 60$

Example 16

3 Evaluate:

a $-3 \times (-16 + 8)$ **b** $-4 + 6 \times (11 - 14)$ **c** $-6 - 18 + 4$

d $-6 \times (-3 + 12)$ **e** $-2 \times (-6 + 16) - 25$ **f** $-15 + 5 \times (-3 + 12)$

g $(-11 + 5) \times 12 + (-15)$ **h** $(-18 - 4) \times 26 - (-12)$

4 Evaluate:

a $-(3 - 27)$ **b** $-(37 - 64)$ **c** $15 + (8 - 16)$

d $-53 + (5 - 11)$ **e** $16 - 21 + 5 \times (-3)$ **f** $-3 \times (86 - 97)$

g $-15 \times (3 - 11)$ **h** $6 \times (14 - 41)$ **i** $-8 \times (11 - 28)$

5 Evaluate:

a $-5 \times (-26 + 8)$ **b** $-4 + 6 \times (15 - 12)$ **c** $-6 - (25 + 4)$

d $-6 \times (-4 + 12)$ **e** $-2 \times (-8 + 16) - 40$ **f** $-99 + 5 \times (-62 + 12)$

g $-81 + 5 \times (51 + (-45))$ **h** $-28 - 4 \times (26 - (-14))$

6 Evaluate:

a $60 \div (-5) \div 12$ **b** $90 \times (-4) \div 10$

c $60 \div 10 \times 3$ **d** $80 \times (-5) \div 25$

7 Evaluate:

a $(-20)^2 + 3 \times (-10)$ **b** $(-20)^2 \times (-10)^3$

c $3 \times (-10)^3 + 10^2$ **d** $(-4) \times (-10)^2 \times (-10)$

8 A shop manager buys 200 shirts at \$26 each and sells them for a total of \$4000. Calculate the total purchase price, and subtract this from the total amount gained from sales.

9 A man puts \$2000 into a bank account every month for 12 months.

a What is his bank balance at the end of 12 months, given that he does not withdraw any money?

b He writes a cheque for \$40 000. What is his new bank balance?

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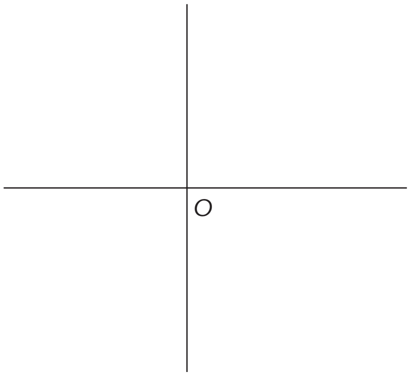
4F The Cartesian plane

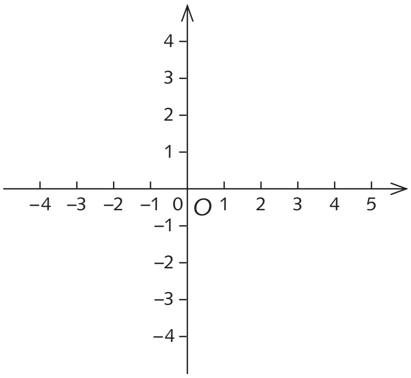
We have previously represented numbers as points on the number line. This idea can be extended by taking pairs of numbers to represent points in a plane. This is called the **Cartesian plane**.

We start with two perpendicular straight lines. They intersect at a point O called the **origin**. We leave the right angle sign out for clarity.



Each of the lines is called an **axis**. The plural of *axis* is *axes*.

Next we mark off intervals of unit length along each axis, and mark each axis as a number line with 0 at the point O. The arrows are drawn to show that the axes extend infinitely in both directions and to indicate which way is the positive direction.



The axes are called the **coordinate axes** or sometimes the **Cartesian coordinate axes**. They are named after the French mathematician and philosopher René Descartes 1596–1650. He introduced coordinate axes to show how algebra could be used to solve geometric problems. Although the idea is simple, it revolutionised mathematics.

Now we imagine adding vertical and horizontal lines to the diagram through the integer points on the axes. We can describe each point where the lines meet by a pair of integers. This pair of integers is called the **coordinates** of the point. The first number is the **horizontal coordinate** and the second number is the **vertical coordinate**.

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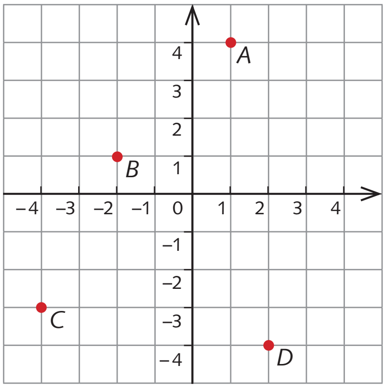
For example, the coordinates of the point labelled A below are (1, 4). This is where the line through the point 1 on the horizontal axis and the line through the point 4 on the vertical axis meet. We move 1 unit to the right of the origin and 4 units up to reach A.



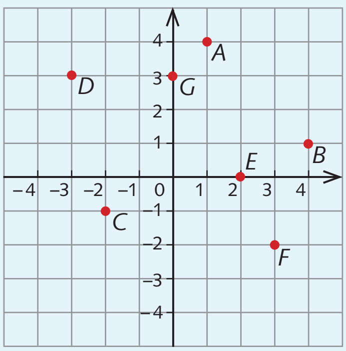
The point D has coordinates (2, −4).
We move 2 units to the right of the origin and 4 units down to get D.

The point B has coordinates (−2, 1).
We move 2 units to the left of the origin and 1 unit up to get B.

The point C has coordinates (−4, −3).
We move 4 units to the left of the origin and 3 units down.

Example 17

Give the coordinates of each of the points marked on the Cartesian plane below.

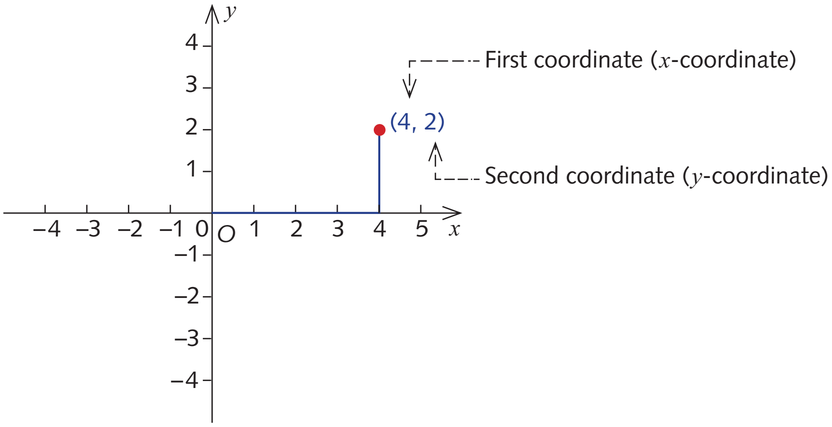


Solution

$A(1,4)$	$B(4,1)$	$C(-2,-1)$	$D(-3,3)$
$E(2,0)$	$F(3,-2)$	$G(0,3)$	

Remember, the first coordinate tells us where to go from the origin in the horizontal direction. If it is negative, we go to the left of the origin; if it is positive, we go to the right of the origin.

The second coordinate tells us where to go from the origin in the vertical direction. If it is negative, we go below the origin; if it is positive, we go above the origin.



The first coordinate is usually called the x-coordinate and the second coordinate is usually called the y-coordinate.

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Exercise 4F

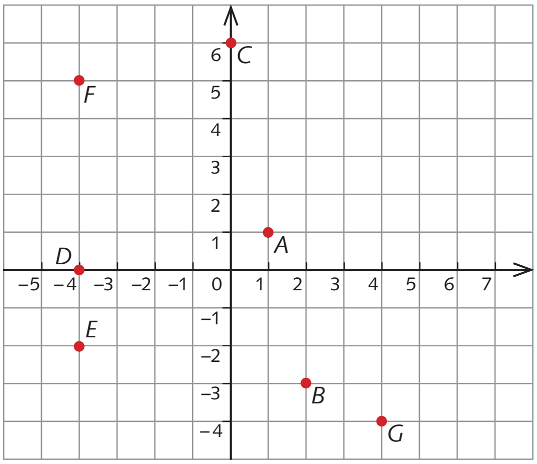
1 On a Cartesian plane, plot each of the following points. The coordinates of the points are given.

a $A(4,1)$ **b** $B(-2,3)$ **c** $C(-2,-2)$ **d** $D(4,-1)$

e $E(1,3)$ **f** $F(0,-2)$ **g** $G(-3,0)$ **h** $H(0,5)$

Example 17

2 **a** Give the coordinates of each of the points A to G below.



b How many of these coordinates have

i a positive x-coordinate?

ii a negative y-coordinate?

3 Do this exercise on graph paper. Set out the coordinate axes by marking intervals of length 1 unit along the horizontal and vertical axes.

a Plot the points $O(0,0), A(3,0), C(3,3)$ and $D(0,3)$, and join the points to form the intervals OA, AC, CD and DO. Describe the shape formed and evaluate its area.

b Plot the points $A(-3,0), B(6,0)$ and $C(0,4)$, and join the points to form the intervals AB, BC and CA. Describe the shape formed and evaluate its area.

c Plot the points $A(-4,-3), B(7,-3), C(7,2)$ and $D(-4,5)$, and join the points to form the intervals AB, BC, CD and DA. Describe the shape formed and evaluate its area.

d Plot the points $A(-4,4), B(1,4)$, and $C(-4,1)$, and join the points to form the intervals AC, CB, BA. Describe the shape formed and evaluate its area.

e Plot the points $O(0,0)$ and $A(1,2)$, and draw the line passing through these points. Plot the points $B(-2,3)$ and $C(0,2)$, and draw the line passing through these points. Describe the relationship between the lines.

f Plot the points $A(-1,1), B(3,3), C(7,3)$ and $D(5,1)$, and join the points to form the intervals AB, BC, CD and DA. Describe the shape formed and evaluate its area.

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4G Negative fractions

Fractions were reviewed in Chapter 2.

We can now introduce negative fractions as well. They lie to the left of 0 on the number line.

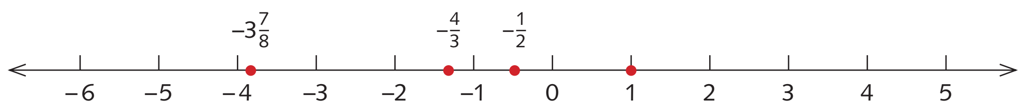
Each positive fraction has an opposite fraction. For example:

$$\frac{5}{4} \text{ has the opposite } -\frac{5}{4} = -1\tfrac{1}{4} = -1.25.$$

A situation where we see negative fractions is in temperatures.

A temperature of $-4.5°\text{C}$ $\left(-\frac{9}{2}°\text{C}\right)$ is $4.5°\text{C}$ below zero.

Several negative fractions are marked on the number line below.



The arithmetic for integers, which we have considered in the previous sections, also extends to fractions.

Addition and subtraction of negative fractions

Example 18

Write the answer to each of these additions and subtractions.

a $-\frac{1}{2}+2$ **b** $-1\frac{1}{2}+2$ **c** $-\frac{1}{3}-\frac{1}{2}$ **d** $-\frac{2}{3}+\frac{1}{5}$

Solution

a $-\frac{1}{2}+2=1\frac{1}{2}$

b $-1\frac{1}{2}+2=\frac{1}{2}$

c $$\begin{aligned}-\frac{1}{3}-\frac{1}{2}&=-\frac{2}{6}-\frac{3}{6}\\&=-\frac{5}{6}\end{aligned}$$

d $$\begin{aligned}-\frac{2}{3}+\frac{1}{5}&=-\frac{10}{15}+\frac{3}{15}\\&=-\frac{7}{15}\end{aligned}$$

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Division by negative integers giving fractions

Example 19

Divide:

a 265 by – 2

b –3765 by –7

Solution

a First divide 265 by 2. The result is $132\frac{1}{2}$.

For $265 \div (-2)$, the signs are different and the result is $-132\frac{1}{2}$.

b First divide 3765 by 7. The result is $537\frac{6}{7}$.

For $-3765 \div (-7)$, the signs are the same and $-3765 \div (-7) = 537\frac{6}{7}$.

Multiplication and division of negative fractions

The methods we have been using with the positive fractions also work with the negative fractions.

Example 20

Give the answer to each of these multiplications and divisions as a fraction.

a $-\frac{1}{2} \times 2$

b $-1\frac{1}{2} \div 2$

c $-\frac{1}{3} \div \frac{1}{2}$

d $-\frac{2}{3} \times \left(-\frac{1}{5}\right)$

Solution

a $-\frac{1}{2} \times 2 = -1$

b $-1\frac{1}{2} \div 2 = -\frac{3}{2} \times \frac{1}{2}$
$= -\frac{3}{4}$

c $-\frac{1}{3} \div \frac{1}{2} = -\frac{1}{3} \times \frac{2}{1}$
$= -\frac{2}{3}$

d $-\frac{2}{3} \times \left(-\frac{1}{5}\right) = \frac{2}{3} \times \frac{1}{5}$
$= \frac{2}{15}$

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Exercise 4G

1 Draw a number line from –5 to 5 and mark on it the positions of

$-3\tfrac{1}{2}, -4\tfrac{3}{4}, -3\tfrac{1}{4}, -1\tfrac{1}{2}, 1\tfrac{1}{2}$.

2 Arrange each of these sets of numbers from smallest to largest.

a $-2, \frac{1}{4}, -1, \frac{-5}{-3}, -\frac{7}{5}, \frac{1}{2}, 1$

b $-\frac{7}{4}, \frac{-11}{-3}, -\frac{11}{5}, \frac{14}{5}, \frac{7}{2}$

c $-\frac{2}{3}, -\frac{4}{5}, -\frac{11}{12}, \frac{12}{13}$

d $-1\tfrac{11}{13}, -\frac{15}{13}, -2, -1$

Example 18

3 Evaluate:

a $-\frac{1}{2}+3$

b $-1\tfrac{1}{2}+4$

c $-\frac{2}{3}-\frac{1}{3}$

d $-\frac{2}{3}+\frac{1}{3}$

e $-\frac{3}{4}-\frac{3}{4}$

f $-\frac{2}{3}+\frac{3}{5}$

g $-\frac{3}{4}+\frac{1}{4}$

h $-\frac{1}{5}-\frac{2}{5}$

i $-\frac{3}{4}+3$

j $-\frac{3}{5}-3$

k $-1\tfrac{1}{2}+\frac{-7}{9}$

l $\frac{3}{7}-2\tfrac{1}{2}$

m $-3\tfrac{1}{2}+3$

n $-2\tfrac{1}{2}+4$

o $-\frac{2}{5}-\frac{1}{3}$

p $-\frac{3}{4}+\frac{1}{3}$

q $-\frac{3}{4}+3\tfrac{2}{9}$

r $-\frac{3}{5}-3\tfrac{2}{7}$

s $-\frac{5}{13}+\left(-3\tfrac{5}{7}\right)$

t $\frac{3}{8}-\left(-\frac{2}{7}\right)$

Example 19

4 Divide:

a 2735 by –2

b –6322 by 3

c –5524 by –7

Example 20 a, d

5 Evaluate:

a $2\times\left(-\frac{3}{8}\right)$

b $-\frac{3}{8}\times\left(-\frac{5}{9}\right)$

c $-\frac{11}{12}\times\left(-\frac{5}{9}\right)$

d $-\frac{11}{5}\times\frac{5}{12}$

e $4\times\left(-\frac{3}{8}\right)\times\frac{5}{9}$

f $5\times\left(-\frac{3}{10}\right)$

g $-\frac{2}{3}\times\frac{5}{6}$

h $-\frac{5}{12}\times\frac{7}{5}$

Example 20 b, c

6 Evaluate:

a $2\div\left(-\frac{1}{3}\right)$

b $\frac{3}{4}\div\left(-\frac{2}{3}\right)$

c $-3\div 5$

d $\frac{3}{4}\div\left(-\frac{11}{12}\right)$

e $-\frac{2}{3}\div\left(-\frac{4}{9}\right)$

f $-\frac{3}{8}\div\frac{5}{12}$

g $-\frac{2}{3}\div\left(-\frac{1}{6}\right)$

h $\frac{5}{7}\div 10$

7 Rewrite each expression in simplest form.

a $\frac{1}{5}-\left(\frac{1}{3}+\frac{1}{2}\right)$

b $-2\tfrac{7}{8}+\left(-\frac{1}{3}\right)\times\frac{4}{3}$

c $\left(-1\tfrac{1}{2}+1\tfrac{3}{4}\right)\times 3\tfrac{1}{2}$

d $-\frac{1}{4}-\frac{1}{3}\times 4$

e $\frac{1}{2}-2\times\frac{3}{8}$

f $-\frac{3}{4}-\left(2-\frac{1}{2}\right)$

g $4-\left(-3\tfrac{1}{2}\right)-2$

h $\frac{1}{6}-\frac{1}{3}+\frac{1}{6}$

i $2\times\left(-\frac{1}{3}\right)-4\tfrac{1}{3}$

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8 Evaluate:

a $-\frac{1}{2}+2\frac{1}{2}-\frac{1}{4}+\frac{3}{4}$

b $-3\frac{1}{4}-\frac{1}{4}+6-2\frac{1}{2}$

c $2\times\left(-\frac{3}{2}\right)\times\left(-\frac{4}{7}\right)+\frac{2}{7}$

d $3\frac{1}{3}\times(-3)+10\frac{1}{2}\div 3$

e $5\frac{1}{4}\div(-7)+\frac{3}{4}$

f $-\frac{1}{2}-\frac{1}{3}-\frac{1}{4}-\frac{1}{5}-\frac{1}{6}$

9 Evaluate:

a $\left(-\frac{1}{2}\right)^2$

b $\left(-\frac{1}{2}\right)^3$

c $\left(-\frac{1}{2}\right)^4$

d $\left(-\frac{1}{2}\right)^5$

e $-16\times\left(-\frac{1}{2}\right)^5$

f $-32\times\left(-\frac{1}{4}\right)^2$

g $24\times\left(-\frac{1}{2}\right)^3$

h $\left(-\frac{1}{3}\right)^3\times -8$

Review exercise

1 Evaluate:

a $25+(-2)$

b $-36+22$

c $-35+50$

d $-51+(-44)$

e $-32+16$

f $-45+(-23)$

g $-160+(-20)$

h $-50+(-10)$

i $110+(-40)$

j $-120+40$

k $-135+(-25)$

l $60-(-25)$

2 In an indoor cricket match, a team has made 25 runs and lost 7 wickets. What is the score of the team? (A run adds 1 to the score and a wicket subtracts 5.)

3 The temperature in June at a base in Antarctica varied from a minimum of –60°C to a maximum of –35°C. What was the value of:

a maximum temperature – minimum temperature?

b minimum temperature – maximum temperature?

4 The temperature in Canberra had gone down to –5°C. The temperature in a heated house was a cosy 22°C. What was the value of:

a inside temperature – outside temperature?

b outside temperature – inside temperature?

5 Evaluate:

a $125\times(-2)$

b -36×11

c -35×50

d $-51\times(-40)$

e -3×16

f $-50\times(-23)$

g $-160\times(-20)$

h $-50\times(-10)$

i $11\times(-40)$

j -120×20

k $-32\times(-4)$

l -25×8

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6 Evaluate:

a $125 \div (-5)$
b $-36 \div 9$
c $-35 \div 5$
d $-51 \div (-3)$
e $-16 \div (-4)$
f $-50 \div (-10)$
g $-160 \div (-20)$
h $-1500 \div (-10)$
i $120 \div (-40)$
j $-120 \div 20$
k $-40 \div (-5)$
l $624 \div (-6)$

7 Evaluate:

a $-4 \times (6 - 7)$
b $7 \times (11 - 20)$
c $-3 \times (5 + 15)$
d $-6 \times (-4 - 6)$
e $-12 \times (-6 + 20)$
f $-(-4)^2$
g $(3 - 7) \times (11 - 15)$
h $(10 - 3) \times (-3 + 10)$
i $(-5 - 10) \times (10 - 4)$

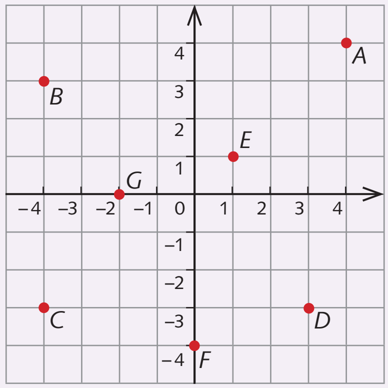
8 Start with the number −5, add 11 and then subtract 20. Multiply the last number you obtained by 4. What is the final result?

9 Start with −100, subtract 200 and then add −300. Divide the result by 100. What is the final result?

10 On a Cartesian plane, plot each of the following points. The coordinates of the points are given.

a $A(1,1)$
b $B(2,-3)$
c $C(0,6)$
d $D(-4,0)$
e $E(-4,-2)$
f $F(-4,5)$

11 Find the coordinates of each of the points A to G below.



12 Evaluate:

a $-\frac{1}{2} + 3$
b $-1\frac{1}{2} + 6$
c $-\frac{9}{13} - \frac{1}{13}$
d $-\frac{7}{8} + \frac{5}{8}$
e $-\frac{5}{12} + \left(-\frac{5}{6}\right)$
f $-\frac{5}{11} + 6$

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13 Evaluate:

a $\frac{2}{3} \times \left(-\frac{1}{4}\right)$ **b** $3\frac{1}{3} \times (-9)$ **c** $-\frac{4}{3} \times \frac{3}{4} \div \frac{7}{8}$

d $-\frac{2}{3} \times \frac{3}{4} \times (-2)$ **e** $\frac{3}{4} \times \left(-\frac{4}{5}\right) \times \frac{1}{2}$ **f** $\frac{1}{2} - \frac{1}{3} \times \frac{1}{2}$

g $-\frac{2}{3} \times \frac{1}{4} \times \frac{3}{5}$ **h** $-\frac{5}{8} \times \frac{1}{2} \times \frac{2}{3}$ **i** $\frac{1}{4} - \frac{2}{3} \div 3$

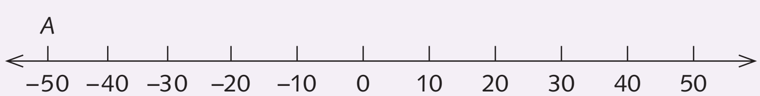
14 AC electricity, which is what you get from a power point, alternates (hence the name alternating current) between positive and negative voltages. The highest positive voltage it reaches is 1.4 times the rated voltage, and the lowest voltage is −1.4 times the rated voltage. If the power point is rated as 230 V AC, what is the difference between the highest and lowest voltages? ('V' stands for the unit of measure 'volts')

15 Evaluate:

a $(-1)^2$ **b** $(-1)^3$ **c** $(-1)^4$ **d** $(-1)^5$ **e** $(-1)^6$

Comment on the results.

16 Johann is asked to walk along a straight pathway that is marked out as a number line, as shown in the diagram. Distances are in metres.



Johann starts at the point A, which is at −50 on the number line.

He is asked to walk along the path and follow these directions:

1 Walk from A to the point −10.
2 Turn through 180°.
3 Walk to the point −20.
4 Turn through 180°.
5 Walk to the point 50.
6 Turn through 180°.
7 Walk back to A.

How far has he walked?

17 The average of a list of numbers is obtained by adding the numbers, and then dividing by the number of numbers. For example, the average of 20, 30, 50 and 60 is given by

$$(20+30+50+60) \div 4 = 160 \div 4 \quad \text{or} \quad \frac{20+30+50+60}{4} = \frac{160}{4}$$
$$= 40 \qquad\qquad = 40$$

Find the average of each of these lists of numbers.

a −20, −30, 40, 60 **b** −6, −1, −10, −20, 22

c −200, −300, −500, −1000 **d** −2, −3, −4, 0, 8, 1

18 Find the average of each of these lists of numbers.

a $-1\frac{1}{4}, 2\frac{1}{2}$ **b** $-1, -1\frac{1}{2}, 0, 6$ **c** $-\frac{3}{4}, -\frac{1}{2}, -\frac{1}{8}, \frac{1}{2}, \frac{1}{4}$

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19 If $a = -1\frac{1}{4}$ and $b = 3\frac{1}{2}$, evaluate:

a $a + b$ **b** $a - b$ **c** $b - a$ **d** ab

e $\dfrac{a}{b}$ **f** $\dfrac{b}{a}$ **g** $\dfrac{a+2}{b}$ **h** $\dfrac{b-a}{b}$

20 Evaluate:

a $(-3)^2 + 2 \times (-3) + 8$ **b** $(-3)^3 - 2 \times (-3) + 10$

c $6 - (-2)^2 - (-2)^3$ **d** $\left(-\dfrac{1}{2}\right)^3 - \left(-\dfrac{1}{2}\right)^2 + \dfrac{1}{2}$

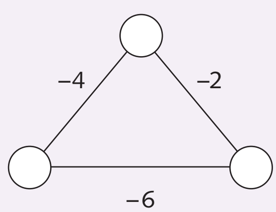
21 Evaluate:

a $1 \div \dfrac{1}{2}$ **b** $1 \div \left(-\dfrac{1}{2}\right)$ **c** $1 \div \left(-\dfrac{3}{2}\right)$ **d** $1 \div \left(-\dfrac{3}{2}\right)^2$

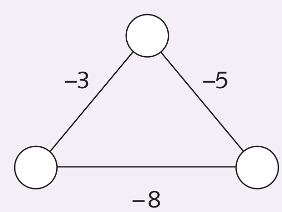
Challenge exercise

1 The integers on the edges of each triangle below are the sums of the integers in the adjoining two circles. Find the numbers in the circles.

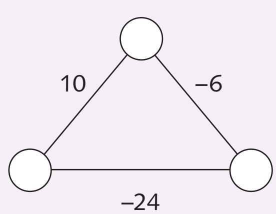
a



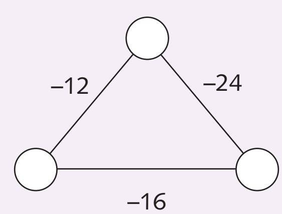
b



c



d



2 Put the three numbers 4, – 2 and –5 into the boxes below

$\square + \square - \square = \_\_\_\_$

so that the answer is:

a 1 **b** 7 **c** –11

3 Put the three numbers 3, – 3 and – 4 into the boxes below

$\square + \square - \square = \_\_\_\_$

so that the answer is:

a 4

b –10

c 2

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4 Which number needs to be placed in the box below to make the following a true statement?

$3 - \square + (-5) = 0$

5 Place brackets in each statement to make it true.

a $2 + -3 \times 3 + 4 = -7$

b $2 + -3 \times 3 + 4 \times 2 = 1$

c $2 - 5 \times 6 + 7 \times 6 - 5 = -71$

6 This is a magic square. All rows, columns and diagonals have the same sum. Complete the magic square.

8	−6	4
		−4

7 **a** Find the value of $1 - 2 + 3 - 4 + 5 - 6$ by:

i working from left to right

ii pairing $(1-2)+(3-4)+(5-6)$

b Evaluate $1 - 2 + 3 - 4 + 5 - 6 + 7 - 8 + \ldots + 99 - 100$.

8 Evaluate $100 + 99 - 98 - 97 + 96 + 95 - 94 - 93 + \ldots + 4 + 3 - 2 - 1$.

9 The average of five given numbers is 2. If the least of these is deleted, the average is 4. What is the smallest number?

10 Find the value of:

a $(2-4)+(6-8)+(10-12)+(14-16)+(18-20)$

b $2 - 4 + 6 - 8 + 10 - 12 + \ldots + 98 - 100$

11 What is the least product you could obtain by multiplying any two of the following numbers: $-7, -5, -1, 1$ and 3?

12 Evaluate:

a $\frac{1}{2\times 3} + \frac{1}{3\times 4} + \frac{1}{4\times 5} + \frac{1}{5\times 6}$

b $\frac{1}{2\times 3} - \frac{1}{3\times 4} + \frac{1}{4\times 5} - \frac{1}{5\times 6}$

c $-\frac{1}{2\times 3} + \frac{1}{3\times 4} - \frac{1}{4\times 5} + \frac{1}{5\times 6}$

d $-\frac{1}{5\times 6} - \frac{1}{6\times 7} - \frac{1}{7\times 8} - \frac{1}{8\times 9}$

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CHAPTER

5

Measurement and Geometry

Review of geometry

The word *geometry* comes from two Greek words meaning 'earth measurement'. Its name explains the origin of the subject. In ancient Egypt, the river Nile regularly overflowed its banks and washed away boundary markers between neighbouring properties. The boundaries had to be found again and this led to the study of the different shapes and sizes of triangles.

Geometry deals with points, lines, planes, angles, circles, triangles, and so on. The ancient Greeks valued its study greatly, and their work established the geometry you will meet in this book.

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5A Angles at a point

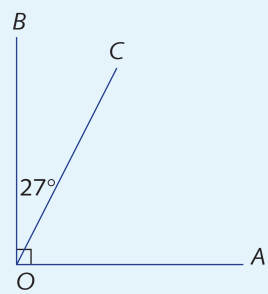
Clear, logical reasoning has always been an important part of geometry. It is very important in geometry to draw diagrams. This makes it easier to construct convincing arguments.

Angles at a point – geometric arguments

- The following can be used in arguments:
  - Adjacent angles can be added.
  - Angles in a revolution add to 360°.
  - Angles in a straight angle add to 180°.
  - Vertically opposite angles are equal.
- Two lines are called **perpendicular** if they meet at right angles.
- The Greek letters α (alpha), β (beta), γ (gamma) and θ (theta) are often used as pronumerals to represent angle size in geometry.

Example 1

Find $\angle AOC$ in the diagram opposite.



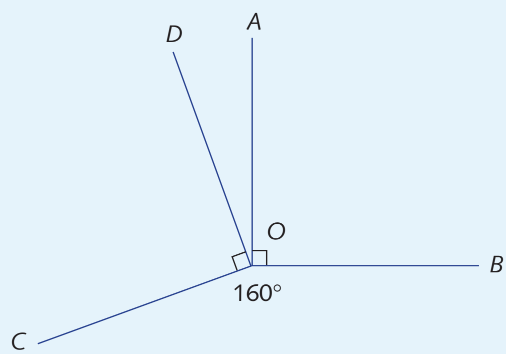
Solution

$\angle AOC + 27° = 90°$ (adjacent angles at O)

so $\angle AOC = 63°$

Example 2

Find $\angle AOD$ in the diagram opposite.



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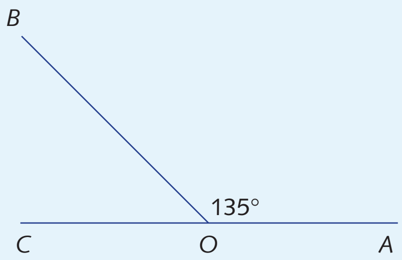
Solution

$\angle AOD + 90° + 160° + 90° = 360°$ (revolution at O)

so $\angle AOD = 20°$

Example 3

Find $\angle BOC$ in the diagram below.



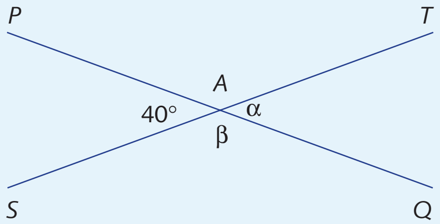
Solution

$\angle BOC + 135° = 180°$ (straight angle $\angle AOC$)

so $\angle BOC = 45°$

Example 4

Find α and β in the diagram below.



Solution

$\alpha = 40°$ (vertically opposite angles at A)

Also, $\beta + 40° = 180°$ (straight angle $\angle PAQ$),

so $\beta = 140°$

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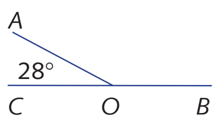
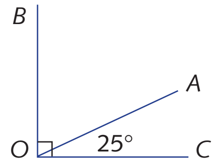
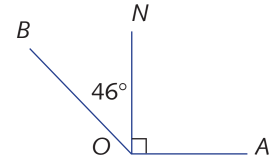
Exercise 5A

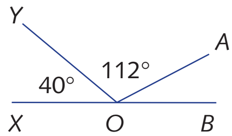
Example 1, 2, 3

1 Find the value of $\angle AOB$ in each diagram below, giving reasons for your answer. In parts **g** and **j**, find the reflex angle $\angle AOB$.

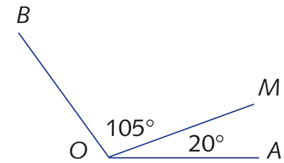
a


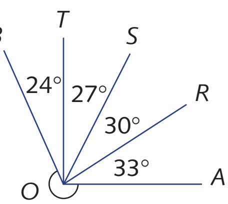
b


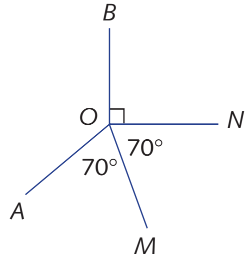
c


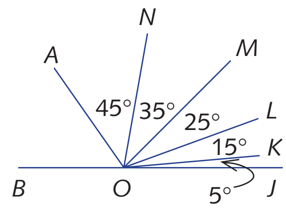
d


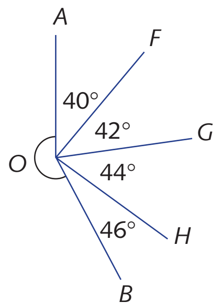
e

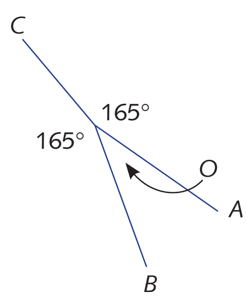

f


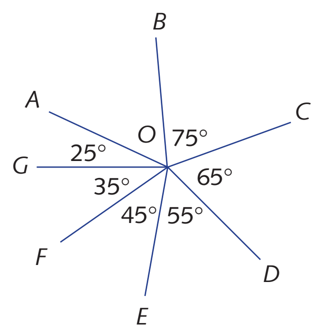
g


h


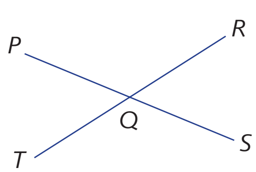
i


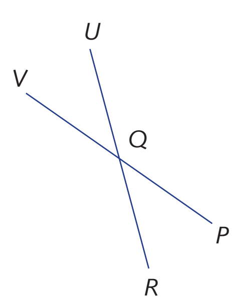
j


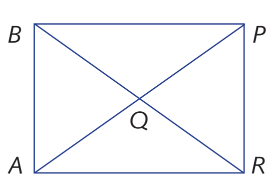
k


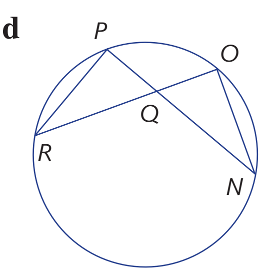
l


2 Name the angle that is vertically opposite to $\angle PQR$ in each diagram below.

a


b


c


d


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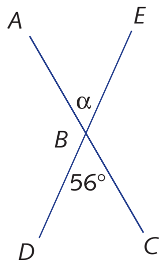
Example 4

3 Find the sizes of the angles denoted by α, β, γ and θ in each diagram below, giving reasons.

a



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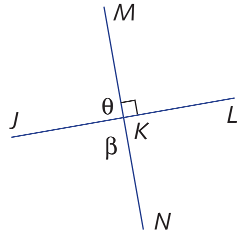

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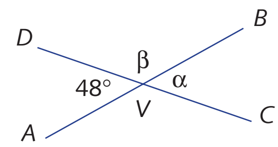

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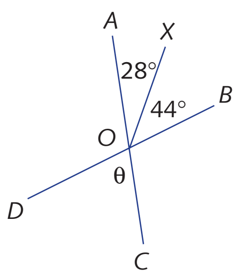

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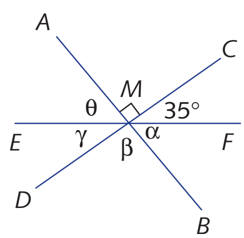

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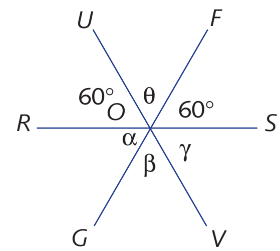

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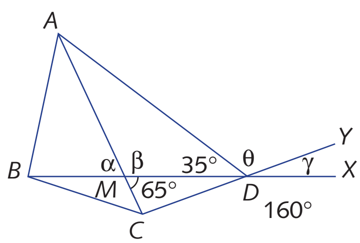

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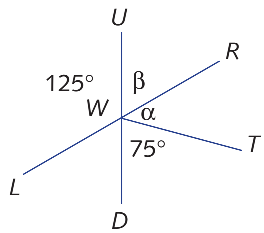
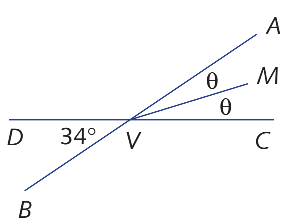


4 In each diagram below, angles marked with the same Greek letter are equal in size. Find the value of each, giving reasons.

a



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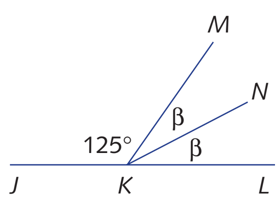

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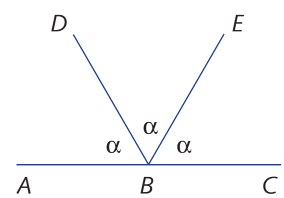

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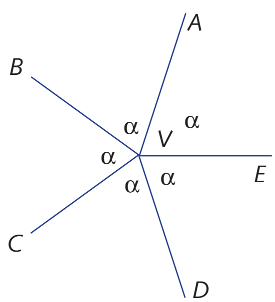

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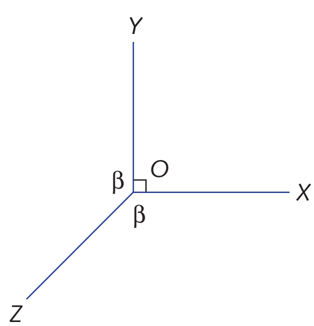

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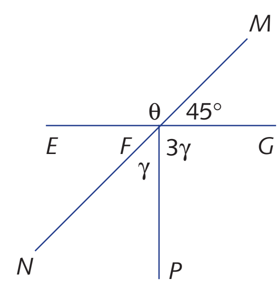
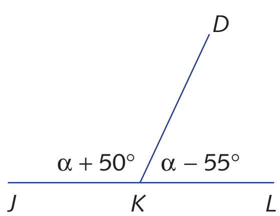


g



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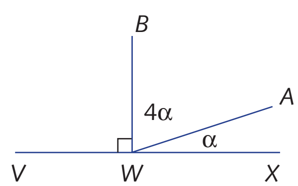

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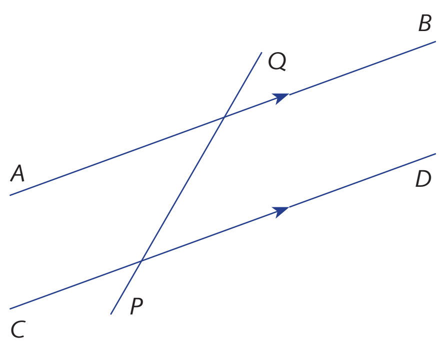
5B Angles associated with transversals

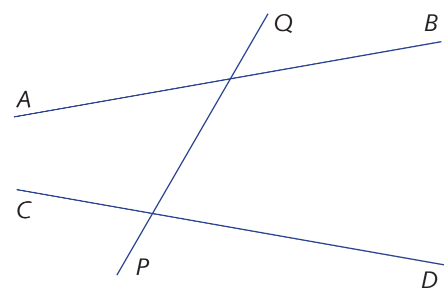
This section and the next one involve the relationships between angles and parallel lines.

As always in geometry, reasons should be as specific as possible. Make sure that you name any parallel lines that you are using in an argument.

Transversal

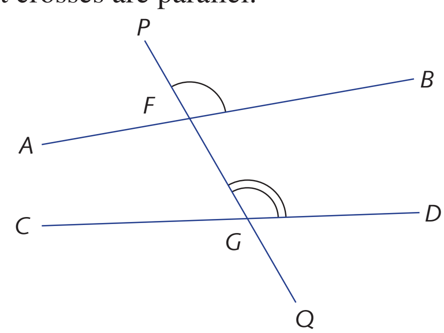
A **transversal** is a line that crosses two other lines. In both diagrams below, the line PQ is a transversal to the lines AB and CD.





Notice that PQ is a transversal whether or not the two lines that it crosses are parallel.

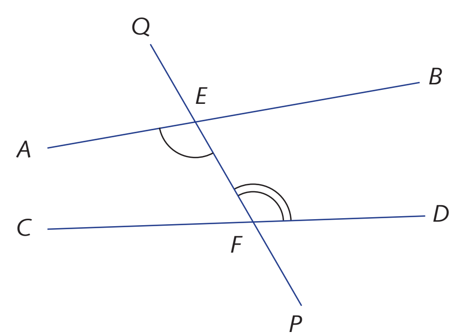
Corresponding angles



The two marked angles in the diagram are called **corresponding angles**, because they are in *corresponding* positions around the two vertices F and G.

There are actually four pairs of corresponding angles in the diagram. Can you name the other three pairs?

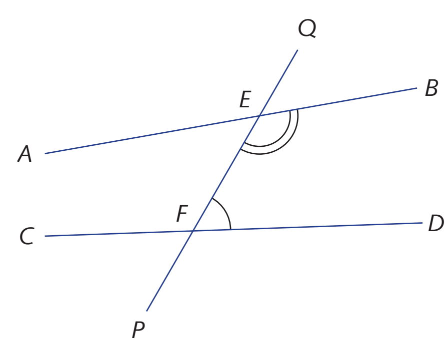
Alternate angles



In the diagram opposite, the two marked angles are called **alternate angles**, because they are on *alternate* sides of the transversal PQ. The two angles must also be inside the two parallel lines.

There is a second pair of alternate angles in the diagram. Can you name the angles of this pair?

Co-interior angles



In the diagram opposite, the two marked angles are called **co-interior angles**, because they are between the two lines and on the same side of the transversal PQ.

There is a second pair of co-interior angles in the diagram. Name the angles of this pair.

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A transversal lying across parallel lines

We learned in Chapter 6 of *ICE-EM Mathematics Year 7* that when a transversal lies across two parallel lines, the alternate angles, the corresponding angles and the co-interior angles are closely related.

Angles associated with transversals

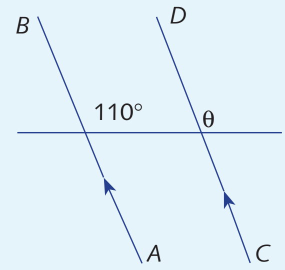
A transversal is a line that crosses two other lines. If the lines crossed by the transversal are parallel, then:

- **corresponding** angles are equal
- **alternate** angles are equal
- **co-interior** angles are supplementary.

Here are three examples that show how to set out your work with corresponding angles, alternate angles and co-interior angles. Your reason should first name the type of angles involved, and then name the relevant pair of parallel lines.

Example 5

Find θ in the diagram opposite.

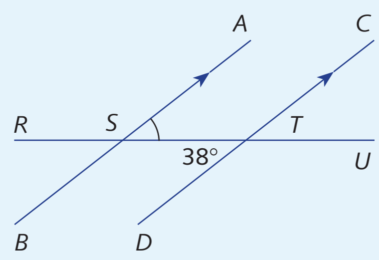


Solution

$\theta = 110°$ (corresponding angles, $AB \parallel CD$)

Example 6

Find $\angle AST$ in the diagram opposite.



Solution

$\angle AST = 38°$ (alternate angles, $AB \parallel CD$)

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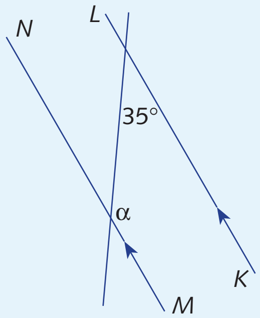
Example 7

Find α in the diagram below.



Solution

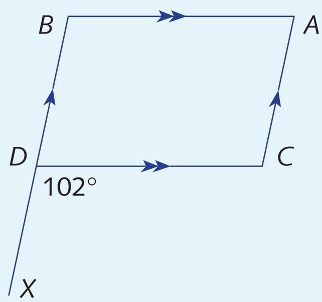
$$\begin{aligned}\alpha + 35° &= 180° \text{ (co-interior angles, } KL \parallel MN)\\ \alpha &= 145°\end{aligned}$$

Two-step solutions

The solution of the problem below needs two steps, with a reason for each step. Notice that the pairs of parallel lines used are different in the two steps.

Example 8

Find $\angle BAC$ in the diagram below.



Solution

First, $\angle DCA = 102°$ (alternate angles, $AC \parallel BD$).

Hence, $\angle BAC = 78°$ (co-interior angles, $AB \parallel CD$).

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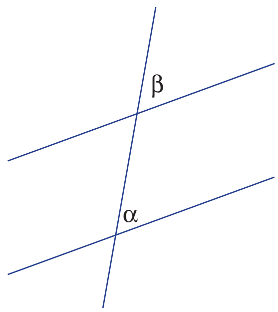
Exercise 5B

1 In each diagram below, identify each pair of angles marked with α and β as corresponding angles, alternate angles or co-interior angles.

a



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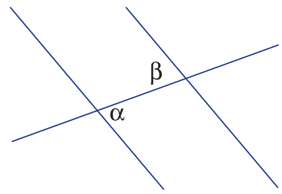

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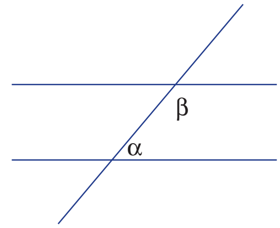

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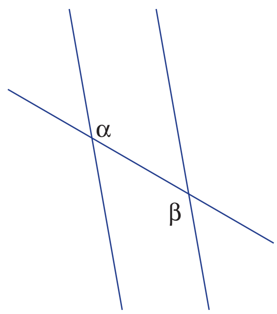
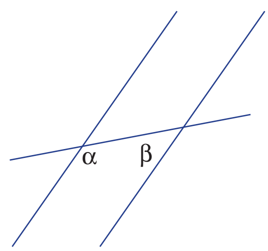


e



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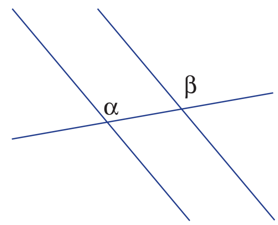

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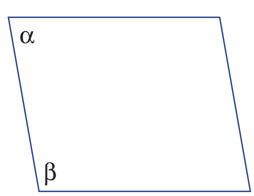

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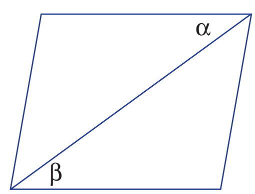

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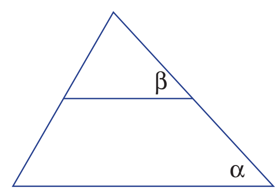

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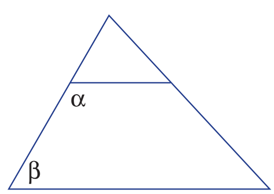

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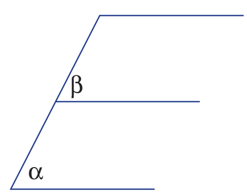

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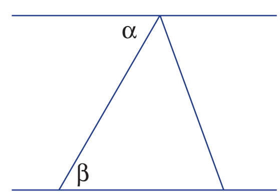
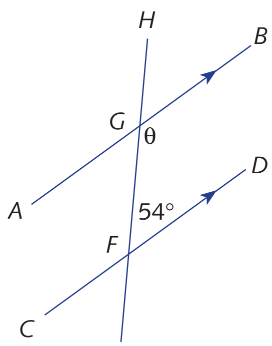


Example 5, 6, 7

2 Find the values of the pronumerals α, β, γ and θ in the diagrams below. Give careful reasons for all your statements, mentioning the relevant parallel lines.

a



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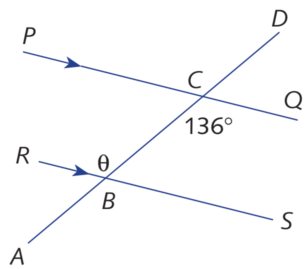

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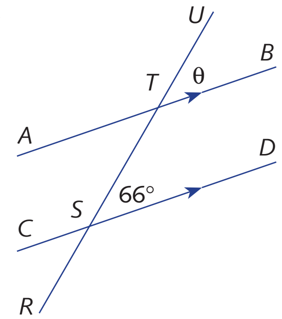

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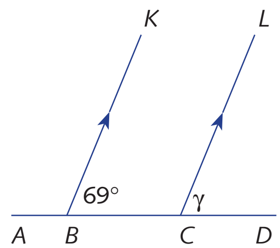
d



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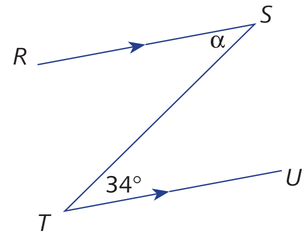

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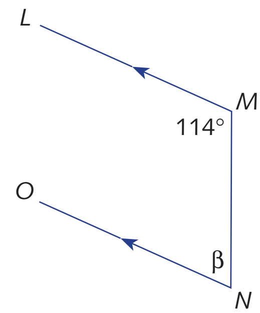

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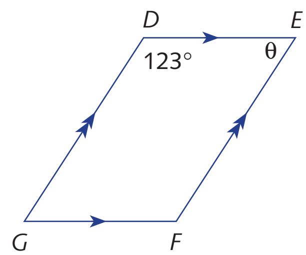

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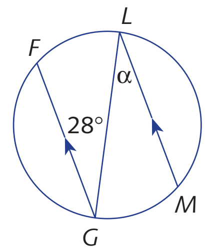

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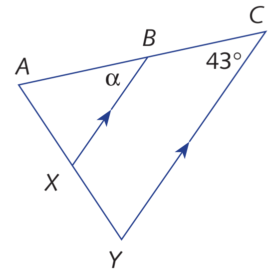

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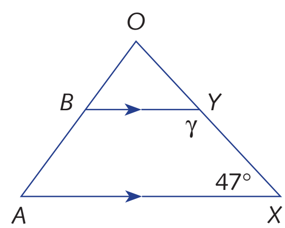

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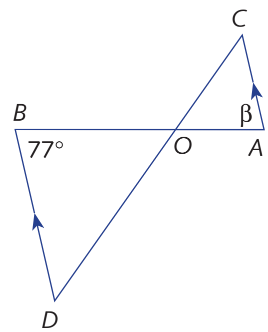

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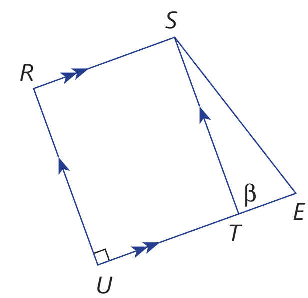
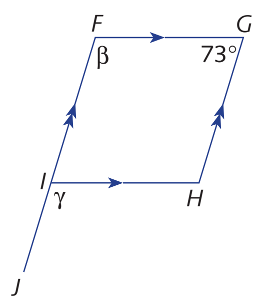


Example 8

3 Find the values of α, β, γ and θ in the diagrams below, giving reasons.

a



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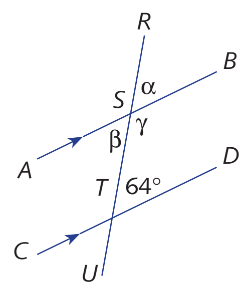

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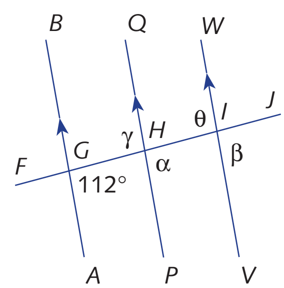

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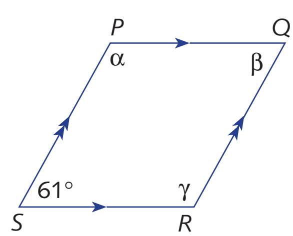
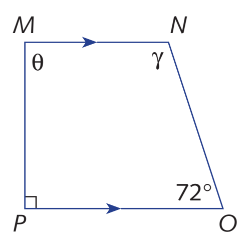


e



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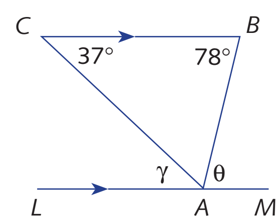

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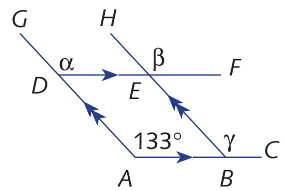

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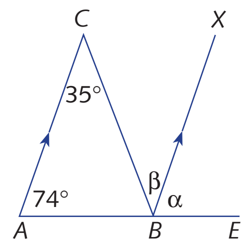

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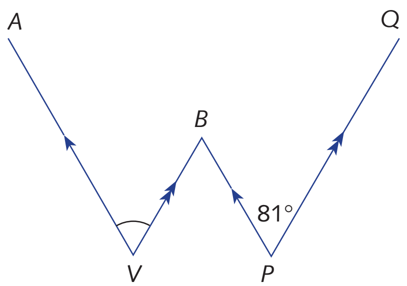
4 Find the size of the marked angle $\angle AVB$ in each diagram below. The solution to each part will require two steps, each with its own reason.

a



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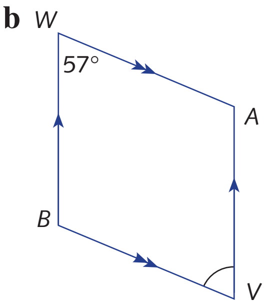

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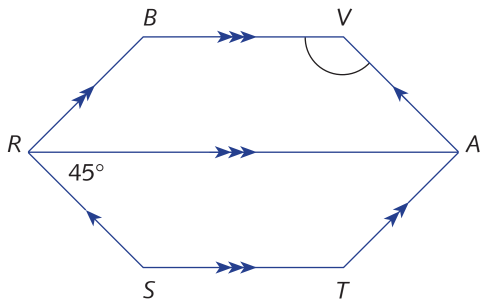
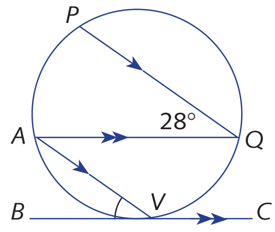


d



5C Further problems involving parallel lines

This section deals with more complicated problems involving parallel lines:

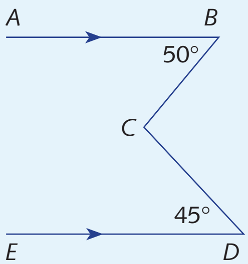
- problems where construction lines need to be added
- problems involving algebra.

Adding construction lines to solve a problem

Some problems cannot be solved until one or more extra lines, called **construction lines**, have been added to the diagram, as in the example below.

Example 9

Find $\angle BCD$ in the diagram opposite.



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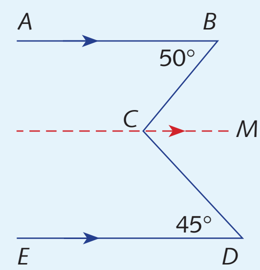
Solution

Construct the line CM through C parallel to AB and ED, as shown in the diagram opposite.

Then $\angle BCM = 50°$ (alternate angles, $AB \parallel CM$),

and $\angle DCM = 45°$ (alternate angles, $ED \parallel CM$).

Hence, $\angle BCD = 95°$ (adjacent angles at C).

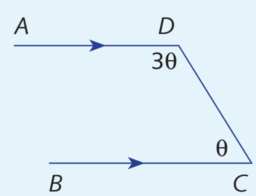


Problems involving algebra

In the example below, the value of θ is found using geometric arguments and algebra.

Example 10

Find θ in the diagram opposite.



Solution

$\theta + 3\theta = 180°$ (co-interior angles, $AD \parallel BC$)

$4\theta = 180°$

$\theta = 45°$

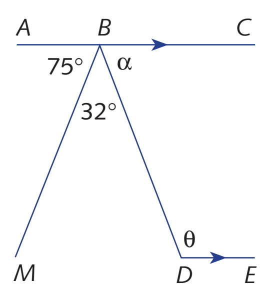
Further problems involving parallel lines

- Extra **construction lines** may need to be added to the diagram.
- Algebra may be required.

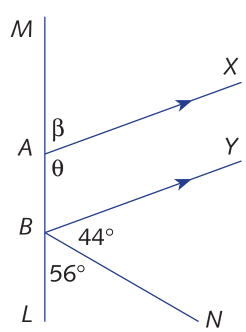
Exercise 5C

1 Find the values of α, β, γ and θ in the diagrams below, giving reasons.

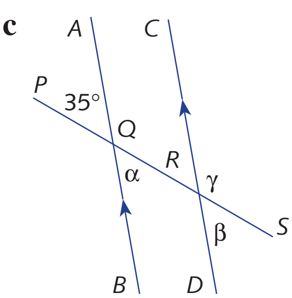
a



b



c



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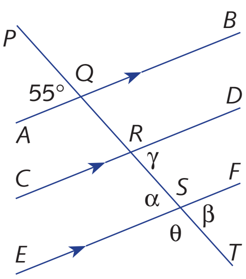
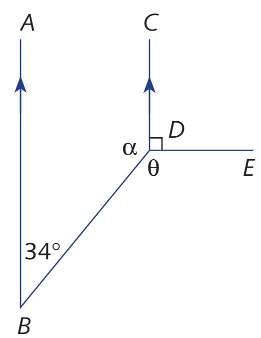
d



e



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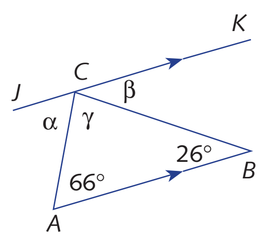

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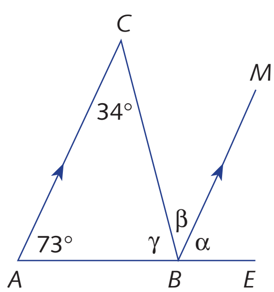
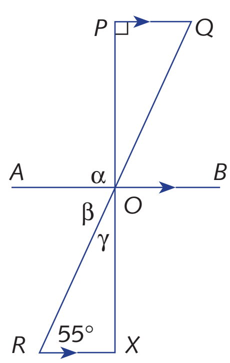
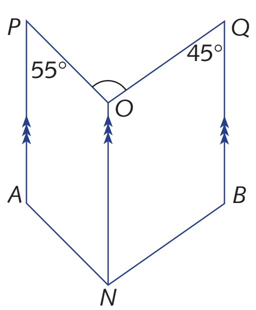


h



2 Find the size of the marked angle $\angle POQ$ in each diagram below. Several steps may be required.

a



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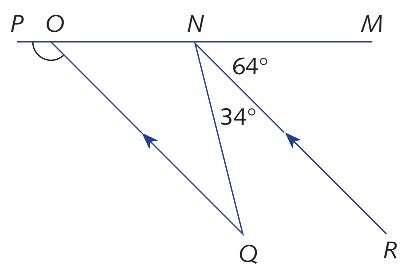
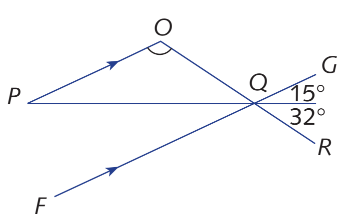


c



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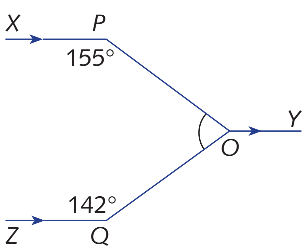
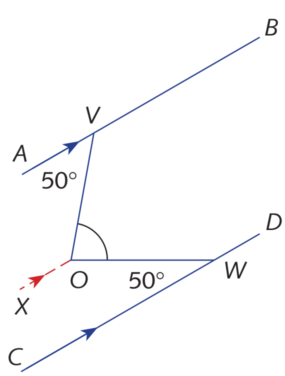


Example 9

3 Copy each diagram, then add a suitable construction line in order to find the size of the marked angle $\angle VOW$ – the first has been done for you. Give reasons.

a



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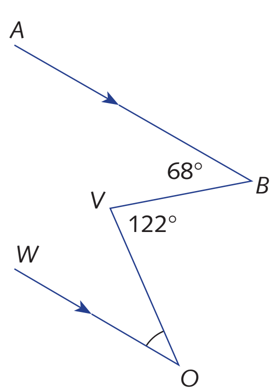

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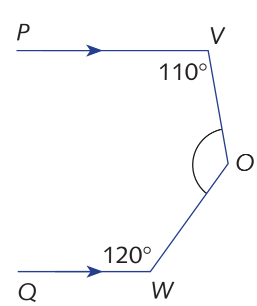

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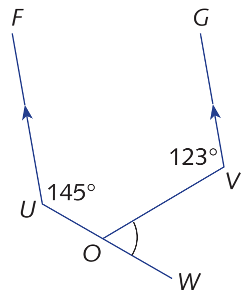
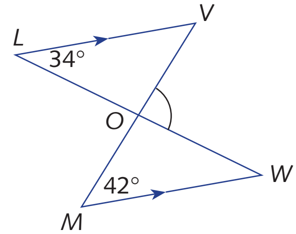
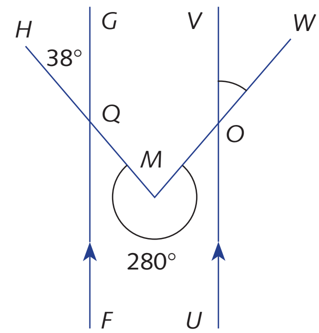
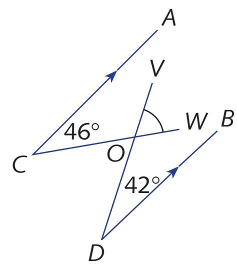
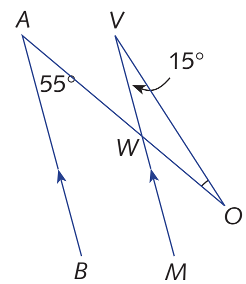
d 

e 

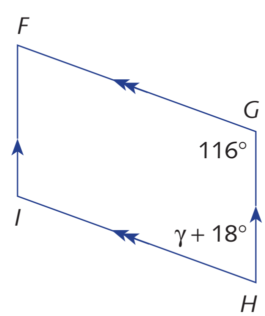
f 

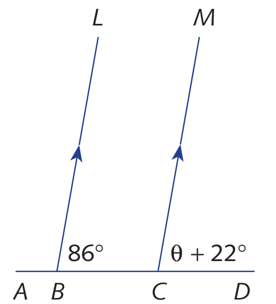
g 

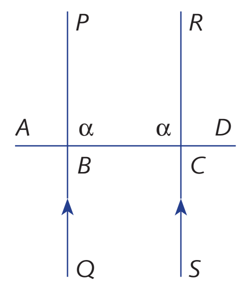
h 

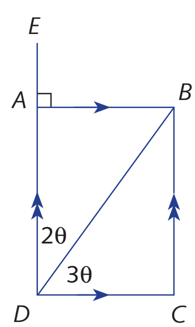
Example 10

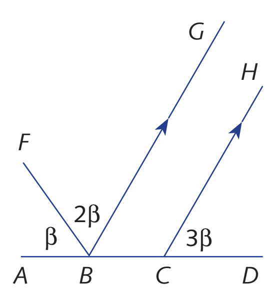
4 Find the values of α, β, γ and θ, giving reasons.

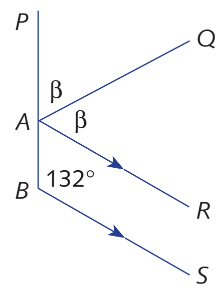
a 

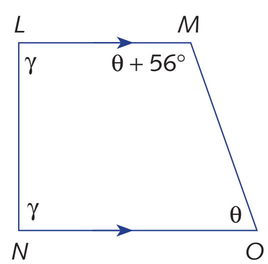
b 

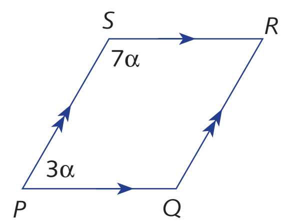
c 

d 

e 

f 

g 

h 

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5D Proving that two lines are parallel

Corresponding, alternate and co-interior angles can be used to prove that two lines are parallel.

Proving that two lines are parallel

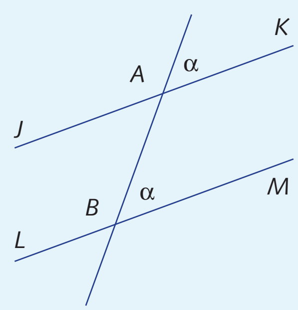
Suppose that a transversal crosses two other lines.

- If the corresponding angles are equal, then the lines are parallel.
- If the alternate angles are equal, then the lines are parallel.
- If the co-interior angles are supplementary, then the lines are parallel.

These three results are called the **converses** of the previous three results, because the logic works in reverse. Here are three examples that show how to set out arguments using them.

Example 11

Find any parallel lines in the diagram opposite.

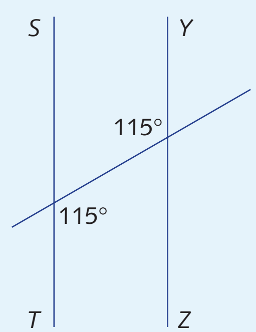


Solution

$JK \parallel LM$ (corresponding angles are equal)

Example 12

Find any parallel lines in the diagram opposite.



Solution

$ST \parallel YZ$ (alternate angles are equal)

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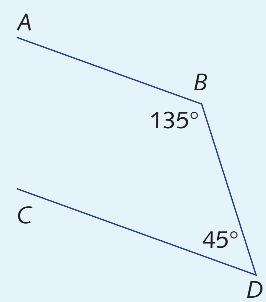
Example 13

Find any parallel lines in the diagram opposite.



Solution

$135° + 45° = 180°$ (co-interior angles are supplementary)

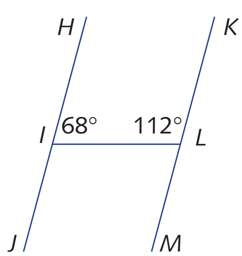
Hence, $AB \parallel CD$

Exercise 5D

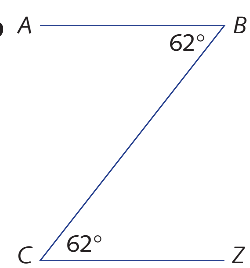
Example 11, 12, 13

1 In each diagram below, name all pairs of parallel lines, giving reasons.

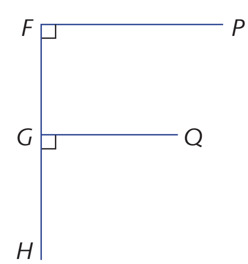
a



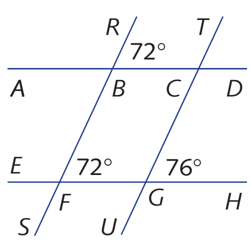
b



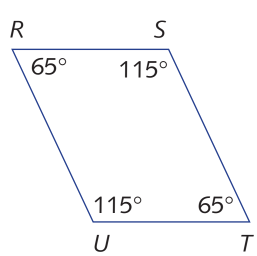
c



d



e



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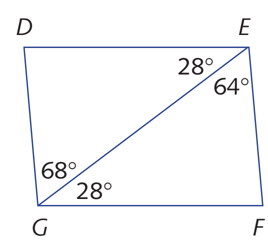

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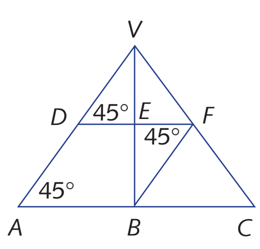

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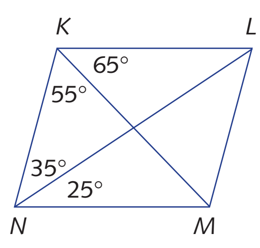

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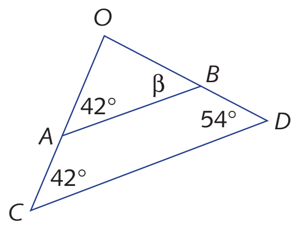
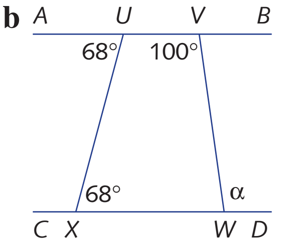
2 In each diagram below, give a reason why $AB \parallel CD$. Hence, find the values of α, β, γ and θ.

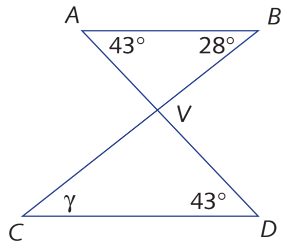
a



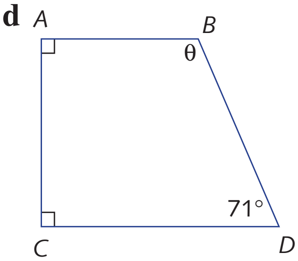
b



c

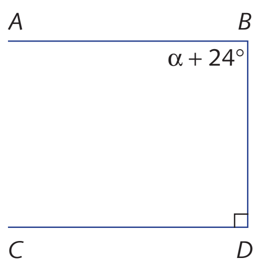


d

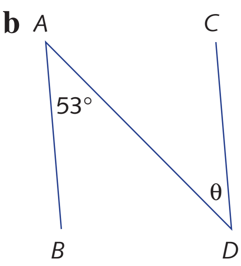


3 In each diagram below, write down the values of α, β, γ and θ that will make AB parallel to CD.

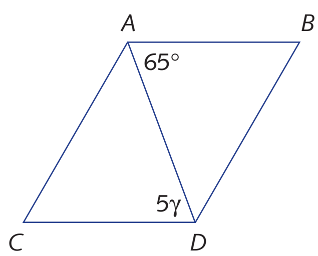
a



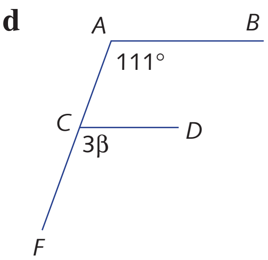
b



c



d

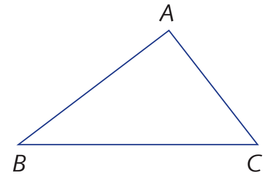


5E Angles in triangles

This section proves and applies two theorems about the angles of any triangle. In mathematics, a true statement and its proof are called a *theorem* – a Greek word meaning 'a thing to be gazed upon' or 'a thing contemplated by the mind'. You may have seen these theorems already, but proving them will probably be new to you.

Triangles

A **triangle** is formed by taking three non-collinear points A, B and C and joining the three intervals AB, BC and CA.



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These intervals are called the **sides** of the triangle, and the three points are called its **vertices**. (The singular of *vertices* is *vertex*.)

Proving the theorem about the angle sum of a triangle

We proved last year that the sum of the interior angles of a triangle is 180°. As we discussed then, checking a theorem many times doesn't prove it – however many triangles you check, there are always more to check. Here is a totally convincing proof that applies to every triangle, whatever its shape.

The theorem and its proof are set out very formally in the manner traditional for geometry. The proof uses only the methods we have already developed about angles at a point and across transversals.

Theorem: The sum of the interior angles of a triangle is 180°.

Proof: Let ABC be a triangle.

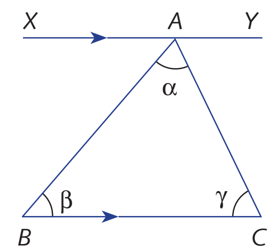
Let $\angle BAC = \alpha$, $\angle ABC = \beta$ and $\angle ACB = \gamma$.

We must prove that $\alpha + \beta + \gamma = 180°$.

Draw the line XY parallel to BC and passing through the vertex A.

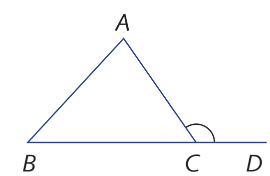
Then $\angle XAB = \beta$ (alternate angles, $XY \parallel BC$), and $\angle YAC = \gamma$ (alternate angles, $XY \parallel BC$).

Hence, $\alpha + \beta + \gamma = 180°$ (straight angle $\angle XAY$).



The exterior angles of a triangle

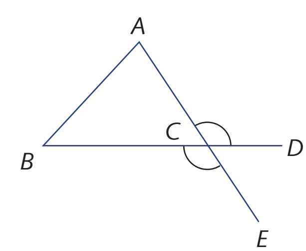
Let ABC be a triangle, with the side BC produced to D. (The word 'produced' means 'extended'.) Then the angle $\angle ACD$ between the side AC and the extension CD is called an **exterior angle** of the triangle.



The angles $\angle CAB$ and $\angle ABC$ are called the **opposite interior angles**, because they are *opposite* the exterior angle at the vertex C.

There are two exterior angles at each vertex, as shown in the lower diagram opposite, and because they are vertically opposite, they are equal in size:

$\angle ACD = \angle BCE$ (vertically opposite at C).



Proving the exterior angle theorem

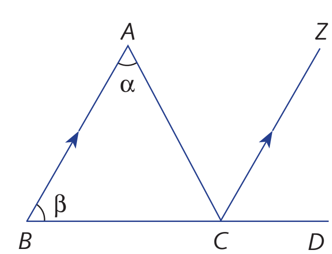
We proved last year that an exterior angle of a triangle is the sum of the opposite interior angles. The following is valid in every situation, whatever shape the triangle may have.

Theorem: An exterior angle of a triangle equals the sum of the two interior opposite angles.

Proof: Let ABC be a triangle, with BC produced to D.

Let $\angle BAC = \alpha$ and $\angle CBA = \beta$.

We must prove that $\angle ACD = \alpha + \beta$.



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Draw the line CZ through C parallel to BA.

$\angle ZCD = \beta$ (corresponding angles, $BA \parallel CZ$), and $\angle ACZ = \alpha$ (alternate angles, $BA \parallel CZ$).

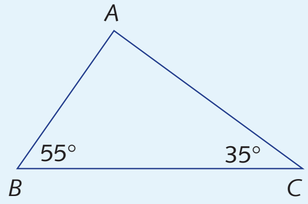
Hence, $\angle ACD = \alpha + \beta$ (adjacent angles at C).

Angles in triangles

- The sum of the interior angles of a triangle is 180°.
- An exterior angle of a triangle equals the sum of the opposite interior angles.

Example 14

Find $\angle A$ in the triangle below.



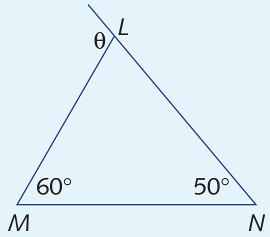
Solution

$\angle A + 55° + 35° = 180°$ (angle sum of ΔABC)

so $\angle A = 90°$

Example 15

Find θ in the triangle below.



Solution

$\theta = 60° + 50°$ (exterior angle of ΔLMN)

so $\theta = 110°$

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Exercise 5E

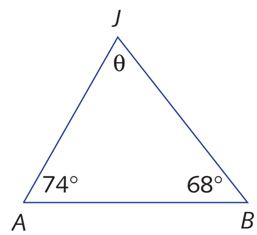
1 Find the third angle of a triangle, two of whose angles are given.

a 60°, 30° **b** 120°, 30° **c** 111°, 11° **d** 62°, 90° **e** 28°, 58°

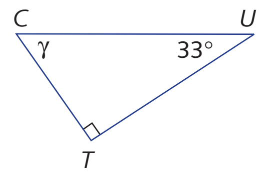
Example 14, 15

2 Find the values of α, β, γ and θ in the diagrams below, giving reasons.

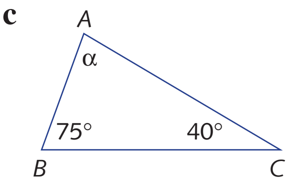
a



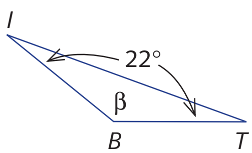
b



c



d



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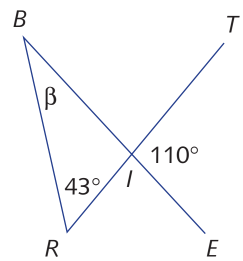

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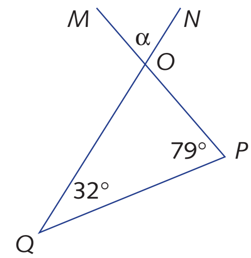

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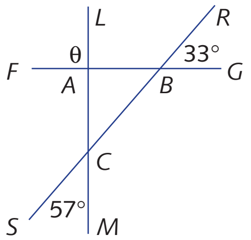

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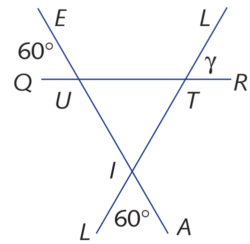

3 Use exterior angles – *not* interior angles – to find α, β, γ and θ in each diagram below. Give reasons.

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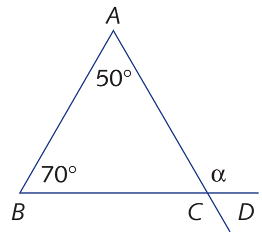

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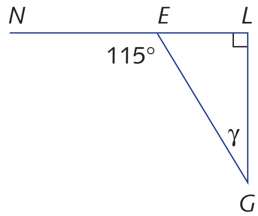

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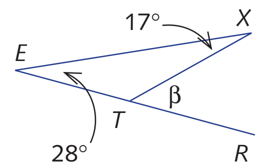

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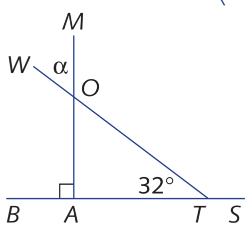

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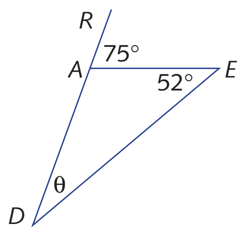

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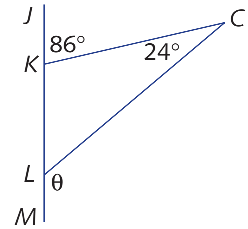

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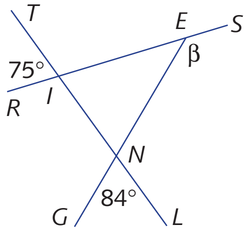

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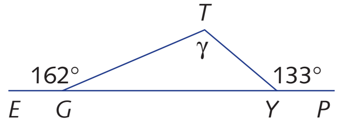

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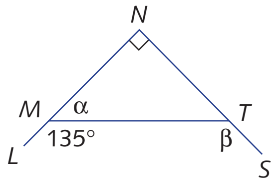
4 Find the values of the pronumerals in each diagram, giving reasons.

a



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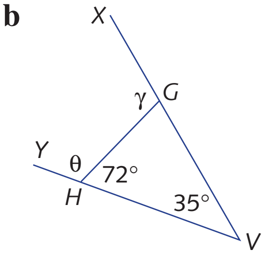

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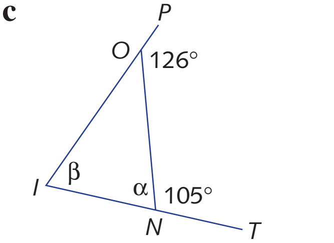

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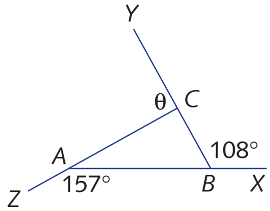

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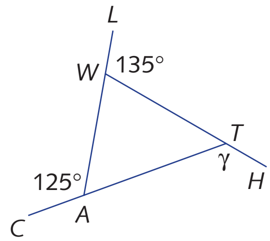

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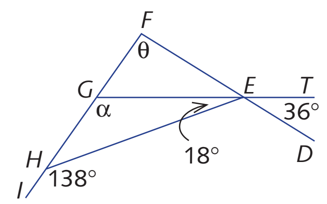

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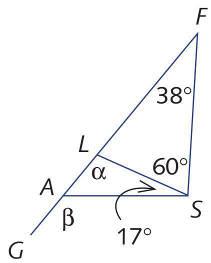

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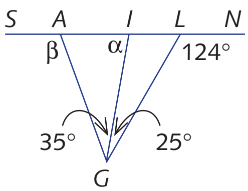
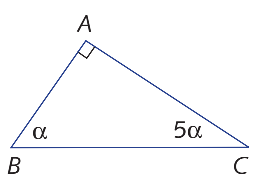


5 Find α, β, γ and θ in these diagrams, giving reasons.

a



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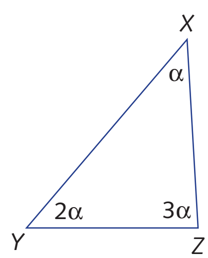

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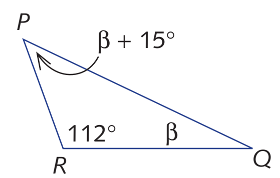

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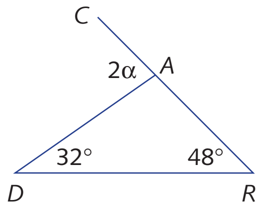

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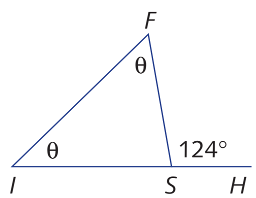

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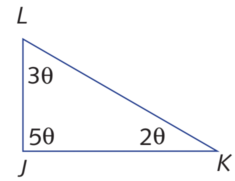

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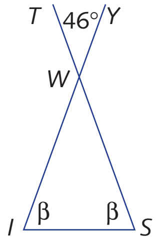

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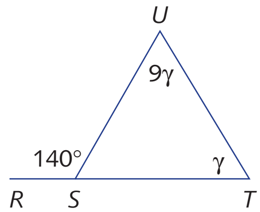

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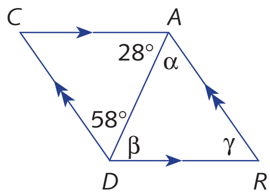
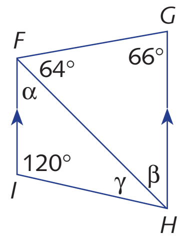
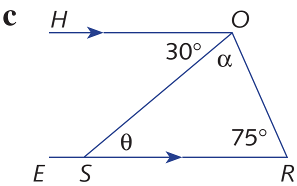
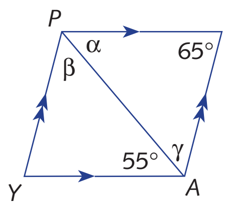
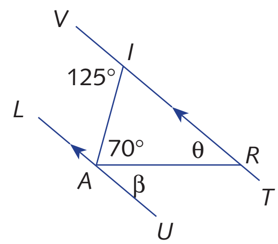
6 Find α, β, γ and θ in these diagrams, giving reasons.

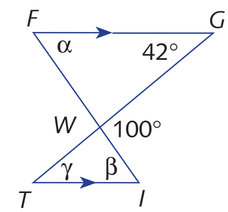
a 

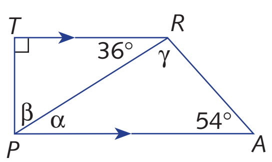
b 

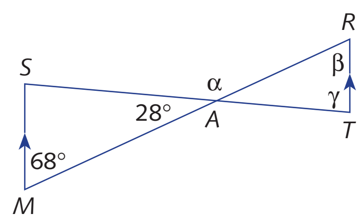
c 

d 

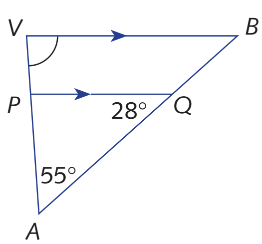
e 

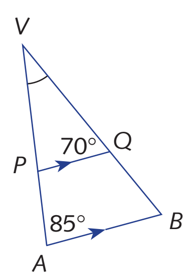
f 

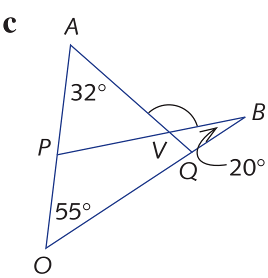
g 

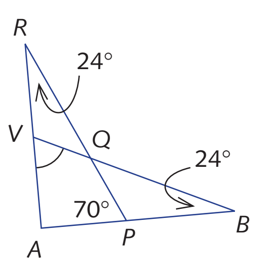
h 

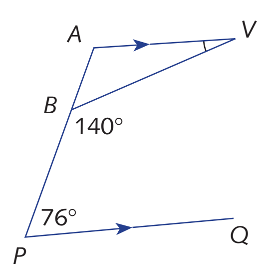
7 Find the size of the marked angle $\angle AVB$ in each diagram, giving reasons.

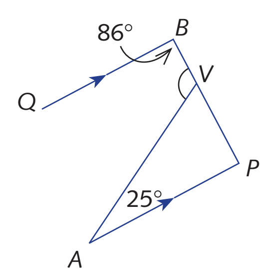
a 

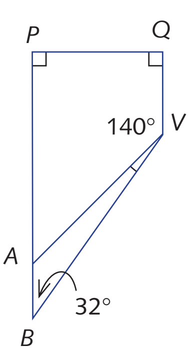
b 

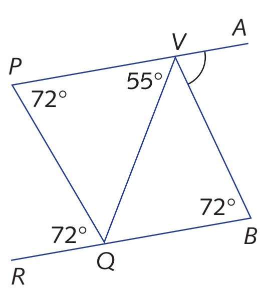
c 

d 

e 

f 

g 

h 

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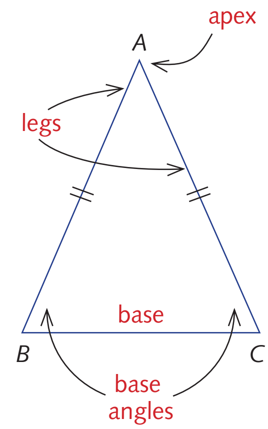
5F Isosceles and equilateral triangles

Isosceles triangles



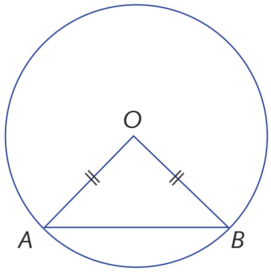
An **isosceles triangle** is a triangle in which two (or more) sides have equal length.

- The equal sides AB and AC in the isosceles triangle ABC to the right are called the **legs**. They have been marked with double dashes to indicate that they are equal in length.
- The vertex A where the legs meet is called the **apex**.
- The third side BC is called the **base**.
- The angles $\angle ABC$ and $\angle ACB$ at the base are called **base angles**.

The word *isosceles* is a Greek word meaning 'equal legs' – *iso* means 'equal', and *sceles* means 'legs'.

Constructing an isosceles triangle using a circle

In the diagram below, the equal radii AO and BO of the circle form the equal legs of the isosceles triangle OAB. The interval AB is the base and the centre O is the apex.



Using two radii of a circle is usually the simplest way to construct an isosceles triangle.

Isosceles triangles and their base angles

Last year, in Chapter 13 of *ICE-EM Mathematics Year 7* we developed in an informal way two results about isosceles triangles. The first is a property that all isosceles triangles have.

- The base angles of an isosceles triangle are equal.
  (This result can also be stated as 'If two sides of a triangle are equal, then the angles opposite those sides are equal.')

The second is a test for a triangle to be isosceles, and is the converse of the first result.

- If two angles of a triangle are equal, then the sides opposite those angles are equal.

In Chapter 12 we will introduce *congruence*, which will allow us to prove these theorems properly.

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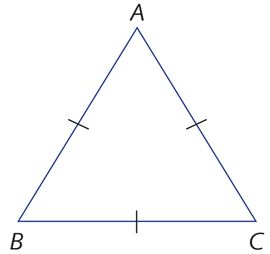
Isosceles triangles

- An **isosceles triangle** is a triangle with two (or more) sides equal.
  - The equal sides are called the **legs**; the legs meet at the **apex**; the third side is the **base**; and the angles opposite the legs are called **base angles**.
  - Two radii of a circle and the interval joining them form an isosceles triangle.
- The base angles of an isosceles triangle are equal.
- Conversely, if two angles of a triangle are equal, then the sides opposite those angles are equal.

Equilateral triangles

An **equilateral triangle** is a triangle in which all three sides have equal length. (The name comes from Latin – *equi* means 'equal', and *latus* means 'side'.)

The diagram below shows an equilateral triangle ABC. Notice how it is an isosceles triangle in three different ways, because the base could be taken as AB, BC or CA.



The interior angles of an equilateral triangle are all 60°

It is not hard to prove that all the angles of an equilateral triangle are 60° provided that we use the earlier theorem that the base angles of an isosceles triangle are equal.

Theorem: The interior angles of an equilateral triangle are all 60°.

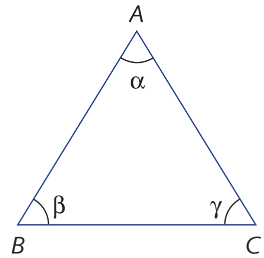
Proof: Let ABC be an equilateral triangle.

Let $\angle BAC = \alpha$, $\angle ABC = \beta$ and $\angle ACB = \gamma$.

We must prove that $\alpha = \beta = \gamma = 60°$.

$\beta = \gamma$ (opposite sides AC and AB are equal), and $\beta = \alpha$ (opposite sides AC and BC are equal), so $\alpha = \beta = \gamma$.

But $\alpha + \beta + \gamma = 180°$ (angle sum of $\angle ABC$), so $\alpha = \beta = \gamma = 60°$.



Equilateral triangles

- An equilateral triangle is a triangle with all three sides equal.
- The angles in an equilateral triangle are all 60°.

Here are examples of arguments using each of the three theorems in this section, and the fact that all the radii of any circle are equal.

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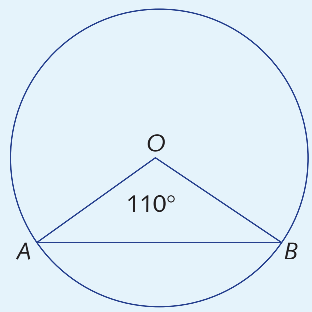
Example 16

Find $\angle OBA$ in the diagram opposite.



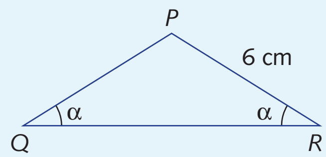
Solution

$AO = BO$ (radii)

Hence, $\angle OBA = 35°$ (base angles of isosceles ΔABO)

Example 17

Find the length of PQ in the diagram opposite.

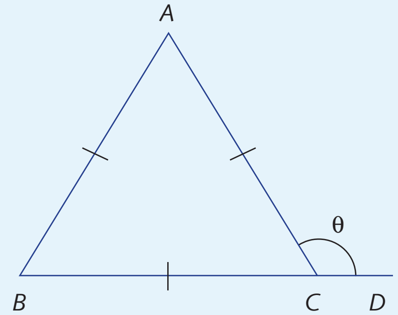


Solution

$PQ = 6$ cm (opposite angles $\angle PQR$ and $\angle PRQ$ are equal)

Example 18

Find θ in the diagram opposite.



Solution

$\angle ABC = 60°$ (equilateral ΔABC)

Hence, $\theta + 60° = 180°$ (straight angle $\angle BCD$)

so $\qquad \theta = 120°$

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Exercise 5F

Example 18

1 Find the value of each pronumeral, giving reasons.

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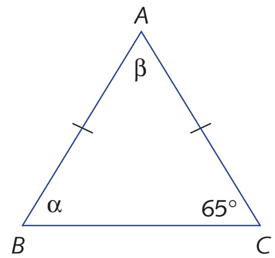

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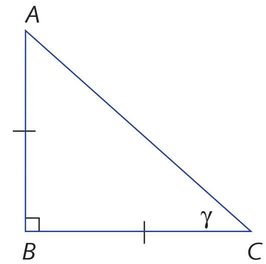

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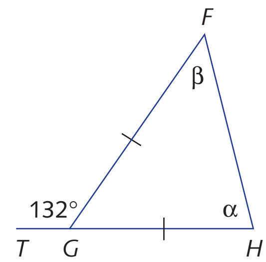

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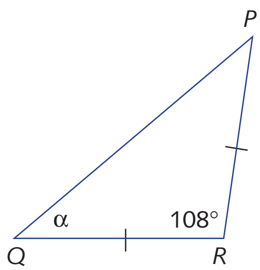

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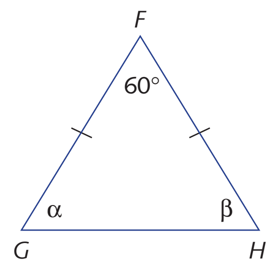

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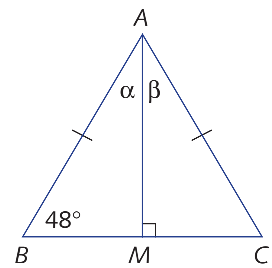
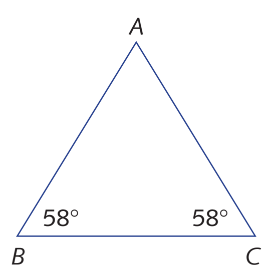


2 Determine whether ΔABC in each part is an isosceles triangle, naming the equal sides. Give reasons.

a


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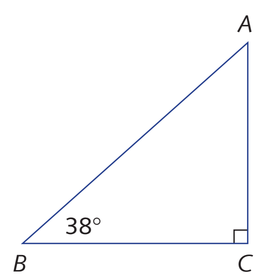

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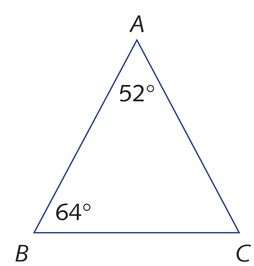

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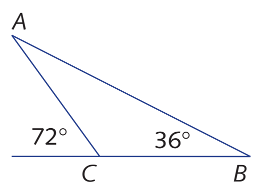

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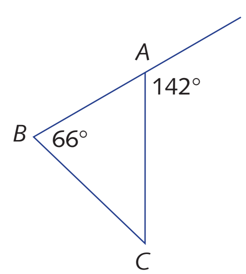

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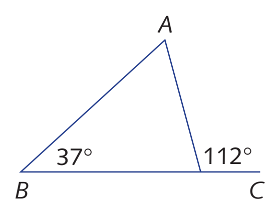
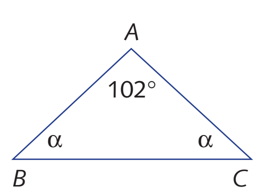


3 **a** Let ΔABC be isosceles, with $AB = AC$. Produce BC to D, and let $\angle ACD = 124°$. Draw a diagram and find $\angle ABC$, giving reasons.

b P and Q are points on a circle with centre O such that $\angle POQ = 48°$. Draw a diagram and calculate the size of $\angle OPQ$.

4 Find the values of the pronumerals in these diagrams, giving reasons.

a


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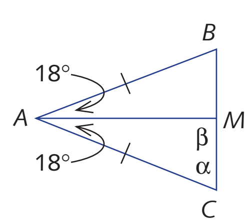

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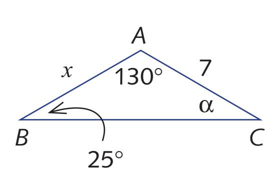

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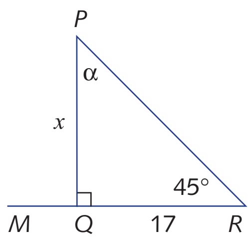
d



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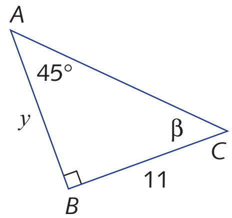

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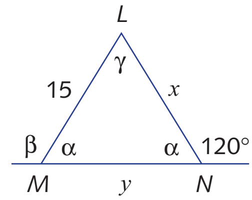
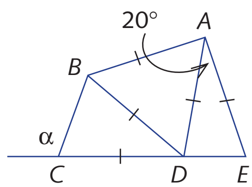


5 Find the values of the pronumerals in these diagrams, giving reasons. In part **d**, O is the centre of the circle.

a



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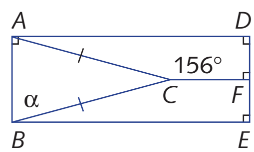

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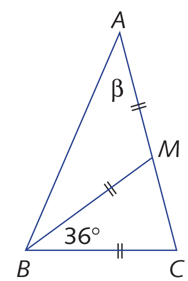

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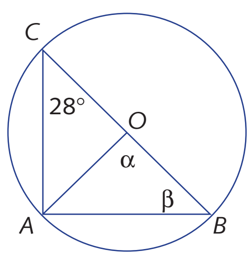

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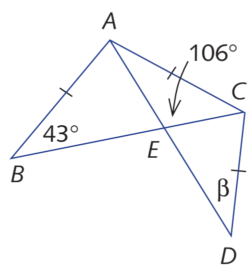

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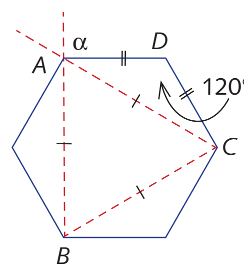
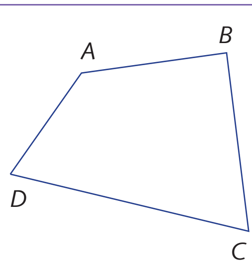
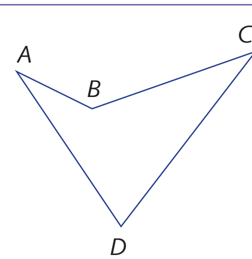


5G Quadrilaterals

Take any four points A, B, C and D in the plane, no three of which are collinear, and join the intervals AB, BC, CD and DA. Provided that no two of these intervals cross over each other, the figure $ABCD$ is called a *quadrilateral* (from Latin – *quadri* means 'four', and *latus* means 'side'). The four points are its **vertices** and the four intervals are its **sides**.



The quadrilateral above is called a **convex quadrilateral** because none of its four interior angles is a reflex angle.



The quadrilateral above is called a **non-convex quadrilateral** because one of its interior angles ($\angle ABC$) is a reflex angle.

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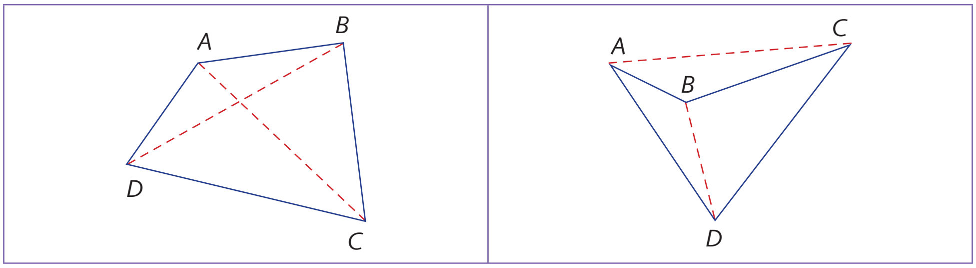
(*Note*: No interior angle can be exactly 180° because the definition requires that no three of the vertices be collinear.)



The intervals AC and BD joining non-adjacent vertices are called the **diagonals** of the quadrilateral. The left-hand quadrilateral above is convex and so both diagonals are inside the quadrilateral. The right-hand quadrilateral above is non-convex and one diagonal is inside the quadrilateral and the other is outside.

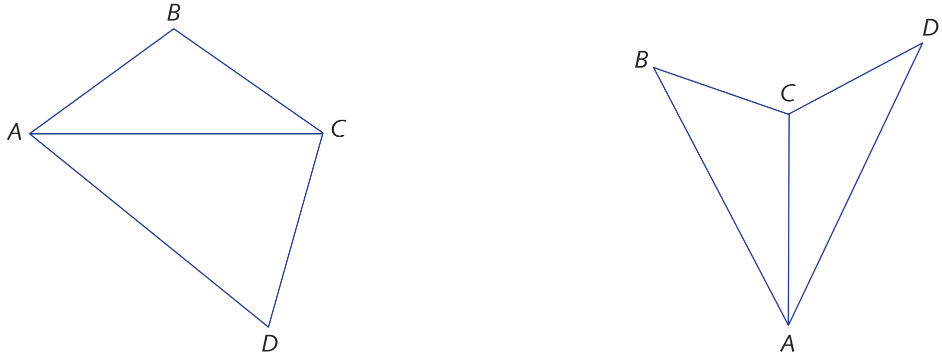
The interior angles of a quadrilateral add to 360°

The interior angles of a quadrilateral always add to 360°.

The general proof divides the quadrilateral into two triangles and then applies the earlier theorem about the angle sum of a triangle.

Theorem: The sum of the interior angles of a quadrilateral is 360°.

Proof: Let $ABCD$ be a quadrilateral, labelled so that the diagonal AC is inside the quadrilateral.
(At least one diagonal lies inside the quadrilateral.)



We must prove that $\angle ABC + \angle BCD + \angle CDA + \angle DAB = 360°$.
Join the diagonal AC.
The interior angles of ΔABC add to 180°.
Similarly, the interior angles of ΔADC add to 180°.
But the sum of the interior angles of $ABCD$ is the sum of the angles of ΔABC and ΔADC.
Hence, the interior angles of $ABCD$ add to 360°.

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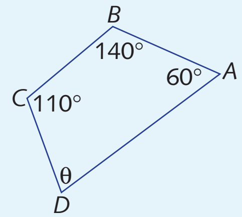
Example 19

Find θ in the diagram opposite.



Solution

$$\begin{aligned}\theta + 60° + 110° + 140° &= 360° \quad \text{(angle sum of quadrilateral } ABCD\text{)}\\ \theta + 310° &= 360°\\ \theta &= 50°\end{aligned}$$

Quadrilaterals

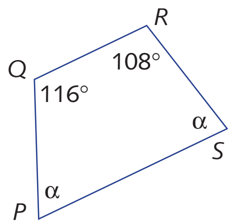
- A **quadrilateral** ABCD has four **vertices**, no three of which are collinear, and four **sides**, no two crossing each other.
- A **convex quadrilateral** has no reflex interior angle. Both **diagonals** of a convex quadrilateral lie inside the figure.
- A **non-convex quadrilateral** has one reflex interior angle. One diagonal of a non-convex quadrilateral lies outside the figure, the other inside.
- The sum of the interior angles of a quadrilateral is 360°.

Exercise 5G

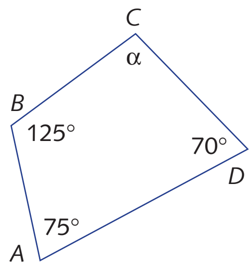
Example 19

1 Find the values of the pronumerals in these diagrams, giving reasons.

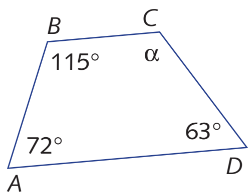
a



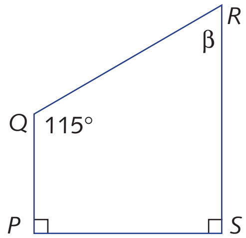
b



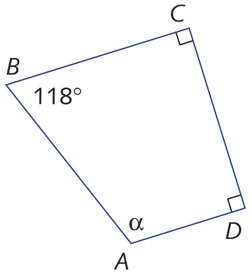
c



d



e



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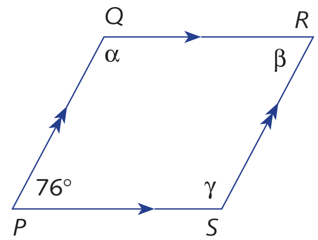
2 In each part, three angles of a quadrilateral are given. Calculate the size of the fourth angle of the quadrilateral. Sketch a quadrilateral with these angle sizes.

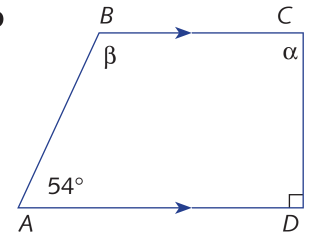
a $90°, 90°, 150°$ **b** $45°, 90°, 135°$ **c** $70°, 160°, 20°$ **d** $40°, 40°, 40°$

3 Find the values of the pronumerals, giving reasons.

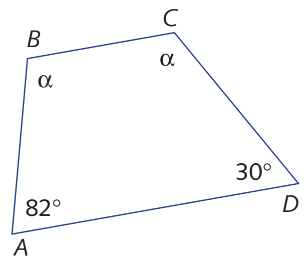
a



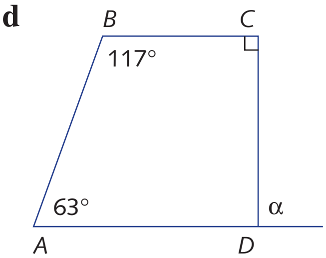
b



c

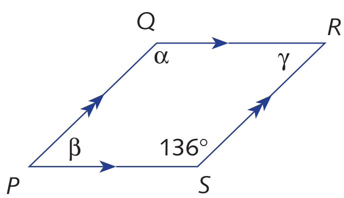


d

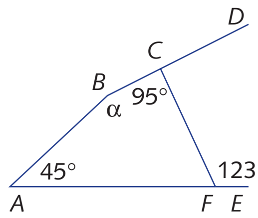


4 Find the values of the pronumerals, giving reasons. In part **f**, O is the centre of the circle.

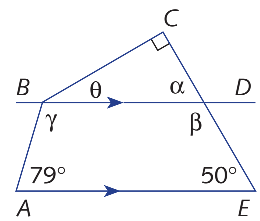
a



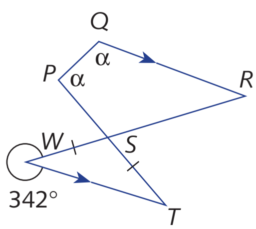
b



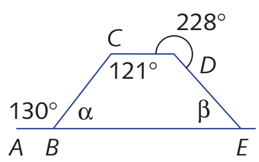
c



d



e



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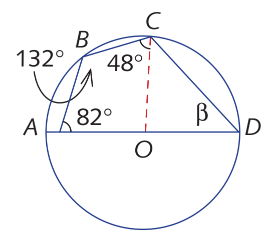
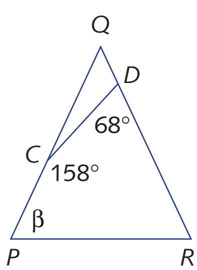


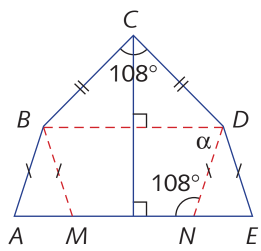
5 Find the values of the pronumerals, giving reasons.

a

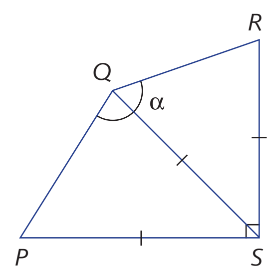


$QP = QR$

b



c



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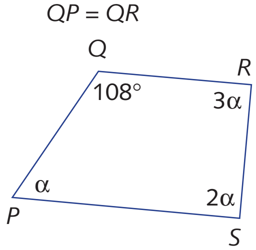

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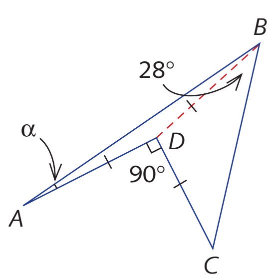

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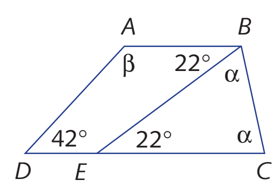

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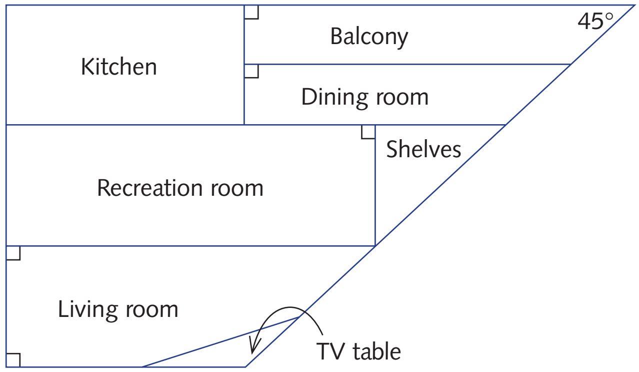
6 The following diagram is a map of the first floor of Tony's house.

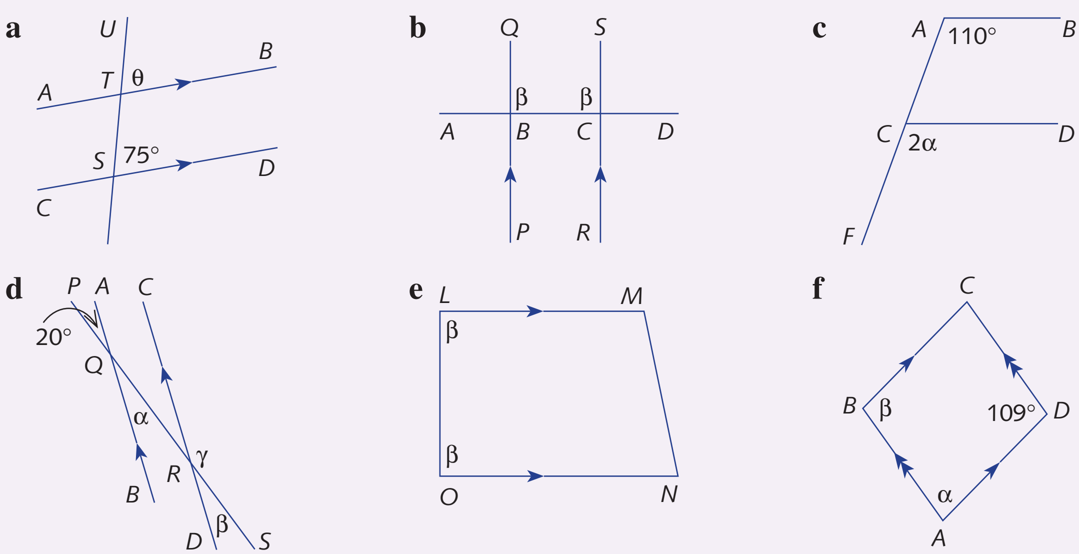


a Tony would like to build some wooden shelves at the end of the recreation room. Each shelf would be an isosceles triangle. Evaluate the internal angles of these shelves.

b Tony would also like to construct a TV table in the shape of an isosceles triangle. Evaluate the internal angles of the TV table.

Review exercise

1 Find the value of the pronumeral in each diagram. Give reasons in each case.



2 State the complement of each angle.

a 30° **b** 63° **c** 74° **d** 84°

3 State the supplement of each angle.

a 127° **b** 76° **c** 134° **d** 15°

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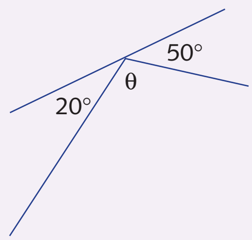
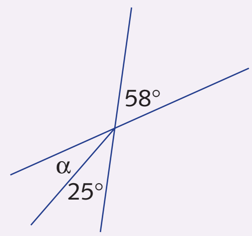
4 Find the value of each pronumeral.

a



b



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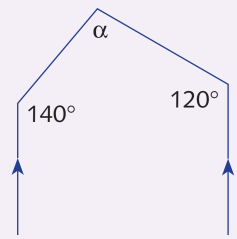

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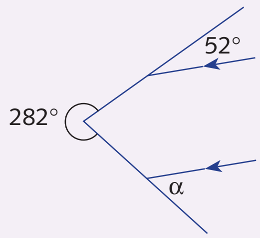

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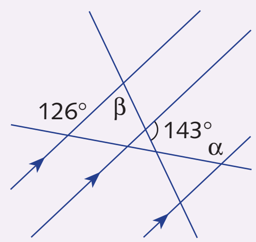

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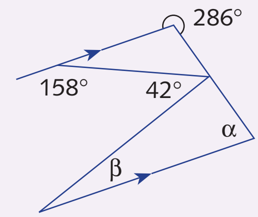
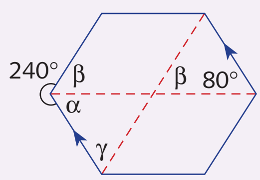


g



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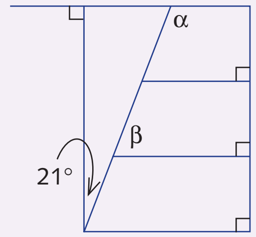

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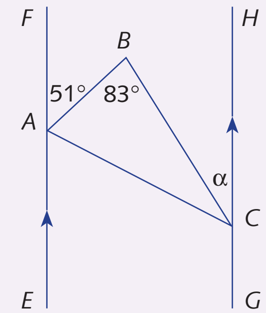
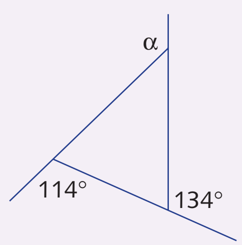
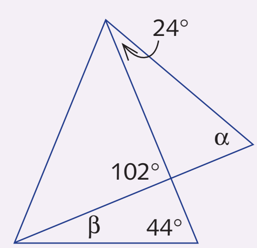


5 Find the values of the pronumerals in these diagrams.

a



b



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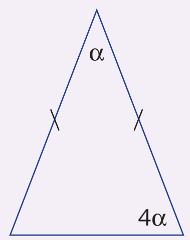
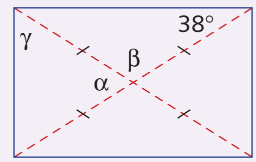
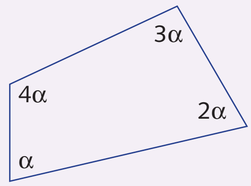


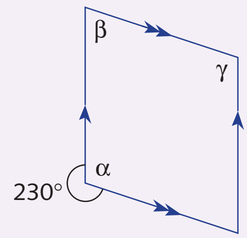
d



e



f



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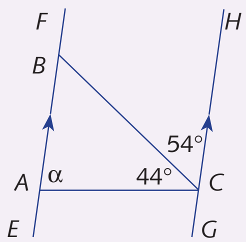
6 Find the value of α in each of these diagrams.

a



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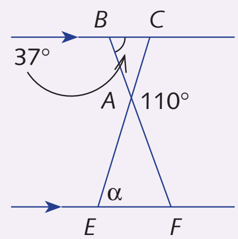
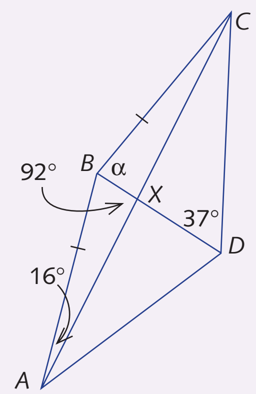


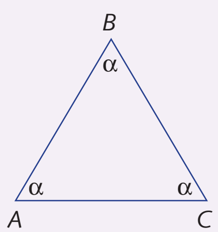
c



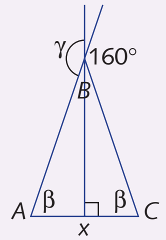
7 Explain why a triangle cannot have a reflex angle as one of its internal angles.

8 Find the values of α, β and γ in the diagrams below, and explain your reasoning.

a



b



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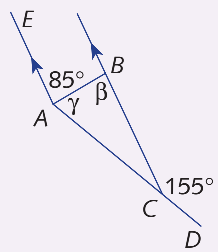
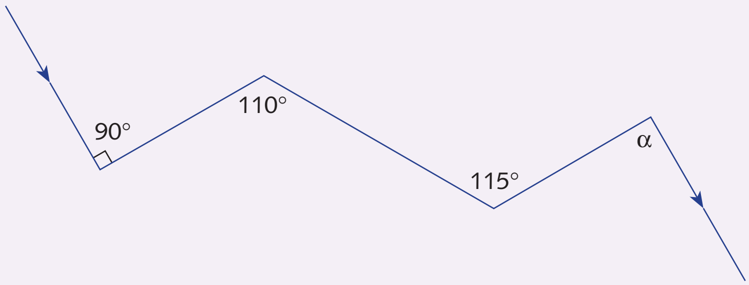
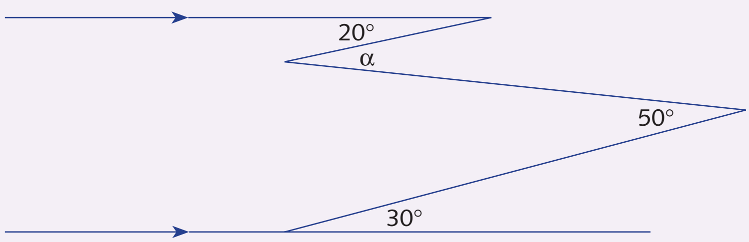


Challenge exercise

1 Calculate the unknown angle.



2 **a** Find the value of α.



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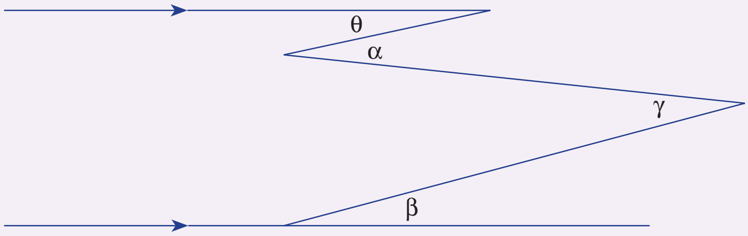
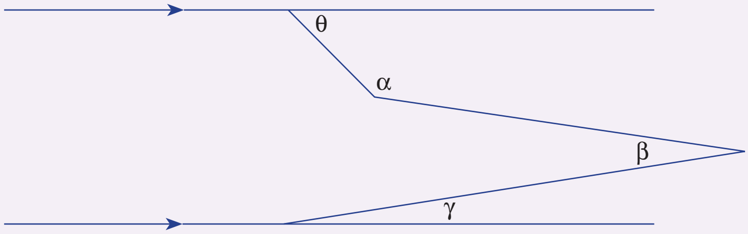
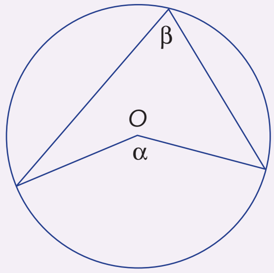
b Find α in terms of β, γ and θ.



c Find α in terms of β, γ and θ.

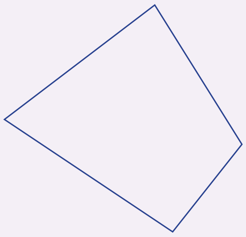


3 The point *O* is the centre of the circle. Find α in terms of β.

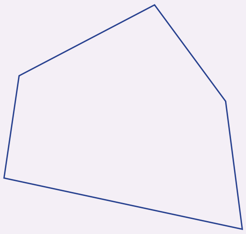


4 The sum of the internal angles of a triangle is 180°. Use this property of a triangle to find the sum of the internal angles in each of the diagrams below. What relationship do you notice?

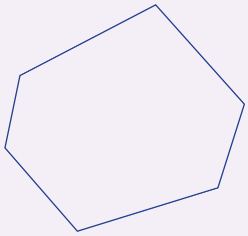
a



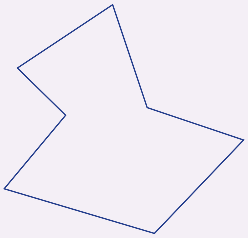
b



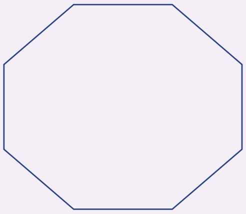
c



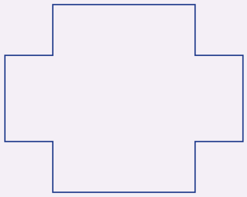
d



e



f



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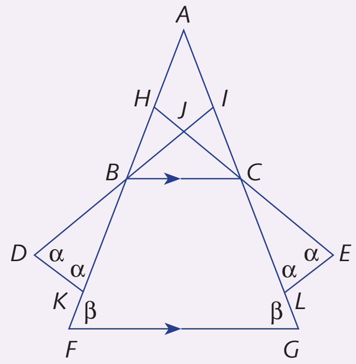
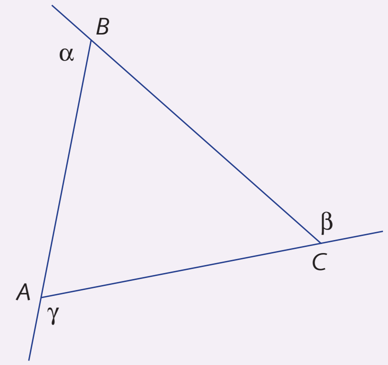
5 Name all the isosceles triangles in the diagram below.



6 Consider the isosceles triangle ABC, with $AB = AC$. Suppose D is on AB, and E is on AC, so that $AD = DE$. If $\angle CED = 142°$, calculate $\angle ABC$.

7 $ABCD$ is a square and AEB is an equilateral triangle. Calculate the two possible values of the size of $\angle DEC$.

8 Find the value of $\alpha + \beta + \gamma$ in the diagram below.



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CHAPTER

6

Number and Algebra

Algebra – part 1

Algebra is an important part of mathematical language. It helps us to state ideas more simply. It also enables us to make general statements about mathematics and solve problems that are difficult to do otherwise.

Algebra was introduced to the Arabs in about 830 CE by Abu Abdullah Muhammad bin Musa al-Khwarizmi. The last part of his name gives us the word algorithm. His work was also influential in the introduction of algebra into Europe in the early thirteenth century.

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6A Notation and substitution

In algebra, letters are used to stand for numbers. For example, if a box contains x stones and you put five more in, then there are $x + 5$ stones in the box. You may or may not know what the value of x is. In algebra, we call the letter that represents a number a **pronumeral**.

Substitution involves giving a value to the letter.

For example, if $x = 20$, then $x + 5 = 25$.

We first recall some conventions and notation in algebra.

Multiplication:	$5 \times x = 5x$	$a \times b = ab$	
Division:	$x \div 5 = \frac{x}{5}$	$a \div b = \frac{a}{b}$	
Powers:	$x \times x = x^2$	$a \times a \times a = a^3$	$z \times z \times z \times z = z^6$

The following table reminds us of the meanings of some commonly occurring algebraic expressions.

$2x + 3$	The number x is multiplied by 2, and 3 is added to the result.
$5x - 3$	The number x is multiplied by 5, and 3 is subtracted from the result.
$3(x - 1)$	One is subtracted from the number x, and the result is multiplied by 3.
$x^2 + 4$	The number x is multiplied by itself, and 4 is added to the result.
$\frac{x}{5} + 6$	The number x is divided by 5, and 6 is added to the result.
$\frac{x + 5}{6}$	5 is added to the number x, and the result is divided by 6.

In algebra we write:

$x^1 = x \qquad 0 + x = x \qquad 1x = x$

Example 1

a For $x = 2$, find $x - 3$.

b For $x = -3$, find $2x + 3$.

Solution

a When $x = 2$:

$$\begin{aligned} x - 3 &= 2 - 3 \\ &= -1 \end{aligned}$$

b When $x = -3$:

$$\begin{aligned} 2x + 3 &= 2 \times (-3) + 3 \\ &= -6 + 3 \\ &= -3 \end{aligned}$$

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Example 2

For $m = -1$ and $n = \frac{1}{2}$, evaluate:

a $m + n$ **b** $m - n$ **c** $2m - n$ **d** $m^2 + n^2$

Solution

a $m + n = -1 + \frac{1}{2}$
$= -\frac{1}{2}$

b $m - n = -1 - \frac{1}{2}$
$= -1\frac{1}{2}$

c $2m - n = 2 \times (-1) - \frac{1}{2}$
$= -2 - \frac{1}{2}$
$= -2\frac{1}{2}$

d $m^2 + n^2 = (-1)^2 + \left(\frac{1}{2}\right)^2$
$= 1 + \frac{1}{4}$
$= 1\frac{1}{4}$

Example 3

For $m = 2$, evaluate:

a m^2 **b** $(-m)^2$ **c** $-m^2$

d m^3 **e** $(-m)^3$ **f** $(3m)^2$

Solution

a $m^2 = 2^2$
$= 4$

b $(-m^2) = (-2)^2$
$= -2 \times -2$
$= 4$

c $-m^2 = -2^2$
$= -4$

d $m^3 = 2^3$
$= 8$

e $(-m)^3 = (-2)^3$
$= (-2) \times (-2) \times (-2)$
$= -8$

f $(3m)^2 = (3 \times 2)^2$
$= 6^2$
$= 36$

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Exercise 6A

Example 1

1 Substitute 2 for x in each expression and evaluate.

a $x+5$ **b** $4x$ **c** $\frac{x}{2}$
d $\frac{7x}{2}$ **e** $3(x+4)$ **f** $2(x-1)$
g $\frac{x+8}{2}$ **h** $\frac{9-x}{4}$

2 Substitute –6 for x in each expression and evaluate.

a $x+3$ **b** $-2x$ **c** $\frac{x}{6}$
d $\frac{x}{2}$ **e** $2(x+5)$ **f** $2(x-5)$
g $\frac{x+8}{2}$ **h** $5(x-4)$

3 Evaluate each expression when $n=-3$.

a n^2 **b** $(-n)^2$ **c** $-n^2$
d n^3 **e** $(-n)^3$ **f** $(3n)^2$
g $3n^2$

Example 2, 3

4 Evaluate each expression when $a=-2$, $b=5$ and $c=-1$.

a $a+b$ **b** a^2+c **c** $a-c$
d c^5 **e** a^2+b^2 **f** $a-b$
g a^2-c^2 **h** $-c$ **i** $\frac{b}{c}$
j $\frac{b}{c^3}$ **k** $(2c)^2$ **l** $(2c)^3$

5 Evaluate each expression when $a=-2$, $b=5$ and $c=-\frac{1}{3}$.

a c^2 **b** a^2-c^2 **c** $a-b$ **d** $b-a$

6 Evaluate each expression when $x=-2$ and $y=-1$.

a $-(2x)$ **b** $-xy$ **c** $2x+3y$ **d** $\frac{x}{y}$
e $\frac{6x}{y}$ **f** x^2+y^2 **g** $x+y$ **h** $-5x+6y$
i $-3y$ **j** x^2y **k** x^3+y^3

7 Evaluate each expression when $a=3$, $b=5$ and $c=-1$.

a $a+b+c$ **b** $a-c+b$ **c** $a^2+b^2+c^2$
d $a^2-b^2-c^2$ **e** $(-a)^2+b^3+c$ **f** $c-a-b$
g $c-b+a^2$ **h** $-(2a)^2+b^3+c$ **i** $(-3a)^3+2a-c$

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6B Solving equations

Joe has a pencil case that contains a number of pencils. He has three other pencils.

We do not know how many pencils there are in the pencil case but we do know he has a total of 11 pencils. Let x be the number of pencils in the pencil case. Then the facts we are given tell us that:

$$x + 3 = 11$$

This statement is called an **equation** and we need to **solve** the equation for x. The solution is $x = 8$.

Reading equations

The equation $2x + 4 = 10$ can be read as 'two times a number x plus 4 is equal to 10'.

The instruction 'Solve the equation $2x + 4 = 10$' can also be read as 'A number x is multiplied by 2, and then 4 is added. The result is 10. Find the number.'

The number in this case is 3. This can be checked by $2 \times 3 + 4 = 10$.

Equivalent equations

Consider these equations

$2x + 3 = 9$ ①

$2x + 5 = 11$ ②

Equation ② is obtained from equation ① by adding 2 to each side of the equation. Equation ① is obtained by subtracting 2 from both sides of equation ②. Equations ① and ② are said to be **equivalent** equations.

Equation ③ below is obtained from equation ② by subtracting 5 from each side of the equation.

$2x = 6$ ③

$x = 3$ ④

Equation ④ is obtained from equation ③ by dividing each side of the equation by 2. You can obtain equation ③ from equation ④ by multiplying each side by 2. All of the above equations are **equivalent**.

Equivalent equations

- If we add the same number to, or subtract the same number from, both sides of an equation, the new equation is **equivalent** to the original equation.
- If we multiply or divide both sides of an equation by the same non-zero number, the new equation is equivalent to the original equation.
- Equivalent equations have exactly the same solutions.

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Example 4

Solve each equation for x and check that LHS = RHS.

a $x + 3 = 5$ **b** $3x = 23$ **c** $2x - 3 = 11$ **d** $\frac{3x}{4} = 10$

Solution

a

$$\begin{aligned} x + 3 &= 5 \\ \boxed{-3} \quad x + 3 - 3 &= 5 - 3 \qquad \text{(Subtract 3 from both sides.)} \\ x &= 2 \end{aligned}$$

Check: Left-hand side (LHS) $= 2 + 3$
$= 5$
$=$ right-hand side (RHS)

b

$$\begin{aligned} 3x &= 23 \\ \boxed{\div 3} \quad \frac{3x}{3} &= \frac{23}{3} \qquad \text{(Divide both sides by 3.)} \\ &= 7\tfrac{2}{3} \end{aligned}$$

Check: $\text{LHS} = 3 \times 7\tfrac{2}{3}$
$= 3 \times \frac{23}{3}$
$= 23$
$= \text{RHS}$

c

$$\begin{aligned} 2x - 3 &= 11 \\ \boxed{+3} \quad 2x - 3 + 3 &= 11 + 3 \qquad \text{(Add 3 to both sides.)} \\ 2x &= 14 \\ \boxed{\div 2} \quad x &= 7 \qquad \text{(Divide both sides by 2.)} \end{aligned}$$

Check: $\text{LHS} = 2 \times 7 - 3$
$= 11$
$= \text{RHS}$

d

$$\begin{aligned} \frac{3x}{4} &= 10 \\ \boxed{\times 4} \quad 3x &= 40 \qquad \text{(Multiply both sides by 4.)} \\ \boxed{\div 3} \quad x &= \frac{40}{3} \qquad \text{(Divide both sides by 3.)} \\ &= 13\tfrac{1}{3} \end{aligned}$$

Check: $\text{LHS} = \frac{3 \times 13\frac{1}{3}}{4}$
$= \frac{40}{4}$
$= 10$
$= \text{RHS}$

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From now on we will not include the box to the left in the solutions.

Example 5

Solve each equations for x.

a $2x - 11 = -7$ **b** $6 - 4x = 22$ **c** $\frac{x}{5} + 7 = 22$

Solution

a $2x - 11 = -7$

$2x = -7 + 11$ (Add 11 to both sides.)

$2x = 4$

$x = 2$ (Divide both sides by 2.)

b $6 - 4x = 22$

$-4x = 16$ (Subtract 6 from both sides.)

$x = -4$ (Divide both sides by -4.)

c $\frac{x}{5} + 7 = 22$

$\frac{x}{5} = 15$ (Subtract 7 from both sides.)

$x = 15 \times 5$ (Multiply both sides by 5.)

$= 75$

Example 6

In each case below, write an equation and solve it.

a A number has 7 added to it and the result is 35.

b A number is divided by 11 and 6 is added to it. The result is 13.

Solution

a Let x be the number.

$x + 7 = 35$

$x = 28$ (Subtract 7 from both sides.)

The number is 28.

b Let x be the number.

$\frac{x}{11} + 6 = 13$

$\frac{x}{11} = 13 - 6$ (Subtract 6 from both sides.)

$\frac{x}{11} = 7$

$x = 77$ (Multiply both sides by 11.)

The number is 77.

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Exercise 6B

Example 4

1 Solve these equations. Check your solutions.

a $m+4=11$ b $2a=6$ c $\frac{x}{5}=10$

d $4x+5=16$ e $5x-3=12$ f $2m+8=12$

g $\frac{2x}{3}=12$ h $\frac{3x}{4}=15$ i $2n+12=20$

Example 5

2 Solve these equations. Check your solutions.

a $2x-6=-12$ b $2x+6=-15$ c $2x-3=-8$

d $6-4x=26$ e $5-2x=12$ f $6-5x=10$

g $\frac{x}{3}+4=22$ h $\frac{x}{4}-5=10$ i $\frac{x}{5}+7=15$

3 Solve each equation for m.

a $m+3=6$ b $m-11=7$ c $m+8=-10$

d $m-5=-11$ e $6-m=10$ f $-8+m=6$

g $m+6=3$ h $8-m=-6$ i $m-11=-5$

4 Solve each equation for n.

a $2n=-6$ b $-3n=9$ c $-5n=25$

d $-3n=16$ e $12n=100$ f $18n=46$

g $5n=17$ h $-6n=-50$ i $3n=17$

5 Solve each equation for x. Check your solutions.

a $\frac{x}{3}=12$ b $\frac{x}{2}=15$

c $\frac{x}{5}=-16$ d $\frac{x}{10}=-2$

e $\frac{x}{3}=-6$ f $\frac{x}{4}=-8$

6 Solve each equation for x. Check your solutions.

a $2x+1=7$ b $5x-1=11$ c $7x+3=17$

d $4x+2=18$ e $1-5x=21$ f $5-20x=100$

g $2-10x=44$ h $5x-11=30$ i $10x+23=100$

7 Solve these equations.

a $x-4=5$ b $3a=36$

c $3z-7=17$ d $11b+4=121$

e $\frac{5x}{4}=30$ f $\frac{x}{7}-2=8$

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Example 6

8 In each case, write an equation, and solve it.

a A number a has 5 added to it, and the result is 21.

b A number x is multiplied by 7, and the result is 35.

c A number z is multiplied by 5, and the result is 37.

d A number m is multiplied by 5, and then 3 is added. The result is 50.

e A number n is divided by 6, and the result is 10.

f A number p is divided by 3, and 5 is subtracted from it. The result is 23.

9 Solve each equation for x.

a $x+5=6$ **b** $5x-2=3$ **c** $2x+7=4$ **d** $\frac{4x}{7}=11$ **e** $\frac{5x}{11}=7$

10 In each case, write an equation and solve it.

a A number x is subtracted from 20, and the result is 10.

b A number m is multiplied by 2, and the result is subtracted from 6. The final result is 20.

c A number n is divided by 8, and 6 is added to the result. The final result is 20.

d A number p is multiplied by 7, and then 10 is added. The result is 60.

e A number x is subtracted from 6, and the result is –10.

f A number y is multiplied by 7, and the result is subtracted from 15. The final result is –6.

g A number k is divided by 10, and 7 is subtracted from the result. The final result is –1.

6C Expanding brackets

The **distributive law for multiplication over addition** implies:

$$\begin{aligned}3\times(x+4)&=3\times x+3\times 4\\&=3x+12\end{aligned}$$

The **distributive law for multiplication over subtraction** implies:

$$\begin{aligned}3\times(x-4)&=3\times x-3\times 4\\&=3x-12\end{aligned}$$

We can describe these results in general by using algebra.

The distributive law

- The distributive law for multiplication over addition:

  $a(b+c)=ab+ac$

- The distributive law for multiplication over subtraction:

  $a(b-c)=ab-ac$

This process of rewriting an expression to remove brackets is usually referred to as **expanding brackets**.

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Example 7

Use the distributive law to expand the brackets.

a $5(x-4)$ **b** $4(3x+2)$ **c** $-2(4+x)$ **d** $-4(5-x)$

Solution

a $5(x-4) = 5 \times x - 5 \times 4$
$= 5x - 20$

b $4(3x+2) = 4 \times 3x + 4 \times 2$
$= 12x + 8$

c $-2(4+x) = -8 - 2x$

d $-4(5-x) = -20 + 4x$

Exercise 6C

Example 7

1 Expand brackets.

a $2(3a-1)$ **b** $5(6p+7)$ **c** $4(3-x)$ **d** $3(a-2)$

e $7(6-2x)$ **f** $3(7-x)$ **g** $-4(5-x)$ **h** $-6(7-2p)$

i $-3(5x-2)$ **j** $-4(6x-3)$ **k** $-3(2-x)$ **l** $-2(3x+1)$

2 First, substitute $x = 5$ in each expression and evaluate it. Next, expand the brackets and again substitute $x = 5$.

a $2(x+3)$ **b** $5(2x+4)$ **c** $2(x-2)$ **d** $6(x-3)$

3 Substitute $a = 2$ in each expression and evaluate.

a $3+2(a-2)$ **b** $6(a-2)$ **c** $3+2(5a-2)$

4 Evaluate each expression when $z = -2$.

a $5z-2(z-4)$ **b** $2-3(2-4z)$ **c** $7z+5(2z-4)$

6D Solving equations with brackets

We will now employ the methods introduced in the last section to solve equations that contain brackets.

Example 8

Solve these equations by first expanding the brackets.

a $2(3x-1) = 15$ **b** $3(1-2x) = -13$

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Solution

a $2(3x-1)=15$

$6x-2=15$ (Expand the brackets.)

$6x=17$ (Add 2 to both sides of the equation.)

$x=\dfrac{17}{6}$ (Divide both sides of the equation by 6.)

$=2\frac{5}{6}$

b $3(1-2x)=-13$

$3-6x=-13$ (Expand the brackets.)

$-6x=-13-3$ (Subtract 3 from both sides of the equation.)

$-6x=-16$

$x=\dfrac{-16}{-6}$ (Divide both sides of the equation by -6.)

$=2\frac{2}{3}$ (Simplify the fraction.)

Example 9

Eight is subtracted from a number, x, and the result is multiplied by 2. The final result is 10. Write an equation and solve it.

Solution

$(x-8)\times 2=10$

$2(x-8)=10$

$2x-16=10$ (Expand the brackets.)

$2x=26$ (Add 16 to both sides.)

$x=13$ (Divide both sides by 2.)

Exercise 6D

Example 8

1 Solve each equation for x.

a $2(x+1)=9$ **b** $5(x-3)=12$ **c** $2(2x+4)=14$

d $3(2x-1)=8$ **e** $5(3+2x)=19$ **f** $5(5-3x)=4$

g $6(x-2)=14$ **h** $4(2-3x)=15$ **i** $5(3x-2)=6$

j $-5(2x+2)=6$ **k** $-3(4x-5)=10$ **l** $-3(2-3x)=4$

Example 9

2 In each case, write an equation and solve it.

a Six is added to a number, x, and the result is multiplied by 3. The final result is -10.

b Six is subtracted from a number, m, and the result is multiplied by 3. The final result is 5.

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c Ten is added to a number, p, and the result is multiplied by 2. The final result is 4.

d Five is subtracted from a number, n, and the result is multiplied by 3. The final result is 14.

e Three is subtracted from a number, x, and the result is multiplied by 2. The final result is −10.

f Three is added to a number, x, and the result is multiplied by −2. The final result is −2.

g Six is subtracted from a number, x, and the result is multiplied by −4. The final result is −10.

6E Collecting like terms and solving equations

Like terms

If a boy has 3 pencil cases with the same number, x, of pencils in each, he has $3x$ pencils in total.

If he is given 2 more pencil cases with x pencils in each of the 5 cases, then he has $3x + 2x = 5x$ pencils in total. This is correct as the number of pencils in each of the five cases is x. $3x$ and $2x$ are said to be **like terms**.

If Janna has x packets of chocolates, each containing y chocolates, then she has $x \times y = xy$ chocolates. If David has twice as many chocolates as Janna, he has $2 \times xy = 2xy$ chocolates. Together they have $2xy + xy = 3xy$ chocolates.

The terms $2xy$ and xy are **like terms**. The pronumerals are the same and have the same exponents in each term.

The distributive law can be used to explain the addition and subtraction of like terms.

$$\begin{aligned} 2xy + xy &= 2 \times xy + 1 \times xy \\ &= (2+1)xy \\ &= 3xy \end{aligned}$$

Example 10

Which of these pairs consist of like terms?

a $3x, 5x$ **b** $4x^2, 8x$ **c** $4x^2y, 12x^2y$

d $ab, 2ba$ **e** $3mn^2, 5nm^2$

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Solution

a $5x$ and $3x$ are like terms.

b These are not like terms because the powers of x differ.

c These are like terms because each is a number times x^2 times y.

d These are like terms because $ab = ba$.

e These are unlike terms because the powers of n and m are different.

Adding and subtracting like terms

Like terms can be added and subtracted, as shown in the example below.

Example 11

Simplify each expression by collecting like terms.

a $4x + 3x - 2x$

b $11m - 2m + 8n - 6n$

c $4m + 2n + 5m + 6n$

d $6m + 2n - 3m - n$

Solution

a $4x + 3x - 2x = 5x$

b $11m - 2m + 8n - 6n = 9m + 2n$

c $\begin{aligned} 4m + 2n + 5m + 6n &= 4m + 5m + 2n + 6n \\ &= 9m + 8n \end{aligned}$

d $\begin{aligned} 6m + 2n - 3m - n &= 6m - 3m + 2n - n \\ &= 3m + n \end{aligned}$

Example 12

Simplify each expression by collecting like terms together.

a $2x^2 + 3x^2 + 5x^2$

b $3xy + 2xy$

c $4x^2 - 3x^2$

d $2x^2 + 3x + 4x$

e $4x^2y - 3x^2y + 3xy^2$

Solution

a $2x^2 + 3x^2 + 5x^2 = 10x^2$

b $3xy + 2xy = 5xy$

c $4x^2 - 3x^2 = 1x^2 = x^2$

d $2x^2 + 3x + 4x = 2x^2 + 7x$

e $4x^2y - 3x^2y + 3xy^2 = x^2y + 3xy^2$

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Example 13

Expand the brackets and collect like terms.

a $2(x-6)+5x$

b $3+3(x-1)$

c $5+x-2(3x-4)$

Solution

a $\begin{aligned} 2(x-6)+5x &= 2x-12+5x \\ &= 7x-12 \end{aligned}$

b $\begin{aligned} 3+3(x-1) &= 3+3x-3 \\ &= 3x \end{aligned}$

c $\begin{aligned} 5+x-2(3x-4) &= 5+x-6x+8 \\ &= 13-5x \end{aligned}$

Example 14

Solve these equations by first collecting like terms.

a $2x+3=5x+1$

b $2(3x-2)-4(x+1)=2$

c $5(3x-2)=6x+3$

Solution

a

$$\begin{aligned} 2x+3 &= 5x+1 \\ 2x+3-2x &= 5x+1-2x && \text{(Subtract } 2x \text{ from both sides.)} \\ 3 &= 3x+1 && \text{(Collect like terms.)} \\ 2 &= 3x && \text{(Subtract 1 from both sides.)} \\ x &= \frac{2}{3} && \text{(Divide both sides by 3.)} \end{aligned}$$

b

$$\begin{aligned} 2(3x-2)-4(x+1) &= 2 \\ 6x-4-4x-4 &= 2 && \text{(Expand the brackets.)} \\ 2x-8 &= 2 && \text{(Collect like terms.)} \\ 2x &= 10 && \text{(Add 8 to both sides.)} \\ x &= 5 && \text{(Divide both sides by 2.)} \end{aligned}$$

c

$$\begin{aligned} 5(3x-2) &= 6x+3 \\ 15x-10 &= 6x+3 && \text{(Expand the brackets.)} \\ 9x &= 13 && \text{(Collect like terms.)} \\ x &= 1\tfrac{4}{9} && \text{(Divide both sides by 9.)} \end{aligned}$$

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Exercise 6E

Example 10

1 Which of these pairs contain like terms and which do not?

a $11a$ and $4a$ **b** $6b$ and $-2b$ **c** $12m$ and $5m$
d $14p$ and $5p$ **e** $7p$ and $-3q$ **f** $-6a$ and $7b$
g $4mn$ and $7mn$ **h** $3pq$ and $-2pq$ **i** $6ab$ and $-7b$
j $4ab$ and $5a$ **k** $11a^2$ and $5a^2$ **l** $6x^2$ and $-7x^2$
m $4b^2$ and $-3b$ **n** $2y^2$ and $2y$ **o** $-5a^2b$ and $9a^2b$
p $6mn^2$ and $11mn^2$ **q** $6\ell^2s$ and $17\ell s^2$ **r** $12d^2e$ and $14de^2$
s $-4x^2y^2$ and $12x^2y^2$ **t** $-5a^2bc^2$ and $8a^2bc^2$

Example 11

2 Simplify these expressions; that is, collect like terms.

a $3x+4x$ **b** $5x-6x$ **c** $4x+7x-8x$
d $4x-7x+8x$ **e** $-11x+10x$ **f** $3x+13x-20x$
g $6x-11x+17x$ **h** $-12m+13m+50m$ **i** $5n+6n-20n$
j $67x-70x+100x$ **k** $4n-5n+10n$ **l** $-10p-30p-5p$
m $5x-10x-15x$ **n** $80p-100p+20p$ **o** $10m+3m+4n+7n$
p $10m-3m+7n-3n$ **q** $3m+2n+4m+6n$ **r** $3x+4y-2x+y$
s $5x+2y-3x-y$ **t** $5m-2n+6m-7n$ **u** $5q-2p+2q-5r$
v $5a-7b+2a-11b$

Example 12

3 Collect like terms.

a $45xy+32xy+16xy$ **b** $19xy+6xy-4xy$
c $8xy^2+9xy^2-y^2x$ **d** $4x+3x+3y+7y$
e $6v-11v+7z-14z$ **f** $6y-11x+10y+15x$
g $7x^3+6x^2+4y^3-x^2$ **h** $8x^2-12x^2+y^2+12y^2$
i $2x^2+x^2-5xy+7yx$ **j** $11x^2-20x^2-5x^2+y^2-10y^2$

Example 13

4 Expand the brackets in each expression, and collect like terms.

a $12+7(x+4)$ **b** $3(3+x)+2x$
c $5(x+2)+2(x+3)$ **d** $2(7+5x)+4(x+6)$
e $9(3-x)+7(x+4)$ **f** $3(2x+7)+2(x-5)$
g $10(x-4)-2(x+1)$ **h** $4(3+x)-3(2x+3)$
i $11(x-1)-5(x-3)$ **j** $7(2x+4)-3(2-4x)$
k $5x+3(2x+7)$ **l** $2x(3a-4)+6(2ax+4x)$

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Example 14

5 Solve each equation for x.

a $3x+2=x-4$

b $5x-4=2x+8$

c $6x+2=2x+8$

d $3x-4=x+6$

e $11x+2=x+10$

f $3x+1=2x-1$

6 Solve each equation for x.

a $3(x-2)+x+4=10$

b $2(x-5)+3x=10$

c $x+5+2(x-4)=9$

d $2x+6+x-1=25$

7 Solve these equations by first expanding the brackets.

a $4(x-2)=10$

b $2(x+3)=x-9$

c $5(x-3)-4(x+8)=2x$

d $3(x+8)+5-3x=2x$

e $13=6(x-4)+(5+3)x$

f $2.5(x+2^2)=6(3x-2)-4(x+1.5)-3$

6F Problem-solving using algebra

The algebra that has been introduced in the previous sections of this chapter can be used to solve problems that are difficult to solve otherwise. In many cases, algebra simplifies the problem into a form to which the methods for solving equations can be applied.

Example 15

Multiplying a number by 4 and adding 6 gives the same result as multiplying the number by 3 and subtracting 4. Find the number.

Solution

Let x be the number. The resulting equation is:

$$4x+6=3x-4$$

$$4x+6-3x=3x-4-3x \qquad \text{(Subtract } 3x \text{ from both sides.)}$$

$$x+6=-4$$

$$x=-10 \qquad \text{(Subtract 6 from both sides.)}$$

Hence, the number is -10.

Check: LHS $=4\times(-10)+6$

$=-40+6$

$=-34$

RHS $=3\times(-10)-4$

$=-34$

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Example 16

Five less than a number, multiplied by 3, is equal to the sum of 2 times the number and 12. Find the number.

Solution

Let x be the number. Then from the information given:

$$\begin{aligned} 3(x-5) &= 2x+12 & & \\ 3x-15 &= 2x+12 & &\text{(Expand the brackets.)} \\ 3x &= 2x+27 & &\text{(Add 15 to both sides.)} \\ x &= 27 & &\text{(Subtract } 2x \text{ from both sides.)} \end{aligned}$$

Hence, the number is 27.

Example 17

John is thinking of getting a mobile phone, but is having trouble deciding between two plans. One costs \$30 a month, includes 10 free minutes, and charges 50 cents a minute after that. The other costs \$10 a month and charges \$1 a minute. After how many minutes do the two plans cost the same?

Solution

Let x be the number of minutes per month at which the cost of the two plans is the same. Then from the information given:

Cost of the first plan per month is $30 + 0.5(x - 10)$

Cost of the second plan per month is $10 + x$

$$\begin{aligned} 30+0.5(x-10) &= 10+x & & \\ 30+0.5x-5 &= 10+x & &\text{(Expand the brackets.)} \\ 25+0.5x &= 10+x & &\text{(Collect like terms.)} \\ 15+0.5x &= x & &\text{(Subtract 10 from both sides.)} \\ 15 &= 0.5x & &\text{(Subtract } 0.5x \text{ from both sides.)} \\ 30 &= x & &\text{(Multiply both sides by 2.)} \end{aligned}$$

The two plans cost the same amount when John has used exactly 30 minutes.

Example 18

The average of eight numbers is 56. A ninth number is added and the average is then 60. What is the ninth number?

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Solution

If the average of eight numbers is 56, then the sum of these numbers is 448.

Let n be the ninth number. Then $\frac{n+448}{9}=60$.

Solving for n:

$$\begin{aligned}\frac{n+448}{9} &= 60 \\ n+448 &= 540 && \text{(Multiply both sides of the equation by 9.)} \\ n &= 92 && \text{(Subtract 448 from both sides of the equation.)}\end{aligned}$$

The ninth number is 92.

Exercise 6F

In each of the following problems introduce a pronumeral, write an equation and solve it.

1 In each case, write an equation in x and solve it.

a A number has 6 added to it, and the result is 24.

b A number is multiplied by 5, and the result is 35.

c A number is multiplied by 3, and the result is 37.

d A number is multiplied by 6, and then 4 is added. The result is 48.

e A number is divided by 6, and the result is 20.

f A number is divided by 7, and 5 is subtracted from it. The result is 33.

Example 15

2 Multiplying a number by 3 and adding 8 gives the same result as multiplying the number by 2 and subtracting 8. Find the number.

Example 16

3 Multiplying a number by 2 and subtracting 6 gives the same result as multiplying the number by 10 and adding 8. Find the number.

Example 17

4 Anthony goes to the fair with his family. They ride together on the Ferris Wheel. They buy one adult and three child tickets for $12. If the child tickets are half the cost of the adult tickets, how much does one child ticket cost?

5 I start with a number, multiply it by 5 and add 7, and I end up with 28. What was the number I started with?

6 The sum of two numbers is 42, and one of the numbers is five times the other. Find the two numbers.

7 The sum of two numbers is twice the difference, and one number is five more than the other. Find the two numbers.

8 Five times a number minus two times the number is equal to six times the same number minus 12. What is the number?

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Example 18

9 In class tests this year, Juan has so far scored 75, 69, 81 and 87. If he wants to get an average of 80 after his next test, what score does Juan need to get?

10 If Amy has an average score of 67 after four tests, and after doing a fifth test her average increases to 72, what score did she get on the fifth test?

11 The local movie theatre sells popcorn in small, medium and large containers. The small size costs $1 less than the medium size, and the large size costs twice as much as the medium size. A group of friends go to the movies and decide to buy one container of each size, and they pay $13. How much does the medium-size popcorn cost?

12 Licia bought her lunch from the school canteen for $3.00. She had a sausage roll, a caramel milkshake and an apple. She paid 60 cents more for the milkshake than for the fruit, and 30 cents more for the sausage roll than for the milkshake. Write an expression for how much she spent, and use it to find the cost of each item.

6G Multiplying and dividing algebraic fractions

The **any-order property of multiplication** was discussed in Chapter 1. In Chapter 3, index notation was used to simplify expressions. For example, $x \times x \times x = x^3$ and $y \times y = y^2$.

Here is another example using index notation.

$$\begin{aligned} 3x \times 2y \times 2xy &= 3 \times x \times 2 \times y \times 2 \times x \times y \\ &= 3 \times 2 \times 2 \times x \times x \times y \times y \\ &= 12x^2y^2 \end{aligned}$$

Example 19

Simplify each expression.

a $5 \times 2a$ **b** $3a \times 2a$ **c** $5xy \times 2xy$ **d** $6x^2y \times 3x$

Solution

a $5 \times 2a = 10a$

b $\begin{aligned} 3a \times 2a &= 3 \times 2 \times a \times a \\ &= 6a^2 \end{aligned}$

c $\begin{aligned} 5xy \times 2xy &= 5 \times 2 \times x \times x \times y \times y \\ &= 10x^2y^2 \end{aligned}$

d $\begin{aligned} 6x^2y \times 3x &= 6 \times 3 \times x \times x \times x \times y \\ &= 18x^3y \end{aligned}$

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In Section 2A we learned to simplify fractions by cancelling common factors. We can do the same dealing with algebraic fractions, provided that the pronumeral we cancel is not equal to zero.

Example 20

Simplify:

a $\dfrac{8x}{2}$ **b** $\dfrac{7xy}{x}$ **c** $\dfrac{56abc}{35c}$

Solution

a $\dfrac{8x}{2} = \dfrac{\cancel{8}^{4}x}{\cancel{2}^{1}}$
$= 4x$

b $\dfrac{7xy}{x} = \dfrac{7\cancel{x}^{1}y}{\cancel{x}^{1}}$
$= 7y$

c $\dfrac{56abc}{35c} = \dfrac{\cancel{56}^{8}ab\cancel{c}^{1}}{\cancel{35}^{5}\cancel{c}^{1}}$
$= \dfrac{8ab}{5}$

Example 21

Simplify:

a $\dfrac{x}{x}$ **b** $\dfrac{x^2}{x}$ **c** $\dfrac{x^3}{x^2}$ **d** $\dfrac{x^2}{x^3}$

Solution

a $\dfrac{x}{x} = \dfrac{\cancel{x}^{1}}{\cancel{x}^{1}}$
$= 1$

b $\dfrac{x^2}{x} = \dfrac{x \times x}{x}$
$= \dfrac{x \times \cancel{x}^{1}}{\cancel{x}^{1}}$
$= x$

c $\dfrac{x^3}{x^2} = \dfrac{x \times x \times x}{x \times x}$
$= \dfrac{x \times \cancel{x}^{1} \times \cancel{x}^{1}}{\cancel{x}^{1} \times \cancel{x}^{1}}$
$= x$

d $\dfrac{x^2}{x^3} = \dfrac{\cancel{x}^{1} \times \cancel{x}^{1}}{x \times \cancel{x}^{1} \times \cancel{x}^{1}}$
$= \dfrac{1}{x}$

Example 22

Simplify:

a $\dfrac{60p^2q}{12p}$ **b** $\dfrac{50x^2y^2}{20xa}$

Solution

a $\dfrac{60p^{\cancel{2}^{1}}q}{12\cancel{p}} = 5pq$

b $\dfrac{50x^{\cancel{2}^{1}}y^2}{20\cancel{x}a} = \dfrac{5xy^2}{2a}$

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Exercise 6G

Example 19

1 Simplify, using index notation:

a $2a \times 6b$ **b** $3x \times 4x$ **c** $5xy \times 10$ **d** $7ab \times 11$

e $5a \times 2a$ **f** $6c \times 11c$ **g** $4m \times 6n$ **h** $7mn \times 11mn$

i $4n^2 \times 3$ **j** $7m^2 \times 6m$ **k** $11m^2 \times 2m^2$ **l** $7a^2b \times 5$

2 Rewrite each expression without brackets.

a $(5n)^2$ **b** $(4z)^2$ **c** $(16z)^2$ **d** $(3c^2)^2$

3 Simplify:

a $2x \times 4x$ **b** $7x \times 3x$ **c** $3x \times 2x$

d $2xy \times 3x$ **e** $4xy \times 2xy$ **f** $6xy \times 2x^2y$

Example 20

4 Simplify:

a $\dfrac{6x}{3}$ **b** $\dfrac{8xy}{y}$ **c** $\dfrac{24xyz}{xz}$ **d** $\dfrac{mnp}{5m}$

e $\dfrac{9zx}{3z}$ **f** $\dfrac{18xy}{y}$ **g** $\dfrac{18xyz}{yz}$ **h** $\dfrac{72abc}{16c}$

Example 21

5 Simplify:

a $\dfrac{4x}{xy}$ **b** $\dfrac{a^2}{a}$ **c** $\dfrac{a^3}{a^2}$ **d** $\dfrac{4ab}{ab}$ **e** $\dfrac{3a^2b}{a}$ **f** $\dfrac{a^3}{a}$

g $\dfrac{a}{a^2}$ **h** $\dfrac{a^2}{a^3}$ **i** $\dfrac{a}{a}$ **j** $\dfrac{a^2b}{ab}$ **k** $\dfrac{a^2b^2}{ab}$ **l** $\dfrac{ab^2}{ab}$

Example 22

6 Simplify:

a $\dfrac{48p^2q}{36p}$ **b** $\dfrac{100a^2b^2}{5ba}$ **c** $\dfrac{200a^3b^2}{10a}$

7 A rectangle has length x cm and width $\dfrac{x}{3}$ cm.

a What is the area of the rectangle? **b** Find the area if $x = 9$.

8 A triangle has base $\dfrac{2x}{3}$ cm and height $\dfrac{x}{4}$ cm.

a What is the area of the triangle in terms of x? **b** Find the area if $x = 24$.

9 A rectangle has length x cm and area $\dfrac{6x^2}{5}$ cm^2. Find its width.

10 A rectangle has length $2x$ cm and width $\dfrac{5x}{3}$ cm.

a What is the area of the rectangle? **b** Find the area if $x = 27$.

11 A square that has side length $8x$ cm is divided into y rectangles of equal area. What is the area of each of these rectangles?

12 A floor of a room is rectangular in shape, has length $10x$ metres and width $7x$ metres. The floor is to be tiled with rectangular tiles each of length $\dfrac{x}{3}$ metres and width $\dfrac{x}{4}$ metres. How many tiles are needed to tile this floor area?

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Review exercise

1 Evaluate each expression for $a = 4$, $b = -2$, $c = -3$ and $d = 5$.

a $d - 3$ **b** $c + 5$ **c** $3d - 8$ **d** $5b + 6$
e $10 - 2b$ **f** $7 - 3a$ **g** $3(a + 2)$ **h** $7(1 - c)$
i $\frac{a}{b}$ **j** $ad - bc$ **k** b^2 **l** $\frac{ac + 7}{5 - 2b}$

2 Substitute $x = -3$ in each expression and evaluate it.

a $2x + 3$ **b** $1 - 2x$ **c** $4x + 3$ **d** $1 + x$
e x^2 **f** x^3 **g** $2 - x^2$ **h** $4 + x^2$
i $-1 + x^3$ **j** $x^3 - 8$ **k** $(2 + x)^3$ **l** $(2 - x)^3$

3 Substitute $m = 2$, $n = -3$ and $p = 11$ in each expression and evaluate it.

a $m^2 + p^2$ **b** $m - n$ **c** $p - n$ **d** $n - p$
e mnp **f** $m^2n^2p^2$ **g** $m - 2p$ **h** $p^2 + m^2 - n$

4 Expand the brackets in each expression.

a $3(a + 4)$ **b** $6(3 - x)$
c $x(2 + y)$ **d** $-2(11 - x)$
e $14(x - 3) - 10(x - 3)$ **f** $3(x + 10) - 4(5 - x)$
g $2a(3 - x) + 3x(a + 7)$ **h** $-5(x + 6) + 3(2x - 4)$
i $-2(x - 3) + 4(3 - x)$ **j** $-7(2x - 4) + 5(4 - x)$

5 Solve each equation. Check your solutions.

a $2x - 3 = -11$ **b** $x + 6 = -12$ **c** $4x - 6 = -10$
d $3 - a = 6$ **e** $2m - 6 = -8$ **f** $10 - 2m = -4$

6 Solve each equation. Check each answer.

a $x + 3x - 5 = 11$ **b** $3(a + 4) = 9$ **c** $2x + 1 = 3(x - 4)$
d $x + 3 = 4x - 1$ **e** $2x + 6 = 3 - x$ **f** $x + 2 = 4(x - 5)$

7 Expand the brackets where necessary and simplify each of these expressions.

a $(x^2y^2) \times (2xy)^3$ **b** $(-2xy)^2 \times x^3y^2$ **c** $\frac{x^2y^2}{xy}$
d $\frac{m^3n^2}{m^2n}$ **e** $(-4x^2y)^2 \times xy$ **f** $\frac{20a^2b^2}{5a}$

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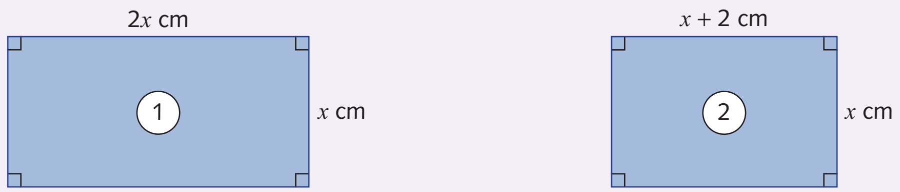
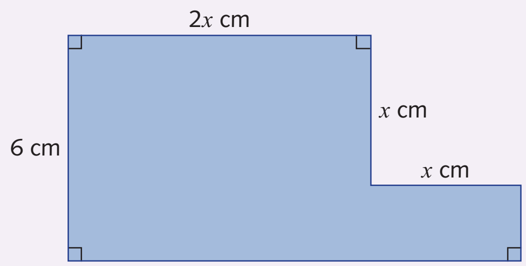
8 In a theme park, the cost of an adult ticket is 1.5 times that of a child ticket. If a family of 2 adults and 3 children paid \$48, how much does an adult ticket cost?

9 In a strict monastery there are 100 monks, either senior or junior. If a senior monk is allowed to have 3 slices of bread for each meal, and a junior monk is only allowed $\frac{1}{3}$ of a slice, and altogether they consume 60 slices of bread during lunch, how many senior monks are there?

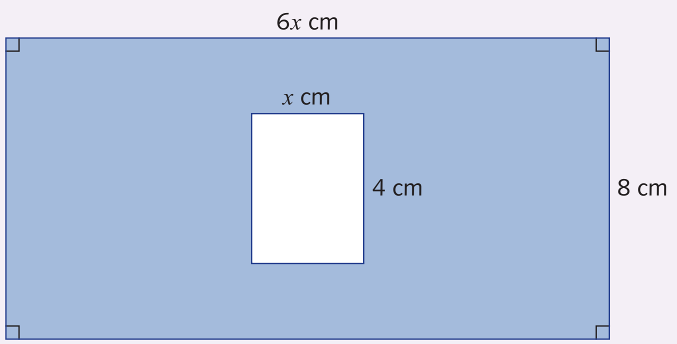
10 The perimeter of rectangle ① is numerically equal to the perimeter of rectangle ②. Find the value of x.



11 The perimeter of the shaded region is 36 cm.
Find the value of x.



12 The area of the shaded region is 121 cm^2.
Find the value of x.



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Challenge exercise

1 **a** Carry out the following operations on the number 5.

- Add 3.
- Multiply the result by 9.
- Subtract 2.
- Add the number you started with (in this case 5).
- Subtract 25.
- Divide by 10.

b Repeat the procedure with another number.

c Let the number be x and apply same the operations. State your result. What is the final number?

2 If a painted wooden cube is cut in half along each edge (making 8 smaller cubes), then each of these cubes is painted on three faces. If a painted wooden cube is cut into thirds along each edge (making 27 smaller cubes), 8 of these cubes are painted on three faces, 12 of these cubes are painted on two faces, 6 are painted on one face only, and one is not painted at all. If a painted wooden cube is cut into four parts along each edge, how many smaller cubes are there, and how many are painted on three, two, one and no faces, respectively? Find an expression for how many cubes are painted on three, two, one and no faces if the large cube is cut into n parts along each edge.

3 Paul, Kate and Sarah's mother left a bowl of jelly beans on their kitchen counter. Paul took one-third of the jelly beans, but then threw back 4 because he didn't like the black ones. Sarah ate 6 jelly beans and then took one-quarter of what remained. Kate took one-third of what remained, and then picked out and ate the last 4 black ones because she really liked them. If after all this there were 6 jelly beans left, how many were there to begin with? If there were 12 jelly beans left, how many were there to begin with? If there were x jelly beans left, how many were there to begin with?

4 A primary school is planning to hold a fete, but needs help in figuring out how much to charge for entry. They intend to have an 'all you can eat' sausage sizzle, and they think each adult will eat about \$3 worth of sausages, and each child \$2 worth. The cost of cleaning after the event will be \$300. They want an adult ticket to cost twice as much as a child ticket, and they expect that, on average, 1.7 parents and 2.3 children will come from each family. How much will they need to charge for adult and child tickets if they are expecting 120 families to attend and they don't want to make a profit or a loss? (Round to the nearest cent.)

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5 Andrew and a group of his friends have decided they want to race go-karts on the weekend, and Andrew wants to work out which of the two local go-kart places will be cheaper. Kart-Kingdom charges \$15 per person to do safety training, and then 50 cents per lap that you do. Go-Mobile charges 80 cents per lap, and the safety training is free. After how many laps does Go-Mobile become more expensive than Kart-Kingdom?

6 Find all the two-digit integers such that when you reverse the digits and add that number to the original number, you get 55.

7 A two-digit number has the property that the sum of the number and its digits is 63. What is the original number?

8 Is there a two-digit integer with the property that 11 times the sum of its digits is equal to itself?

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CHAPTER

7

Number and Algebra

Percentages

We encounter percentages often, and they can be very useful in many aspects of our lives. For example, we see discounts offered in shops everywhere – as smart shoppers, we definitely need to know how to do percentage calculations if we want the best deal.

The word 'percentage' comes from the Latin *per centum*, meaning 'per hundred'. A percentage is another way of writing a fraction with a denominator of 100. The symbol for percentage is %. For example:

$$8\% = \frac{8}{100}, \quad 25\% = \frac{25}{100}, \quad 99\% = \frac{99}{100}, \quad 150\% = \frac{150}{100}$$

In this chapter, you will learn how to perform several different types of useful calculations involving percentages.

Also, we review the previous study of percentages and consider more applications of percentages, such as discounts, profits and loss, interest and GST.

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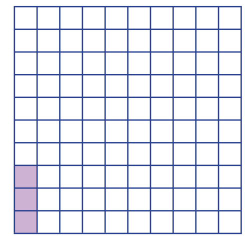
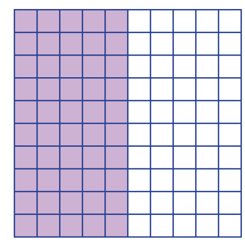
7A Review of percentages

To understand the basic idea of percentages, we need only look at the meaning of ‘out of a hundred’.

A square that has been cut into 100 smaller squares can be used to model percentages. If we colour in three of them, we say that ‘three out of a hundred’ or ‘three per cent’ are coloured in.



If we colour 50 of them, we say that ‘50 out of a hundred’ or ‘50 per cent’ are coloured in. Half of the squares are coloured in.



Converting percentages to fractions

A percentage is a fraction with a denominator of 100. To convert a percentage to its fraction equivalent, write it as a fraction with a denominator of 100 and then simplify. For example:

$$\begin{aligned} 65\% &= \frac{65}{100} \\ &= \frac{13}{20} \end{aligned} \qquad\qquad \begin{aligned} 150\% &= \frac{150}{100} \\ &= 1\tfrac{1}{2} \end{aligned}$$

For percentages such as $12\frac{1}{2}\%$ or $33\frac{1}{3}\%$, an extra step is required to convert the numerators to whole numbers:

$$\begin{aligned} 12\tfrac{1}{2}\% &= \frac{12\frac{1}{2}}{100} \\ &= \frac{25}{200} && \text{(Multiply the numerator and denominator of the fraction by 2.)} \\ &= \frac{1}{8} \end{aligned}$$

$$\begin{aligned} 33\tfrac{1}{3}\% &= \frac{33\frac{1}{3}}{100} \\ &= \frac{100}{300} && \text{(Multiply the numerator and denominator of the fraction by 3.)} \\ &= \frac{1}{3} \end{aligned}$$

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Converting percentages to decimals

A percentage can be converted to a decimal by writing it as a fraction with a denominator of 100 and then converting to a decimal.

$$\begin{aligned}65\% &= \frac{65}{100}\\ &= 0.65\end{aligned} \qquad \begin{aligned}150\% &= \frac{150}{100}\\ &= 1.5\end{aligned}$$

For a percentage such as 37.5%, divide 37.5 by 100 in one step:

$$\begin{aligned}37.5\% &= \frac{37.5}{100}\\ &= 0.375\end{aligned}$$

Converting fractions and decimals to percentages

We know from the meaning of the word 'percentage' that fractions with a denominator of 100 convert directly to percentages. For example:

$$\frac{2}{100} = 2\% \qquad \frac{37}{100} = 37\%$$

$$\frac{175}{100} = 175\% \qquad \frac{100}{100} = 1 = 100\%$$

Equivalent fractions can be used for some fractions whose denominators are not 100 to find the corresponding percentage:

$$\begin{aligned}\frac{2}{10} &= \frac{20}{100}\\ &= 20\%\end{aligned} \qquad \begin{aligned}\frac{3}{5} &= \frac{60}{100}\\ &= 60\%\end{aligned}$$

$$\begin{aligned}\frac{3}{20} &= \frac{15}{100}\\ &= 15\%\end{aligned} \qquad \begin{aligned}\frac{3}{2} &= \frac{150}{100}\\ &= 150\%\end{aligned}$$

More generally, to convert a fraction to a percentage, multiply by 100%, which is the same as multiplying by 1.

$$\begin{aligned}\frac{2}{5} &= \frac{2}{5} \times \frac{100}{1}\%\\ &= 40\%\end{aligned} \qquad \begin{aligned}\frac{2}{3} &= \frac{2}{3} \times \frac{100}{1}\%\\ &= 66\tfrac{2}{3}\%\end{aligned}$$

To convert a decimal to a percentage, the procedure is the same – multiply by 100%, which is the same as multiplying by 1.

$$\begin{aligned}0.6 &= 0.6 \times 100\%\\ &= 60\%\end{aligned} \qquad \begin{aligned}3.2 &= 3.2 \times 100\%\\ &= 320\%\end{aligned}$$

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Here are some commonly used percentages and their fraction equivalents. It can be very useful to know these. Try to learn them in both directions, but you can quickly work them out if you forget them.

For example, $\frac{1}{5} = 20\%$ and 37.5% is $\frac{3}{8}$.

Fraction	Percentage
$\frac{1}{2}$	50%
$\frac{1}{4}$	25%
$\frac{1}{3}$	$33\frac{1}{3}\%$
$\frac{3}{4}$	75%
$\frac{2}{3}$	$66\frac{2}{3}\%$
$\frac{1}{5}$	20%
$\frac{2}{5}$	40%

Fraction	Percentage
$\frac{3}{5}$	60%
$\frac{4}{5}$	80%
$\frac{1}{8}$	$12\frac{1}{2}\%$
$\frac{3}{8}$	$37\frac{1}{2}\%$
$\frac{5}{8}$	$62\frac{1}{2}\%$
$\frac{7}{8}$	$87\frac{1}{2}\%$
$\frac{1}{1}$	100%

Percentage of a quantity

We often talk about a percentage of a particular quantity. For example:

- 20% of all suitcases are brown.
- 52% of voters in a recent election voted for a particular political party.
- 80% of the students at a particular school access the internet at least once a day.

In such cases, we may want to know the actual number of objects or people involved. The following example shows how to do calculations of this type.

Example 1

32% of people interviewed watched the Australian Open tennis final. Calculate how many of the people interviewed watched the final if:

a 300 people were interviewed

b 250 people were interviewed

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Solution

a If 300 people were interviewed, then 32 of every 100 people interviewed watched the final. That is, $32 \times 3 = 96$ people watched it. This can be set out as follows:

32% of 300 people interviewed watched the final.

$$\begin{aligned}\text{Number of people who watched} &= 32\% \text{ of } 300\\ &= \frac{32}{100} \times \frac{300}{1}\\ &= 96\end{aligned}$$

b 32% of 250 people interviewed watched the final.

$$\begin{aligned}\text{Number of people who watched} &= 32\% \text{ of } 250\\ &= \frac{32}{100} \times \frac{250}{1}\\ &= \frac{8}{25} \times \frac{250}{1}\\ &= 80\end{aligned}$$

Expressing one quantity as a percentage of another

Sometimes we want to describe one quantity as a percentage of another.

Example 2

There are 50 people in a swimming club and 35 of them go to squad training. Calculate the percentage of the club members who go to squad training.

Solution

35 out of the 50 or $\frac{35}{50}$ go to squad training.

$$\begin{aligned}\text{Percentage going to squad training} &= \frac{35}{50} \times 100\%\\ &= 70\%\end{aligned}$$

Thus 70% of the swimming club members go to squad training.

Comparing fractions using percentages

One useful way to compare your results in a French test, in which you scored $\frac{16}{20}$, and a German test, in which you scored $\frac{42}{50}$, is to change both scores to percentages.

In this case, $\frac{16}{20} = 80\%$ and $\frac{42}{50} = 84\%$, so you did better in the German test.

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Percentages

- A **percentage** is a fraction with denominator of 100. For example:
  $5\% = \frac{5}{100}$
- To convert a percentage to a fraction, write the percentage as a fraction with a denominator of 100 and then simplify.
- To convert a percentage to a decimal, write the percentage as a fraction with a denominator of 100 and then convert to a decimal.
- To convert a fraction or a decimal to a percentage, multiply by 100%.
- To express one quantity as a percentage of another, write the first quantity as a fraction of the second, and then convert to a percentage by multiplying by 100%.
- A useful way to compare two fractions is to convert them both to percentages.

Example 3

a Convert 45% to a fraction.

b Convert 135% to a fraction.

c Convert 45% to a decimal.

d Convert 135% to a decimal.

Solution

a $$45\% = \frac{45}{100} = \frac{9}{20}$$

b $$135\% = \frac{135}{100} = 1\tfrac{7}{20}$$

c $$45\% = \frac{45}{100} = 0.45$$

d $$135\% = \frac{135}{100} = 1.35$$

Example 4

Express each fraction as a percentage.

a $\frac{27}{100}$

b $\frac{127}{1000}$

Solution

a $$\frac{27}{100} = 27\%$$

b $$\frac{127}{1000} = \frac{127}{1000} \times \frac{100}{1}\% = \frac{127}{10}\% = 12.7\%$$

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Example 5

Express each fraction as a percentage.

a $\frac{4}{5}$ **b** $\frac{37}{50}$ **c** $2\frac{4}{5}$

Solution

a
$$\begin{aligned}\frac{4}{5} &= \frac{4}{5} \times \frac{100}{1}\% \\ &= \frac{4}{\cancel{5}^{1}} \times \frac{\cancel{100}^{20}}{1}\% \\ &= 80\%\end{aligned}$$

b
$$\begin{aligned}\frac{37}{50} &= \frac{37}{50} \times \frac{100}{1}\% \\ &= \frac{37}{\cancel{50}^{1}} \times \frac{\cancel{100}^{2}}{1}\% \\ &= 74\%\end{aligned}$$

c
$$\begin{aligned}2\tfrac{4}{5} &= \frac{14}{5} \times \frac{100}{1}\% \\ &= \frac{14}{\cancel{5}^{1}} \times \frac{\cancel{100}^{20}}{1}\% \\ &= 280\%\end{aligned}$$

Example 6

Write $7\frac{1}{4}\%$ as a fraction.

Solution

$$\begin{aligned}7\tfrac{1}{4}\% &= \frac{7\frac{1}{4} \times 4}{100 \times 4} \\ &= \frac{29}{400}\end{aligned}$$

Example 7

a What is 20% of 415? **b** What is 7% of 197?

Solution

a
$$\begin{aligned}20\% \text{ of } 415 &= \frac{20}{100} \times \frac{415}{1} \\ &= \frac{1}{5} \times \frac{415}{1} \\ &= 83\end{aligned}$$

b
$$\begin{aligned}7\% \text{ of } 197 &= \frac{7}{100} \times \frac{197}{1} \\ &= \frac{1379}{100} \\ &= 13.79\end{aligned}$$

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Example 8

There are 200 Year 8 students in a school. Of these, 112 are girls. What percentage of the students are girls?

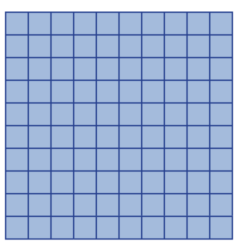
Solution

$$\begin{aligned}\text{Percentage of girls in the school} &= \frac{112}{200} \times 100\% \\ &= \frac{112}{2}\% \\ &= 56\%\end{aligned}$$

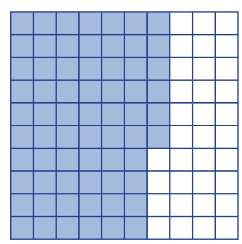
Exercise 7A

1 What percentage of each square is shaded?

a



b



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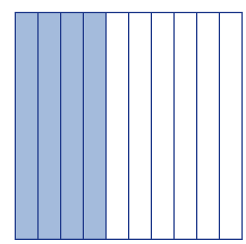

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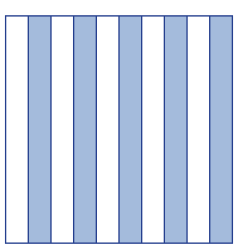

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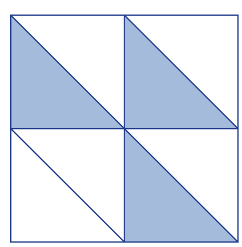
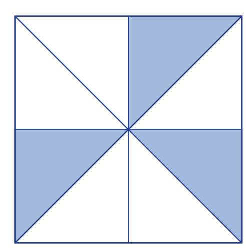


f



Example 3a, b

2 Convert each percentage to a fraction.

a 10% **b** 25% **c** 85%

d 50% **e** 40% **f** 20%

g 125% **h** 630% **i** 205%

j 138% **k** 496% **l** 43125%

Example 3c, d

3 Convert each percentage to a decimal.

a 55% **b** 37% **c** 19%

d 80% **e** 30% **f** 10%

g 250% **h** 148% **i** 3912%

j 3% **k** 7% **l** 1%

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Example 4

4 Express each fraction as a percentage.

a $\frac{23}{100}$ **b** $\frac{99}{100}$ **c** $\frac{11}{100}$ **d** $\frac{150}{100}$ **e** $\frac{230}{100}$

f $\frac{999}{100}$ **g** $\frac{42}{10}$ **h** $\frac{67}{10}$ **i** $\frac{14}{10}$ **j** $\frac{9}{10}$

k $\frac{5}{10}$ **l** $\frac{1}{10}$ **m** $\frac{670}{1000}$ **n** $\frac{810}{1000}$ **o** $\frac{330}{1000}$

p $\frac{999}{1000}$ **q** $\frac{123}{1000}$ **r** $\frac{702}{1000}$ **s** $\frac{9999}{1000}$ **t** $\frac{1234}{1000}$

Example 5

5 Express each fraction as a percentage.

a $\frac{2}{4}$ **b** $\frac{49}{50}$ **c** $\frac{4}{25}$ **d** $\frac{1}{20}$ **e** $\frac{3}{5}$

f $\frac{19}{25}$ **g** $1\frac{1}{4}$ **h** $\frac{1}{40}$ **i** $3\frac{17}{25}$ **j** $\frac{6}{5}$

k $\frac{10}{4}$ **l** $\frac{25}{20}$ **m** $\frac{3}{40}$ **n** $2\frac{2}{5}$ **o** $\frac{17}{40}$

6 Write each decimal as a percentage.

a 0.65 **b** 0.23 **c** 0.75 **d** 0.5 **e** 0.3

f 0.6 **g** 1.5 **h** 1.1 **i** 1.8 **j** 1

k 0.07 **l** 0.02 **m** 1.05 **n** 4.09 **o** 7.03

Example 6

7 Write each percentage as a fraction.

a $2\frac{1}{2}\%$ **b** $7\frac{1}{2}\%$ **c** $8\frac{1}{4}\%$ **d** $37\frac{1}{2}\%$

e $12\frac{1}{2}\%$ **f** $67\frac{1}{2}\%$ **g** $87\frac{1}{2}\%$ **h** $66\frac{2}{3}\%$

8 Complete the table below, showing the decimal, fraction and percentage equivalents in each row.

Decimal	Fraction	Percentage
0.5		50%
	$\frac{1}{4}$	
		75%
0.4		
		100%
0.367		
	$\frac{29}{100}$	
0.403		
	$\frac{3}{8}$	
	$\frac{5}{4}$	
2.75		
		200%

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9 Express each quantity as a percentage of the total.

a 30 cents out of a dollar

b 250 m out of 1 km

c 160 women in a train carrying 200 people

d 16 teeth filled out of 24

e \$10 discount on a \$200 jumper

f 18 out of 45 people in a restaurant ordering pizza

10 Calculate these amounts.

a 50% of 468 **b** 25% of 144

c $12\frac{1}{2}\%$ of 1360 **d** $33\frac{1}{3}\%$ of 8730

e 20% of 500 **f** 40% of 455

g $37\frac{1}{2}\%$ of 192 **h** $87\frac{1}{2}\%$ of 960

i $66\frac{2}{3}\%$ of 525

Example 7

11 Calculate:

a 20% of 100 **b** 20% of 200

c 20% of 1000 **d** 50% of 300

e 5% of 30 **f** 15% of 300

g 63% of 100 **h** 78% of 500

i 57% of 30 **j** 200% of 100

k 200% of 118 **l** 200% of 3.5

12 **a** Find 10% of 340. **b** Find 30% of 340. **c** Find 70% of 340.

d Find 5% of 340. **e** Find $2\frac{1}{2}\%$ of 340. **f** Find $7\frac{1}{2}\%$ of 340.

13 **a** Find 25% of 320. **b** Find 75% of 320. **c** Find $12\frac{1}{2}\%$ of 320.

14 **a** Find 10% of 456. **b** Find 30% of 456.

c Find 10% of 912. **d** Find 5% of 912.

15 Use fractions to calculate:

a 20% of 48 m **b** 75% of 80 lollies **c** 30% of 270 children

16 Use decimals to find:

a 25% of 28 songs **b** 40% of 280 stories **c** 15% of 160 students

Example 8

17 Express \$1.80 as a percentage of \$5.00.

18 Express \$24.40 as a percentage of \$30.50.

19 Mike bought 72 stamps. He bought 27 more stamps than Carla. Express the number of stamps Carla bought as a percentage of Mike's.

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7B Changing fractions to percentages and percentages to fractions (revisited)

In this section, we look at some harder examples of changing percentages to fractions and fractions to percentages. The emphasis here is on converting a fraction with a denominator that does not divide into a power of 10 to a percentage.

Example 9

Write $\frac{2}{7}$ as a percentage.

Solution

$$\begin{aligned}\frac{2}{7} &= \frac{2}{7} \times 100\% \\ &= \frac{200}{7}\% \\ &= 28\tfrac{4}{7}\%\end{aligned}$$

Changing percentages to fractions

We now look at how to change a percentage to a fraction. For example:

$$\begin{aligned}45\tfrac{1}{2}\% &= \frac{45\frac{1}{2}}{100} \\ &= \frac{45\frac{1}{2} \times 2}{100 \times 2} \quad \text{(Multiply numerator and denominator by 2.)} \\ &= \frac{91}{200}\end{aligned}$$

Example 10

Write each percentage as a fraction.

a $23\tfrac{1}{3}\%$ **b** $32\tfrac{4}{5}\%$ **c** $5\tfrac{1}{7}\%$

Solution

a
$$\begin{aligned}23\tfrac{1}{3}\% &= \frac{23\frac{1}{3}}{100} \\ &= \frac{23\frac{1}{3} \times 3}{100 \times 3} \\ &= \frac{70}{300} \\ &= \frac{7}{30}\end{aligned}$$

b
$$\begin{aligned}32\tfrac{4}{5}\% &= \frac{32\frac{4}{5}}{100} \\ &= \frac{32\frac{4}{5} \times 5}{100 \times 5} \\ &= \frac{164}{500} \\ &= \frac{41}{125}\end{aligned}$$

c
$$\begin{aligned}5\tfrac{1}{7}\% &= \frac{5\frac{1}{7}}{100} \\ &= \frac{36}{700} \\ &= \frac{9}{175}\end{aligned}$$

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Expressing one quantity as a percentage of another

We have already looked at how to express one quantity as a percentage of another in the previous section. Here are some examples in which the resulting percentages involve fractions.

Example 11

Express 55 as a percentage of 120.

Solution

$$\begin{aligned}\frac{55}{120} &= \frac{55}{120} \times \frac{100}{1}\% \\ &= \frac{55}{6} \times \frac{5}{1}\% \\ &= \frac{275}{6}\% \\ &= 45\tfrac{5}{6}\%\end{aligned}$$

Example 12

Two bakeries, Brownie and Best Bake, both bake bread. On a particular day, 27 out of 40 loaves at the Brownie Bakery were baked the previous day. At the Best Bake Bakery, 57 out of 90 loaves were baked the previous day. Which bakery was selling the greater percentage of loaves baked the previous day?

Solution

The percentage of loaves baked the previous day at the Brownie Bakery is:

$$\begin{aligned}\frac{27}{40} \times 100\% &= \frac{2700}{40}\% \\ &= \frac{135}{2}\% \\ &= 67\tfrac{1}{2}\%\end{aligned}$$

The percentage of loaves baked the previous day at the Best Bake Bakery is:

$$\begin{aligned}\frac{57}{90} \times 100\% &= \frac{570}{9}\% \\ &= 63\tfrac{1}{3}\%\end{aligned}$$

Hence, the Brownie Bakery had the greater percentage of loaves baked a day earlier.

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Example 13

Write 0.32 out of 4.8 as a percentage.

Solution

$$\begin{aligned}\text{First } \frac{0.32}{4.8} &= \frac{32}{480}\\ &= \frac{32}{480}\times 100\%\\ &= \frac{2}{30}\times 100\%\\ &= \frac{2}{3}\times 10\%\\ &= 6\tfrac{2}{3}\%\end{aligned}$$

Exercise 7B

Example 9

1 Express each fraction as a percentage. Your answers will contain fractions.

a $\frac{2}{3}$ **b** $\frac{1}{6}$ **c** $\frac{5}{6}$

d $\frac{7}{9}$ **e** $\frac{11}{12}$ **f** $\frac{5}{7}$

Example 10

2 Express each percentage as a fraction.

a $12\frac{1}{5}\%$ **b** $6\frac{3}{4}\%$ **c** $5\frac{1}{4}\%$ **d** $15\frac{1}{6}\%$

e $6\frac{2}{3}\%$ **f** $5\frac{1}{12}\%$ **g** $12\frac{1}{6}\%$ **h** $21\frac{3}{4}\%$

i $42\frac{2}{7}\%$ **j** $6\frac{2}{7}\%$ **k** $44\frac{2}{3}\%$ **l** $7\frac{5}{12}\%$

3 Express each percentage as a fraction in simplest form.

a 200% **b** 1.5% **c** 124% **d** 210% **e** 0.5% **f** $125\frac{1}{2}\%$

4 Express each percentage first as a decimal and then as a fraction in simplest form.

a 12.25% **b** 6.75% **c** 5.75% **d** 8.06%

e 12.05% **f** 227% **g** 11 790% **h** 5711%

Example 11

5 Express the first number in each pair as a percentage of the second.

a 23, 69 **b** 35, 125 **c** 45, 135

d 456, 2000 **e** 34, 134 **f** 56, 400

6 In his vocabulary tests last term, Mick scored 21 out of 25, 13 out of 15, and 17 out of 20. By expressing his scores as percentages, rank Mick's efforts on the tests from best to worst.

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Example 12

7 The total population of a state is 2 500 000. The population of the capital city is 1 500 000.

a What percentage of the population of the state lives in the capital city?

b What percentage of the population of the state lives outside the capital city?

8 The number of people who enter Luna Park on a particular Sunday is 3200. Of these, 1250 purchase a ride on the rollercoaster. What percentage of the 3200 people purchase a ride on the rollercoaster?

9 In medieval Europe, the Black Death plague had an extremely high fatality rate. Calculate the percentage of those who died from the disease if the number of survivors in a village of 300 people infected with the disease was 15.

10 A drug for treating cholera is 98.5% effective.

a If 1200 infected people are treated with the drug, how many would you expect to recover?

b A different drug was administered, and 78 out of 80 people recovered. Using percentages, work out which drug is more effective.

11 **a** How much larger is $\frac{1}{3}$ than 33%, as a percentage?

b How much larger is $\frac{1}{3}$ than 0.3, as a percentage?

12 A survey was conducted at three different schools to find how many students travel to school by bus.

- School A reported that $\frac{2}{7}$ of its students are bus travellers.
- School B reported that 110 of its 400 students are bus travellers.
- School C reported that 28.9% of its students travel by bus.

Rank the three schools in terms of the percentage of students who travel by bus.

13 The bread sales on Saturday in a supermarket were:

- white bread, 135 loaves
- wholemeal, 84 loaves
- multigrain, 117 loaves
- raisin bread, 24 loaves

Express the number of loaves of each type sold as a percentage of total sales.

14 Express each of these situations as a percentage.

a In a population of 24 million people, 0.06 million work in the automobile industry.

b In a chess game, 5 out of 32 pieces were captured and removed from the board.

15 The average percentage, by weight, of body fat is 17% for men and 23% for women. Calculate the amount of fat, in kilograms, for:

a a woman weighing 76 kg

b a man weighing 72 kg

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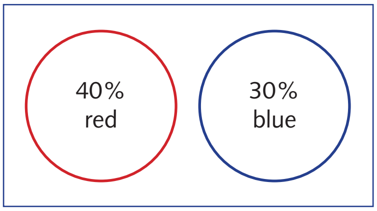
7C Sums, differences and products of percentages

Sums and differences of percentages

In some situations, but not all, percentages can be added and subtracted. There are two conditions that must be satisfied before addition or subtraction of percentages can be undertaken.

- They are percentages of the same whole.
- There is no overlap among the groups being considered.

For example, 200 people are asked their favourite colour (only one colour is allowed). 40% reply blue and 30% reply red, so $40\% + 30\% = 70\%$ have red or blue as their favourite colour.



Also, $40\% - 30\% = 10\%$ more of the people surveyed have blue rather than red as their favourite colour. In this situation both percentages are percentages of the same 200 people and there is no overlap between the groups.

Example 14

30% of the students in a school of 1200 have blond hair and 25% have black hair. What percentage of the students have either blond or black hair?

Solution

The percentage is $30\% + 25\% = 55\%$.

We could also solve the problem by working out the actual numbers involved.

30% of $1200 = 360$

25% of $1200 = 300$

Total $= 660$

$$\text{Percentage} = \frac{660}{1200} \times 100\%$$
$$= \frac{660}{12}\%$$
$$= 55\%$$

However, there is no need to do it in this way because the two conditions above are satisfied. Both the percentages given in the question are percentages of the number of students in the school, and we can safely assume that no one has both black and blond hair.

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Example 15

60% of people on a large railway platform are reading a newspaper and 24% are reading a novel. The other people on the platform are not reading. What is the percentage of people on the platform who are:

a reading? **b** not reading?

Solution

a $60\% + 24\% = 84\%$

84% of people on the platform are reading.

b Since all of the people (either reading or not reading) on the platform $= 100\%$,

$100\% - 84\% = 16\%$.

Hence, 16% of people on the platform are not reading.

Note that in the last example there is once again no overlap between the people on the platform who are reading and those who are not reading.

In the next example, we cannot add the percentages because they are percentages of different wholes. The first, wrong, solution shows what happens if you try to do this.

Example 16

Two football teams, the Sharks and the Raiders, play a match. The Sharks club has 10 000 members and the Raiders club has 12 000 members. 15% of the Sharks club members and 20% of the Raiders club members attend the match. What percentage of the combined membership of the clubs attend the match?

Solution (Wrong!)

$15\% + 20\% = 35\%$

35% of the combined membership of the clubs attend the match.

Solution (Correct)

Number of Sharks attending $= 15\%$ of $10\,000 = 1500$

Number of Raiders attending $= 20\%$ of $12\,000 = 2400$

$$\begin{aligned}\text{Number of supporters attending the match} &= 1500 + 2400\\ &= 3900\end{aligned}$$

$$\begin{aligned}\text{Combined membership of the clubs} &= 10\,000 + 12\,000\\ &= 22\,000\end{aligned}$$

$$\begin{aligned}\text{Percentage of combined membership at match} &= \frac{3900}{22\,000} \times 100\%\\ &= \frac{195}{11}\%\\ &= 17\tfrac{8}{11}\%\end{aligned}$$

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Multiplication of percentages

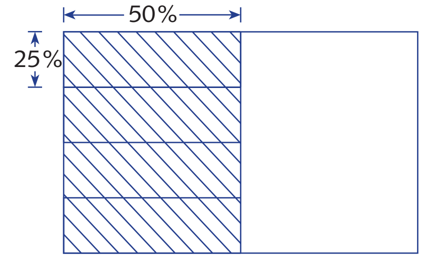
Multiplication of percentages arises in situations such as the following.

50% of a class are boys, and 25% of them have blue eyes. Hence, the percentage of students in the class who are boys with blue eyes is 25% of 50%.

We can draw a diagram to illustrate this. The large cross-hatched rectangle represents the boys in the class, and the smaller blue rectangle represents the 25% of the boys who have blue eyes.

The percentage of students who are boys and have blue eyes is:

$$\begin{aligned} 25\% \text{ of } 50\% &= \frac{1}{4} \text{ of } \frac{1}{2} \\ &= \frac{1}{8} \\ &= 12\tfrac{1}{2}\% \end{aligned}$$



Example 17

Find 20% of 70%.

Solution

$$\begin{aligned} 20\% \text{ of } 70\% &= \frac{20}{100} \times \frac{70}{100} \\ &= \frac{1}{5} \times \frac{7}{10} \\ &= \frac{7}{50} \\ &= 14\% \end{aligned}$$

Example 18

There are 65 000 spectators in a football stadium of whom 55% barrack for the Reds. Of those who barrack for the Reds, 80% are male. Find the percentage of spectators who are male and barrack for the Reds.

Solution

Percentage of spectators who are male and barrack for the Reds

$$\begin{aligned} &= 80\% \text{ of } 55\% \\ &= \frac{80}{100} \times 55\% \\ &= \frac{4}{5} \times \frac{11}{20} \\ &= \frac{44}{100} \\ &= 44\% \end{aligned}$$

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Exercise 7C

1 In each part, add the percentages and express the result as a fraction.

a $5\% + 20\% + 30\%$

b $0.5\% + 24\% + 65.5\%$

c $2\% + 10\% + 50\%$

d $12.5\% + 22.5\% + 2.25\%$

Example 17

2 Calculate:

a 20% of 50%

b 15% of 30%

c 60% of 80%

d 34% of 90%

e $12\frac{1}{2}\%$ of 80%

f $33\frac{1}{3}\%$ of 25%

g 40% of 35%

h 80% of 15%

i $66\frac{2}{3}\%$ of 50%

3 a Find 30% of 20%

b Find 20% of 30%.

Example 15

4 65% of a group of 4300 teenagers said they like healthy food.

a How many of the teenagers said they like healthy food?

b What is the percentage of the teenagers who said they do not like healthy food?

5 In a particular group of 600 people:

- 15% watch only the Channel Nine news every night
- 40% watch only the Channel Seven news every night
- 25% watch only the Channel Two news every night.

a How many people watch:

i the Channel Nine news?

ii the Channel Seven news?

iii the Channel Two news?

b How many people watch none of these news broadcasts?

6 When a group of 500 people were asked to name their favourite sport 34% responded ‘only football’ and 26% responded ‘only cricket’.

a How many people responded ‘only football’?

b How many people responded ‘only cricket’?

c How many people did not respond ‘only cricket’ or ‘only football’?

Example 18

7 15% of a herd of goats are male and 6% of the male goats are older than 4 years of age. What percentage of the total number of goats are male and older than 4 years of age?

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8 A jar contains dark chocolate balls and milk chocolate balls. 35% of the chocolate balls are dark chocolate and 15% of the dark chocolate balls are wrapped in gold foil. What percentage of the chocolate balls are:

a milk chocolate?

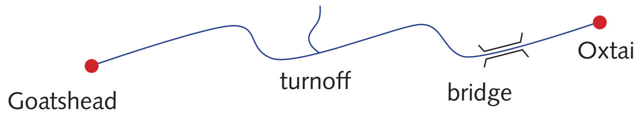
b dark chocolate in gold foil?

9 60% of a group of students are boys, and 30% of the boys have blue eyes. What percentage of the class are boys with blue eyes?

10 80% of a herd of cattle are Herefords, and 15% of the Herefords are under one year of age. What percentage of the cattle are Herefords under one year of age?

11 40% of a group of students are going to a concert, and 23% of the students going to the concert are also going to a party afterwards. What percentage of students go to both the concert and the party afterwards?

12 If you travel from Goatshead to Oxtail, you have covered 45% of the total distance between the two towns when you reach the turnoff. The bridge is 60% of the way from the turnoff to Oxtail. What percentage of the total trip still remains when you get to the bridge?



13 Explain why the following reasoning is incorrect. 20% of the students going to one school have blond hair. 30% of the students going to a nearby school have blond hair. Therefore, $20\% + 30\% = 50\%$ of the students in the two schools have blond hair.

14 Explain why the following reasoning is incorrect. 60% of the Queensland Reds' supporters wear red shirts. 40% of the Queensland Reds' supporters are female. Therefore, 40% of $60\% = 24\%$ of Queensland Reds' supporters are female and wear red shirts.

7D Percentage increase and decrease

Percentages provide a convenient way to express an increase or decrease in a quantity.

In the newspapers or on television, you will often hear such news as:

- A company's profits for the year have increased by 20%.
- The population of a city has increased by 7% over the past two years.
- The grain production in a certain area is down by 25%.

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Percentage increase

Example 19

a Increase 600 by 5%.

b Increase 850 by $6\frac{1}{2}\%$.

Solution

a 5% of $600 = \frac{5}{100} \times 600$
$= 30$

Therefore, 600 increased by $5\% = 600 + 30$
$= 630$

b $6\frac{1}{2}\%$ of $850 = \frac{6\frac{1}{2}}{100} \times \frac{850}{1}$

$= \frac{13}{\cancel{200}^{4}} \times \frac{\cancel{850}^{17}}{1}$

$= \frac{13 \times 17}{4}$

$= \frac{221}{4}$

$= 55\frac{1}{4}$

Therefore, 850 increased by $6\frac{1}{2}\% = 850 + 55\frac{1}{4}$
$= 905\frac{1}{4}$

Example 20

A house was valued at \$200 000 in January 2004 and at \$324 000 in January 2011.

Find the percentage increase in the value of the house.

Solution

Increase in value of the house $= 324\,000 - 200\,000$
$= 124\,000$

Percentage increase in value $= \frac{124\,000}{200\,000} \times 100\%$
$= 62\%$

We often use percentage increase to describe such things as population increases. For example, we may read in the newspaper that the population of Queensland increased by 8% over the past five years.

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Example 21

The population of a particular mob of kangaroos in a given area is increasing by 5% every year. If the population on 1 January 2005 was 660, what was the population of kangaroos on 1 January 2006?

Solution

If the population at 1 January 2005 was 660, then the population on 1 January 2006 would be 660 + (5% of 660).

$$\begin{aligned}5\% \text{ of } 600 &= \frac{1}{20} \times 660\\ &= \frac{66}{2}\\ &= 33\end{aligned}$$

The total population of kangaroos on 1 January 2006 was 660 + 33 = 693.

Example 22

A shop finds its sales for February have increased by 24% from its January figures. In January, the sales figure was \$32 000. Find the sales for February.

Solution

$$\begin{aligned}\text{Increase in sales} &= 24\% \text{ of } 32\,000\\ &= \frac{24}{100} \times 32\,000\\ &= \$7680\end{aligned}$$

$$\begin{aligned}\text{February sales} &= 7680 + 32\,000\\ &= \$39\,680\end{aligned}$$

Percentage decrease

Everyday examples of percentage decrease are also common. For example:

- The number of pelicans in a particular breeding colony has decreased by 20% from the previous year.
- The numbers of supporters at the matches of the Gunarrong Football Club are down by 30% from the previous season.
- The number of tourists visiting the Big Turnip tourist site has decreased by 15% from the previous year.

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Example 23

a Decrease 600 by 8%.

b Decrease 800 by $12\frac{1}{4}\%$.

Solution

a 8% of $600 = \dfrac{8}{100} \times \dfrac{600}{1}$
$= 48$

Decreased amount $= 600 - 48$
$= 552$

b $12\frac{1}{4}\%$ of $800 = \dfrac{12\frac{1}{4}}{100} \times \dfrac{800}{1}$
$= \dfrac{49}{\cancel{400}^{1}} \times \dfrac{\cancel{800}^{2}}{1}$
$= 98$

Decreased amount $= 800 - 98$
$= 702$

Example 24

The population of Gopedope was 80 000 on 1 February 1845. By 1 February 1901, the population had fallen by 45%. Find the population on 1 February 1901.

Solution

Drop in population of Gopedope $= 45\%$ of $80\,000$
$= \dfrac{45}{100} \times 80\,000$
$= 45 \times 800$
$= 36\,000$

Population of Gopedope on 1 February $1901 = 80\,000 - 36\,000$
$= 44\,000$

Example 25

Rabbit Island is home to two types of rabbits.

On 1 January 2005, there were 600 grey rabbits and 400 pink-nosed rabbits.

On 1 January 2006, there were 1260 grey and 550 pink-nosed rabbits.

a For each type of rabbit, find the percentage increase in population.

b Find the percentage increase in the total rabbit population.

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Solution

a Grey rabbits increased by $1260 - 600 = 660$.

$$\begin{aligned}\text{Percentage increase} &= \frac{660}{600} \times 100\% \\ &= \frac{660}{6}\% \\ &= 110\%\end{aligned}$$

Pink-nosed rabbits increased by $550 - 400 = 150$.

$$\begin{aligned}\text{Percentage increase} &= \frac{150}{400} \times 100\% \\ &= 37.5\%\end{aligned}$$

b The total number of rabbits, increased from $600 + 400 = 1000$ by $660 + 150 = 810$.

$$\begin{aligned}\text{Percentage increase} &= \frac{810}{1000} \times 100\% \\ &= 81\%\end{aligned}$$

Discount

Discount is an important example of the use of percentages. Shops offer discounts as incentives to buy. You will see signs such as these written on shop windows such as ‘15% off all track pants’.

The method for calculating a discounted price is shown in the following example.

Example 26

Gloria goes to the Mighty Good Emporium to buy a bicycle. The shop is offering a 20% discount on all bicycles. The bicycle Gloria wants is marked at \$460. How much does she pay for it?

Solution

$$\begin{aligned}20\% \text{ of } \$460 &= \frac{20}{100} \times 460 \\ &= \frac{1}{5} \times 460 \\ &= 92\end{aligned}$$

Gloria obtains a discount of \$92. She pays $(460 - 92) = \$368$.

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Exercise 7D

Example 19

1 Find the value when:

a 800 is increased by 7%

b 1000 is increased by 6%

c 750 is increased by 8%

d 270 is increased by 10%

e 4500 is increased by 9%

f 6000 is increased by $6\frac{1}{2}\%$

2 Calculate the percentage change (increase or decrease) that occurs when:

a 20 becomes 40

b 25 becomes 100

c 2 becomes 10

d 10 becomes 11

e 8 becomes 10

f 110 becomes 100

g 90 becomes 120

h 100 becomes 80

i 48 becomes 36

3 Find the new amount after each percentage change.

a \$45 000 is increased by 20%

b \$60 000 is decreased by 15%

c \$32 000 is increased by $12\frac{1}{2}\%$

d \$42 000 hectares is increased by 24%

Example 20

4 The circulation of a newspaper on 1 January 2009 was 720 000. On 1 January 2010, the circulation was 800 000. Find the percentage increase in circulation.

Example 22

5 The farmers in a particular region of China planted 45 000 hectares of farmland with sorghum in 2005. By 2010, the area had increased by 24%. Find the area planted with sorghum in 2010.

Example 23

6 Find the value when:

a 500 is decreased by 10%

b 6250 is decreased by 8%

c 5500 is decreased by 6%

d 7000 is decreased by $6\frac{1}{2}\%$

Example 26

7 Calculate the discounted price in each case.

a A discount of 10% on a purchase of \$3210

b A discount of 12% on a purchase of \$5600

c A discount of 5% on a purchase of \$68

d A discount of 8% on a purchase of \$56

e A discount of 23% on a purchase of \$78.50

Example 24

8 The population of a city was 92 000 on 1 February 1920. By 1 February 1950, the population had fallen by 45%. Find the population at this second date.

9 The circulation of a newspaper on 1 January 2008 was 840 000. On 1 January 2009, the circulation was 640 000. Find the percentage decrease in circulation.

10 If the annual increase in the population of a country is 2.5% and the present number of inhabitants is 2 624 000, what will the population be in a year's time?

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11 The population of a city was 1 000 000 and it increased by $2\frac{1}{2}\%$ of this value in each of 3 succesive years.

a What was the population at the end of the first year?

b What was the population at the end of 3 years?

Example 20

12 A house was bought for \$300 000 in 2000. It was sold again in 2005 for \$475 000. What was the percentage increase in the value of the house?

13 A man buys a car for \$7500 and sells it two years later for \$5000. What is his percentage loss?

14 Two different machines are used to produce chocolates in a chocolate shop.

Machine A produces 520 g of chocolate from 650 g of raw materials. Machine B produces 450 g of chocolates from 600 g of raw materials. The rest of the raw materials stick to the machines.

a What percentage, by weight, of the raw materials is turned into chocolates by machine A?

b What percentage, by weight, of the raw materials is turned into chocolates by machine B?

c Which of the machines is the more efficient?

15 A floor covering shop made a profit of 6% on its total costs last year. If the total costs were \$1 240 000, what was the profit?

16 A shoe shop had total costs of \$5 200 000 on salaries, shoes etc., and their total sales was \$4 628 000.

a What was the loss?

b What was the loss expressed as a percentage of the total costs?

17 Find the profit or loss for these situations.

a 10% loss on \$76 000

b 8% profit on \$80 000

c 50% profit on \$1 000 000

d 25% loss on \$420 000

18 Find the percentage profit or loss on costs in these situations.

a Costs \$12 000 and sales \$13 920

b Costs \$52 000 and sales \$56 680

c Costs \$11 200 and sales \$100 800

d Costs \$22 000 and sales \$200 900

19 A shop has a discount sale that marks down all prices by 20%. If a customer buys a jacket originally marked at \$90, how much does he pay?

20 Gaye buys three novels and two textbooks at a shop that is advertising 25% discount on all books. The novels are marked at \$36 each and the textbooks are marked at \$72 and \$96. What is the total price after the discount has been given?

21 A shopkeeper paid \$200 each for two handbags. He sold the first handbag for a profit of 25% of the cost price. He sold the second at a loss of 15% of the cost price. How much profit did the shopkeeper make on the two bags?

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7E The unitary method

The unitary method was introduced in Section 2D. Here is a reminder of how it works.
The unitary method is very useful for doing the problems of the previous section in reverse.

Example 27

If 8 oranges cost \$5.60, how much do 3 oranges cost?

Solution

$\boxed{\div 8}$ 8 oranges cost \$5.60

1 orange costs $\dfrac{5.60}{8} = 0.7$

$= 70\text{c}$

$\boxed{\times 3}$ 3 oranges cost $3 \times 70\text{c} = 210\text{c}$

$= \$2.10$

Notice that we first worked out the cost of one orange. The word 'unit' is another word for 'one' (of some object). This is why the method is called the *unitary* method.

The same idea can be used to find 100% of an amount if you are given a percentage of that amount.

Example 28

7% of an amount of money is \$84. What is the sum of money?

Solution

7% of the amount is 84.

$\boxed{\div 7}$ 1% of the amount is 12.

$\boxed{\times 100}$ 100% of the amount is \$1200.

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Example 29

8% of an amount of money is \$6000. How much is 3%?

Solution

8% of the amount is 6000.

$\boxed{\div 8}$ 1% of the amount is 750.

$\boxed{\times 3}$ 3% of the amount is \$2250.

Example 30

A 15% discount is offered on all goods at a clothing shop. A suit has a discounted price of \$425. What was the original price of the suit?

Solution

$$\begin{aligned}\text{Discounted price} &= (100 - 15)\% \text{ of the original price}\\ &= 85\% \text{ of the original price}\end{aligned}$$

85% of the original price = 425

$$\boxed{\div 85} \qquad \begin{aligned}1\% \text{ of the original price} &= \frac{425}{85}\\ &= 5\end{aligned}$$

$$\boxed{\times 100} \qquad 100\% \text{ of the original price} = 500$$

The original price of the suit was \$500.

Note: We could have used 5% as the unit here.

Alternative method

Let x be the original price.

$$\begin{aligned}85\% \text{ of } x &= 425\\ \frac{85}{100} \times x &= 425\\ x &= 425 \times \frac{100}{85}\\ &= 500\end{aligned}$$

The original price is \$500.

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Example 31

A population of ants increases by 20% in a week and the new population is 125 400. What was the population at the beginning of the week?

Solution

We can use 1% as the unit in this problem.

120% of the original population is 125 400.

$\boxed{\div 120}$ 1% of the original population is 1045.

$\boxed{\times 100}$ 100% of the original population is 104 500.

The population at the beginning of the week was 104 500.

Sometimes the method can be modified to make our calculations simpler.

Alternative method using 20% as the unit

120% of the original population is 125 400.

$\boxed{\div 6}$ 20% of the original population is 20 900.

$\boxed{\times 5}$ 100% of the original population is 104 500.

The population at the beginning of the week was 104 500.

Example 32

John spends 24% of his weekly wage on rent and he spends 25% of the remainder in repayments on a motorbike. He still has $228 left. How much does he earn in one week?

Solution

$$\begin{aligned}\text{Percentage of wage left after payment of rent} &= 100\% - 24\% \\ &= 76\%\end{aligned}$$

$$\begin{aligned}\text{Percentage of wage spent in repayments on motorbike} &= 25\% \text{ of } 76\% \\ &= \frac{1}{4} \text{ of } 76\% \\ &= 19\%\end{aligned}$$

$$\begin{aligned}\text{Percentage of wage remaining} &= 76\% - 19\% \\ &= 57\% \\ 57\% \text{ of John's wage} &= 228 \\ 1\% \text{ of of John's wage} &= \frac{228}{57} \\ &= 4\end{aligned}$$

Hence, 100% of John's wage is $400.

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GST (goods and services tax)

The GST on an item is calculated as 10% of the pre-GST price of that item and is included in the price. Therefore, the price including GST is 110% of the pre-GST price.

Example 33

The price of a suit before GST is added is \$440. What is the price after it is added?

Solution

100% of the pre-GST price is \$440.

$\boxed{\div 10}$ 10% of the price is \$44.

$\boxed{\times 11}$ 110% of the price is \$484.

Example 34

The price of a shirt, including GST, is \$61.60. What was the original price of the shirt before GST was added?

Solution

110% of the original price is 61.60.

$\boxed{\div 11}$ 10% of the original price is \$5.60.

$\boxed{\times 10}$ 100% of the original price is \$56.

Exercise 7E

Example 28

1 20% of the children in a swimming pool are boys. There are 8 boys. How many children are there in the swimming pool?

Example 29

2 **a** 20% of an amount of money is \$46 890. What is 80% of the amount of money?

b 5% of an amount of money is \$42 000. What is 20% of the amount of money?

c $12\frac{1}{2}\%$ of an amount of money is \$64 800. What is the total amount of money?

d $33\frac{1}{3}\%$ of an amount of money is \$45 000. What is 60% of the amount of money?

Example 30

3 The price of a car has been discounted by 10%. The discounted price is \$22 500. What was the price before the discount?

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4 A park contains paved areas, lawns and garden beds. 8% of the area of the park is garden bed. There are 22 000 m^2 of garden bed.

a What is the total area of the park?

b The lawns are 80% of the park. How many square metres of lawn are there?

5 Mr Ho invests an amount of money for one year at 5% per annum interest. The bank tells him that at the end of the year he will have \$126 000. How much does Mr Ho invest?

Example 31

6 The number of students at a school increased by 12% from 2010 to 2011. In 2011, there are 1680 students. How many students were there in 2010?

7 a 105% of x is 568 911. Find x.

b 10% of x is 45 670. Find x.

c 7% of w is 8470. Find w.

d 120% of a is 14 460. Find a.

Example 32

8 Raylee gives 60% of her weekly wage to her mother and 25% of the remainder to her brother. She still has \$240 left. How much does she earn in one week?

9 Felicity spent 20% of her savings on a bicycle and 15% of the remainder on a book. What percentage of her savings did she have left?

10 A discount of 15% is offered on the price of a book. The discounted price is \$136. What was the original price?

Example 33

11 The price of a bike before GST is added is \$320. What is the price after GST is added?

Example 34

12 For each item, the cost including GST is given. What was the cost of each item before GST was added?

a A shirt priced at \$88

b A book priced at \$68.20

c A model aeroplane priced at \$253

d A chess set priced at \$93.50

Review exercise

1 Write these fractions as percentages. Use fractions in your answers if necessary.

a $\frac{23}{25}$ b $\frac{3}{4}$ c $\frac{3}{8}$ d $\frac{7}{8}$ e $\frac{3}{5}$ f $\frac{2}{3}$

g $\frac{45}{60}$ h $\frac{110}{100}$ i $\frac{48}{72}$ j $\frac{12}{96}$ k $\frac{31}{25}$ l $\frac{300}{150}$

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2 Write each percentage as a mixed numeral with the fraction in simplest form.

a 50% **b** 85% **c** 43% **d** 200%

e 340% **f** 185% **g** 12.5% **h** 624%

3 Calculate:

a 40% of 20 **b** 25% of 18 **c** 23% of 200

d 18% of 150 **e** 97% of 200 **f** 3% of 600

g 85% of 4080 **h** 110% of 340 000 **i** 125% of 36

4 In a city, 33% of vehicles are utes, 18% are four-wheel drives, 27% are sedans and 22% are station wagons. If there are 29 700 vehicles in the city, how many of each type of vehicle are there?

5 When the kindergarten children arrived yesterday, $\frac{1}{4}$ of them were wearing skirts, $\frac{2}{5}$ were wearing shorts, $\frac{3}{10}$ were wearing dresses and the rest of the children were wearing jeans. What percentage of the children were wearing jeans?

6 What is the weight of fat, in grams, in one 200 g container of yoghurt, if 3% of the yoghurt, by weight, is fat?

7 Find:

a 12.5% of 3000 **b** 3.5% of 6000 **c** $33\frac{1}{3}$% of 4200

d 5.6% of 9000 **e** 62.5% of 4200 **f** 12.5% of 320 000

g 5.5% of 200 **h** 2.4% of 1000 **i** 0.1% of 78

j 15% of 140 **k** 99.95% of 30 000 **l** 1.375% of 240

8 The label on a 750 mL bottle of soft drink states that it contains 5% pure lemon juice.

a How many millilitres of pure lemon juice does it contain?

b If pure lemon juice costs 50 cents per 100 mL, what is the value of the lemon juice in the bottle?

9 In a group of 500 people, 22% are left-handed. How many of the group are left-handed?

10 In a flock of 2650 sheep, 32% have black wool. How many black sheep are there?

11 A breakfast cereal contains $23\frac{1}{2}$% oats, by weight. What is the weight of oats in a package of cereal containing 1500 g of the cereal?

12 There are 16 540 high-school students in Sunview, and 60% of them attend state high schools. What is the number of state high-school students in Sunview?

13 It is known that watermelons are 92% water. A particular watermelon weighs 3600 g. How much does the water in this watermelon weigh?

14 A meat pie weighs 320 g. It is found that 73% of the pie is meat. What is the weight of the meat in the pie?

15 A packet of shortbread biscuits weighs 280 g. It is known that the biscuits are 12% butter, by weight. Find the weight of butter in one packet of biscuits.

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16 In total, 14 carbon, hydrogen and nitrogen atoms make up one toxic molecule. If the number of carbon atoms is 5 more than the number of nitrogen atoms, and $42\frac{6}{7}\%$ of the atoms are carbon, find the number of each type of atom.

17 The price of a shirt including GST is \$110. What is the pre-GST price of the shirt?

18 A hotel charges a fee of 2% for a guest using a credit card. If the hotel bill is \$1250 before the fee is applied, how much is the fee?

19 A shop is offering a discount of 10% on all goods. If a garden table is priced at \$550, what is the discount?

20 The label on a 750 mL bottle of soft drink states that it contains 15% pure mango juice.

a How many millilitres of mango juice does it contain?

b If pure mango juice costs 25 cents per 100 mL, what is the value of the mango juice in the bottle?

21 In a clothing factory, 28% of the employees are female. There are 672 female employees. How many employees are there in total?

22 A sheep station has 1872 merino sheep. This is 36% of all the sheep on the station. How many sheep are there on the station?

23 A farm grows both barley and wheat. 30% of the barley is grown organically and 60% of the wheat is grown organically. 70% of the grain produced by the farm is wheat and the remainder is barley. What percentage of the farm's grain is organically grown?

24 On a test containing 30 true/false questions, David gave 50% more right answers than wrong ones. How many questions did he answer correctly?

25 A car salesyard is offering 15% off all new cars. If a car is priced at \$32 500, what is the discount being offered?

Challenge exercise

1 Fred Callum earns a salary of \$450 per week for a 40-hour week. His weekly salary is increased by 10% and his number of hours worked is decreased by 10%. Calculate his new hourly salary.

2 The length of a rectangle is increased by 15% and the width decreased by 10%. If the area of the original rectangle is 100 cm^2, find the area of the new rectangle.

3 In a lottery, only 0.002% of tickets won prizes. If there were two prizes, how many tickets were sold?

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4 An illness has struck the town of Abelanne. On 1 January, 20% of the residents have the illness. On 1 February, 10% of the people who had the disease have recovered but 40% of those who did not have the disease now have it. What percentage of the population has the disease on 1 February?

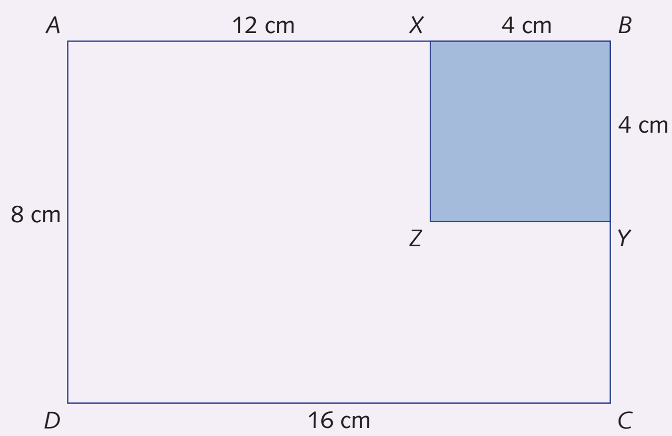
5 In an election in a small island nation of 10 000 people, 80% of the population voted. Of those who voted, 63% voted for the New Liberation Party. How many people did not vote for the New Liberation Party?

6 A dairy farmer found that the milk supplied by his cows was 5% cream and 95% skimmed milk. He wanted to know how much skimmed milk he should add to a litre of milk to reduce the percentage of cream to 4%. (*Recall*: 1 L = 1000 mL). What is the answer?

7 A girl makes up a cordial drink so that 10% of the drink is cordial concentrate. The rest is water. The girl has 350 mL of the drink in a jug. She now decides that she wants the cordial concentrate content to be only 8%. How much extra water does she need to put in the jug?

8 **a** **i** Find the area of the shaded region *XBYZ*.

ii Find the area of the unshaded region *AXZYCD*.



b Find 20% of the area of rectangle *ABCD*.

c Find 80% of the shaded area + 20% of the area of *AXZYCD*.

d Express the answer to part **c** as a percentage of the area of rectangle *ABCD*.

9 35% of students in a class have brown hair. Five brown-haired students join the class so that 48% now have brown hair. How many students were previously in the class?

10 What two-digit numbers increase by 75% when their digits are reversed?

11 The number 12 321 is a **palindrome** since it is unchanged when its digits are reversed. What percentage of five-digit numbers are palindromes?

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CHAPTER

8

Measurement and Geometry

Pythagoras' theorem

Triangles have been a source of fascination for people over many millenniums. You will learn some of the intriguing facts about triangles in this and subsequent chapters.

Pythagoras' theorem is an amazing result that relates the lengths of the sides of a right-angled triangle. It is one of the most widely used results in all of mathematics. In this chapter, we will learn about why it is so useful and important, and how to prove it.

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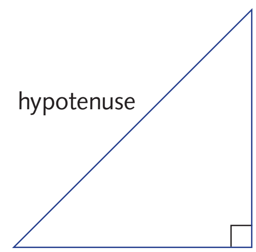
Introduction



Among the set of all triangles, there is a special class, known as **right-angled triangles** or **right triangles**. A right-angled triangle has one and only one right angle, and the other two angles are complementary acute angles.

The longest side in a right triangle is called the **hypotenuse**. This name is derived from the Greek word ‘teino’, which is related to the word ‘tension’ in English, and has the basic meaning of *to stretch*.

It was well known early in the ancient world that if the two sides adjacent to the right angle have lengths 3 units and 4 units, then the hypotenuse has a length of 5 units.

A number of questions arise:

- Is there anything special about the numbers 3, 4 and 5?
- Are there other sets of three numbers that form the side lengths of a right-angled triangle?
- Is there some relationship between the lengths of the sides of a right-angled triangle?

These were questions that the Greeks examined in some detail. They found and proved a remarkable statement that relates the lengths of the three sides of a right-angled triangle. This work was done by the Pythagorean school of Greek thinkers, and so is known today as **Pythagoras’ theorem**.

A **theorem** is a statement of an important mathematical truth.

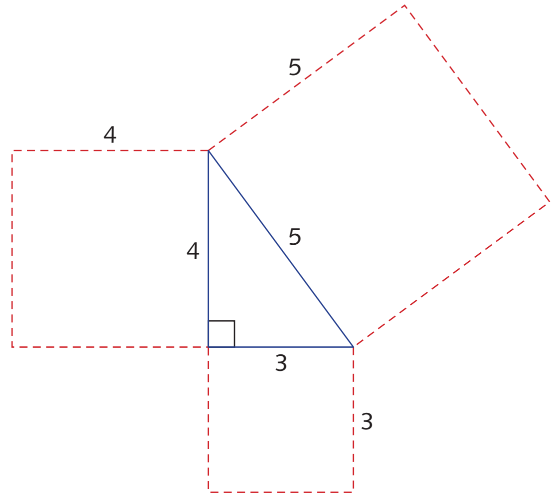
Exercise

Draw a line AB of length 12 cm, leaving yourself some space above the line you have drawn. Using a set square, draw a line AC at right angles to AB, of length 5 cm. Now join C and B to form a right-angled triangle and measure the length of the hypotenuse. You should find that the answer is 13 cm.

Exercise

Repeat the above construction, this time with $AB = 8$ cm and $AC = 6$ cm. Measuring the third side, you should find it is 10 cm. This is because this triangle is an enlargement of the 3, 4, 5 triangle.

To understand what Pythagoras and the Greeks proved, we need to look at the *squares* of the lengths of the sides of a right-angled triangle.



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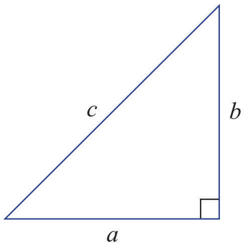
Lengths	3	4	5	Lengths	5	12	13	Lengths	6	8	10
Squares	9	16	25	Squares	25	144	169	Squares	36	64	100

Can you see the pattern?

- In the 3, 4, 5 triangle, $9+16=25$, that is, $3^2+4^2=5^2$.
- In the 5, 12, 13 triangle, $25+144=169$, that is, $5^2+12^2=13^2$.
- And in the 6, 8, 10 triangle, $36+64=100$, that is, $6^2+8^2=10^2$.

Pythagoras' theorem

- The square of the length of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the lengths of the other two sides.
- In symbols, if we call the length of the hypotenuse c, and call the lengths of the other two sides a and b, then $a^2+b^2=c^2$.

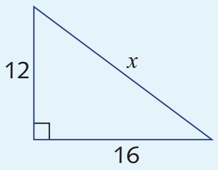


8A Applying Pythagoras' theorem

We can use Pythagoras' theorem to find the length of the hypotenuse of a right-angled triangle, without measuring, if we know the two shorter sides.

Example 1

Find the length of the hypotenuse in the right-angled triangle opposite.



Solution

Let x be the length of the hypotenuse.

Pythagoras' theorem says that:

$$\begin{aligned} x^2 &= 12^2+16^2 \\ &= 144+256 \\ &= 400 \end{aligned}$$

so $x=20$

Remember that 400 is the square of the length of the hypotenuse, so we have to take the square root in the final step to find the correct value of x.

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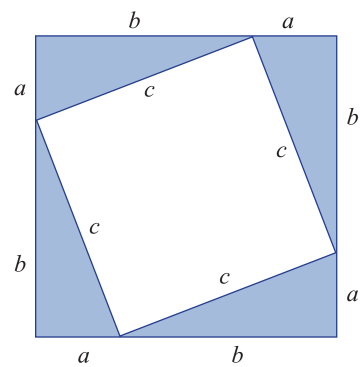
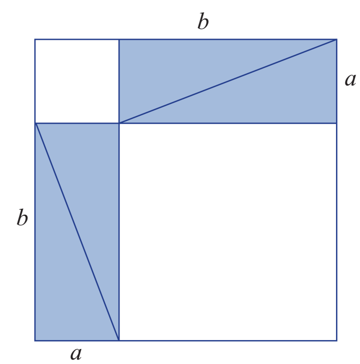
Why the theorem is true

To establish a theorem, we must give a valid proof. There are many ways to prove Pythagoras' theorem, but some are easier than others. Here is a simple geometric demonstration of why the result is true.

The diagram below shows four identical right-angled triangles, each with hypotenuse length c and the other sides of length a and b. They have been arranged within a large square of side length $a+b$, leaving uncovered the white square of side length c.



The triangles and the square, with side length c, can be rearranged within the same large square. The square at the top left has area a^2 and the square at the bottom right has area b^2.

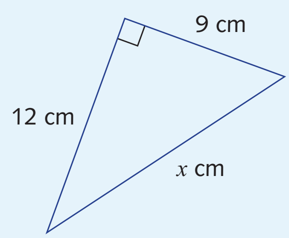


The area of the white square in the first diagram must be equal to the sum of the areas of the two white squares in the second diagram.

From this we can see that $a^2 + b^2 = c^2$.

Example 2

Use Pythagoras' theorem to find the value of x in the triangle below.



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Solution

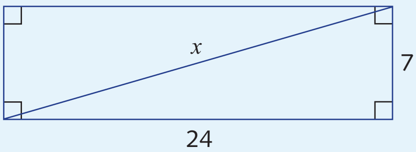
Pythagoras' theorem says that:

$$\begin{aligned} x^2 &= 9^2 + 12^2 \\ &= 81 + 144 \\ &= 225 \end{aligned}$$

so $x = 15$

Example 3

Find the length of the diagonal in the rectangle below.



Solution

The diagonal divides the rectangle into two right-angled triangles. Pythagoras' theorem tells us that:

$$\begin{aligned} x^2 &= 7^2 + 24^2 \\ &= 49 + 576 \\ &= 625 \end{aligned}$$

so $x = 25$

Exercise 8A

1 Write down these squares.

a 9^2 **b** 11^2

c 7^2 **d** 15^2

2 Find the square root of each of these numbers.

a 64 **b** 144

c 196 **d** 900

3 Calculate:

a $7^2 + 24^2$ **b** $9^2 + 12^2$

c $20^2 - 16^2$ **d** $17^2 - 15^2$

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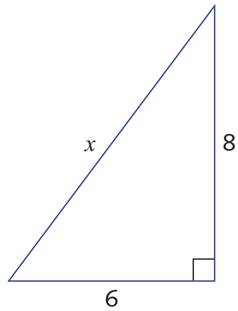
Example 1, 2

4 Use Pythagoras' theorem to find the length of the missing side in each of these triangles.

a



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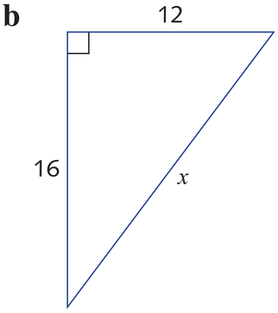

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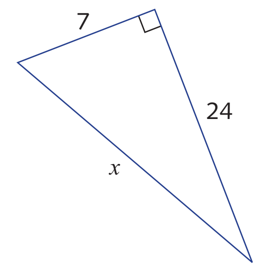
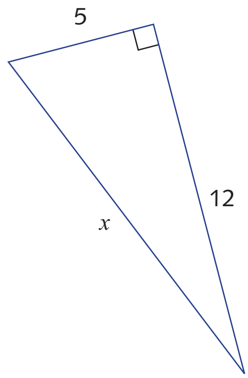


d



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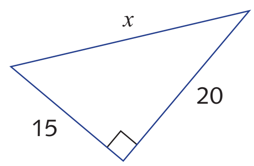

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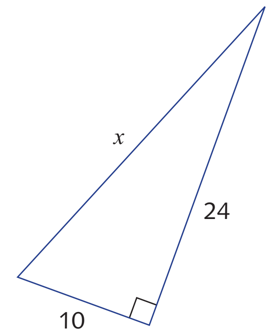
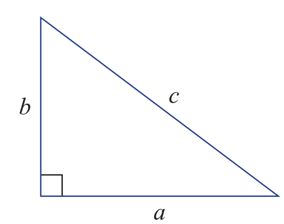
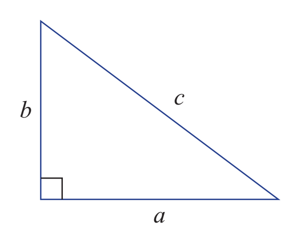


5 The triangle opposite is a right-angled triangle with hypotenuse of length c. The other two sides have lengths a and b. Apply Pythagoras' theorem to complete the table below. The first row is done for you. What pattern do you notice?



Row	a	b	a^2	b^2	$c^2 = a^2 + b^2$	c
1	3	4	9	16	25	5
2	6	8				
3	9	12				
4	12	16				
5	15	20				

6 The triangle opposite is a right-angled triangle with hypotenuse of length c. The other two sides have lengths a and b. Apply Pythagoras' theorem and complete the table below.
What pattern do you notice?



Row	a	b	a^2	b^2	$c^2 = a^2 + b^2$	c
1	3	4				
2	5	12				
3	7	24				
4	9	40				
5	11	60				

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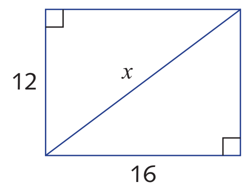
Example 3

7 Find the values of x, y and z in these diagrams.

a



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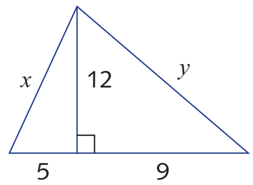

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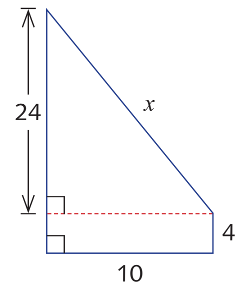

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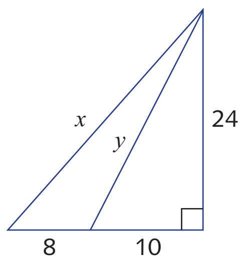

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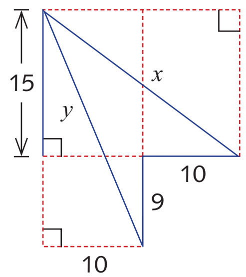

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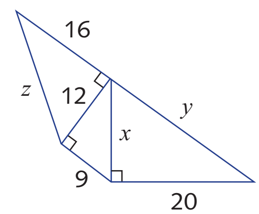
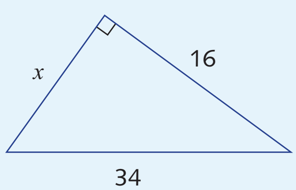


8B Finding a shorter side

We can use Pythagoras' theorem to find the length of one of the shorter sides in a right-angled triangle, given the other two sides.

Example 4

Find the length of the missing side, marked in the diagram as x.



Solution

Applying Pythagoras' theorem

$$
\begin{aligned}
x^2 + 16^2 &= 34^2 \\
x^2 + 256 &= 1156 \\
x^2 &= 1156 - 256 \\
&= 900
\end{aligned}
$$

so $\quad x = 30$ (taking the square root)

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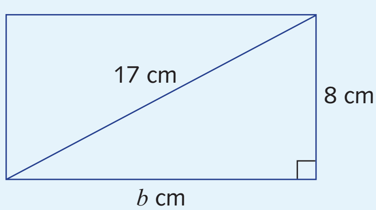
To make such calculations easier, here is a table of squares from 13 to 40. You should know the squares from 1 to 12 already.

Number	Square	Number	Square
13	169	27	729
14	196	28	784
15	225	29	841
16	256	30	900
17	289	31	961
18	324	32	1024
19	361	33	1089
20	400	34	1156
21	441	35	1225
22	484	36	1296
23	529	37	1369
24	576	38	1444
25	625	39	1521
26	676	40	1600

Example 5

A rectangle has width 8 cm and diagonal 17 cm.

What is its length?



Solution

Let b be the length, measured in cm. Then:

$$\begin{aligned} b^2 + 8^2 &= 17^2 \quad \text{(Pythagoras' theorem)} \\ b^2 + 64 &= 289 \\ b^2 &= 289 - 64 \\ &= 225 \\ \text{so} \quad b &= 15 \end{aligned}$$

The length of the rectangle is 15 cm.

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Converse of Pythagoras' theorem

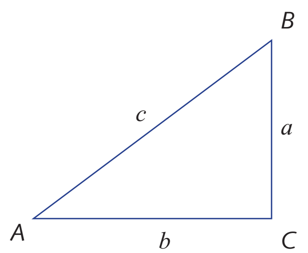
We have seen that in a right-angled triangle, the square of the length of the hypotenuse equals the sum of the squares of the lengths of the other two sides.

Now suppose that we have a triangle in which the square of the length of the longest side equals the sum of the squares of the lengths of the other two sides. Does it follow that the triangle is right-angled? The answer is yes, and this result is called the **converse** of Pythagoras' theorem.

We will be able to prove this after we have studied congruence in Chapter 12.

Theorem

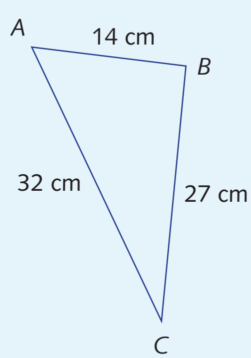
If $c^2 = a^2 + b^2$, then $\angle ACB$ is a right angle.



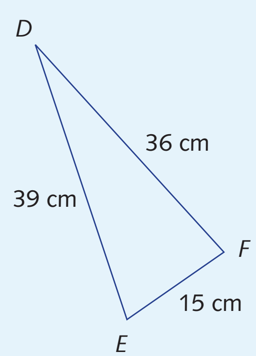
Example 6

Which of the triangles below are right-angled triangles? Name the right angle in each case.

a



b



Solution

In each case, let a and b be the lengths of the two shorter sides of the triangle and let c be the length of the longest side.

If $a^2 = b^2 + c^2$, then the triangle is a right-angled triangle, with the right angle opposite the hypotenuse.

a $14^2 + 27^2 = 925$

$32^2 = 1024$

$14^2 + 27^2 \neq 32^2$

The triangle is not right-angled.

b $15^2 + 36^2 = 1521$

$39^2 = 1521$

$15^2 + 6^2 = 39^2$

Triangle DEF is right-angled, with $\angle DFE = 90°$.

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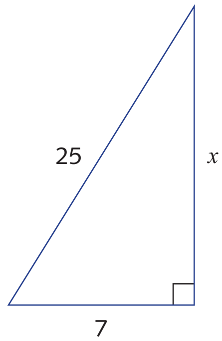
Exercise 8B

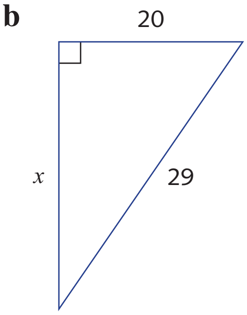
Example 4

1 Use Pythagoras' theorem to find the value of x in each triangle.

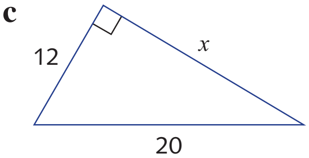
a



b



c



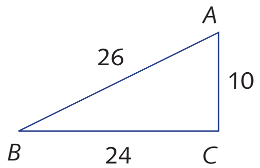
Example 5

2 A rectangle has one side of length 20 cm and its diagonal has length 25 cm. Find the length of the other side.

Example 6

3 Decide whether each of the triangles below is right-angled. If it is, name the right angle.

a



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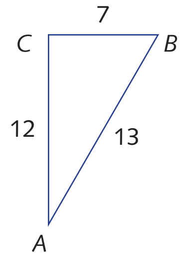

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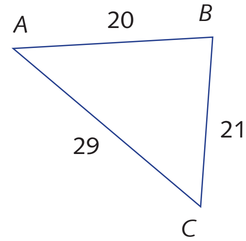
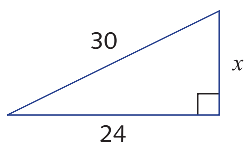


4 Find the values of the pronumerals in these diagrams.

a



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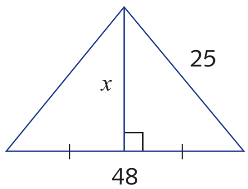

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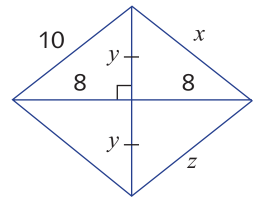

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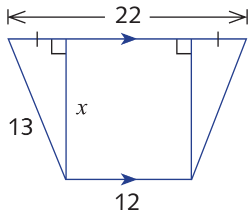

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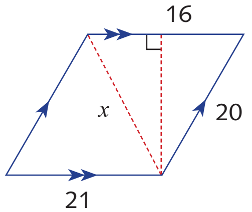

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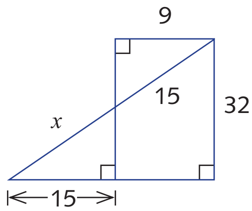

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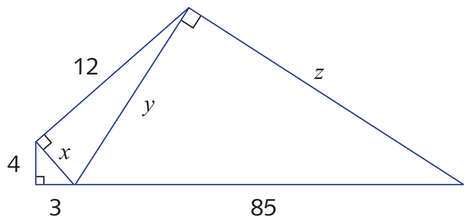

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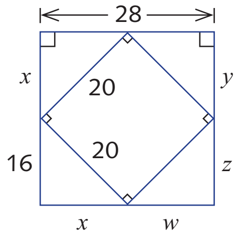

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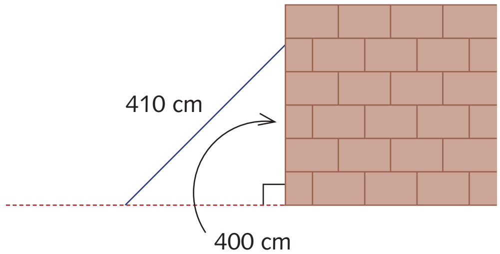
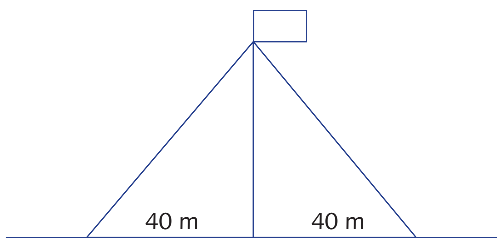
5 The sides of a rectangular table are 150 cm and 360 cm. What is the length of the diagonal?

6 A ladder of length 410 cm is leaning against a wall. It touches the wall 400 cm above the ground. What is the distance between the foot of the ladder and the wall?

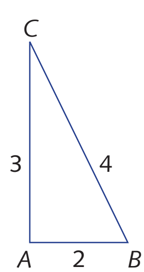


7 A flagpole is supported by two cables of equal length as shown below. The total length of the cables is 100 m. Each cable is attached to a point on level ground 40 m away from the foot of the flagpole. The dimensions of the flag are 1.2 m $\times$ 2.4 m. Calculate the height to the top of the flag.

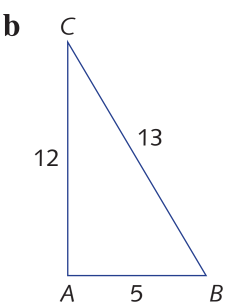


8 Which of the triangles below are right-angled triangles?

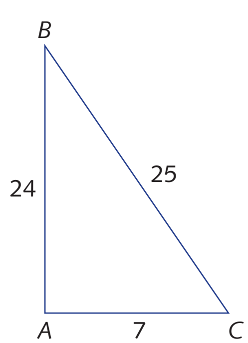
a



b



c



9 A ship sails 32 kilometres north and then 24 kilometres east. How far is it from its starting point?

10 **a** Find the square of the length of the diagonal of a square of side 1 cm.

b Find the square of the length of the diagonal of a square of side length:

i 2 cm **ii** 3 cm **iii** 5 cm **iv** n cm

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8C Irrational numbers

You may have noticed that in the final step in each example in Section 8B we arrived at a perfect square, such as 25, 169 or 625. We were then able to take the square root and arrive at a whole-number answer. In many cases this will not happen.

The square root of a number that is not a perfect square cannot be written as a fraction, and is an example of what is called an **irrational number**.

The square root of 2 is an example of an irrational number. Since $1^2 = 1$ and $2^2 = 4$, the square root of 2, which is written as $\sqrt{2}$, must be somewhere between 1 and 2.

Although we cannot express the square root of 2 in exact decimal form, we can find an **approximation** to it. In fact, $\sqrt{2} \approx 1.414$, correct to 3 decimal places, and $\sqrt{2} \approx 1.41$ correct to 2 decimal places.

When we use Pythagoras' theorem to solve a problem, we often need to find the square root of a number that is not a perfect square. We can do this approximately by using a calculator, or by looking up a table.

Here is a list of square roots of the numbers from 2 to 29. When the square root is not a whole number, we have given an approximation correct to 2 decimal places.

Number	Square root (2 dec. pl.)	Number	Square root (2 dec. pl.)
2	1.41	16	4
3	1.73	17	4.12
4	2	18	4.24
5	2.24	19	4.36
6	2.45	20	4.47
7	2.65	21	4.58
8	2.83	22	4.69
9	3	23	4.80
10	3.16	24	4.90
11	3.32	25	5
12	3.46	26	5.10
13	3.61	27	5.20
14	3.74	28	5.29
15	3.87	29	5.39

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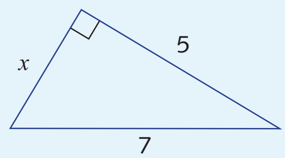
Example 7

Find the length, correct to 2 decimal places, of the missing side in the right-angled triangle.



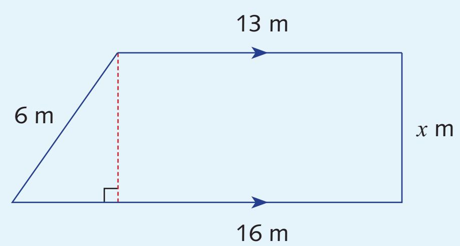
Solution

By Pythagoras' theorem:

$$\begin{aligned} x^2 + 5^2 &= 7^2 \\ x^2 + 25 &= 49 \\ x^2 &= 49 - 25 \\ &= 24 \\ x &= \sqrt{24} \\ &\approx 4.90 \text{ (using the table)} \end{aligned}$$

Example 8

Find the height of the trapezium below.



Solution

By Pythagoras' theorem:

$$\begin{aligned} x^2 + (16 - 13)^2 &= 6^2 \\ x^2 + 9 &= 36 \\ x^2 &= 36 - 9 \\ &= 27 \\ x &= \sqrt{27} \\ &\approx 5.20 \text{ (using the table)} \end{aligned}$$

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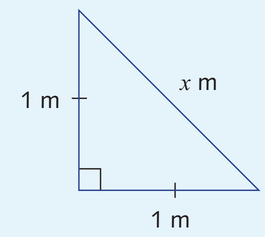
Example 9

Find the length of the hypotenuse in the triangle to the right, correct to 2 decimal places. What type of triangle is it?



Solution

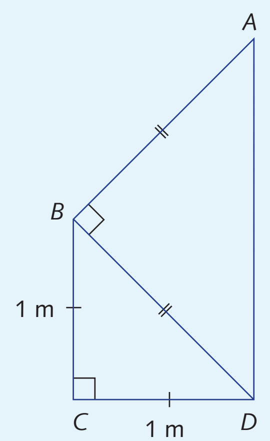
By Pythagoras' theorem:

$$\begin{aligned} 1^2 + 1^2 &= x^2 \\ x^2 &= 2 \\ x &= \sqrt{2} \\ &\approx 1.41 \text{ (to 2 decimal places)} \end{aligned}$$

This triangle is an isosceles right-angled triangle. The length of the hypotenuse is 1.41 m, correct to 2 decimal places.

Example 10

Find the length of AD in the diagram to the right.



Solution

By Pythagoras' theorem:

$$\begin{aligned} 1^2 + 1^2 &= BD^2 \\ \text{so} \quad BD^2 &= 2 \\ \text{Also,} \quad BD^2 + AB^2 &= AD^2 \\ \text{so} \quad 2 \times BD^2 &= AD^2 \quad \text{(isosceles } \Delta ABD\text{)} \\ \text{Hence,} \quad 2 \times 2 &= AD^2 \\ AD^2 &= 4 \\ \text{so} \quad AD &= 2 \end{aligned}$$

The length of AD is 2 m.

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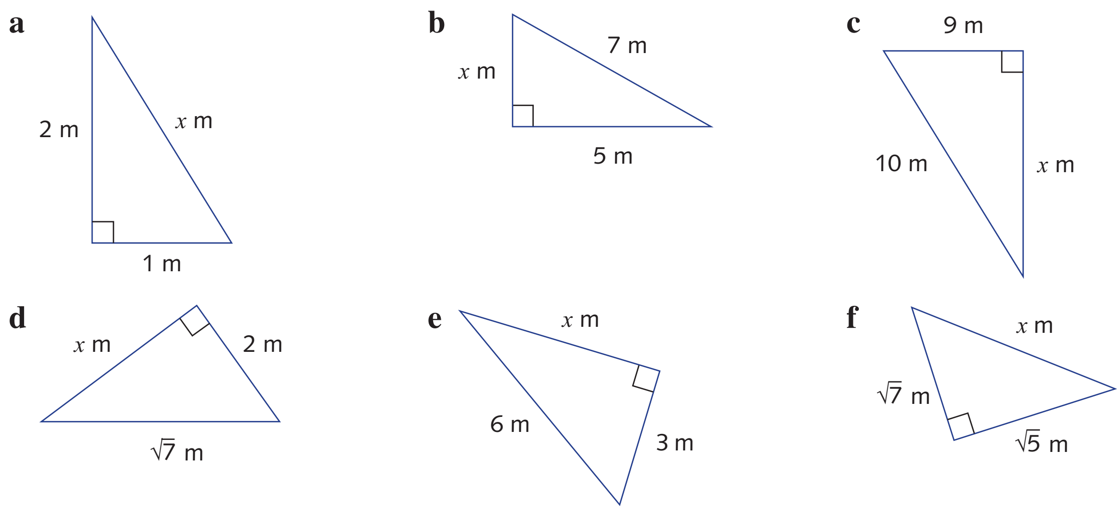
Exercise 8C

Note: In this exercise, you should use the table on page 206 to work out any square roots you need.

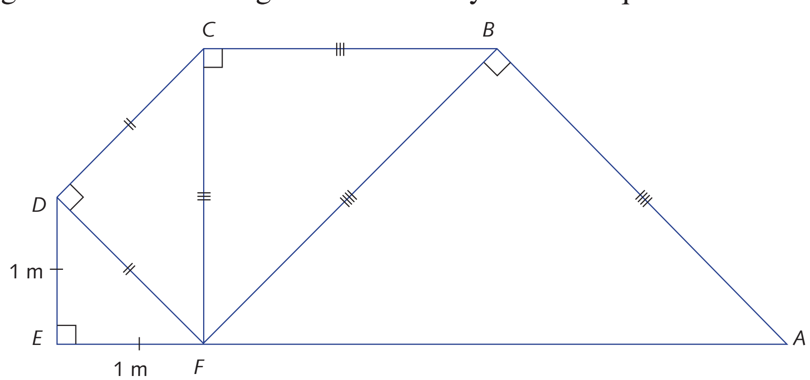
Example 7, 9

1 Use Pythagoras' theorem to find the length of the unknown side in each diagram below. Round your answers correct to 2 decimal places.



Example 10

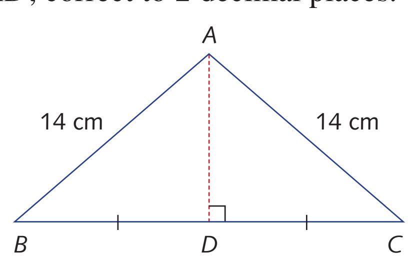
2 Find the length of AF in the diagram below. Only take the square root in the final step.



3 Find the length of the diagonal of a rectangle with side lengths 2 cm and 5 cm. Round off your answer correct to 2 decimal places.

4 Find the perimeter of a square with diagonal length 6 cm. Round your answer to 2 decimal places.

5 An isosceles triangle ABC has equal sides $AB = AC = 14$ cm. The side BC is 26 cm. A line AD is drawn at right angles from A to BC. This divides BC into two intervals of equal length. Find the length of AD, correct to 2 decimal places.



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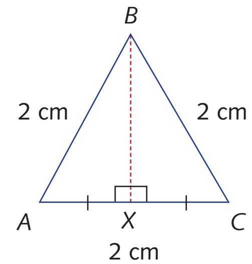
6 Triangle ABC is equilateral, with each side of length 2 cm. Find the length of BX.



7 The equal sides of a right-angled isosceles triangle are 3 cm in length. What is the length of the hypotenuse?

8 A cross-country runner runs 3 km west, then 2 km south and then 8 km east. How far is she from her starting point? Calculate your answer in kilometres, correct to 2 decimal places.

9 A rod 2.5 m long is leaning against a wall. The bottom of the rod is 1.5 m from the wall. How far up the wall does the rod reach?

10 A man starts from a point A and walks 2 km due north to a point B and then 5 km due west to a point C. How far is C from A? Calculate your answer in kilometres, correct to 2 decimal places.

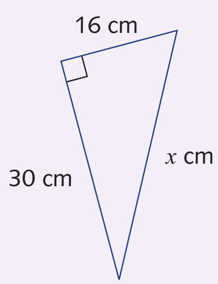
11 Amelia starts from a point A and walks 6 km due south to B. Amelia then walks due east to a point C which is 7 km from A. How far is C from B? Calculate your answer in kilometres, correct to 2 decimal places.

12 A right-angled isosceles triangle has a hypotenuse of length 6 cm. Find the lengths of the other two sides in centimetres, correct to 2 decimal places.

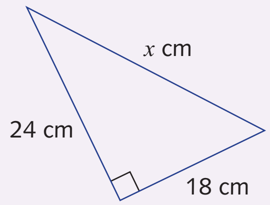
Review exercise

1 Find the value of x in each of these diagrams.

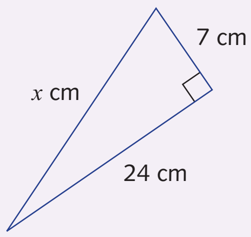
a



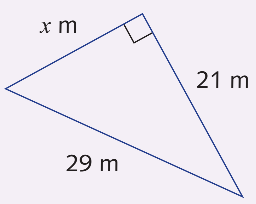
b



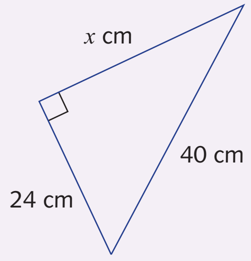
c



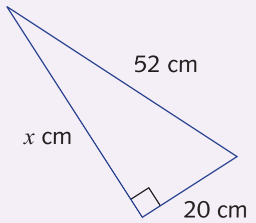
d



e



f



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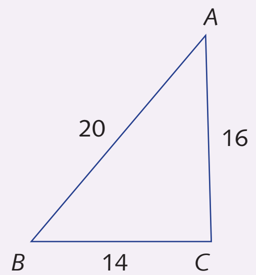
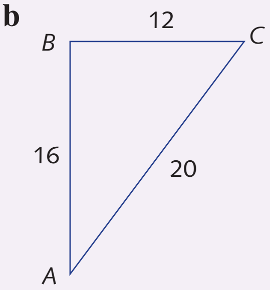
2 Decide whether each of the triangles below is right-angled. If it is, name the right angle.

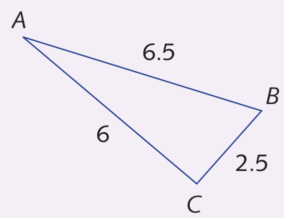
a



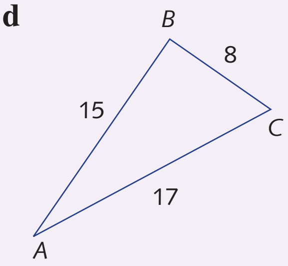
b



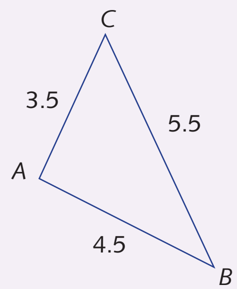
c



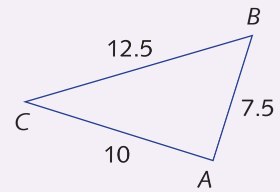
d



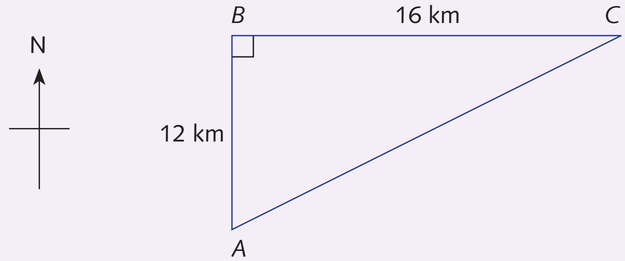
e



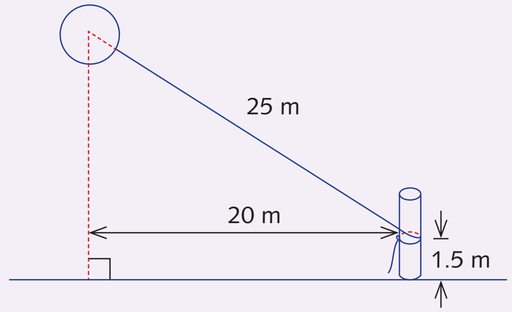
f



3 John starts at a point A and cycles 12 km due north. He turns east at point B, cycles 16 km and reaches point C. How far is he from his starting point?



4 How high is the balloon off the ground if it is tied to a wooden pole at a height of 1.5 m by a cord of length 25 m?



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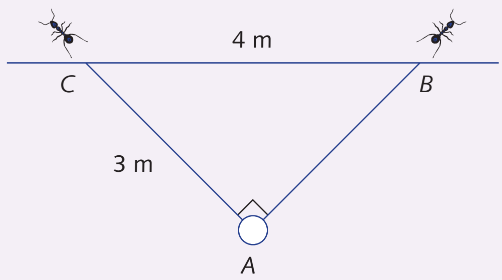
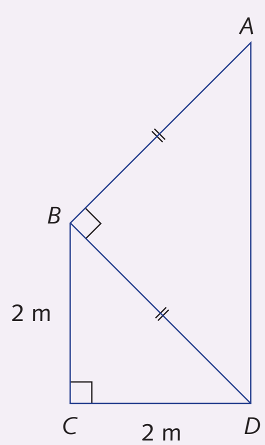
5 Two ants take different paths CA and BA, respectively, while walking back to their nest at point A. How much further does one ant walk than the other? Round your answer to 2 decimal places.



6 **a** A square has side length 3 cm. Find the length of the diagonal correct to 2 decimal places.

b The diagonal length of a square is 4 cm. Find the side length of the square.

7 Find the length of AD in the diagram below.



Challenge exercise

1 Show that the diagonal of a square of side length a is $a\sqrt{2}$.

2 What is the length of the longest diagonal in a cube with edges of length x m?

3 What is the area of an equilateral triangle with sides of length x m?

Hint: The height also divides the base into two equal parts.

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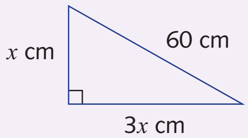
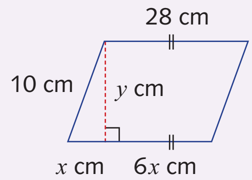
4 Find the exact values of x and y in the following diagrams.

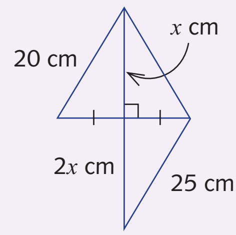
a



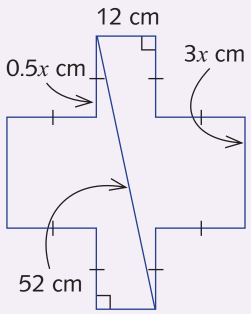
b



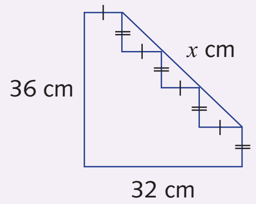
c



d



e

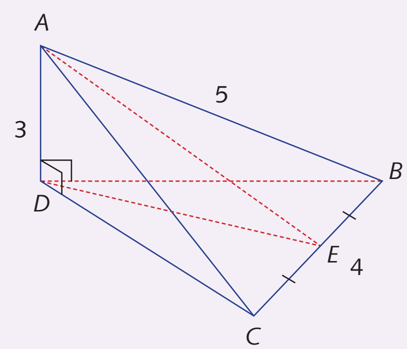


5 A right-angled triangle has hypotenuse of length 41 cm and one of the sides is 9 cm. Find the area of the triangle.

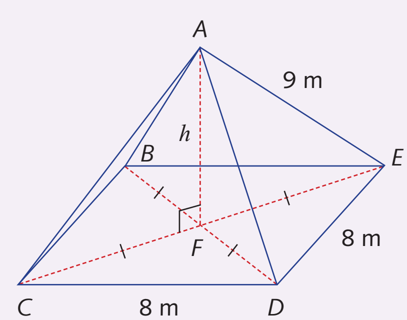
6 Draw an equilateral triangle of side length 4 cm. Find, correct to 2 decimal places, the height of this triangle and its area.

7 In the diagram to the right, ΔABC and ΔDBC are isosceles triangles, with $AB = AC$ and $DB = DC$.

AE and DE are the perpendicular bisectors of ΔABC and ΔDBC, respectively, so $\angle AEB = \angle DEB = 90°$. Find AE, DE and DB, correct to 2 decimal places.



8 A pyramid has a square base $BCDE$ of side length 8 m. The sloping edges of the pyramid are of length 9 m. Find the height h.



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CHAPTER

9

Review and problem-solving

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9A Review

Chapter 1: Whole numbers

1 The population of an ant colony was 14 000 at the start of January. By the end of the month, 256 ants had died, 89 ants had moved into the colony and 123 ants had moved out. The population at the end of January was 14 298. How many ants were born between the start and the end of the month?

2 If each of 37 students in 24 classes ate 5 pies, how many pies were eaten in total?

3 The cost of 20 packets of chips is $9.50, while each packet costs 65 cents when sold individually. If you want to buy 40 packets of chips, which option is cheaper? How much do you save by taking the cheaper option?

4 The quotient arising from the division of a number by 53 is 29 and the remainder is 23. What is this number?

5 The product of two numbers is 67 267, and the smaller of them is 137. Find the sum of the two numbers.

6 Andrew has 347 marbles and Bashir has 135. Bashir wins 106 marbles from Andrew. How many marbles does each boy have now?

7 What number multiplied by 23 will give the same product as 391 multiplied by 37?

8 How many times must 43 be added to 1649 to give 4186?

9 At an election, there are three candidates, A, B and C. Candidate A obtains 19 878 more votes than C, and B obtains 12 435 more votes than C. Candidates A and B together obtain 137 187 votes. How many voted for C?

Chapter 2: Fractions and decimals

1 Simplify:

a $\dfrac{125}{35}$ **b** $\dfrac{29}{145}$ **c** $\dfrac{289}{136}$ **d** $\dfrac{8}{36}$ **e** $1\frac{15}{9}$

2 Evaluate the following additions and subtractions. Write each answer as a mixed numeral, with the fractional part in simplest form.

a $4\frac{1}{2}+3\frac{2}{5}$ **b** $7\frac{2}{3}-5\frac{1}{4}$ **c** $6\frac{5}{8}+2\frac{5}{6}$

d $9\frac{3}{10}-\dfrac{3}{4}$ **e** $5\frac{7}{12}+3\frac{4}{9}$ **f** $8\frac{2}{9}-6\frac{2}{3}$

3 Evaluate:

a $120\times\dfrac{75}{100}$ **b** $120\div\dfrac{75}{100}$ **c** $5\frac{1}{4}\times\dfrac{2}{3}$ **d** $5\frac{1}{4}\div\dfrac{2}{3}$

e $2\frac{7}{9}\times7\frac{1}{2}$ **f** $2\frac{7}{9}\div7\frac{1}{2}$ **g** $8\div1\frac{5}{7}$ **h** $8\times1\frac{5}{7}$

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4 A rectangular garden uses a straight river bank as its longer side. The other sides are fenced. If the longer side is 25 m, and the shorter side is $\frac{1}{3}$ of the longer side, what is the total length of fencing material?

5 If a year (365 days) is divided into lunar months of 28 days, how many months will there be in a year? Express your answer as a mixed numeral.

6 There were 60 people at a party. If $\frac{1}{5}$ of the people wore red, $\frac{1}{4}$ wore blue and the rest wore white, how many people wore white?

7 Horatia has saved \$1500. If she spends $\frac{2}{5}$ of the money on rent and shopping, and $\frac{2}{9}$ of the remaining money on petrol, how much does she have left?

8 Order each set of fractions from smallest to largest.

a $\frac{2}{3}, \frac{5}{6}, \frac{3}{4}, \frac{1}{2}, \frac{4}{5}$ **b** $\frac{11}{25}, \frac{4}{5}, \frac{13}{20}, \frac{1}{4}, \frac{6}{10}$

9 A balloon shop had 120 balloons in the morning, 24 of which were filled with helium.

a What fraction of the balloons were filled with helium?

b During the day, $\frac{1}{8}$ of the helium balloons were sold and 12 of the ordinary balloons were accidentally popped. What fraction of the balloons left at the end of the day were filled with helium?

10 If Alice can swim 100 m backstroke in 1 minute 20 seconds and she can swim butterfly at $\frac{9}{8}$ times her backstroke speed, and freestyle at $\frac{10}{9}$ times her butterfly speed, how fast can she swim 100 m freestyle?

11 Calculate:

a 1.2×3.8 **b** $1.3 + 2.5 \times 0.6$ **c** $7.1 + 4.5 \div 1.5$

d $13.6 - 7.85$ **e** $12.5 \times 3.1 - 0.8742$ **f** $92.8 \times 87.1 + 0.51$

Chapter 3: Review of factors and indices

1 Find all the factors of:

a 12 **b** 17 **c** 270 **d** 135 **e** 58 **f** 144

2 List all the prime numbers between 100 and 160.

3 Find the lowest common multiple (LCM) of:

a 5 and 35 **b** 18 and 48 **c** 170 and 66 **d** 12, 25 and 75

4 Find the highest common factor (HCF) of:

a 17 and 25 **b** 96 and 39 **c** 150 and 24 **d** 380, 190 and 650

5 Find the result of multiplying the HCF and the LCM of each pair of numbers below. Compare the result in each case to the product of the original pair. What do you notice?

a 24 and 76 **b** 102 and 54

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6 Write in the box the number needed to make each statement true.

a $2^8 \div 2^5 = 2^{\square}$ **b** $5^3 \times 5^2 = 5^{\square}$ **c** $4^3 \times 4^2 = 2^{\square}$

d $6^7 \div 3^7 = \square^7$ **e** $(3^2)^2 = \square$ **f** $\square^5 \div 6^5 = 1$

g $3^3 \times 3^{\square} = 81$ **h** $(5^{\square})^2 = 5^{16}$ **i** $(7^{13} \div 7^8)^{\square} = 7^{15}$

7 Evaluate:

a 8^2 **b** $\sqrt{121}$ **c** $\sqrt{56^2}$ **d** $\sqrt{5^4}$

e $(8^2)^2$ **f** $\sqrt{2209}$ **g** 25^3

8 Copy and fill in the missing digits so that the numbers in each set are:

a divisible by 3: 2567__, 199__4, __56

b divisible by 6: 7365__, 1__08, 222__

c divisible by 3, 4 and 5: 10__0, 12 198__, 59__20

9 Fill in the spaces so that the number __4__20 is divisible by 11.

10 Find the prime factorisations of:

a 210 **b** 5040 **c** 436 **d** 1386

11 **a** Evaluate:

i $2^2 \times 7^3$ **ii** 11×13^2 **iii** $2^3 \times 7 \times 9 \times 15$

b Write the product in part **a iii** as a product of powers of primes.

12 Using prime factorisation, find the HCF and LCM of:

a 12 960 and 3600 **b** 1750 and 1176

13 Calculate mentally:

a 25×16 **b** 125×32 **c** 75×40 **d** 5×874

Chapter 4: Negative numbers

1 Calculate:

a $-15 + 23$ **b** $25 - (-61)$ **c** $37 - 90 + 23$

d $23 - 28 - 6$ **e** $-213 - (-1129) + 81$ **f** $9 - (16 + (-28))$

2 Evaluate:

a $3 \times (-5 + 9) - 6$ **b** $-3 \times -9 - 12$

c $-21 \div 3 + 9 \times -2$ **d** $-12 \times (-4) - (-8 - 2) \times 5$

e $\dfrac{-95 - 6 \times 8}{8 - (-2)}$ **f** $\dfrac{28 \times 8}{5} - \dfrac{8 \times (-12)}{6}$

g $-\dfrac{6}{13} + \dfrac{5}{8}$ **h** $\dfrac{7}{12} \times \left(-\dfrac{9}{7}\right) + \dfrac{5}{3}$

i $2\dfrac{3}{4} - 6\dfrac{2}{5}$ **j** $12.8 - (-10.6) \times \dfrac{2}{3}$

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3 The maximum temperature for Thredbo today was -8°C, while Mount Kosciusko had a top temperature of -24.3°C. How much warmer than Mount Kosciusko was Thredbo today?

4 Plot these points on the Cartesian plane.

$A(-4, 2)$ $\quad B(3, -10)$ $\quad C(9, 8)$ $\quad D(-7, 0)$

5 Points A, B and C have coordinates (1, 0), (3, 2) and (5, 0), respectively. Plot the points and join them with straight lines. What is the area of the triangle ABC?

6 Arrange these numbers in increasing order.

$-\frac{9}{8}, \frac{12}{8}, 0.2, -0.02, -\frac{7}{9}, \frac{1}{0.4}$

7 Sam started the week with a debt of \$360.50. He worked hard and earned \$3100, but had \$250 stolen. He won \$80.20 in the lottery and had to pay an electricity bill of \$210. After paying his debt and his electricity bill, how much did Sam have left?

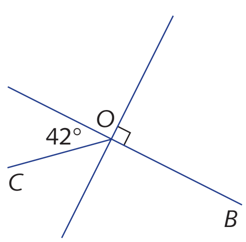
8 Mauna Kea is Hawaii's tallest volcano and the world's highest mountain when measured from its base on the sea floor. The height above sea level is 2405 metres. The base of Mauna Kea on the sea floor is 7286 metres below sea level. What is the height of Mauna Kea from sea floor to summit?

9 In a maths competition, Harry scored 20. Each correct answer was worth 10 marks. For each incorrect answer, 2.5 marks were deducted. If Harry answered 5 questions correctly, how many incorrect answers did he have?

Chapter 5: Review of geometry

1 Find the value of $\angle BOC$ in each diagram below.

a



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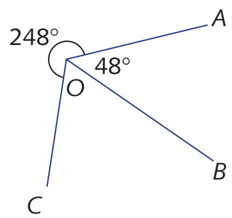
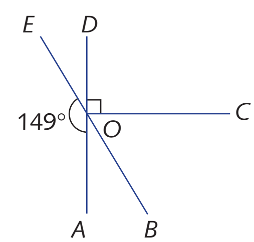
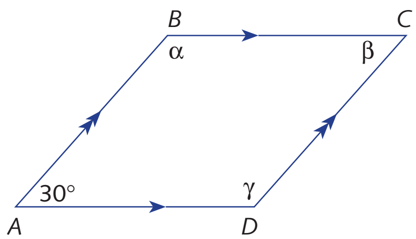


c



2 Find the values of α, β and γ in these diagrams.

a



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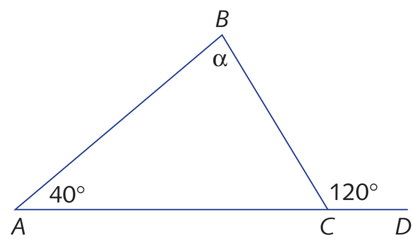

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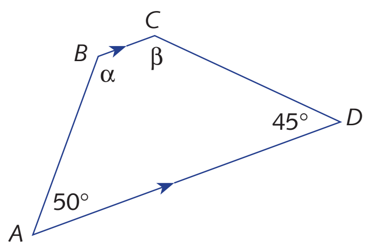

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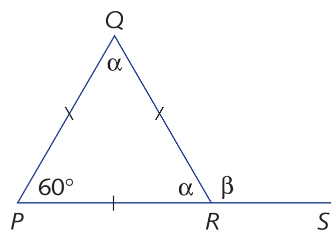

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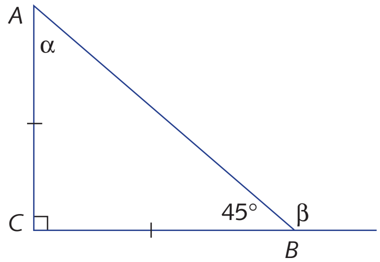
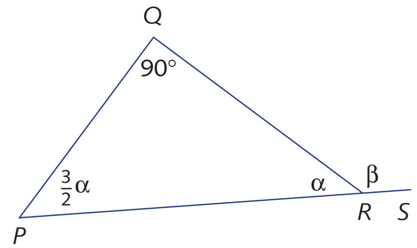
e

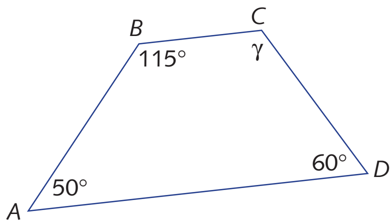


f



3 For each diagram below, write down an equation involving the pronumerals and then solve the equation.

a



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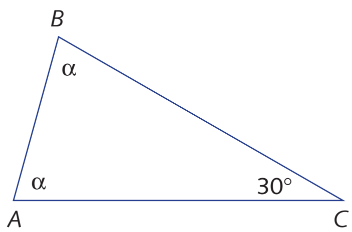
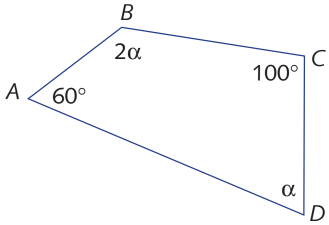
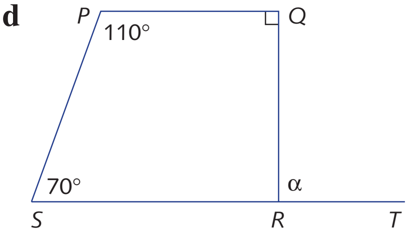


c



d



Chapter 6: Algebra – part 1

1 Evaluate each expression for $a = 2$, $b = 1$ and $c = 3$.

a $a + b + c$ **b** abc **c** $\dfrac{a+b}{c}$ **d** $\dfrac{a^2 + b^2}{a^2 - b^2}$

2 Substitute $x = -1$ for each expression and evaluate it.

a x^3 **b** x^5 **c** $(2x)^2$ **d** $(2x)^3$

e $-x$ **f** $x^3 + 5$ **g** $x^2 + 5$ **h** $\dfrac{2-x}{x^2+2}$

3 Solve:

a $m + 3 = 8$ **b** $m - 3 = 11$ **c** $2m = 8$

d $\dfrac{m}{4} = 6$ **e** $2x - 4 = 11$ **f** $\dfrac{x}{4} - 8 = 7$

g $5 - x = 11$ **h** $2x - 4 = -6$ **i** $\dfrac{x}{5} + 8 = -7$

4 Expand brackets.

a $2(x - 3)$ **b** $-3(x + 4)$ **c** $5(x - 2)$

d $a(x + 3)$ **e** $-3(x - 4)$ **f** $-4(3 - x)$

5 Collect like terms to simplify each of these expressions.

a $2x - 3y + 5x + 2y$ **b** $x - y + 2x + 4y$

c $5x - 7x + 6x$ **d** $3x - 11x + 8x$

e $2xy - 8xy + 6xy$ **f** $6x^2y - 2yx^2 + x^2y$

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6 Evaluate each expression for $a = -3$, $b = \frac{2}{5}$ and $c = 10$.

a $a + b - c$ **b** abc **c** $b^2(c + a)$ **d** $(c^2 + a^2) - 2b$

e $\frac{ca^2}{30}$ **f** $\frac{a^2 b}{c - b}$ **g** $a^2 - b^2 - c$ **h** $\frac{a}{b}(a - c)$

7 Solve each equation.

a $2x + 32 = 6$ **b** $5b - 12 = 3$ **c** $53 + 4a = 97$

d $\frac{14}{b} + 23 = 10$ **e** $\frac{6x}{5} = 120$ **f** $\frac{7x + 2}{5} = 6$

8 Expand brackets in each of these expressions and collect like terms.

a $23x + 7(x + y)$ **b** $\frac{1}{4}(52y + x) + 2x$

c $3y(4x + 2) + 8x(y + 3)$ **d** $27 - 5x + x(4x + 6)$

e $\frac{x}{3}(9 - x) + 6x(x + 2)$ **f** $32x + 7x(y - 3) + 2x - 5y(x - 1)$

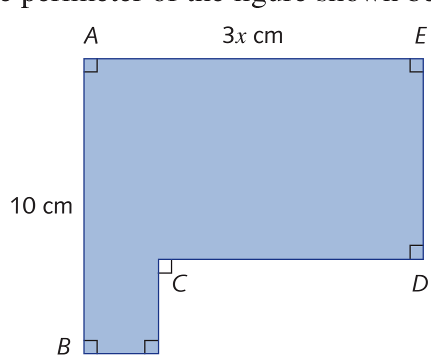
g $a(x - 10 + 3y) - 2a(x + y)$ **h** $4xy(3 + x) - 3yx(6 + 5y)$

9 Select any whole number and add 14. Double the result and then subtract 8 from the number. Divide by 2 and then subtract the original number. Prove that the result is always 10.

10 The entry fee for the Melbourne Show is \$20 per adult and \$8 per child. The cost of each ride is \$5 per person. Stephen's family (2 parents and 3 children) went to the Show and spent \$164 on rides and entrance tickets. If the whole family always went on rides together, how many rides did they go on?

11 Adrian has a habit of reading a book before he sleeps. He reads the same number of pages every night, Monday to Friday. He reads three times more pages on Saturday night than on a weeknight. He does not read any pages on Sunday night. If Adrian took 7 weeks to finish a book that had 280 pages, how many pages does he read on a weeknight, and how many on a Saturday night?

12 The perimeter of the figure shown below is 43 cm. Find the value of x.



Chapter 7: Percentages

1 Express each fraction as a percentage.

a $\frac{2}{5}$ **b** $\frac{5}{6}$ **c** $\frac{7}{8}$ **d** $\frac{7}{9}$ **e** $\frac{5}{7}$ **f** $\frac{3}{40}$

2 Express each percentage as a fraction.

a $8\frac{1}{2}\%$ **b** $16\frac{2}{3}\%$ **c** $15\frac{1}{3}\%$ **d** $8\frac{6}{9}\%$ **e** $4\frac{2}{7}\%$ **f** $5\frac{1}{2}\%$

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3 Express each percentage as a fraction in simplest form.

a 150% **b** 2.5% **c** 136% **d** 204% **e** 0.25% **f** 5.25%

4 Complete each percentage addition and express the result as a fraction.

a $15\% + 25\% + 14\%$ **b** $45\% - 10\% + 7\%$

5 20% of a class of 30 are aged 13 years or under. How many of the class are over 13 years of age?

6 Find:

a 25% of 5000 **b** 12.5% of 16 000 **c** $66\frac{2}{3}\%$ of 4200

d 6.6% of 9000 **e** 87.5% of 4200 **f** 7% of 320 000

7 Find the new amount after each of the percentage changes.

a \$155 000 increased by 20% **b** \$60 000 decreased by 5%

c \$32 000 increased by 15% **d** 108 000 hectares increased by 24%

8 Calculate the discounted price in each case.

a A discount of 10% on a purchase of \$4520

b A discount of 15% on a purchase of \$27

c A discount of 5% on a purchase of \$86

d A discount of $12\frac{1}{2}\%$ on \$240

9 Calculate:

a 30% of 50% **b** 20% of 30% **c** 50% of 80% **d** 20% of 90%

10 **a** 20% of a number is 1020. Find the number.

b 25% of a number is 600. Find the number.

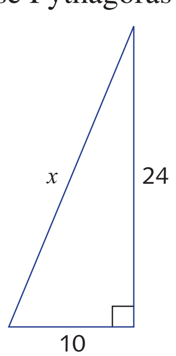
c 15% of a number is 330. Find the number.

11 The price of a book pre-GST is \$30. Assume a GST rate of 10%. What is the price of the book including GST?

Chapter 8: Pythagoras' theorem

1 Use Pythagoras' theorem to find the missing side lengths in the triangles below.

a



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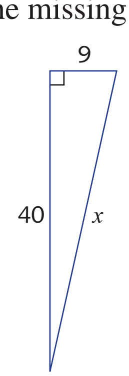

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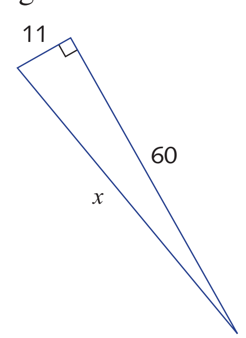

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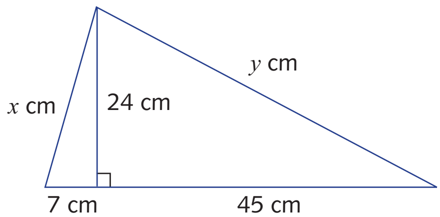
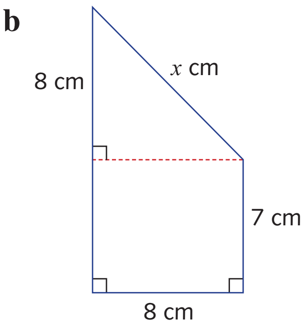
2 Find the values of x and y in the diagrams below.

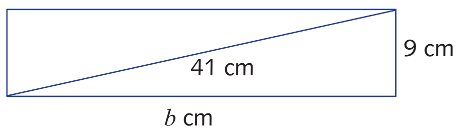
a



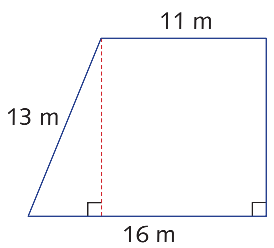
b



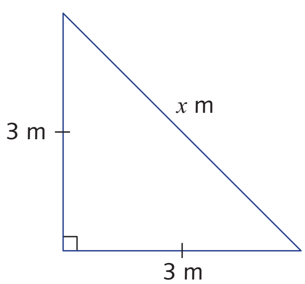
3 A rectangle has width 9 cm and diagonal 41 cm as shown below. What is its length?



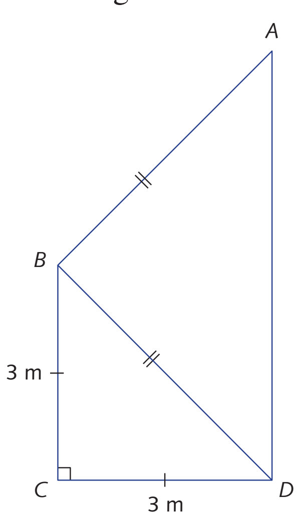
4 Find the height of the trapezium shown below.



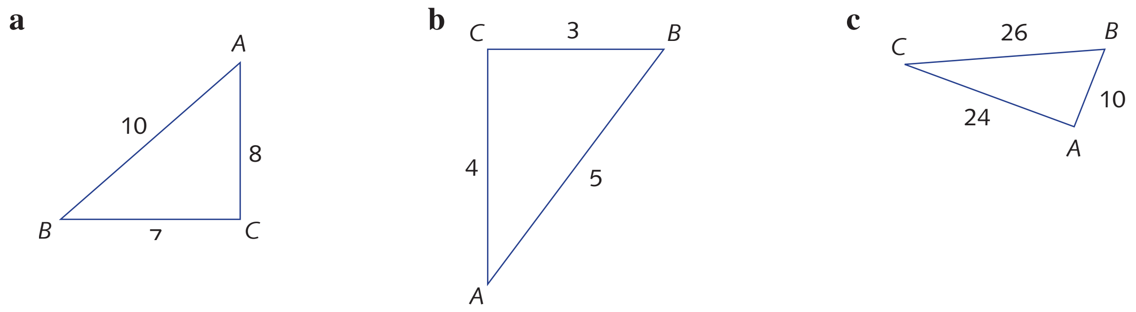
5 Find the length of the hypotenuse, correct to 2 decimal places, in the triangle below.



6 Find the length of the side AD in the diagram below.



7 For each triangle shown below, decide whether it is a right-angled triangle. If it is, name the right angle.



9B Problem-solving

Kaprekar's routine

Choose a set of three digits. Repeats are allowed, but the digits are not *all* allowed to be the same (so 176 and 838 are okay but 444 is not). Now carry out the following steps.

Step 1: Make the largest three-digit number you can out of them.

Step 2: Then reverse this number to get the smallest number possible. This may involve putting a zero at the front of a number.

Step 3: Then subtract the smaller number from the larger one. If the result of the subtraction is two digits, then a 0 is placed to the left of the number.

Now go back to Step 1 with this number and repeat the process until you notice something special happening. For example, if you choose 680, the first step gives 860, step 2 gives 068, step 3 gives $860 - 68 = 792$ and then you start again with $792 \ldots 972 - 279 = \ldots$

1 Describe what eventually happens.

2 What do you notice about the numbers produced on the way?

We call the number at which you finally arrive a **self-producing integer**. With some sets of three digits, you get it straight away. Other sets take a few iterations but never more than six.

If the set you start with contains *four* digits instead of three, you get a different self-producing integer.

3 Find what it is.

4 Find out what happens if you start with a set of *five* digits. (Don't be in too much of a hurry to report on your findings here, because there is a bit more to it than meets the eye. It might be useful to have a partner to help you. Also, if you have been doing all your subtractions without the aid of a calculator up until now, well, now might be a good time to get one out!)

5 *Challenge*: There are three self-producing integers with 10 digits. This is probably a job for a computer program if you feel like having a bash at it.

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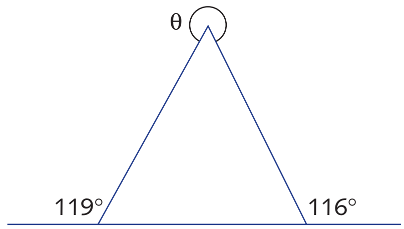
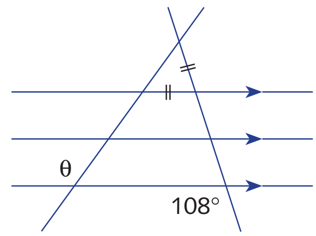
Angle chasing

1 Find θ in each diagram below.

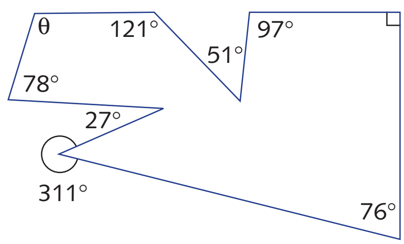
a



b



c



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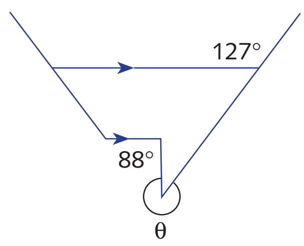
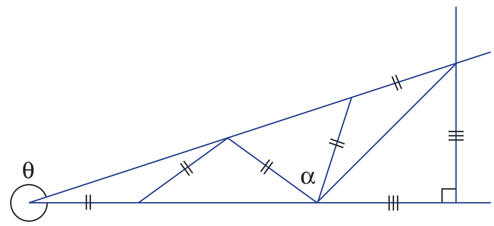


2 What pairs of whole number values can θ and α take in the diagram below? Explain the procedure you followed to obtain your answer.



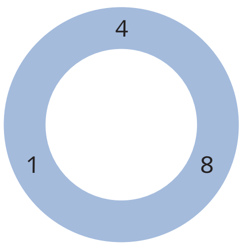
Multiples

1 Take a three-digit multiple of 37:

$13 \times 37 = 481$

Arrange the digits of 481 clockwise in a circle.

Reading around the circle clockwise, we obtain the numbers 481, 814 and 148.



a Show that each of these is a multiple of 37.

b Find out what happens for the three-digit multiples of 37 below:

185, 259, 296, 740, 814 and 925

2 Now look at a multiple of 41 that has five digits:

$1679 \times 41 = 68\,839$

a Show that each of the numbers 68 839, 88 396, 83 968, 39 688 and 96 883 is a multiple of 41.

b Try some more five-digit multiples of 41 to see what happens.

3 **a** Work out each of these multiplications.

i $37 \times 5 \times 3 = \square$ **ii** $37 \times 2 \times 3 = \square$ **iii** $37 \times 7 \times 3 = \square$

What do you notice?

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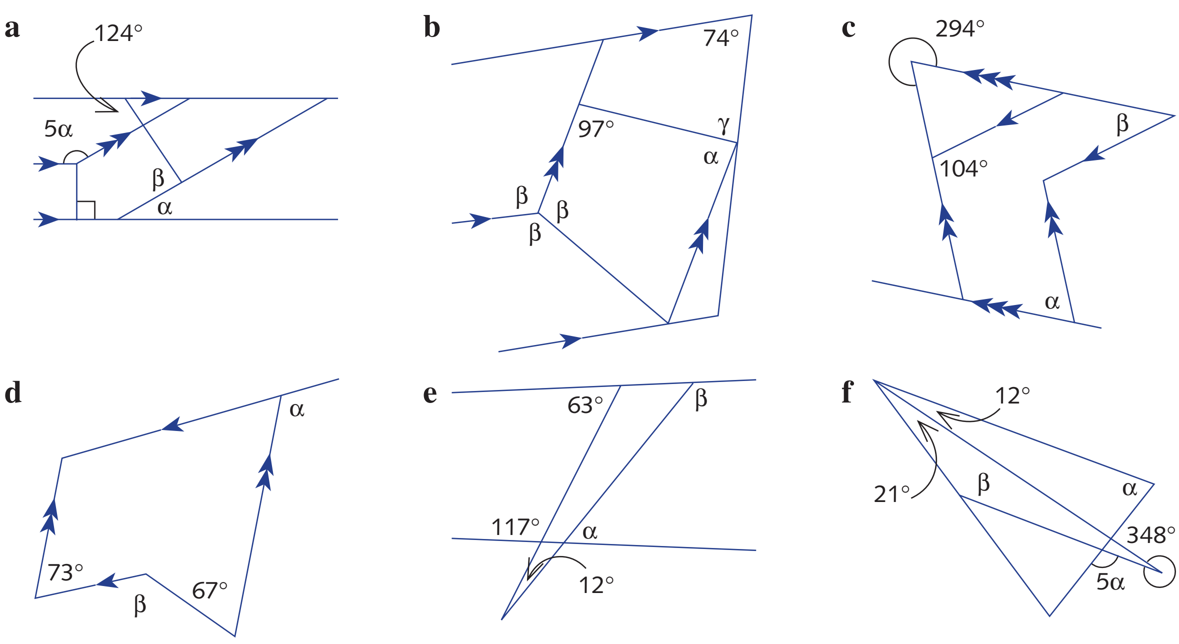
b Can you predict the value of $37 \times 9 \times 3$? Can you explain what happens?

c Now try $37 \times 13 \times 3$. Can you predict the value of $37 \times 17 \times 3$? Can you explain what happens?

d Show that 41 is a factor of 11 111. Can you make up a similar problem to part **c** based on this? (*Hint*: Use the form $41 \times a \times b =$ ____. Replace b with a suitable number and then choose other numbers in turn to replace a.)

Geometry challenge

1 Find the values of the pronumerals in each of these diagrams.



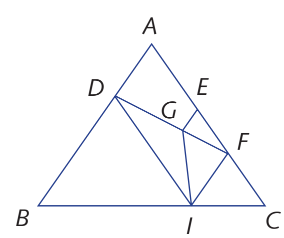
2 The triangle ABC opposite is an isosceles triangle, with $AB = AC$ and $\angle ABC = 55°$. $AD \parallel EG \parallel FI$ and $DI \parallel AF$.

a Find $\angle BAC$ and $\angle ACB$.

b Find $\angle FIC$ and prove that ΔFIC is isosceles.

c Prove that $DB = DI$.

d If $GD = GI$ and $\angle FGI = 34°$, find $\angle GIF$ and $\angle EFG$.



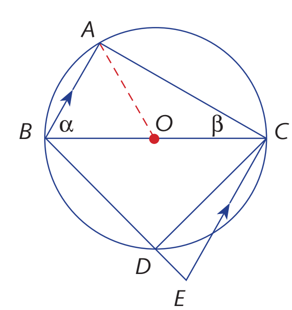
3 O is the centre of the circle shown opposite.

a Let $\angle ABC$ be α. Express $\angle AOC$ in terms of α.

b Let $\angle ACB$ be β. Express $\angle AOB$ in terms of β.

c Prove that ΔABC is a right-angled triangle.
Hence, name two other right-angled triangles in the diagram.

d If $\angle ECD = 25°$, find $\angle ABD$.



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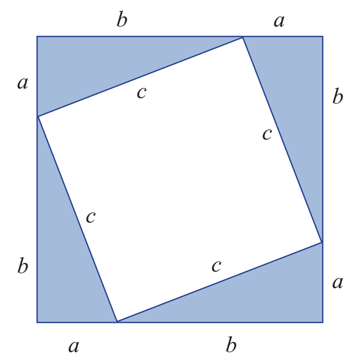
Pythagoras' theorem

In Chapter 8 we gave a proof of Pythagoras' theorem that relied on rearranging triangles and comparing areas in two diagrams.
We now give a proof using just one diagram.



a Show that $(a+b)^2 = a^2 + 2ab + b^2$.

b Show that the total area of the four triangles is $2ab$.

c Hence, show $c^2 = a^2 + b^2$.

Pythagorean triples

In Chapter 8 we encountered a number of right-angled triangles with side lengths that are positive integers. Two such triangles are the ones with side lengths 3, 4 and 5, and 5, 12 and 13.

The numbers 3, 4, 5 are an example of a **Pythagorean triple**. This means that:

$$5^2 = 3^2 + 4^2$$

In a similar way, 5, 12, 13 is a Pythagorean triple because:

$$13^2 = 5^2 + 12^2$$

In general, the set of numbers a, b, c is a Pythagorean triple if a, b and c are positive integers and $a^2 + b^2 = c^2$. The name comes from the fact that a right-angled triangle can be formed with side lengths a, b and c, where c is the length of the hypotenuse.

It is said that Pythagoras invented the following method for finding some of these triples.

Step 1: Take an odd number as the length of one side.

Step 2: Square it and subtract 1.

Step 3: Halve the result (this gives the second side).

Step 4: Add 1 to this result to get the length of the hypotenuse.

For example:

Step 1: Take 11 as the length of the first side.

Step 2: Square it to obtain 121 and subtract 1 to obtain 120.

Step 3: Halve the result to obtain 60 (the length of the second side).

Step 4: Add 1 to get 61 (the length of the hypotenuse).

Check that $11^2 + 60^2 = 61^2$.

Produce some more Pythagorean triples using this method, starting with some other odd numbers.

Not all Pythagorean triples can be produced in this way – an example is the triple 15, 8, 17. Can you see why this triple does not fit the pattern?

Plato, the Greek philosopher and mathematician, recorded the following method for producing Pythagorean triples.

Step 1: Start with a number divisible by 4 as the first side length.

Step 2: Square it, divide by 4 and subtract 1 (this is the second side).

Step 3: Square the original number, divide by 4 and add 1 (this is the hypotenuse).

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For example:

Step 1: Start with 8.

Step 2: Square it to obtain 64, divide by 4 to obtain 16, and subtract 1 to obtain 15.

Step 3: Square 8 to obtain 64, divide by 4 to obtain 16, and add 1 to obtain 17.

Generate some other triples using Plato's method, starting with $12, 16, 20, \ldots$

Again, not all Pythagorean triples can be produced in this way. For example, $77, 36, 85$ is a Pythagorean triple. Can you see why it does not fit Plato's method or Pythagoras' method?

The complete story

Here is a remarkable fact: there is a procedure for producing all possible Pythagorean triples!

To start the explanation, notice that if we have a Pythagorean triple, such as $3, 4, 5$, we can get infinitely many others, all related to it, by multiplying each of the numbers by the same factor. For example, $9, 12, 15$ (multiplying by 3) is also a Pythagorean triple.

A Pythagorean triple is said to be **primitive** if the three numbers have no common factor other than 1. For example, $3, 4, 5$ is primitive but $6, 8, 10$ is not. Do the above methods produce primitive Pythagorean triples?

Generating all possible Pythagorean triples

The procedure that produces all of the primitive Pythagorean triples a, b, c uses algebra.

Here it is:

Start with any two positive integers u and v, where $v > u$, u and v have no common factor other than 1, and one of them is odd and the other is even. Now calculate:

$$a = v^2 - u^2$$
$$b = 2uv$$
$$c = u^2 + v^2$$

Then a, b, c is a primitive Pythagorean triple, and every primitive Pythagorean triple can be produced in this way.

Use this method to reproduce all of the primitive Pythagorean triples you created using Pythagoras' method and Plato's method.

Sports percentages

It is suitable to use a calculator or spreadsheet for this exercise.

A common use of percentages is to decide how sporting teams are placed on a ladder or league table. When you look at a ladder, you expect to see the teams with the most wins at the top and those with fewest wins near the bottom. If matches are drawn, half the points for a win are given to both teams involved. The points for wins and draws are called **premiership points**. A win is often worth four points and a draw is worth two.

A problem arises when two or more teams have the same number of premiership points. They cannot both occupy the same rung on the ladder, so a way of deciding which team is to be placed higher is needed. This is where percentages come in.

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The total number of points (or goals) scored by a team in the season to date is expressed as a percentage of the total number of points its opponents have scored against it. The formula is:

$$\text{percentage} = \frac{\text{points for}}{\text{points against}} \times 100\%$$

When a team has scored more points than its opponents have scored against it to date, its percentage at that stage will be more than 100%. If it has been outscored by its opponents overall, its percentage will be less than 100%.

This method does not favour teams that are high-scoring, but that allow their opponents to score highly against them, over low-scoring teams with good defensive set-ups. It is commonly used in some football codes, netball, basketball, ten-pin bowling and volleyball. For other sports, this method is less appropriate due to the scoring systems involved. For this reason, it is not usually used for hockey, soccer, cricket or softball.

The table below is a netball ladder after five games of a season have been played. Only two of the percentages have been calculated.

	TEAM	Wins	Losses	Draws	Goals For	Goals Against	Percentage	Prem. Points
1	Pelicans	5	–	–	115	80	143.8%	20
2	Cassowaries	4	1	–	126	90		16
3	Jabirus	3	2	–	104	96		12
4	Hawks	2	3	–	130	104		8
5	Brolgas	2	3	–	105	112		8
6	Kookaburras	2	3	–	90	120		8
7	Emus	1	4	–	77	105	73.3%	4
8	Eagles	1	4	–	85	125		4

Activity 1

Your first task is to calculate the percentages of the other six teams, and confirm that the teams are in the right order. Remember, if a team has more premiership points, it will be placed above a team with fewer premiership points, even though its percentage may not be as high.

The teams then play the sixth round of games. The results are:

- Brolgas (27) beat Jabirus (16)
- Emus (20) beat Kookaburras (15)
- Hawks (30) beat Eagles (28)
- Cassowaries (31) beat Pelicans (27).

These results change the teams' numbers of wins, losses, goals for and goals against for the season, and their premiership points. It may also change their places on the ladder.

Activity 2

Update the ladder. This means you will need to work out all the new figures and percentages for each team. For example, the Brolgas' Goals For will increase by 27 and their Goals Against will increase by 16. The Jabirus' Goals For will go up by 16 and their Goals Against will go up by 27. A calculator may help you work out the percentages.

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Activity 3

1 Which team's percentage did not change? Can you explain why?

2 How many of the teams that won had their percentages go down? Which ones were they?

3 Is it possible to lose and have your percentage increase? Give an example of a team that had this happen.

4 Add up the Goals For column. Add up the Goals Against column. Explain your results.

9C Modular arithmetic and congruence

Arithmetic modulo 7 means that you divide a number by 7 and just look at the remainder. The quotient gets discarded. For example, $15 = 7 \times 2 + 1$, so we write:

$15 \equiv 1 \pmod 7$

Read this as '15 is congruent to 1, modulo 7'. Such an equation is called a **congruence.**

Thinking in modulo 7 can relate to the days of the week.

Today is Monday. What day of the week will it be in 15 days time?

In 14 days time it will be Monday again, so in 15 days time it will be Tuesday.

Today is Monday 4 May. What day of the week will it be on 4 June?

There are 31 days in May, and $31 = 7 \times 4 + 3$. In 28 days time it will be Monday again, so in 31 days time it will be Thursday.

There are seven possible remainders 0, 1, 2, 3, 4, 5 and 6 after division by 7, and there are seven days in the week.

Arithmetic modulo 5 means that you divide the number by 5 and just look at the remainder. Someone counted on the fingers of one hand the people coming into the Wombat Heights Football Ground, but forgot to count how many times he'd gone around his hand:

1, 2, 3, 4, 0, 1, 2, 3, 4, 0, 1, 2, 3, … 2, 3

'I don't know how many people went into the Football Ground today, but I do know that the number ended in a 3 or an 8.'

Arithmetic modulo 12 and arithmetic modulo 24 are related to counting the hours around a 12-hour clock and around a 24-hour clock.

Dividing 90 by 12 and by 24 gives

$90 \equiv 6 \pmod{12}$ and $90 \equiv 18 \pmod{24}$

'It is now 8:30 p.m. What will my 12-hour watch and my 24-hour clock say in 90 hours time?'

'My 12-hour watch will say 2:30, and my 24-hour clock will say 14:30.'

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Arithmetic modulo *n* where n is any whole number greater than one is possible.

There are three possible remainders 0, 1, 2 after division by 3. Similarly, there are ten possible remainders 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 after division by 10.

In each case every whole number is congruent to one of the remainders. The plural of modulus is moduli.

Here are some more congruences, using various moduli:

$$\begin{array}{ll} 20 \equiv 2 \pmod{3} & 67 \equiv 7 \pmod{10} \\ 42 \equiv 0 \pmod{7} & 35 \equiv 1 \pmod{2} \\ 100 \equiv 1 \pmod{11} & 80 \equiv 8 \pmod{9} \end{array}$$

We can use any whole number greater than 1 as the **modulus**.

Congruence with negative integers

Here is how to divide -10 by 7 and get a whole number remainder less than 7:

$$-10 = 7 \times -2 + 4, \text{ so } -10 \equiv 4 \pmod{7}$$

Here are two everyday interpretations of this idea:

Arithmetic modulo 7 can also be used with negative integers.

Today is Monday. What day of the week was it 10 days ago?

Fourteen days ago it was also Monday, so 10 days ago it was Friday.

Today is Monday 4 May. What day of the week was it on 4 April?

There are 30 days in April, and $-30 = 7 \times (-5) + 5$. Thirty-five days ago it was also Monday, so 30 days ago it was Saturday.

Arithmetic modulo 12 and **arithmetic modulo 24** can be used with negative integers.

Here is an example with 12-hour and 24-hour clocks.

'It is now 8:30 p.m. What did my 12-hour watch and my 24-hour clock say 31 hours ago?'

Dividing -31 by 12 and by 24 gives:

$$-31 = 12 \times -3 + 5 \quad \text{and} \quad -31 = 24 \times -2 + 17$$

Writing this in congruences gives:

$$-31 \equiv 5 \pmod{12} \quad \text{and} \quad -31 \equiv 17 \pmod{24}$$

'My 12-hour watch said 1:30, and my 24-hour clock said 13:30.'

Here are some more congruences involving negative integers and various moduli:

$$\begin{array}{ll} -20 \equiv 1 \pmod{3} & -67 \equiv 3 \pmod{10} \\ -42 \equiv 0 \pmod{7} & -35 \equiv 1 \pmod{2} \\ -100 \equiv 10 \pmod{11} & -80 \equiv 1 \pmod{9} \end{array}$$

Congruence modulo a whole number

The integers 9 and 23 are both congruent to 2 modulo 7, so we say that they are congruent to each other:

$$9 \equiv 23 \pmod{7}$$

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We could have checked this directly simply by subtracting 9 from 23:

$23 - 9 = 14$, which is a multiple of 7

Working with days of the week this becomes:

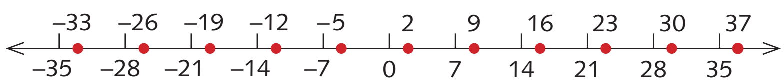
'It will be the same day of the week in 9 days time and in 23 days time.'

This provides us with the standard definition of congruence modulo 7. Two integers are called **congruent modulo 7** if their difference is a multiple of 7. By the division algorithm, every integer is congruent modulo 7 to one, and only one, of the seven remainders 0, 1, 2, 3, 4, 5 and 6.

The integers 9 and −33 are also congruent modulo 7, because

$9 - (-33) = 9 + 33 = 42$, which is a multiple of 7.
'It will be the same day of the week in 9 days time as it was 33 days ago.'

On the number line below, we have marked both the multiples of 7 and all the integers congruent to 2 modulo 7.



Each integer congruent to 2 modulo 7 is 2 more than a multiple of 7, no matter whether the integer is positive or negative.

Here are some more congruences. Check them by subtracting the integers:

$10 \equiv 40 \pmod 3$	$-27 \equiv -67 \pmod{10}$
$-42 \equiv 21 \pmod 7$	$30 \equiv 514 \pmod 2$
$-100 \equiv -540 \pmod{11}$	$30 \equiv -51 \pmod 9$

Modular arithmetic

- Two integers are called **congruent modulo 7** if their difference is a multiple of 7.
- Every integer is congruent modulo 7 to exactly one whole number less than 7; that is, to 0, 1, 2, 3, 4, 5 or 6.
- The **modulus** can be any whole number greater than 1.

Exercise 9C

1 Copy and complete:

a $27 = 7 \times \_\_\_ + \_\_\_$, hence $27 \equiv \_\_\_ \pmod 7$

b $359\,412\,294 \equiv 10 \times \_\_\_ + \_\_\_$, hence $359\,412\,294 \equiv \_\_\_ \pmod{10}$

c $53 = 12 \times \_\_\_ + \_\_\_$, hence $53 \equiv \_\_\_ \pmod{12}$

d $3 = 8 \times \_\_\_ + \_\_\_$, hence $3 \equiv \_\_\_ \pmod 8$

2 Copy and complete:

a $36 \equiv \_\_\_ \pmod 2$

b $100 \equiv \_\_\_ \pmod{12}$

c $156 \equiv \_\_\_ \pmod{25}$

d $94 \equiv \_\_\_ \pmod 9$

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3 Find the first number over 50 that is congruent to 1:

a modulo 2 **b** modulo 8 **c** modulo 10 **d** modulo 3

e modulo 19 **f** modulo 15 **g** modulo 30 **h** modulo 66

4 Roger says 'She loves me, she loves me not, she loves me, she loves me not. . .' as he counts the petals of a flower. The flower has 47 petals. Is happiness assured for Roger or not?

5 Six children – Adam, Bessie, Charles, Doreen, Edwin and Florence – choose chocolates in turn from a box of 80 chocolates. Adam starts, and they keep going in alphabetical order. Who gets the last chocolate?

6 It is 4 p.m. now. What will the time be in 110 hours?

7 If the first of November is a Sunday, what dates of November are Wednesdays? What day of the week is the 27 November?

8 In what years between 1890 and 1920 could one have truthfully said, 'Next year is a leap year'?

9 **a** What is 52 348 181 modulo 4? Is there an easier way than dividing?

b What is 52 348 181 modulo 100? Is there an easier way than dividing?

c What is 52 348 181 modulo 50? Is there an easier way than dividing?

d What is 52 348 181 modulo 25? Is there an easier way than dividing?

10 **a** What is the time 27 hours after 11 a.m.?

b What day of the week is 176 days after a Wednesday?

c A man is facing north. He turns through 90° in a clockwise direction 57 times. In which direction is he facing now?

11 **a** Write -13 as a multiple of 5 plus remainder, where the remainder is 0, 1, 2, 3 or 4.

b Hence, copy and complete, '$-13 \equiv$ ___ (mod 5).'

c Mark the multiples of 5 between -26 and 14 on a number line. Mark also -13 and all integers between -26 and 14 congruent to -13 modulo 5.

12 **a** Mark on the number line all integers between -10 and 5 congruent to 2 modulo 5.

b Mark on the number line all integers between -10 and 5 congruent to 3 modulo 5.

13 Copy and complete:

a $-20 = 7 \times$ ___ + ___, hence $-20 \equiv$ ___ (mod 7)

b $-30 = 8 \times$ ___ + ___, hence $-30 \equiv$ ___ (mod 8)

c $-65 \equiv$ ___ (mod 11) **d** $-70 \equiv$ ___ (mod 9)

e $-100 \equiv$ ___ (mod 11) **f** $-72 \equiv$ ___ (mod 6)

14 **a** Divide the numbers $-16, -11, -1, 0, 4, 15$ and 18 by 4. Hence, decide which of the numbers are congruent mod 4.

b Divide the numbers $-14, -30, 5, 18, 45, 53$ and 73 by 12. Hence, decide which of the numbers are congruent mod 12.

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9D Addition, multiplication and powers in modular arithmetic

Addition

The wonderful thing about adding in modular arithmetic is that you can simply add the remainders. For example:

$$31 = 7 \times 4 + 3 \text{ and } 30 = 7 \times 4 + 2, \text{ so } 31 + 30 = 7 \times 8 + 5$$

and writing this in modulo 7 arithmetic:

$$30 + 31 \equiv 3 + 2 \equiv 5 \pmod 7$$

Check this by adding the integers first: $30 + 31 = 61 \equiv 5 \pmod 7$.

Today is Monday 4 May. What day of the week will it be on 4 July? May has 31 days and June has 30 days, and $31 + 30 \equiv 5 \pmod 7$, so it will be Saturday.

When the remainders add to more than 7, we reduce the remainder modulo 7:

$$31 + 30 + 31 \equiv 3 + 2 + 3 \equiv 8 \equiv 1 \pmod 7$$

This is because $31 + 30 + 31 = 7 \times 12 + 8 = 7 \times 13 + 1$.

Today is Monday 4 May. What day of the week will it be on 4 August?
Since $31 + 30 + 31 \equiv 1 \pmod 7$, it will be Tuesday.

Here are some more additions with various moduli.

$10 + 25 \equiv 1 + 1 \equiv 2 \pmod 3$	$26 + 39 \equiv 6 + 9 \equiv 5 \pmod{10}$
$9 + 75 \equiv 2 + 5 \equiv 0 \pmod 7$	$-11 + 21 \equiv 1 + 1 \equiv 0 \pmod 2$
$-100 + 12 \equiv 10 + 1 \equiv 0 \pmod{11}$	$30 + 50 \equiv 3 + 5 \equiv 8 \pmod 9$

Check them by adding the integers first. For example, $10 + 25 \equiv 35 \equiv 2 \pmod 3$.

Multiplication

Multiplication of integers is repeated addition, so we can use the same method for multiplying in modular arithmetic. For example:

$$10 \times 12 \equiv 3 \times 5 \equiv 1 \pmod 7 \text{ and } 65 \times 12 \equiv 2 \times 5 \equiv 3 \pmod 7$$

Check the first calculation by multiplying the integers first: $10 \times 12 = 120 \equiv 1 \pmod 7$.
You can check the second if you feel the need to do so.

A mine organises its shifts in 12-day cycles, with the first cycle beginning on a Monday. On what days of the week will the 11th cycle and the 66th cycle begin?

Since $10 \times 12 \equiv 1 \pmod 7$, the 11th cycle will begin on a Tuesday.

Since $65 \times 12 \equiv 3 \pmod 7$, the 66th cycle will begin on a Thursday.

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Here are some more multiplications with various moduli.

$$\begin{aligned}
10 \times 25 &\equiv 1 \times 1 \equiv 1 \pmod 3\\
25 \times 38 &\equiv 5 \times 8 \equiv 0 \pmod{10}\\
9 \times 75 &\equiv 2 \times 5 \equiv 3 \pmod 7\\
-10 \times 25 &\equiv 0 \times 1 \equiv 0 \pmod 2\\
-100 \times 12 &\equiv 10 \times 1 \equiv 10 \pmod{11}\\
30 \times 50 &\equiv 3 \times 5 \equiv 6 \pmod 9
\end{aligned}$$

Check at least some of them by multiplying the integers first.

Powers

Taking powers is repeated multiplication, so once again we can use the same method for taking powers in modular arithmetic. For example:

$$10^3 \equiv 36^3 \equiv 27 \equiv 6 \pmod 7 \text{ and } 30^6 \equiv 2^6 \equiv 64 \equiv 1 \pmod 7$$

Here are some similar calculations:

$$14^4 \equiv 2^4 \equiv 16 \equiv 1 \pmod 3 \qquad 25^3 \equiv 5^3 \equiv 125 \equiv 5 \pmod{10}$$

$$4^{10} \equiv (4^2)^5 \equiv 16^5 \equiv 0^5 \equiv 0 \pmod 8 \qquad (-11)^6 \equiv 1^6 \equiv 1 \pmod 2$$

$$(-3)^8 \equiv 9^4 \equiv 4^2 \equiv 5 \pmod{11} \qquad (-2)^9 \equiv (-8)^3 \equiv 1^3 \equiv 1 \pmod 9$$

Fibonacci sequences in modular arithmetic

Fibonacci was a mediaeval Italian mathematician who studied, among other things, a curious sequence of numbers which is now named after him. The Fibonacci sequence starts with 1 and 1, then every term is the sum of the two previous terms:

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, \ldots$$

The numbers soon get very big and take a bit of calculating. But now try writing out the sequence in arithmetic modulo 8. The rules are the same – the first two terms are 1 and 1, then every term is the sum modulo 8 of the two previous terms:

$$1, 1, 2, 3, 5, 0, 5, 5, 2, 7, 1, 0, 1, 1, \ldots$$

We get a zero every 6 terms, and the whole thing repeats itself after 12 terms. The final questions in the following exercises continue this experimentation.

Exercise 9D

1 Perform these additions in modulo arithmetic. It is best to reduce the numbers *before* you add them, but do the ones marked ✓ both ways for comparison.

✓ **a** $16 + 2 \pmod 5$

✓ **b** $902 + 30 \pmod 9$

✓ **c** $7 + 7 \pmod 2$

✓ **d** $6 + 8 + 3 + 15 \pmod 3$

e $4 + 4 + 8 + 7 + 6 \pmod 5$

f $1 + 2 + 3 + 4 + 5 \pmod 4$

✓ **g** $1 + 3 + 5 + 7 + 9 \pmod 5$

h $79 + 79 + 79 + 79 \pmod{80}$

i $243 + 527 + 384 + 22 + 3 \pmod 5$

j $3\,628\,598 + 21\,242\,522 + 376\,554\,004 \pmod{100}$

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2 Reduce the numbers first before you multiply, but do those marked ✓ both ways.

✓**a** $7 \times 8 \pmod{5}$

✓**b** $11 \times 12 \pmod{8}$

✓**c** $4 \times 4 \times 4 \pmod{3}$

d $9 \times 8 \pmod{12}$

e $247 \times 3482 \pmod{10}$

f $552 \times 661 \times 776 \pmod{5}$

3 If you have a string of numbers to multiply, reduce them all first, but also reduce the product at each step as you go along.

a $26 \times 13 \times 17 \times 35 \times 62 \pmod{3}$

b $10 \times 23 \times 59 \times 44 \times 33 \pmod{6}$

c $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \pmod{8}$

d $1 \times 3 \times 5 \times 7 \times 9 \pmod{8}$

4 Reducing as you go along is really important when taking powers. You can also group the factors in pairs, or in threes, if that will make the calculations easier.

a $5 \times 5 \times 5 \times 5 \times 5 \times 5 \pmod{7}$

b $4 \times 4 \times 4 \times 4 \pmod{7}$

c $16 \times 16 \times 16 \times 16 \times 16 \pmod{7}$

d $23^8 \pmod{5}$

e $5^6 \pmod{11}$

f $4^7 \pmod{12}$

g $2^{12} \pmod{12}$

h $9^{26} \pmod{10}$

5 These questions involve both sums and products. Reduce the numbers to start with, then keep reducing the numbers after each step so they do not get too big.

a $14 \times 5 + 21 \times 4 \pmod{3}$

b $15 \times 4 + 25 \times 23 \pmod{6}$

c $10^5 + 2^5 \pmod{11}$

d $3 + 3^2 + 3^3 + 3^4 \pmod{5}$

6 Perform these additions, then explain what general result they illustrate.

a $2 + 2 + 2 \pmod{3}$

b $1 + 1 + 1 + 1 \pmod{4}$

c $3 + 3 + 3 + 3 + 3 \pmod{5}$

d $5 + 5 + 5 + 5 + 5 + 5 \pmod{6}$

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7 My watch has 6 functions, which the silver button goes through in order when I push it. If I push the button 74 times, how many more times must I push the button to get it back to the start? Use modulo arithmetic.

8 A satellite goes around the Earth once every 7 hours. If it is 9 a.m. now, what time will it be in 247 revolutions? (Don't calculate 7×247, use modulo arithmetic.)

9 It is Wednesday today. Christmas is 280 days away. Then 25 days holiday at the beach. Then 211 days at work. Then overseas for 143 days. What day of the week is the last day overseas? Use modulo arithmetic.

10 Eric has a packet of 37 Smarties. He counts them out on his five fingers, over and over again, 19 times in fact. What finger does he end on? (Don't calculate 37×19, use modulo arithmetic.)

11 A dancer turns 60° nine times, then 40° twelve times, then 90° thirteen times. What angle does he make with his original position? Use modulo arithmetic.

12 **a** Prove that every power of 10 is equal to 1 modulo 9.

b Hence, prove that every whole number is congruent modulo 9 to the sum of its digits.

c Hence, prove that every whole number is congruent modulo 3 to the sum of its digits.

13 **a** Prove that every odd power of 10 is congruent to -1 modulo 11, and that every even power of 10 is congruent to 1 modulo 11.

b Hence, prove that every whole number such as 123 456 is congruent modulo 11 to the alternating sum of its digits, where alternating sum means $6-5+4-3+2-1$ with the units taken as positive.

14 Write out the Fibonacci sequences using the following moduli. Continue each sequence until it repeats itself. In each case, say how often the zeros occur, and how long it takes for the whole thing to repeat itself.

a modulo 3 **b** modulo 2 **c** modulo 4 **d** modulo 5

e modulo 6 **f** modulo 10 **g** modulo 9 **h** modulo 11

i modulo 10 (this one takes quite a while – don't give up)

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CHAPTER

10

Number and Algebra

Rates and ratios

Paul travels at 60 kilometres per hour.

The Smith household uses 500 litres of water per day.

These are examples of rates. Rates are used here to measure speed and water consumption, and we encounter many different kinds of rates in everyday life. For example, Mary earns $15 an hour working at the supermarket.

Ratios are usually used to compare two related quantities. For example, salad dressing may be made using a ratio of one part vinegar to two parts oil.

We will see that many practical problems can be solved by working with ratios in appropriate ways.

Many problems about rates and ratios can be solved by using a familiar and simple idea – the unitary method.

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10A Review of the unitary method

The unitary method was introduced in Chapter 2, and was used again in Chapter 7 where we looked at percentages. In this section, we review the unitary method, with particular attention to the setting out of problems.

Here is an example about comparing prices. Examples like this occur whenever we go shopping

Example 1

If 12 mangoes cost \$24, how much do 7 mangoes cost?

Solution

The unit in this example is 1 mango.

12 mangoes cost \$24.

$\boxed{\div 12}$ 1 mango costs \$2.

$\boxed{\times 7}$ 7 mangoes cost \$14.

Sometimes the numbers involved are nicely related. This enables us to take a shortcut, as shown in the following two examples.

Example 2

If 12 mangoes cost \$27, how much do 8 mangoes cost?

Solution

The easiest unit to choose here is 4 mangoes, because 4 is the highest common factor (HCF) of 12 and 8.

12 mangoes cost \$27.

$\boxed{\div 3}$ 4 mangoes cost \$9.

$\boxed{\times 2}$ 8 mangoes cost \$18.

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Example 3

If 14 apples cost \$12, how many apples can I buy for \$30?

Solution

The easiest unit to choose is \$6, because 6 is the HCF of 12 and 30.

\$12 buys 14 apples.

$\boxed{\div 2}$ \$6 buys 7 apples.

$\boxed{\times 5}$ \$30 buys 35 apples.

Example 4

Ginger beer costs \$3.35 for a cartoon of 6 cans. If the shop is prepared to split up its cartons, how much will they charge for 11 cans? Give your answer correct to the nearest 5 cents.

Solution

6 cans cost 335 cents.

$\boxed{\div 6}$ 1 can costs $\frac{335}{6}$ cents.

$\boxed{\times 11}$ 11 cans cost $11 \times \frac{335}{6}$ cents $= \frac{3685}{6}$ cents (Leave as an improper fraction.)

$\approx$ \$6.15 (correct to the nearest 5 cents)

Exercise 10A

Example 1, 2

1 **a** If 3 kg of potatoes cost \$3.60, find the cost of 4 kg.

b If 5 pens cost \$15, find the cost of 6 pens.

c If 8 tennis balls cost \$15, find the cost of 20 tennis balls.

d If 9 billiard balls weigh 1440 g, how much do 6 billiard balls weigh?

e If the length of a row of 12 seats is 480 cm, how long will a row of 21 seats be?

f The donuts of a certain donut manufacturer contain 25 g of fat ‘per standard serve’. If one standard serve is 2 donuts, how much fat, in grams, is consumed by a person who eats 5 donuts?

Example 3, 4

2 **a** If 10 bananas cost \$4, how many can I buy for \$14?

b How many shares can I buy for \$5400 if 1000 shares cost \$14?

c At a sale, exercise books cost \$8 per dozen. How many can I buy for \$22?

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d A caterer must buy 4 lettuces to make sandwiches for 50 people. How many lettuces will he need to buy to feed 225 people?

e The first two pages of a student's essay contain 800 words. How many pages are needed if the essay is to meet the requirement of 5000 words? (Assume that each page contains the same number of words.)

3 **a** On a map, 4 cm represents 2 km. How far apart are two suburbs A and B if they are 15 cm apart on the map?

b On a map, 10 cm represents a distance of 50 km. What distance on the map would represent 60 km?

c At a certain time of day, a man 1.8 m tall casts a shadow 2 m long. What is the height of a person who casts a shadow of length 1.8 m?

d If a tree of height 12 m casts a shadow of length 16.8 m, how long a shadow will a tree of height 9 m cast?

e A chef's favourite cake recipe uses 8 eggs and 500 g of flour. If he scales the recipe up to use a dozen eggs, how many grams of flour should he use?

4 **a** If 100% of a school's population is 840, what is 75% of the population?

b If 60% of an amount is \$540, what is 40% of the amount?

c I own 32 copies of the 4 cent postage stamp issued by Jaga Jaga Island in 1930, and this is 20% of the total number of these stamps in existence. How many more of these stamps must I buy in order to own 25% of the total?

5 *Note*: These problems may require some written calculations.

a If 2 kg of tomatoes cost 5, how many kilograms of tomatoes can I buy for \$8?

b If a dozen apples costs \$5, how much do 17 apples cost?

c If 350 g of tinned pineapple contains 210 g of fruit, how much fruit will 500 g contain?

d If 750 mL of soft drink contains 288 g of sugar, how much sugar will 1 litre contain?

6 *Hint*: These problems can be done choosing a 'unit' that is a fraction.

a Three-fifths of an amount is \$243. Find two-fifths of the amount.

b A man can cut down 12 trees in three-quarters of an hour. How many trees can he cut down in half an hour, working at the same rate?

c One-third of the class can plant 28 plants during one period. How many plants could half of the class plant in the same time?

d 100 g of crispbread contains 1.2 g of saturated fat. How much crispbread contains 3 g of saturated fat?

e The label on a certain cat medicine states that 3.2 mL should be given for every kilogram of body weight. How much medicine should be given to a kitten weighing 2.5 kg?

7 If you are shopping, which is the better buy?

a 400 g of tinned peaches costing \$6.40, or 550 g costing \$9.00

b 500 g of lychees costing \$5.50, or 300 g costing \$3.40

c 140 g of toothpaste costing \$1.68, or 180 g costing \$2.16

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8 If a box of cereal is selling on special at \$3.60 for a 375 g packet, how much is 500 g of the cereal at this price? If there is actually a 500 g box selling for \$4.50, which is the cheaper way to buy 500 g of cereal, and by how much?

9 A ship has taken 15 days to travel $\frac{3}{7}$ of its journey. How much longer will it take to complete the journey?

10 If 2 painters can paint 3 rooms in 9 hours, how long does it take for 3 painters to paint 4 rooms, working at the same rate?

10B Solving problems using the unitary method

Many problems that reverse a previous operation can be solved quickly using the unitary method.

Example 5

The price of a shirt has been discounted by 20%, and the discounted price is \$64. What was the original price?

Solution

We know that 80% of the price is \$64, and we want to find 100% of the price.

80% is \$64.

$\boxed{\div 4}$ 20% is \$16.

$\boxed{\times 5}$ 100% is \$80.

Calculations involving rates

Quantities such as 600 L per hour, 35 km/h \$15 per m^2 and \$25 per litre are called **rates**. The unitary method usually makes rate questions quite straightforward.

Example 6

Water is flowing into a dam at a constant rate of 600 L per hour.

a How much water flows into the dam in 2 hours?

b How much water flows into the dam in 3.5 hours?

c How long does it take for 12 000 L of water to flow into the dam?

d How long does it take for 10 000 L of water to flow into the dam?

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Solution

a 600 L flows into the dam in 1 hour.

$\boxed{\times 2}$ 1200 L flows into the dam in 2 hour.

b 600 L flows into the dam in 1 hour.

$\boxed{\div 2}$ 300 L flows into the dam in half an hour.

$\boxed{\times 7}$ 2100 L flows into the dam in 3.5 hours.

c 600 L flows into the dam in 1 hour.

$\boxed{\times 20}$ 12 000 L flows into the dam in 20 hours.

d 600 L flows into the dam in 1 hour.

$\boxed{\div 6}$ 100 L flows into the dam in 10 minutes.

$\boxed{\times 100}$ 10 000 L flows into the dam in 1000 minutes.

Hence, it takes 16 hours and 40 minutes for 10 000 L to flow into the dam.

Alternative method

d Number of hours for 10 000 L to flow into the dam $= 10\,000 \div 600$

$$= \frac{50}{3}$$

$$= 16\tfrac{2}{3}$$

Hence, it will take 16 hours and 40 minutes for 10 000 L to flow into the dam.

Example 7

Edward can paint one house in 2 days. James can paint one house in 3 days. Working at these rates, how long does it take to paint a house if they work together?

Solution

Edward: 1 house, 2 days

$\boxed{\times 3}$ 3 houses, 6 days

James: 1 house, 3 days

$\boxed{\times 2}$ 2 houses, 6 days

Together: 5 houses, 6 days

$\boxed{\div 5}$ 1 house, $\frac{6}{5}$ days

Hence, it will take $1\frac{1}{5}$ days to paint one house if they work together.

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Changing units

Careful use of successive one-step conversions allows the units to be changed, as in the previous example. Here is another example involving changes of two units.

Example 8

Van is earning \$18 per hour making packages. How much is he earning in cents per second?

Solution

Van earns \$18 per hour.

Van earns 1800 cents per hour.

Van earns $\frac{1800}{60} = 30$ cents per minute.

Van earns $\frac{30}{60}$ cents per second = half a cent per second.

Exercise 10B

Example 5, 6

1 **a** The price of a dress has been discounted by 20% and the discounted price is \$96. What was the original price?

b Water is flowing out of a dam at a constant rate of 200 litres per hour. How much water has flowed out of the dam in 3 hours?

c If 10% of a mine's production in a day is 2100 tonnes, what is the total production for the day?

d If 70% of an amount is \$847, what is the whole amount?

e A car travels 96 km on 12 L of petrol. How much petrol would it use in travelling 200 km?

2 **a** A car travelling at a constant speed travels 130 km in 2 hours. How far will it travel in 3 hours?

b A truck travels 55 km on 10 L of petrol. How far will it travel on 36 L?

c If 100 US dollars (US\$100) is worth 180 Australian dollars (A\$180), how much is US\$40 worth in Australian dollars?

d A watch loses 3 minutes every 6 months. How much time will it lose in 8 months?

e A car travels 80 km on 12 L of petrol. How far could it travel on 300 L of petrol?

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3 **a** If 20% of a farmer's crop is 620 tonnes, what is the total crop?

b Nina is in hospital and has drunk 840 mL of water, which is 70% of her daily allowance. What is her daily allowance?

c If 120% of an amount is $156, what was the original amount?

d Hail destroyed 58% of a crop of plums, and only 10 500 kg was sent to market. How much would have been sent to market if there had been no hailstorm?

e A farmer has used 6000 L of petrol harvesting his crop, and has so far harvested 35%. How much more petrol, correct to the nearest 100 L, will he need to harvest the whole crop?

Example 8

4 **a** A guesthouse has used 630 L of milk over a 30-day period. At what rate, in litres per week, is it using milk?

b A firm pays its workers $1800 per 40-hour week. What rate of pay is this in dollars per minute?

c 350 megalitres of water has flowed into a dam in two and a half days. What is the average flow rate, in megalitres per day?

d A car has travelled 240 km on 48 L of petrol. At what rate, in litres per 100 km, is the car using petrol?

e A log of wood weighs 3 kg and has a volume of 2400 cm^3. What is its density, in kilograms per cm^3?

f A woman has been paid $3800 for 250 hours of work. At what weekly rate is she being paid, assuming there are 40 hours in a normal working week?

g A battered old truck has driven the 740 km from Wagga Wagga to Wee Waa in $18\frac{1}{2}$ hours. What was its average speed, in kilometres per hour?

5 **a** A large tank is leaking water at 20 mL per second. Express this rate in litres per hour.

b Every student in a school of 800 throws away an average of 200 g of rubbish a day. If there are 200 days in the school year, what is the rate at which the students throw away rubbish, in kilograms per year?

c The water stored in Warragamba Dam went from 41.6% of the dam's capacity to 40.4% in a 30-day period. At what weekly rate is the water being used?

d A car travels 100 km on 15 L of petrol. At what rate is the car using petrol, in units of kilometres per litre?

e At a certain time, one Australian dollar bought 72 US cents.

- **i** Express A$12 in US dollars.
- **ii** Express US$18 in Australian dollars.

f At a certain time, one Australian dollar bought 40 British pence. Express in Australian dollars the price of a jug that cost 55 British pounds. (There are 100 British pence in a British pound.)

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6 **a** A truck has driven the 1050 km from Adelaide to Innamincka in $13\frac{1}{2}$ hours. What was its average speed, in kilometres per hour? Give your answer correct to the nearest kilometre per hour.

b A bullet is fired at a speed of 800 metres per second. What is its speed in kilometres per hour?

c At a certain time, one Australian dollar bought 72 US cents.

i Express A$21.06 in US dollars, correct to the nearest US cent.

ii Express US$21 in Australian dollars, correct to the nearest Australian cent.

7 **a** If 20% of the marbles in a box are blue and there are 34 blue marbles, how many marbles are there in the box?

b If 110% of an amount is $220, what was the original amount?

Example 7

8 Natalie can paint one house in 4 days, and Erin can paint one house in 5 days. Working at these rates, how long does it take if they work together?

10C Speed

Speed is one of the most familiar rates of all. It is a measure of how far something goes for each given period of time.

Constant speed

If the speed of an object does not change over time, we say that it is travelling with **constant speed**. For example, a car travelling at a constant speed of 60 kilometres per hour would travel 60 km in 1 hour. It would travel 120 km in 2 hours and 150 km in $2\frac{1}{2}$ hours.

Looking at things the other way around, if I travel at a constant speed for 100 km and it takes me 1 hour to complete the journey, then my speed is 100 kilometres per hour. This is usually written as 100 km/h. If I take 2 hours to travel the 100 km (at a constant speed), then my speed is only 50 km/h.

If the distance is measured in metres and the time is measured in seconds, the speed is measured in metres per second. If an athlete runs 100 metres in 10 seconds, her speed is 10 metres per second. This is usually written as 10 m/s.

The speed of a moving object is thus the distance travelled divided by the time the object takes to travel that distance:

$$\text{speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

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Example 9

Mary travelled at a constant speed for 60 km and it took 4 hours to complete the journey. What was her speed?

Solution

$$\begin{aligned}\text{speed} &= \frac{\text{distance}}{\text{time}}\\ &= \frac{60}{4}\text{ km/h}\\ &= 15\text{ km/h}\end{aligned}$$

or

60 km in 4 h

$\boxed{\div 4}$ 15 km in 1 h

Speed = 15 km/h

Average speed

When we drive a car or ride a bike, it is very rare for our speed to remain the same for a long period of time. Most of the time, especially in the city, we are slowing down or speeding up, so our speed is not constant. If we travel 20 km in 1 hour, then we say that our **average speed** is 20 km/h, even though we may have travelled much faster than this at some times in that hour, and come to a complete stop at others. When we calculate speed, we often mean average speed, and in all our unitary method calculations in this chapter we make the assumption of constant speed.

Example 10

Paul rides 6 km on his bike in three-quarters of an hour. What is his average speed?

Solution

$$\begin{aligned}\text{average speed} &= \frac{\text{distance travelled}}{\text{time taken}}\\ &= 6 \div \frac{3}{4}\\ &= \frac{{}^{2}\cancel{6}}{1} \times \frac{4}{\cancel{3}}\\ &= 8\end{aligned}$$

Paul's average speed is 8 km/h.

Alternatively, we can use unitary method to work out speeds.

Paul rides 6 km in $\frac{3}{4}$ of an hour.

$\boxed{\div 3}$ Paul rides 2 km in $\frac{1}{4}$ of an hour.

$\boxed{\times 4}$ Paul rides 8 km in 1 hour.

Paul's average speed is 8 km/h.

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Example 11

Anthony travelled at an average speed of 50 km/h for 4 hours and 30 minutes. How far did he travel?

Solution

A speed of 50 km/h means that: in 1 hour, Anthony travelled 50 km
in 4 hours, Anthony travelled 200 km
in $\frac{1}{2}$ hour, Anthony travelled 25 km.

Hence, in $4\frac{1}{2}$ hours, Anthony travelled $(200 + 25)$ km $= 225$ km.

Alternative solution

We can take the formula:

$$\text{speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

and rearrange it as:

$$\text{distance travelled} = \text{speed} \times \text{time taken}$$

$$\begin{aligned}\text{Hence, the distance Anthony travelled} &= 50 \text{ km/h} \times 4\tfrac{1}{2} \text{ hours}\\ &= 225 \text{ km}\end{aligned}$$

Example 12

A couple go for a walk of 10 km. They walk at an average speed of 2.5 km/h. How long does it take them to complete the walk?

Solution

In each hour, they walk 2.5 km.

$$\begin{aligned}\text{The number of hours needed to walk 10 km} &= \text{the number of lots of 2.5 in 10}\\ &= 10 \div 2.5\\ &= 4\end{aligned}$$

Hence, the couple take 4 hours to complete the walk.

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Example 13

A cyclist completes a circuit of 15 km. She cycles at an average speed of 12 km/h. How long does it take her to complete the circuit?

Solution

In each hour, she cycles 12 km.

$$\begin{aligned}\text{The number of hours needed to cycle 15 km} &= \text{the number of lots of 12 in 15}\\ &= 15 \div 12\\ &= \frac{5}{4}\\ &= 1\tfrac{1}{4}\end{aligned}$$

Hence, the cyclist takes $1\frac{1}{4}$ hours or 1 hour and 15 minutes to complete the circuit.

Alternatively, it takes 1 hour to go 12 km

$\frac{1}{4}$ hour to go 3 km

Hence, it takes $1\frac{1}{4}$ hours to go 15 km.

We see from the last two examples that in general:

$$\text{time taken} = \frac{\text{distance travelled}}{\text{speed}}$$

Be careful that you are consistent with units. For example, if speed is given in kilometres per hour but distance is given in metres, you need to convert the distance to kilometres before calculating the time taken in hours.

Speed

- The **speed** of an object moving at a constant speed is the distance travelled divided by the time the object takes to travel that distance:

  $$\text{speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

- Depending on the information we are given, we can rewrite this as:

  distance travelled = speed × time taken

  or as:

  $$\text{time taken} = \frac{\text{distance travelled}}{\text{speed}}$$

- Alternatively, we can use the unitary method.
- It is important to be consistent with units when calculating speed, distance travelled or time taken.

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Exercise 10C

Example 9, 10

1 A train takes 4 hours to complete a journey of 360 km. What is the average speed of the train?

Example 11

2 A car travelled at 100 km/h for 4 hours. How far did it go?

Example 12, 13

3 A car travels a distance of 240 km at an average speed of 60 km/h. How long does the journey take?

4 Jonathon walks 12 km in 3 hours. If he walks at the same speed, how far will he walk in:

a 1 hour? **b** 2 hours? **c** 4 hours? **d** 7 hours? **e** $5\frac{1}{2}$ hours?

5 A car travels 320 km in 5 hours. If it travels at the same speed, how far will it travel in:

a 1 hour? **b** 2 hours? **c** 10 hours? **d** 7 hours? **e** $5\frac{1}{2}$ hours?

6 Complete the table.

	Speed	Distance	Time
a		100 km	2 hours
b		30 m	6 minutes
c		15 km	$\frac{1}{2}$ hour
d	30 m/s		4 seconds
e	55 km/h		11 hours
f	60 km/h		$\frac{1}{3}$ hour

7 A boy cycles for 2 hours and 20 minutes at 18 km/h. How far does he go?

8 An aircraft travels 4230 km in $4\frac{1}{2}$ hours. What is its average speed?

9 A train travels for 48 minutes at 80 km/h. How far does it go?

10 A plane is flying at a speed of 640 km/h. How far will it travel between 10:30 a.m. and 11:15 a.m. the same day?

11 Henry travels 48 km by train in $\frac{3}{4}$ of an hour and then cycles 12 km in $\frac{1}{2}$ of an hour.

a How long is he travelling in total?

b What is his average speed during the train trip?

c What is his average speed during the bike trip?

d What is his average speed over the whole trip?

12 A standard (or Olympic) triathlon race involves 1500 m of swimming, followed by a 40-km bike ride, and finally a 10-km run. If a competitor took 20 minutes for the swim, 55 minutes for the ride and 35 minutes for the run, what was her average speed throughout the event?

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13 Michael swims 10 laps of a 50 m pool in 8 minutes. He rests for 10 minutes and then swims a further 15 laps at 68 seconds per lap. What is his average speed in m/s, correct to two decimal places, for:

a the first 10 laps? **b** the last 15 laps?

c the whole swim, excluding rest time?

14 A family are travelling in a car at a steady speed of 85 km/h, and have covered 320 km since they left home at 8 a.m. They plan to have lunch at the next town, which is 422 km from home. When, to the nearest minute, will they have lunch?

15 David and Renae stand 120 m apart. David begins running towards Renae at a speed of 4 m/s. At the same time, Renae runs towards David at a speed of 6 m/s. How long does it take for them to meet?

10D Ratios

Ratios provide a way of comparing two or more related quantities. Ratios are closely connected to fractions, but in many problems they are more convenient to use than fractions.

Suppose that I have 5 red jelly beans and 7 yellow jelly beans. The ratio of the number of red jelly beans to the number of yellow jelly beans is written as:

number of red jelly beans : number of yellow jelly beans = 5 : 7

Example 14

A bag of bread rolls contains 13 wholemeal rolls and 9 multigrain rolls. Write down:

a the ratio of the number of wholemeal rolls to the number of multigrain rolls
b the ratio of the number of multigrain rolls to the number of wholemeal rolls
c the ratio of the number of wholemeal rolls to the total number of rolls

Solution

a Number of wholemeal rolls : number of multigrain rolls = 13 : 9
b Number of multigrain rolls : number of wholemeal rolls = 9 : 13
c Number of wholemeal rolls : total number of rolls = 13 : 22

Now suppose that I mix 200 mL of cordial and 700 mL of water in a jug. The mixture then contains two parts of cordial to every seven parts of water. This is written as.

cordial : water = 2 : 7 (Read this as 'The ratio of cordial to water is 2 to 7.')

or as:

water : cordial = 7 : 2 (Read this as 'The ratio of water to cordial is 7 to 2.')

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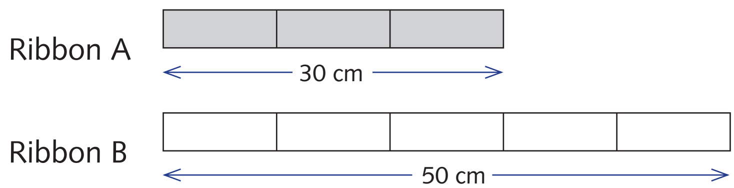
In this example, 'one part' is 100 mL. Another mixture of identical strength could be made by taking 'one part' to be 1 L, and mixing 2 L of cordial with 7 L of water. This idea of **parts** is very useful in dealing with problems involving ratios.

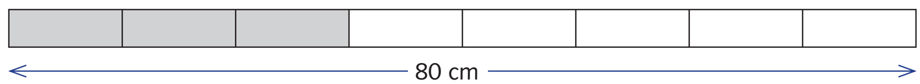
Now consider an example involving ratios of lengths. The next diagram shows two ribbons, a grey one that is 30 cm long and a white one 50 cm long.



Taking 'one part' to be 10 cm:

length of grey ribbon : length of white ribbon = 3 : 5

Now suppose that we join the two ribbons together to make a single ribbon of length 80 cm, as shown.



Then:

length of grey ribbon : total length = 3 : 8

and:

length of white ribbon : total length = 5 : 8

We can also write that the grey section is $\frac{3}{8}$ of the total length, and the white section is $\frac{5}{8}$ of the total length.

Example 15

A mixture contains 200 mL of milk and 500 mL of water. What is the ratio of milk to water?

Solution

Take 1 part to be 100 mL.

200 mL of milk = 2 parts

500 mL of water = 5 parts

Hence, the ratio of milk to water = 2 : 5

Ratios and fractions

If you cut a rope into two equal lengths, the ratio of the two parts is 1 : 1. In this case, each piece is half the total length. Can you see a connection between the ratio 1 : 1 and the fraction $\frac{1}{2}$?

In the following examples, we will explore this link between ratios and fractions.

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Example 16

An alloy of gold and silver contains 2 parts of gold to 5 parts of silver by mass.

a What fraction of the alloy is gold?

b What fraction of the alloy is silver?

c How much of each does 700 g of the alloy contain?

Solution

a There are $2 + 5 = 7$ parts in the alloy, 2 of which are gold.

Hence, the fraction of gold in the alloy $= \dfrac{2}{7}$

b There are 5 parts of silver in the alloy. Hence, the fraction of silver in the alloy $= \dfrac{5}{7}$

c There are 200 g of gold and 500 g of silver.

Example 17

One-tenth of the population has red hair. What is the ratio of the number of redheads to the rest of the population?

Solution

$$\begin{aligned}\text{Rest of the population} &= 1 - \frac{1}{10}\\ &= \frac{9}{10} \text{ of the whole population}\end{aligned}$$

Since $\dfrac{1}{10}$ of the population has red hair, divide the population into 10 parts, each of size $\dfrac{1}{10}$.

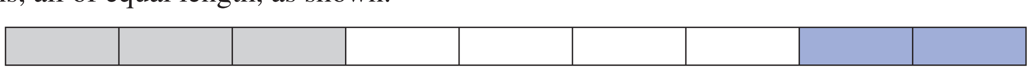
Then 1 part = redheads

There 9 parts = rest of the population
so redheads : the rest of the population is $1 : 9$.

Ratios with three or more terms

We often have a mixture of more than two things. For example, when making a cake, we sometimes mix flour, sugar, butter and milk.

Suppose that a streamer is made by joining three grey ribbons, four white ribbons and two blue ribbons, all of equal length, as shown.



There are three parts of grey, four parts of white and two parts of blue, so we say that the ratio of grey to white to blue is:

grey : white : blue $= 3 : 4 : 2$

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Example 18

A batch of concrete is made from 10 kg of sand, 2 kg of cement, 5 kg of water and 2 kg of gravel. Express the parts as a ratio and express each ingredient as a fraction of the whole.

Solution

The ratio is sand : cement : water : gravel = $10:2:5:2$

The concrete contains 10 parts of sand to 2 parts of cement to 5 parts of water to 2 parts of gravel. There are 19 parts altogether, each part being 1 kg of material, so:

- $\frac{10}{19}$ of the concrete is sand
- $\frac{2}{19}$ of the concrete is cement
- $\frac{5}{19}$ of the concrete is water
- $\frac{2}{19}$ of the concrete is gravel.

Reducing a ratio to simplest form

A ratio involving whole numbers can be reduced to simplest form – just like a fraction – by dividing all the terms in the ratio by their highest common factor (HCF). Sometimes doing this can make it easier to understand the situation we are looking at, as in the next example.

Example 19

A mixture contains 6 parts of oil, 2 parts of insecticide and 10 parts of water by volume. Express the ratio of oil : insecticide : water in simplest form.

Solution

$$\begin{aligned} \text{oil} : \text{insecticide} : \text{water} &= 6:2:10 \\ \boxed{\div 2} \qquad &= 3:1:5 \end{aligned}$$

Equivalent ratios behave like equivalent fractions.

$1:3 = 2:6 = 3:9 = 4:12$, and so on, just as $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12}$, and so on.

In other cases, we may need to find a common denominator for a set of fractions, so that we can easily compare them.

We first multiply the fraction by the lowest common denominator and then simplify the ratios if necessary.

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Example 20

A mixture contains $\frac{1}{10}$ sand, $\frac{1}{5}$ soil and $\frac{7}{10}$ rocks. Work out the ratio sand : soil : rocks.

Solution

$$\begin{aligned} \text{sand}:\text{soil}:\text{rocks} &= \frac{1}{10}:\frac{1}{5}:\frac{7}{10} \\ &= 1:2:7 \end{aligned}$$

Example 21

Eliminate any fractions in these ratios, then reduce them to simplest form.

a $4\frac{1}{2}:3$

b $\frac{1}{2}:\frac{2}{3}:\frac{5}{6}$

Solution

a $$\begin{aligned} 4\tfrac{1}{2}:3 &= 9:6 \\ &= 3:2 \end{aligned}$$

b $\frac{1}{2}:\frac{2}{3}:\frac{5}{6} = 3:4:5$

Exercise 10D

Express all ratios in simplest form.

Example 14

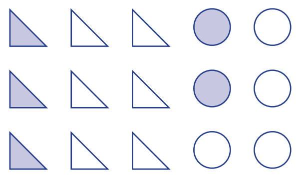
1 In a bowl of fruit, there are 6 oranges and 7 apples. Write down:

a the ratio of the number of apples to the number of oranges

b the ratio of the number of oranges to the number of apples

c the ratio of the number of oranges to the number of pieces of fruit

2 For the diagram opposite, write down:



a the ratio of the number of triangles to the number of circles

b the ratio of the number of blue circles to the number of white circles

c the ratio of the number of blue triangles to the number of white triangles

Example 15

3 A jug of orange juice is made up of 100 mL of pure orange juice and 900 mL of water. What is the ratio of pure orange juice to water?

Example 16

4 In a park, there are 23 native trees for every 40 exotic trees.

a What is the fraction of native trees in the park?

b What is the fraction of exotic trees in the park?

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Example 17 **5** One-third of the flowers in a garden are blue. What is the ratio of the number of blue flowers to the number of other-coloured flowers?

Example 18 **6** A rope is cut into sections so that the resulting lengths are in the ratio $1:2$.

a Express the length of the shorter piece of rope as a fraction of the total length of rope.
b Express the length of the longer piece of rope as a fraction of the total length.

7 A bowl contains green and blue marbles. One-fifth of the marbles are green. What is the ratio of the number of green marbles to the number of blue marbles?

8 In a bus, $\frac{2}{7}$ of the passengers are male. What is the ratio of the number of male passengers to the number of female passengers?

Example 19 **9** Reduce each ratio to simplest form.

a $2:4$ **b** $12:18$ **c** $320:480$ **d** $96:144$
e $12:4$ **f** $36:24$ **g** $512:32$ **h** $243:27$

10 The number of students enrolled at a school is 1200. Of these, 625 are male.

a What is the ratio of the number of male students to the number of female students?
b What fraction of the school population is male?

11 Jonathon has 27 CDs and 60 DVDs in his collection. Write down the ratio of the number of CDs to the number of DVDs in his collection.

Example 20 **12** A salad dressing consists of $\frac{3}{8}$ vinegar, $\frac{1}{2}$ oil and $\frac{1}{8}$ lemon juice by volume. Find the ratio vinegar : oil : lemon juice.

Example 21 **13** Eliminate any fractions in these ratios, then reduce them to simplest form.

a $2\frac{1}{2}:7$ **b** $3\frac{1}{3}:8$ **c** $2.5:4.5$ **d** $6\frac{1}{5}:4\frac{1}{4}$
e $\frac{3}{4}:\frac{5}{8}$ **f** $\frac{1}{3}:\frac{2}{9}$ **g** $5\frac{1}{4}:3\frac{1}{2}$ **h** $\frac{7}{8}:\frac{7}{24}$

14 In a school of 1029 students, 504 are boys. What is the ratio of the number of boys to the number of girls?

15 A rectangle has a length of 6 cm and a width of 4 cm. A second rectangle has length 9 cm and width 6 cm. Find the ratio of:

a the lengths of the rectangles
b the widths of the rectangles
c the perimeters of the rectangles
d the areas of the rectangles

16 A rectangle has a length of 6 cm and a width of 4.5 cm. A second rectangle has length 9 cm and width 1.5 cm. Find the ratio of:

a the lengths of the rectangles
b the widths of the rectangles
c the perimeters of the rectangles
d the areas of the rectangles

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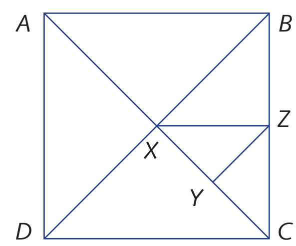
17 The ratio of the number of times the pedals are turned on a bicycle to the number of times the rear wheel revolves is called the *gear ratio*. If the gear ratio is $9:4$, how many times will the rear wheel turn if I pedal:

a 72 times? **b** 108 times? **c** 12 times?

18 A 250 mL packet of milk contains 295 mg calcium and 110 mg of phosphorus. Give the ratio of calcium to phosphorus in simplest form.

19 The lengths of a triangle are in the ratio $5:12:13$. If the perimeter of the triangle is 36 cm, what is the length of each side?

20 In the diagram opposite, $ABCD$ is a square with diagonals AC and BD meeting at X, Z is the midpoint of BC and Y is the midpoint of XC. Find the following ratios of areas.



a area $(\Delta\, AXD)$: area $(\Delta\, AXB)$

b area $(\Delta\, AXD)$: area $(\Delta\, ZXB)$

c area $(\Delta\, AXD)$: area $(\Delta\, ZYC)$

d area $(\Delta\, DBC)$: area $(\Delta\, XYZ)$

21 The cost of a new office building is to be shared between three people in the ratio $4:5:6$. If the office building costs \$450 000, how much does each person have to pay?

22 Express each ratio in simplest form.

a $33:9:12$ **b** $100:65:35$ **c** $2\frac{1}{2}:5:1\frac{1}{4}$

d $1\frac{1}{3}:2\frac{1}{2}:3\frac{1}{4}$ **e** $7:4\frac{1}{2}:\frac{1}{6}$ **f** $11\frac{1}{2}:20\frac{1}{2}:22\frac{1}{4}$

23 An alloy is made by combining 6.25 kg of metal A, 3.75 kg of metal B and 8.75 kg of metal C. Find the ratio of metal A to metal B to metal C.

24 At an agricultural show in the 1950s, freshly hatched chickens were dyed yellow, red and blue. The ratio of the numbers of the different-coloured chickens was yellow : red : blue = $6:4:5$. If there were 45 chickens in total, how many were there of each colour?

10E Using ratios in problems

Ratio problems are usually best solved using the language of parts. The rest of the working is just another form of the unitary method.

For example in a bookshop, the ratio of the number of novels to the number of textbooks is $4:7$. There are 480 novels. How many textbooks are there?

We can use the language of parts to solve this question.

There are 480 novels, so:

$$\begin{aligned} 4 \text{ parts} &= 480 \\ 1 \text{ part} &= 120 \end{aligned}$$

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Thus:

$7 \text{ parts} = 840$

Hence, there are 840 textbooks.

The ratio $4:7 = 480:840$

Example 22

The ratio of boys to girls in a school is $2:3$. If there are 264 boys:

a how many girls are there?

b how many students are there altogether?

Solution

a

$$\begin{aligned} 2 \text{ parts} &= 264 \text{ students} \\ \boxed{\div 2} \quad 1 \text{ part} &= 132 \text{ students} \\ \boxed{\times 3} \quad 3 \text{ parts} &= 396 \text{ students} \end{aligned}$$

There are 396 girls in the school.

b $5 \text{ parts} = 660$ students (Multiply the second line above by 5.)

Thus there are 660 students altogether.

Dividing a quantity in a given ratio

A very common application of ratio is the division of a quantity in a given ratio. The key step here is to add the parts.

Example 23

A man divides his estate of \$360 000 in the ratio $4:3:3$ amongst his daughter and his two sons. How much does each receive?

Solution

There are $4 + 3 + 3 = 10$ parts in total.

$$\begin{aligned} 10 \text{ parts} &= \$360\,000 \\ \boxed{\div 10} \quad 1 \text{ part} &= \$36\,000 \\ \boxed{\times 3} \quad 3 \text{ parts} &= \$108\,000 \\ 4 \text{ parts} &= \$144\,000 \end{aligned}$$

(Multiply through the second line above by 4.)

Hence, the daughter receives \$144 000 and each son receives \$108 000.

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Exercise 10E

Example 22

1 The ratio of the number of boys to the number of girls in a class is 5 : 4. If there are 15 boys, how many girls are there?

2 A bowl contains green and red glass balls. The ratio of the number of green balls to the number of red balls is 2 : 3. If there are 18 red balls, how many green balls are there?

3 A ribbon is cut so that the lengths of the parts are in the ratio 8 : 5. If the shorter piece is 15 cm in length, what was the length of the original ribbon?

4 The ratio of the cost of a shirt to the cost of a tie is 8 : 5. If the shirt costs $48 more than the tie, find the cost of the shirt and the cost of the tie.

5 Two sums of money are in the ratio 5 : 8. The smaller amount is $65. Find the larger amount.

6 **a** Divide $45 in the ratio 4 : 5. **b** Divide 720 kg in the ratio 5 : 3.

7 **a** Divide 96 m in the ratio 9 : 7. **b** Divide 144 cm in the ratio 5 : 1.

8 **a** Divide $72 in the ratio 1 : 2 : 5. **b** Divide 95 kg in the ratio 5 : 6 : 8.

Example 23

9 Three friends decide to divide $12 000 amongst them in the ratio 1 : 2 : 3. How much does each receive?

10 An interval AB is 6 cm in length. A point C is a point on AB such that the ratio of the length AB to the length CB is 2 : 1. Find the lengths of AC and CB.

11 Jane and Anthony run a business. They have decided that all profits will be divided between them in the ratio 5 : 4, with Jane receiving the larger share. In 2005, the business made $81 900. How much does each person receive?

12 There are 24 children in a class. The ratio of the number of boys to the number of girls is 3 : 5. How many boys and how many girls are there?

13 160 mm of snow contains as much water as 15 mm of rain. A town within the Arctic Circle receives about 4250 mm of snow a year. If the snow had fallen as rain, what would the equivalent rainfall for the year have been?

14 The angles of a triangle are in the ratio 6 : 5 : 7. Find the size of each of the angles.

15 A piece of string 253 cm long is to be divided in the ratio 2 : 3 : 6. How long is each part?

16 Students in a school are told to choose one out of three sports options: tennis, basketball or swimming. Given that the pupils choose the options in the ratio 4 : 2 : 3 and that 120 choose tennis, find:

a the number of pupils in the school

b the number of students who choose swimming

17 In a class of 30 students, the ratio of boys to girls is 2 : 3. If 6 boys join the class, find the new ratio of boys to girls in the class.

18 The ratio of length to width of a rectangle is 2 : 3. If its area is 54 cm^2, find its perimeter.

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10F Scale drawings

A **scale drawing** is used where an object being illustrated is too large to be shown at full size on the page. For example, a scale drawing might be used to show:

- the plan of a building
- a map of a suburb or a country
- a photograph of a distant galaxy.

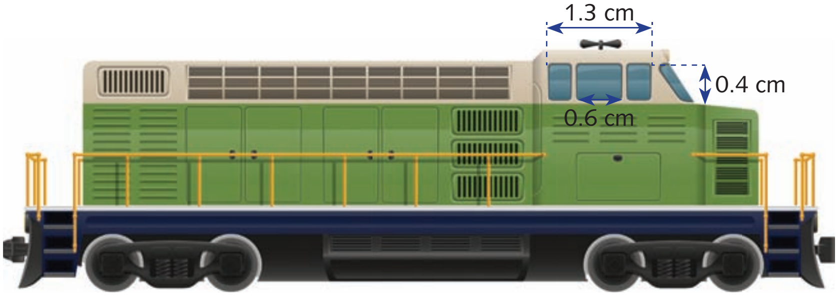
Scale drawings are also used when very small objects are to be shown, such as:

- a diagram of a cross-section of skin or hair
- an enlargement of an image on a computer screen
- a magnified photograph of an insect.

A scale drawing has the same shape as the original object, but a different size. All the lengths of the original object are reduced or magnified in the drawing in exactly the same ratio. This ratio is called the **scale** of the drawing.

Scale = length on the drawing : length on the actual object

The scale of a drawing can be given as the ratio of two numbers. For example, in the image of the side of an engine, the scale is $1:200$.



scale = 1 : 200

This means that a length of 1 cm on the image corresponds to a length of 200 cm, or 2 m, on the actual engine. Thus, the scale can also be written as:

scale = 1 cm : 2 m

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Example 24

a Measure the overall length of the engine on the previous page (including the couplings). Then use the scale to find the approximate length (in metres) of the actual engine.

b Find the approximate width and height of the window panel (in metres), and the area of the panel.

Solution

a

$$\begin{aligned} \text{Engine length in photograph} &= 11 \text{ cm} \\ \boxed{\times 200} \quad \text{Actual engine length} &= 2200 \text{ cm} \\ &= 22 \text{ m} \end{aligned}$$

b

$$\begin{aligned} \text{Width of window panel in photograph} &= 0.6 \text{ cm} \\ \boxed{\times 200} \quad \text{Actual width of window panel} &= 120 \text{ cm} \\ &= 1.2 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Height of window panel in photograph} &= 0.4 \text{ cm} \\ \boxed{\times 200} \quad \text{Actual height of window panel} &= 80 \text{ cm} \\ &= 0.8 \text{ cm} \\ \text{Area of window panel} &= 1.2 \times 0.8 \\ &= 0.96 \text{ m}^2 \end{aligned}$$

In general we express a scale in simplest form. For example, a scale of 3 : 9 is written as 1 : 3.

Scale drawings

- A **scale drawing** of an object has the same shape as the object, but a different size.
- The **scale** of the drawing is the ratio:

  length on the drawing : length on the actual object.
- A scale can be written as the ratio of two numbers, or as the ratio of two lengths. For example:

  scale = 1 : 500 or scale = 1 cm : 5 m

Example 25

Convert the two measurements in each scale to the same unit. Hence, convert the scale to a ratio of two numbers.

a 1 cm : 4 m

b 3 cm : 1 mm

Solution

a

$$\begin{aligned} 1 \text{ cm} : 4 \text{ m} &= 1 \text{ cm} : 400 \text{ cm} \\ &= 1 : 400 \end{aligned}$$

b

$$\begin{aligned} 3 \text{ cm} : 1 \text{ mm} &= 30 \text{ mm} : 1 \text{ mm} \\ &= 30 : 1 \end{aligned}$$

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Example 26

Convert each scale to a ratio of lengths in the units indicated.

a $1:250\,000 = 1\text{ cm} : \_\_\_\_ \text{ km}$

b $200:1 = 1\text{ cm} : \_\_\_\_ \text{ mm}$

Solution

a
$$\begin{aligned} 1:250\,000 &= 1\text{ cm} : 250\,000\text{ cm} \\ &= 1\text{ cm} : 2500\text{ m} \\ &= 1\text{ cm} : 2.5\text{ km} \end{aligned}$$

b
$$\begin{aligned} 200:1 &= 200\text{ mm} : 1\text{ mm} \\ &= 20\text{ cm} : 1\text{ mm} \\ &= 1\text{ cm} : 0.05\text{ mm} \end{aligned}$$

Problems involving scale drawing

Problems involving scale drawings can be solved in exactly the same way as the ratio problems discussed earlier.

Example 27

On a map with a scale of $1:500\,000$ two towns are 12 cm apart. How far apart are the actual towns?

Solution

1 cm represents 500 000 cm, which is 5000 m, which is 5 km

so 1 cm represents 5 km

$\boxed{\times 12}$ 12 cm represents 60 km

Thus the towns are 60 km apart.

Example 28

Eleni has photographed a tiny bug at a scale of $80:1$. If a leg on the photograph measures 4 mm, how long is the actual leg?

Solution

80 mm represents 1 mm, so:

$\boxed{\div 10}$ 8 mm represents 0.1 mm

$\boxed{\div 2}$ 4 mm represents 0.05 mm

Thus the bug's leg is 0.05 mm long.

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Exercise 10F

Use appropriate units in your answers to Questions **3–14**.

Example 25

1 Convert the two measurements in each scale to the same unit. Hence, convert the scale to a ratio of two numbers.

a 4 cm : 32 cm

b 15 cm : 55 cm

c 1 cm : 2 m

d 10 cm : 4.5 m

e 1 cm = 3 km

f 5 cm = 200 km

g 3 cm : 6 mm

h 20 cm : 5 mm

i 1 cm = 0.4 mm

Example 26

2 Copy and complete each scale conversion.

a 2 : 15 = 1 cm : ___ cm

b 3 : 10 = 12 cm : ___ cm

c 1 : 700 = 1 cm : ___ m

d 1 : 4600 = 1 cm : ___ m

e 1 : 300 000 = 1 cm : ___ km

f 1 : 1600 000 = 12 cm : ___ km

g 10 : 1 = 1 cm : ___ mm

h 50 : 1 = 1 cm : ___ mm

i 200 : 1 = 1 cm : ___ mm

j 5000 : 1 = 1 cm : ___ mm

Example 27

3 A map of a country is drawn to a scale of 1 : 12 500 000. Find the actual distance between two points whose separation on the map is:

a 1 cm

b 8 cm

c 1.2 cm

d 1 mm

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4 A map of a city is drawn to a scale of 1 cm : 2 km. Find the actual distance between two points whose separation on the map is:

a 3 cm
b 11 cm
c 7.5 cm
d 4.8 cm
e 5 mm
f 1 mm
g 1.5 mm
h 0.2 mm

5 A plan of a rectangular block of land is drawn to a scale of 1 : 1000. The side lengths of the block on the plan are 8 cm and 15 cm.

a Calculate the side lengths of the actual block.
b Calculate the area of the actual block.

6 A plan of a large ornamental doorway is drawn to a scale of 1 : 50. The doorway is 8 cm wide on the plan. Above the door is a rectangular panel, whose base equals the width of the door, and whose height is 2 cm on the plan.

a Calculate the width of the actual door and the height of the actual panel.
b Calculate the area of the actual panel.

7 A plan of a house is drawn to a scale of 1 : 200. Find the measurement on the plan of:

a the hall, which has length 6 m
b the bedroom, which has width 3 m
c the walls, which have height 2.8 m
d the side of the house, which has length 11.2 m
e the kitchen table, which has width 1.2 m

8 Find the scale factor of each diagram, map or photograph described below. Give each answer as a ratio of two numbers.

a Two towns 3600 km apart are 12 cm apart on a map.
b A tree that is 80 m tall measures 4 cm on a photograph.
c A flea that is 3 mm long measures 15 cm on a microscope photograph.
d A hair that is 0.03 mm thick is 2 cm thick on a microscope photograph.
e A model aeroplane has a wingspan of 3.2 m, and the actual aeroplane has a wingspan of 80 m.
f A galaxy that is 120 light years across measures 10 cm on a telescope photograph. (You may take 1 light year to be 10^{16} m.)

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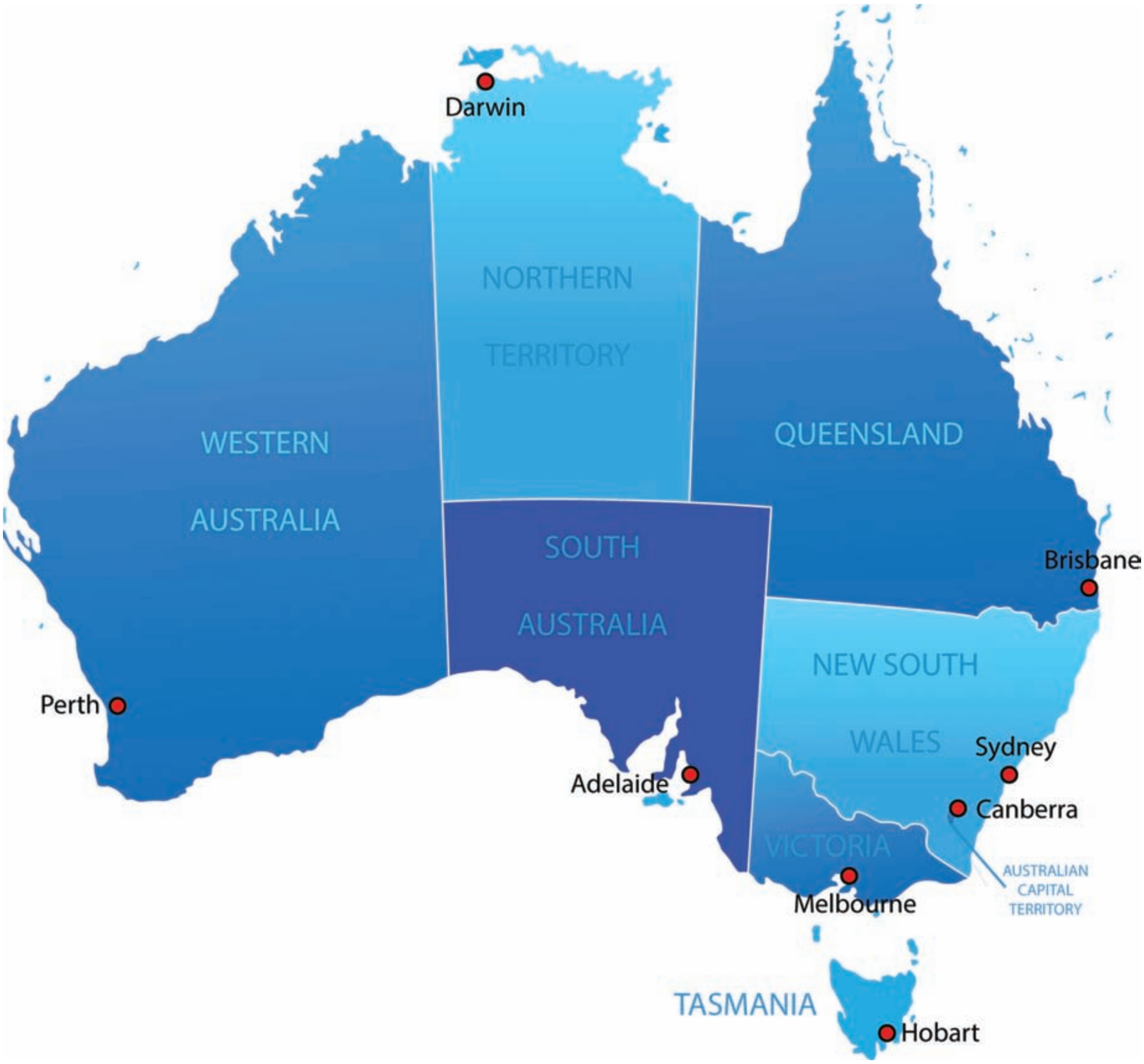
Example 24

9 The map of Australia shown below has a scale of 1: 28 000 000. Use your ruler to measure each of the following straight-line distances on the map, then calculate the actual distance.



scale = 1: 28 000 000

a Perth to Canberra

b Adelaide to Canberra

c Melbourne to Canberra

d Hobart to Canberra

e Sydney to Canberra

f Brisbane to Canberra

g The longest east–west distance lying entirely within the country

h The longest north–south distance lying entirely within the country (including Tasmania)

i The length of the border between South Australia and Western Australia

j The shortest distance across Bass Strait

10 Referring to the map in Question **9**, the shape of South Australia can be approximated by a rectangle.

a Draw a rectangle whose area is a reasonable approximation of the area of South Australia.

b Measure its side lengths, and hence calculate the approximate area of the state.

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11 The illustration of a fly shown below has a scale of 10 : 1.

scale = 10 : 1

a Measure the length of the fly in the illustration, correct to the nearest millimetre, then calculate its actual length.

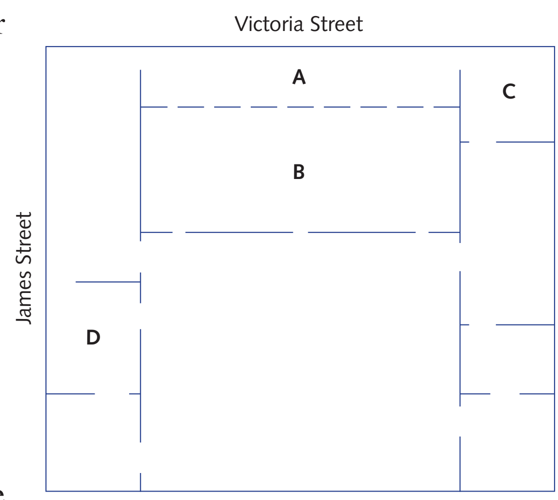
b Measure the length of the wing in the illustration, correct to the nearest millimetre, then calculate its approximate length.

12 The diagram below shows the basic floor plan of the first floor of a public building. Note that the dimensions of the Mawson Long Room are given.

a Use these dimensions, together with your own measurements of the plan, to work out, approximately, the scale of the drawing.

b Hence, find the lengths of the frontages to James Street and Victoria Street, correct to the nearest metre.

c Find the dimensions of the meeting room, correct to the nearest metre, and hence calculate its area, correct to the nearest square metre.



A Terrace
B Mawson Long Room
C Meeting Room
D Kitchen

Mawson Long Room dimensions:

length	20 m
height	4 m
width	7.7 m

13 **Investigation activity:** Draw a plan of your bedroom (looking down from above) showing the positions of any doors and windows, and all items of furniture.

a First measure the dimensions of your room, and work out a suitable scale that will allow you to draw the plan on a page of your exercise book.

b Then measure the dimensions and position of each item in the room, reduce them to scale, and draw each item on the plan.

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Review exercise

Note: Some of these questions can be done mentally. Do as many as you can in your head.

1 **a** Divide 735 in the ratio 3 : 2.
b Divide 735 in the ratio 4 : 5 : 6.
c Divide 342 in the ratio 3 : 2 : 1.
d Divide 600 in the ratio 5 : 6.
e Divide 660 in the ratio 3 : 8.
f Divide 6600 in the ratio 2 : 3 : 6.

2 Reduce each ratio to simplest form:

a 6 : 3
b 45 : 105
c 16 : 96
d 64 : 108
e 441 : 270 : 300
f 78 : 156 : 222
g $2\frac{1}{2} : 3\frac{1}{4}$
h $1\frac{1}{2} : 1\frac{1}{8}$

3 Find the cost of:

a 6 magazines, if 8 magazines cost \$100 (assuming that all the magazines have the same price)

b 20 packets of rice, if 30 packets cost \$82.50

4 A shirt manufacturer decides that he can supply 280 shirts in 4 weeks using 7 machinists. How long would it take for 15 machinists to produce 1000 shirts?

5 If apples are sold at 10 for \$3, find the number of apples that can be bought with:

a \$36
b \$22.50

6 Sally is knitting jumpers for newly born babies.

a If she knits at a rate of 2 rows per minute, and each row of knitting contains 45 stitches, how many stitches per second does she knit?

b If it takes Sally 5 hours to knit a jumper, and there are 150 rows of knitting in each jumper, how many rows per minute does she knit?

c If it takes Sally 4 hours to knit a jumper, and there are 144 rows of knitting in each jumper with, on average, 50 stitches per row, how many stitches per minute does she knit?

7 Joe walks every morning to keep fit.

a If he walks a distance of 10 km at an average speed of 6 km/h, how long, in minutes, does his morning walk take him?

b If he walks a distance of 7 km and it takes him 80 minutes, what is his average speed, in km/h?

c If he walks at an average speed of 5 km/h for 140 minutes, how far does he walk?

8 Divide \$15 in the ratio 1 : 4 : 5.

9 A 250 mL packet of milk contains 8 g of protein and 12.5 g of carbohydrate. Give the ratio of protein to carbohydrate in simplest form.

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10 A rope is cut into three pieces whose lengths are in the ratio $1:3:6$. Given that the length of the longest piece is 36 m, find:

a the length of the original rope

b the length of the shortest piece of rope

11 The costs of material, labour and administration for an advertising campaign are in the ratio $8:5:2$. If the total cost of the campaign is \$35 700, find the cost of the labour.

12 A machine makes 720 bottles in 12 hours. How many bottles does it make in 40 minutes?

13 The total amount of prize money in a photography competition is \$19 600. Given that the prize money is divided among the first, second and third prizes in the ratio $7:5:2$, find the amount each prize winner receives.

14 A rectangle has length 8 cm and width 4 cm. Another rectangle has length 12 cm and width 6 cm. Find:

a the ratio of the lengths

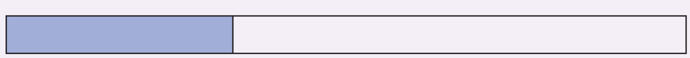
b the ratio of the widths

c the ratio of the perimeters

d the ratio of the areas

15 A hockey team played 27 matches in a season. The ratio of losses to wins was $4:5$. How many games did the team win and how many did it lose?

16 The diagram below shows a ribbon. One-third of the ribbon is blue. Give the ratio (length of blue section):(length of white section).



17 In the diagram below, the ratio $XA : AB : BY = 2:3:1$.

Find the following ratios.

a $XB : BY$

b $XA : XY$

c $AB : XY$

d $XA : AY$

18 Three numbers are in the ratio $2:5:3$. If the largest number is 120, find the other two numbers.

19 The sides of a right-angled triangle are 5 cm, 12 cm and 13 cm. A rectangle has sides 3 cm and 5 cm. Find the ratio of the area of the triangle to the area of the rectangle.

20 The scale of a map reads $1:350\,000$. Find the distance apart of two towns that are:

a 3 cm apart on the map

b 35 mm apart on the map

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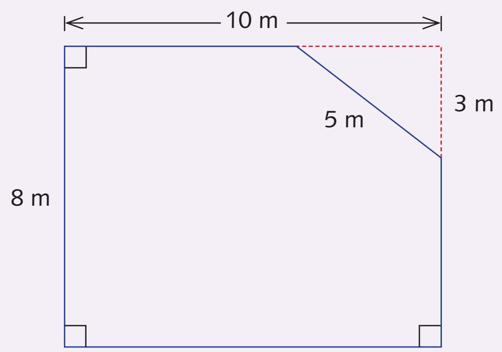
Challenge exercise

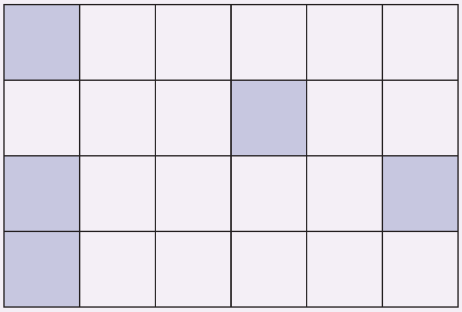
1 It took David an hour to ride 20 km from his house to the nearest town. He then spent 40 minutes on the return journey. What was his average speed?

2 The diagram below shows a metal plate that is to be painted.

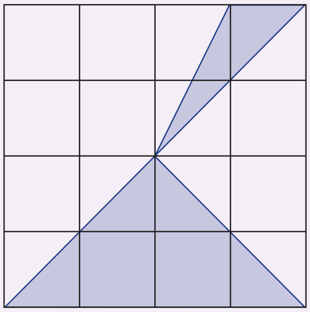


How much will it cost to paint the plate if paint costs $2.50 per square metre?

3 Some of the squares in the diagram below are shaded. More squares need to be shaded if the ratio (number of unshaded squares) : (number of shaded squares) is to be 2 : 1. How many more squares need to be shaded?



4 In the diagram below, find the ratio of the shaded area to the unshaded area.



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5 Four points A, B, C and D are marked in order on a line such that $AB : AC = 3 : 5$ and $BD : CD = 7 : 2$. If CD is 10 cm long, find the length of AB.

6 A container is filled with 64 L of orange juice, then 12 L of juice are removed and the container is topped up with lemon juice. The juice is thoroughly mixed before 12 L of the mixture is removed and the container is again topped up with lemon juice. What is the ratio of orange juice to lemon juice in the final mixture?

7 Two equal-sized vats, A and B, each containing 1000 L of oil, are being drained at a constant rate. It takes 4 hours to drain vat A completely and 5 hours to drain vat B completely. Find the time at which the amount of oil in vat B is four times the amount of oil in vat A.

8 **a** Two containers each contain 1 L of orange cordial. The first container contains cordial concentrate and water in the ratio $1 : 9$, and the second container contains cordial concentrate and water in the ratio $1 : 8$. The contents of both containers are tipped into a 2-litre bottle. What is the ratio of cordial concentrate to water in the 2-litre bottle?

b A bottle contains 1 L of cordial. The ratio of cordial concentrate to water is $1 : 5$. Another bottle contains 0.5 L of cordial. The ratio of cordial concentrate to water in this bottle is $1 : 4$. The contents of both bottles are tipped into a third container. What is the ratio of cordial concentrate to water in this third container?

c A bottle contains one-third of a litre of cordial, which consists of water and cordial concentrate in the ratio $1 : 4$. Two-thirds of a litre of water are then poured in. What is the ratio of concentrate to water now?

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CHAPTER

11

Number and Algebra

Algebra – part 2

We review and extend the use of negative numbers and fractions in algebra. In the course of this chapter, we will introduce new ways of solving problems from topics you have studied arising in the year, such as percentages, speed and ratios.

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11A Expanding brackets and collecting like terms

We learned how to expand brackets in Chapter 6. We now review that work, including expressions with negative integers and negative fractions.

Example 1

Simplify:

a $-3a \times (-4)$

b $-(-3a)$

c $6a - (-a)$

d $-7a \times 6a$

Solution

a $-3a \times (-4) = 12a$

b $-(-3a) = 3a$

c $6a - (-a) = 6a + a = 7a$

d $-7a \times 6a = -42a^2$

We recall two distributive laws introduced earlier:

$a(b + c) = ab + ac$ and $a(b - c) = ab - ac$

These are used in the following examples.

Example 2

Expand the brackets in each expression.

a $3(2x - 5)$

b $-4(2x - 6)$

c $-\frac{1}{2}(4x - 18)$

d $-(4 - 5a)$

Solution

a $3(2x - 5) = 6x - 15$

b $-4(2x - 6) = -8x + 24$

c $-\frac{1}{2}(4x - 18) = -2x + 9$

d $-(4 - 5a) = -4 + 5a$

Example 3

Expand the brackets and collect like terms.

a $3 - (6 - 5x)$

b $2x - (8 - 10x)$

c $3(-2x + 5) + 4(3x + 6)$

d $-4(-2x + 5) - 6(4 - 2x)$

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Solution

a $3-(6-5x) = 3-6+5x$
$= -3+5x$

b $2x-(8-10x) = 2x-8+10x$
$= 12x-8$

c $3(-2x+5)+4(3x+6) = -6x+15+12x+24$
$= -6x+12x+15+24$
$= 6x+39$

d $-4(-2x+5)-6(4-2x) = 8x-20-24+12x$
$= 8x+12x-20-24$
$= 20x-44$

Exercise 11A

Example 1

1 Simplify:

a $-3a\times 2$ **b** $-6b\times(-5)$ **c** $-(-6m)$ **d** $2\times(-3a)$
e $-5a\times(-3b)$ **f** $-(-5h)$ **g** $-7\times(-3a)$ **h** $-m\times 3$
i $2a\times(-3a)$ **j** $-4x\times(-3x)$ **k** $-3a\times 5a$

Example 2

2 Expand the brackets in each case.

a $3(x+2)$ **b** $5(2x+3)$ **c** $6(6x+3)$
d $-5(4x+7)$ **e** $7(5a-6)$ **f** $-4(5+2b)$
g $7(3z+5)$ **h** $6(2a-b)$ **i** $-11(6a-5)$
j $-7(6-x)$ **k** $-6(-3x-4)$ **l** $-5(2a-3b)$
m $-3(-6m+5)$ **n** $-3(-m+8)$ **o** $-7(-7m-3)$
p $-4(2-3c)$ **q** $\frac{1}{3}(2x-9)$ **r** $-\left(10+\frac{1}{4}x\right)$

Example 3a, b

3 Expand the brackets and collect like terms in each case.

a $3(2x-5)+5x$ **b** $6(x-3)+10$ **c** $4(2x-3)-5$
d $6(x-7)+6x$ **e** $-4(n-6)+4n$ **f** $7(2m-5)+8$
g $-4(8k+7)+60k$ **h** $7(-8m-3)+5m$ **i** $-6(-h+3)-18$
j $-4(-2x+3)-8x$ **k** $-4(3-3k)-12k+12$ **l** $-5(-7-7m)-20m$

Example 3c, d

4 Expand the brackets and collect like terms.

a $4(2x+6)+5(3x+2)$ **b** $2(5x+3)+3(x+4)$
c $6(x-7)+6(x-5)$ **d** $5(2m-3)+6(3m+4)$
e $2(3m-4)-5(m+4)$ **f** $6(4k-2)-(k+6)$
g $7(z+6)-2(3z+4)$ **h** $7(5a+2)-(4a-3)$
i $7(2x+7)-5(6x-2)$ **j** $7(p-2)-8(3-p)$
k $8(q-5)-2(q+3)$ **l** $6(m-4)-6(3m-1)$

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11B Addition and subtraction of algebraic fractions

Recall how to add and subtract fractions. When the denominators of two fractions are the same, we use the common denominator and add the numerators. For example:

$$\frac{3}{7}+\frac{2}{7}=\frac{3+2}{7}=\frac{5}{7}$$

When the denominators of the fractions to be added are different, we first find equivalent fractions with the lowest common denominator. We then proceed as for fractions with the same denominator. For example:

$$\begin{aligned}\frac{3}{7}+\frac{2}{5}&=\frac{15}{35}+\frac{14}{35}\\&=\frac{29}{35}\end{aligned}\quad\text{or}\quad\begin{aligned}\frac{3}{7}+\frac{2}{5}&=\frac{15+14}{35}\\&=\frac{29}{35}\end{aligned}$$

We will use the second notation most of the time.

Subtraction of two fractions is undertaken in a similar way.

We use the same procedures that we previously used to add and subtract algebraic fractions that are like terms.

Example 4

Express as a single fraction.

a $\frac{x}{7}+\frac{4x}{7}$ **b** $\frac{2x}{5}+\frac{x}{3}$ **c** $\frac{2z}{11}+\frac{z}{2}$ **d** $-\frac{m}{3}+\frac{2m}{5}$

Solution

a $$\begin{aligned}\frac{x}{7}+\frac{4x}{7}&=\frac{x+4x}{7}\\&=\frac{5x}{7}\end{aligned}$$

b $$\begin{aligned}\frac{2x}{5}+\frac{x}{3}&=\frac{6x+5x}{15}\\&=\frac{11x}{15}\end{aligned}$$

c $$\begin{aligned}\frac{2z}{11}-\frac{z}{2}&=\frac{4z-11z}{22}\\&=\frac{-7z}{22}\\&=-\frac{7z}{22}\end{aligned}$$

d $$\begin{aligned}-\frac{m}{3}+\frac{2m}{5}&=\frac{-5m+6m}{15}\\&=\frac{m}{15}\end{aligned}$$

(It is better to put the sign in front of the entire fraction.)

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Example 5

Expand the brackets and collect like terms.

a $2\left(\frac{x}{3}+4\right)+\frac{x}{3}$

b $5\left(\frac{4x}{7}+6\right)+\frac{2x}{3}$

Solution

a
$$\begin{aligned} 2\left(\frac{x}{3}+4\right)+\frac{x}{3} &= \frac{2x}{3}+8+\frac{x}{3} \\ &= \frac{3x}{3}+8 \\ &= x+8 \end{aligned}$$

b
$$\begin{aligned} 5\left(\frac{4x}{7}+6\right)+\frac{2x}{3} &= \frac{20x}{7}+30+\frac{2x}{3} \\ &= \frac{60x+14x}{21}+30 \\ &= \frac{74x}{21}+30 \end{aligned}$$

Exercise 11B

Example 4

1 Simplify.

a $\frac{2x}{5}+\frac{x}{5}$ **b** $\frac{2x}{7}+\frac{3x}{7}$ **c** $\frac{5z}{11}-\frac{2z}{11}$

d $\frac{3m}{11}-\frac{2m}{11}$ **e** $\frac{2x}{3}+\frac{x}{2}$ **f** $\frac{4x}{5}+\frac{x}{2}$

g $\frac{5x}{7}-\frac{3x}{4}$ **h** $\frac{3a}{7}+\frac{a}{2}$ **i** $\frac{5x}{6}-\frac{4x}{7}$

j $\frac{5x}{7}+\frac{3x}{2}-\frac{5x}{4}$ **k** $\frac{6b}{11}+\frac{2b}{3}-\frac{b}{2}$ **l** $\frac{5x}{7}-\frac{x}{3}$

m $\frac{3x}{4}+\frac{3x}{2}-\frac{5x}{4}$ **n** $\frac{7x}{24}+\frac{2x}{3}-\frac{x}{4}$ **o** $\frac{5c}{3}-\frac{c}{5}$

p $\frac{7x}{24}+\frac{5x}{6}-\frac{x}{3}$ **q** $\frac{x}{6}-\frac{2x}{3}-\frac{x}{4}$ **r** $\frac{x}{24}+\frac{2x}{3}-\frac{3x}{4}$

s $\frac{7x}{12}+\frac{5x}{6}-\frac{2x}{3}$ **t** $-\frac{7x}{24}-\frac{5x}{6}-\frac{5x}{4}$

2 Collect like terms.

a $\frac{x}{3}+\frac{x}{4}+\frac{y}{3}+\frac{y}{4}$ **b** $\frac{x}{3}-\frac{x}{4}+\frac{y}{3}-\frac{y}{4}$ **c** $\frac{x}{3}+\frac{y}{4}+\frac{y}{3}+\frac{y}{5}$

d $\frac{y}{3}-\frac{x}{4}+\frac{y}{3}+\frac{x}{2}$ **e** $\frac{2x}{3}-\frac{x}{5}+\frac{2y}{3}-\frac{3y}{4}$ **f** $-\frac{2x}{3}+\frac{2y}{3}+\frac{4x}{5}-\frac{5y}{8}$

g $\frac{2x}{3}-\frac{3x}{7}+\frac{3y}{4}-\frac{5y}{8}$ **h** $-\frac{5x}{3}+\frac{11x}{6}-\frac{7y}{4}-\frac{5y}{8}$ **i** $\frac{2x}{5}-\frac{3x}{10}+\frac{y}{4}-\frac{11y}{8}$

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Example 5

3 Expand the brackets and collect like terms.

a $4\left(\frac{x}{5}+6\right)+\frac{x}{5}$

b $5\left(\frac{4x}{7}+6\right)+\frac{2x}{7}$

c $5\left(\frac{2x}{3}+\frac{1}{2}\right)+\frac{4x}{7}$

d $-3\left(\frac{3x}{5}+2\right)-\frac{4x}{11}$

e $12\left(\frac{4x}{3}+\frac{1}{2}\right)+\frac{3x}{7}$

f $-7\left(\frac{7x}{12}+2\right)-\frac{4x}{3}$

g $-4\left(-\frac{x}{5}+8\right)+\frac{x}{5}+12$

h $-5\left(\frac{3x}{7}+12\right)-\frac{4x}{7}+30$

4 Expand the brackets and collect like terms.

a $2\left(3x+\frac{1}{2}\right)+x+\frac{1}{2}$

b $3\left(5x+\frac{3}{2}\right)+2x+\frac{1}{2}$

c $5\left(3x-\frac{3}{5}\right)-4x+\frac{1}{10}$

d $-2\left(3x+\frac{3}{2}\right)+6x+\frac{11}{2}$

e $-2\left(3x-\frac{11}{2}\right)+7x+4\frac{1}{2}$

f $6\left(-3x+\frac{3}{4}\right)-4x+\frac{11}{2}$

11C Solving equations

Setting up and solving equations is central to the study and application of mathematics. Solving equations was introduced in Chapter 6. We first review these methods, and then consider equations that involve adding or subtracting algebraic fractions.

Example 6

Solve each equations.

a $x+3=-2$

b $-2x=10$

c $\frac{x}{3}=-5$

d $2x+5=-6$

e $5-11x=-7$

Solution

a
$$\begin{aligned} x+3 &= -2 \\ \boxed{-3} \quad x &= -2-3 \\ &= -5 \end{aligned}$$

(Subtract 3 from both sides of the equation.)

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b

$$\begin{aligned} -2x &= 10 \\ \boxed{\div(-2)} \quad x &= \frac{10}{-2} \\ &= -5 \end{aligned}$$

(Divide both sides of the equation by –2.)

or

$$\boxed{\times(-1)} \quad 2x = -10$$

(Multiply both sides of the equation by –1.)

$$\boxed{\div(-2)} \quad x = -5$$

c

$$\frac{x}{3} = -5$$

$$\boxed{\times 3} \quad x = -15$$

(Multiply both sides by 3.)

d

$$2x + 5 = -6$$

$$\boxed{-5} \quad 2x = -11$$

(Subtract 5 from both sides of the equation.)

$$\begin{aligned} \boxed{\div 2} \quad x &= \frac{-11}{2} \\ &= -5\tfrac{1}{2} \end{aligned}$$

(Divide both sides by 2.)

e

$$5 - 11x = -7$$

$$\boxed{-5} \quad -11x = -12$$

(Subtract 5 from both sides of the equation.)

$$\begin{aligned} \boxed{\div -11} \quad x &= \frac{-12}{-11} \\ &= 1\tfrac{1}{11} \end{aligned}$$

(Divide both sides by –11.)

or

$$11x = 12$$

(Multiply both sides of the equation by –1.)

$$x = 1\tfrac{1}{11}$$

(Divide both sides by 11.)

Example 7

Collect like terms and solve.

a $5x + 3x + 4 = 36$

b $2(3z + 4) + 5 = 20$

c $5x + 5 = 2x - 4$

d $-5x + 3 = 2x - 11$

Solution

a

$$5x + 3x + 4 = 36$$

$$8x + 4 = 36$$

(Collect like terms.)

$$\boxed{-4} \quad 8x = 32$$

(Subtract 4 from both sides.)

$$\boxed{\div 8} \quad x = 4$$

(Divide both sides by 8.)

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b

$$
\begin{aligned}
 & & 2(3z+4)+5 &= 20 & & \\
 & & 6z+8+5 &= 20 & & \text{(Expand brackets.)} \\
 & & 6z+13 &= 20 & & \text{(Collect like terms.)} \\
 & \boxed{-\ 13} & 6z &= 7 & & \text{(Subtract 13 from both sides.)} \\
 & \boxed{\div\ 6} & z &= 1\tfrac{1}{6} & & \text{(Divide both sides by 6.)}
\end{aligned}
$$

c

$$
\begin{aligned}
 & & 5x+5 &= 2x-4 & & \\
 & \boxed{-\ 2x} & 3x+5 &= -4 & & \text{(Subtract } 2x \text{ from both sides of the equation.)} \\
 & & 3x &= -9 & & \text{(Subtract 5 from both sides of the equation.)} \\
 & \boxed{\div\ 3} & x &= -3 & & \text{(Divide both sides by 3.)}
\end{aligned}
$$

d

$$
\begin{aligned}
 & & -5x+3 &= 2x-11 & & \\
 & \boxed{+\ 5x} & 3 &= 7x-11 & & \text{(Add } 5x \text{ to both sides.)} \\
 & \boxed{+\ 11} & 14 &= 7x & & \text{(Add 11 to both sides.)} \\
 & \boxed{\div\ 7} & 2 &= x & & \text{(Divide both sides by 7.)}
\end{aligned}
$$

We can also use the methods from the previous section to solve equations involving fractions. We consider two ways of solving the equation $\frac{m}{5}+\frac{m}{3}=1$.

Method 1

Simplify the left-hand side.

$$
\begin{aligned}
 & & \frac{m}{5}+\frac{m}{3} &= 1 & & \\
 & & \frac{3m+5m}{15} &= 1 & & \text{(Obtain a common denominator.)} \\
 & & \frac{8m}{15} &= 1 & & \\
 & \boxed{\times 15} & 8m &= 15 & & \text{(Multiply both sides of the equation by 15.)} \\
 & \boxed{\div 8} & m &= 1\tfrac{7}{8} & & \text{(Divide both sides of the equation by 8.)}
\end{aligned}
$$

Method 2

Multiply through by the lowest common multiple.

$$
\begin{aligned}
\frac{m}{5}+\frac{m}{3} &= 1 & & \\
3m+5m &= 15 & & \text{(Multiply both sides of the equation by 15.)} \\
8m &= 15 & & \left(\frac{m}{5}\times 15 = 3m \text{ and } \frac{m}{3}\times 15 = 5m\right) \\
m &= 1\tfrac{7}{8} & & \text{(Divide both sides of the equation by 8.)}
\end{aligned}
$$

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Example 8

Solve each equation for m.

a $\frac{m}{6} - \frac{m}{7} = 10$

b $\frac{2m}{5} - \frac{m}{4} = 21$

Solution

a We shall use method 2.

$$\frac{m}{6} - \frac{m}{7} = 10$$

$$7m - 6m = 420 \qquad \text{(Multiply both sides of the equation by 42.)}$$

$$m = 420$$

Check: LHS $= \frac{m}{6} - \frac{m}{7}$

$= \frac{240}{6} - \frac{420}{7}$

$= 70 - 60$

$= 10$

$=$ RHS

b We shall use method 1.

$$\frac{2m}{5} - \frac{m}{4} = 21$$

$$\frac{8m - 5m}{20} = 21 \qquad \text{(Obtain a common denominator.)}$$

$$\frac{3m}{20} = 21$$

$\boxed{\times 20}$ $\quad 3m = 420$ (Multiply both sides of the equation by 20.)

$\boxed{\div 3}$ $\quad m = 140$ (Divide both sides of the equation by 3.)

Exercise 11C

Example 6

1 Solve these equations.

a $x + 7 = -12$

b $x - 11 = -14$

c $x + 7 = 7$

d $5x = -12$

e $-9x = -27$

f $-8x = 8$

g $\frac{x}{5} = -7$

h $\frac{-5x}{2} = -6$

i $3x + 1 = -8$

j $5x - 7 = -14$

k $-2x + 11 = -12$

l $4x - 6 = -8$

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Example 7

2 Solve these equations.

a $2x+7x+5=41$ **b** $6x-3x+5=3$ **c** $z-5z+10=13$
d $5(x+3)+4=39$ **e** $3(2z+5)+5=28$ **f** $-4(3x+1)+7=22$
g $6x+4=7x+5$ **h** $-3y+11=y+13$ **i** $7x+2=3x+1$
j $5x-2=8x+1$ **k** $x+14=4x+10$ **l** $13x+4=2x+7$
m $3(2x-4)=x$ **n** $5(2x-4)=11x$ **o** $\frac{p}{3}-5=p$
p $\frac{1}{2}(6-2x)=x-3$

3 Solve these equations.

a $\frac{x}{3}-4=11$ **b** $\frac{2x}{5}+5=25$ **c** $\frac{5x}{7}+2=11$
d $5-\frac{4x}{5}=7$ **e** $\frac{5x}{7}-2=30$ **f** $\frac{7x}{11}-1=6$

Example 8

4 Solve these equations.

a $\frac{x}{5}+\frac{2x}{5}=1$ **b** $\frac{4m}{7}-\frac{2m}{7}=10$ **c** $\frac{3m}{4}-\frac{m}{4}=20$ **d** $\frac{5m}{2}+\frac{2m}{3}=6$
e $\frac{3x}{4}+\frac{2x}{5}=6$ **f** $\frac{3x}{5}+\frac{x}{7}=5$ **g** $\frac{3x}{11}+\frac{3x}{4}=8$ **h** $\frac{5x}{8}-\frac{x}{4}=72$
i $\frac{5x}{6}-\frac{3x}{4}=40$ **j** $\frac{x}{7}-\frac{x}{8}=1$ **k** $3\left(\frac{m}{6}-\frac{m}{9}\right)=21$ **l** $\frac{3x}{8}+\frac{3x}{4}+12=8$

5 Is there a solution to the equation $x+3=x+5$? Explain your answer.

6 Trang can run up a hill at 3 m/s and down the hill at 7 m/s. It takes her 60 seconds in total to run up and then down the hill. What distance does she cover going from the bottom to the top of the hill?

11D Problem-solving with equations

The next three examples demonstrate the usefulness of equations in solving different kinds of problems.

Example 9

I think of a number, multiply it by 3 and add 4 to the result. The number I obtain is 10 more than the number I first thought of. What was my original number?

Solution

Let x be the number.

$3x+4 = x+10$
$2x+4 = 10$ (Subtract x from both sides of the equation.)
$2x = 6$ (Subtract 4 from both sides of the equation.)
$x = 3$ (Divide both sides of the equation by 2.)

The number I first thought of was 3.

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Example 10

Josephine bought a desktop computer and a printer at a total cost of \$1560. The desktop computer cost $5\frac{1}{2}$ times as much as the printer. Write an equation and find the cost of the computer and the cost of the printer.

Solution

Let \$$x$ be the cost of the printer.

The computer cost $\$\left(5\frac{1}{2}\times x\right)=\$\frac{11x}{2}$

The total cost = \$1560

$$\begin{aligned} \text{So } \frac{11x}{2}+x &= 1560 \\ \frac{13x}{2} &= 1560 \\ x &= \frac{3120}{13} \\ &= 240 \end{aligned}$$

Thus the printer cost \$240 and the computer cost \$1320.

Example 11

Grant runs half the distance to school and walks for the remainder of the journey. He runs at 3 m/s but slows to 2 m/s for the second half of his trip. He takes 50 minutes to complete the trip. Find the distance Grant has to travel to school.

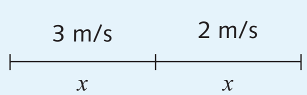
Solution

We will use metres and seconds as our units. Let x m be half the distance to the school in metres. Recall that:

distance = speed × time

and therefore:

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$



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$$\begin{aligned}\text{Then time spent running at 3 m/s} &= \frac{x}{3}\text{ seconds}\\ \text{and time spent walking at 2 m/s} &= \frac{x}{2}\text{ seconds}\\ \text{Also, time taken for the entire trip} &= 50\text{ minutes}\\ &= 3000\text{ seconds}\end{aligned}$$

Hence:

$$\begin{aligned}\frac{x}{3}+\frac{x}{2} &= 3000\\ 2x+3x &= 18\,000 && \text{(Multiply both sides of the equation by 6.)}\\ 5x &= 18\,000 && \text{(Divide both sides of the equation by 5.)}\\ \boxed{\div 5}\qquad x &= 3600\end{aligned}$$

Thus half the distance Grant travels from home to school is 3600 m, so the full distance he travels is 7200 m or 7.2 km.

Exercise 11D

Example 9

1 For each problem, write an equation and solve it to find the unknown number.

a Six is added to a number, x, and the result is multiplied by –3. The result of this is –72.

b One-third of a number, x, and one-quarter of the same number are added, and the result is 25.

c Three-fifths of a number, z, and one-seventh of the same number are added and the result is 30.

d Two-thirds of a number, m, is subtracted from three-fifths of the same number, and the result is 1.

e The result of multiplying a number, a, by –2 and adding 6 is the same as the result of multiplying a by 3 and subtracting 4.

2 For each problem, write an equation and find the unknown number.

a When half of a number is subtracted from two-thirds of the same number, the result is 10.

b Ten is subtracted from a number, and the result is multiplied by –5. The result of this is 30.

c Six is added to two-thirds of a number, and the result is –10.

d Two-thirds is subtracted from half of a number, and the result is –1.

Example 10

3 Chloe buys x kg of bananas for \$3 a kilogram, and x kg of apples for \$2 a kilogram. The total cost is \$15.50. Find the value of x.

4 The length of a rectangle is twice its width. The perimeter is 13 cm. Let x cm be the width. Write an equation for x and find the width and length.

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5 The average of three numbers is -56. Two of the numbers are 20 and -10. The third number is x. Write an equation for x and solve it.

6 The sum of eight numbers is x, and the sum of a different set of nine numbers is $2x$. The average of the smaller set of numbers is one more than the average of the larger set of numbers. Write an equation for x and solve it.

7 Five hundred dollars more than 10% of an amount of money $\$m$ is \$1280. Find m.

8 Trevor invests \$10 000 for a year. He obtains 8% per annum for a part of the money and 10% per annum for the remainder. At the end of the year, he will receive \$950 in interest. How much does he invest at 10%?

9 Anthony is twice as old as Julian. Five years ago, he was three times as old. Let x be Julian's present age. Write an equation for x and find the present ages of both Anthony and Julian.

Example 11

10 Giorgia cycles to school every day. She manages to go half the distance at 4 m/s but slows to 3 m/s for the second half of her trip. She takes 35 minutes to complete the trip. Find the distance she travels, to the nearest metre.

11 Frances travels between two places, Akville and Bracktown. She walks half the distance at 5 km/h and runs the other half at 10 km/h. If the total time for her journey is 3 hours, what is the distance from Akville to Bracktown?

12 When one-third of a number is added to three-quarters of a number, the result is 10 more than the number. Find the number.

13 If $\frac{3}{4}$ of one of the acute angles of a right-angled triangle is $15\frac{1}{4}^\circ$ larger than $\frac{1}{6}$ of the other, find the acute angles.

Review exercise

1 Expand the brackets.

a $2(5x+7)$ b $4(2b+3)$ c $6(7-3x)$

d $-5(4x+9)$ e $-3(3a-2)$ f $-4(5-8x)$

2 Expand the brackets and collect like terms.

a $5(x-2)+7$ b $3(4x+1)-5x$

c $3(5y+3)+4$ d $5(x-40)+3x$

e $6(x+2)+7$ f $-5(2x-3)+11x$

g $-4(1+5x)-2$ h $-3(4y-7)+10y$

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3 Expand the brackets and collect like terms.

a $2(2x+3)+3(x+4)$ **b** $6(x+5)+7(x-3)$

c $3(x-2)-4(x+2)$ **d** $2(3y+4)+4(4y-1)$

e $5(x+2)-3(7+x)$ **f** $7(5+x)-2(3+x)$

g $-5(a+2)-4(2a+2)$ **h** $-2(x-5)-3(x-6)$

4 Express each expression as a single fraction.

a $\frac{3x}{8}+\frac{x}{8}$ **b** $\frac{4x}{5}-\frac{2x}{5}$ **c** $\frac{2x}{7}-\frac{5x}{7}$ **d** $\frac{4x}{5}+\frac{x}{2}$

e $\frac{6x}{5}-\frac{x}{4}$ **f** $\frac{2x}{7}-\frac{2x}{3}$ **g** $\frac{2x}{3}+\frac{x}{2}-\frac{3x}{4}$ **h** $\frac{3x}{5}-\frac{2x}{3}+\frac{x}{2}$

5 Expand the brackets and collect like terms.

a $2\left(\frac{x}{3}+5\right)+\frac{x}{3}$ **b** $3\left(3-\frac{x}{4}\right)+\frac{x}{4}$ **c** $-2\left(\frac{x}{5}+3\right)-\frac{x}{5}$

d $2\left(\frac{3x}{5}+1\right)+\frac{x}{4}$ **e** $-\left(4+\frac{2x}{7}\right)-\frac{3x}{5}$ **f** $2\left(\frac{2x}{5}-3\right)+\frac{x}{4}$

6 Solve each equation for x.

a $x+3=5$ **b** $x+2=-7$ **c** $x-2=3$ **d** $x-4=-2$

e $x+5=-9$ **f** $x-1=-5$ **g** $6x=36$ **h** $5x=-20$

i $-7x=28$ **j** $-6x=-30$ **k** $2x+5=9$ **l** $7x+3=24$

m $2x-3=5$ **n** $4x-8=-12$ **o** $8x-2=30$ **p** $\frac{x}{5}=-20$

q $\frac{x}{2}+3=-9$ **r** $2-\frac{x}{5}=8$ **s** $3x+5=4$ **t** $5+4x=-8$

7 Solve each equation for x.

a $3x+2x+6=21$ **b** $3(x+2)+3=15$ **c** $-2(3x+2)+4=5$

d $3x=x+4$ **e** $2x=x-6$ **f** $3x=8-4x$

g $8x+2=7x-3$ **h** $5x-7=-x+9$ **i** $2x-4=5x+1$

j $15x+4=3x-2$ **k** $2x-5=-4x+6$ **l** $x-21=5x-24$

8 Solve these equations.

a $\frac{2x}{7}+\frac{x}{7}=6$ **b** $\frac{5m}{9}-\frac{2m}{9}=1$ **c** $\frac{x}{5}+\frac{2x}{5}=1$

d $\frac{3x}{7}+\frac{x}{5}=2$ **e** $\frac{m}{3}-\frac{2m}{5}=1$ **f** $\frac{4x}{9}-\frac{4x}{5}=1$

9 Four people each paid \$2 towards the cost of a raffle ticket, which won a prize of \$$x$. The prize was shared equally amongst the four, and each person made a profit of \$123. What was the amount of the prize?

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10 For each problem, write an equation and solve it to find the unknown number.

a Ten is added to a number, x, and the result is multiplied by $\frac{1}{2}$. The result of this is -32.

b One-quarter of a number, x, and one-fifth of the same number are added. The result is 60.

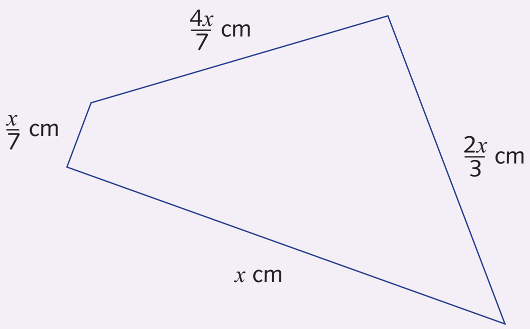
c The result of multiplying a number, a, by -6 and adding 3 is the same as the result of multiplying a by 10 and subtracting 20.

d Two-sevenths of a number, z, is subtracted from three-fifths of the same number, and the result is 10.

e Six is added to seven-tenths of a number, z, and the result is the same as subtracting 4 from z.

11 One thousand dollars more than 20% of an amount of money $\$m$ is $\$16800$. Find m.

12 The perimeter of this quadrilateral is 100 cm. Find the value of x and the lengths of the four sides.



Challenge exercise

1 Solve each equation for x.

a $\frac{x-4}{5}+\frac{2x-5}{6}=3$

b $\frac{x-5}{4}+\frac{5x-3}{10}=-5$

2 Think of a number, add 30 to it, multiply the result by 5, add 5 times the number you first thought of, subtract 50, divide the result by 10 and subtract 10. Use algebra to show that you always get the number you first thought of.

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3 Two towns are 50 km apart. Tom starts from town A and travels at 50 km/h towards town B. Clarrie starts from town B at the same time that Tom started, and travels at 40 km/h towards town A.

a After how many minutes do they meet?

b How far from A are they when they meet?

4 Elizabeth travels from Crocville to Barratown. For the first d km she can travel at 80 km/h, but after that she has to reduce her speed to 60 km/h. It takes Elizabeth 3 hours to travel the 200 km from Crocville to Barratown. How far does Elizabeth drive at 80 km/h?

5 I think of a number. I add one-third of that number and one-quarter of the same number to get 5 more than the number I first thought of. What is this number?

6 David travels from town A to town B. He walks half the distance at 3 km/h and runs the other half at 10 km/h. The total time for the journey is 4 hours. What is the distance from A to B?

7 A sum of money is divided equally between three friends, Harry, Larry and Carrie. An equal amount is divided between Anne, Leslie and Bronwyn in the ratio $2:3:5$. If Harry receives \$28 more than Anne, how much does Bronwyn receive?

8 David buys \$$2x$ worth of grapes at \$5 a kilogram, and \$$x$ worth of peaches at \$7 a kilogram. He buys a total of 10 kg of peaches and grapes. How much of each fruit does he buy?

9 If x is subtracted from both the numerator and denominator of $\frac{3}{4}$, the result is $\frac{7}{10}$. Find x.

10 A school has 1025 students. A total of 400 students cannot swim. This consists of $\frac{1}{5}$ of the boys and $\frac{4}{7}$ of the girls. If x boys can swim, write an equation for x and solve it. How many boys are there in the school?

11 Solution A contains 40% of concentrated acid and solution B contains 60% of the same concentrated acid. How many cubic centimetres of each type are needed to produce 1000 cm^3 of mixture containing 55% concentrated acid?

12 Water flows from tank A to tank B at a rate of 2 litres per minute. Initially tank A has 200 litres in it and tank B has 100 litres in it. Water drains from tank B at 0.5 litres per minute. After how many minutes are there equal volumes of water in the two tanks?

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CHAPTER

12

Measurement and Geometry

Congruent triangles

Figures are copied everywhere – a photocopier can produce a copy of a figure, craft workers use copies of figures to create designs or patterns, copies of figures can be obtained by tracing, and so on.

The idea of a 'copy' also arises in geometry and is called '**congruence**'.

You have previously met the three kinds of geometrical transformation – translation, reflection and rotation. Two figures are called **congruent** if one is the image of the other when you apply one or more of these transformations.

We will see that two triangles are congruent when the sides and angles of one triangle are equal to the sides and angles of the other.

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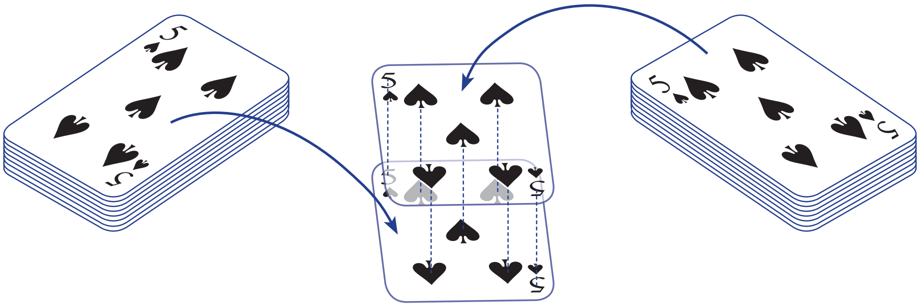
12A Congruence of figures in the plane

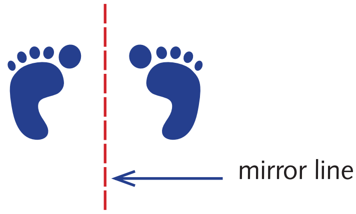
If we take the five of spades from each of two identical decks of cards, they look exactly the same. We can move one card and place it on top of the other one so that the pictures on the two cards coincide exactly, as shown below.



Two objects like this are called **congruent**. The word 'congruent' comes from Latin and means 'in agreement' or 'in harmony'. Here is a more precise definition:

> Two plane figures are called **congruent** if one figure can be moved on top of the other, by a sequence of translations, rotations and reflections, so that they coincide exactly. Thus congruent figures have the same size and the same shape.

For example, the two footprints opposite are congruent because the footprint on the right is a reflection of the footprint on the left.



Pairing the parts of congruent figures

When two figures are congruent, we can pair every part of one figure with the corresponding part of the other by finding a sequence of translations, rotations and reflections which moves one figure exactly on top of the other. For example, the two figures below are congruent. Some of the pairings of points with points, intervals with intervals, and angles with angles are shown in the list below.

Can you see how to transform one figure to the other? There are several ways of doing it. Once you have done this you will see the following pairings. We use the notation P' for the image of the point P under transformation, as introduced in Chapter 18 of *ICE-EM Mathematics Year 7*.

$A \leftrightarrow A'$	$D \leftrightarrow D'$	$BC \leftrightarrow B'C'$	$\angle EAB \leftrightarrow \angle E'A'B'$
$B \leftrightarrow B'$	$E \leftrightarrow E'$	$BD \leftrightarrow B'D'$	$\angle BCD \leftrightarrow \angle B'C'D'$
$C \leftrightarrow C'$	$AB \leftrightarrow A'B'$	$BE \leftrightarrow B'E'$	$\angle DBE \leftrightarrow \angle D'B'E'$

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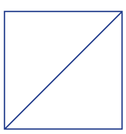
If two figures are congruent, then paired intervals have the same length, paired angles have the same size, and paired regions have the same area.

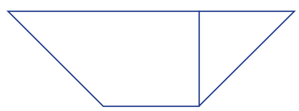
Congruent figures

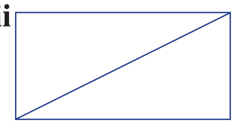
- Two plane figures are called **congruent** if one figure can be moved on top of the other, by a sequence of translations, rotations and reflections, so that they coincide exactly.
- Congruent figures have the same shape and the same size.
- When two figures are congruent, we can find a transformation that pairs every part of one figure with the corresponding part of the other, so that:
  - paired angles have the same size
  - paired intervals have the same length
  - paired regions have the same area.

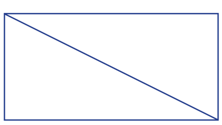
Exercise 12A

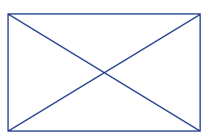
1 List the figures in the collection below that are congruent to each of the figures **i**, **ii** and **iii**.

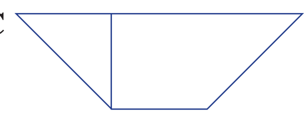
i


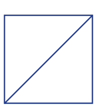
ii


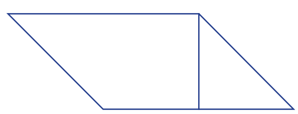
iii


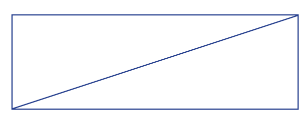
A


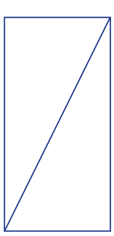
B


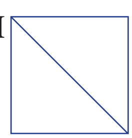
C


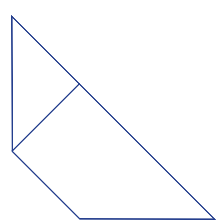
D


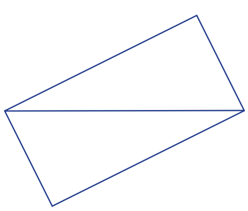
E


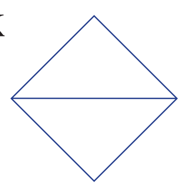
F


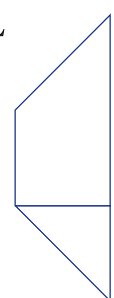
G


H


I


J


K


L


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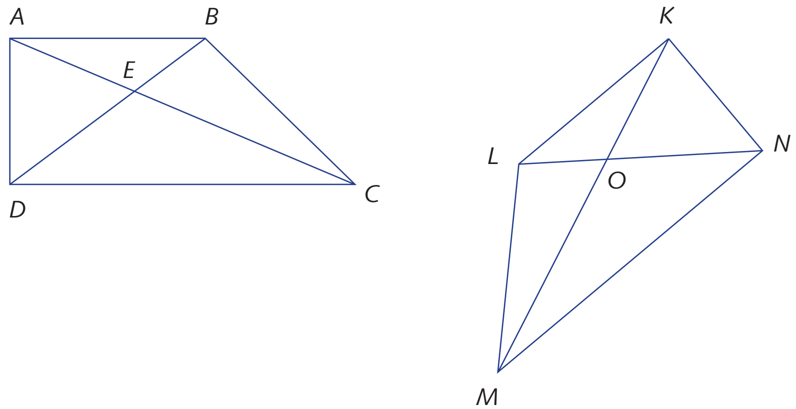
2 Complete the pairings of vertices, sides and angles of these two congruent figures.



a $A \leftrightarrow$ **b** $B \leftrightarrow$ **c** $C \leftrightarrow$

d $D \leftrightarrow$ **e** $E \leftrightarrow$ **f** $AB \leftrightarrow$

g $AC \leftrightarrow$ **h** $BD \leftrightarrow$ **i** $ED \leftrightarrow$

j $\angle ABC \leftrightarrow$ **k** $\angle EAB \leftrightarrow$ **l** $\angle DAB \leftrightarrow$

3 Discussion questions

a A page of writing is reflected in a mirror. Is the image congruent to the real page?

b Think of two large jacaranda trees that you have seen. Are they congruent?

c Is a model of the Sydney Harbour Bridge congruent to the real bridge?

d Two identical twins stand at attention in army uniform. Are they congruent?

e Think of two cumulus clouds of about the same size. Are they congruent?

f Are two copies of the same photograph congruent?

g What lower-case letter, apart from b itself, is congruent to b?

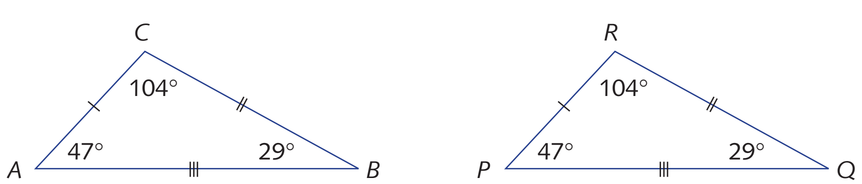
h What digit, apart from 9 itself, is congruent to 9?

12B Congruent triangles

Most geometrical reasoning about congruence that we are going to do involves only congruent triangles.

Translations

Here are two congruent triangles. The sides AB and PQ are on the same line.



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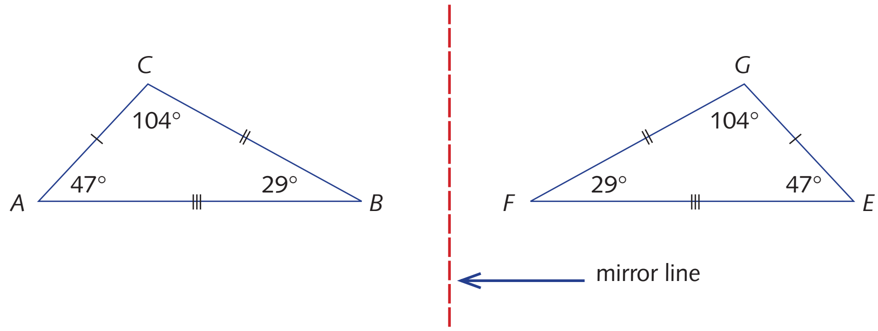
The translation that takes P to A also takes Q to B and R to C. Thus the two triangles are congruent because ΔABC is the image of ΔPQR under a translation.

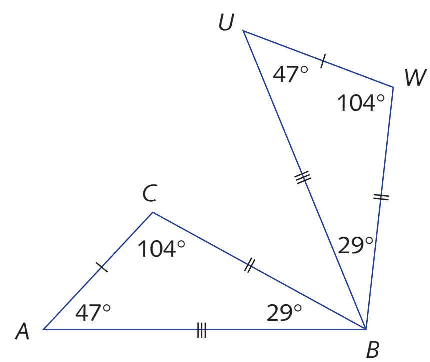
Reflections



The triangle FGE is the reflection of the triangle BCA. Thus the two triangles are congruent.

Rotations

Triangle UBW is a rotation of triangle ABC about B. Thus the two triangles are congruent.



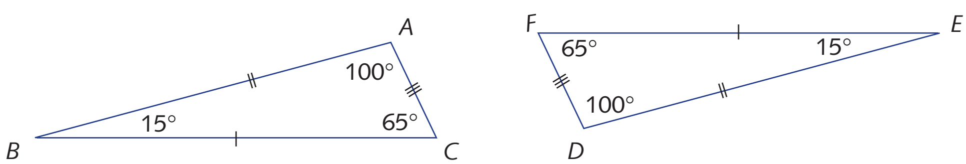
When are two triangles congruent?

We can see from the examples above that when two triangles are congruent, paired sides have the same length and paired angles have the same size.

The rest of this chapter will develop four tests for two triangles to be congruent. At this stage, however, we simply note that:

> If the vertices of two triangles can be paired up so that paired sides have the same length and paired angles have the same size, then they are congruent.

To demonstrate this, cut out one triangle and move it so that it lies exactly on top of the other triangle. This process involves only translations and rotations, and a reflection if you have to turn the triangle over.



Thus the two triangles above are congruent. You should be able to image how you would transform one triangle so that it lies on top of the other.

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Writing a congruent statement with vertices in matching order

When writing symbolic congruence statements, we use the symbol ≡ for ‘is congruent to’.

For example, the triangles in the diagram on the previous page are congruent. This is written as:

$\Delta ABC \equiv \Delta DEF$ (Read this as ‘triangle ABC is congruent to triangle DEF.’)

It is vitally important *to write the vertices of the two triangles in matching order*. In the statement above, we wrote the two triangles as ΔABC and ΔDEF because the paired vertices are:

$A \leftrightarrow D$ and $B \leftrightarrow E$ and $A \leftrightarrow F$

This attention to detail is useful when it comes to paired sides. We can read off the paired sides from the congruence statement $\Delta ABC \equiv \Delta DEF$ in the natural way:

$AB \leftrightarrow DE$ and $BC \leftrightarrow EF$ and $CA \leftrightarrow FD$

Congruent triangles

- If the vertices of two triangles can be paired up so that paired angles have equal size and paired sides have equal length, then they are congruent.
- When writing a **congruence statement**, always write the vertices of the two congruent triangles in matching order. We can then read off paired angles and paired sides so that the pairings can be read off in the natural way. For example, the statement $\Delta ABC \equiv \Delta DEF$ means that:

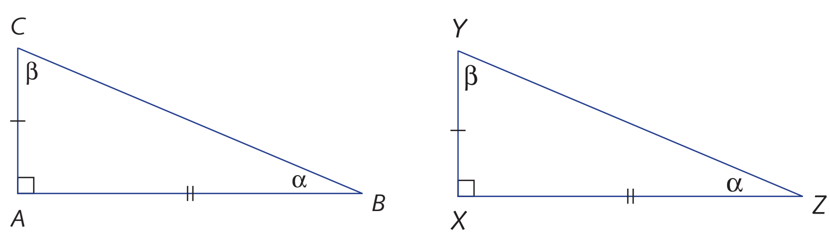
  $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$,

  $AB = DE$, $BC = EF$, $CA = FD$.

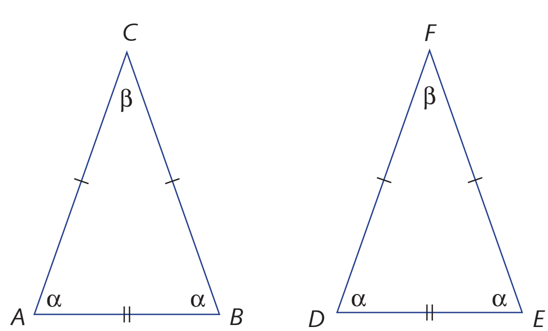
Exercise 12B

1 Write a congruence statement for each pair of congruent triangles.

a



b



ΔABC is isosceles with $CA = CB$.

ΔDEF is isosceles with $FD = FE$

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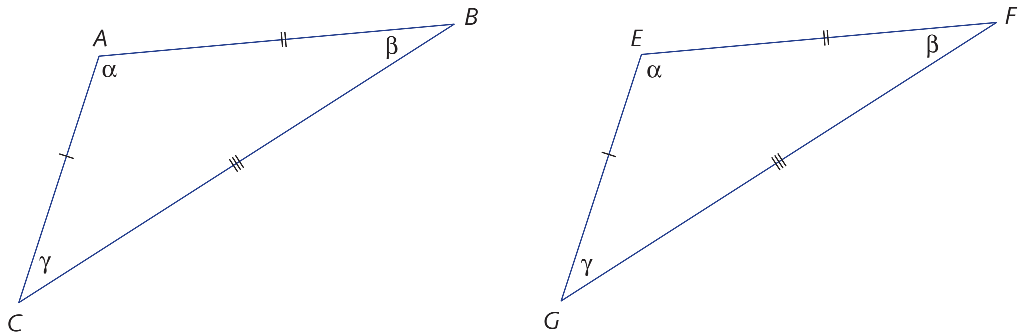
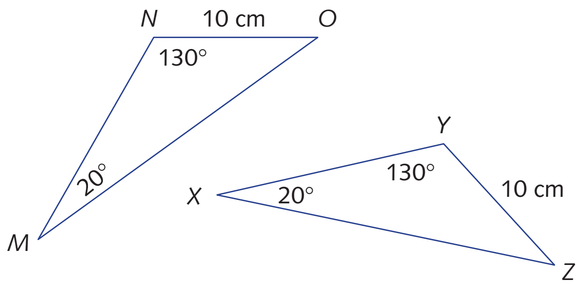
c



d

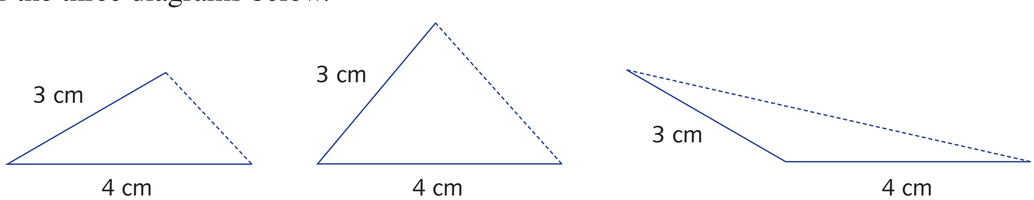


12C Congruent triangles: The SSS and AAS tests

The SSS congruence test

To prove that two triangles are congruent, it is not necessary to prove that they have three pairs of equal sides and three pairs of equal angles. In this section and the next, we will investigate the minimum amount of information needed to establish that two triangles are congruent.

Consider the three diagrams below.



The three triangles shown are clearly not congruent because the 3 cm and 4 cm can flap about. This shows that just knowing that two pairs of sides are equal is not enough information to establish congruence. Knowing that *three* pairs of sides are equal, however, is enough to establish congruence, as the discussion below will demonstrate.

This test for congruence is called the **SSS congruence test**. The initials **SSS** stand for **S**ide, **S**ide, **S**ide.

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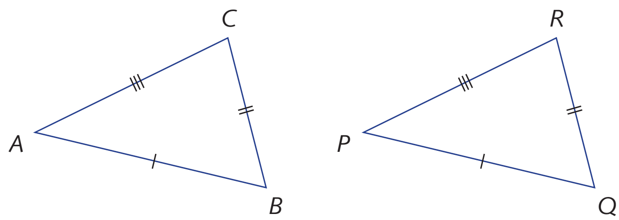
SSS congruence test

If the three sides of one triangle are equal to the three sides of another, then the two triangles are congruent.

For example, in the diagram below, $\Delta ABC \equiv \Delta PQR$ (SSS).



Thus the SSS congruence test tells us that the angles of any triangle are determined by the three sides.

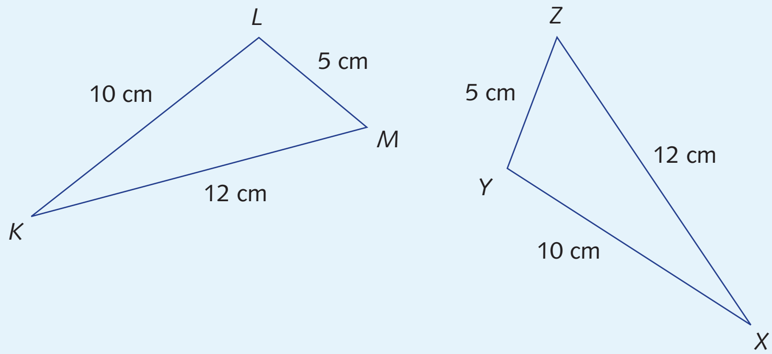
We can now conclude from the congruence that:

$\angle A = \angle P$
$\angle C = \angle R$
$\angle B = \angle Q$

You should always write the initials of the test used to demonstrate the congruence after the congruence statement.

Example 1

Write down a statement that the two triangles below are congruent, giving the appropriate congruence test as a reason.



Solution

$\Delta KLM \equiv \Delta XYZ$ (SSS)

We review the construction of triangles with given side lengths.

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Constructing a triangle given three sides

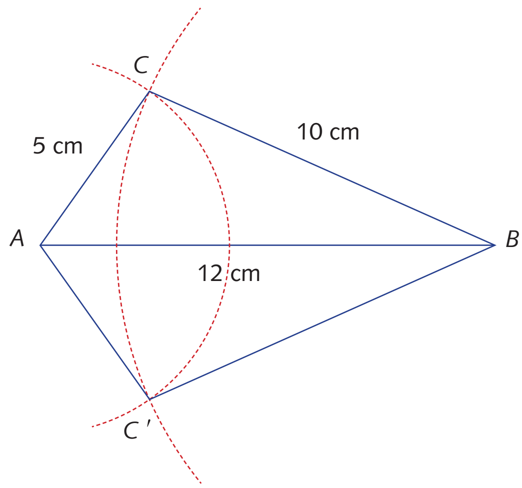
We will construct a triangle with three given side lengths. Here is an example using the side lengths 12 cm, 10 cm and 5 cm.

Step 1: Draw an interval AB of length 12 cm.

Step 2: Draw a circle with centre A and radius 5 cm.

Step 3: Draw a circle with centre B and radius 10 cm crossing the first circle at C and C'.

Step 4: Join up the triangle ABC and the triangle ABC'.



The two triangles ABC and ABC' are reflections of each other in AB and so are congruent.

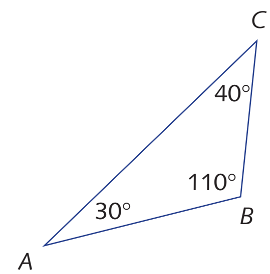
Note: The three angles of any triangle are determined by the lengths of the three sides.

The AAS congruence test

We first consider two important ideas about triangles.

Sides opposite angles

Here is a triangle with the angles marked.



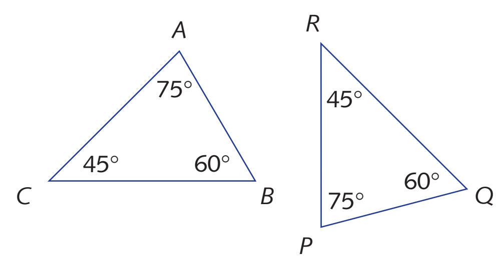
We say that the side AB is **opposite** the angle 40° at C.

The side BC is opposite the angle 30° at A.

The side AC is opposite the angle 110° at B.

Matching sides and angles

Here are two triangles with the same three angles in both. We see that A matches P, B matches Q and C matches R.



We also say that AB and PQ are matching sides because:

- AB is opposite the angle 45° in ΔABC, and
- PQ is opposite the angle 45° in ΔPQR.

Similarly, BC and QR are matching sides because they are both opposite 75°.

Also CA and RP are matching sides because they are both opposite 60°.

Matching sides

When two triangles each have the same three angles, we say that a side of one triangle **matches** a side in the second if the two sides are opposite the same angle.

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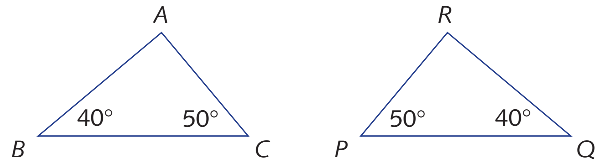
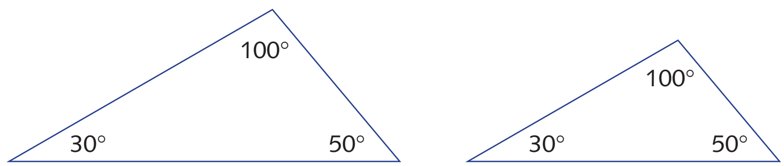
Notice that if two angles of one triangle are equal to two angles of another, then the third pair of angles are also equal. This is because the angle sum of a triangle is 180°. For example, the third angle in each triangle below is 90°.



Thus BC and PQ are matching sides in this situation also. (They are matching because they are opposite equal angles.)

Now consider the diagrams below.



The two triangles above both have angles of 30°, 50° and 100°, but they are not congruent, because they have different sizes. This shows that just knowing that three pairs of angles are equal is not enough to establish congruence.

If, however, we know also that a pair of matching sides are equal, then the two triangles are congruent. This test for congruence is called the **AAS congruence test**. The initials **AAS** stand for **A**ngle, **A**ngle, **S**ide.

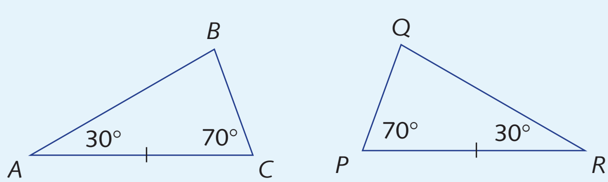
AAS congruence test

If two angles and one side of one triangle are equal to two angles and the matching side of another triangle, then the two triangles are congruent.

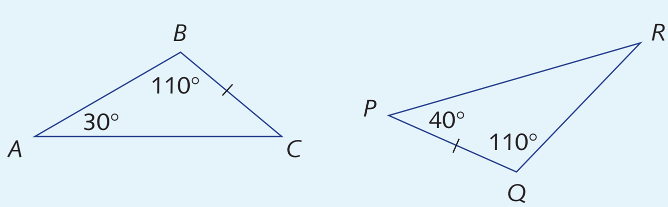
Example 2

In each part, write down a statement that the two triangles are congruent, giving the appropriate congruence test as a reason.

a



b



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Solution

a In the triangles ABC and PQR:

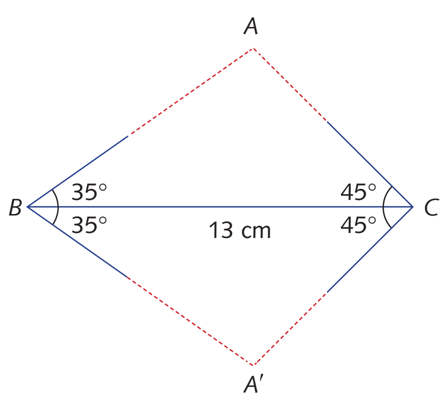
$$\begin{aligned} \angle A &= \angle R = 30^\circ \\ \angle C &= \angle P = 70^\circ \\ AC &= PR \text{ (matching side)} \\ \text{so } \Delta ABC &\equiv \Delta RQP \text{ (AAS)} \end{aligned}$$

(Note that the second triangle is named in matching order.
A matches R, B matches Q and C matches P.)

b In the triangles ABC and PQR:

$$\begin{aligned} \angle B &= \angle Q = 110^\circ \text{ (given)} \\ \angle C &= 40^\circ \text{ (angle sum of } \Delta ABC) \\ \text{so } \angle C &= \angle P \\ BC &= PQ \text{ (given)} \\ \text{so } \Delta ABC &\equiv \Delta RQP \text{ (AAS)} \end{aligned}$$

To see why the AAS test is valid, we look at a particular case of a triangle ABC with base $BC = 13$ cm and base angles equal to $B = 35^\circ$ and $C = 45^\circ$. How many such triangles are there? Are they all congruent to one another? We can construct two such triangles by drawing the rays shown and extending them to meet at the points A and A'.



Note: $\Delta A'BC$ is the reflection of ΔABC in the line BC because reflection preserves angles. So these two triangles are congruent.

Using congruence to find lengths and angles

Once the congruence of two triangles has been established, we can draw conclusions about the remaining pairs of matching angles or matching sides. This allows us to draw conclusions about lengths and angles.

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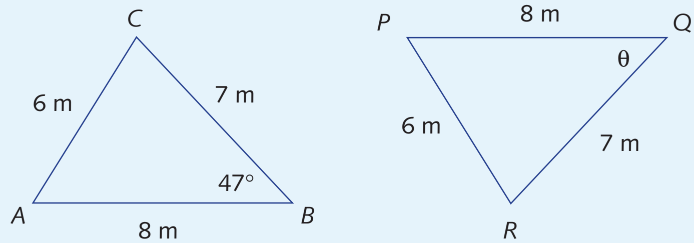
Example 3

Use congruence to find the value of θ.



Solution

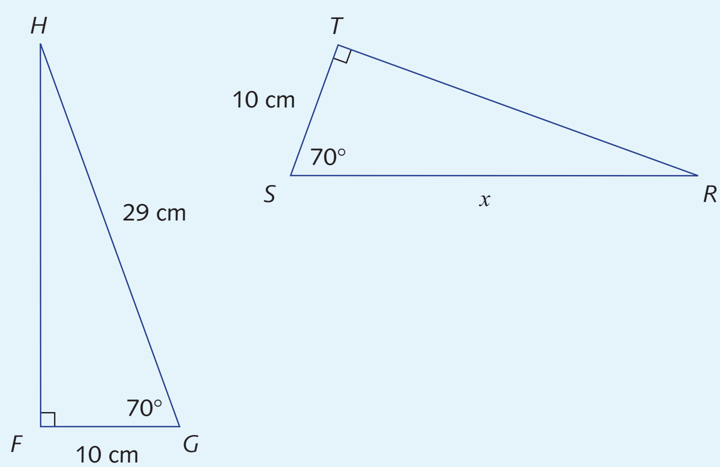
From the diagram, $\Delta ABC \equiv \Delta PQR$ (SSS)

so $\theta = \angle B$ (matching angles of congruent triangles)

$= 47°$

Example 4

Use congruence to find the value of x.



Solution

From the diagram, $\Delta FGH \equiv \Delta TSR$ (AAS)

so $x = HG$ (matching sides of congruent triangles)

$= 29$ cm

You may have noticed that the left-hand diagrams in Examples 3 and 4 have too much information. In both triangles, at least one measurement must be only approximate. All of this can safely be ignored at this stage.

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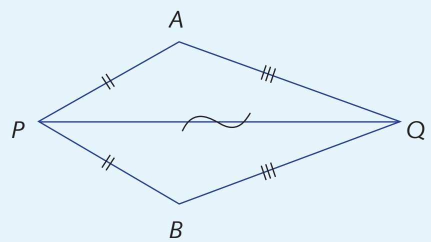
Using congruence to show that two lengths or two angles are equal

Congruence can be used to show that two matching lengths or two matching angles are equal.

Example 5

Use congruence to show that $\angle PAQ = \angle PBQ$.



Solution

From the diagram, $\Delta PAQ \equiv \Delta PBQ$ (SSS)

so $\Delta PAQ = \Delta PBQ$ (matching angles of congruent triangles)

Note: The side PQ is called **common** to both triangles, and is marked on the diagram with the wavy symbol ~. In the above example it provides the third pair of equal sides.

Exercise 12C

Question **1** is for class discussion.

1 **a** Three metal rods, of lengths 2 m, 3 m and 4 m, are bolted together with hinges at the corners. What congruence test explains why the structure is rigid?

b A rope is attached to the top of a vertical pole. The rope is then tied down to the ground so that it makes an angle of 20° to the vertical. Another identical pole nearby is secured in the same way. Do the two ropes necessarily have the same length? Why or why not?

2 **a** Using a ruler and compasses only, construct a triangle with side lengths of 6 cm, 8 cm and 5 cm.

b **i** What happens when you try to construct a triangle with side lengths of 4 cm, 5 cm and 12 cm?

ii Copy and complete: 'The longest side of a triangle …'

c Jack says that in the Great Outback there are three towns A, B and C. Town B is 34 km from A and 68 km from C. Town A is 110 km from C. (All distances are as the crow flies.) Comment on Jack's claim.

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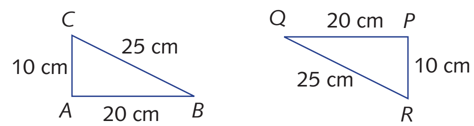
3 **a** Using a ruler and compasses only, construct a triangle ABC in which $AB = 12$ cm, $\angle A = 45°$ and $\angle B = 60°$. Measure the length of AC. Which congruence test tells you that your neighbour gets the same length?

b Ajun claims to have constructed a triangle in which one side is 10 cm and the angles at each end of this side are 105° and 80°. Comment on Ajun's claim.

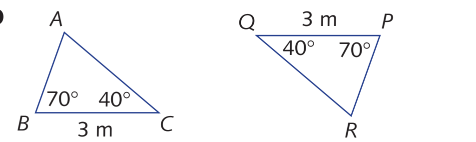
Example 1, 2

4 In each part below, say whether the two triangles are congruent. If they are congruent, write a congruence statement, including the appropriate congruence test.

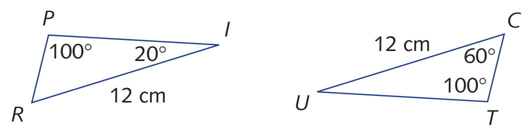
a



b



c



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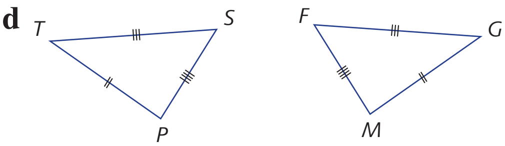

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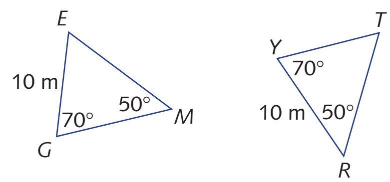

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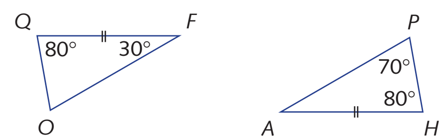
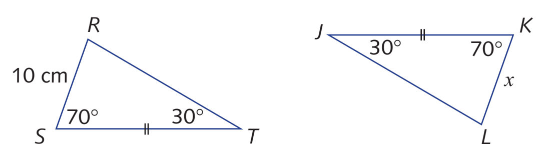


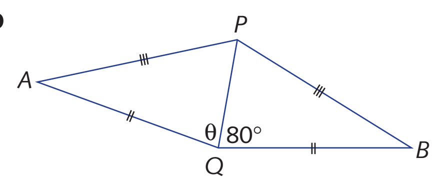
Example 3, 4

5 If possible, write down a congruence statement (with the appropriate congruence test) in each part. Then find the value of x or θ, giving reasons. Keep the vertices written in matching order.

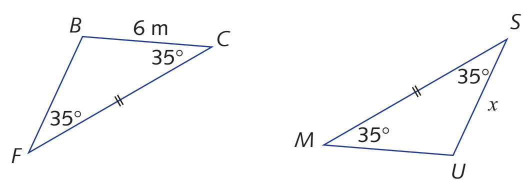
a



b



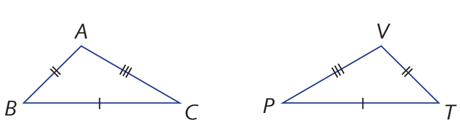
c



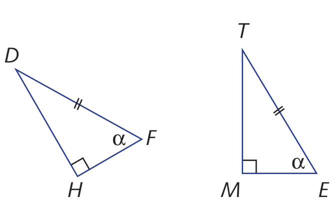
Example 5

6 In each part, first write a congruence statement. Then prove the required result, giving all reasons.

a Prove that $\angle ACB = \angle VPT$.



b Prove that $FH = EM$.



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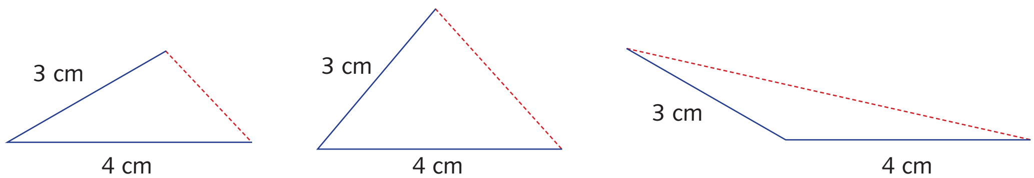
12D Congruent triangles: The SAS and RHS tests

We saw in the preceding section that having two pairs of equal sides is not enough to establish that two triangles are congruent, because the two sides can flap about freely, like the blades of a pair of scissors.



The two tests discussed in this section stop the two sides flapping about by specifying one angle of the triangle as well.

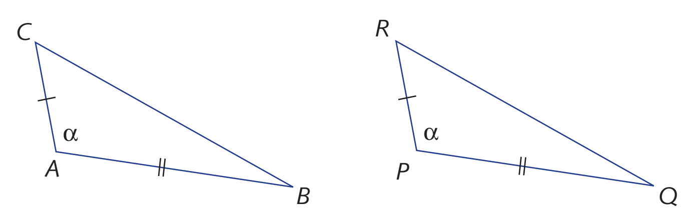
The SAS congruence test

The easiest way to stop the two sides flapping is to specify the angle between them. This angle is called the **included angle**. ('Included' means 'shut in'.) The resulting test is called the **SAS congruence test**. The letters SAS stand for **S**ide, **A**ngle, **S**ide.

SAS congruence test

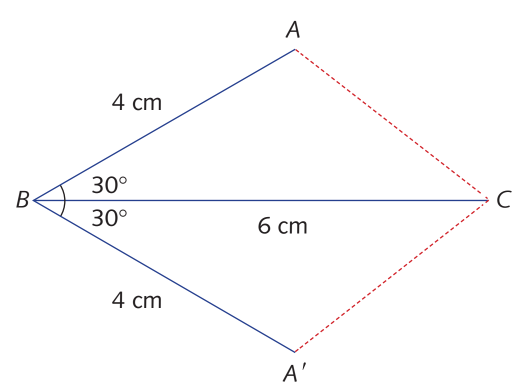
If two sides of one triangle are respectively equal to two sides of another triangle, and the angles included between the sides in each pair are equal, then the triangles are congruent.

Be careful! For the test to work, the angles *must* be the ones included between the sides in each pair.



In the diagram above, $\Delta ABC \equiv \Delta PQR$ (SAS).

To see why this is true, take a triangle ABC with side lengths $AB = 4$ cm, $BC = 6$ cm and included angle, $\angle B = 30°$. Its reflection in the line BC is also shown. Triangle ABC is congruent to triangle $A'BC$ because reflection in BC moves A to A'.



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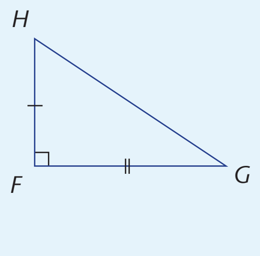
Check for yourself that any other triangle in the plane with the given side lengths and included angle is one of the triangles we have drawn with BC in a different position and orientation.

Example 6

Write down a statement that the two triangles below are congruent, giving the appropriate congruence test as a reason.



Solution

$\Delta HFG \equiv \Delta RTS$ (SAS)

The RHS congruence test

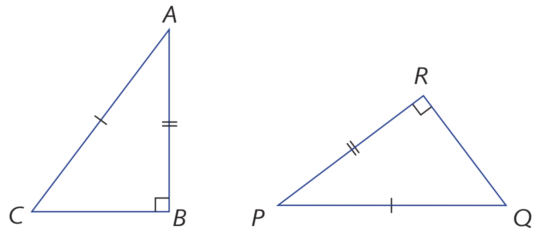
A useful test that can be applied to right-angled triangles, and is easy to apply, is the **RHS congruence test**. (The letters RHS stand for **R**ight angle, **H**ypotenuse, **S**ide.)

RHS congruence test

If two right-angled triangles have equal hypotenuses, and another pair of equal sides, then the triangles are congruent.

Proof of the RHS congruence test

$$
\begin{aligned}
BC^2 &= AC^2 - AB^2 \text{ (Pythagoras' theorem in } \Delta ABC\text{)}\\
&= PQ^2 - PR^2 \text{ (equal hypotenuse and equal side)}\\
&= QR^2 \text{ (right-angled } \Delta PQR\text{)}
\end{aligned}
$$



So $BC = QR$ and the two triangles have equal sides. Hence, they are congruent by the SSS congruence test.

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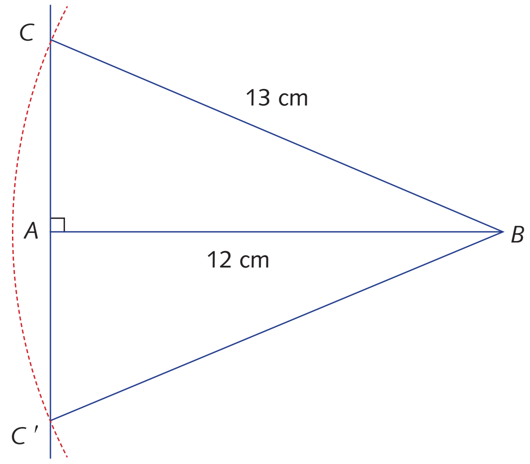
Constructing a right-angled triangle with a given hypotenuse and side

Construct a right-angled triangle ABC with $\angle A = 90°$, $BC = 13$ cm and $AB = 12$ cm.



The side BC is the hypotenuse of the right-angled triangle. By Pythagoras' theorem, it must be the longest side.

Step 1: In the middle of your page, draw an interval AB of length 12 cm.

Step 2: Draw a line through A at right angles to AB.

Step 3: With centre B and radius 13 cm, draw a circle crossing the vertical line at two points C and C'.

Step 4: Join up the triangle ABC and the triangle ABC'.

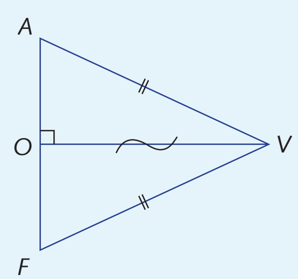
The two triangles you have drawn are congruent, being reflections of each other in AB.

Example 7

Write down a statement that the two triangles below are congruent, giving the appropriate congruence test as a reason.

Solution

$\Delta AOV \equiv \Delta FOV$ (RHS)



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The four standard congruence tests for triangles

Two triangles are **congruent** if:

SSS: the three sides of one triangle are respectively equal to the three sides of the other triangle, **or**

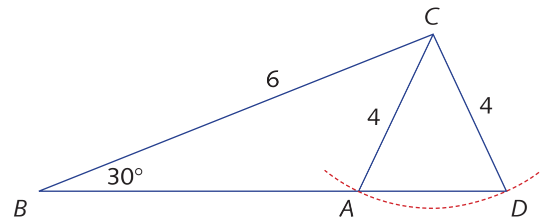
AAS: two angles and one side of one triangle are respectively equal to two angles and the matching side of the other triangle, **or**

SAS: two sides and the included angle of one triangle are respectively equal to two sides and the included angle of the other triangle, **or**

RHS: the hypotenuse and one side of one right-angled triangle are respectively equal to the hypotenuse and one side of the other right-angled triangle.

The case of the non-included angle

In the diagram, triangles BCA and BCD have a common side BC, a common angle $\angle B$, and $AC = DC$. As you can see, the triangles are not congruent.



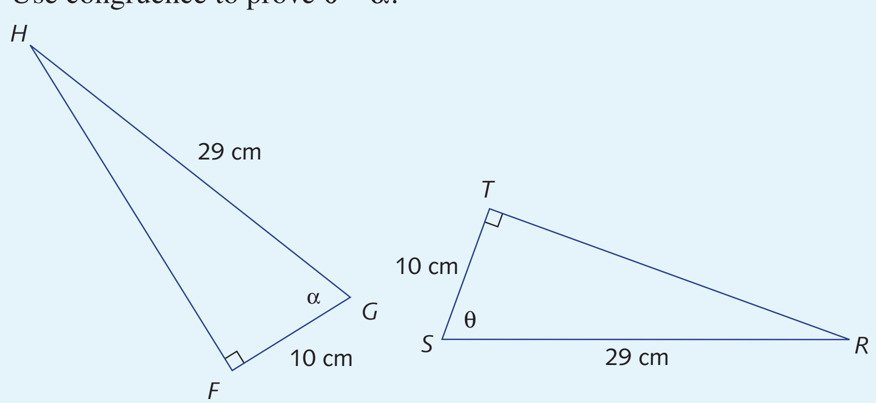
Therefore two sides and a non-included angle are not sufficient to determine congruency. Further discussion of the case of the non-included angle is to be found in Question **6** of Exercise 12D and Question **8** of the Challenge exercise.

Using congruence to find sides or angles

When two triangles are congruent, and we are given side lengths and/or angles of one of them, we can find the matching sides and/or angles of the other.

Example 8

Use congruence to prove $\theta = \alpha$.



Solution

From the diagram, $\Delta FGH \equiv \Delta TSR$ (RHS)

so $\theta = \alpha$ (matching angles of congruent triangles)

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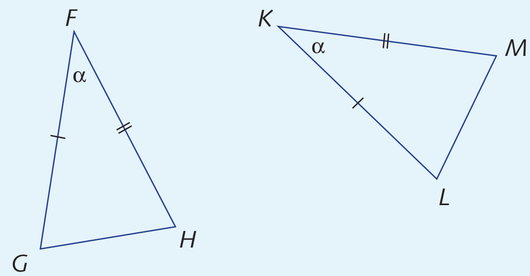
Example 9

Use congruence to show that $GH = LM$.



Solution

From the diagram, $\Delta GFH \equiv \Delta LKM$ (SAS)

so $GH = LM$ (matching sides of congruent triangles)

Exercise 12D

Question **1** is suitable for discussion.

1 **a** Two pairs of compasses, each with legs 12 cm long, are opened so that the arms make an angle of 25°. What congruence test explains why the distance between the point and the pencil lead is the same in both compasses?

b Two 6-metre ladders on horizontal ground rest against a wall, with each ladder reaching 5 metres up the wall. What congruence test explains why the angle between the ladder and the ground is the same for both ladders?

c Pradap attached a rope 9 metres up a vertical pole, then stretched the rope and secured it to the ground near the pole. He measured the length of the rope to be 8.5 metres. Comment on this situation.

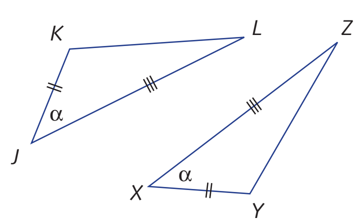
2 **a** Using a ruler and compasses only, construct an isosceles triangle with legs that are 8 cm each and with an apex angle that is 45°. Measure the length of the base.

b Using ruler and compasses only, construct a right-angled triangle ABC in which $\angle A$ is a right angle, $AB = 6$ cm and the hypotenuse $BC = 8$ cm. Use your protractor to measure the size of $\angle B$.

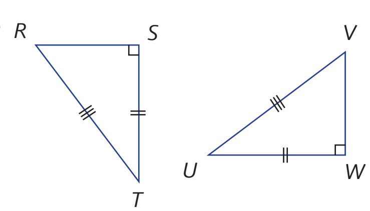
Example 6, 7

3 In each part below, if the two triangles are congruent, write down the congruence statement, giving the appropriate congruence test as a reason.

a



b



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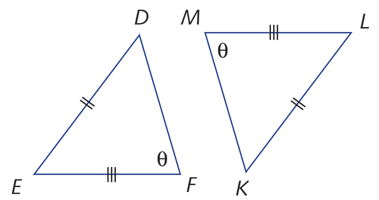
c



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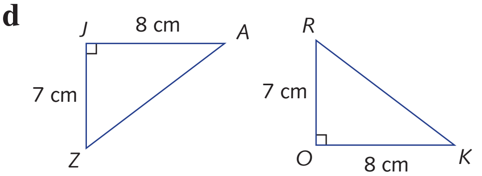
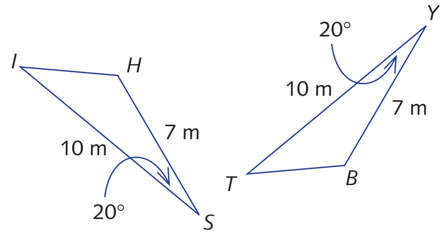


e



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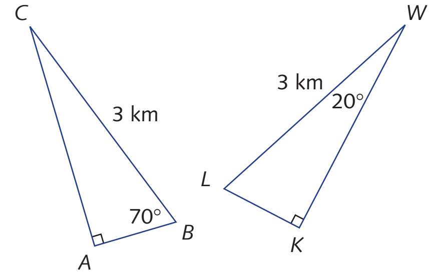
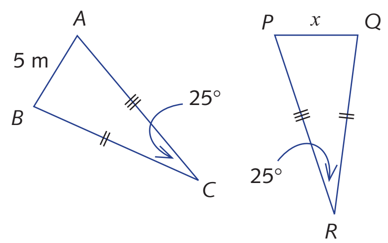


4 If possible, write down a congruence statement (with the appropriate congruence test). Then find the value of x or θ, giving the reason.

a



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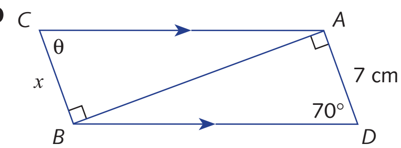

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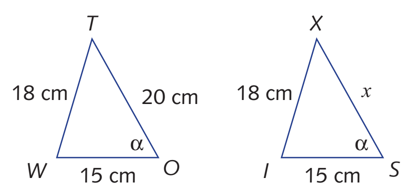
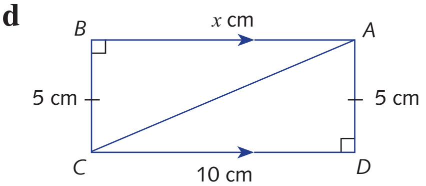


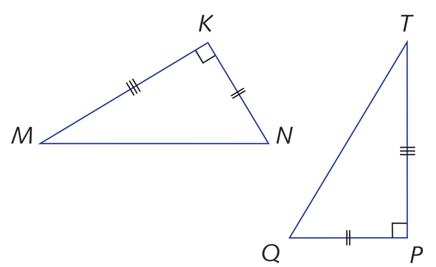
d



Example 8, 9

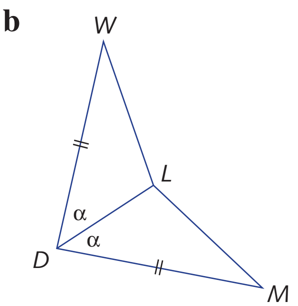
5 In each part, first write a congruence statement. Then prove the required result, giving all reasons.

a



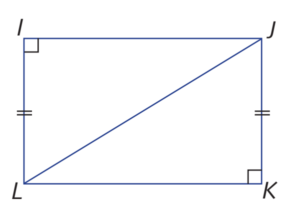
Prove that $\angle M = \angle T$.

b



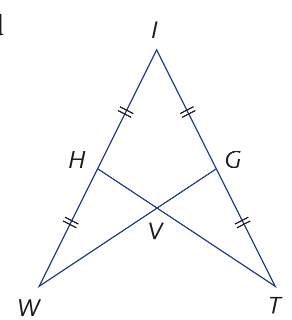
Prove that $WL = ML$.

c



Prove that $IJ = KL$.

d



Prove that $HT = GW$.

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6 The SAS congruence test requires that the angle be the angle included between the two sides. Here is an example where the angle is not included – the construction will produce two triangles that are not congruent. Construct a triangle ABC with $AC = 8$ cm, $BC = 6$ cm and $\angle A = 30°$ as follows.

Step 1: Draw a long horizontal line AX near the bottom of a new page in your exercise book.

Step 2: Construct an angle of 30° at A.

Step 3: Mark C on the sloping arm so that $AC = 8$ cm.

Step 4: With centre C and radius 6 cm, draw an arc cutting the horizontal line at B and B'.

Step 5: Join CB and CB'.

Then ΔABC and $\Delta AB'C$ both fulfil the conditions, but they are clearly not congruent.

12E Using congruence in geometrical problems

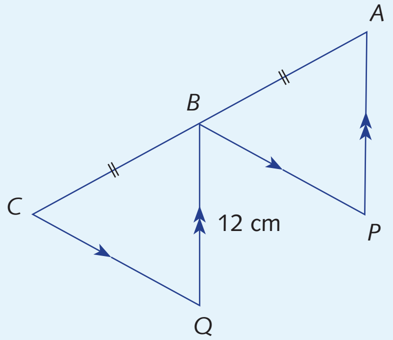
This section shows how to use congruence in geometrical reasoning and problems.

Setting out congruence proofs

Here are three examples showing a recommended way of writing out a solution to a problem involving congruence. Attention to detail is critical.

Example 10

Find the length of AP in the diagram below.



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Solution

In the triangles ABP and BCQ:

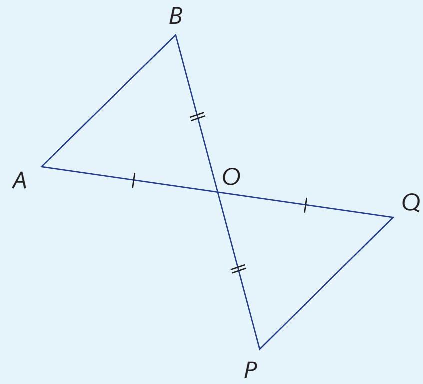
$AB = BC$ (given)
$\angle BAP = \angle CBQ$ (corresponding angles, $AP \parallel BQ$)
$\angle ABP = \angle BCQ$ (corresponding angles, $BP \parallel CQ$)
so $\Delta ABP \equiv \Delta BCQ$ (AAS)
Hence, $AP = 12$ cm (matching sides of congruent triangles)

Proving that two lines are parallel

Congruence is often used to prove that two alternating angles or two corresponding angles are equal. It then follows that the associated lines are parallel.

Example 11

Prove that $AB \parallel PQ$ in the diagram below.



Solution

In the triangles AOB and QOP:

$AO = QO$ (given)
$BO = PO$ (given)
$\angle AOB = \angle QOP$ (vertically opposite at O)
so $\Delta AOB \equiv \Delta QOP$ (SAS)
Hence $\angle A = \angle Q$ (matching angles of congruent triangles)
so $AB \parallel PQ$ (alternate angles are equal)

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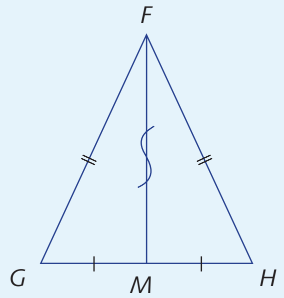
Proving that two lines are perpendicular

If two angles are equal and add to 180°, they must both be right angles. This idea is often used in problems to prove that two lines are perpendicular.

Example 12

In the diagram opposite, FM joins the apex F of the isosceles ΔFGH to the midpoint M of its base. Use congruence to prove that $FM \perp GH$; that is, FM is perpendicular to GH.



Solution

In the triangles GFM and HFM:

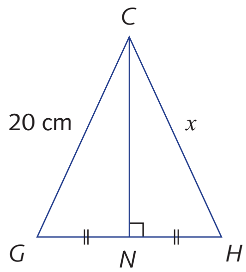
$$\begin{aligned}
FM &= FM \text{ (common)}\\
FG &= FH \text{ (given)}\\
GM &= HM \text{ (given)}\\
\text{so } \Delta GFM &\equiv \Delta HFM \text{ (SSS)}\\
\text{Hence } \angle GMF &= \angle HMF \text{ (matching angles of congruent triangles)}\\
\text{But } \angle GMF + \angle HMF &= 180^\circ \text{ (straight angle at } M\text{)}\\
\text{so } \angle GMF &= \angle HMF\\
&= 90^\circ
\end{aligned}$$

Exercise 12E

Example 10

1 In each part below, prove that two triangles are congruent. Hence, find the value of x or θ. The point O is always the centre of the circle.

a



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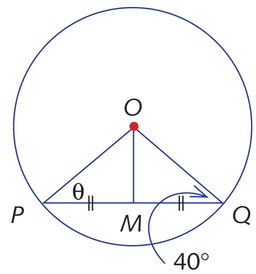

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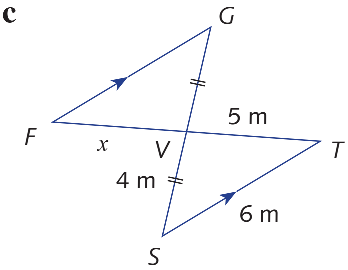

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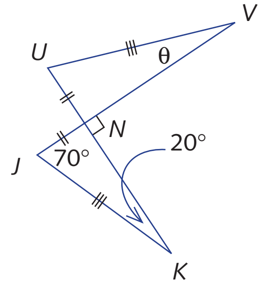

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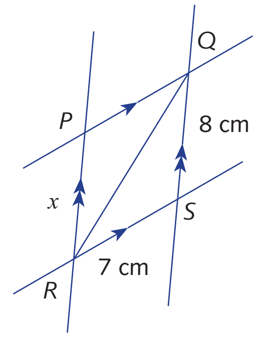

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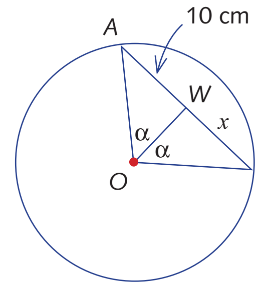
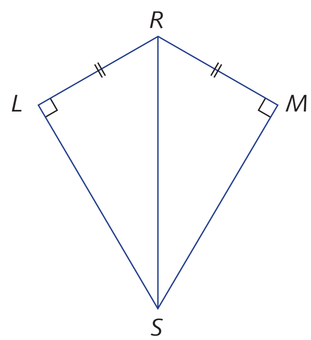


2 In these problems, the points O and Z are always the centres of circles.

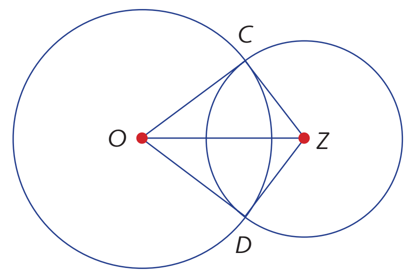
a



i Prove that $\Delta RLS \equiv \Delta RMS$.

ii Hence, show that $LS = MS$.

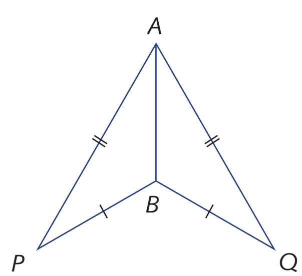
b



i Prove that $\Delta COZ \equiv \Delta DOZ$.

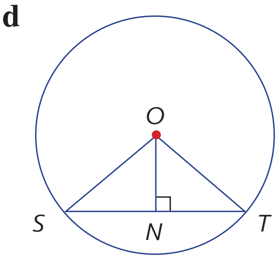
ii Hence, show that $\angle OCZ = \angle ODZ$.

c



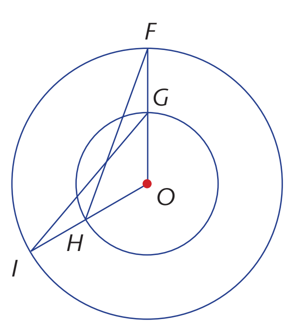
Prove that $\angle P = \angle Q$.

d



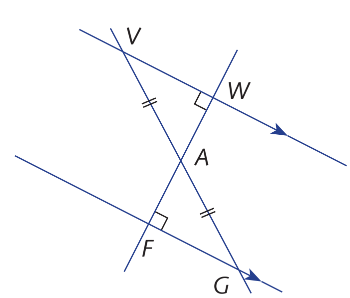
Prove that $SN = TN$.

e



Prove that $\angle OFH = \angle OIG$.

f



Prove that $VW = FG$.

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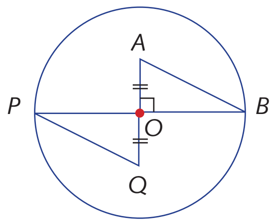
Example 11

3 In each part below, use congruent triangles to prove that the lines AB and PQ are parallel. The point O is always the centre of the circle.

a



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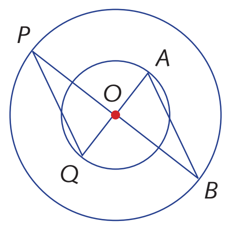

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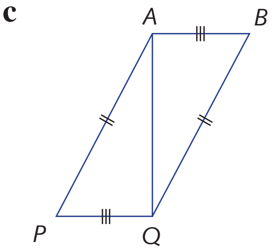
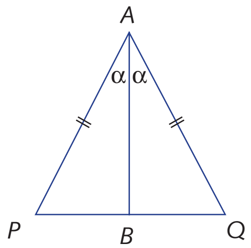


Example 12

4 In each part below, use congruent triangles to prove that the lines AB and PQ are perpendicular. The point O is always the centre of the circle.

a



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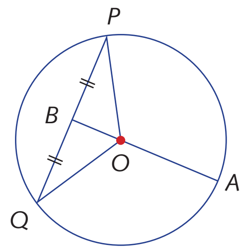

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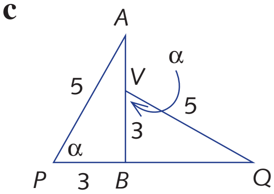
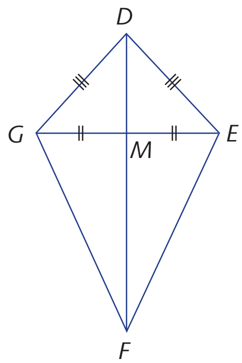


5 Each of these problems requires two applications of congruence. The points O and Z are the centres of circles.

a



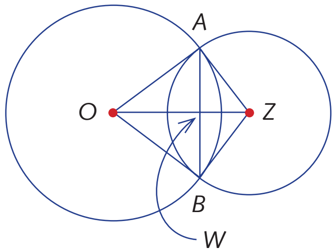
i Prove that $\Delta GDM \equiv \Delta EDM$.

ii Hence, show that $DM \perp GE$.

iii Prove that $\Delta GMF \equiv \Delta EMF$.

iv Hence, show that $GF = EF$. $DEFG$ is called a kite.

b



i Prove that $\Delta OAZ \equiv \Delta OBZ$.

ii Hence, show that $\angle AOW = \angle BOW$.

iii Prove that $\Delta AOW \equiv \Delta BOW$.

iv Hence, show that OZ is the perpendicular bisector of AB.

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12F Congruence and special triangles

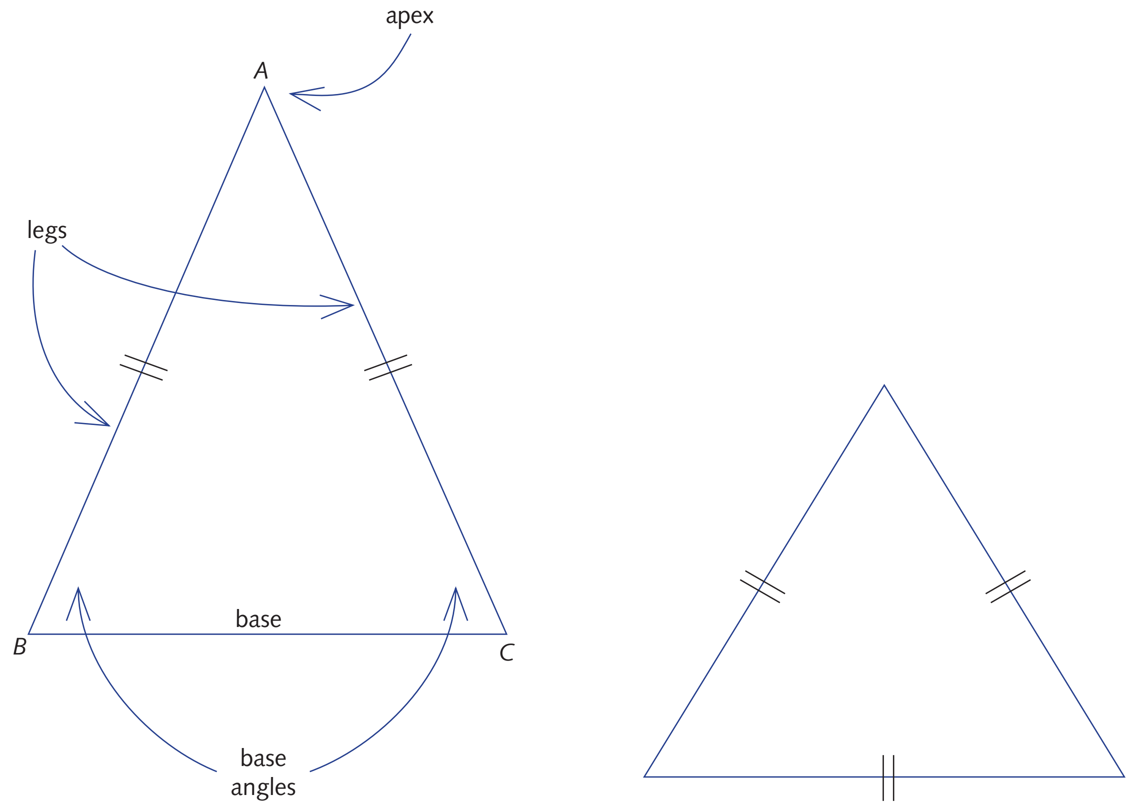
Isosceles and equilateral triangles were introduced in Chapter 13 of *ICE-EM Mathematics Year 7*. Congruence allows us to give proper proofs of three basic theorems about these special triangles.

First, here is a reminder of the definitions of isosceles and equilateral triangles.

Isosceles and equilateral triangles

- An **isosceles triangle** is a triangle with two (or more) sides equal.
  - The equal sides are called the **legs** and the third side is called the **base**.
  - The legs meet at the **apex** and the other two angles are the **base angles**.
- An **equilateral triangle** is a triangle with all three sides equal.

The word 'isosceles' comes from Greek and means 'equal legs'. The word 'equilateral' comes from Latin and means 'equal sides'.



Isosceles triangle

Equilateral triangle

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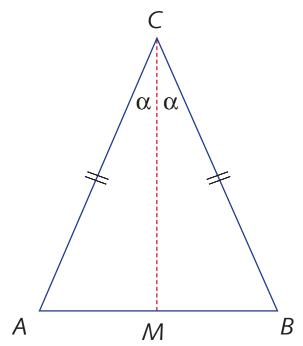
The base angles of an isosceles triangle are equal

Notice that the proof is based on congruent triangles.



Theorem: The base angles of an isosceles triangle are equal.

Proof: Let ΔABC be isosceles, with $CA = CB$.

We need to prove that $\angle A = \angle B$.

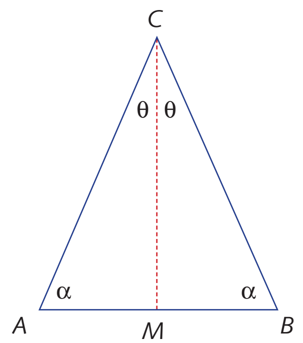
Draw the bisector of $\angle ACB$, and let it meet AB at M.

In the triangles ACM and BCM:

$CA = CB$ (given)
$CM = CM$ (common side)
$\angle ACM = \angle BCM$ (construction)
so $\Delta ACM \equiv \Delta BCM$ (SAS)
Hence $\angle A = \angle B$ (matching angles of congruent triangles)

If two angles of a triangle are equal, then the sides opposite those angles are equal

This proof is also based on congruence.



Theorem: If two angles of a triangle are equal, then the sides opposite those angles are equal. (That is, the triangle is isosceles.)

Proof: Let ΔABC be a triangle with $\angle A = \angle B$.

We need to prove that $CA = CB$.

Draw the bisector of $\angle ACB$, and let it meet AB at M.

In the triangles AMC and BMC:

$\angle CAM = \angle CBM$ (given)
$\angle ACM = \angle BCM$ (construction)
$CM = CM$ (common side)
so $\Delta AMC \equiv \Delta BMC$ (AAS)
Hence $CA = CB$ (matching sides of congruent triangles)

That is, the triangle is isosceles.

This second theorem is the **converse** of the first theorem.

- The first theorem says: 'If two sides of a triangle are equal, then the angles opposite those sides are equal'.
- The second theorem says: 'If two angles of a triangle are equal, then the sides opposite those angles are equal'.

We have now proved that both the theorem and its converse are true.

Isosceles triangles

- The base angles of an isosceles triangle are equal.
- Conversely, if two angles of a triangle are equal, then the sides opposite those angles are equal. (That is, the triangle is isosceles.)

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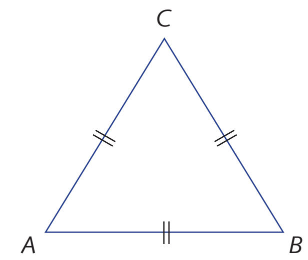
The angles of an equilateral triangle are all 60°

We can prove this result, by applying the earlier result that the base angles of an isosceles triangle are equal. This result is a simple corollary of the earlier result that the base angles of an isosceles triangle are equal.



Theorem: The interior angles of an equilateral triangle are all 60°.

Proof:
$AC = AB$ (given)
so $\angle C = \angle B$ (angles opposite equal sides)
Also $CA = CB$ (given)
so $\angle A = \angle B$ (angles opposite equal sides)

Therefore $\angle A = \angle B = \angle C$

Since the angles add to 180°, each angle must be $180 \div 3 = 60°$.

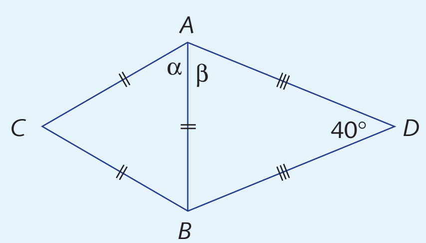
Equilateral triangles

- Each interior angle of an equilateral triangle is 60°.
- Conversely, if all the angles of a triangle are equal, then the triangle is equilateral.

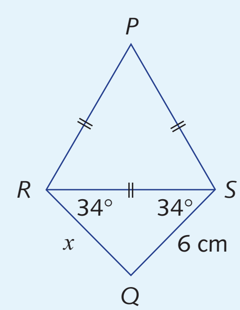
The last dot point is proven in Question **2** of Exercise 12F.

Example 13

a Find α and β in the diagram below.



b Find x and $\angle PSQ$ in the diagram below.



Solution

a First, $\alpha = 60°$ (equilateral ΔABC)
Also, $\angle DBA = \beta$ (base angles of isosceles ΔABD)
so $\beta + \beta + 40° = 180°$ (angle sum of ΔDAB)
Hence $\beta = 70°$

b First, $\angle PSR = 60°$ (equilateral ΔABC)
Also, $\angle PSQ = 94°$ (adjacent angles)
so $RQ = SQ$ (opposite sides of ΔQRS are equal),
Hence $x = 6$ cm

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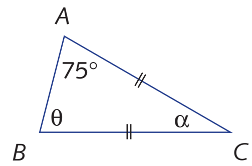
Exercise 12F

Example 13

1 Use the results of this section about isosceles and equilateral triangles to find the values of x, y, α, β, γ and θ. Give all reasons in your solutions. The point O in part **n** is the centre of the circle.

a



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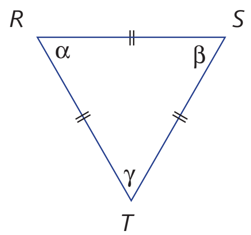

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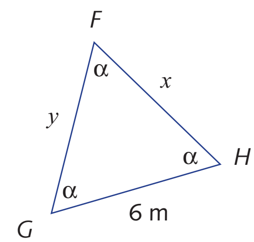

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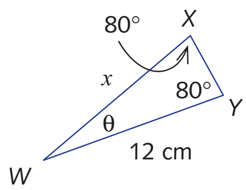

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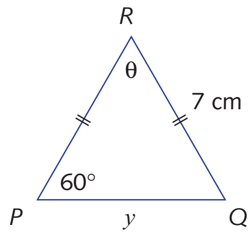

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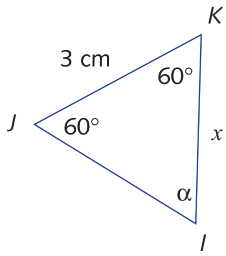

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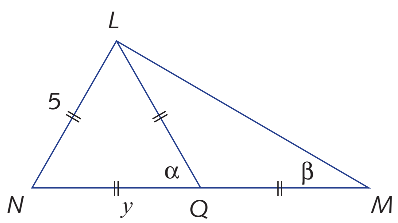

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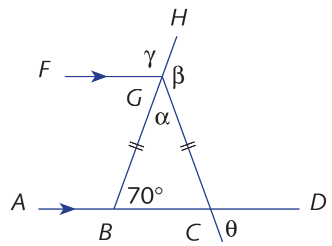

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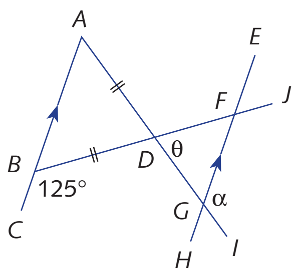

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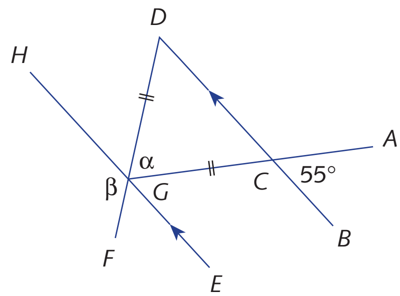

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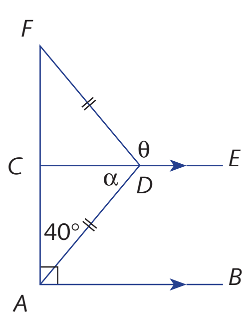

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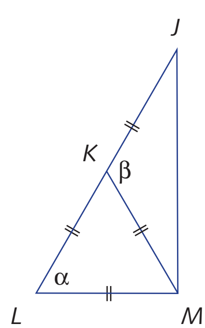

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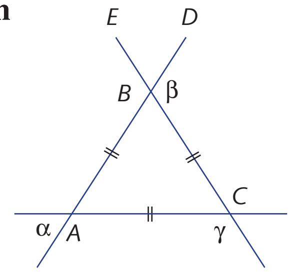
m



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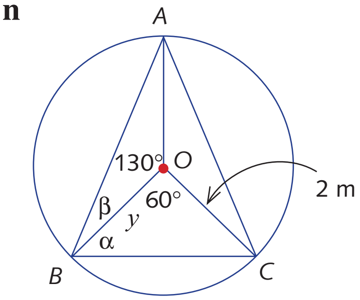
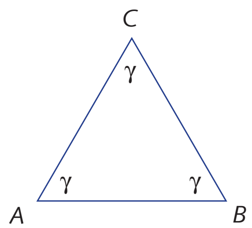


2 In ΔABC opposite, all angles are equal.

a Explain why all the angles are 60°.

b Explain why the triangle is equilateral.

You have now proven a test for an equilateral triangle:
'*If all angles of a triangle are equal, then the triangle is equilateral*'.

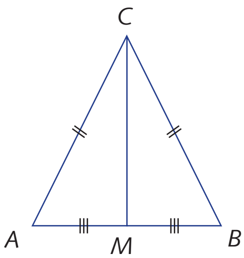


3 Let ΔABC be isosceles, with $AC = BC$, and join CM, where M is the midpoint of the base AB.

a Show that $\Delta AMC \equiv \Delta BMC$.

b Hence, show that $\angle A = \angle B$.

This is a second proof of '*the base angles of an isosceles triangle are equal*'.

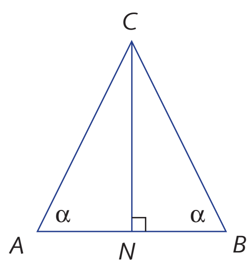


4 Let ABC be a triangle with $\angle A = \angle B$. We will prove that $AC = BC$.

Construct the line through C perpendicular to the base AB.
Let this line meet AB at N.

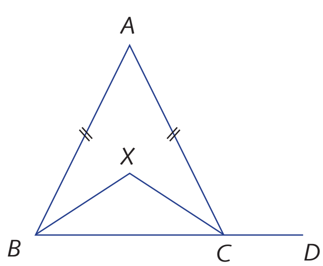
a Show that $\Delta ANC \equiv \Delta BNC$.

b Hence, show that $AC = BC$.



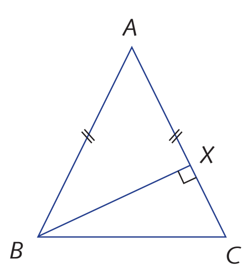
This is another proof of the result '*If the two base angles of a triangle are equal then the sides opposite those angles are equal*'.

5 ΔABC is an isosceles triangle with $AB = AC$.
The angle bisector of $\angle B$ intersects the angle bisector of $\angle C$ at X.
Prove that $\angle BXC = \angle ACD$.



6 ΔABC is an isosceles triangle with $AB = AC$. X is the point on AC such that BX is perpendicular to AC.

Prove that $\angle XBC = \frac{1}{2}\angle BAC$.



7 The bisector of an angle of a triangle cuts the opposite side at right angles. Prove that the triangle is isosceles.

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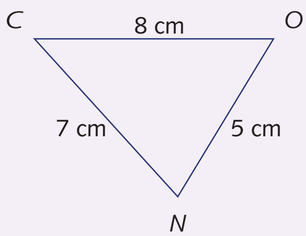
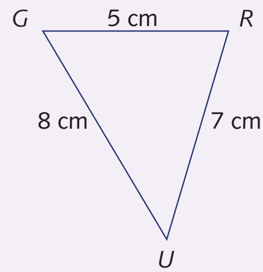
Review exercise

1 In each part write a congruence statement and give a reason.

a





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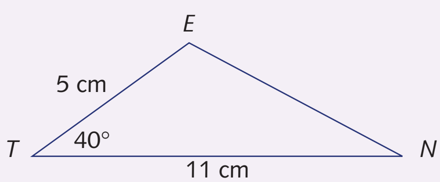

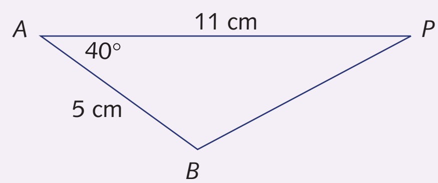

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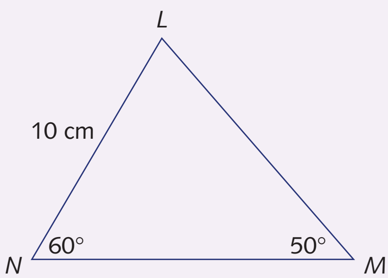
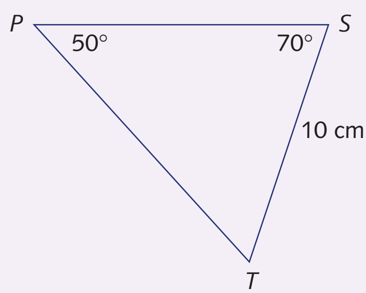
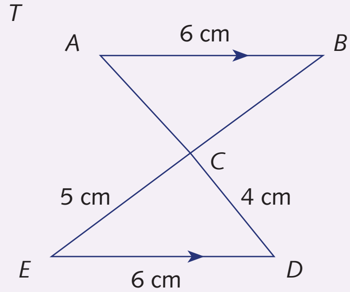




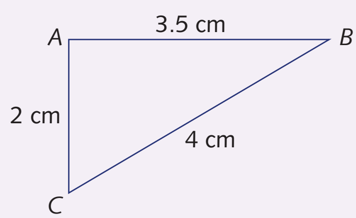
2 **a** In the diagram opposite, name the two congruent triangles and explain why they are congruent.

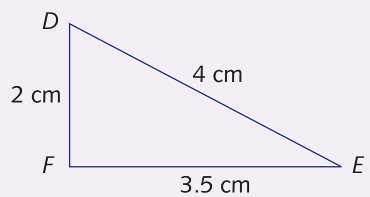
b Find AC.



3 For each pair, write a congruence statement and give a reason.

a





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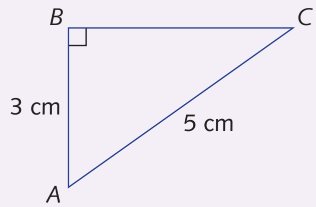

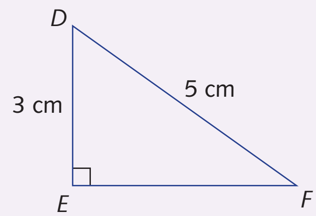

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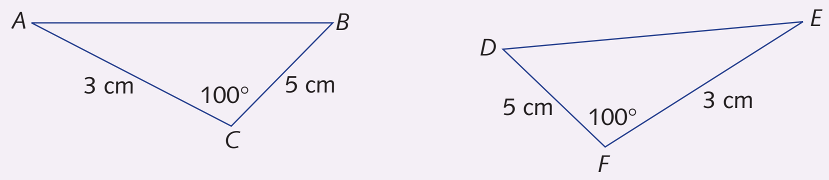
c



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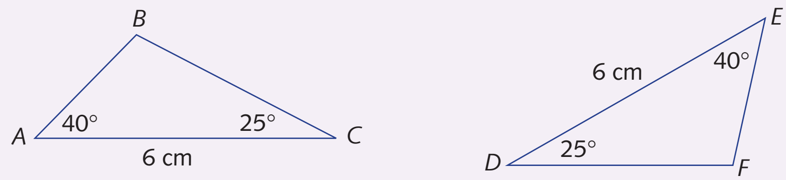
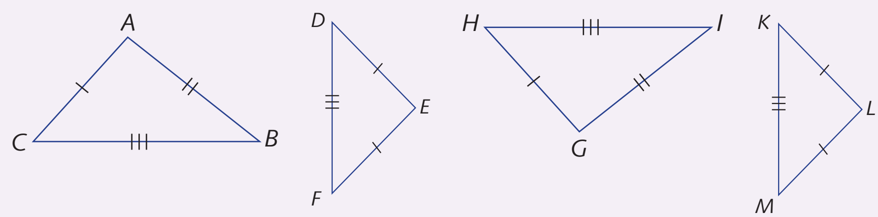


4 In each part, name the triangle congruent to ΔABC.

a



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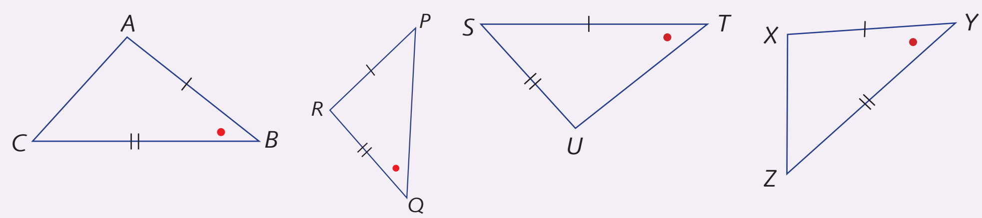

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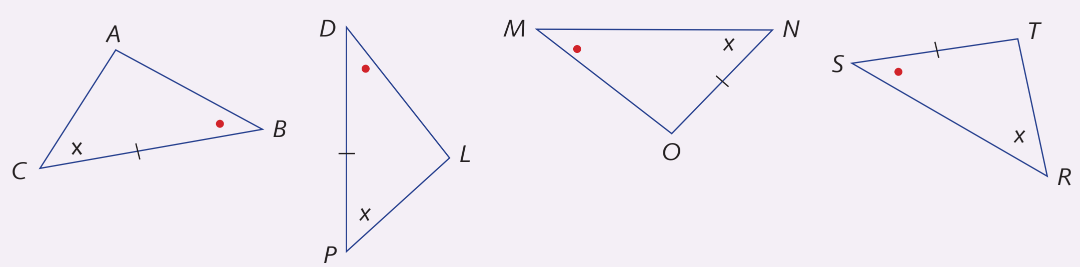
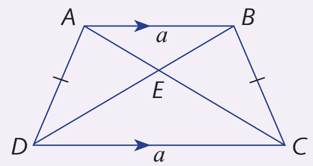


5 In each part, name all the pairs of congruent triangles, giving the abbreviated reason.

a



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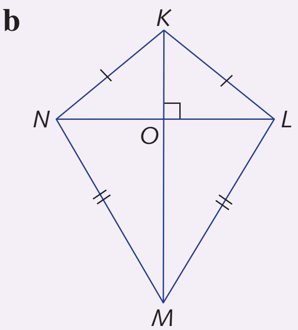

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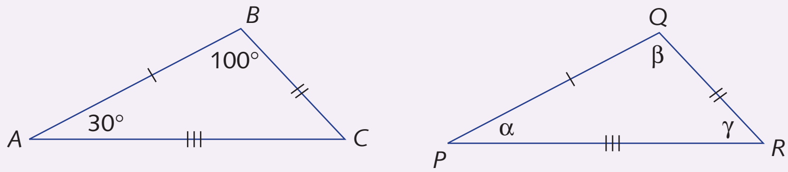
6 Find the values of the pronumerals in each.

a



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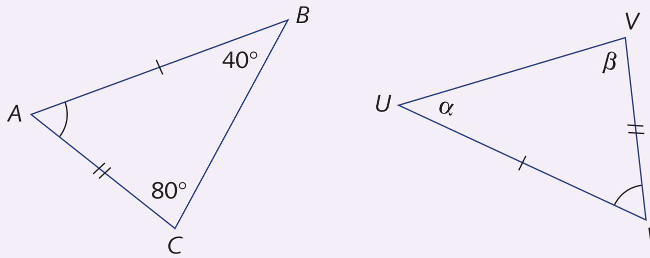

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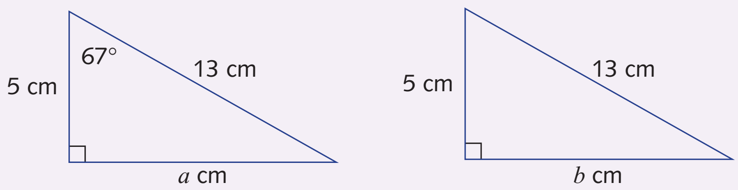

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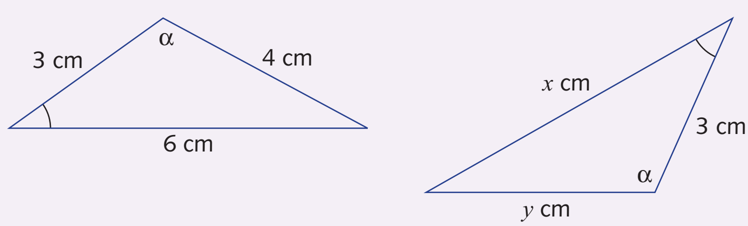

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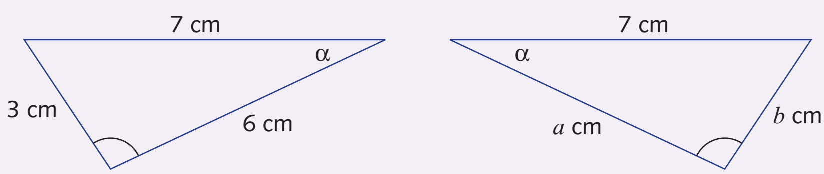


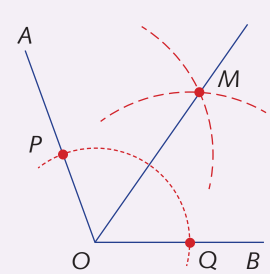
Challenge exercise

1 This question provides a proof of the angle bisector construction. The arcs intersecting at M are centred at P and Q.

a Copy the diagram and join the intervals PM and QM.

b Prove that $\Delta PMO \equiv \Delta QMO$.

c Hence explain why OM is the bisector of $\angle AOB$.



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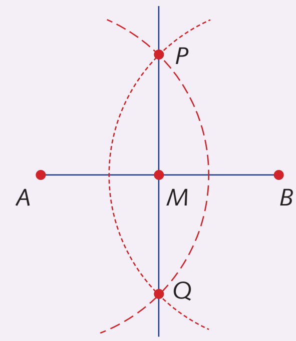
2 This question provides a proof of the construction of the perpendicular bisector of an interval. The arcs are of the same radius and centred at A and B.



a Copy the diagram and join the intervals AP, BP, AQ and BQ.

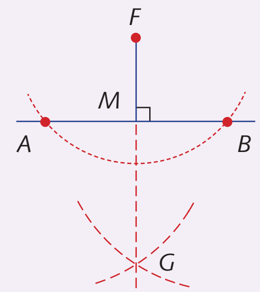
b Prove that $\Delta APQ \equiv \Delta BPQ$.

c Hence prove that $\angle APQ = \angle BPQ$.

d Prove that $\Delta APM \equiv \Delta BPM$.

e Hence prove that $AM = BM$ and $PQ \perp AB$.

3 This question provides a proof of the construction of the perpendicular from a given point to a given line.



a Copy the diagram and join the intervals AF, BF, AG, BG and FG.

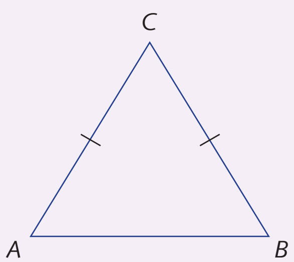
b Prove that $\Delta AFG \equiv \Delta BFG$.

c Hence prove that $\angle AFG = \angle BFG$.

d Prove that $\Delta AFM \equiv \Delta BFM$.

e Hence prove that $FG \perp AB$.

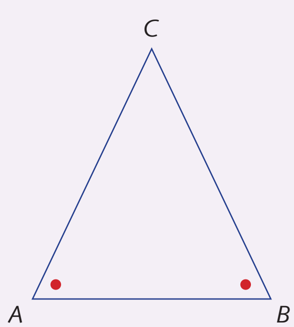
4 In this question, we prove that the base angles of an isosceles triangle are equal by proving that the triangle is congruent to itself. The proof is reputedly due to the Greek mathematician Pappus, who lived in Alexandria from about 290 CE to about 350 CE. Let ΔABC be isosceles, with $AC = BC$.



a Prove that $\Delta ACB \equiv \Delta BCA$. (Note the changed order of the vertices.)

b Hence show that $\angle A = \angle B$.

5 In this question, we give another proof that a triangle with equal base angles is isosceles. This proof involves no construction – the triangle is proven to be congruent to itself. Let ABC be a triangle with $\angle A = \angle B$. We just prove that $AC = BC$.



a Prove that $\Delta ABC \equiv \Delta BAC$.

b Hence show that $AC = BC$.

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6 Explain why the supposed 'proof' outlined below is invalid.

Let ABC be a triangle with $\angle A = \angle B$.

Mark the midpoint M of the base AB, and join CM.

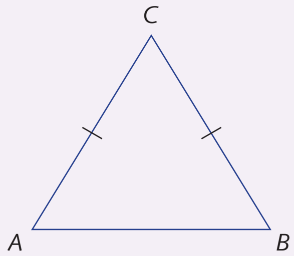
Then $\Delta AMC \equiv \Delta BMC$.

Hence $AC = BC$.

7 The result 'proven' below is clearly false.
Explain why the reasoning is invalid.

Let ΔABC be an isosceles triangle with $AC = BC$.

Let P be any point on the base AB, and join PC.



In the triangles ACP and BCP:

$CP = CP$	(common)
$CA = CB$	(given)
$\angle CAP = \angle CBP$	$\left(\text{base angles of isosceles } \Delta ABC\right)$
so $\Delta ACP \equiv \Delta BCP$	(SAS)
Hence $AP = BP$	(matching sides of congruent triangles)

so P is the midpoint of AB.

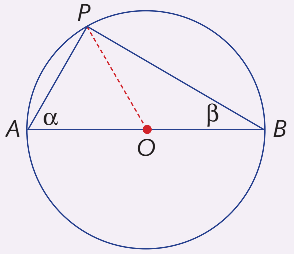
8 There is a valid 'OSS congruence test' (where O stands for 'obtuse angle'):

'If two sides and an obtuse non-included angle of one triangle are equal to two sides and a matching obtuse non-included angle of another triangle, then the triangles are congruent.'

Explain why this result is true.

9 The Greeks said that the oldest geometrical theorem stated and proved was Thales' theorem. Thales' theorem states:

'An angle in a semicircle is a right angle.'



a Let AOB be a diameter of a circle with centre O. Let P be any other point on the circumference. Join AP, BP and OP, and let $\angle PAB = \alpha$ and $\angle PBA = \beta$.

b Use isosceles triangles to prove that $\angle APB = 90°$.

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10 The lines through each vertex of a triangle perpendicular to the opposite side are called the **altitudes** of a triangle. They are concurrent at the **orthocentre** of the triangle. (This can be proved using theorems that you will meet in Year 10.) The orthocentre may lie outside the triangle.

a On a large sheet of paper, draw an obtuse-angled triangle, then construct its orthocentre. You will need to produce two of the sides to perform this construction.

b Let H be the orthocentre of ΔABC. Prove that each of the four points A, B, C and H is the orthocentre of the triangle formed by the other three points. This is true in both cases – when ΔABC is acute-angled and when ΔABC is obtuse-angled.

11 The perpendicular bisectors of the sides of a triangle are concurrent at a point called the **circumcentre**, and this point is the centre of the circle through all three vertices. Here is an outline of a proof. Check the details.

a Take a triangle ABC. Let P be the midpoint of BC, Q be the midpoint of CA, and R be the midpoint of AB. Let the perpendicular bisectors of AB and AC meet at M. Join MA, MB and MC.

b Prove that $\Delta AMR \equiv \Delta BMR$ and that $\Delta AMQ \equiv \Delta CMQ$.

c Hence prove that the circle with centre M and radius MA passes through B and C.

d Prove that MP is perpendicular to BC.

e Under what circumstances does the circumcentre lie outside the triangle?

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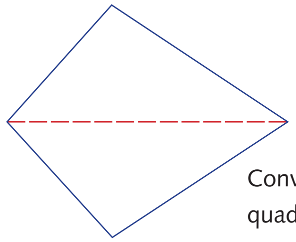
CHAPTER

13

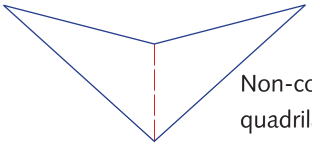
Measurement and Geometry

Congruence and special quadrilaterals

In any quadrilateral, whether it is convex or non-convex, the sum of the interior angles is 360º.



Convex quadrilateral



Non-convex quadrilateral

To show this, we divide the quadrilateral into two triangles and use the fact that the angle sum of each triangle is 180°.

The sum of the exterior angles is 360°.

However, if the quadrilateral has extra properties then new features emerge. Special quadrilaterals include squares, rectangles, parallelograms, rhombuses, trapeziums, kites and cyclic quadrilaterals. In this chapter we will begin to investigate the first four of these and prove special results about them using parallel lines, isosceles triangles and congruence.

We will study the trapezium and the kite in *ICE-EM Mathematics Year 9*, Chapter 7 and cyclic quadrilaterals in *Year 10*, Chapter 13.

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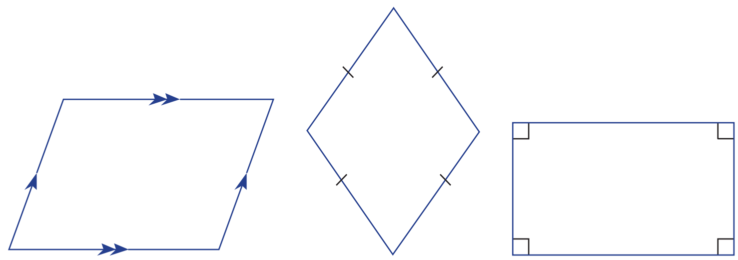
Investigation on diagonals

A **parallelogram** is a quadrilateral with two pairs of parallel sides, a **rhombus** has four equal sides and a **rectangle** has four right angles.

Draw accurate diagrams of a parallelogram, a rhombus and a rectangle in your workbook.



Parallelogram Rhombus Rectangle

Join the diagonals of each quadrilateral.

1 In which figures are the diagonals of equal length?

2 In which figures do the diagonals bisect each other?

3 In which figures do the diagonals meet at right angles to each other?

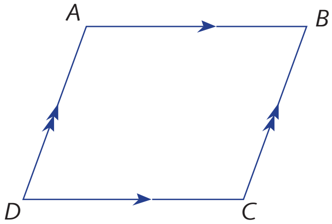
4 A **square** is a quadrilateral that is both a rectangle and a rhombus. What properties do its diagonals have?

In the following sections, we will give geometric proofs of the results you have just discovered by measurement, and we will do much more.

13A Parallelograms and their properties

A **parallelogram** is a quadrilateral with opposite sides that are parallel. Thus the quadrilateral $ABCD$ shown opposite is a parallelogram because $AB \parallel DC$ and $DA \parallel CB$.

The word 'parallelogram' comes from Greek words meaning 'parallel lines'.



Definition of a parallelogram

A **parallelogram** is a quadrilateral with opposite sides that are parallel.

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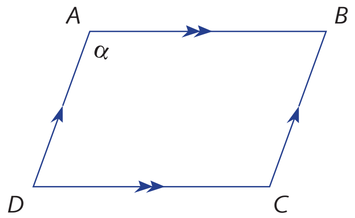
Properties of a parallelogram

First property: the opposite angles of a parallelogram are equal



Theorem: The opposite angles of a parallelogram are equal.

Proof: Let $ABCD$ be a parallelogram.
Let $\angle A = \alpha$.
We need to prove that $\angle C = \alpha$.

$\angle D = 180° - \alpha$ (co-interior angles, $AB \parallel DC$)

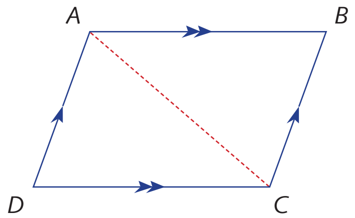
$\angle C = 180° - \angle D$ (co-interior angles, $AD \parallel BC$)

$= 180° - (180° - \alpha)$

$= \alpha$

Hence, the opposite angles of a parallelogram are equal.

Second property: the opposite sides of a parallelogram are equal



Theorem: The opposite sides of a parallelogram are equal.

Proof: Let $ABCD$ be a parallelogram.

We need to prove that $AB = BC$ and $AD = BC$.

Join the diagonal AC.

In the triangles ABC and CDA:

$\angle BAC = \angle DCA$ (alternate angles, $AB \parallel DC$)

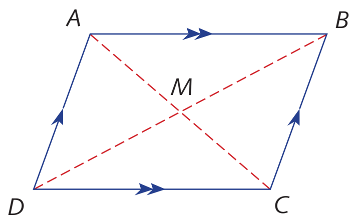
$\angle BCA = \angle DAC$ (alternate angles, $AD \parallel BC$)

AC is common.

So, $\Delta ABC \equiv \Delta CDA$ (AAS)

Hence, $AB = CD$ and $BC = AD$ (matching sides of congruent triangles)

Third property: the diagonals of a parallelogram bisect each other



Theorem: The diagonals of a parallelogram bisect each other.

Proof: Let $ABCD$ be a parallelogram.

Let the diagonals AC and BD meet at M.

We need to prove that $AM = CM$ and $DM = BM$.

In the triangles ABM and CDM:

$\angle BAM = \angle DCM$ (alternate angles, $AB \parallel DC$)

$\angle ABM = \angle CDM$ (alternate angles, $AB \parallel DC$)

$AB = CD$ (opposite sides of parallelogram)

Hence, $\Delta ABM \equiv \Delta CDM$ (AAS)

So $\Delta AM = CM$ and $DM = BM$ (matching sides of congruent triangles)

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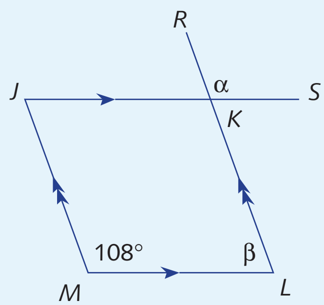
Properties of a parallelogram

- The opposite angles of a parallelogram are equal.
- The opposite sides of a parallelogram are equal.
- The diagonals of a parallelogram bisect each other.

We have just proved three properties of a parallelogram. You can now use these properties to help you solve problems, as shown in the examples below. Where it is possible to do so, name the particular parallelogram being used.

Example 1

Find α and β in the diagram below.



Solution

$\angle JKL = 180°$ (opposite angles of parallelogram $JKLM$)

so $\alpha = 108°$ (vertically opposite angles at K)

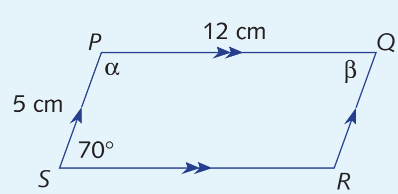
$\beta + 108° = 180°$ (co-interior angles, $JM \parallel KL$)

so $\quad \beta = 72°$

Example 2

a Calculate the perimeter of the parallelogram opposite.

b Find the values of α and β.



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Solution

a $RS = 12$ cm (opposite sides of parallelogram $PQRS$)

and $QR = 5$ cm (opposite sides of parallelogram $PQRS$)

Hence, perimeter $= 12 + 12 + 5 + 5$

$= 34$ cm

b $\alpha + 70° = 180°$ (co-interior angles, $PQ \parallel SR$)

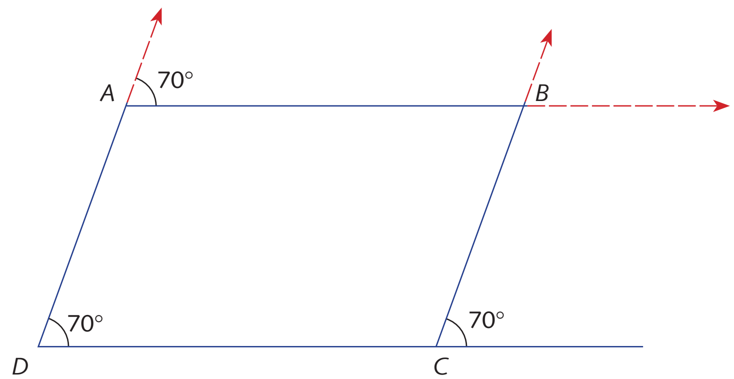
$\alpha = 110°$

$\beta = 70°$ (opposite angles of parallelogram $PQRS$)

The converses of each of the properties are true. They are proved in the exercises.

Exercise 13A

1 Construct a parallelogram $ABCD$ as follows. Start with the interval DC.



Step 1: Use your protractor to construct parallel rays upwards and to the right from D and from C at an angle of 70°. (We have chosen 70°, but any reasonable angle will do in place of 70°.)

Step 2: Choose a point A on the left-hand ray some distance from D.

Step 3: Use your protractor to construct another angle of 70° at the point A in a position corresponding to $\angle D$. This will give a line parallel to DC.

Step 4: Extend this line through A to form the quadrilateral $ABCD$.

a Why is $ABCD$ a parallelogram?

b Check that the opposite sides are equal in length.

c Draw the diagonals AC and BD, and check that their intersection M is the midpoint of each diagonal.

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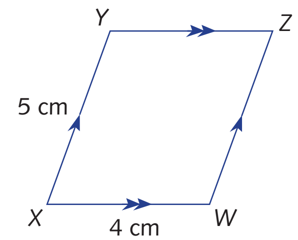
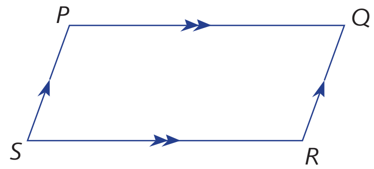
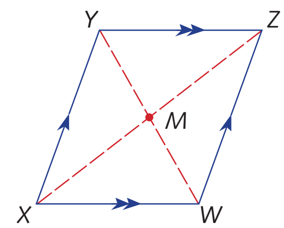
2 In the diagram to the right, $XYZW$ is a parallelogram. What are the lengths of WZ and YZ? Give reasons for your answer.



3 In the diagram to the right, $PQRS$ is a parallelogram. Prove that $\angle S = \angle Q$ and $\angle P = \angle R$, as presented earlier.



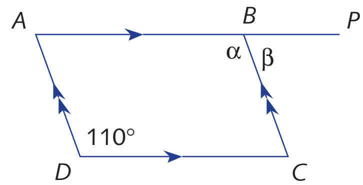
4 In the diagram to the right, $XYZW$ is a parallelogram. M is the point of intersection of XZ and YW. Prove that $XM = ZM$, as presented earlier.



Example 1

5 Use the properties of a parallelogram to find the values of x, y, z, α and β in the diagrams below. Give reasons.

a



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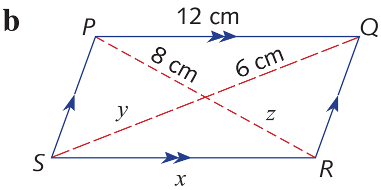

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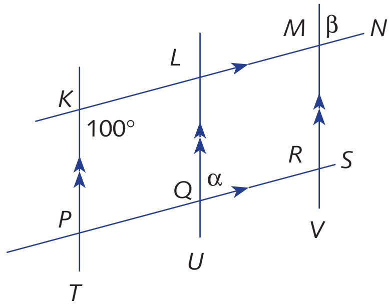

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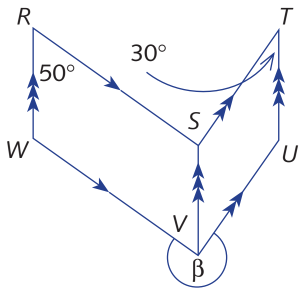

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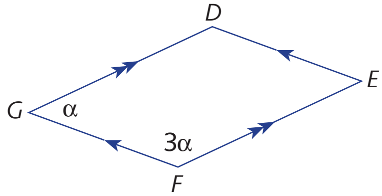

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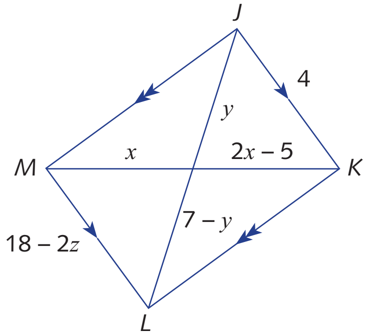

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Example 2

6 Find the perimeter of a parallelogram with side lengths that are:

a 12 cm and 22 cm

b 6.5 km and 12.5 km

c $5\frac{1}{2}$ m and 7 m

d 13 cm and 23 cm

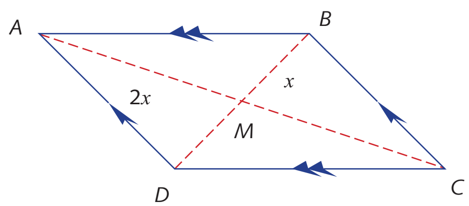
7 Draw a parallelogram using the following steps. First, choose two horizontal lines on your page as opposite sides. Then place your ruler obliquely across the page and use the two edges of the ruler to draw the other two sides. Label the parallelogram *ABCD*.

a Draw the diagonals *AC* and *BD*, and label their intersection *M*.

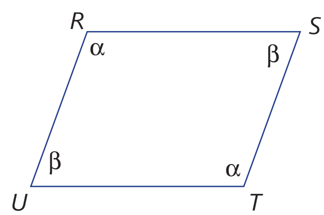
b Draw the circle with centre *M* passing through *A*. Does it go through *C*? What property of the parallelogram does this illustrate?

c Draw a second circle with centre *M* passing through *B*. Does it go through *D*? Why or why not?

8 Show that the diagonal *AC* in the diagram below has twice the length of the diagonal *BD*.



9 In the quadrilateral *RSTU* below, the angles are equal.



Let $\angle R = \angle T = \alpha$ and $\angle S = \angle U = \beta$.

a Use the fact that the angle sum of a quadrilateral is 360° to prove that $\alpha + \beta = 180°$.

b Hence, prove that *RSTU* is a parallelogram.

You have now proved the following test for a parallelogram:

'*If the opposite angles of a quadrilateral are equal, then the quadrilateral is a parallelogram.*'

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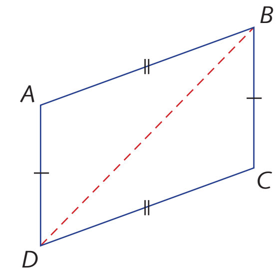
10 In the quadrilateral $ABCD$ to the right, the opposite sides are equal. Join the diagonal BD.



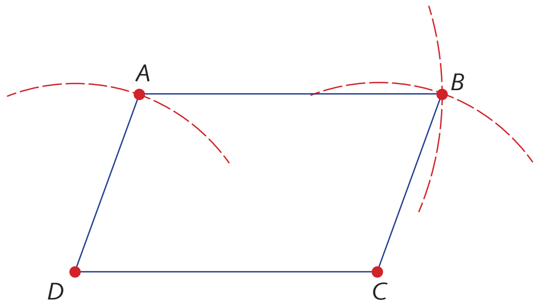
a Prove that $ABD \equiv CDB$.

b Hence, show that $AB \parallel DC$ and $AD \parallel BC$, and hence show that $ABCD$ is a parallelogram.

You have now proved the following test for a parallelogram:

'If the opposite sides of a quadrilateral are equal, then the quadrilateral is a parallelogram.'

11 Here is another construction of a parallelogram. Start with the interval DC. Make it 8 cm long.



Step 1: Draw a circle with centre D and radius 5 cm. Mark a point A on the circle above DC.

Step 2: Draw a circle with centre A and radius 8 cm (this is the length of DC).

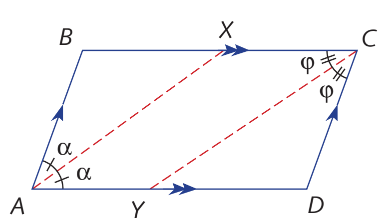
Step 3: Draw a circle with centre C and radius 5 cm. Label as B the point of intersection of the circle from step 2 and the circle from step 3.

Step 4: Join up the quadrilateral $ABCD$.

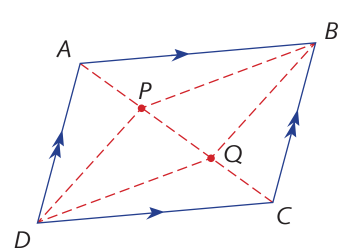
a Explain why $ABCD$ is a parallelogram.

b Use your protractor to check that opposite angles are equal.

12 In the diagram below, $ABCD$ is a parallelogram. The bisector of $\angle A$ meets BC at X. The bisector of $\angle C$ meets AD at Y. Prove AX is parallel to CY.



13 In the diagram to the right, $ABCD$ is a parallelogram. AC is a diagonal and P and Q are points on AC such that $AP = CQ$. Prove that $PBQD$ is a parallelogram.



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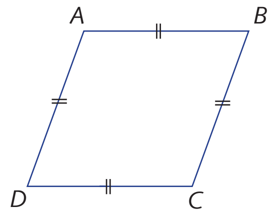
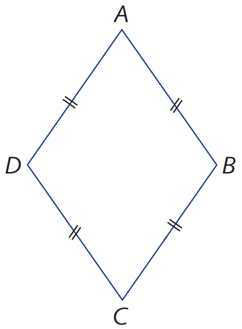
13B Rhombuses and their properties

A **rhombus** is a quadrilateral in which all four sides are equal. Thus the two quadrilaterals $ABCD$ below are both rhombuses because $AB = BC = CD = DA$.





The first rhombus looks like a 'pushed-over square'. You may already be familiar with this description – it is a common-sense way of thinking about a rhombus.

The second rhombus has been rotated so that it looks like the diamond in a pack of cards. It is usually better to think of this as the characteristic shape of a rhombus.

The Greeks took the word *rhombos* from the shape of a piece of wood that was whirled about the head, like a bullroarer, in religious ceremonies.

Definition of a rhombus

A **rhombus** is a quadrilateral in which all four sides are equal.

This section begins with an investigation of some important properties of the diagonals of a rhombus.

A rhombus is a parallelogram

Theorem: A rhombus is a parallelogram.

Proof: Let $ABCD$ be a rhombus.

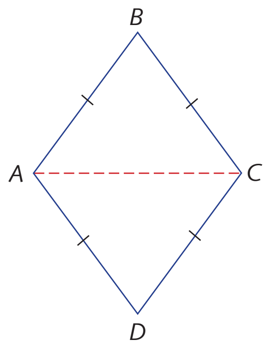
$AB = BC = CD = DA$

Join the diagonal AC.

Since AC is common,

$\Delta ABC \equiv \Delta ADC$ (SSS),

so $\Delta DAC = \angle BCA$ (matching angles of congruent triangles)



Hence, $AD \parallel BC$ (alternate angles)

A similar argument shows that $AB \parallel CD$.

Thus $ABCD$ is a parallelogram.

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Diagonal properties of a rhombus

Because a rhombus is a parallelogram, its diagonals bisect each other. The diagonals of a rhombus have two further properties.

Theorem: **a** Each diagonal of a rhombus bisects the two interior angles through which it passes.

b The diagonals of a rhombus bisect each other at right angles.

Proof: **a** To prove the first result, join the diagonal AC.

Since AC is common,

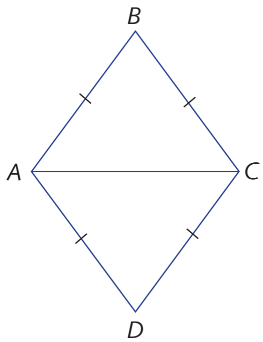
$\Delta ABC \equiv \Delta ADC$ (SSS)

Therefore, $\angle BAC = \angle DAC$ (matching angles of congruent triangles)

Therefore, the diagonal AC bisects the angle at the vertex A.

Similarly, $\angle BCA = \angle DCA$ (matching angles of congruent triangles)

So the diagonal AC bisects the angle at the vertex C.



b To prove the second result, join the other diagonal, BD, and let them meet at M. We must show $\angle BMA = 90°$. Let $\angle BAC = \alpha$ and $\angle ABD = \beta$

Then $\angle DBC = \alpha$ (by part **a**)

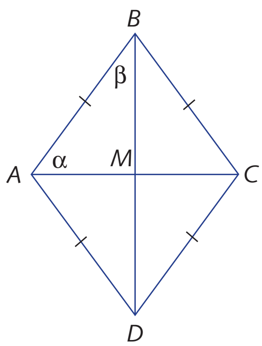
and $\angle BCA = \alpha$ (base angles of isosceles ΔABC)

Thus $\quad 2\alpha + 2\beta = 180°$ (angle sum of ΔABC)

$\alpha + \beta = 90°$

Therefore, $\angle BMA = 90°$ (angle sum of ΔAMB)

Therefore, the diagonals of rhombus bisect each other at right angles.



These properties can be proved in other ways, such as using congruent triangles.

Properties of a rhombus

- A rhombus is a parallelogram.
- The diagonals of a rhombus bisect each other at right angles.
- Each diagonal of a rhombus bisects the two interior angles through which it passes.

You can now use these diagonal properties of a rhombus to help you solve problems, as shown in the following examples. Where it is possible to do so, name the particular rhombus being used.

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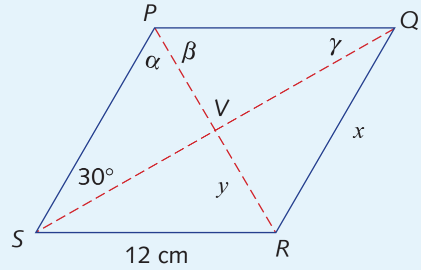
Example 3

Find α, β, γ, x and y in the diagram below, given that $PQRS$ is a rhombus.



Solution

First, $\angle PVS = 90°$ (diagonals of rhombus $PQRS$ meet at right angles)
so $\alpha = 60°$ (angle sum of ΔPVS)
and $\beta = 60°$ (diagonals of rhombus $PQRS$ bisect interior angles)
Second, $\gamma = 30°$ (base angles of isosceles ΔSPQ)
and $x = 12$ cm (sides of rhombus $PQRS$)
Third, $\angle SRP = 60°$ (base angles of isosceles ΔSPR)
so ΔSPR is equilateral, since all angles are 60°.
Hence, $PR = SR = 12$ cm
and $y = 6$ cm (diagonals of rhombus $PQRS$ bisect each other)

Exercise 13B

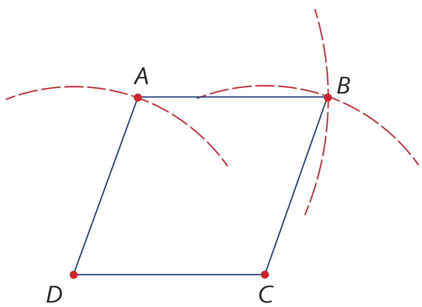
1 Construct a rhombus as follows. Start by drawing an interval DC, 5 cm long.

Step 1: Draw a circle with centre D and radius DC. Mark a point A on the circle above DC.

Step 2: Draw a circle with centre A and the same radius DC.

Step 3: Draw a circle with centre C and the same radius. Label as B the point of intersection of this circle and the circle from step 2.

Step 4: Join up the points A, B, C and D.



Explain why this figure $ABCD$ is a rhombus.

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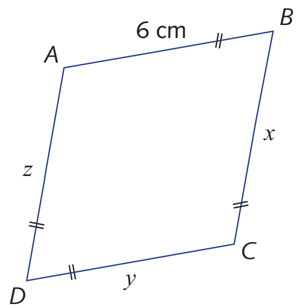
Example 3

2 Find the values of x, y, z, α, β, γ and θ. Give reasons.

a



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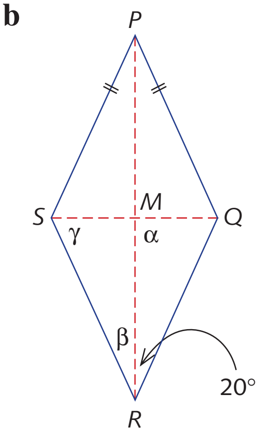

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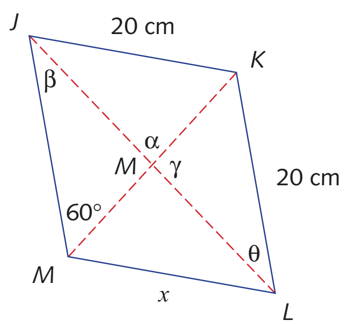

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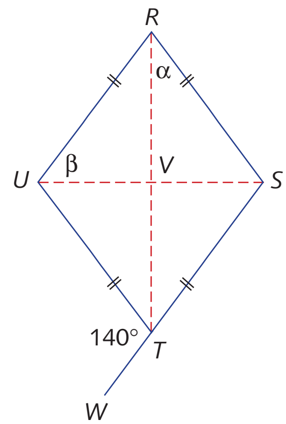

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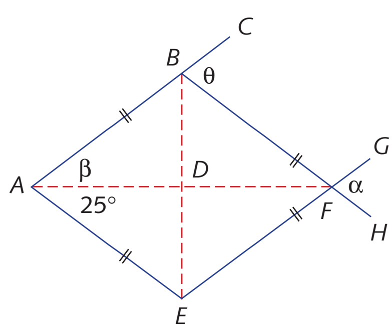

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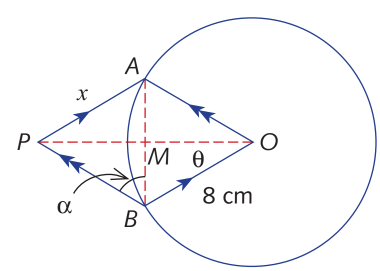
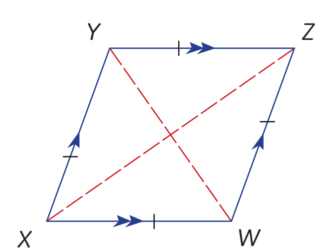


(The point O is the centre of the circle.)

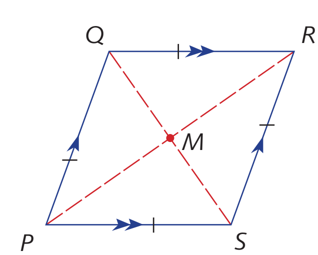
3 In the diagram to the right, $XYZW$ is a rhombus.

Prove that XZ bisects $\angle X$ as presented earlier.



4 In the diagram to the right, $PQRS$ is a rhombus. Let M be the point of intersection of the diagonals QS and PR.

Prove that $QM = MS$, $PM = MR$ and $\angle QMR$ is a right angle, as presented earlier.



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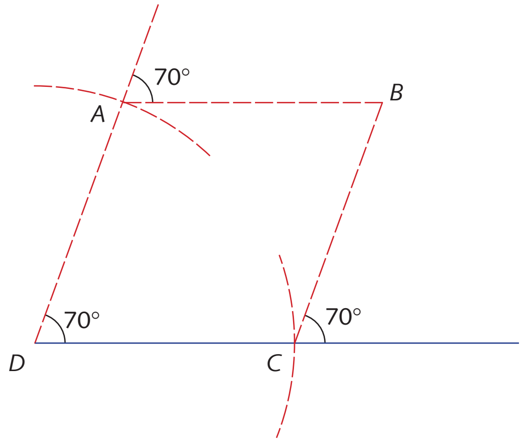
5 Construct a rhombus $ABCD$ as follows. Start with the interval DC.



a Why is $ABCD$ a rhombus?

b Join the diagonals and check with a protractor that they bisect the vertex angles and are perpendicular.

Step 1: Use your protractor to construct parallel rays upwards and to the right from D and from C at an angle of 70°. (We have chosen 70°, but any reasonable angle will do in place of 70°.)

Step 2: Place the point of your compasses on D. Draw an arc that passes through C and cuts the ray at a point A.

Step 3: Use your protractor to construct another angle of 70° at the point A in a position corresponding to $\angle D$. This will give a line parallel to DC.

Step 4: Extend this line through A to form the rhombus $ABCD$.

6 **a** Join a rhombus as follows. First, use the two edges of your ruler to draw two parallel lines. Then rotate your ruler to draw a second pair of parallel lines. Label the rhombus $ABCD$.

b Draw the diagonals AC and BD, and label their intersection M.

c Use your protractor to check that the diagonals are perpendicular.

d Check with your protractor that the diagonals bisect the vertex angles through which they pass.

e Draw the circle with centre M passing through A, then draw the circle with centre M passing through B. Do these circles go through C and D, respectively? What property of a rhombus do you need to know to answer these questions?

f Prove that this construction does indeed produce a rhombus.

(This is a difficult question.)

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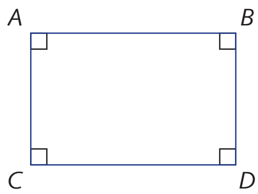
13C Rectangles and squares and their properties

Rectangles

A **rectangle** is a quadrilateral in which all four angles are right angles. The quadrilateral $ABCD$ is a rectangle because $\angle A = \angle B = \angle C = \angle D = 90^\circ$.

A rectangle $ABCD$ is a parallelogram because each pair of co-interior angles is supplementary.



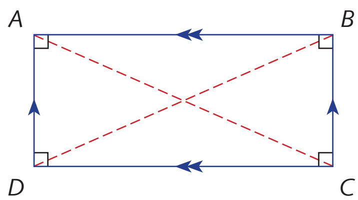
Definition of a rectangle

A **rectangle** is a quadrilateral in which all four angles are right angles.

The word 'rectangle' comes from Latin and means 'right-angled'.

Since a rectangle is a parallelogram, it has all the properties of a parallelogram. This means that:

- the opposite sides are equal, and
- the diagonals bisect each other.



The diagonals of a rectangle, however, have another important property – they are equal in length.

The proof below uses Pythagoras' theorem.

Theorem: The diagonals of a rectangle are equal.

Proof: Let $ABCD$ be a rectangle, with diagonals AC and BD.
Let $AB = x$ and $BC = y$.

We need to prove that $AC = BD$.

First, $DC = x$ (opposite sides of rectangle)

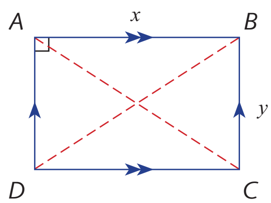
Using Pythagoras' theorem in ΔABC:

$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ &= x^2 + y^2 \end{aligned}$$

Using Pythagoras' theorem in ΔBCD:

$$\begin{aligned} BD^2 &= DC^2 + CB^2 \\ &= x^2 + y^2 \end{aligned}$$

Hence, $AC = BD$



An alternative proof using congruence is developed in question **7** of Exercise 13C.

Diagonal properties of a rectangle

The diagonals of a rectangle are equal and bisect each other.

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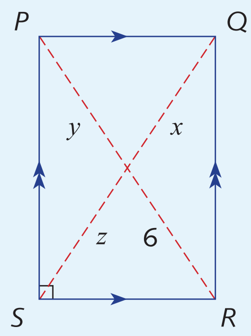
You can now use these diagonal properties of a rectangle to help you solve problems, as shown in the example below.

Example 4

Find the values of x, y and z.



Solution

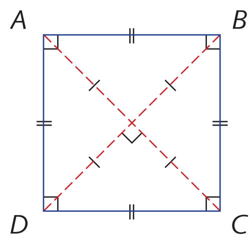
PQRS is a rectangle.

So $x = y = z = 6$ (diagonals of a rectangle are equal and bisect each other)

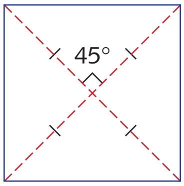
Squares

A **square** is a rectangle with all sides equal. That is, a square is a quadrilateral that is both a rhombus and a rectangle.

Because a square is a rectangle, it has all the properties of a rectangle. Thus its diagonals are equal and bisect each other.



Because a square is also a rhombus, its diagonals meet at right angles. Also, each diagonal bisects the right angles at the vertices through which it passes, and so meets each side at 45°.



Squares and their properties

A **square** is a rectangle with all sides equal. Thus a square is both a rectangle and a rhombus, so:

- all sides are equal and all angles are right angles
- the opposite sides are parallel
- the diagonals are equal in length, the diagonals bisect each other at right angles, and each diagonal meets each side at 45°.

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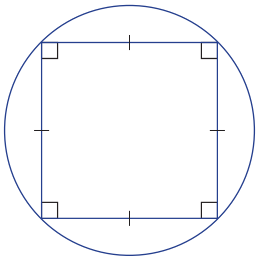
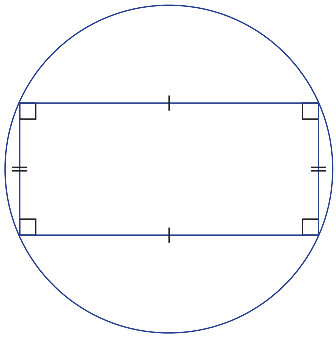
Because the diagonals of both rectangles and squares are equal in length and bisect each other, a circle can be drawn with centre the point of intersection of the diagonals and that passes through the four vertices. This is called the **circumcircle** of the rectangle or square.

The word 'square' comes originally via the Old French from the Latin word *quattuor*, meaning 'four'.



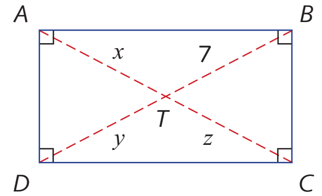


Exercise 13C

Example 4

1 Use the properties of rectangles and squares to find the values of x, y, z, t, α, β, γ and θ.

a



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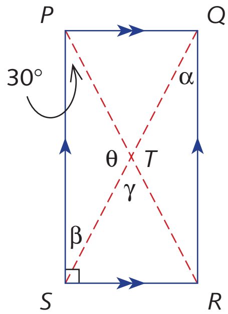

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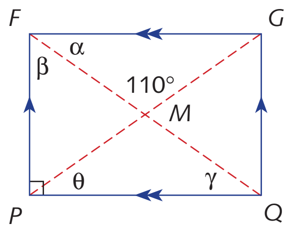

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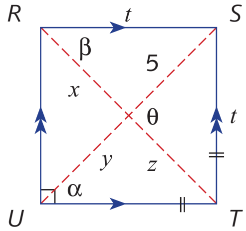

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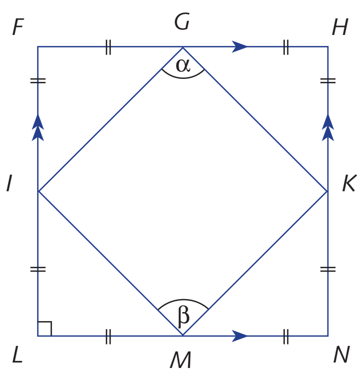

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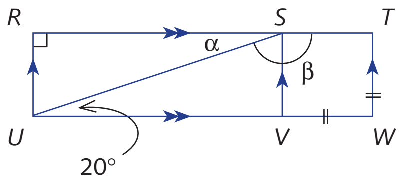

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2 Draw a rectangle using the following steps.

a First, use two horizontal lines on your page as opposite sides. Then construct two vertical lines which form the other two sides. Call the rectangle $ABCD$.

b Draw the two diagonals and label their intersection M.

c Construct the circle with centre M passing through A. What property of a rectangle tells us that it must pass through B, C and D?

3 Draw a square using the following steps.

a First, draw a horizontal line. Construct a vertical line meeting the horizontal line at D. With centre D and radius 5 cm, draw a circle meeting the vertical line at A and the horizontal line at C. With centres A and C, draw arcs of radius 5 cm meeting at B. Join AB and BC. Explain why $ABCD$ is a square.

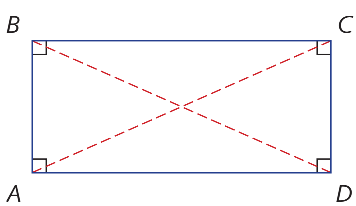
b Draw the two diagonals, and label their intersection M.

c Use your protractor to check that the diagonals intersect at right angles, and that they each meet the sides at 45º.

d Construct the circle with centre M passing through A. Notice that it goes through B, C and D. What property of a square tells us that it must pass through B, C and D?

4 Prove that if a parallelogram has one right angle then it is a rectangle.

5 Here is an alternative proof, using congruence, that the diagonals of a rectangle are equal. Let $ABCD$ be a rectangle, with diagonals AC and BD.



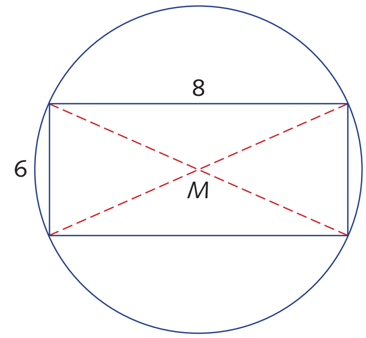
a Prove that $\Delta ACD \equiv \Delta BDC$.

b Hence, prove that $AC = DB$.

6 Find the radius of the circumcircle of:

a a rectangle with side length 6 and 8

b a square of side length 7



7 A quadrilateral has all four sides equal and two equal diagonals. Prove it is a square.

8 **a** $ABCD$ is a square, Q is a point on DC, and R is a point on BC. Suppose that AQ is perpendicular to DR. Prove $AQ = DR$.

b Suppose that two lines m and n are perpendicular and that each line intersects opposite edges of the square $ABCD$. Show that the intervals cut out by the square on m and n are equal in length.

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Review exercise

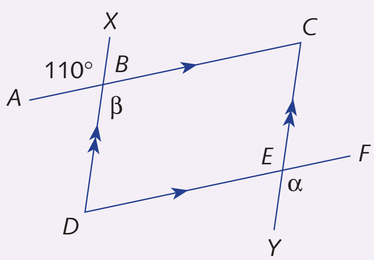
1 List all the properties of the diagonals of:

a a parallelogram **b** a rhombus

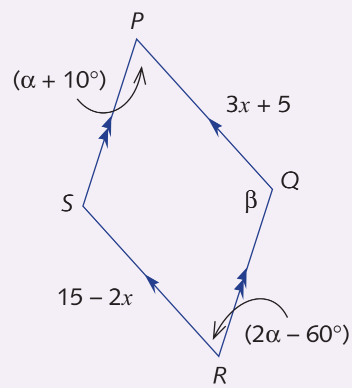
c a rectangle **d** a square

2 Use the properties of a parallelogram to find the values of x, y, z, α and β in the diagrams below. Give reasons.

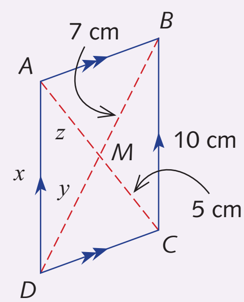
a



b

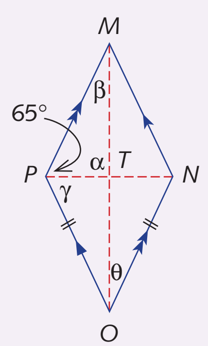


c

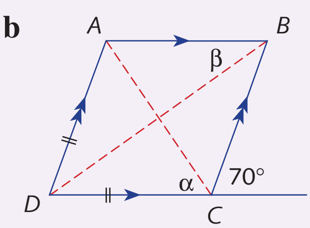


3 Use the properties of a rhombus to find the values of x, α, β, γ and θ in the diagrams below, giving reasons.

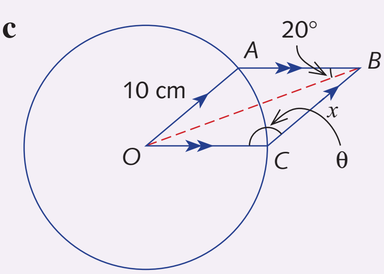
a



b



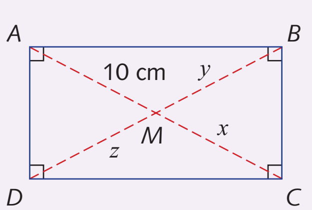
c



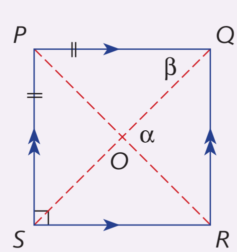
(The point O is the centre of the circle.)

4 Use the properties of rectangles and squares to find the values of x, y, z, α, β, γ and θ in the diagrams below, giving reasons.

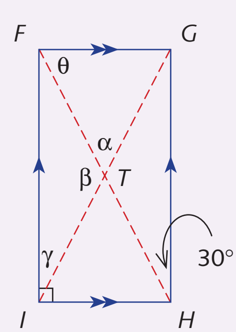
a



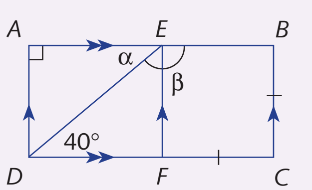
b



c



d



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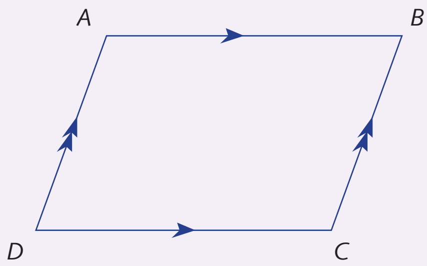
Note: The next three questions are intended as reviews of the proofs in this chapter.

5 The figure $ABCD$ below is a parallelogram.

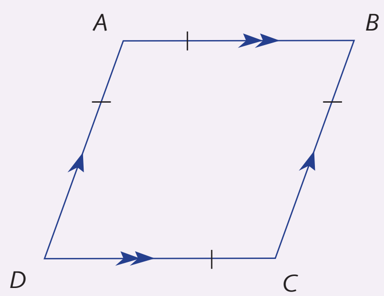


a Prove that the opposite angles are equal.

b Prove that the opposite sides are equal.

c Prove that the diagonals bisect each other.

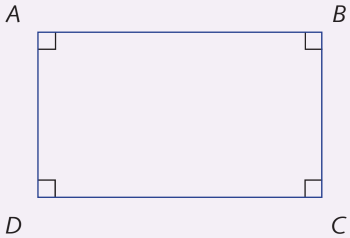
6 The figure $ABCD$ below is a rhombus.



a Prove that each diagonal bisects the two vertices through which it passes.

b Prove that the diagonals meet at right angles.

7 The figure $ABCD$ below is a rectangle.



Prove that the diagonals are equal.

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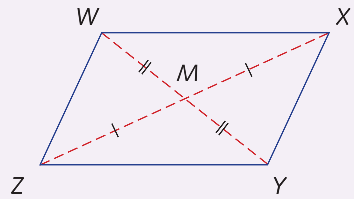
Challenge exercise

1 In the quadrilateral $WXYZ$ below, the diagonals bisect each other at M.



a Prove that $\Delta WMX \equiv \Delta YMZ$.

b Hence show that $WX \parallel ZY$.

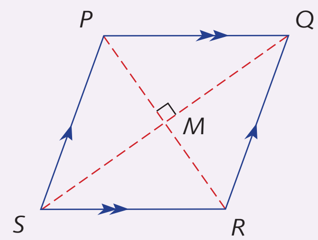
c Prove that $\Delta WMZ \equiv \Delta YMX$.

d Hence show that $WZ \parallel XY$, and hence that $WXYZ$ is a parallelogram.

You have now proved the following test for a parallelogram:

'*If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.*'

2 In the parallelogram $PQRS$ below, the diagonals meet at right angles at M.



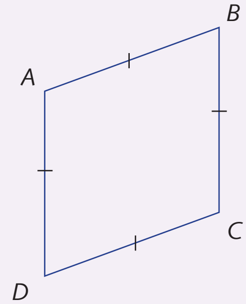
a Prove that $\Delta PMQ \equiv \Delta RMQ$.

b Hence show that $PQ = RQ$, and so $PQRS$ is a rhombus.

You have now proved the following test for a rhombus:

'*If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.*'

3 All sides of the quadrilateral $ABCD$ below are equal.



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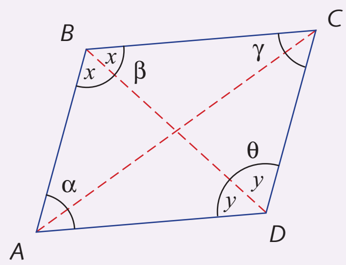
a Prove that $\Delta ADB \equiv \Delta CBD$.

b Hence explain why $AB \parallel DC$ and $AD \parallel BC$, and so $ABCD$ is a parallelogram.

You have now proved that every rhombus is a parallelogram.

4 The diagonals of the quadrilateral $ABCD$ bisect each vertex angle, as shown in the diagram below.



a Use the angle sums of ΔABD and ΔCBD to prove that $\alpha = \gamma$.

b Use a similar argument to prove that $\beta = \theta$.

c Hence explain why $AB \parallel DC$ and $AD \parallel BC$, and so $ABCD$ is a rhombus.

You have now proved the following test for a rhombus:

'*If each diagonal of a quadrilateral bisects the two vertex angles through which it passes, then the quadrilateral is a rhombus.*'

5 A parallelogram $ABCD$ has diagonals of equal length.

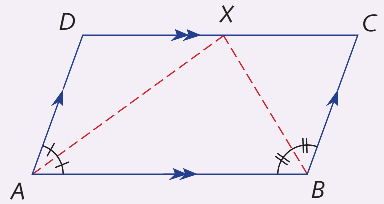
a Prove that $\Delta ABD \equiv \Delta BAC$.

b Hence prove that $\angle A = \angle B = 90°$, and so $ABCD$ is a rectangle.

You have now proved the following test for a rectangle:

'*If the diagonals of a parallelogram are equal, then the parallelogram is a rectangle.*'

6 In the diagram below, $ABCD$ is a parallelogram such that the bisector of $\angle A$ meets the bisector of $\angle B$ on DC at X. Prove $AB = 2BC$.



7 $ABCD$ is a parallelogram, P is the midpoint of BC, and DP and AB are extended to meet at Q. Prove that $AQ = 2AB$.

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CHAPTER

14

Measurement and Geometry

Circles

Squares, rectangles and triangles are all geometrical shapes that have straight edges. The circle is the first geometrical object we come across that has a curved edge.

The points on a circle each lie a fixed distance from the point O, called the **centre**. This distance is called the **radius**.

In this chapter we shall obtain formulas for the circumference of a circle and also for the area of a circle. These formulas use an amazing number called **pi** (π), which is approximately 3.14.

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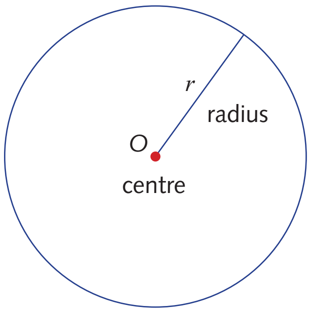
14A Features of the circle

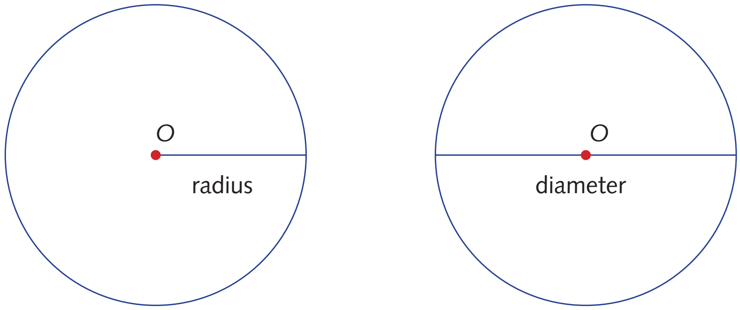
A **circle** is formed by all points that lie a fixed distance r, called the **radius**, from a fixed point O, called the **centre**.



Given a circle, any interval drawn from the centre to any point on the circle is called a **radius** of the circle. (The plural of the word 'radius' is radii.) Thus we use the word 'radius' in two senses: it means an interval joining the centre to a point on the circle, and it also means the length of such an interval.

Since the distance from the centre to any point on the circle is always the same, all radii of a given circle have the same length. This is how compasses work.

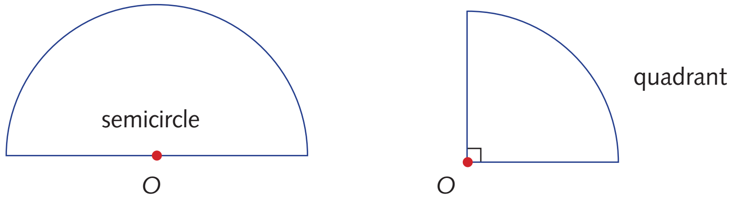
Any interval joining two points on the circle and passing through the centre is called a **diameter** of the circle. Any two diameters of a given circle have the same length.



Notice that the diameter of a circle is equal to twice its radius, and so the radius of a circle is half the diameter.

A diameter divides the circle into two equal parts. Each part is called a **semicircle**.

If we draw a radius that cuts the semicircle into two equal parts, then each part is called a **quadrant**. The Latin root *quad* means 'four'. A circle can be cut into four quadrants.



Any two radii divide the circle into two (not necessarily equal) pieces. Each piece is called a **sector**. The word 'sector' comes from the Latin word *secare*, meaning 'to cut'. When you cut up a pizza, you normally cut the pizza into sectors.

The angle between two radii is called the **angle contained in the sector**.

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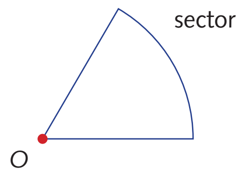

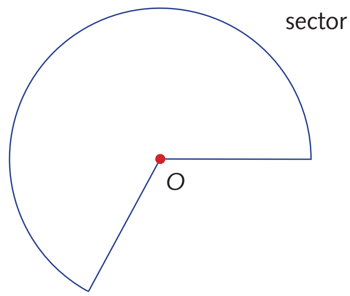

A quadrant and a semicircle are special kinds of sectors for which the angle of the sectors are 90° and 180° respectively.

Features of the circle

- A circle is formed by all the points that lie a fixed distance r from a fixed point O.
- Any interval drawn from the centre to a point on the circle is called a **radius** of the circle.
- Any interval joining two points on the circle and passing through the centre is called a **diameter** of the circle.
- A diameter divides the circle into two equal parts. Each part is called a **semicircle**.
- If a radius is drawn cutting a semicircle into two equal parts, then each part is called a **quadrant**.
- Any two radii divide the circle into two pieces. Each piece is called a **sector**.

Exercise 14A

1 Use your compasses to draw a circle with:

a radius 5 cm

b radius 7 cm

c diameter 12 cm

2 Use your protractor and compasses to draw a sector with:

a radius 5 cm and containing an angle of 45°

b diameter 14 cm and containing an angle of 150°

3 Complete these statements.

a A quadrant is a sector containing an angle of ______°.

b A semicircle is a sector containing an angle of ______°.

c Two quadrants can be joined to form a ______.

4 **a** If you cut a circle into 3 equal sectors, what is the angle of the sector?

b Repeat for 5, 8 and 10 equal sectors.

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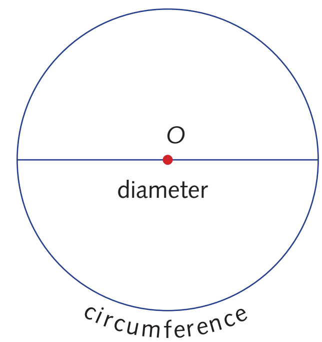
14B Circumference of a circle

Suppose we have drawn a circle of radius 2 cm. An obvious question to ask is 'What is the distance around the circle?' In other words, imagine that our circle is made of string. If we pull it tight and lay it on our ruler, what is the length of the string?



The distance around the edge of a circle (that is, the perimeter of the circle) is called its **circumference**. This word comes from the Latin words *circum*, meaning 'around', and *ferre*, which means 'to carry'.

The Greeks noticed that if you double the diameter, you double the circumference, and if you triple the diameter, you triple the circumference. In other words, the ratio of the circumference to the diameter is always the same.

Here are some (approximate) measurements of the circumferences and diameters of some circles.

Diameter (cm)	Circumference (cm)	$\frac{\text{Circumference}}{\text{Diameter}}$
2	6.3	3.15
3	9.4	3.13
4	12.6	3.15
5	15.7	3.14

You will notice from the table that, even though we measure different diameters and matching circumferences, the *ratio* of the two is always (approximately) the same. It is measurement and rounding error that makes the ratios appear slightly different each time.

This constant ratio is given the symbol π (pronounced 'pie') in mathematics. It is the Greek letter *pi*, and is equivalent to our 'p'. Thus, in any circle:

$$\frac{\text{circumference}}{\text{diameter}} = \pi$$

Using C for circumference and d for diameter, we can write the formula for the circumference of a circle as:

$$C = \pi d$$

Since the diameter is twice the radius, we can rewrite this formula, using the letter r for the radius, as:

$$C = 2\pi r$$

These formulas should be memorised.

The number π is an example of a decimal that does not terminate or repeat. We can display the first few places of the number π by writing:

$$\pi = 3.141\,592\,653\,58\ldots$$

We will normally round π to two decimal places, and take π as 3.14. When greater accuracy is required, we can take more decimal places, for example, $\pi \approx 3.1416$.

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There is also a fraction that is close (but *not equal*) to π. This is the fraction $3\frac{1}{7} = \dfrac{22}{7}$.
Note that $\dfrac{22}{7} = 3.142\,857\ldots$ while $\pi = 3.141\,592\,6\ldots$, so these numbers agree only to two decimal places.

We will write $\pi \approx 3.14$ or $\pi \approx \dfrac{22}{7}$ to express the *approximate* equality of the two sides. In fact π is slightly closer to $\dfrac{22}{7}$ than to 3.14.

Numbers such as π that are neither terminating nor recurring decimals are called **irrational numbers**. The number $\sqrt{2}$, which you met in Chapter 8 on Pythagoras' theorem, is similar to this. You will learn more about irrational numbers later in your study of mathematics.

The number π, in particular, is one that you will learn more and more about as you progress in mathematics. It is a truly amazing number!

Example 1

Find the circumference of a circle:

a with diameter 14 cm

b with radius 21 cm

Give each answer:

i in terms of π

ii as an approximate value, using $\pi \approx \dfrac{22}{7}$

Solution

a i
$$\begin{aligned} C &= \pi d \\ &= 14\pi \text{ cm} \end{aligned}$$

ii
$$\begin{aligned} C &= 14\pi \\ &\approx 14 \times \frac{22}{7} \\ &= 44 \text{ cm} \end{aligned}$$

b i
$$\begin{aligned} C &= 2\pi r \\ &= 2 \times \pi \times 21 \\ &= 42\pi \text{ cm} \end{aligned}$$

ii
$$\begin{aligned} C &= 42\pi \\ &\approx 42 \times \frac{22}{7} \\ &= 132 \text{ cm} \end{aligned}$$

Note that the fractional answers in part **ii** are only approximate, because we have used an approximation for π.

Example 2

Find the circumference of a circle:

a with diameter 5 cm

b with radius 10 cm

Give each answer:

i in terms of π

ii as an approximate value, using $\pi \approx 3.14$

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Solution

a i $C = \pi d$
$= 5\pi$ cm

ii $C = 5\pi$
$\approx 5 \times 3.14$
$= 15.70$ cm

b i $C = 2\pi r$
$= 2 \times \pi \times 10$
$= 20\pi$ cm

ii $C = 20\pi$
$\approx 20 \times 3.14$
$= 62.80$ cm

Circumference of the circle

In any circle:

- the ratio of the circumference to the diameter is $\dfrac{\text{circumference}}{\text{diameter}} = \pi$
- circumference $= \pi \times$ diameter, or $C = \pi d$
- circumference $= 2\pi \times$ radius, or $C = 2\pi r$.

Exercise 14B

1 Take a large round object and try to measure (approximately) its circumference by rolling the object along a ruler or measuring tape. Measure the diameter of the object as carefully as you can and see how close the ratio of the circumference to the diameter is to π.

2 Use your ruler to measure (approximately) the diameter of a 20c coin in millimetres. Write down the radius and find the circumference of the coin. Repeat the exercise with a 10c coin and a 5c coin.

Example 1i

3 Find the circumference of a circle with the given radius. Leave your answers in terms of π.

a 14 cm **b** 7 cm **c** $3\frac{1}{2}$ mm **d** 42 m

Example 1ii

4 Use $\pi \approx \dfrac{22}{7}$ to find the approximate value of the circumference of a circle with radius:

a 14 cm **b** 7 cm **c** $3\frac{1}{2}$ mm **d** 42 m

5 Find the circumference of a circle with the given diameter. Leave your answers in terms of π.

a 14 cm **b** 7 cm **c** $3\frac{1}{2}$ mm **d** 42 m

6 Use $\pi \approx \dfrac{22}{7}$ to find the approximate value of the circumference of a circle with diameter:

a 14 cm **b** 7 cm **c** $3\frac{1}{2}$ mm **d** 42 m

Example 2i

7 Find the circumference of a circle with the given radius. Leave your answers in terms of π.

a 10 cm **b** 5 cm **c** 20 mm **d** 15 m

Example 2ii

8 Use $\pi \approx 3.14$ to find the approximate value of the circumference of a circle with radius:

a 10 cm **b** 5 cm **c** 20 mm **d** 15 m

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9 Find the circumference of a circle with the given diameter. Leave your answers in terms of π.

a 10 cm **b** 5 cm **c** 20 mm **d** 15 m

10 Use $\pi \approx 3.14$ to find the approximate value of the circumference of a circle with diameter:

a 10 cm **b** 5 cm **c** 20 mm **d** 15 m

11 If the radius of a circle is doubled, what happens to the circumference?

12 A circle has circumference 66 cm. Using $\pi = \frac{22}{7}$, find the approximate value of the diameter of the circle.

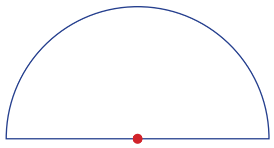
13 A circle has circumference 62.8 cm. Using $\pi \approx 3.14$, find the approximate value of the radius of the circle.

14 If the circumference of a circle is halved, what happens to the diameter?

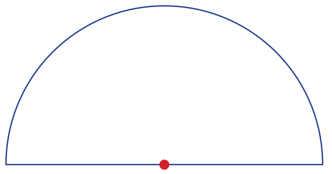
15 The perimeter of a semicircle is the distance around the semicircle (which includes the diameter).

a What is the perimeter of a semicircle with radius 35 cm?

Leave your answer in terms of π. (The dot indicates the centre of the full circle.)



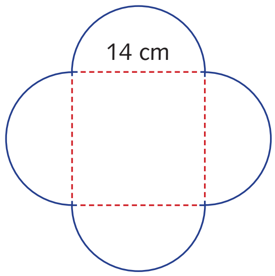
b What is the approximate value of the perimeter of a semicircle with radius 42 cm if we use $\pi \approx \frac{22}{7}$?



16 Four semicircles are drawn along the edges of a square with side length 14 cm.

a Find the perimeter of the region, giving your answer in terms of π.

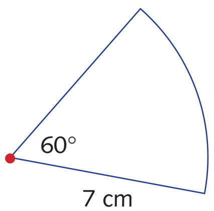
b Find the approximate value of the perimeter, using $\pi \approx \frac{22}{7}$.



17 The perimeter of a sector is the length of the arc plus twice the radius.

a Find the perimeter of the sector with radius 7 cm in which the angle at the centre is 60°. Leave your answer in terms of π.

b Now use $\pi \approx \frac{22}{7}$ to find the approximate value of the perimeter in part **a**.

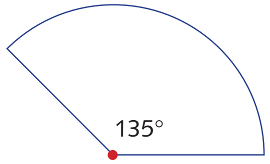


18 Use $\pi \approx 3.14$ to find the approximate value of the perimeter of the sector with:

a radius 10 m, containing an angle of 30°

b radius 9 cm, containing an angle of 10°

c radius 24 m, containing an angle of 135°



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14C Area of a circle

We know that the area of a rectangle is the length times the width. In earlier chapters, we used this idea to find simple formulas for the areas of squares, rectangles and triangles. We will now describe a way to find the **area of a circle**. By this we mean the area *inside* the circle. This problem is much more difficult than finding the area of a polygon (a straight-edged figure) such as a triangle or a quadrilateral.

The ancient Greek mathematician Archimedes came up with a number of clever ways to find a formula for the area of a circle in terms of the radius. The Greeks discovered that if you double the radius of a circle, then the area increases by a factor of 4. If you triple the radius, then the area increases by a factor of 9. The area of a circle appears to be related to the square of the radius.

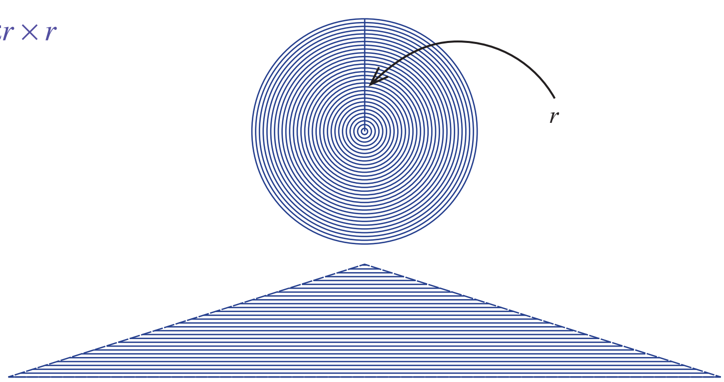
Here are two ways to dissect a circle to find a formula for its area.

Dissection using concentric circles

First, suppose that we have a circle of radius r, and imagine that the inside of the circle is made up of a large number of concentric circular pieces of very thin string. The circles are then cut along a radius, straightened and laid one on top of the other, as shown in the diagram below.

These strings will now form a figure that is approximately a triangle. (The thinner the pieces of string, the closer we come to a triangle.) The base of the triangle is the circumference of the original circle, which is πd or $2\pi r$, and the height of the triangle is the radius r of the original circle, so the area inside the circle is approximately:

$$\begin{aligned}\frac{1}{2}\times \text{base}\times\text{height} &= \frac{1}{2}\times 2\pi r\times r\\ &= \pi r^2\end{aligned}$$



Dissection by sectors

The second way of finding the area is to dissect the circle into sectors, just as you would a pizza. These sectors can then be arranged alternately, as in the diagram on the next page, to form a shape that is approximately a rectangle with a width that is r and with a length that is half the circumference – that is, πr. The smaller the sectors, the closer the shape is to being a rectangle. Thus the area is approximately:

$$\begin{aligned}\text{length}\times\text{width} &= \pi r\times r\\ &= \pi r^2\end{aligned}$$

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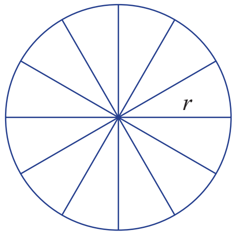

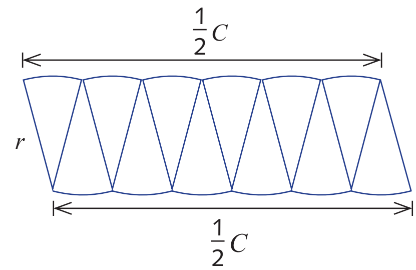

The formula for the area of a circle

The dissections shown above strongly suggest that the correct formula for the area of a circle with radius r is:

$$A = \pi r^2$$

A formal proof of this result requires an understanding of the concept of limits, which you will study in senior mathematics.

This formula should be memorised.

Example 3

Find the area of a circle:

a with radius 7 cm

b with diameter 7 cm

Give each answer:

i in terms of π

ii as an approximate value, using $\pi \approx \frac{22}{7}$

Solution

a i $A = \pi r^2$

$= 49\pi \text{ cm}^2$

ii $A = 49\pi$

$\approx 49 \times \frac{22}{7}$

$= 154 \text{ cm}^2$

b i Since the diameter is 7 cm, the radius is $\frac{7}{2}$ cm.

$A = \pi r^2$

$= \frac{49}{4}\pi \text{ cm}^2$

ii $A = \frac{49}{4}\pi$

$\approx \frac{49}{4} \times \frac{22}{7}$

$= 38\frac{1}{2} \text{ cm}^2$

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Remember that these answers are only approximate, since we have used an approximation for π.

Example 4

Using $\pi \approx 3.14$, find the approximate value of the area of a circle:

a with radius 10 cm

b with diameter 30 cm

Solution

a $A = \pi r^2$
$\approx 3.14 \times 10 \times 10$
$= 314 \text{ cm}^2$

b Since the diameter is 30 cm, the radius is 15 cm.

$A = \pi r^2$
$\approx 3.14 \times 15 \times 15$
$= 706.5 \text{ cm}^2$

Area of a circle

The area of a circle is given by the formula

area $= \pi \times (\text{radius})^2$, or $A = \pi r^2$

Exercise 14C

1 Use the measurements you made in question **2** of Exercise 14B to find (approximately) the area of a 20c coin, in square millimetres. Repeat the exercise with a 10c coin and a 5c coin.

Example 3i

2 Find the area of a circle with the given radius. Give each area in terms of π.

a 14 cm **b** 7 cm **c** $3\frac{1}{2}$ mm **d** 42 m

Example 3ii

3 Use $\pi = \dfrac{22}{7}$ to find the approximate value of the area of a circle with radius:

a 14 cm **b** 7 cm **c** $3\frac{1}{2}$ mm **d** 42 m

4 Find the area of a circle with the given diameter. Give each area in terms of π.

a 14 cm **b** 7 cm **c** $3\frac{1}{2}$ mm **d** 42 m

5 Use $\pi \approx \dfrac{22}{7}$ to find the approximate value of the area of a circle with diameter:

a 14 cm **b** 7 cm **c** $3\frac{1}{2}$ mm **d** 42 m

6 Find the area of a circle with the given radius. Give each area in terms of π.

a 10 cm **b** 5 cm **c** 20 mm **d** 15 m

Example 4

7 Use $\pi \approx 3.14$ to find the approximate value of the area of a circle with radius:

a 10 cm **b** 5 cm **c** 20 mm **d** 15 m

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8 Find the area of a circle with the given diameter. Give each area in terms of π.

a 10 cm **b** 5 cm **c** 20 mm **d** 15 m

9 Use $\pi \approx 3.14$ to find the approximate value of the area of a circle with diameter:

a 10 cm **b** 5 cm **c** 20 mm **d** 15 m

10 If the diameter of a circle is doubled, what happens to its area?

11 A circle has area 1386 cm^2. Using $\pi \approx \frac{22}{7}$, find the approximate value of the radius of the circle.

12 A circle has an area of 314 cm^2. Using $\pi \approx 3.14$, find the approximate value of the radius of the circle.

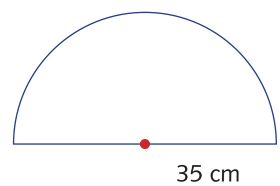
13 If the area of a circle is divided by 9, what happens to the radius? What happens to the circumference?

14 If the area of a circle is divided by 16, what happens to the diameter? What happens to the circumference?

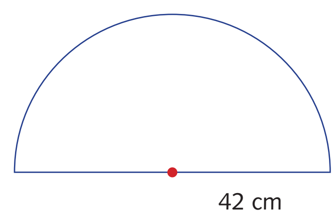
15 **a** What is the area inside a semicircle if the radius of the semicircle is 35 cm? Give your answer in terms of π.

b Find an approximate answer, using $\pi \approx \frac{22}{7}$.

(Note that the dot indicates the centre of the full circle.)



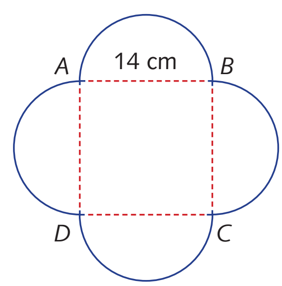
16 What is the approximate value of the area of a semicircle with radius 42 cm if we use $\pi \approx \frac{22}{7}$?



17 Four semicircles are drawn along the edges of a square with side length 14 cm.

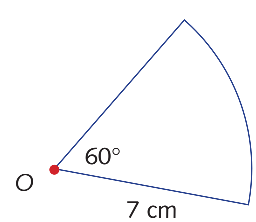
a Find the area of the entire region, giving your answer in terms of π.

b Find the approximate value of the area, using $\pi \approx \frac{22}{7}$.



18 **a** Find the area of the sector with radius 7 cm that contains an angle of 60°. Leave your answer in terms of π.

b Now use $\pi \approx \frac{22}{7}$ to find the approximate value of the area in part **a**.



19 Use $\pi \approx 3.14$ to find the approximate value of the area of a sector with:

a radius 10 m, containing an angle of 30°

b radius 9 cm, containing an angle of 10°

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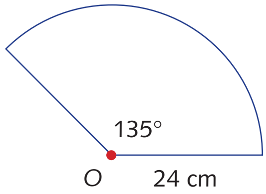
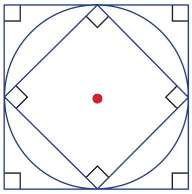
20 Find the area of the sector shown opposite. Leave your answer in terms of π.



21 A circle has radius r. Find the area of the circle and the two squares to show that $2 < \pi < 4$.

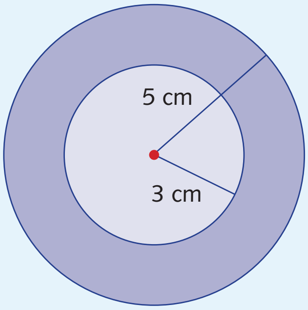


14D Areas of composite figures

We can now find the areas of more complicated figures, using the ideas of addition and subtraction of areas that we have previously used for such calculations involving rectangles and triangles.

Example 5

Find the shaded area enclosed between the circles in the diagram below. The smaller circle has radius 3 cm while the larger has radius 5 cm.



Solution

The area enclosed between the two circles is simply the area of the larger circle minus the area of the smaller one.

$$\begin{aligned}\text{Area} &= (\pi \times 5^2) - (\pi \times 3^2)\\ &= 25\pi - 9\pi\\ &= 16\pi\ \text{cm}^2\end{aligned}$$

We can leave the area in terms of π, or approximate it using $\pi \approx \frac{22}{7}$, giving:

$$\begin{aligned}\text{Area} &= 16\pi\\ &\approx 16 \times \frac{22}{7}\\ &= \frac{352}{7}\\ &= 50\tfrac{2}{7}\ \text{cm}^2\end{aligned}$$

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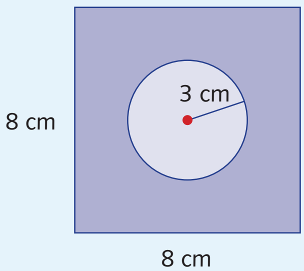
The region between two circles with a common centre is called an **annulus**, which is the Latin word for 'ring'.

Example 6

A circle of radius 3 cm is cut from a square with side length 8 cm.

Find the area of the remaining region:



a in terms of π

b using $\pi \approx \dfrac{22}{7}$

Solution

a Area of square $= 8 \times 8$

$= 64\text{ cm}^2$

Area of circle $= \pi \times 3^2$

$= 9\pi\text{ cm}^2$

Shaded area $= (64 - 9\pi)\text{ cm}^2$

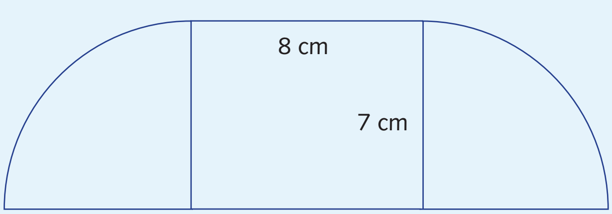
b Shaded area $\approx 64 - 9 \times \dfrac{22}{7}$

$= \dfrac{250}{7}$

$= 35\frac{5}{7}\text{ cm}^2$

Example 7

The figure below, consisting of a rectangle of length 8 cm and width 7 cm and two quarter-circles of radius 7 cm, is cut from a piece of cardboard.



a Find the area of the figure in terms of π.

b Find the approximate area of the figure, using $\pi \approx 3.14$.

Solution

a Area of rectangle $= 8 \times 7$

$= 56 \text{ cm}^2$

Area of quarter-circle $= \frac{1}{4} \times \pi \times 7^2$

$= \frac{49}{4}\pi \text{ cm}^2$

Area of figure $= 56 + 2 \times \frac{49}{4}\pi$

$= 56 + \frac{49}{2}\pi \text{ cm}^2$

b Area of figure $\approx 56 + \frac{49}{2} \times 3.14$

$= 132.93 \text{ cm}^2$

Area of an annulus

The area of an annulus is given by the formula

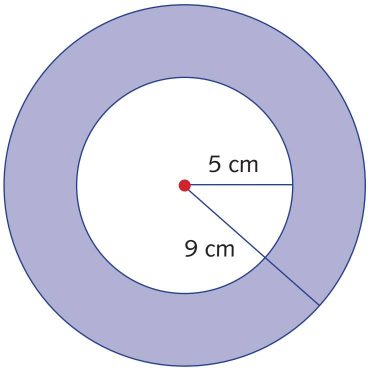
$A = \pi (R^2 - r^2)$,

where R is the radius of the outer circle, and r is the radius of the inner circle.

Exercise 14D

Example 5

1 Find the approximate value of the area of the annulus formed by a circle of radius 9 cm and one of radius 5 cm. Use $\pi \approx \frac{22}{7}$.



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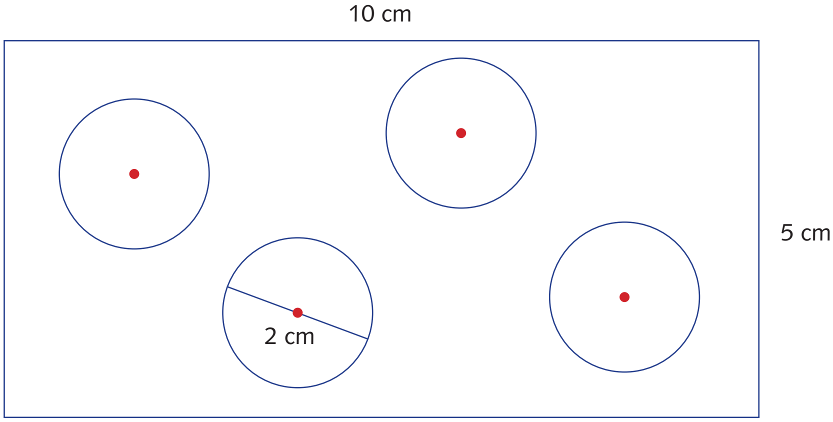
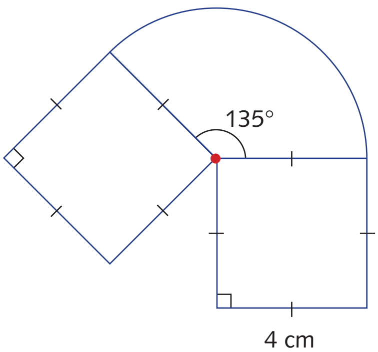
Example 6

2 Four coins, each with diameter 2 cm, are cut from a metal rectangle with side lengths that are 10 cm and 5 cm, as shown below. What is the remaining area? Give your answer in terms of π and then find an approximate value using $\pi \approx \frac{22}{7}$.



Example 7

3 The diagram to the right shows two squares and a sector. Find the area of this figure, leaving your answer in terms of π.



4 Jim's mum used a pastry cutter to cut 12 circles of dough, each of diameter 7 cm, from a rectangular slab of pastry that was 85 cm by 15 cm. What is the approximate value of the area of dough that was left? Use $\pi \approx 3.14$.

5 Which has the larger area:

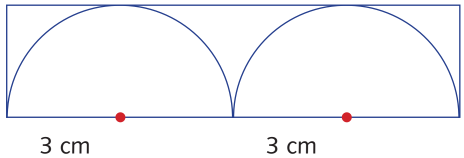
a a circle of diameter 10 cm or a square of side length 8 cm?

b a circle of radius 4 cm or a square of side length 7 cm?

c a circle of diameter 12 m or a triangle with base 13 m and height 17 m?

6 A square of side length 14 mm can just fit inside a circle of diameter 20 mm. Find the approximate value of the area of the circle that is not covered by the square. Use $\pi \approx 3.14$.

7 Two semicircles, each of radius 3 cm, are cut from a rectangle of length 12 cm, as shown below.



a What is the width of the rectangle?

b What is the area remaining? Leave your answer in terms of π.

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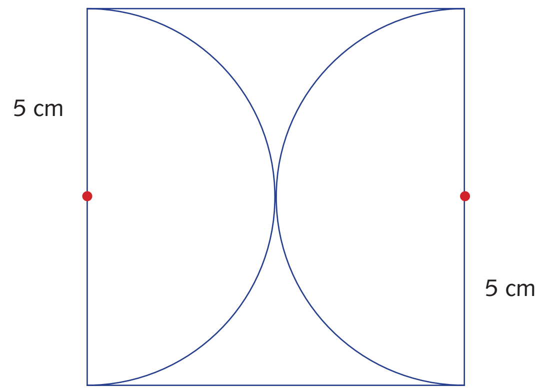
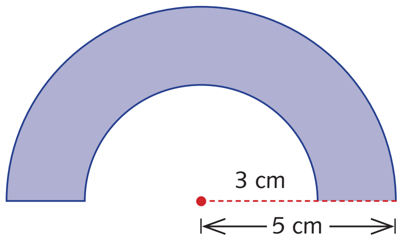
8 Two semicircles, each of radius 5 cm, are cut from a square, as shown below.

a What is the side length of the square?

b What is the approximate value of the remaining area? Use $\pi \approx 3.14$.



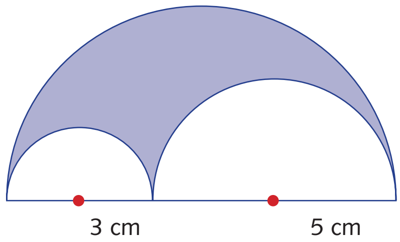
9 Find the approximate value of the shaded area shown opposite. Use $\pi \approx \frac{22}{7}$.



10 Two semicircles, of radii 3 cm and 5 cm, are cut from a larger semicircle, as shown opposite.

a What is the radius of the largest semicircle?

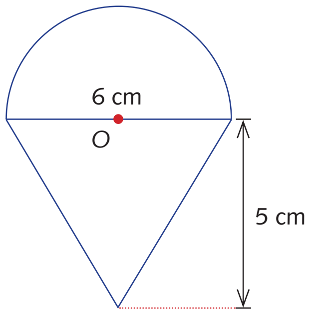
b What is the approximate value of the remaining area? Use $\pi \approx 3.14$.



11 An athletics track is made up of two parallel straight sections that are 102 metres long and 62 metres apart, with two semicircular ends. Calculate the approximate distance travelled by an athlete who runs one lap of this track. Use $\pi \approx \frac{22}{7}$.

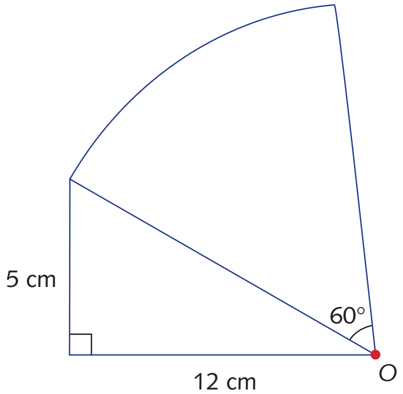
12 Find the area of the following figures. Give your answers in terms of π.

a



A semicircle on a triangle.

b



A sector of a circle, centre *O*, on a right-angled triangle.

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Review exercise

1 Use your compasses, ruler and protractor to draw:

a a circle with radius 3 cm

b a circle with diameter 8 cm

c a sector with radius 5 cm and an angle of 110° at the centre

d a semicircle with radius 4 cm

e a quadrant with radius 5 cm

2 Find the circumference of each of these circles. Leave your answers in terms of π.

a Radius 2 m

b Diameter 86 mm

c Radius 12 mm

d Radius 187 cm

3 Use $\pi \approx 3.14$ to find the approximate value of the circumference of each circle in Question **2**.

4 Use $\pi \approx \frac{22}{7}$ to find the approximate value of the circumference of each of these circles.

a Radius 14 mm

b Diameter 35 cm

c Radius 42 m

5 Find the approximate value of the perimeter of each figure, using $\pi \approx 3.14$.

a A semicircle with diameter 9 cm

b A quadrant with radius 2 cm

c A quadrant with radius 5 cm

d A semicircle with radius 8 mm

e A sector with radius 14 cm containing an angle of 120°

6 Find the area of each of the following circles, giving your answers in terms of π.

a Radius 21 mm

b Diameter 14 m

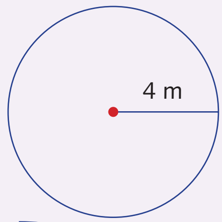
c Radius 63 cm

d Radius 35 m

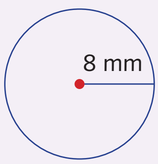
7 Use $\pi \approx \frac{22}{7}$ to find the approximate value of the area of each of the circles in Question **6**.

8 Find the approximate value of the area of each figure shown below. Use $\pi \approx 3.14$.

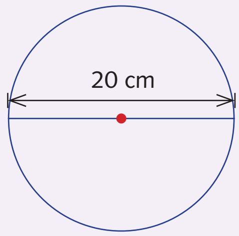
a



b



c



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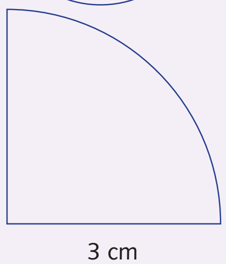

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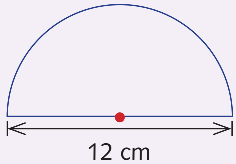

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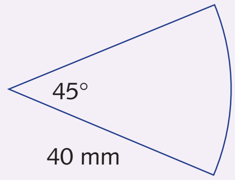

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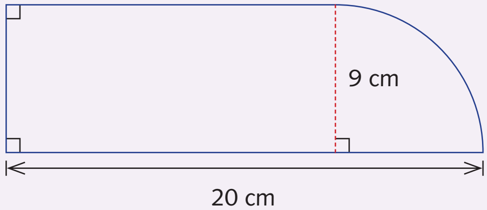
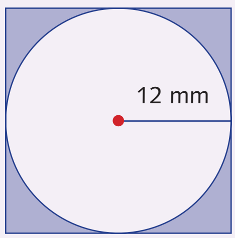
9 Find the approximate value of the area of the annulus formed by a circle of radius 10 cm and a circle radius 4 cm. Use $\pi \approx \frac{22}{7}$.

10 Find the area of the figure shown below, leaving your answer in terms of π.

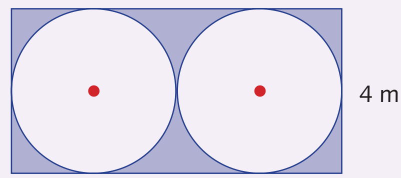


11 Find the approximate value of the shaded area for each of these figures. Use $\pi \approx 3.14$.

a



b



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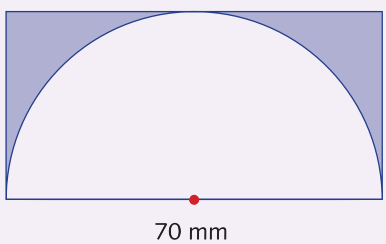

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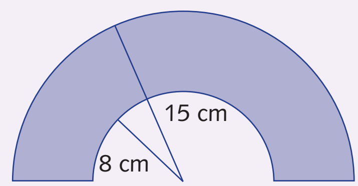
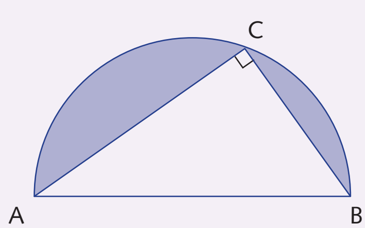


12 A tyre has a diameter of 52 cm. Calculate the circumference of the tyre. Give your answer in terms of π.

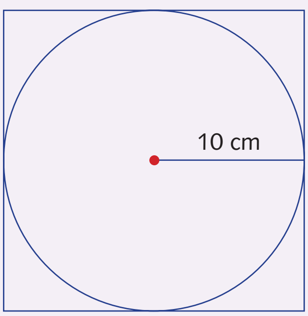
13 Meredith has a circular table with diameter 1.2 m. What is the approximate area of a circular tablecloth that would cover the table and have 5 cm hanging over the edge? Use $\pi \approx 3.14$.

14 A CD has diameter 11.8 cm, and the hole in the centre has diameter 1.4 cm. Find the approximate area of the CD, using $\pi \approx 3.14$.

15 AB is the diameter of the semicircle shown opposite. If $AC = 4$ cm, $CB = 3$ cm and $\angle ACB = 90°$, find the area of the shaded region. Give your answer in terms of π.



16 In the diagram opposite, the circle with radius 10 cm touches the insides of the square. Find the ratio of the area of the square to the area of the circle. Give your answer in terms of π.



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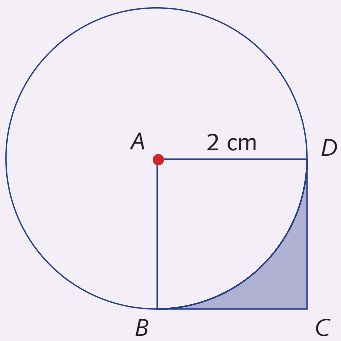
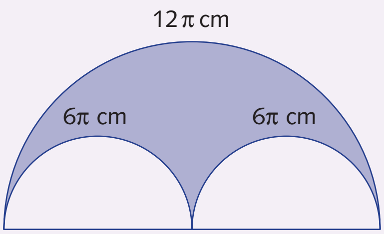
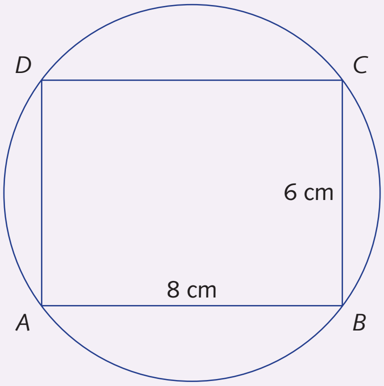
17 The circle in the diagram has centre A and radius 2 cm. If $ABCD$ is a square, find the area of the shaded region. Give your answer in terms of π.



18 The arc lengths of three semicircles are given. Find the area of the shaded region.

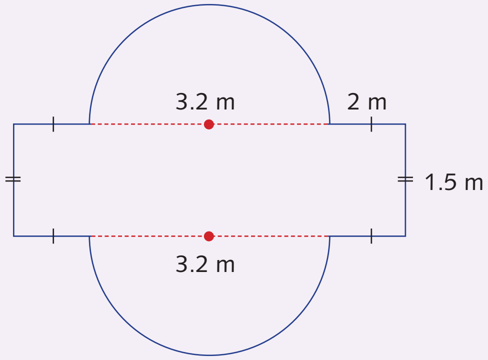


19 In the diagram to the right, $ABCD$ is a rectangle in which $AB = 8$ cm and $BC = 6$ cm. Find the area of the circle in cm^2.

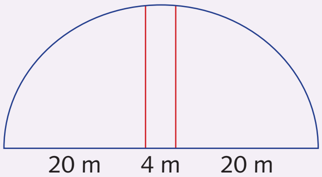


20 A Milo tin has a diameter of 10 cm. How far will it move if it is rolled through six revolutions?

21 Find the approximate value of the perimeter of the figure to the right, using $\pi \approx 3.14$.



22 A hockey goal 'circle' is made up of a rectangle and two quadrants, as shown below. Find its approximate area, using $\pi \approx 3.14$.



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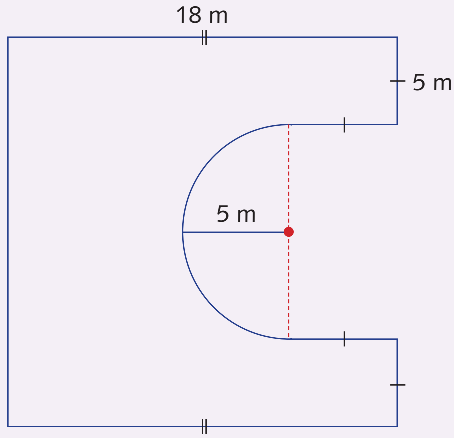
Challenge exercise

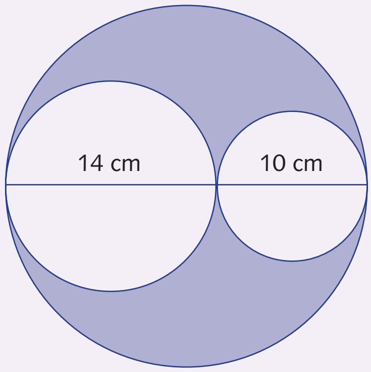
1 For the figure shown below, use $\pi \approx 3.14$ to find the approximate value of:

a the perimeter

b the area



2 Find the shaded area of the figure below, leaving your answer in terms of π.



3 A circular swimming pool has a diameter of 10 m and is 1.3 m deep. The interior of the pool was covered with square tiles of side length 2 cm. Approximately how many tiles were required?

4 Suppose we roll a circle of radius 1 cm around a larger circle of radius 4 cm without slipping.

a How many times has the small circle rotated in completing exactly one revolution of the larger circle?

b What happens when the smaller circle rolls on the inside of the larger circle?

5 The hypotenuse of a right-angled triangle is 15 cm. A circle of radius 2 cm is drawn inside the triangle so that it touches each of the three sides. What is the perimeter of the triangle?

6 By how much does the circumference of a circle increase if we increase its diameter by π units?

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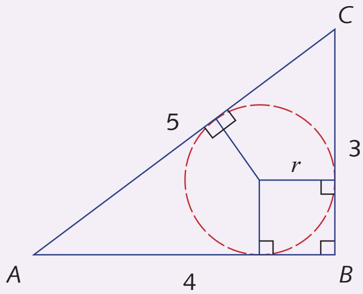
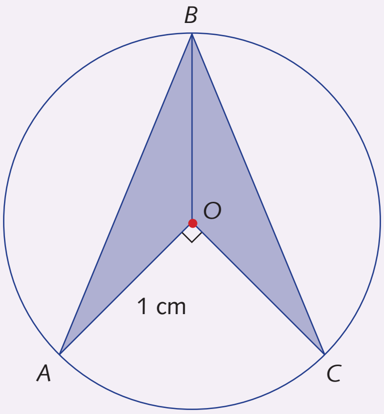
7 An arc of a circle C_1 subtends an angle of 60° at the centre, and has the same length as an arc of a circle C_2 subtending an angle of 45° at the the centre. Find the ratio of the area of C_1 to C_2.

8 **a** *ABC* is a right-angled triangle with a right angle at *B*. The circle shown is the 'incircle' of the triangle. Find the radius, *r*, of the incircle.

b Find the radius of the incircle of a right-angled triangle with sides 5, 12 and 13.



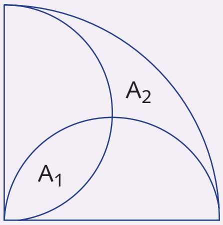
9 Triangles *BOC* and *BOA* in the diagram on the right are congruent and $\angle AOC = 90°$. The radius of the circle centre *O* is 1 cm. Find the area of the shaded region.



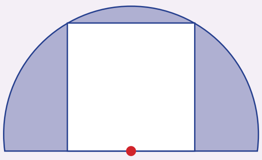
10 The vertices of a square lie on a circle of radius 4 cm. Find the ratio of the area of the circle to the area of the square in terms of π.

11 Two semicircles are drawn in a quadrant of radius 2 cm.

Find the ratio of the area (A_1) : area (A_2).



12 The vertices of a square lie on the boundary of a semicircle of radius 10 cm. Find the shaded area shown in terms of π.



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CHAPTER

15

Measurement and Geometry

Areas, volumes and time

The **area** of a plane figure is a measure of the amount of the size of the interior region. Calculating areas is an important skill used by many people in their daily work. Builders and tradespeople often need to work out the areas and dimensions of the things they are building, and so do architects, designers and engineers.

In this chapter, we will revise and extend the range of figures whose areas we can calculate. This will include parallelograms, kites, trapeziums and rhombuses. Our basic approach is to dissect these figures into simpler ones whose areas we can already calculate.

When we consider solids, we use the idea of **volume** to measure the size of the interior region. We will learn how to find the volumes of a range of solids known as **prisms**, and also how to find the volume of a cylinder. Finally, we will look at how to find the surface area of a prism by looking at each of its faces.

In this chapter, we develop a number of fundamental formulas for areas and volumes. You should commit these to memory. You should also know where these formulas come from and how they work.

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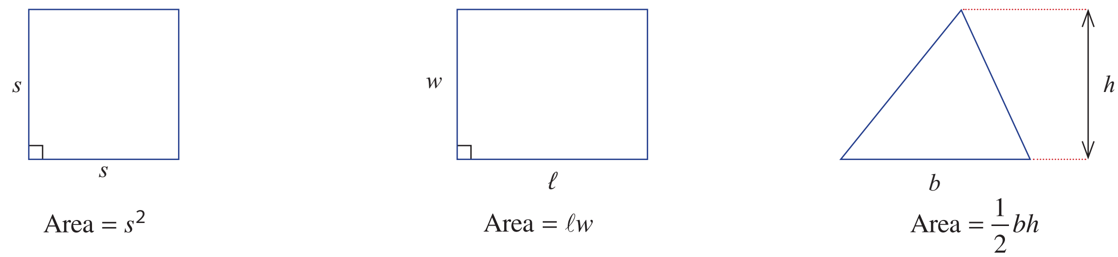
15A Review of area and length

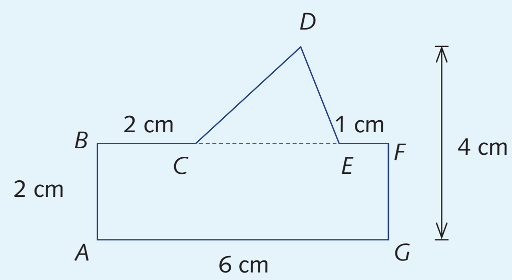
We have already met the formulas for the areas of squares, rectangles and triangles, as shown below.



We can use these basic shapes to find the areas of more complicated figures, just as we did in the previous chapter. If a region can be broken into non-overlapping rectangular or triangular pieces, then the area of the region is the sum of the areas of the pieces.

Example 1

The region below consists of a triangle on top of a rectangle. Find the area of the region.



Solution

Triangle CBE has base $CE = 3$ cm and height 2 cm.

$$\begin{aligned}\text{Area of triangle } CDE &= \frac{1}{2} \times 3 \times 2\\ &= 3\text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area of rectangle } ABFG &= 6 \times 2\\ &= 12\text{ cm}^2\end{aligned}$$

Total area of the figure is 15 cm^2.

Sometimes a region is obtained by removing a piece (or pieces) from a larger one. In this case, we subtract the areas of the removed parts.

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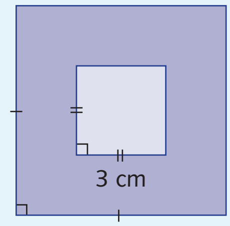
Example 2

Find the area of the figure enclosed between the two squares.



Solution

$$\begin{aligned}\text{Area} &= 7^2 - 3^2\\ &= 49 - 9\\ &= 40\ \text{cm}^2\end{aligned}$$

Area of a composite figure

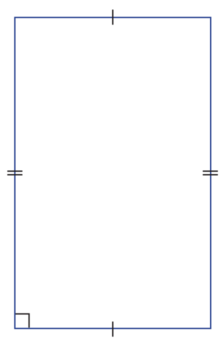
- To find the area of a composite figure, we dissect it into simpler figures and add the areas of the these pieces.
- We can also find areas by subtraction.

Exercise 15A

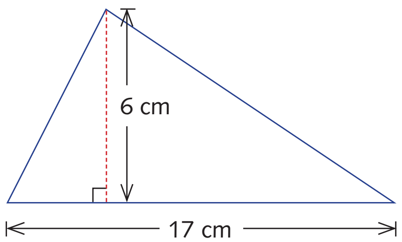
Example 1

1 Find the area of each figure.

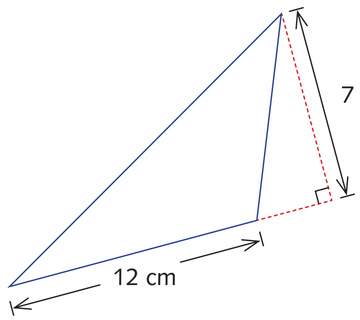
a



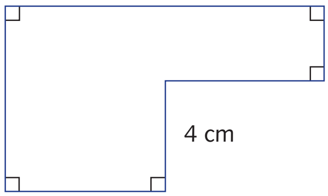
b



c



d



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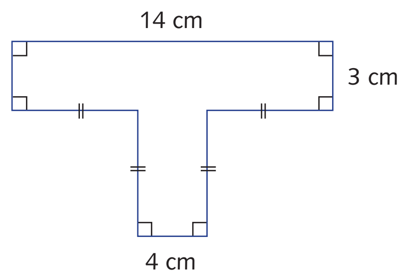
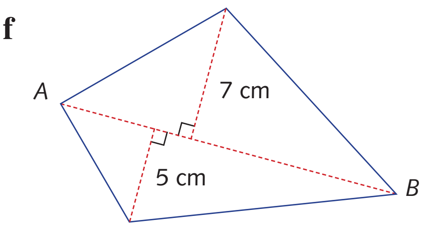
e



f

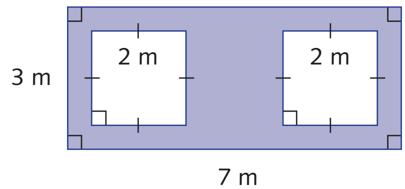


$AB = 18$ cm

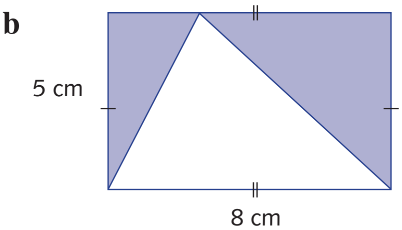
Example 2

2 Find the area of the shaded region in each figure.

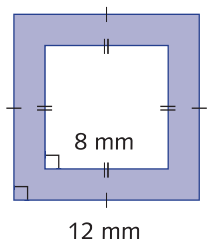
a



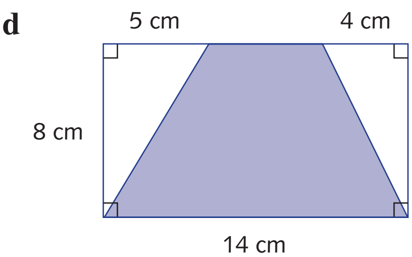
b



c



d

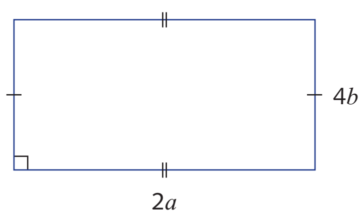


3 A rectangular orchard is 520 m long and 300 m wide. Find the area of the orchard in hectares. (Recall that 1 hectare = 10 000 m^2.)

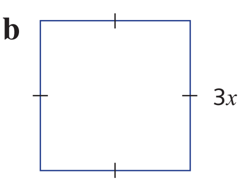
4 The base and height of a triangle are whole numbers and its area is 12. Find all values of the base and height.

5 Write down an algebraic formula, in simplest form, for the area of each figure.

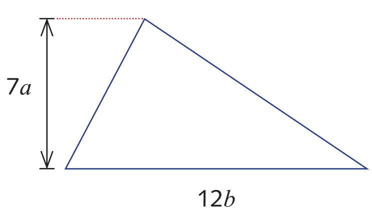
a



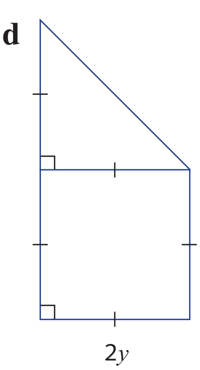
b



c



d



6 A room has 4 walls and 2 doorways. The walls each have dimensions 7 m by 5 m, and the doorways have dimensions $\frac{3}{4}$ m by 2 m. If the inside walls of the room must be painted, how much will this cost if the rate for painting is \$5 per square metre?

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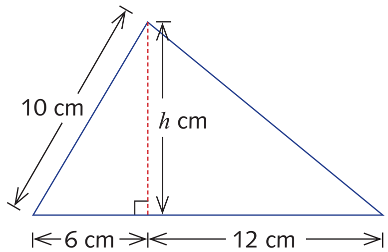
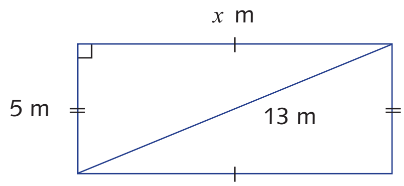
7 Use Pythagoras' theorem to find h, and hence find the area of the triangle.

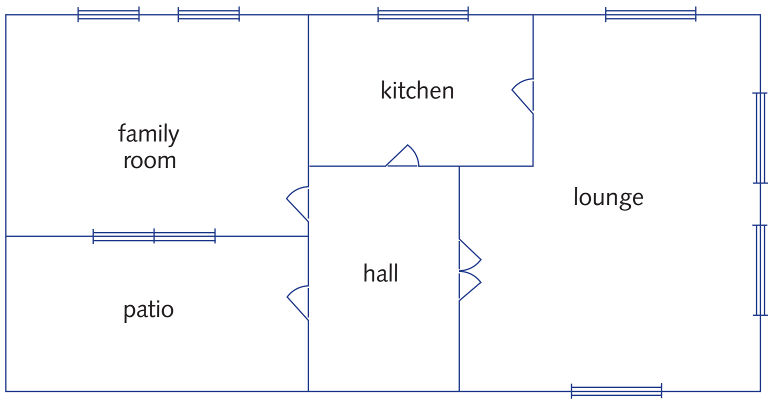


8 Find x and hence find the area of the rectangle.



9 Find the area of the floor of the room that is left uncovered if a carpet measuring 4 m by 3 m is laid in a room 5 metres square.

10 This diagram is the ground floor plan of a house drawn to a scale of 1 cm to represent 2 metres. (Take width to be left to right.)
Take measurements and answer the questions below.



a What is the length of the house?

b What is the width of the house?

c Find the length and width of the family room.

d What is the area of the kitchen?

e What is the width of the lounge at its widest point?

f What is the area of the patio?

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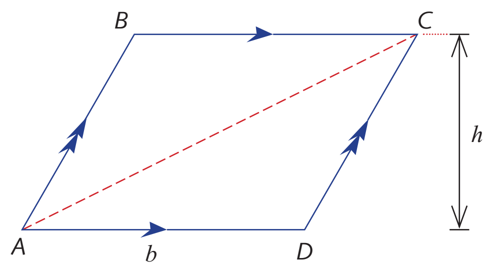
15B Areas of special quadrilaterals

Area of a parallelogram

A **parallelogram** has each pair of its opposite sides parallel, as shown below.



To find the area of this figure, first draw the diagonal AC to form triangles ABC and ADC.

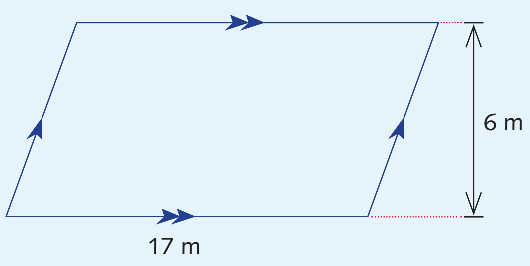
The area of $\Delta ABC = \frac{1}{2} \times b \times h$. Similarly, the area of $\Delta ADC = \frac{1}{2} \times b \times h$.

$$\begin{aligned} \text{The area of the parallelogram } ABCD &= \text{area of } \Delta ADC + \text{area of } \Delta ABC \\ &= \frac{1}{2}bh + \frac{1}{2}bh \\ &= bh \end{aligned}$$

Area of a parallelogram = base × height = bh

Example 3

Find the area of the parallelogram shown below.



Solution

$$\begin{aligned} \text{Area} &= \text{base} \times \text{height} \\ &= 17 \times 6 \\ &= 102\ \text{m}^2 \end{aligned}$$

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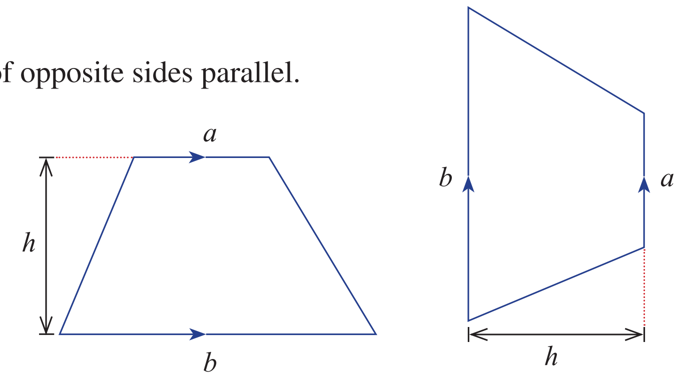
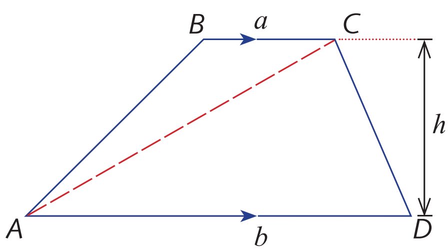
Area of a trapezium

A **trapezium** is a quadrilateral with one pair of opposite sides parallel.



To find the area of a trapezium, we need to know the lengths of the two parallel sides and the perpendicular distance between these two sides, which we will call the **perpendicular height** of the trapezium.

To find the formula for the area of a trapezium first draw the diagonal AC to form triangles ABC and ADC. The height of both triangles is h.

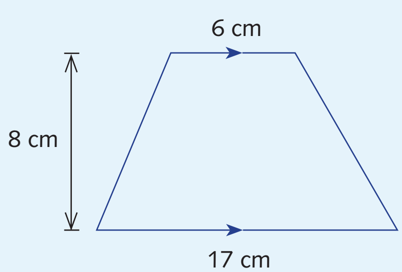


$$
\begin{aligned}
\text{The area of the trapezium} &= \text{area of } \Delta ABC + \text{area of } \Delta ADC \\
&= \frac{1}{2} \times a \times h + \frac{1}{2} \times b \times h \\
&= \frac{1}{2}ah + \frac{1}{2}bh \\
&= \frac{1}{2}h(a+b)
\end{aligned}
$$

$$\text{Area of a trapezium} = \frac{1}{2}h(a+b)$$

Example 4

Find the area of the trapezium shown opposite.



Solution

$$
\begin{aligned}
\text{Area} &= \frac{1}{2}h(a+b) \\
&= \frac{1}{2} \times 8 \times (6+17) \\
&= 92 \text{ cm}^2
\end{aligned}
$$

Note: Every parallelogram is a trapezium, but not every trapezium is a parallelogram.

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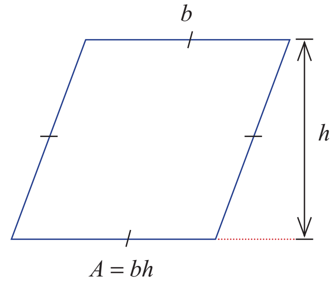
Area of a rhombus and a kite

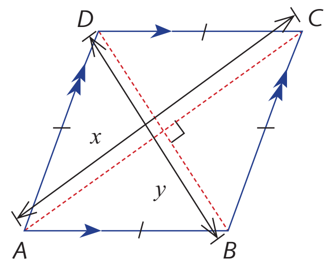
A **rhombus** is a quadrilateral with all four sides equal.



Since a rhombus is also a parallelogram, the area A is given by:

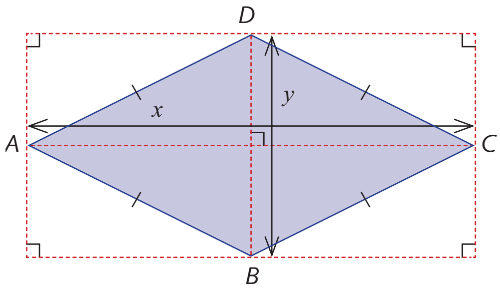
$$A = bh$$

It was shown in Chapter 13 that the diagonals of a rhombus bisect each other at right angles.



There is a formula for the area of a rhombus in terms of the product of the diagonals.

Take a rhombus and stand it on one corner. The two diagonals cut the rhombus into four right-angled triangles, which can be completed to form four rectangles inside a larger rectangle.



Since the eight triangles have the same area, the area of the rhombus is half the area of the large rectangle, which is $x \times y$. Hence if x and y are the lengths of the diagonals of a rhombus, then

$$\text{area of a rhombus} = \frac{1}{2}xy$$

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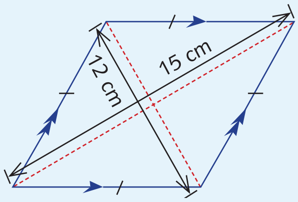
Example 5

a Find the area of the rhombus with diagonals 12 cm and 15 cm.



b A rhombus has area 144 cm^2 and one of the diagonals has length 12 cm. What is the length of the second diagonal?

Solution

a $\text{Area} = \frac{1}{2}xy$ (where x and y are the lengths of the diagonals)

$= \frac{1}{2} \times 12 \times 15$

$= 90 \text{ cm}^2$

b $144 = \frac{1}{2} \times 12 \times y$ (where y is the length of the other diagonal)

$144 = 6y$

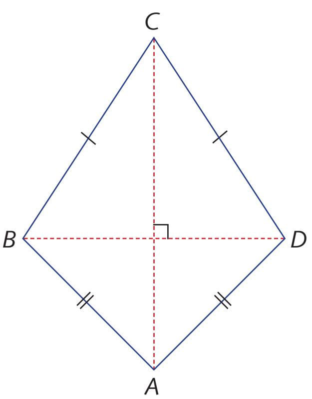
$24 = y$

The other diagonal has length 24 cm.

A **kite** is a quadrilateral that has two pairs of adjacent equal sides.

Note: A rhombus is clearly a kite but a kite is not necessarily a rhombus.

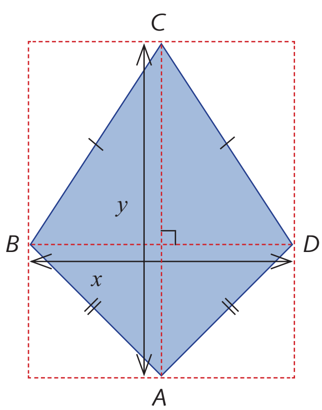
The diagonals AC and BD are perpendicular – can you show this using congruence?



As we did for the rhombus, we can complete the kite to form a rectangle whose area is twice that of the kite.

Hence for a kite with diagonals x and y

$$\text{area of a kite} = \frac{1}{2}xy$$



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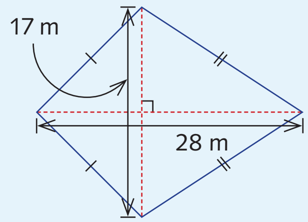
Example 6

a Find the area of a kite with diagonals 17 m and 28 m.



b A kite has area 256 cm^2 and one of its diagonals has length 8 cm. What is the length of the other diagonal?

Solution

a

$$\text{Area} = \frac{1}{2}xy$$
$$= \frac{1}{2} \times 17 \times 28$$
$$= 238 \text{ m}^2$$

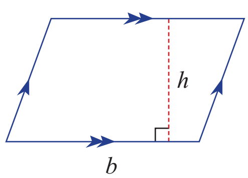
b

$$256 = \frac{1}{2} \times 8 \times y$$
$$256 = 4y$$
$$64 = y$$

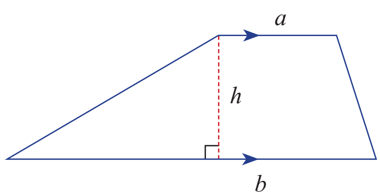
The length of the other diagonal is 64 cm.

Areas of special quadrilaterals

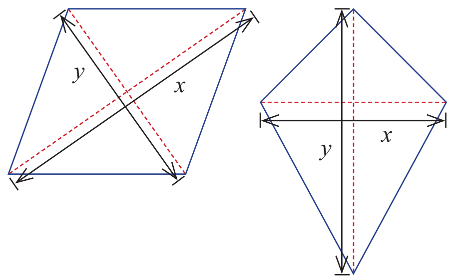
- Area of a parallelogram $= bh$



- Area of a trapezium $= \frac{1}{2}h(a+b)$



- Area of a rhombus or a kite $= \frac{1}{2}xy$



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Exercise 15B

1 The table below gives the bases, heights and areas of various parallelograms. Fill in the missing entries.

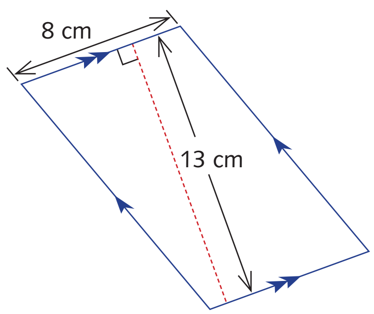
	Base	Height	Area
a	7 cm	6 cm	
b	9 cm		27 cm^2
c		17 m	85 m^2
d	12 km		144 km^2

	Base	Height	Area
e	500 km		30 000 km^2
f		50 m	6000 m^2
g	55 m		5500 m^2
h	16 cm		256 cm^2

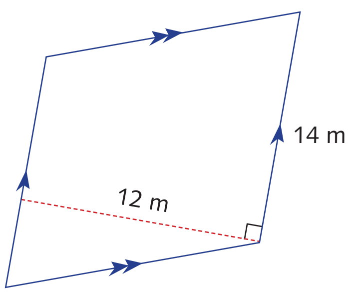
Example 3, 4

2 Find the areas of these regions.

a



b



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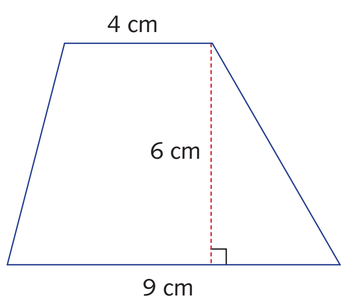

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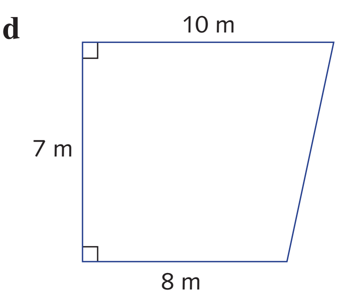

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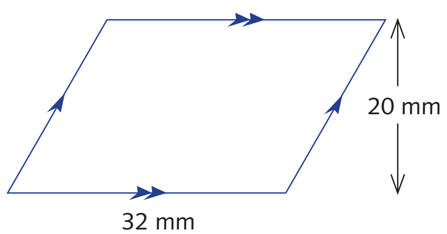

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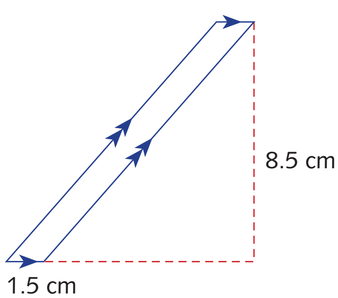

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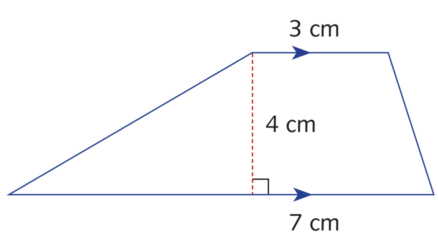

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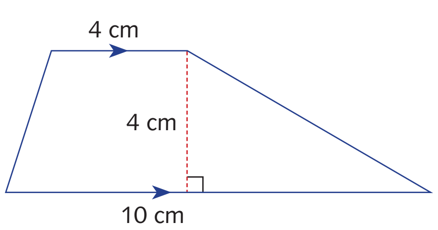

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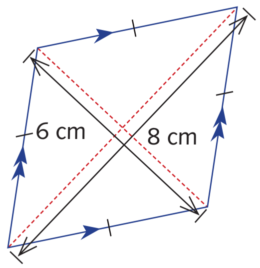
Example 5, 6

3 Find the area of these regions.

a



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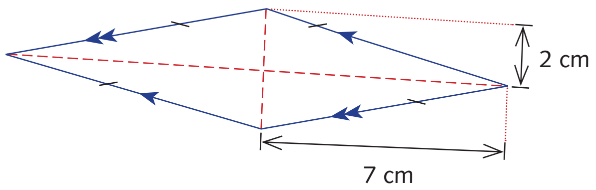

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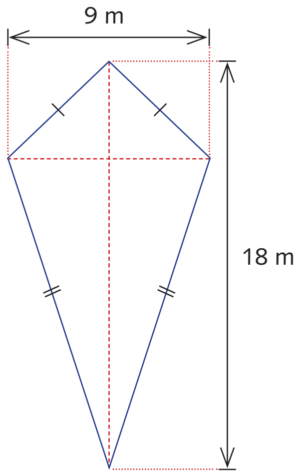

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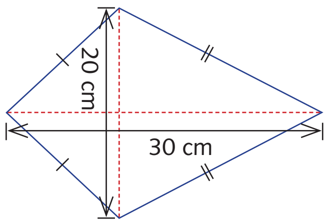
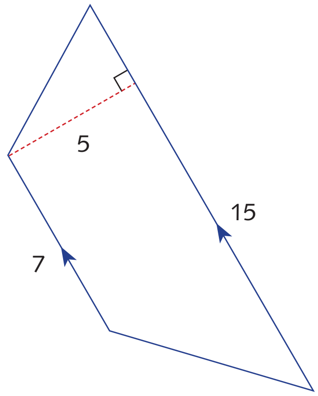


4 Find the areas of these trapeziums. All measurements are in centimetres.

a



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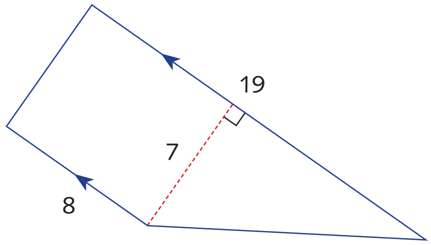

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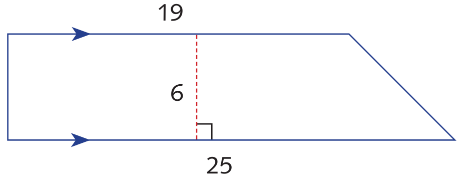

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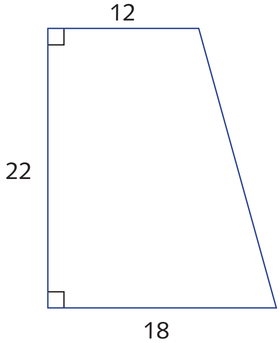

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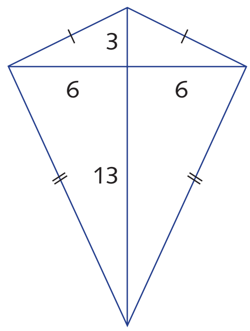
5 Find the area of each figure below.

a



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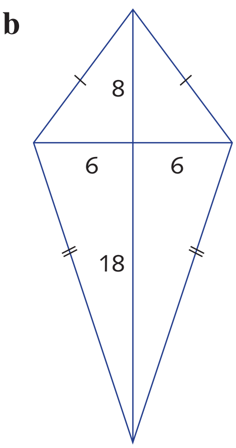
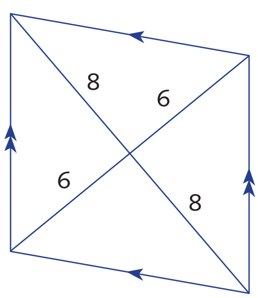


c



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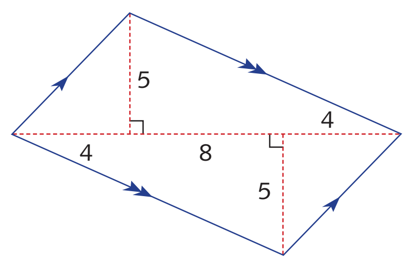
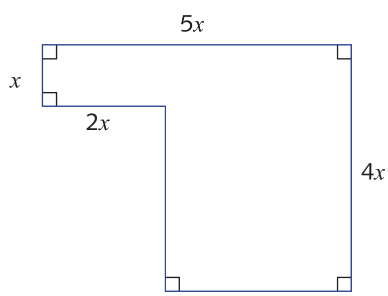
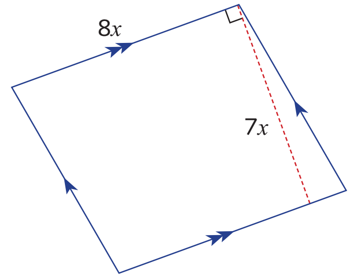


6 Find an algebraic expression, in simplest form, for the area of each figure below.

a



b



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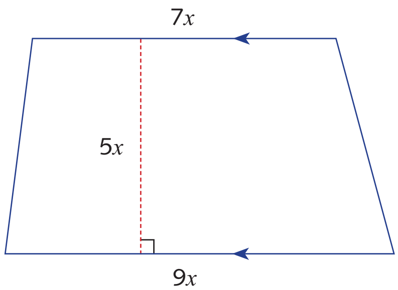

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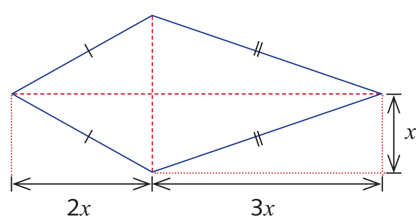
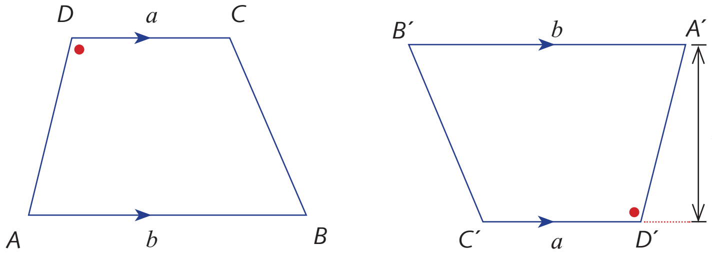


7 Prove that the area of a trapezium is $\frac{1}{2}h(a+b)$ by pasting together two identical trapeziums $ABCD$ and $A'B'C'D'$.



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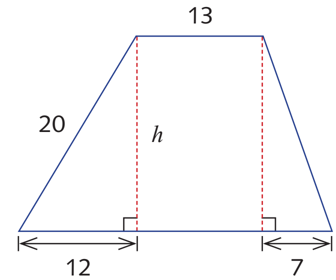
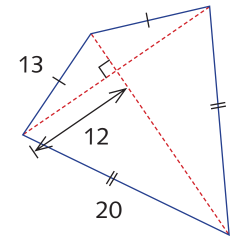
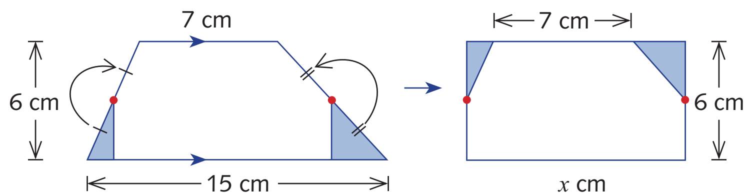
8 Use Pythagoras' theorem to find the height of this trapezium, and hence find its area.



9 Find the lengths of the diagonals of this kite, and hence find its area.



10 Here is another way of dissecting a trapezium to find its area. Take the midpoints of the two non-parallel sides and cut off the triangles, as shown. These are then moved up to form a rectangle.

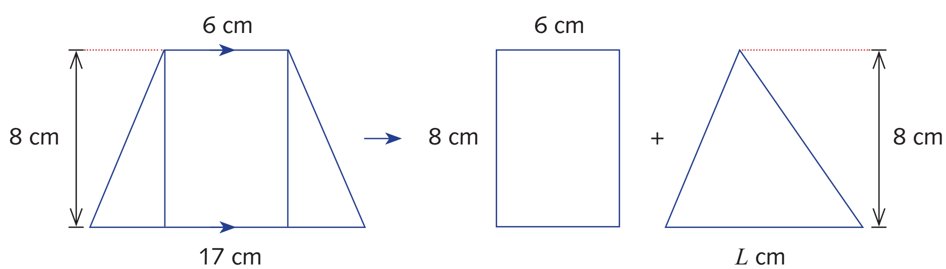


a What is the length x cm of the rectangle?

b Find the area of the trapezium, using the rectangle.

c Use the usual formula for the area of a trapezium in the first diagram to confirm your answer in part **b**.

11 Here is yet another way of dissecting a trapezium to find its area. Cut the two triangles from the end of the trapezium and rearrange them to form a rectangle and a triangle, as shown below.



a What is the value of L?

b Find the area of the trapezium.

c Can you draw a trapezium in which this method of dissection will not work?

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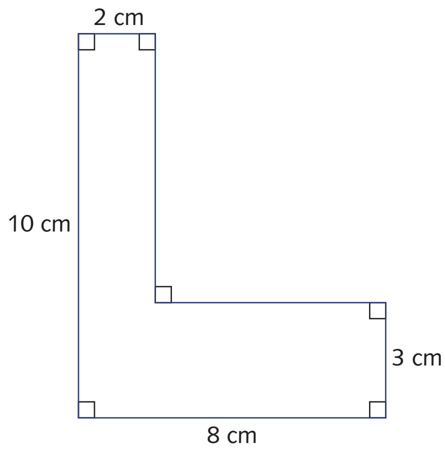
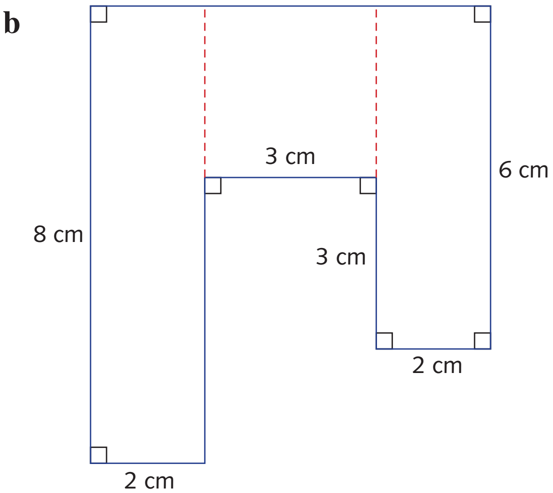
12 Find the area of the following shapes by first dividing them into rectangles.

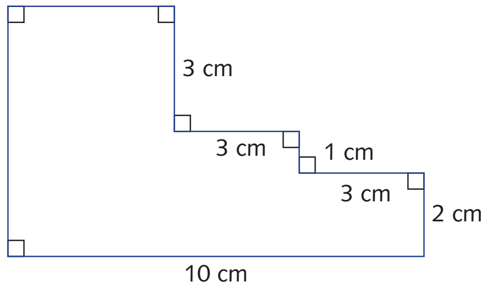
a



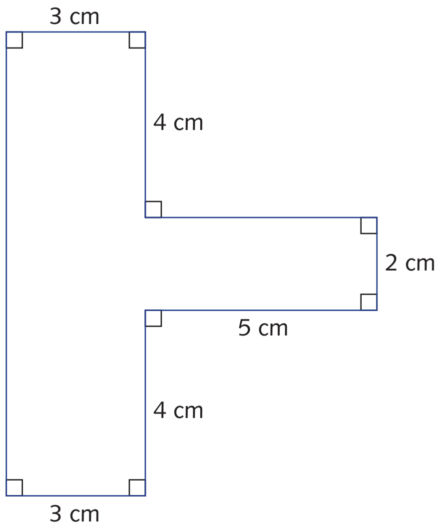
b



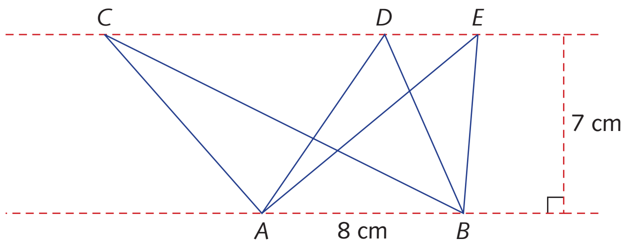
c



d



13 Write down the area of each of the triangles with base *AB*.



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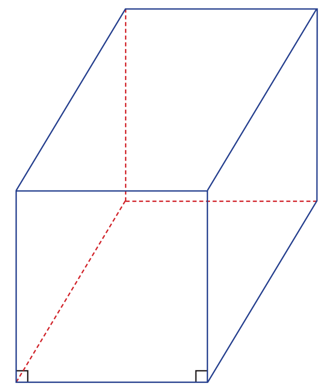
15C Volume of a rectangular prism

Look around the room you are in. Most rooms have a horizontal floor, and the ceiling is horizontal as well. The four walls are vertical. Here is a sketch of what the room might look like.



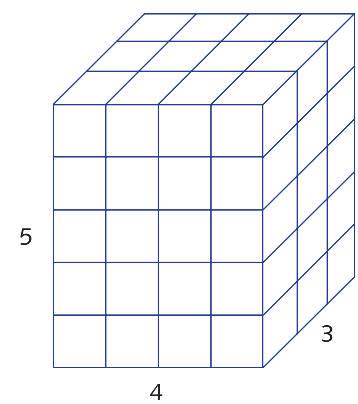
This is an example of a **right-rectangular prism**. Note that:

- the base is a rectangle
- if we slice it through any horizontal plane, the cross-section is a rectangle congruent to the base
- there are three right angles at each corner.

(The word 'right' is used here as a compact and convenient way of saying 'the walls are vertical'.) If the faces are all congruent then we call the solid a **cube**. An example of such a solid is a shoe box.

The **volume** of a rectangular prism is a measure of the space inside the prism.

Consider a rectangular prism of side lengths 5, 4 and 3. We call these numbers **dimensions** of the prism. (They could also be called the **length**, **width** and **height** of the prism, but since we can rotate the prism, these words are used rather loosely, so it is often better to talk about the dimensions.)



We can cut the prism up into cubes, each of side length 1, as shown. We say that the volume of each of these cubes is 1, which we read as '1 cubic unit'.

Altogether there are $5 \times 3 \times 4 = 60$ cubes, each of volume 1, so we say that the volume of the prism is 60.

In practice, the dimensions could be 3 cm, 4 cm and 5 cm. The unit cube will have volume 1 cm^3 and the box will have volume 60 cm^3.

In general, the volume of a rectangular prism is given by:

$$\begin{aligned}\text{volume of a rectangular prism} &= \text{length} \times \text{width} \times \text{height}\\ &= \ell wh\end{aligned}$$

Example 7

Find the volume of a rectangular prism with dimensions 4 cm, 16 cm and 3 cm.

Solution

$$\begin{aligned}\text{Volume} &= \ell wh\\ \text{Then } V &= 4 \times 16 \times 3\\ &= 192 \text{ cm}^3\end{aligned}$$

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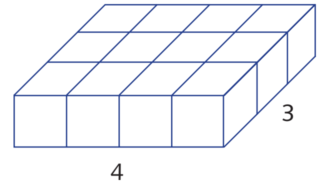
Slices and area



We can see that the rectangular prism is made up of five copies of the $4 \times 3 \times 1$ slice shown. The base area of the slice is 12 and the height is 1. We have 5 such slices in the original prism so we obtain volume $= 12 \times 5 = 60$ as before.

Thus the volume of a rectangular prism equals Ah, where A is the area of the base.

Note: The base can be any one of the three different faces of the prism.

The formulas above are still valid even when the dimensions are not whole numbers.

Right-rectangular prism

- A **right-rectangular prism** is a polyhedron in which:
  - the base is a rectangle
  - each cross-section parallel to the base is a rectangle congruent to the base
  - there are three right angles at each corner.
- Volume of a right-rectangular prism = length × width × height
  $= \ell wh$
- Volume of a right-rectangular prism = Area of base × height
  $= Ah$

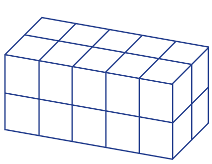
Exercise 15C

Example 7

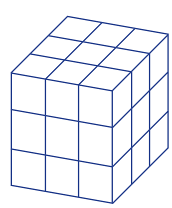
1 Find the volume of a rectangular prism whose dimensions are 12 mm, 10 mm and 7 mm.

2 If  is a 1-unit cube, find the volume of each of these figures in unit cubes.

a



b



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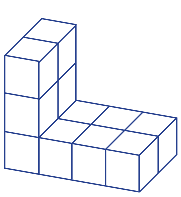

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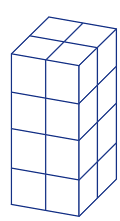

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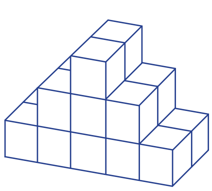

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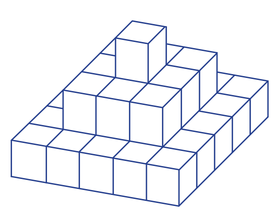

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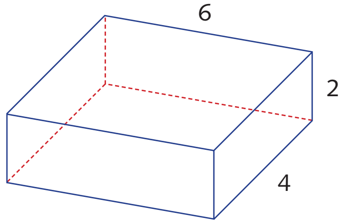
3 The table below gives the lengths, breadths, heights and volumes of various rectangular prisms. Fill in the missing entries.

	Length	Breadth	Height	Volume
a	3 cm	7 cm	6 cm	
b	5 cm	9 cm	4 cm	
c		3 m	2 m	18 m^3
d	3 m	12 m		144 m^3

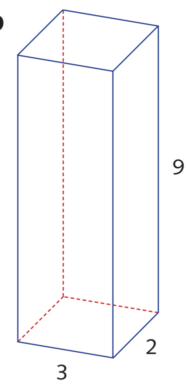
	Length	Breadth	Height	Volume
e	56 m	40 m	70 m	
f	2 mm	8 mm	16 mm	
g	32 m	4 m		1024 m^3
h	8 cm	8 cm		256 cm^3

4 Give the volume of each prism. All measurements are in cm.

a



b



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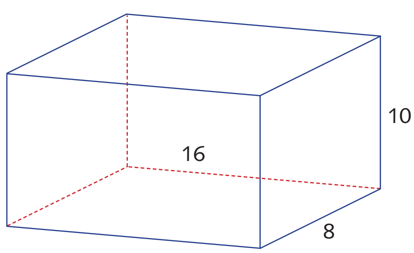

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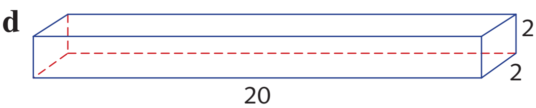
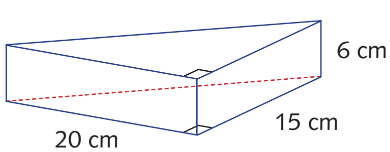


5 A swimming pool has dimensions 10 m by 12 m and is 5 m deep. What is its volume?

6 A rectangular water tank, with a base that is 1.7 m by 1.4 m, holds water to a depth of 2 m. What is the volume of the tank, in cubic metres?

7 A Rubik's cube has side length 7 cm. What is its volume?

8 A piece of cheese sold in the shops is obtained by cutting a rectangular block of cheese in two. What is the volume of the cheese?



9 A tank in the form of a rectangular prism has base dimensions 2.3 m by 1.4 m and holds water to a depth of 2 m. If the depth is increased by 3 m, what is the increase in the volume of the water?

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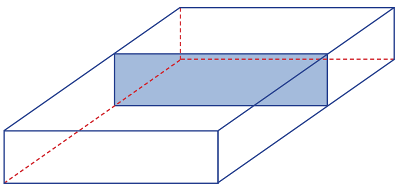
15D Volumes of other prisms

Recall that a **polyhedron** is a solid bounded by **polygons**. A **right prism** is a polyhedron that has two congruent and parallel faces called the base and top, and all its remaining faces are rectangles. A prism has **uniform cross-section**. This means that it is possible to take slices through the solid parallel to the base so that the area of each slice is always the same.

The name 'prism' comes from the Greek word *prizein*, which means 'to saw'. The idea is that if we 'saw' through a prism, the cross-sections are always the same.

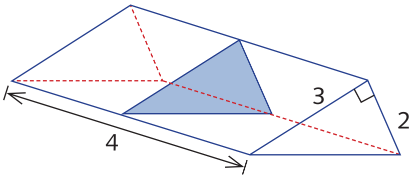
In a **rectangular prism**, the cross-section is always a rectangle.



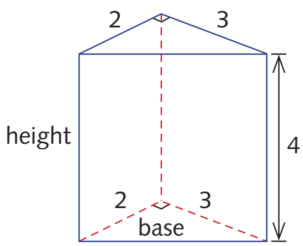
Volume of a triangular prism

In a **triangular prism**, the cross-section is always a triangle.

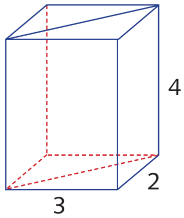
Here is a triangular prism with a cross-section that is a right-angled triangle.



We can redraw the prism as shown here.



This prism can be thought of as the rectangular prism with dimensions $2 \times 3 \times 4$ cut in half.



$$\begin{aligned}\text{Volume of triangular prism} &= \frac{1}{2} \times 2 \times 3 \times 4 \\ &= 12\end{aligned}$$

In this case the base of the prism is the right-angled triangle with shorter side lengths 2 and 3.

The area of this triangle $= \frac{1}{2} \times 2 \times 3$.

The volume of the prism is given by the area of the base of the prism multiplied by the height.

$$\begin{aligned} V &= 3 \times 4 \\ &= 12 \end{aligned}$$

The volume of any triangular prism whose base has area A and whose height (or depth) is h is given by:

volume of a prism $= Ah$

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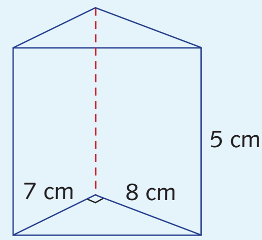
Example 8

Find the volume of the prism shown on the right.



Solution

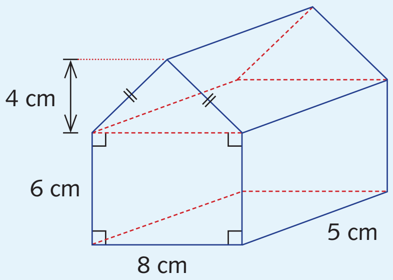
This is a triangular prism with area of base $A = \frac{1}{2} \times 8 \times 7$

$= 28\ \text{cm}^2$

$$\begin{aligned}\text{Volume} &= Ah\\ &= 28 \times 5\\ &= 140\ \text{cm}^3\end{aligned}$$

Example 9

Find the volume of the prism shown in the diagram.



Solution

The cross-section is the front face of the prism, a triangle on a rectangle.

$$\begin{aligned}A &= \left(\frac{1}{2} \times 8 \times 4\right) + (8 \times 6)\\ &= 64\ \text{cm}^2\end{aligned}$$

$$\begin{aligned}\text{Volume} &= Ah\\ &= 64 \times 5\\ &= 320\ \text{cm}^3\end{aligned}$$

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The base of the prism can be any polygon. Any polygonal prism can be be divided up into triangular prisms.

Volumes of other prisms

- A **right prism** is a polyhedron that has two congruent and parallel faces and all its remaining faces are rectangles.
- A prism has uniform cross-section.
- The volume of any prism whose base has area A and whose height is h is given by:

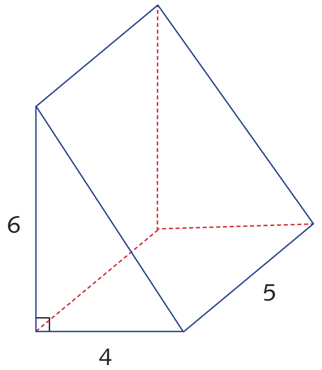
  volume of a prism = Ah

Exercise 15D

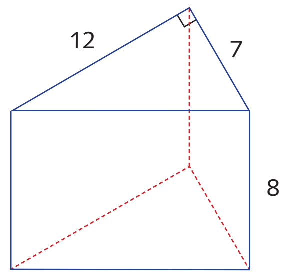
Example 8, 9

1 Find the volume of each prism. All measurements are in centimetres.

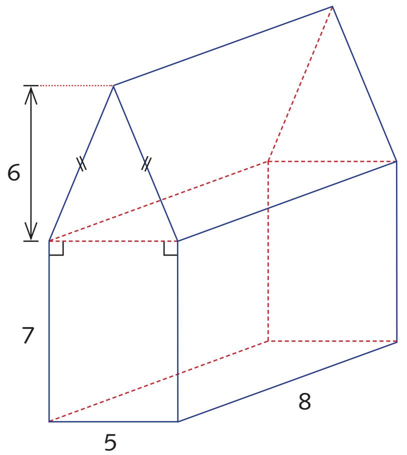
a



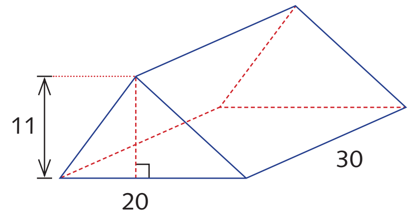
b



c



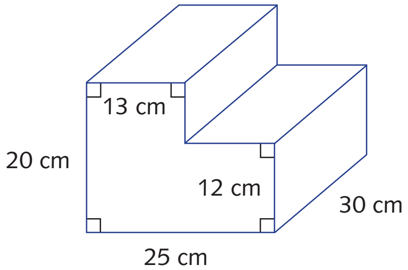
d



2 A small step has the shape shown opposite.

a Find the area of the front face.

b Find the volume of the step.



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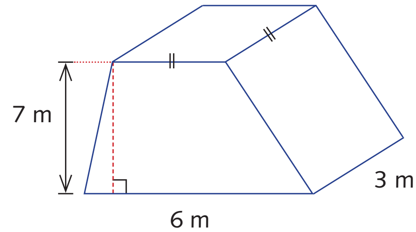
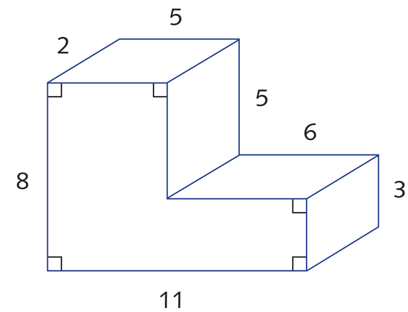
3 A large pedestal is in the shape of a prism whose front face is a trapezium.

a Find the area of the front face.

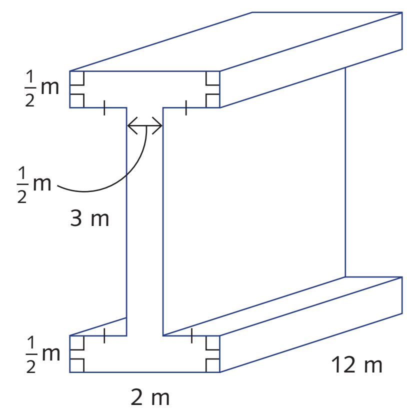
b Find the volume of the pedestal.



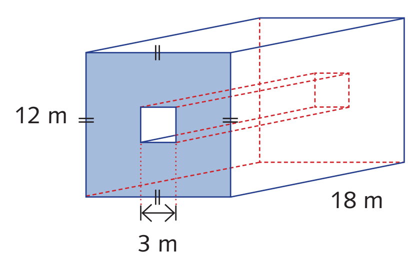
4 Find the volume of the stone block shown opposite, using the given measurements, which are in metres.



5 A steel girder has measurement in metres as shown. What is its volume?



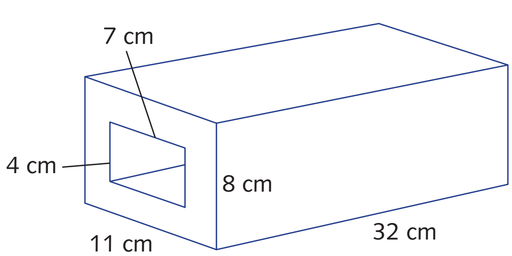
6 A 3 m by 3 m square prism is removed from a 12 m by 12 m square prism with depth 18 m. What is the volume of the remaining solid?



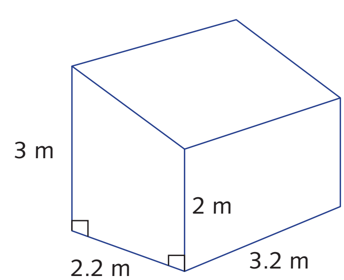
7 The front face of a prism is a parallelogram with base 8 cm and height 6 cm. If the depth is 12 m, find the volume.

8 Find the volume of each prism.

a



b



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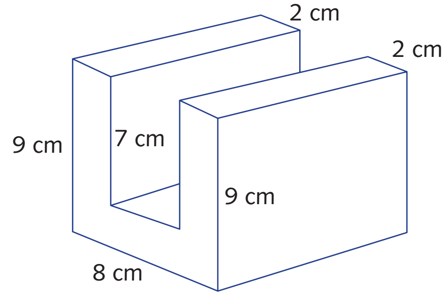
9 The volume of the prism is 792 cm^3.

Find:

a the area of the cross-section

b the length of the prism



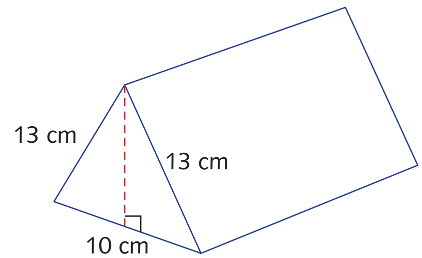
10 The volume of the prism is 4800 cm^3.

Find:

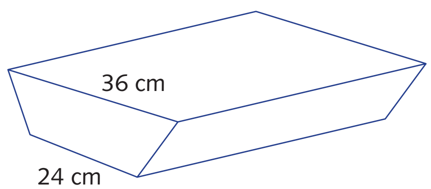
a the height of the triangle

b the area of the cross-section

c the length of the prism



11 The cross-section of the prism is a trapezium of height 5 cm. The volume of the prism is 3900 cm^3. Find the length of the prism.



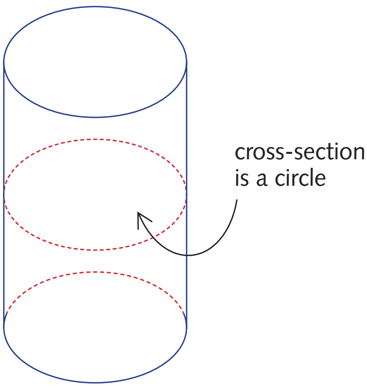
15E Volume of a cylinder

If you look in the kitchen cupboard, you will probably see a number of cans of food. In mathematics, we call these solids **cylinders**, from a Greek word meaning 'to roll'.

If we slice a cylinder parallel to its base, then each cross-section is a circle of the same size as the top and bottom.

We can use the same formula we found before to find the volume of a cylinder. Its volume is the area of the base, which is a circle, multiplied by the perpendicular height. We cannot prove this formula at this stage.

Informally, we can approximate the base by a polygon and use the formula for the area of a prism.



cylinder

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If the base circle of the cylinder has radius r, then we saw in Chapter 14 that the area of the circle is πr^2. If the height is h, then we say that the volume is:

$$\text{volume} = \pi r^2 \times h = \pi r^2 h$$

Hence:

$$\text{volume of a cylinder} = \pi r^2 h,$$

where r is the radius of the circular base and h is the perpendicular height.

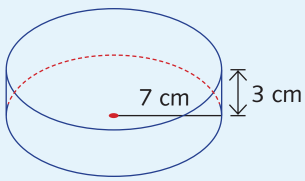
Example 10

A cylinder has base radius 7 cm and height 3 cm. Find:

a the exact volume, in terms of π

b an approximate value for the volume, using $\pi \approx \dfrac{22}{7}$

Solution



a $V = \pi r^2 h$

$= \pi \times 49 \times 3$

$= 147\pi \text{ cm}^3$

b $V = \pi r^2 h$

$\approx \dfrac{22}{7} \times 49 \times 3$

$= 462 \text{ cm}^3$

Exercise 15E

Example 10

1 For each cylinder, find:

i the exact volume, in terms of π

ii an approximate value for the volume, using $\pi \approx \dfrac{22}{7}$

a Radius 14 m, height 20 m

b Diameter 7 cm, height 16 cm

c Radius 21 mm, height 12 mm

2 A jam jar is in the shape of a cylinder. It has a base radius of $3\frac{1}{2}$ cm and a height of 15 cm. Use $\pi \approx \dfrac{22}{7}$ to find an approximate value for its volume.

3 Use $\pi \approx 3.14$ to find the approximate value of the volume of a can with base diameter 60 mm and height 20 mm.

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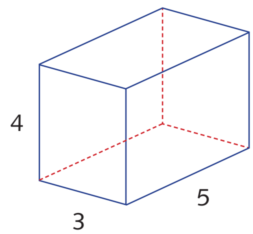
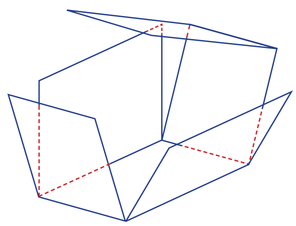
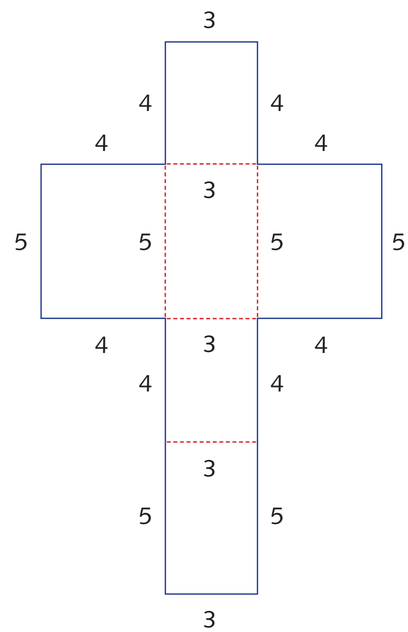
15F Surface area of a prism

Suppose we take a rectangular prism whose dimensions are 3 by 4 by 5, and open it out as shown below.







We can find the area of the flattened box by adding up the areas of the six rectangular faces. There are:

- two rectangular faces with area $3 \times 4 = 12$
- two rectangular faces with area $3 \times 5 = 15$
- two rectangular faces with area $4 \times 5 = 20$.

In total, this gives an area of $2 \times (12 + 15 + 20) = 94$.

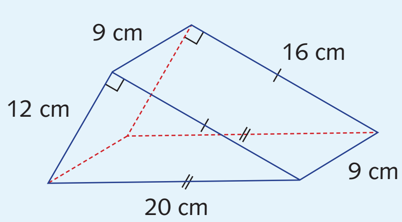
This is called the **surface area** of the box.

In practice, the dimensions will be measured in centimetres and the surface area in cm^2. The surface area of a prism is the sum of the areas of its faces. A rectangular prism with dimensions a, b and c has surface area:

surface area of a rectangular prism $= 2(ab + ac + bc)$

Example 11

Find the surface area of the triangular prism shown opposite.



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Solution

Area of front $= \frac{1}{2} \times 12 \times 16 = 96\text{ cm}^2$

Area of back $= 96\text{ cm}^2$

Area of the three rectangular faces $= (9 \times 20) + (9 \times 12) + (9 \times 16)$

$= 432\text{ cm}^2$

Total surface area $= 96 + 96 + 432$

$= 624\text{ cm}^2$

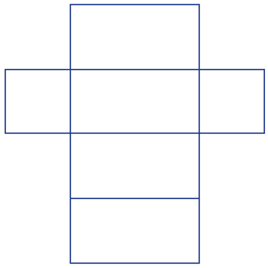
Surface area of a prism

The surface area of a prism is the sum of the areas of its faces.

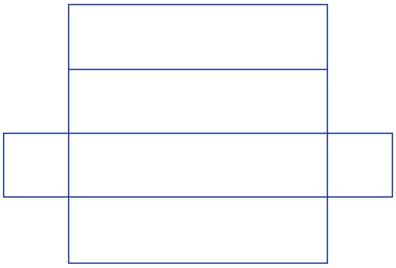
Exercise 15F

1 In each case below, the surface of a solid has been cut open and opened up as shown. Draw the original solid.

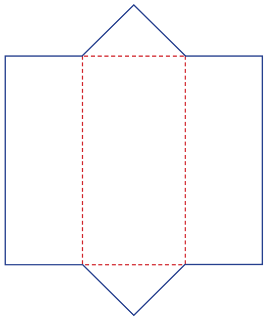
a



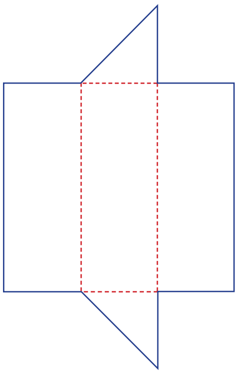
b



c



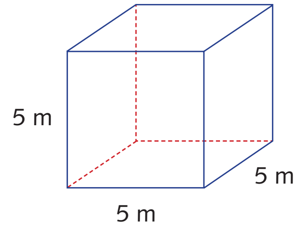
d



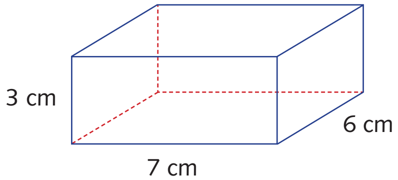
Example 11

2 Find the surface area of each rectangular prism.

a



b



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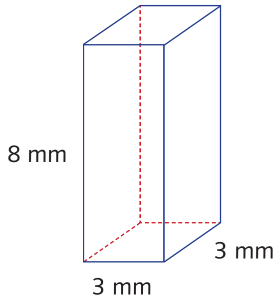
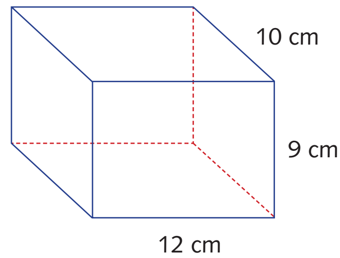
c



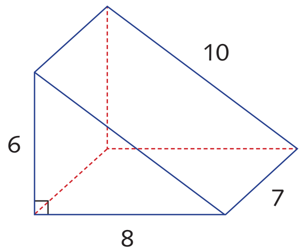
d



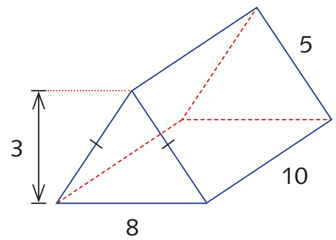
Example 11

3 Find the surface area of each triangular prism. All measurements are in centimetres.

a

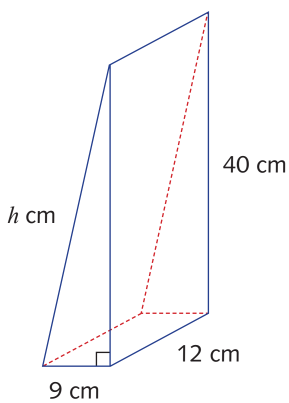


b

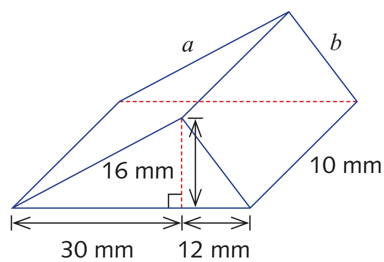


4 Use Pythagoras' theorem to find the unknown lengths in the figures below and then calculate their surface areas.

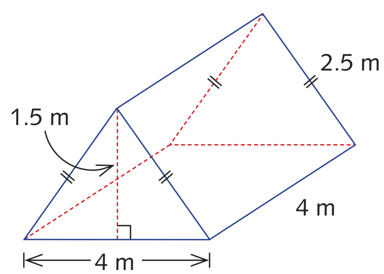
a



b

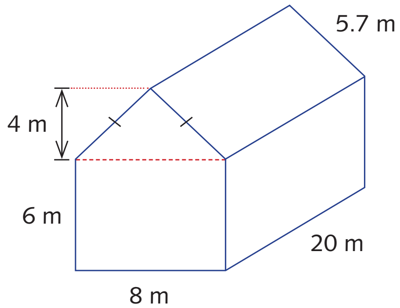


5 A tent made from calico, including the ground sheet, is in the shape of a triangular prism, with dimensions as shown. How much calico is needed to make the tent?

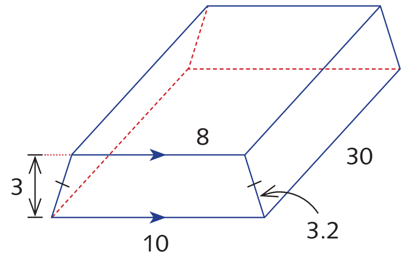


6 Find the surface area of each prism.

a



b



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15G Conversion of units

We often need to convert from one standard unit to another.

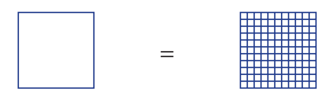
We recall:

$1\text{ cm} = 10\text{ mm} \qquad 1\text{ m} = 100\text{ cm} \qquad 1\text{ km} = 1000\text{ m}$

Just as lengths can be converted from one unit to another, so can areas and volumes. We can use the basic length conversions above to obtain area and volume conversions.

Area conversions

A square of side length 1 cm has area 1 cm^2.
Since $1\text{ cm} = 10\text{ mm}$, the area of this square is also $10 \times 10 = 100\text{ mm}^2$.
Hence $1\text{ cm}^2 = 100\text{ mm}^2$



1 cm2 　 10 mm × 10 mm = 100 mm2

Similarly, we have:

$$1\text{ m}^2 = 100^2\text{ cm}^2 = 10\,000\text{ cm}^2 = 1\,000\,000\text{ mm}^2$$
$$1\text{ km}^2 = 1000^2\text{ m}^2 = 1\,000\,000\text{ m}^2$$

Hence, to obtain the conversion factor for areas, we square the corresponding conversion factor for lengths.

Length

$$\begin{tikzcd} \text{mm} \arrow[r, bend left, "\div 10"] & \text{cm} \arrow[l, bend left, "\times 10"] \arrow[r, bend left, "\div 100"] & \text{m} \arrow[l, bend left, "\times 100"] \arrow[r, bend left, "\div 1000"] & \text{km} \arrow[l, bend left, "\times 1000"] \end{tikzcd}$$

Area

$$\begin{tikzcd} \text{mm}^2 \arrow[r, bend left, "\div 10^2"] & \text{cm}^2 \arrow[l, bend left, "\times 10^2"] \arrow[r, bend left, "\div 100^2"] & \text{m}^2 \arrow[l, bend left, "\times 10^4"] \arrow[r, bend left, "\div 1000^2"] & \text{km}^2 \arrow[l, bend left, "\times 10^6"] \end{tikzcd}$$

Volume

$$\begin{tikzcd} \text{mm}^3 \arrow[r, bend left, "\div 10^3"] & \text{cm}^3 \arrow[l, bend left, "\times 10^3"] \arrow[r, bend left, "\div 100^3"] & \text{m}^3 \arrow[l, bend left, "\times 10^6"] \arrow[r, bend left, "\div 1000^3"] & \text{km}^3 \arrow[l, bend left, "\times 10^9"] \end{tikzcd}$$

Example 12

Convert each of the following measurements to cm^2.

a 2.5 m^2

b 3600 m^2

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Solution

a
$$1 \text{ m}^2 = 100^2 \text{ cm}^2$$
$$= 10\,000 \text{ cm}^2$$
$$\text{so } 2.5 \text{ m}^2 = 2.5 \times 10\,000 \text{ cm}^2$$
$$= 25\,000 \text{ cm}^2$$

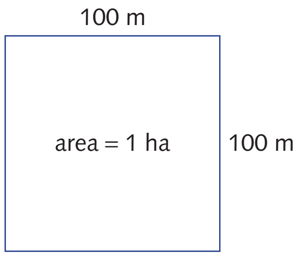
b
$$1 \text{ cm}^2 = 10^2 \text{ mm}^2$$
$$= 100 \text{ mm}^2$$
$$\text{so } 3600 \text{ mm}^2 = \frac{3600}{100} \text{ cm}^2$$
$$= 36 \text{ cm}^2$$

Hectares

The metric system was introduced in Europe by Napoleon in the early nineteenth century, and was adopted by Australia in 1966. Up until that time the Imperial system was used and land was measured in acres. To include a similar sized unit of area in the metric system, the following unit of area is defined.

A **hectare** (ha) is the area enclosed by a square with side length 100 m.

That is,
$$1 \text{ ha} = (100 \times 100) \text{ m}^2$$
$$= 10\,000 \text{ m}^2$$
$$= 10^4 \text{ m}^2$$
$$100 \text{ ha} = 100 \times 10^4 \text{ m}^2$$
$$= 10^6 \text{ m}^2$$
$$= 1 \text{ km}^2$$



Example 13

The area of a cattle station in outback Australia is 200 000 ha.

Calculate:

a the area in m^2

b the area in km^2

c the dimensions of the station, if it is a square, to one decimal place

d the dimensions of the station, if it is a rectangle and one side length is 50 km

Solution

a
$$1 \text{ ha} = (100 \times 100) \text{ m}^2$$
$$= 10\,000 \text{ m}^2$$
$$\text{so } 200\,000 \text{ ha} = (200\,000 \times 10\,000) \text{ m}^2$$
$$= 2\,000\,000\,000 \text{ m}^2$$
$$= (2 \times 10^9) \text{ m}^2$$

b Since $1 \text{ km}^2 = 100$ ha

$200\,000 \text{ ha} = 2000 \text{ km}^2$

c Let x km be the side length of the square,

so $\quad x^2 = 2000$

Hence, $\quad x \approx 44.7 \quad$ (to one decimal place)

So the station is approximately 44.7 km by 44.7 km.

d $\dfrac{2000}{50} = 40$

The station is 40 km by 50 km.

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Volume conversions

A cube of side length 1 cm has volume 1 cm^3 (this is read as, 1 cubic centimetre). Since 1 cm = 10 mm, the volume of this cube is also $10 \times 10 \times 10 = 1000\ \text{mm}^3$.

Hence $1\text{ cm}^3 = 1000\text{ mm}^3$.

Similarly, we have :

$$1\text{ m}^3 = 100^3\text{ cm}^3 = 1\,000\,000\text{ cm}^3 = 10^6\text{ cm}^3$$
$$1\text{ km}^3 = 1000^3\text{ m}^3 = 1\,000\,000\,000\text{ m}^3 = 10^9\text{ m}^3$$

Hence, to obtain the conversion factor for volumes, we cube the corresponding conversion factor for lengths.

Example 14

Convert each of the following measurements to the units indicated in the brackets.

a 2760 mm^3 (cm^3) **b** 0.27 m^3 (cm^3)

c $256\,000\text{ cm}^3$ (m^3) **d** 0.59 cm^3 (mm^3)

Solution

a
$$\begin{aligned} 10\text{ mm} &= 1\text{ cm} \\ \text{so } 10^3\text{ mm}^3 &= 1\text{ cm}^3 \\ 1000\text{ mm}^3 &= 1\text{ cm}^3 \end{aligned}$$
$$\begin{aligned} \text{Hence, } 2760\text{ mm}^3 &= \frac{2760}{1000}\text{ cm}^3 \\ &= 2.76\text{ cm}^3 \end{aligned}$$

b
$$\begin{aligned} 1\text{ m} &= 100\text{ cm} \\ \text{so } 1\text{ m}^3 &= 100^3\text{ cm}^3 \\ 1\text{ m}^3 &= 1\,000\,000\text{ cm}^3 \\ &= 10^6\text{ cm}^3 \end{aligned}$$
$$\begin{aligned} \text{Hence, } 0.27\text{ m}^3 &= (0.27 \times 10^6)\text{ cm}^3 \\ &= 2.7 \times 10^5\text{ cm}^3 \\ &= 270\,000\text{ cm}^3 \end{aligned}$$

c
$$\text{From } \mathbf{b}, 10^6\text{ cm}^3 = 1\text{ m}^3$$
$$\begin{aligned} \text{Hence, } 256\,000\text{ cm}^3 &= \frac{256\,000}{1\,000\,000}\text{ cm}^3 \\ &= 0.256\text{ m}^3 \end{aligned}$$

d
$$\text{From } \mathbf{a}, 1\text{ cm}^3 = 1000\text{ mm}^3$$
$$\begin{aligned} \text{Hence, } 0.59\text{ cm}^3 &= 0.59 \times 1000\text{ mm}^3 \\ &= 590\text{ mm}^3 \end{aligned}$$

Litres

The following units of volume are used when measuring liquids.

A **litre** (1 L) is equal to 1000 cm^3.

$$\begin{aligned} \text{That is, } 1\text{ L} &= 1000\text{ cm}^3 \\ 1\text{ m}^3 &= 10^6\text{ cm}^3 \\ &= 1000\text{ L} \end{aligned}$$

A **millilitre** (1 mL) is equal to $\frac{1}{1000}$ of a litre and is thus equal to 1 cm^3.

That is, $1\text{ mL} = \frac{1}{1000}\text{L} = 1\text{ cm}^3$.

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Example 15

A large water trough in the shape of a rectangular prism has internal dimensions of 3 m by 0.6 m by 0.5 m. How many litres of water does the trough hold when full?

Solution

Since $1000\ \text{cm}^3 = 1$ litre, the volume is best calculated in cubic centimetres.

Now $100\ \text{cm} = 1\ \text{m}$

$$\begin{aligned}\text{Volume} &= 300\ \text{cm} \times 60\ \text{cm} \times 50\ \text{cm}\\ &= 900\,000\ \text{cm}^3\\ &= 900\ \text{litres}\end{aligned}$$

Hence, the water trough holds 900 litres.

Conversion of units

Length

$1\ \text{cm} = 10\ \text{mm}$	$1\ \text{m} = 100\ \text{cm}$	$1\ \text{km} = 1000\ \text{m}$

Area

$1\ \text{cm}^2 = 100\ \text{mm}^2$	$1\ \text{m}^2 = 10^4\ \text{cm}^2$	$1\ \text{km}^2 = 10^6\ \text{m}^2$

Volume

$1\ \text{cm}^3 = 1000\ \text{mm}^3$	$1\ \text{m}^3 = 10^6\ \text{cm}^3$	$1\ \text{km}^3 = 10^9\ \text{m}^3$

Area of land

$1\ \text{ha} = 10^4\ \text{m}^2$	$1\ \text{km}^2 = 100\ \text{ha}$

Litres (measurement of liquids)

1 litre = 1000 millilitres	$1\ \text{mL} = 1\ \text{cm}^3$	$1\ \text{m}^3 = 1000\ \text{L}$

Exercise 15G

Example 12

1 Convert each measurement to cm^2.

a $300\ \text{mm}^2$ **b** $0.7\ \text{m}^2$ **c** $90\ \text{mm}^2$ **d** $3.1\ \text{m}^2$

2 Convert each measurement to mm^2.

a $2\ \text{cm}^2$ **b** $0.5\ \text{m}^2$ **c** $0.6\ \text{cm}^2$ **d** $2.3\ \text{m}^2$

3 Convert each measurement to m^2.

a $2.4\ \text{km}^2$ **b** $36\,000\ \text{cm}^2$ **c** $0.36\ \text{km}^2$ **d** $2800\ \text{cm}^2$

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4 A table top measures 900 mm × 1150 mm.

a Calculate the area of the table top in square millimetres.

b Express your answer to part **a** in square centimetres.

c Give the dimensions of the table top in centimetres.

d Using the dimensions found in **c**, calculate the area of the table top in square centimetres.

e Do your answers to part **b** and part **d** agree?

f Express the area of the table top in square metres.

g Give the dimensions of the table top in metres.

h Using the dimensions found in part **g**, calculate the area of the table top in square metres.

i Check that your answers to part **f** and part **h** agree.

Example 13

5 A rectangular piece of land measures 260 m by 430 m. Calculate the area of the land in:

a m^2

b hectares

6 A rectangular piece of land has an area of 2.7 ha. If the block of land is 135 metres wide, how long is the block of land?

7 A cattle station has an area of 260 km^2. What is this area in hectares?

8 A farm has an area of 480 ha. Calculate the area in:

a m^2

b km^2

9 One acre is approximately 0.4 hectares. A rectangular block of land measures 50 m × 150 m. Calculate the area of the block of land in:

a hectares

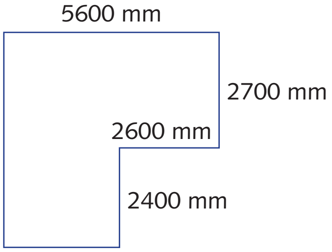
b acres (approximately)

10 A classroom measures 6400 mm × 7800 mm. Carpet is to be laid on the floor at a cost of $45 per square metre. Calculate:

a the floor area in square metres

b the cost of the carpet for the classroom

11 Robyn intends to paint the ceiling of her living room. A plan of the room is drawn with measurements. One litre of paint will cover 12 m^2. Calculate:



a the area of the ceiling in square metres

b the amount of paint needed to put one coat of paint on the ceiling

12 Convert 1 m^2 to:

a cubic centimetres

b litres

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Example 14 **13** Convert each of the following measurements to the units indicated in the brackets.

a $5760 \text{ mm}^3 (\text{cm}^3)$ **b** $0.56 \text{ m}^3 (\text{cm}^3)$

c $756\,000 \text{ cm}^3 (\text{m}^3)$ **d** $0.59 \text{ cm}^3 (\text{mm}^3)$

14 Convert each measurement to the units in the brackets.

a 0.62 m^3 (L) **b** 2600 cm^3 (L) **c** $52\,000 \text{ mm}^3$ (mL)

d $2.7 \text{ L} (\text{cm}^3)$ **e** $960 \text{ L} (\text{m}^3)$ **f** $26 \text{ mL} (\text{mm}^3)$

Example 15 **15** A large tank in the shape of a rectangular prism has internal dimensions $3 \text{ m} \times 2 \text{ m} \times 1.5 \text{ m}$. How many litres does the tank hold when full?

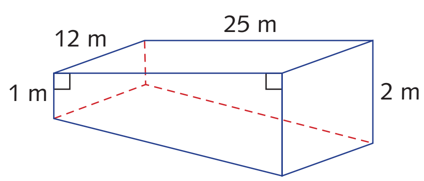
16 A cylindrical water tank has a diameter of 3 m and a height of 2 m. Calculate, to the nearest 100 mL, the volume of the tank in:

a m^3

b litres

17 A cylindrical water tank has diameter 4 m and a height of 2.5 m. If the tank is initially full, for how many days can a family use this water tank if their daily consumption of water is 600 litres? (Assume no water enters the tank during the time period.)

18 A school swimming pool has dimensions as shown. How long would it take to fill this pool if the pump can deliver 1500 litres of water a minute?



19 A medicine bottle is in the shape of a cylinder with base diameter 50 mm and height 80 mm. If the normal dosage of medicine is 15 mL, how many dosages can be obtained from the bottle? (Assume that initially the bottle is completely full.)

20 A swimming pool is in the shape of a rectangular prism. The pool is 12 metres long, 4 metres wide and 1.5 metres deep. The pool is to be lined with tiles.

a How many tiles are needed to line the pool if each tile is:

i $100 \text{ mm} \times 100 \text{ mm}$?

ii $200 \text{ mm} \times 200 \text{ mm}$?

iii $200 \text{ mm} \times 100 \text{ mm}$?

b A path 1 metre wide is to be put around the pool. How many pavers are needed to make the path if each of the pavers is of size:

i $1 \text{ m} \times 1 \text{ m}$?

ii $500 \text{ mm} \times 500 \text{ mm}$?

iii $250 \text{ mm} \times 250 \text{ mm}$?

c When the pool is completed, it is filled at 400 litres/hour. How long will it take to fill the pool to 10 cm below the top?

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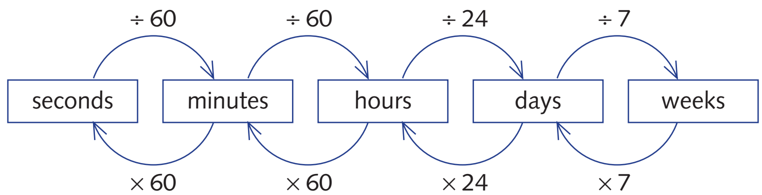
15H Time

Time is different from other measurements because it is not based on powers of 10.

- There are 60 seconds in 1 minute.
- There are 60 minutes in 1 hour.
- There are 24 hours in 1 day.
- There are 7 days in a week.



There are two ways of recording the time of day:

- using **a.m.** and **p.m**. For example, 5:30 p.m. means 5 hours 30 minutes after midday, and 12:30 a.m. means 30 minutes after midnight. Remember that a.m. stands for *ante meridiem*, which is Latin for 'being before noon', and p.m. stands for *post meridiem*, which is Latin for 'being after noon'.
- using the **24-hour system**. For example, 2235 means 22 hours and 35 minutes after midnight. This is the same as 10:35 p.m. 0530 means 5 hours 30 minutes after midnight or 5:30 a.m. 1730 is the same as 5:30 p.m. Note that there is no need to indicate morning by using a.m., or afternoon by using p.m., when using 24-hour time.

Example 16

A truck driver drove for 5 hours 45 minutes on Monday, 4 hours 50 minutes on Tuesday and 6 hours 30 minutes on Wednesday. What was the total time she spent driving?

Solution

$$\begin{array}{rr} 5\text{ h} & 45\text{ m} \\ +4\text{ h} & 50\text{ m} \\ +6\text{ h} & +30\text{ m} \\ \hline 15\text{ h} & 125\text{ m} \\ \hline \end{array}$$

(Add the minutes, add the hours.)

$$\begin{aligned} \text{Total time spent driving} &= 15 \text{ hours} + 2 \text{ hours} + 5 \text{ minutes} \\ &\quad \text{(Convert minutes total to hours and minutes.)} \\ &= 17 \text{ hours } 5 \text{ minutes} \end{aligned}$$

Duration or elapsed time

Calculating elapsed time is a skill that is made interesting by the fact that time is based on the numbers 24 and 60.

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Example 17

a Jane left home at 3:54 p.m. to travel to her aunt's house. She arrived at 5:40 p.m. How long did her journey take?

b Jane's father drove from their home to her aunt's house and it took him 48 minutes. How much longer did Jane take to arrive?

Solution

a There are 6 minutes to 4 p.m., and 1 hour and 40 minutes to 5:40 p.m.

$$\begin{aligned}\text{So Jane's total travel time} &= 6 \text{ minutes} + 1 \text{ hour } 40 \text{ minutes}\\ &= 1 \text{ hour } 46 \text{ minutes}\end{aligned}$$

b We need to work out the difference between Jane's travel time of 1 hour 46 minutes and her father's time of 48 minutes.

12 minutes are needed to build up from 48 minutes to 1 hour, and then there are 46 minutes after that.

$$\begin{aligned}\text{Total} &= 12 + 46\\ &= 58 \text{ minutes}\end{aligned}$$

Jane took 58 minutes longer to arrive than her father.

Example 18

Marathon runners take around 3 hours to run 42 km. If a runner started at 11:23:00 a.m. (23 minutes past 11, no seconds) and finished at 2:40:49 p.m., what was the time taken to complete the marathon?

Solution

The time taken to complete the marathon is calculated by building up to the next whole minute or hour.

	Time	Hour	Minutes	Seconds
Start time	11:23:00 a.m.			
Build up to	12:00:00 p.m.		37 minutes	
Build up to	2:00:00 p.m.	2 hours		
Build up to	2:40:49 p.m.		40 minutes	49 seconds
Total		2 hours	77 minutes	49 seconds
Total time taken to complete the marathon (convert minutes to hours and minutes).		3 hours	17 minutes	49 seconds

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Time

- Measurement of time is based on the numbers 7, 24 and 60.
- Building up to whole minutes, hours or days is helpful when calculating time differences and when adding times.

Exercise 15H

1 Convert each of these times to 24-hour time.

a 4:30 p.m. **b** 11 a.m.

c 6:49 p.m. **d** Six thirty-five in the evening

e Twenty-five to five in the morning **f** Five past four in the afternoon

g Three minutes to midnight

2 Write each of these 24-hour times in 12-hour time using a.m. and p.m.

a 1400 **b** 0800

c 0835 **d** 2230

Example 16

3 Phil worked for 25 days 18 hours in January, 23 days 22 hours in February and 24 days 11 hours in March. What was the total time he worked?

Example 17

4 David travelled for 5 days 23 hours by boat, then 16 hours by plane and finally 3 days 22 hours by train to be home for his mother's birthday. How long did his journey take?

5 What is the elapsed time, in days, hours and minutes, between:

a 12 noon and 4:45 p.m.?

b 6 p.m. and 11:30 p.m.?

c 1100 and 1830?

d ten past seven in the morning and four-thirty in the afternoon of the same day?

e 3:35 a.m. on Tuesday and 6:20 a.m. on the next Wednesday?

f 1:32 a.m. on Friday and 2:50 p.m. on the following Sunday?

g 2 p.m. on Thursday and 7:25 a.m. on the following Tuesday?

6 If I started a train trip at 1530 and finished it at 0725 the next day, how long did the journey take?

Example 18

7 Three people ran the New York marathon; their times were 2:45:42 (2 hours 45 minutes 42 seconds), 2:43:57 and 2:42:15. What were the time differences between the first and second competitors, and between the second and third?

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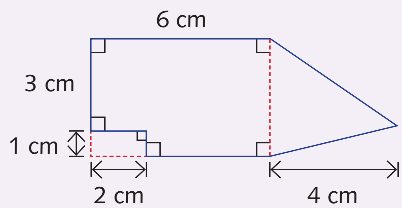
8 Suppose a child is born at 2249 on one day and another child is born at 0318 the next day. How much older is the first child?

9 At the end of 2010, the men's world record time for the 100 m sprint was 9:58 seconds, while the women's record was 10:49 seconds. How much faster was the men's record?

10 Sarah takes 4 hours and 55 minutes to complete a 200-page novel, while Derek takes 5 hours and 12 minutes. Reading at the same speeds, how much faster is Sarah than Derek, in seconds, if they both read a 300-page novel?

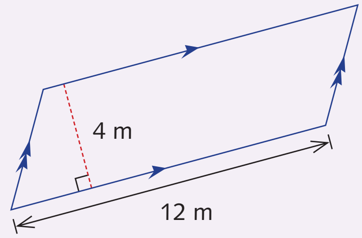
Review exercise

1 Find the area of the region shown below.

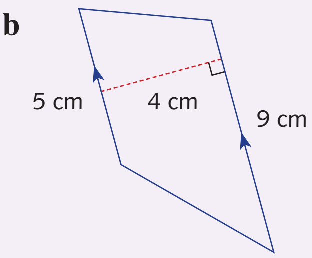


2 Find the area of each region.

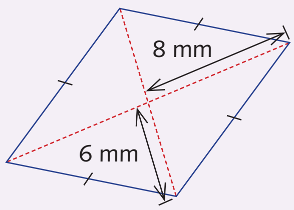
a



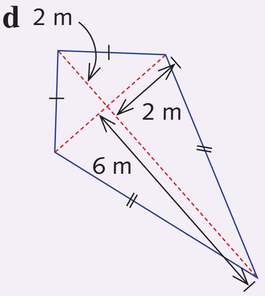
b



c



d



3 What is the volume of a rectangular fish tank with dimensions 90 cm, 45 cm and 60 cm?

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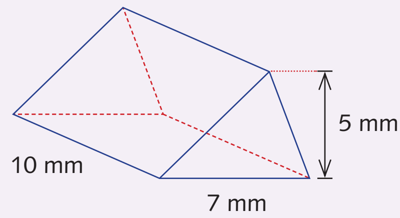
4 Find the volume of each prism.

a



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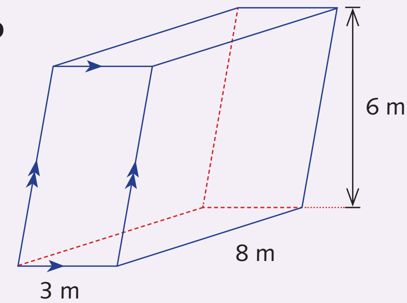

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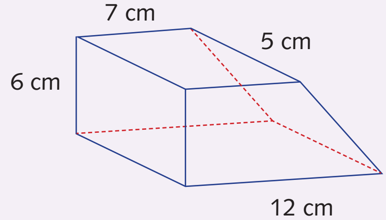

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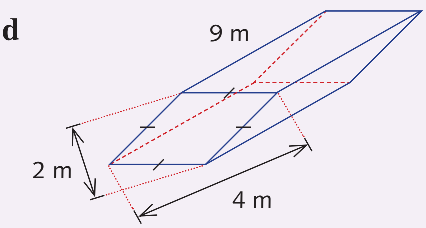
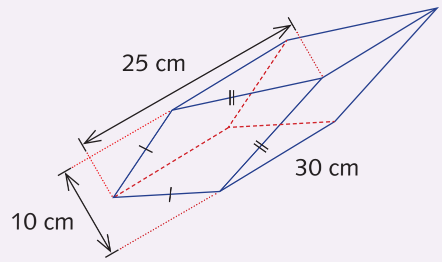


e



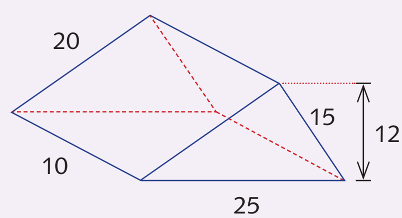
5 What are the exact volumes of the cylinders with the following measurements?

a Radius 5 m, height 4 m

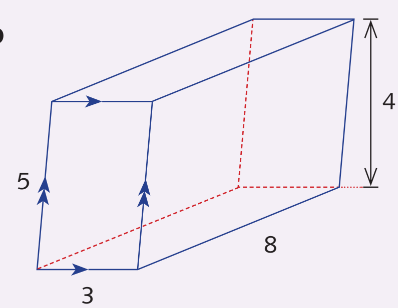
b Diameter 6 cm, height 7 cm

6 Find the surface area of each prism.

a



b



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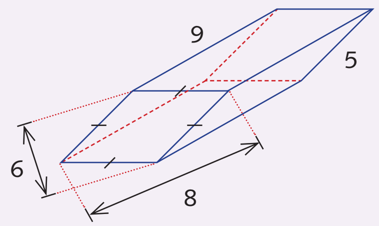
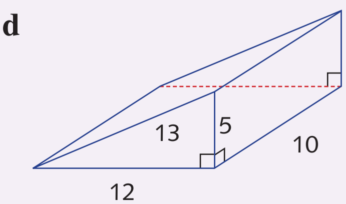


d



7 Convert each measurement to cm^2.

a $3.5\ m^2$ **b** $7200\ mm^2$ **c** $4\ m^2$ **d** $67\,000\ mm^2$

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8 Convert 450 000 hectares to:

a m^2 **b** km^2

9 How many litres does a cylinder of radius 20 cm and height 1 m hold?

10 How many litres does a rectangular prism with dimensions 20 cm × 80 cm × 50 cm hold?

11 What is the elapsed time, in days, hours and minutes, between:

a 12 noon and 7:45 p.m.?

b 1 p.m. and 11:30 p.m.?

c 1115 and 1830?

d twenty past seven in the morning and five-thirty in the afternoon of the same day?

e 2:35 a.m. on Tuesday and 7:35 a.m. on the next Wednesday?

f 2:36 a.m. on Friday and 2:51 p.m. on the following Sunday?

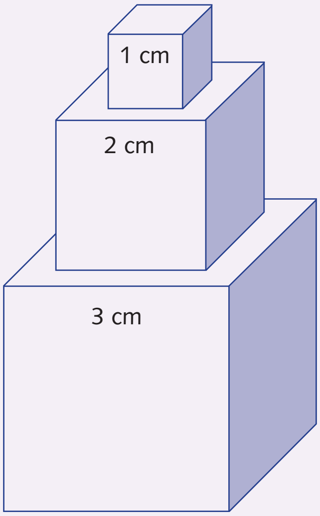
g 1 p.m. on Thursday and 7:36 a.m. on the following Tuesday?

Challenge exercise

1 The areas of three faces of a rectangular prism are 10 cm^2, 14 cm^2 and 35 cm^2. Find the volume of the rectangular prism.

2 A cylindrical container of diameter 80 cm and height h cm is filled with water. How many cylinders of diameter 20 cm and height h cm would you need to hold the same amount of water?

3 Three cubes, with side lengths of 1 cm, 2 cm and 3 cm, are glued together as shown opposite. Find the surface area of the object.



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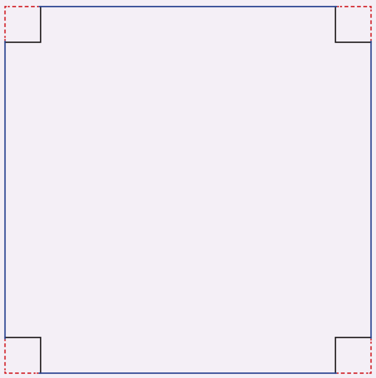
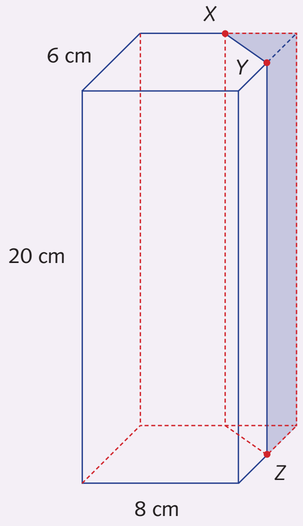
4 A square piece of card has area 49 cm^2. Squares of side length 1 cm are cut from each corner of the card, as shown opposite. Find the volume of the open box formed by turning up the flaps and gluing.

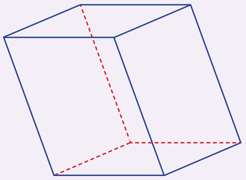


5 In the diagram below, points X, Y and Z are the midpoints of the edges they lie on. A triangular prism is formed by cutting through the points X, Y and Z. Find the volume of the remaining solid.



6 A cube with edge length 6 cm is made from cubes with edge length 1 cm. The 1-cm cubes making up the larger cube are then rearranged to form two rectangular prisms. The base of one rectangular prism is 8 cm $\times$ 5 cm and the base of the other is 6 cm $\times$ 4 cm. What is the height of each rectangular prism?

7 Prisms whose bases are horizontal but whose sides are not vertical are called **oblique** prisms. An oblique rectangular prism is an example of a **parallelepiped**.

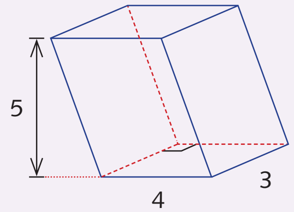


The formula we found earlier:

$$\text{volume} = \text{area of base} \times (\text{perpendicular}) \text{ height}$$

works for oblique prisms as well.

Find the volume of the parallelepiped shown opposite.



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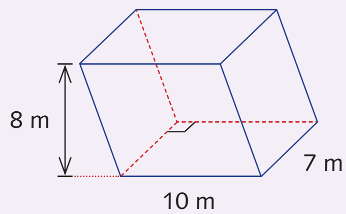
8 Find the volume of the parallelepiped shown below.



9 A cube of side length 3 cm is made from 27 cubes with sides of length 1 cm. Three cubes are removed so that there is a hole from one face to the opposite face. What is the surface area of the resulting object?

10 How many solid rectangular prisms of dimensions $1\text{ cm} \times 2.5\text{ cm} \times 1.6\text{ cm}$ can be cast from 1 m^3 of steel?

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CHAPTER

16

Statistics and Probability

Probability

Probability deals with how likely it is that something will happen. It is an area of mathematics with many diverse applications. Probability is used in areas ranging from weather forecasting and insurance – where it is used to calculate risk factors and premiums – to predicting the risks of new medical treatments and forecasting the effects of global warming.

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16A An introduction to probability

There are many situations in which it would be useful to be able to measure how likely (or unlikely) it is that an event will occur. We can do this in mathematics by using the idea of **probability**, which we define as a number between 0 and 1 that we assign to any **event** we are interested in. Then:

- a probability of 1 represents an event that is 'certain' or 'guaranteed to happen'
- a probability of 0 represents an event that we would describe as 'impossible' or one that 'cannot possibly occur'
- an event that has a probability $\frac{1}{2}$ is as likely to occur as not to occur
- an event that has a probability close to 0 is unlikely to occur
- an event that has a probability close to 1 is likely to occur.

In this chapter we look at methods for determining probabilities.

Sample space

A box contains 12 identical marbles numbered from 1 to 12. The box is shaken and a marble is randomly taken from it and its number noted. This is an example of doing a random **experiment**. The numbers $1, 2, \ldots, 12$ are called the **outcomes** of this experiment. The outcomes for this experiment are **equally likely**. The probability of each outcome is $\frac{1}{12}$. The complete set of possible outcomes (or sample points) for any experiment is called the **sample space** of that experiment. For example, we can write down the sample space ξ for this experiment as:

$$\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

In this chapter, all of the experiments have finite sample spaces with equally likely outcomes. For a sample space with n equally likely outcomes the probability of each outcome is $\frac{1}{n}$.

Events

An **event** is a collection of sample points. An **event** is a subset of the sample space.

Suppose that for the experiment above we are interested in getting a prime number. In this case 'the number is prime' is the event that interests us. Some of the outcomes will give rise to this event. For instance, if the outcome is 2, then the event 'the number is prime' takes place. We say that the outcome 2 is **favourable to the event** 'the number is prime'. If the outcome is 4, then the event 'the number is prime' does not occur. The outcome 4 is **not favourable to the event**.

Of the 12 possible outcomes, these are the ones that are favourable to the event 'the number is prime':

$$\{2, 3, 5, 7, 11\}$$

In many situations, 'success' means 'favourable to the event' and 'failure' means 'not favourable to the event'.

Events are named by capital letters. For instance, we talk about:

B the event 'a prime is obtained' from the experiment described above.

C the event 'obtaining an even number' when a die is tossed.

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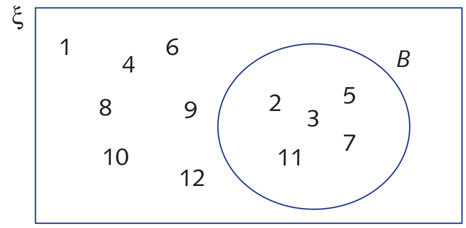
An outcome is favourable to an event if it is a member of that event. For example:

$2 \in B$ and $2 \in C$

These sample spaces and events can be illustrated with **Venn diagrams**. Venn diagrams were introduced in the context of sets in Year 7.

Here is the sample space ξ and the event B. In this context ξ is the universal set for the experiment of withdrawing a marble and observing the number on it, as described previously.



Probability of an event

The probability of the event A is written $P(A)$.

Probabilities are assigned to events in such a way that:

$0 \le P(A) \le 1$ for all events A

That is, the probability of an event is a number between 0 and 1 inclusive.

For an experiment in which all of the outcomes are equally likely:

$$\text{probability of an event} = \frac{\text{number of outcomes favourable to that event}}{\text{total number of outcomes}}$$

For the event B described on the previous page, $P(B) = \frac{5}{12}$.

In general, the probability of an event is the sum of the probabilities of the outcomes that are favourable to that event.

The total probability is 1

The sum of the probabilities of the outcomes of an experiment is 1.

For the experiment of withdrawing a marble discussed above, each outcome has probability $\frac{1}{12}$. The sum of these probabilities is 1.

The words 'random' and 'randomly'

In probability, we frequently hear these words, as in the following situation:

A classroom contains 23 students. A teacher comes into the room and chooses a student at random to answer a question about history.

What does this mean? It means that the teacher chose the student as if she knew nothing at all about the students. Another way of interpreting this is to imagine that the teacher had her eyes closed and had no idea who was in the class when she chose a student. Each student is equally likely to be chosen.

The probability of a particular student being chosen is $\frac{1}{23}$.

Example 1

A die is rolled once. Draw a Venn diagram for the experiment and circle the outcomes favourable to the event 'the number is greater than 4'. What is the probability of a number greater than 4 being obtained?

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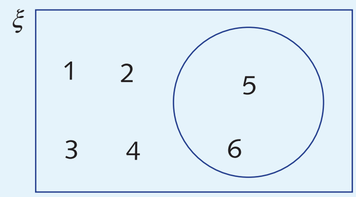
Solution

The outcomes favourable to the event are a 5 or a 6 appearing.

$$P(\text{number greater than 4}) = \frac{2}{6} = \frac{1}{3}$$



Example 2

A standard pack, or simply a pack, of playing cards consists of four suits: Hearts, Diamonds, Clubs and Spades. Each suit has 13 cards consisting of Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen and King. The pack is shuffled and a card is drawn. What is the probability of drawing:

a a King?

b a Heart?

Solution

a Let K be the event 'a King is drawn'.

$$P(K) = \frac{4}{52} = \frac{1}{13}$$

b Let H be the event 'a Heart is drawn'.

$$P(H) = \frac{13}{52} = \frac{1}{4}$$

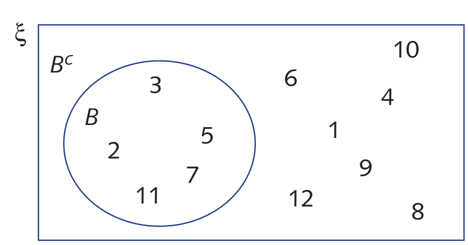
The complement of an event

Sometimes, rather than being interested in seeing if the event A happens, we are interested in the event 'not A'. For instance, we might want to know the probability that it does not rain tomorrow, or that a particular team does not win a premiership, or that the number 40 is not drawn out in a lottery draw.

The event '**not A**' consists of every possible outcome that is not in A; that is, 'not A' is everything in the sample space ξ that is outside A. The event 'not A' is called the **complement** of A and is denoted by A^c.

Recall our experiment of drawing a marble from a bag of 12 marbles, with B the event 'a prime is obtained'.

$$B^c = \{1, 4, 6, 8, 9, 10, 12\} \text{ and } P(B^c) = \frac{7}{12}$$



Notice that $P(B) = \dfrac{5}{12}$ and $P(B) + P(B^c) = 1$.

Every outcome of a sample space is contained in one of A or A^c, but not both.

Therefore $P(A) + P(A^c) = 1$ and $P(A^c) = 1 - P(A)$.

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The complement of an event

The complement of an event A is everything in the sample space ξ that does not lie in A. It is denoted by A^c and is called the event 'not A'.

$$P(A^c) = 1 - P(A)$$

Example 3

A dice is thrown and the value on the uppermost face observed. Find the probability of obtaining:

a 5 or 6

b 1, 2, 3 or 4

Solution

a Let A be the event $\{5 \text{ or } 6\}$.

$$P(A) = \frac{2}{6} = \frac{1}{3}$$

b The event $\{1, 2, 3, 4\}$ is A^c. Therefore:

$$\begin{aligned} P(A^c) &= 1 - P(A) \\ &= 1 - \frac{1}{3} \\ &= \frac{2}{3} \end{aligned}$$

When we wish to find the probability of an event, it is sometimes a smart strategy to calculate the probability of the complement, and subtract the answer from 1.

Example 4

A number is chosen from the first 100 whole numbers. That is, $\xi = \{0, 1, \ldots, 99\}$. What is the probability that the number chosen is not divisible by 7?

Solution

Let A be the event 'a number is divisible by 7'. There are 15 such numbers in the first 100 whole numbers.

$$P(A) = \frac{15}{100} = \frac{3}{20}$$

A^c is the event 'a number is not divisible by 7'.

$$P(A^c) = 1 - \frac{3}{20} = \frac{17}{20}$$

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Exercise 16A

1 One letter is chosen at random from the word SALE. What is the probability that it is A?

2 What is the probability of choosing a prime number from the numbers $5, 6, 7, 8, 9, 10$?

3 What is the probability of randomly picking the most expensive car from a range of six new and differently priced cars in a showroom?

Example 1

4 What is the probability of choosing an integer that is exactly divisible by 5 from the set $\{5, 6, 7, 8, 9, 10, 11, 12\}$?

5 In a raffle, 200 tickets are sold. If you have bought one ticket, what is the probability that you will win first prize?

6 One card is chosen at random from a pack of 52 ordinary playing cards. What is the probability that it is the Ace of Hearts?

7 A number is chosen from the first 15 positive whole numbers. What is the probability that it is exactly divisible by both 3 and 4?

Example 2

8 One card is drawn at random from a pack of playing cards. What is the probability that it is:

a an Ace? **b** a red card? **c** a Heart? **d** a picture card?

(A picture card is a King, Queen or Jack.)

9 A book of 150 pages has a picture on 20 of its pages. If one page is chosen at random, what is the probability that it has a picture on it?

10 One counter is picked at random from a bag containing 15 red counters, 5 white counters and 5 yellow counters. What is the probability that the counter removed is:

a red? **b** yellow? **c** not red?

11 If you bought 10 raffle tickets and a total of 400 were sold, what would be the probability that you won first prize?

Example 3

12 The numbers on a roulette wheel go from 0 to 36. The wheel is spun. What is the probability that when it stops it will be pointing to:

a an even number? **b** an odd number?

c a number less than 10, excluding zero?

Example 4

13 A number is chosen at random from the first 20 positive whole numbers. What is the probability that it is not a prime number?

14 A card is drawn at random from a pack of cards pack of 52 playing cards. What is the probability that it is not a 2?

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15 One letter is chosen from the letters of the alphabet. What is the probability that it is not a vowel?

16 A box of 60 coloured crayons contains a mixture of colours. Only 10 of the crayons are red. If one crayon is chosen at random, what is the probability that it is not red?

17 A number is chosen at random from the first 10 positive whole numbers. What is the probability that it is not exactly divisible by 3?

18 One letter is chosen at random from the word ALPHABET. What is the probability that it is not a vowel?

19 In a raffle, 500 tickets are sold. If you bought 20 tickets, what is the probability that you will not win first prize?

20 If you roll an ordinary six-sided die, what is the probability that you will not get a score of 5 or more?

21 There are 200 packets hidden in a lucky dip. Five packets contain \$1 and the rest contain 5 cents. What is the probability that you will not draw out a packet containing \$1?

22 When a pack of playing cards is cut, what is the probability that the card showing is not a picture card (picture cards being Jacks, Queens and Kings)?

23 A letter is chosen at random from the letters of the word SUCCESSION. What is the probability that the letter is:

a N? **b** S? **c** a vowel? **d** not S?

24 A card is drawn at random from a pack of playing cards. What is the probability that it is:

a an Ace? **b** a Spade? **c** not a Club? **d** not a 7 or an 8?

25 A bag contains a set of snooker balls (15 red balls and one each of white, yellow, green, brown, blue, pink and black). What is the probability that one ball selected at random is:

a red? **b** not red? **c** black? **d** neither red nor white?

26 There are 60 cars in a station car park. Of the cars, 22 are Australian-made, 28 are Japanese-made and the rest are European. What is the probability that the first car to leave is:

a Japanese? **b** not Australian? **c** European? **d** American?

27 A whole number is chosen from the first 30 positive whole numbers. What is the probability that:

a it is divisible by 5? **b** it is divisible by 5 and 3?

c it is divisible by 5 but not by 3? **d** it is even and divisible by 5?

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16B 'Or' and 'and'

Sometimes, rather than just considering a single event, we are interested in two or more events happening. Suppose a number is chosen at random from the first 20 positive whole numbers.

Let A be the event 'the number is even'.

Let B be the event 'the number is divisible by 5'.

In this experiment the sample space is:

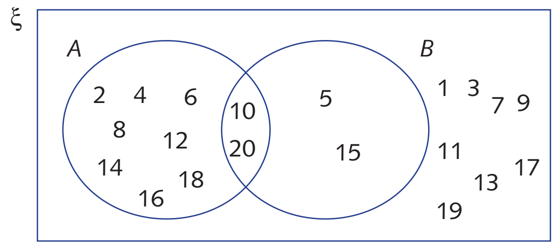
$$\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$$

and the events we are looking at are:

$$A = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$$

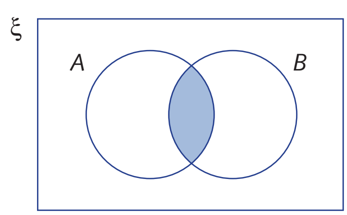
$$B = \{5, 10, 15, 20\}$$

The Venn diagram illustrating these events is as shown. Venn diagrams were introduced in Year 7.

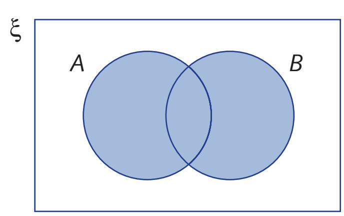


The event 'a number that is even **and** divisible by 5 is chosen' is the **intersection** of the sets A and B and is written as $A \cap B$. The event $A \cap B$ is often called 'A and B'.

The event 'an even number **or** a number divisible by 5 is chosen' is the **union** of the sets A and B and is written as $A \cup B$. The event $A \cup B$ is often called 'A or B'.



$A \cap B$ is shaded



$A \cup B$ is shaded

For an outcome to be in the event $A \cup B$, it must be in **either** the set of outcomes for A or the set of outcomes for B. Of course, it can be in both sets.

For an outcome to be in the event $A \cap B$, it must be in **both** the set of outcomes for A and the set of outcomes for B.

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Example 5

Consider the experiment 'rolling a 10-sided die numbered 1, 2,…, 10'. Let A be the event 'the number obtained is greater than 6' and let B be the event 'the number obtained is even'. Show the events on a Venn diagram, and find their intersection.

Solution

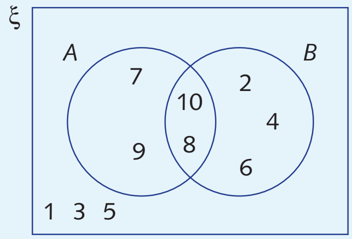
A consists of the outcomes 7, 8, 9, 10.

That is, $A = \{7, 8, 9, 10\}$

B consists of the outcomes 2, 4, 6, 8, 10.

That is, $B = \{2, 4, 6, 8, 10\}$

So, $A \cap B = \{8, 10\}$



Example 6

A person is asked to think of a number between 1 and 10 (inclusive). What is the probability that the number chosen is:

a even?

b greater than 7?

c less than 5?

d greater than 7 or less than 5?

e even or greater than 7?

f even and greater than 7?

g even and less than 5?

h greater than 7 and less than 5?

Solution

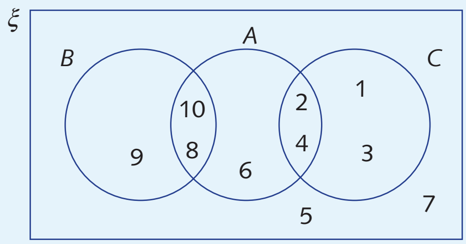
Let A be the event 'number chosen is even'

B be the event 'number chosen is greater than 7'

C be the event 'number chosen is less than 5'.

Then the outcomes that are favourable to these events are:

$A = \{2, 4, 6, 8, 10\}$ $\quad B = \{8, 9, 10\}$ $\quad C = \{1, 2, 3, 4\}$



a $P(A) = \frac{5}{10} = \frac{1}{2}$ $\quad$ **b** $P(B) = \frac{3}{10}$ $\quad$ **c** $P(C) = \frac{4}{10} = \frac{2}{5}$

(*continued over page*)

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d $B \cup C = \{1, 2, 3, 4, 8, 9, 10\}$

Thus $P(B \cup C) = \frac{7}{10}$ (There is no overlap of B and C.)

e $A \cup B = \{2, 4, 6, 8, 9, 10\}$

Hence, $P(A \cup B) = \frac{6}{10} = \frac{3}{5}$

f $A \cap B = \{10, 8\}$

Hence, $P(A \cap B) = \frac{2}{10} = \frac{1}{5}$

g $A \cap C = \{2, 4\}$

Hence, $P(A \cap C) = \frac{2}{10} = \frac{1}{5}$

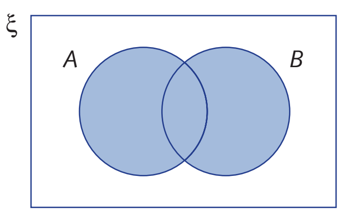
h There are no numbers common to both B and C.

Hence, $P(B \cap C) = 0$

That is, it is impossible to find a number that is both greater than 7 and less than 5 at the same time.

Note: The previous example shows that $P(A \cup B)$ may not be the same as $P(A) + P(B)$. This is because A and B may overlap.

The problem can be done without a Venn diagram, but a Venn diagram makes it easier to list the set.



We recall that for a finite set S, the symbol $|S|$ stands for the number of elements in S. The number of elements in $A \cup B$ is:

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$|A \cap B|$ is subtracted as the sum $|A| + |B|$ includes the elements in $A \cap B$ twice.

Divide both sides of this equation by $|\xi|$, the number of elements in the sample space.

$$\frac{|A \cup B|}{|\xi|} = \frac{|A|}{|\xi|} = \frac{|B|}{|\xi|} - \frac{|A \cap B|}{|\xi|}$$

For a sample space with equally likely outcomes this means:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

This result is true for all sample spaces.

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'Or' and 'and'

- For an outcome to be in the event $A \cup B$, it must be in either the set of outcomes for A or the set of outcomes for B, and of course it can be in both sets. $A \cup B$ is called the union of A and B.
- For an outcome to be in the event $A \cap B$, it must be in both the set of outcomes for A and the set of outcomes for B. $A \cap B$ is called the intersection of A and B.
- For any two events, A and B:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Exercise 16B

Example 5

1 Consider the random experiment of rolling a fair six-sided die. Let A be the event 'an even number is rolled'. Let B be the event 'a number less than 3 is rolled'. Illustrate with a Venn diagram.

2 A number is chosen at random from the numbers 1, 2, 3, …, 20. Let A be the event 'the number chosen is a multiple of 3' and B be the event 'the number chosen is greater than 12'. Illustrate with a Venn diagram.

3 A fair six-sided die is thrown and the uppermost number noted. Let A be the event 'the number is odd' and B be the event 'the number is greater than 3'. Illustrate with a Venn diagram.

Example 6

4 A person is asked to think of a number between 1 and 10 (inclusive). Find the probability that the number is:

a odd
b greater than 6
c less than 4
d greater than 6 or less than 4
e odd or greater than 6
f odd and greater than 6
g odd and less than 4
h greater than 6 and less than 4

5 A fair six-sided die is thrown and the uppermost number is noted. Find the probability that the number is:

a even
b a 6
c less than or equal to 3
d even and a 6
e even or a 6
f less than or equal to 3 and a 6
g less than or equal to 3 or a 6
h even and less than or equal to 3
i even or less than or equal to 3

6 In a certain experiment, two events A and B satisfy $P(A) = 0.6$, $P(B) = 0.3$ and $P(A \cap B) = 0.2$. Find $P(A \cup B)$.

7 For two events C and D, $P(C) = 0.7$, $P(C \cap D) = 0.3$ and $P(C \cup D) = 0.9$. Find $P(D)$.

8 For two events E and F, $P(E) = 0.85$, $P(F) = 0.72$ and $P(E \cup F) = 0.95$. Find $P(E \cap F)$.

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16C Two-way tables

Students in a city school carried out a survey of the students in their school, investigating whether or not they ate breakfast. The survey presented the question 'Do you eat breakfast on a regular basis?' and the students recorded the results along with gender of each respondent.

The results were organised in a table.

	Male	Female	Total
Eat breakfast regularly	320	410	730
Do not eat breakfast regularly	300	200	500
Total	620	610	1230

This table is called a **two-way table**. It is a very good format for answering questions that involve 'and' and 'or'.

For example, for a student selected at random from the school:

$$P(\text{male } \textit{and} \text{ 'eats breakfast regularly'}) = \frac{320}{1230} = \frac{32}{123}$$

For a two-by-two table, the two-way table takes the following form in general.

	A	A^c	Total
B	$\lvert A \cap B \rvert$	$\lvert A^c \cap B \rvert$	$\lvert B \rvert$
B^c	$\lvert B^c \cap A \rvert$	$\lvert B^c \cap A \rvert$	$\lvert B^c \rvert$
Total	$\lvert A \rvert$	$\lvert A^c \rvert$	

Read this table comparing it with the table on eating breakfast.

It is worth constructing the associated probability table for this.

	A	A^c	Total
B	$P(A \cap B)$	$P(A^c \cap B)$	$P(B)$
B^c	$P(B^c \cap A)$	$P(B^c \cap A)$	$P(B^c)$
Total	$P(A)$	$P(A^c)$	1

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Example 7

A school has 1200 students. The table below gives information about whether or not each student plays a musical instrument.

	Boys	Girls
Plays a musical instrument	325	450
Does not play an instrument	225	200

Each student has a school number between 1 and 1200.

Each student's number is written on a card and the 1200 cards are placed in a hat. One card is randomly pulled out of the hat. What is the probability that the number pulled out belongs to:

a a boy?

a a girl?

c a student who plays a musical instrument?

d a boy who plays a musical instrument?

Solution

a Number of students $= 1200$

Number of boys $= 325 + 225$

$= 550$

$$P(\text{boy's number}) = \frac{550}{1200} = \frac{11}{24}$$

b Number of students $= 1200$

Number of girls $= 450 + 200$

$= 650$

$$P(\text{girl's number}) = \frac{650}{1200} = \frac{13}{24}$$

c Number of students who play a musical instrument $= 325 + 450$

$$P(\text{musician's number}) = \frac{775}{1200} = \frac{31}{48}$$

d $$P(\text{boy musician's number}) = \frac{325}{1200} = \frac{13}{48}$$

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Example 8

A survey of 200 people was carried out to determine hair and eye colour. The results are shown in the table below.

	Fair	Brown	Red	Black
Blue	25	9	6	18
Brown	16	16	18	22
Green	15	17	22	16

What is the probability that a person chosen at random from this group has:

a fair *or* brown hair?

b blue *or* brown eyes?

c red hair *and* green eyes?

Solution

a $P(\text{fair } or \text{ brown hair}) = \dfrac{25+16+15+9+16+17}{200} = \dfrac{98}{200} = \dfrac{49}{100}$

b $P(\text{blue } or \text{ brown eyes}) = \dfrac{25+9+6+18+16+16+18+22}{200} = \dfrac{65}{100} = \dfrac{13}{20}$

c $P(\text{red hair } and \text{ green eyes}) = \dfrac{22}{200} = \dfrac{11}{100}$

Exercise 16C

Example 7

1 The patrons in a cinema each receive a numbered ticket when entering. The following two-way table describes the audience in terms of the categories Child–Adult and Male–Female.

	Male	Female	Total
Child	15	23	38
Adult	24	30	54
Total	39	53	92

A ticket is drawn at random from a hat. The winner will receive free popcorn and soft drink. What is the probability that the number drawn is held by:

a a male?

b a female?

c a child?

d a male child?

e a female child?

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2 Five thousand drivers were questioned and classified according to age and number of accidents in the last year. The results are in the table below.

	Younger than 28	28 or older	Total
No accidents	650	850	1500
One or more accidents	2500	1000	3500
Total	3150	1850	5000

A driver from the group of 5000 is chosen at random. What is the probability that the driver:

a is younger than 28?

b had no accidents and is younger than 28?

c is 28 years or older and has had one or more accidents?

3 The eye colour and gender of 300 people were recorded. The results are shown in the table below.

Eye colour / Gender	Blue	Brown	Green	Grey
Male	40	50	10	20
Female	80	70	10	20

What is the probability that a person chosen at random from the sample:

a has blue eyes?

b is male?

c is male and has green eyes?

d is female and does not have blue eyes?

e has blue eyes or is female?

f is male or does not have green eyes?

4 A bowl contains green and red normal jelly beans and green and red double-flavoured jelly beans. The number of each type is given in the following table.

	Green	Red
Normal jelly bean	39	54
Double-flavoured jelly bean	27	24

A jelly bean is randomly taken out of the bowl. Find the probability that:

a it is a double-flavoured jelly bean

b it is a green jelly bean

c it is a green normal jelly bean

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Example 8

5 A survey of 400 people was carried out to determine hair and eye colour. The results are shown in the table below.

Hair colour / Eye colour	Fair	Brown	Red	Black
Blue	50	18	12	36
Brown	32	32	36	44
Green	30	34	44	32

What is the probability that a person chosen at random from this group has:

a blue eyes?

b red hair?

c fair *or* brown hair?

d blue *or* brown eyes?

e red hair *and* green eyes?

f eyes that are not green?

g hair that is not red?

h fair hair *and* blue eyes?

i eyes that are not blue *or* hair that is not fair?

6 The 330 subjects volunteering for a medical study are classified by gender and blood pressure (high, normal and low). The results are shown in the table below.

	H	N	L
M	176	44	20
F	22	44	24

If a subject is selected at random, find:

a $P(\text{N})$

b $P(\text{F} \cap \text{H})$

c $P(\text{F} \cup \text{H})$

7 A survey of 3000 people was carried out to determine which type of breakfast they had. The results are shown in the table below.

Breakfast	Qld	NSW	Vic
Only toast	500	300	120
Only cereal	320	320	360
Cooked breakfast	300	340	440

What is the probability that a person chosen at random from this group:

a eats only toast for breakfast *and* comes from Victoria?

b eats only toast *or* only cereal?

c eats a cooked breakfast *and* comes from Queensland?

d eats only toast?

e comes from NSW?

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16D Further uses of Venn diagrams

Sometimes, instead of listing the outcomes of the experiment in the appropriate events on a Venn diagram, the number of outcomes in each event is written on the diagram. Consider the following examples.

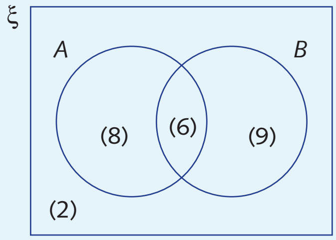
Example 9

In a group of 25 families, 14 own a Playstation, 15 own an Xbox, and 6 own both a Playstation and an Xbox. Represent this information on a Venn diagram.

Solution

Let A be the event 'family owns a Playstation' and B be the event 'family owns an Xbox'. There are:

- 6 elements in $A \cap B$
- $14 - 6 = 8$ elements in A but not B
- $15 - 6 = 9$ elements in A^c but not B
- 2 elements not in A or B since $6 + 8 + 9 + 2 = 25$



Example 10

In a class of 28 students, 20 study French and 15 study chemistry. Each student in the class studies either French or chemistry.

a Represent this information on a Venn diagram.

b One student is selected at random from the group. What is the probability that the student studies:

i both French and chemistry? **ii** chemistry but not French?

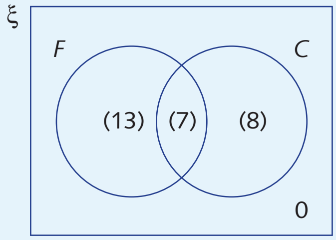
Solution

a Let F be the event 'student studies French' and C be the event 'student studies chemistry'. Twenty students study French and 15 study chemistry in a class of 28.

$$|F \cup C| = |F| + |C| - |F \cap C|$$

$$28 = 15 + 20 - |F \cap C|$$

Therefore, $|F \cap C| = 7$



Then $20 - 7 = 13$ students study French but not chemistry, and $15 - 7 = 8$ students study chemistry but not French.

b **i** Using the Venn diagram, $P(F \cap C) = \frac{7}{28} = \frac{1}{4}$

ii Using the Venn diagram, $P(F^c \cap C) = \frac{8}{28} = \frac{2}{7}$

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Exercise 16D

Example 9

1 In a group of 40 children, 12 like both heavy metal and hip-hop; 8 children like heavy metal but not hip-hop; 14 children like hip-hop but not heavy metal; and the rest like neither. Represent this information on a Venn diagram.

2 A survey of 50 families showed that 28 owned a cat, 30 owned a dog and 15 owned both a dog and a cat.

a Represent this information on a Venn diagram.

b One family is selected at random from the group. What is the probability that the family owns:

i a cat but not a dog?

ii a dog but not a cat?

iii neither a dog nor a cat?

Example 10

3 In a class of 26 students, 18 study chemistry and 12 study economics. Each student studies either chemistry or economics.

a Represent this information on a Venn diagram.

b One student is selected at random. What is the probability that the student studies:

i chemistry and economics?

ii only economics?

iii only chemistry?

4 In a youth club of 50 people, 18 play basketball, 28 play soccer and 10 play neither sport.

a Represent this information on a Venn diagram.

b One person is selected at random. What is the probability that the person chosen plays:

i basketball and soccer?

ii soccer but not basketball?

iii only basketball?

5 In a group of 50 students, 30 study mathematics, 25 study physics and 20 study both.

a Represent this information on a Venn diagram.

b One student is selected at random from the group. What is the probability that the student studies:

i mathematics but not physics?

ii physics but not mathematics?

iii neither physics nor mathematics?

6 In a group of 100 students, 50 study history, 30 study English literature and 20 study both.

a Represent this information on a Venn diagram.

b If a student is selected at random from the group, what is the probability that the student studies:

i at least one of these subjects?

ii history but not English literature?

iii history, given that the student also studies English literature?

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Review exercise

1 Alex has seven different caps in the bottom of his wardrobe. Four of them are black, two of them are blue and one is red. If he pulls out a cap at random, what is the probability it is blue?

2 A nurseryman has azaleas at a special low price because they have lost their labels and they are not yet in flower. From a trolley that has 15 pink azaleas and eight red azaleas, what is the probability of choosing:

a a pink azalea? **b** a red azalea?

3 Chelsea has 12 ribbons in her top drawer. Four of them are pink, three are blue, three are green and two are white. If she chooses one ribbon at random, what is the probability that it is green?

4 A debating team consists of four boys and eight girls. If one of the team is chosen at random to be the leader, what is the probability that the leader is a girl?

5 A basketball team consists of five players: Adams, Brown, Cattogio, O'Leary and Nguyen. If a player is chosen at random, what is the probability that his name starts with a vowel?

6 Slips of paper numbered 1, 2, 3, ..., 10 are placed in a hat and one is drawn at random. What is the probability that the number on the slip of paper is not a multiple of three?

7 Heidi chooses, at random, one shape from the following set: equilateral triangle, square, parallelogram, rectangle, circle. What is the probability that the chosen shape has exactly four axes of symmetry?

8 A basket contains the following balls: two AFL footballs, three soccer balls, a basketball, a rugby ball and two tennis balls. If Liam chooses a ball to play with, at random, what is the probability that he chooses a round ball?

9 A jar contains 27 balls. Twenty of the balls have a star on them and 10 of the balls have an elephant printed on them. Every ball has at least one of these symbols on it.

a Draw a Venn diagram to illustrate this.

b If one ball is withdrawn at random, what is the probability of choosing:

i a ball with an elephant printed on it?

ii a ball with a star and an elephant printed on it?

iii a ball with an elephant printed once but without a star?

10 In a group of 200 students, 100 study geography, 60 study mathematics and 40 study both.

a Represent this information on a Venn diagram.

b If a student is selected at random from the group, what is the probability that the student studies:

i at least one of these subjects? **ii** geography but not mathematics?

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11 A card is drawn at random from a well-shuffled pack of playing cards. Find the probability that the card chosen:

a is a Heart

b is a court card (that is, a King, Queen or Jack)

c has a face value between 2 and 8 inclusive

d is a Heart or a court card

e is a Heart and a court card

f has a face value between 2 and 8 inclusive and is a court card

g has a face value between 2 and 8 inclusive or is a court card

12 A survey of 150 people was carried out to determine eye colour and gender. The results are shown in the table below.

Eye colour	Male	Female
Blue	20	40
Brown	25	35
Green	15	15

What is the probability of a person chosen at random:

a having blue eyes?

b being male?

c being male *and* not having blue eyes?

d being female *and* not having blue eyes?

e having blue eyes *or* being female?

f being male *or* not having green eyes?

13 A survey of 100 people was carried out to determine which hand they preferred to use and their gender. The results are shown in the table below.

Preferred hand	Male	Female
Left	15	22
Right	33	30

What is the probability of person chosen at random being:

a left-handed?

b female?

c male *and* right-handed?

d male *or* right-handed?

e left-handed *and* female?

f left-handed *or* female?

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Challenge exercise

1 Seventy-five students all own a dog, a cat or a bird. They were asked which pets they own. The replies gave the following information:
37 own a bird, 33 own a cat, 40 own a dog, 16 own both a cat and a bird, 11 own both a dog and a cat, 12 own both a bird and a dog.

a Draw a Venn diagram representing this information.

b How many students own a bird, a dog and a cat?

c A student is chosen at random and asked which pets they own. What is the probability that they own:

i a cat and a dog only?

ii a bird and a dog only?

iii a bird, a dog and a cat?

iv a bird but not a cat?

v a dog but not a bird?

2 A bag contains 1000 balls, some of which are red, some blue and the rest yellow.

The probability of drawing a red ball is $\frac{1}{25}$ and the probability of drawing a yellow ball is $\frac{7}{20}$. Find:

a the number of yellow balls

b the number of blue balls

c the number of yellow balls that need to be removed to make the probability of drawing a red ball $\frac{1}{20}$

d the number of yellow balls (from the original 1000) that need to be removed so that the probability of drawing a yellow ball is $\frac{1}{6}$

3 A box contains 3 black and 1 yellow ball. A second box contains 2 black and 2 yellow balls. A ball is taken from the first box and put in the second. A ball is then withdrawn from the second box. Find the probability that the ball taken from the second box is:

a black

b yellow

4 A three-digit number is chosen at random.

a Describe the sample space and draw a Venn diagram.

Find the probability that the number is divisible by:

b 5

c 7

d 3

e 5 and 7

f 5 or 7

g 3 and 5 and 7

h 3 and 5 but not 7

i 5 and 7 but not 3

j 3 or 5 or 7

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5 A bag contains 120 balls, some of which are red and the rest black. The probability of a ball, drawn at random, being red is $\frac{2}{15}$. Find:

a the number of red balls in the bag

b the number of black balls in the bag

c the number of red balls that should be added to the bag to change the probability of obtaining a red ball to $\frac{1}{2}$

6 Find the probability that a three-digit number chosen at random is divisible by:

a 5 b 3 c 15 d 3 or 5

7 A bag contains 120 balls, some of which are red, some blue and the rest yellow. The probability of drawing a blue ball is $\frac{1}{12}$ and the probablity of drawing a red ball is $\frac{2}{5}$. Find:

a the number of yellow balls in the bag

b the number of yellow balls that should be removed from the bag so that the probability of drawing a yellow ball is $\frac{1}{3}$

8 A game is played in which a counter is placed on the square C.

A	B	C	D	E

A coin is tossed. If it comes down 'head', the counter is moved one square to the right. If the coin comes down 'tail', the counter is moved one square to the left. Find the probability that, after two tosses of the coin, the counter will be on square:

a A b C c E

9 Jim has $512 and plays a game in which he has an equal chance of winning or losing. He wins four times and loses four times in random order. He bets half the money he has left at each stage. A win gives him twice what he bets. How much does Jim have at the end?

10 In a school of 650 everybody studies French (F), German (G) or Indonesian (I). All but 41 study French, 12 study French and Indonesian but not German. Thirteen study Indonesian and German but not French and the same number study German only. Twice as many study French and German but not Indonesian as study all three. The number studying Indonesian only is the same as the total studying both French and German. What is the probability that a student chosen at random studies French only?

11 A teacher presents three problems to a class. Sixty per cent of them solve problem 1. Forty per cent solve problem 2 and 55% solve problem 3. Also,

- 15% solve problems 1 and 2 only
- 35% solve problems 1 and 3 only
- 15% solve problems 2 and 3 only
- 5% solve all three problems.

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A student is chosen at random. What is the probability that they solve:

a exactly two of the problems?

b none of the problems?

12 The 28 students in a class all buy music CDs. The categories they choose are rock (R), folk (F) and classical (C). One student buy classical CDs only. Nine students buy folk CDs only and five students but rock CDs only. Also,

$|R \cap C| = 12$, $|C \cap F \cap R| = |F^c|$ and $|F \cap R| > |F \cap C|$

A student is selected at random. Find the probability that the student studies both classical and folk.

13 Weather records for July 2010 in Sydney showed that the month had no hot, calm, dry days. Of the 31 days, 7 were wet and cold but not windy, 4 were wet and windy but not cold, 8 were cold and windy but dry. 16 days were windy, 22 were cold and 2 were wet but not cold or windy. If a day is chosen at random. What is the probability that the day was:

a cold, wet and windy?

b cold but calm and dry?

14 Use Venn diagrams to simplify each of the following events or express in a different form.

a $(A \cap B^c) \cup (A \cap B)$

b $(A \cup B)^c$

c $(A \cap B)^c$

d $(A^c \cap B) \cap (A \cap B^c)$

e $((A^c \cap B^c) \cup (A \cap B^c))^c$

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CHAPTER

17

Number and Algebra

Formulas and factorisation

Formulas are widely used in many fields of work and study, including science, medicine, finance and engineering. We also commonly use formulas when we are working with spreadsheets on a computer.

In the course of this chapter, topics in algebra that have been studied earlier will be revised. Substitution into formulas often results in an equation that needs to be solved.

We introduce another important technique, factorisation, which is the opposite of expanding of brackets.

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17A Formulas

Formulas have been used in earlier chapters. For example:

- the area A of a rectangle is given by the formula $A = \ell w$, where ℓ and w are the length and width of the rectangle
- the volume V of a cylinder is given by the formula $V = \pi r^2 h$, where r is the radius of the cylinder and h the height of the cylinder.

The subject of a formula

In the first formula above, A is the **subject** of the formula; in the second formula, V is the subject. It is called the subject as it is expressed in terms of the other pronumerals.

In Example 1, the value of the subject of the formula is to be found, given the values of the other pronumerals in the formula.

Example 1

Given the formula $P = 2\ell + 2w$, find the value of P if $\ell = 10$ and $w = 5$.

Solution

$$\begin{aligned} P &= 2\ell + 2w \\ &= 2 \times 10 + 2 \times 5 \\ &= 30 \end{aligned}$$

If we are given a set of values that includes the value of the subject of the formula, we will need to solve an equation to determine the unknown value.

Example 2

Given the formula $y = mx + c$, find:

a the value of x if $y = 3$, $m = 2$ and $c = -4$

b the value of m if $y = 13$, $x = 2$ and $c = 3$

Solution

a $y = mx + c$

$3 = 2x - 4$ (Substitute $y = 3$, $m = 2$ and $c = -4$.)

Solve the equation for x.

$2x = 7$ (Add 4 to both sides of the equation.)

$x = 3\frac{1}{2}$ (Divide both sides of the equation by 2.)

(*continued over page*)

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b $y = mx + c$

$13 = 2m + 3$ (Substitute the values.)

Solve the equation for m.

$2m = 10$ (Subtract 3 from both sides of the equation.)

$m = 5$ (Divide both sides of the equation by 2.)

Example 3

Given the formula $P = 2(\ell + w)$, find the value of w if $P = 21$ and $\ell = 5$.

Solution

$$\begin{aligned}
P &= 2(\ell + w) && \\
21 &= 2(5 + w) && \text{(Substitute the values.)}\\
21 &= 10 + 2w && \text{(Expand the brackets.)}\\
2w + 10 &= 21 && \\
2w &= 11 && \text{(Subtract 10 from both sides of the equation.)}\\
w &= \frac{11}{2} && \text{(Divide both sides of the equation by 2.)}\\
&= 5\tfrac{1}{2} &&
\end{aligned}$$

Example 4

The formula for the surface area S cm^2 of a rectangular prism is $S = 2(\ell w + \ell h + hw)$, where ℓ, w and h are the length, width and height of the prism, in cm. If $S = 60$, $\ell = 5$ and $w = 4$, find h, the height of the prism.

Solution

$$\begin{aligned}
60 &= 2(5 \times 4 + 5 \times h + 4 \times h) && \text{(Substitute in values.)}\\
60 &= 40 + 10h + 8h && \\
20 &= 10h + 8h && \text{(Subtract 40 from both sides of the equation.)}\\
20 &= 18h && \\
\frac{10}{9} &= h && \text{(Divide both sides of the equation by 18.)}\\
h &= 1\tfrac{1}{9} &&
\end{aligned}$$

The height of the rectangular prism is $1\frac{1}{9}$ cm.

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Exercise 17A

Example 1

1 The area of a triangle, A cm^2, is given by the formula $A = \frac{bh}{2}$, where b cm is the length of the base and h cm is the height of the triangle.

a Find the area when $b = 6$ and $h = 7$.

b Find the length of the base if the area is 72 cm^2 and the height is 8 cm.

Example 2

2 Given the formula $y = mx + c$:

a find y if $m = 6$, $x = 3$ and $c = -10$

b find y if $m = -2$, $x = 3$ and $c = 7$

c find x if $m = 8$, $y = 20$ and $c = 5$

d find x if $m = -4$, $y = 10$ and $c = 2$

3 Temperature is measured using two different scales. The formula for changing the temperature in degrees Fahrenheit (°F) to degrees Celsius (°C) is $C = \frac{5}{9}(F - 32)$, where C is the temperature in °C and F is the temperature in °F.

Find the temperature in °C corresponding to a temperature of:

a 100°F

b 212°F

c 0°F

d –40°F

Example 3

4 Given the formula $T = a + (n - 1)d$:

a find T if $a = 6$, $n = 10$ and $d = 2$

b find T if $a = 6$, $n = 10$ and $d = -2$

c find a if $T = 300$, $n = 5$ and $d = 2$

d find n if $T = 49$, $d = 3$ and $a = 10$

e find d if $T = 35$, $n = 6$ and $a = 5$

f find d if $T = -35$, $n = 6$ and $a = -5$

5 The average a of two numbers m and n is given by the formula $a = \frac{m + n}{2}$.

a Find a if $m = 2$ and $n = 8$.

b Find a if $m = 6$ and $n = -10$.

c Find m if $a = 6$ and $n = 4$.

d Find a if $m = -2$ and $n = -6$.

e Find m if $a = 0$ and $n = -9$.

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6 The formula for the interior angle I of a regular polygon with n sides is $I = \dfrac{180n - 360}{n}$, where I is measured in degrees.

a Find the size of each interior angle of:

i a pentagon (5 sides) **ii** a hexagon (6 sides) **iii** a dodecagon (12 sides)

b Find the number of sides of a polygon for which the size of each interior angle is:

i 135° **ii** 144°

7 If $s = 2(a - b)$, find:

a s when $a = 5$ and $b = -2$

b a when $s = 10$ and $b = 6$

c b when $a = -2$ and $s = 6$

8 Given that $A = \dfrac{PRT}{100}$, find:

a A when $P = 200$, $R = 4$ and $T = 6$

b P when $A = 1200$, $R = 3$ and $T = 10$

c T when $A = 14\,000$, $P = 12\,000$ and $R = 5$

9 The area of a rectangle is x cm^2. The length of the rectangle is 4 cm.

a Write an expression in terms of x for the width of the rectangle.

b Write a formula for the perimeter P cm of the rectangle in terms of x.

c Find P if:

i $x = 10$ **ii** $x = 24$

d Find x if:

i $P = 20$ **ii** $P = 40$

Example 4

10 The formula for the surface area S cm^2 of a rectangular prism is $S = 2(\ell w + \ell h + hw)$, where ℓ, w and h are the length, width and height of the prism, measured in cm. If $S = 80$, $\ell = 5$ and $w = 2$, find h, the height of the prism.

11 If $c = a^2 + b^2$, find:

a the value of c when $a = 2$ and $b = 1$

b the value of c when $a = 3$ and $b = 4$

c the value of c when $a = 5$ and $b = 12$

12 If $c = \sqrt{a^2 + b^2}$, find the value of c when:

a $a = 5$ and $b = 12$

b $a = 3$ and $b = 4$

c $a = 24$ and $b = 7$

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17B Expansion and factorisation

We have learned how to expand brackets. For example:

$2(x+3)=2x+6$ and $3(x-4)=3x-12$

Example 5

Expand the brackets.

a $3(3x+5)$ **b** $-4(7-2x)$ **c** $3x(x+2)$

d $ab(2a+b)$ **e** $-3x^2(x+3)$

Solution

a $3(3x+5)=9x+15$

b $-4(7-2x)=-28+8x$

c $3x(x+2)=3x^2+6x$

d $ab(2a+b)=2a^2b+ab^2$

e $-3x^2(x+3)=-3x^3-9x^2$

There are many situations where one needs to reverse this process.

For example:

$2x+6=2(x+3)$ and $3x-12=3(x-4)$

The expression $2x+6$ has two **terms**, $2x$ and 6. The common factor of these terms is 2.

We write:

$$\begin{aligned}2x+6 &= 2\times x+2\times 3\\ &= 2(x+3)\end{aligned}$$

- To go from the left-hand side of this equation to the right-hand side, you **factorise** $2x+6$.
- To go from the right-hand side of this equation to the left-hand side, you **expand the brackets**.

The new procedure is called **factorisation** and it is intrinsically harder than expanding brackets.

Highest common factor

The expression $12x^2+9x$ has two terms, $12x^2$ and $9x$.

- They have a common factor 3.
- They have a common factor x.
- They have a common factor $3x$.
- They have no other common factors.

In this case, $3x$ is the highest common factor (HCF), and we write $12x^2+9x=3x(4x+3)$.

Similarly we say that $3ab^2$ is the highest common factor of $21a^2b^2$ and $15ab^2$ since 3 is the HCF of 21 and 15, a is the highest power of a which divides both expressions and b^2 is the highest power of b which divides both expressions.

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Example 6

Find the highest common factor of each pair of terms.

a $3x, 5x$ **b** $2x, 8x$ **c** $8x, 12$ **d** $15x, 20y$

e $2x^2, 6x$ **f** $12ab^2, 6b$ **g** $3x^3, 4x^2$

Solution

a The highest common factor of $3x$ and $5x$ is x.

b $8x = 4 \times 2x$

The highest common factor of $2x$ and $8x$ is $2x$.

c $8x = 4 \times 2x$ and $12 = 4 \times 3$

The highest common factor of $8x$ and 12 is 4.

d $15x = 5 \times 3x$ and $20y = 5 \times 4y$

The highest common factor of $15x$ and $20y$ is 5.

e $2x^2 = 2x \times x$ and $6x = 2x \times 3$

The highest common factor of $2x^2$ and $6x$ is $2x$.

f $12ab^2 = 6b \times 2ab$

The highest common factor of $12ab^2$ and $6b$ is $6b$.

g The highest common factor of $3x^3$ and $4x^2$ is x^2.

Example 7

Factorise:

a $3x + 12$ **b** $4z^2 + 3z$ **c** $-4x + 8$

Solution

a
$$\begin{aligned} 3x + 12 &= 3 \times x + 3 \times 4 \\ &= 3(x + 4) \end{aligned}$$

b
$$\begin{aligned} 4z^2 + 3z &= z \times 4z + z \times 3 \\ &= z(4z + 3) \end{aligned}$$

c $-4x + 8 = 4(-x + 2)$ *or* $-4x + 8 = -4(x - 2)$

(Either of these answers is acceptable.)

Example 8

Factorise:

a $2x + 4y$ **b** $2xy - 4y$ **c** $6ab^2 - ab$

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Solution

a $2x + 4y = 2 \times x + 2 \times 2y$
$= 2(x + 2y)$

b $2xy - 4y = 2y \times x - 2y \times 2$
$= 2y(x - 2)$

c $6ab^2 - ab = ab \times 6b - ab \times 1$
$= ab(6b - 1)$

Example 9

Factorise:

a $abc + 4a^2bc$

b $2x^3y - 6y + 14xy^2$

Solution

a $abc + 4a^2bc = abc \times 1 + abc \times 4a$
$= abc(1 + 4a)$

b $2x^3y - 6y + 14xy^2 = 2y \times x^3 - 2y \times 3 + 2y \times 7xy$
$= 2y(x^3 - 3 + 7xy)$

Exercise 17B

Example 5

1 Expand the brackets.

a $9(2x + 5)$ **b** $2(5 + 3x)$ **c** $7(2x - 11)$ **d** $5(2x + 10)$

e $-4(11 + 2x)$ **f** $-6(3x - 5)$ **g** $-5(x - 1)$ **h** $7(-3x - 4)$

i $5x(x + 2)$ **j** $3x(x - 6)$ **k** $-7x(2x - 11)$ **l** $5x(10 - x)$

m $ab(2a + b)$ **n** $3x^2(x + 3)$ **o** $-4x^2(3 - 2x)$ **p** $-5x(2x + 3)$

q $ab^2(a + 3)$ **r** $pq(p + q)$ **s** $-xy(3x + 2y)$ **t** $xy^2(2x - 2y)$

Example 6

2 Find the highest common factor of each pair of terms.

a $4, 12$ **b** $18x, 27x$ **c** $12x^2, 6x$ **d** $24ab^2, 3b$

e $15x, 18$ **f** $14x^2, 70$ **g** $30x, 10y$ **h** $56a, 84$

i $15x, 4y$ **j** $16x^3, 5x^2$ **k** $20xy, y^2$

3 Factorise:

a $2a + 4$ **b** $-2a + 4$ **c** $-2x - 4$ **d** $20a + 30$

e $-56x + 96$ **f** $5 - 15x$ **g** $20 - 25x$ **h** $100 - 25x$

i $-56x - 80$ **j** $18 - 90z$

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Example 7

4 Factorise:

a $5x+60$ **b** $6m^2+3m$ **c** $-3x+6$ **d** $6x^2+3x$

e $8-4x$ **f** $8x-16x^2$ **g** $-5x+10x^2$ **h** $70+10a$

i $3x+30$ **j** $5m^2+15m$ **k** $-3x^2+6x$ **l** $4x^2+12x$

m $25-5x^2$ **n** $4x-16x^2$ **o** $-m+10m^2$ **p** $-70n+10n^2$

q $4x^3-60x$ **r** $18m^3+3m^2$ **s** $-3x+6x^2$ **t** $-6x^2-3x$

u $8x^2-4x$ **v** $3x-15x^2$ **w** $-15x+10x^2$ **x** $70a^2+10a$

Example 8

5 Factorise:

a $3x+9y$ **b** $xy+4x$ **c** $2x-8y$ **d** $6x+8xy$

e $3a-12ab$ **f** $10m+8mn$ **g** $12xy+10x^2y$ **h** $pq+qr$

Example 9

6 Factorise:

a $3xyz-12xy^2z$ **b** $16a^2bc+4ab^3c$

c $8m^2np+20m^2n^3p^4$ **d** $9p^2q-3pq+12pq^2$

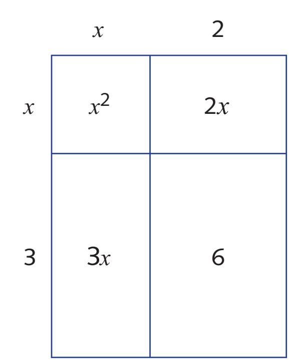
e $5a^2b+20abc+15ab^2c$

17C Binomial products

How do we expand an expression of the form $(x+2)(x+3)$?

An expression such as $(x+2)(x+3)$ is called a **binomial product**, as each factor of the product has two terms in it. To expand a binomial expression, we multiply each term in the second bracket by each term in the first, and then collect like terms.

$$\begin{aligned}(x+2)(x+3) &= x(x+3)+2(x+3)\\ &= x^2+3x+2x+6\\ &= x^2+5x+6\end{aligned}$$



The expansion can be illustrated using an area diagram, as shown above.

Example 10

Expand and simplify:

a $x(x+4)$ **b** $(x+3)(x+4)$ **c** $(2x+5)(x+4)$

d $(x-5)(x+6)$ **e** $(x-3)^2$ **f** $(x-4)(x+4)$

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Solution

a $x(x+4) = x^2 + 4x$

b $(x+3)(x+4) = x(x+4) + 3(x+4)$
$= x^2 + 4x + 3x + 12$
$= x^2 + 7x + 12$

c $(2x+5)(x+4) = 2x(x+4) + 5(x+4)$
$= 2x^2 + 8x + 5x + 20$
$= 2x^2 + 13x + 20$

d $(x-5)(x+6) = x(x+6) - 5(x+6)$
$= x^2 + 6x - 5x - 30$
$= x^2 + x - 30$

e $(x-3)^2 = (x-3)(x-3)$
$= x(x-3) - 3(x-3)$
$= x^2 - 3x - 3x + 9$
$= x^2 - 6x + 9$

f $(x-4)(x+4) = x(x+4) - 4(x+4)$
$= x^2 + 4x - 4x - 16$
$= x^2 - 16$

Exercise 17C

Example 10

1 Expand and simplify:

a $x(x-6)$ **b** $x(2x+1)$ **c** $2x(x-4)$
d $(x+2)(x-3)$ **e** $(x+8)(x-7)$ **f** $(x+10)(x-5)$
g $(m+2)(m-3)$ **h** $(z+6)(z-6)$ **i** $(a+10)(a+2)$
j $(n-8)(n+5)$ **k** $(z+6)(z+2)$ **l** $(2c+3)(5c+6)$
m $(2a-5)(a+10)$ **n** $(2y-7)(y+10)$ **o** $(2s-6)(s+8)$
p $(y-9)^2$ **q** $(x+5)^2$ **r** $(2x-3)^2$

2 Expand and simplify:

a $(x+2)^2$ **b** $(x-4)^2$ **c** $(2x-7)^2$
d $(4a-2)^2$ **e** $(3m-5)^2$ **f** $(4x-6)^2$
g $(3m-6)^2$ **h** $(4-h)^2$ **i** $(3m-4)^2$

3 Expand and simplify:

a $(a-6)(a+6)$ **b** $(x-5)(x+5)$ **c** $(2x-4)(2x+4)$
d $(5s-3)(5s+3)$ **e** $(6m-2)(6m+2)$ **f** $(3n-7)(3n+7)$
g $(6c-7)(6c+7)$ **h** $(7v+3)(7v-3)$ **i** $(5-m)(5+m)$

4 Expand and simplify:

a $(m+7)(m-7)$ **b** $(7b+6)(4b+3)$ **c** $(a+2b)(a-2b)$
d $(m-n)(m-2n)$ **e** $(m-n)(m+n)$ **f** $(a+b)^2$
g $(2a-b)(a+3b)$ **h** $(g-3h)(g+3h)$ **i** $(a-b)^2$

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17D Factorisation of simple quadratics

A **monic quadratic** expression is an expression of the form $x^2 + bx + c$, where b and c are given numbers. When we expand $(x + 3)(x + 4)$, we obtain a monic quadratic:

$$\begin{aligned} x(x+4) + 3(x+4) &= x^2 + 4x + 3x + 12 \\ &= x^2 + 7x + 12 \end{aligned}$$

We want to develop a method for reversing this process.

In the expansion $(x + 3)(x + 4) = x^2 + 7x + 12$, notice that the coefficient of x is $3 + 4 = 7$. The term that is independent of x, the constant term, is $3 \times 4 = 12$. This suggests a method of factorising.

In general, when we expand $(x + p)(x + q)$, we obtain

$$x^2 + px + qx + pq = x^2 + (p + q)x + pq$$

The coefficient of x is the sum of p and q, and the constant term is the product of p and q.

Factorisation of simple quadratics

To factorise a simple quadratic, look for two numbers that add to give the coefficient of x, and that multiply together to give the constant term.

You can check your answer by expansion.

To factorise $x^2 + 8x + 15$, we look for two numbers that multiply to give 15 and add to give 8. Of the pairs that multiply to give 15(15×1, 5×3, $-5 \times (-3)$ and $-15 \times (-1)$), only 5 and 3 add to give 8.

Therefore, $x^2 + 8x + 15 = (x + 3)(x + 5)$.

The result can be checked by expanding $(x + 3)(x + 5) = x(x + 5) + 3(x + 5)$.

Example 11

Factorise:

a $x^2 + 6x + 8$ **b** $x^2 - 6x + 8$ **c** $x^2 - 3x - 18$

Solution

a We are looking for two numbers with product 8 and sum 6. The numbers are 2 and 4.
$x^2 + 6x + 8 = (x + 2)(x + 4)$

b We are looking for two numbers with product 8 and sum -6. The numbers are -2 and -4.
$x^2 - 6x + 8 = (x - 2)(x - 4)$

c We are looking for two numbers with product -18 and sum -3. The numbers are -6 and 3.
$x^2 - 3x - 18 = (x - 6)(x + 3)$

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Example 12

Factorise $x^2 - 36$.

Solution

We are looking for two numbers with product –36 and sum 0. The numbers are –6 and 6.

Thus $x^2 - 36 = (x-6)(x+6)$

Example 13

Factorise $x^2 + 8x + 16$.

Solution

We are looking for two numbers with product 16 and sum 8. The numbers are 4 and 4.

Thus $x^2 + 8x + 16 = (x+4)(x+4)$
$= (x+4)^2$

Exercise 17D

Example 11a

1 Factorise these quadratic expressions.

a $x^2 + 4x + 3$	**b** $x^2 + 5x + 4$	**c** $x^2 + 7x + 6$
d $x^2 + 5x + 6$	**e** $x^2 + 11x + 24$	**f** $x^2 + 13x + 36$
g $x^2 + 10x + 16$	**h** $x^2 + 11x + 30$	**i** $x^2 + 12x + 20$
j $x^2 + 14x + 48$	**k** $x^2 + 16x + 63$	**l** $x^2 + 18x + 80$
m $x^2 + 11x + 28$	**n** $x^2 + 16x + 55$	**o** $x^2 + 12x + 32$

Example 11b

2 Factorise these quadratic expressions.

a $x^2 - 6x + 5$	**b** $x^2 - 7x + 12$	**c** $x^2 - 8x + 15$
d $x^2 - 9x + 14$	**e** $x^2 - 10x + 24$	**f** $x^2 - 12x + 35$
g $x^2 - 12x + 32$	**h** $x^2 - 9x + 18$	**i** $x^2 - 13x + 12$
j $x^2 - 15x + 14$	**k** $x^2 - 13x + 42$	**l** $x^2 - 17x + 72$
m $x^2 - 15x + 56$	**n** $x^2 - 19x + 90$	**o** $x^2 - 17x + 52$

Example 11c

3 Factorise these quadratic expressions.

a $x^2 - x - 6$	**b** $x^2 + 2x - 15$	**c** $x^2 + 3x - 10$
d $x^2 - 6x - 16$	**e** $x^2 - 4x - 12$	**f** $x^2 + 2x - 24$
g $x^2 + x - 30$	**h** $x^2 - 3x - 18$	**i** $x^2 - 7x - 18$
j $x^2 + x - 20$	**k** $x^2 + 3x - 40$	**l** $x^2 - 4x - 5$
m $x^2 - 4x - 45$	**n** $x^2 + x - 42$	**o** $x^2 - x - 20$

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Example 12

4 Factorise these quadratic expressions.

a $x^2 - 4$　b $y^2 - 16$　c $x^2 - 1$

d $m^2 - 25$　e $p^2 - 9$　f $x^2 - 144$

Example 13

5 Factorise these quadratic expressions.

a $x^2 + 14x + 49$　b $a^2 + 10a + 25$　c $x^2 - 6x + 9$

d $x^2 - 12x + 36$　e $a^2 - 10a + 25$　f $x^2 + 6x + 9$

6 Factorise these quadratic expressions.

a $x^2 + 13x + 36$　b $x^2 + 15x + 36$　c $x^2 + 20x + 36$

d $x^2 + 12x + 36$　e $x^2 - 15x + 36$　f $x^2 - 13x + 36$

g $x^2 - 2x - 24$　h $x^2 + 5x - 24$　i $x^2 + 10x - 24$

j $x^2 - 23x - 24$　k $x^2 - x - 42$　l $x^2 - x - 30$

m $x^2 + x - 132$　n $x^2 + x - 30$　o $x^2 - 2x - 48$

p $x^2 + 8x - 48$　q $x^2 + 22x - 48$　r $x^2 - 47x - 48$

s $x^2 - 14x + 48$　t $x^2 + 16x + 48$　u $x^2 + 14x + 40$

v $x^2 - 22x + 40$　w $x^2 - 3x - 40$　x $x^2 + 18x - 40$

y $x^2 - 3x - 28$　z $x^2 + 12x - 28$

Review exercise

1 a Given the formula $C = \frac{5}{9}(F - 32)$:

i find C if $F = 27$　ii find F if $C = 40$

b Given the formula $A = \frac{1}{2}(a + b)h$:

i find A if $a = 2$, $b = 5$ and $h = 3$　ii find h if $A = 20$, $a = 3$ and $b = 7$

2 Find the highest common factor of each pair of terms.

a $10, 5$　b $7x, 7y$　c $12x, 28x$　d $21y, 42y^2$

3 Factorise:

a $2x + 6$　b $7x - 49$　c $-3 + 42x$　d $100x + 5$

e $-14x + 28$　f $8x^2 - x$　g $30x^2 + 12$　h $-20y + 25y^2$

4 Factorise:

a $4x + 12y$　b $xy + 6x$　c $15x - 10y$　d $7x + 14xy$

e $12a - 4ab$　f $6m + 9mn$　g $14xy + 4x^2y$　h $10pq + 8qr$

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5 Factorise:

a $4xyz - 6xy^2z$ **b** $25a^2bc + 10ab^3c$ **c** $9m^2np + 27m^2n^3p^4$

d $22p^2q - 2pq + 4pq^2$ **e** $6a^2b + 8abc + 4ab^2c$ **f** $4x^2yz - 8xy^2z - 12xyz^2$

6 Expand the brackets and simplify in each case.

a $x(x+15)$ **b** $x(6x+7)$ **c** $5x(2x-1)$

d $(x+6)(x+4)$ **e** $(x+4)(x-2)$ **f** $(2x-1)(x+5)$

g $(x-4)(4x+1)$ **h** $(3x-2)(5x+1)$ **i** $(x-6)(2x-3)$

7 Expand the brackets and simplify in each case.

a $(x+1)^2$ **b** $(x+2)^2$ **c** $(x+9)^2$

d $(x-1)^2$ **e** $(x-5)^2$ **f** $(x-4)^2$

g $(2x+1)^2$ **h** $(3x-2)^2$ **i** $(9-x)^2$

8 The formula used in an experiment is $E = \dfrac{w}{w+x}$.

Find the value of E when $w = 30$ and $x = 18$.

9 For the formula $s = ut + \dfrac{1}{2}at^2$, find:

a the value of s, when $u = 4$, $t = 3$ and $a = 6$

b the value of u, when $s = 100$, $t = 5$ and $a = 4$

c the value of a, when $s = 200$, $t = 10$ and $u = 6$

10 Factorise:

a $x^2 + 8x + 12$ **b** $x^2 + 9x + 18$ **c** $x^2 + 11x + 30$

d $x^2 + 11x + 28$ **e** $x^2 - 11x + 24$ **f** $x^2 - 10x + 24$

g $x^2 - 14x + 24$ **h** $x^2 - 25x + 24$ **i** $x^2 + x - 20$

j $x^2 - 2x - 48$ **k** $x^2 - 4x - 12$ **l** $x^2 + 3x - 40$

m $x^2 - 7x - 8$ **n** $x^2 - x - 132$ **o** $x^2 + 15x - 100$

11 Factorise these quadratic expressions.

a $x^2 - 64$ **b** $y^2 - 81$ **c** $x^2 - 121$

d $m^2 - 169$ **e** $p^2 - 225$ **f** $x^2 - 196$

12 Factorise these quadratic expressions.

a $x^2 + 10x + 25$ **b** $a^2 + 40a + 400$ **c** $x^2 - 16x + 64$

d $x^2 - 2x + 1$ **e** $a^2 - 40a + 400$ **f** $x^2 + 16x + 64$

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Challenge exercise

1 **a** Expand $(e+4)^2$.

b Use this result to expand:

i $(e+4)^3$ **ii** $(e+4)^4$

2 Expand:

a $(a+b+1)(a+b-1)$ **b** $(x+a-3)(x-a-3)$

c $(x^2+x+1)(x^2-x+1)$ **d** $(x-1)(x-1)(x^2+1)$

e $(x-1)(x^2+x+1)$ **f** $(x+1)(x^2-x+1)$

3 Draw diagrams (involving rectangles) to show that:

a $(x+4)^2 = x^2+8x+16$ **b** $(a+b)^2 = a^2+2ab+b^2$

4 **Heron's formula** for the area A of a triangle with side lengths a, b and c is given by

$A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \dfrac{a+b+c}{2}$

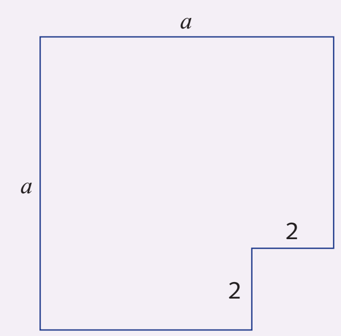
a Find A for $a = 3$, $b = 4$ and $c = 5$.

b Find A for $a = 13$, $b = 14$ and $c = 15$.

c Find A for $a = 193$, $b = 194$, $c = 195$.

Hint: Factorise A^2 into its prime factors.

5 **a** The figure opposite represents a square piece of paper with a corner cut out. Make a single straight cut to reshape the paper to form a rectangle with length $a+2$ and width $a-2$. This illustrates $a^2-4 = (a+2)(a-2)$.



b Similarly, illustrate the identity $a^2-b^2 = (a-b)(a+b)$.

6 In Question **5b** we saw that $(a-b)(a+b) = a^2-b^2$. We can use this identity to perform arithmetic short cuts. For example:

$$\begin{aligned} 49 \times 51 &= (50-1)(50+1) \\ &= 50^2 - 1^2 \\ &= 2500 - 1 \\ &= 2499 \end{aligned}$$

Use this trick to find:

a 19×21 **b** 29×31 **c** 48×52

d 67×73 **e** 99×101 **f** 202×198

7 **a** Expand and simplify $(n+1)^2 - n^2$.

b Using your answer, write the number 51 as the difference of two square numbers.

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CHAPTER

18

Number and Algebra

Graphing straight lines

Graphing straight lines is contained in a topic in mathematics called **coordinate geometry**. Coordinate geometry is one of the most important and exciting ideas of mathematics. It provides a connection between algebra and geometry through graphs of lines and curves. This enables geometric problems to be solved algebraically and provides geometric insights into algebra. An immediate illustration of this will be shown in this chapter through the relation between solving linear equations and straight-line graphs.

Straight-line graphs can be used in many different practical situations. It will be seen how straight-line graphs can be used to help to understand problems involving constant rate.

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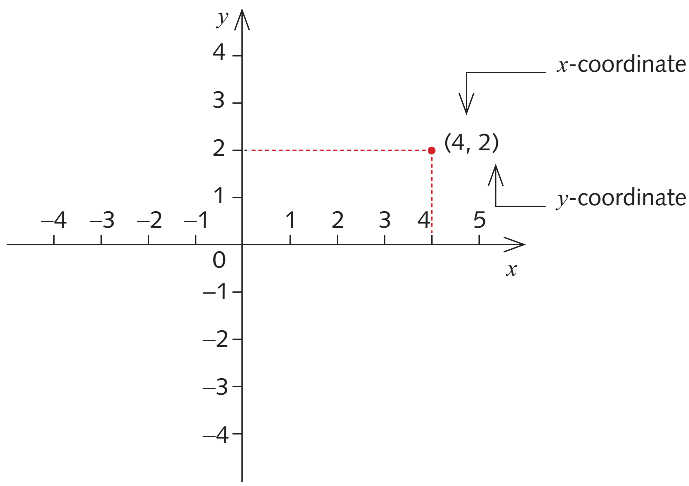
18A The Cartesian plane



The number plane was introduced in Chapter 4. This is known as the **Cartesian plane**. In the diagram above, we have labelled the horizontal axis the x-axis and the vertical axis the y-axis. With this labelling, the first coordinate of a point is the x-coordinate, and the second coordinate is the y-coordinate. For example, the point $A(4, 2)$ has x-coordinate 4 and y-coordinate 2.

Suppose that we are given a rule connecting the x and y-coordinates of a point. We can make up a table of values and plot the points on the Cartesian plane.

In the example below, what do you notice about the points? You should check, using your ruler and eye, that they lie on a line.

We can follow the same kind of procedure for any given rule – a table can be formed and the corresponding points plotted, although they do not always lie on a line.

Example 1

For each given rule, complete the table, list the coordinates of the points, and plot the points on the Cartesian plane. Draw a line through them.

a $y = -x$

x	–3	–2	–1	0	1	2	3
y							

b $y = x + 1$

x	–3	–2	–1	0	1	2	3
y							

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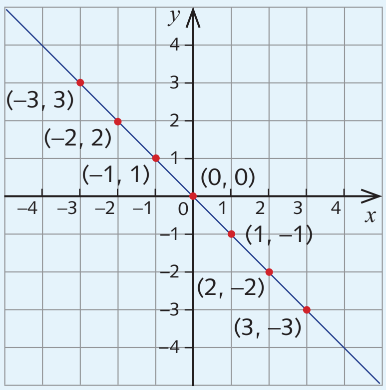
Solution

a $y = -x$

x	–3	–2	–1	0	1	2	3
y	3	2	1	0	–1	–2	–3

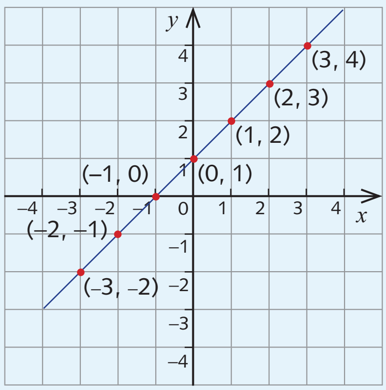
The points are $(-3,3)$, $(-2,2)$, $(-1,1)$, $(0,0)$, $(1,-1)$, $(2,-2)$ and $(3,-3)$.



b $y = x + 1$

x	–3	–2	–1	0	1	2	3
y	–2	–1	0	1	2	3	4

The points are $(-3,-2)$, $(-2,-1)$, $(-1,0)$, $(0,1)$, $(1,2)$, $(2,3)$ and $(3,4)$.



Exercise 18A

Example 1

1 For each given rule, complete the table, list the coordinates, and plot the corresponding set of points on a number plane. Draw a line through each set of points.

a $y = 3x$

x	–3	–2	–1	0	1	2	3
y							

b $y = -2x$

x	–3	–2	–1	0	1	2	3
y							

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c $y = x - 2$

x	−3	−2	−1	0	1	2	3
y							

d $y = x + 2$

x	−3	−2	−1	0	1	2	3
y							

e $y = 2x + 1$

x	−3	−2	−1	0	1	2	3
y							

f $y = 1 - x$

x	−3	−2	−1	0	1	2	3
y							

2 For each given rule, complete the table, list the coordinates, and plot the corresponding set of points on a number plane. Draw a line through each set of points.

a $y = x + \frac{1}{2}$

x	−3	−2	−1	0	1	2	3
y							

b $y = x - \frac{1}{2}$

x	−3	−2	−1	0	1	2	3
y							

c $y = 2x + \frac{1}{2}$

x	−3	−2	−1	0	1	2	3
y							

d $y = -x + \frac{1}{2}$

x	−3	−2	−1	0	1	2	3
y							

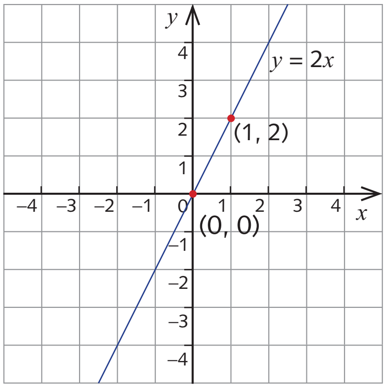
18B Drawing straight lines by plotting two points

Straight-line graphs that pass through the origin

We have seen that when we determine a table of values from a rule such as $y = 2x$ (or $y = 3x$ or $y = -x$), the points lie on a line. That is, we can draw a line through the points. The line is said to be the **graph** of $y = 2x$ (or $y = 3x$ or $y = -x$).

Since two points determine a line uniquely, we require two coordinate pairs that satisfy the equation. For $y = 2x$, the values $x = 0$ and $x = 1$ are suitable.

x	0	1
y	0	2



The coordinates $(0, 0)$ and $(1, 2)$ are plotted and the line is drawn through the points.

It is wise to use a third point as a check.
For example, $x = 2, y = 4$.

The point $(2, 4)$ lies on the line.

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Example 2

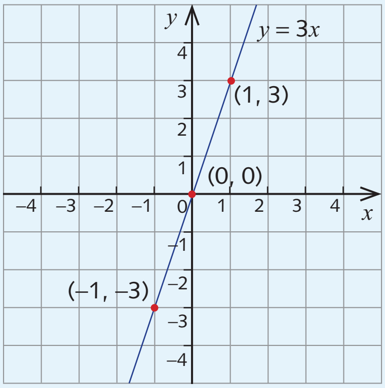
Draw the graph of $y = 3x$.

Solution

First make a table of values. Only two points are required but a third point is calculated as a check.

x	–1	0	1
y	–3	0	3

Now draw the line through the points.



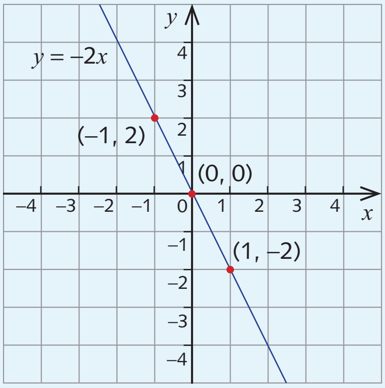
Example 3

Draw the graph of $y = -2x$.

Solution

First make a table of values. Only two points are required but a third point is calculated as a check.

x	–1	0	1
y	2	0	–2



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Straight-line graphs that do not pass through the origin

Not all straight-line graphs pass through the origin. In Section 18A, you plotted points for graphs such as $y = x + 1$ and $y = x + \frac{1}{2}$. Look back at those diagrams and check that the lines through the plotted points do not pass through the origin.

Scales

In the previous examples, we have chosen to use 0.5 cm to represent 1 unit. (We have done this to save space, but you may find it more convenient to use 1 cm to represent 1 unit.) We say that we have used the **scale** '0.5 cm represents 1 unit'. Up until now we have used the same scale on both the x- and y-axes, but sometimes it is helpful to use different scales on the two axes. Here is an example to show this.

Example 4

Draw the graph of $y = 5x + 10$.

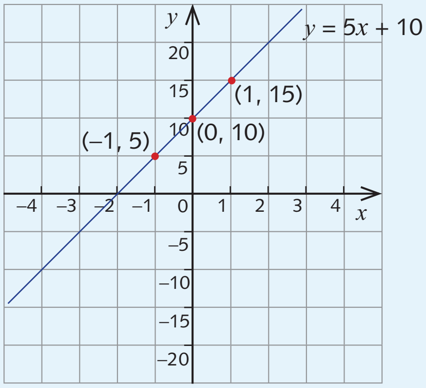
Solution

First make a table of values.

x	–1	0	1
y	5	10	15

We choose the scale '0.5 cm represents 1 unit' on the x-axis, and the scale '0.5 cm represents 5 units' on the y-axis. (*Reason*: The y values change by 5 when the x values change by 1.)

Here is the straight-line graph.



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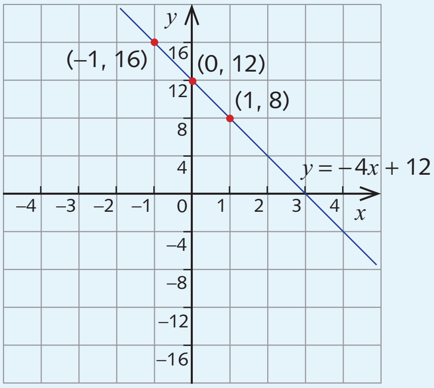
Example 5

Draw the graph of $y = -4x + 12$. Choose suitable scales for the two axes.

Solution

x	–1	0	1
y	16	12	8

We choose the scale '0.5 cm represents 1 unit' on the x-axis, and the scale '0.5 cm represents 4 units' on the y-axis.

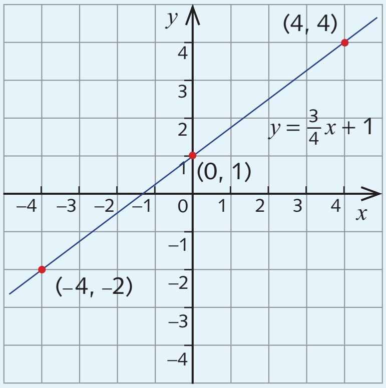


Example 6

Draw the graph of $y = \frac{3}{4}x + 1$. Use the standard scale of '0.5 cm represents 1 unit' on both the x- and y-axes.

Solution

x	–4	0	4
y	–2	1	4



Exercise 18B

Example 2, 3

1 On a single set of axes, draw the graphs of:

a $y = x$ **b** $y = 2x$ **c** $y = 4x$ **d** $y = -3x$ **e** $y = -4x$

2 On a single set of axes, draw the graphs of:

a $y = -x$ **b** $y = 3x$ **c** $y = 5x$ **d** $y = \frac{1}{2}x$ **e** $y = -\frac{1}{2}x$

(*Hint*: For parts **d** and **e**, use the x-values –2, 0 and 2.)

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Example 4

3 **a** Use the same set of axes with the same scales to draw the graphs of:

i $y = 2x$ **ii** $y = 2x - 1$ **iii** $y = 2x + 1$

iv $y = 2x + 4$ **v** $y = 2x + 3$

b What is the relationship between these straight-line graphs?

Example 5, 6

4 **a** Use the same set of axes to draw the graphs of:

i $y = 2x + 2$ **ii** $y = -x + 2$ **iii** $y = \frac{1}{2}x + 2$

iv $y = -\frac{1}{4}x + 2$ **v** $y = 3x + 2$ **vi** $y = 2 - 3x$

vii $y = 2 - \frac{1}{3}x$

b What is the relationship between these straight-line graphs?

18C Points and lines

We will now use the ideas developed in the algebra chapters of this book to answer some questions about straight-line graphs. We will use both substitution and solving equations to discover some further facts about straight-line graphs.

Finding the y-coordinate given the x-coordinate

We can find the y-coordinate of any point on the graph, corresponding to a given x value. To do this, we substitute the x-coordinate into the equation of the line, as shown in the following example. This will give us the coordinates of a point on the line.

Example 7

Find the coordinates of the points on the graph for the given x value for the straight-line graph of $y = 4x - 5$.

a $x = -1$ **b** $x = 0$ **c** $x = 20$

Solution

a $y = 4 \times (-1) - 5$
$= -4 - 5$
$= -9$

The y-coordinate is -9.

The coordinates are $(-1, -9)$.

b $y = 4 \times 0 - 5$
$= -5$

The y-coordinate is -5.

The coordinates are $(0, -5)$.

c $y = 4 \times 20 - 5$
$= 75$

The y-coordinate is 75.

The coordinates are $(20, 75)$.

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Finding the x-coordinate given the y-coordinate

This involves solving an equation, as shown in the following example.

Example 8

For the straight-line graph with equation $y = 4x - 5$, find the coordinates of the points on the line with y-coordinate:

a $y = 11$ **b** $y = 0$

Solution

a For $y = 11$, $4x - 5 = 11$

$\boxed{+5}$ $4x = 16$ (Add 5 to both sides of the equation.)

$\boxed{\div 4}$ $x = 4$ (Divide both sides of the equation by 4.)

The x-coordinate is 4.

The coordinates are $(4, 11)$.

b For $y = 0$, $4x - 5 = 0$

$\boxed{+5}$ $4x = 5$ (Add 5 to both sides of the equation.)

$\boxed{\div 4}$ $x = \frac{5}{4}$ (Divide both sides of the equation by 4.)

The x-coordinate is $\frac{5}{4}$.

The coordinates are $\left(\frac{5}{4}, 0\right)$.

Checking that a point lies on the graph

We check that a point lies on a line by seeing if the coordinates satisfy the equation.

Example 9

Check whether or not each of the following points lie on the line with equation $y = 2x + 3$.

a $(3, 9)$ **b** $(-2, 7)$ **c** $\left(-\frac{2}{3}, 0\right)$

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Solution

a When $x = 3,\ y = 2 \times 3 + 3$
$= 9$

So the point (3,9) lies on the line.

b When $x = -2,\ y = 2 \times -2 + 3$
$= -1$

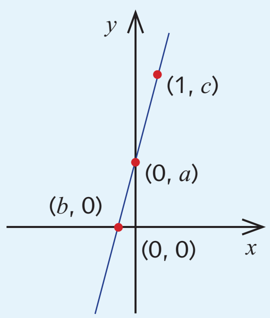
This does not agree with the given y value, so the point (–2, 7) does not lie on the line $y = 2x + 3$.

c For $x = -\frac{2}{3},\ y = 2 \times \frac{-2}{3} + 3$
$= 1\frac{2}{3}$

This does not agree with the given y value, so the point $\left(-\frac{2}{3}, 0\right)$ does not lie on the line $y = 2x + 3$.

Example 10

The graph of $y = 4x + 3$ is shown opposite.
Find the values of a, b and c.



Solution

When the x-coordinate is 1,

$y = 4 \times 1 + 3$
$= 7$

so $c = 7$

When the x-coordinate is 0,

$y = 4 \times 0 + 3$
$= 3$

so $a = 3$

When the y-coordinate is 0,

$4x + 3 = 0$

$4x = -3$ (Subtract 3 from both sides.)

$x = -\frac{3}{4}$ (Divide both sides of the equation by 4.)

so $b = -\frac{3}{4}$

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Exercise 18C

Example 7

1 For the straight-line graph of $y = 2x - 3$, find the coordinates of the point on the line with x-coordinate:

a $x = -1$ **b** $x = 0$ **c** $x = 20$

2 For the straight-line graph of $y = -4x + 5$, find the coordinates of the point on the line with x-coordinate:

a $x = 1$ **b** $x = 0$ **c** $x = 2$

3 For the straight-line graph of $y = x - 5$, find the coordinates of the point on the line with x-coordinate:

a $x = -1$ **b** $x = 0$ **c** $x = 20$

4 For the straight-line graph of $y = -2x - 3$, find the coordinates of the point on the line with x-coordinate:

a $x = -1$ **b** $x = 0$ **c** $x = 15$

Example 8

5 For the straight-line graph with equation $y = 3x - 2$, find the x-coordinate corresponding to each of these y-coordinates.

a $y = 4$ **b** $y = 0$ **c** $y = 19$ **d** $y = -34$ **e** $y = -50$

6 For the straight-line graph with equation $y = -3x + 2$, find the x-coordinate corresponding to each of these y-coordinates.

a $y = 6$ **b** $y = 0$ **c** $y = 26$ **d** $y = -34$ **e** $y = -50$

Example 9

7 Check whether or not each point lies on the line with equation $y = 2x - 1$.

a $(3, 9)$ **b** $(-2, -5)$ **c** $(-1, -3)$ **d** $(4, 10)$

8 Check whether or not each point lies on the line with equation $y = -2x + 3$.

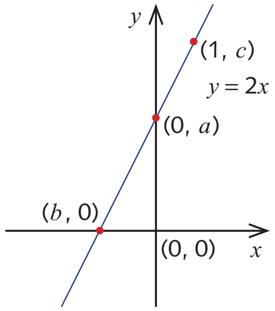
a $(3, 9)$ **b** $(-2, 7)$ **c** $(-1, 5)$ **d** $(4, -5)$

9 Check whether or not each point lies on the line with equation $y = -6x$.

a $(0, 0)$ **b** $(1, 6)$ **c** $(-1, 6)$ **d** $(4, 11)$

Example 10

10 The graph of $y = 2x + 3$ is shown below. Find the values of a, b and c.



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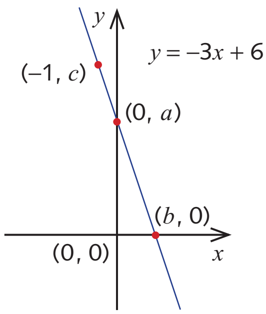
11 The graph of $y = -3x + 6$ is shown below. Find the values of a, b and c.



12 The x-coordinate of a particular point on the line $y = 5 - 3x$ is 10. Write down the y-coordinate.

13 The y-coordinate of a particular point on the line $y = 10 + 2x$ is 72. Write down the x-coordinate.

14 If the points $(-1, a)$, $(b, 15)$, and $(c, -20)$ lie on the line with equation $y = -5x$, find the values of a, b and c.

15 If the points $(3, a)$, $(-12, b)$ and $(c, -12)$ lie on the line with equation $y = -\frac{2}{3}x$, find the values of a, b and c.

18D The y-intercept and the gradient of a line

The y-intercept

The **y-intercept** of a line is the y-value of the point where the line cuts the y-axis. If a point is on the y-axis, then its x-coordinate is 0. This means we can find the y-intercept by substituting $x = 0$ into the equation.

Example 11

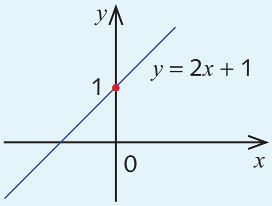
Find the y-intercept of the line with equation $y = 2x + 1$.

Solution

To find the y-intercept, we put $x = 0$.

$$\begin{aligned} \text{Then } y &= 2 \times 0 + 1 \\ &= 1 \end{aligned}$$

Hence the y-intercept is 1.



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Example 12

Find the y-intercept of the line with equation $y = 3x - 5$.

Solution

Substitute $x = 0$ into the equation.

Then $y = 3 \times 0 - 5$
$= -5$

The y-intercept is -5.

Gradient

Here are two pictures of a car going up a hill.

We can see that the first hill is steeper than the second one. How can we measure the steepness of a hill?

There is a simple way to do this. We measure how far up in metres the car moves in the vertical direction (rise) for each 1 m that it moves in the horizontal direction (run).

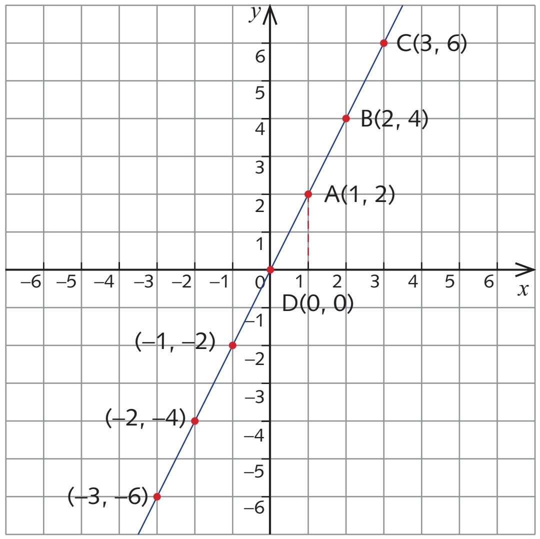
We then say the gradient of the hill $= \dfrac{\text{rise}}{\text{run}}$.

Using this idea we can easily see that the gradient of the left-hand hill is greater than the right-hand hill.

We use a similar idea to measure the slope of a straight-line graph.

Lines through the origin with positive gradient

The graph shown is $y = 2x$.



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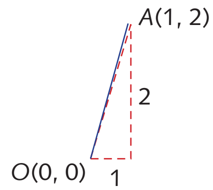
Look at the interval joining $O(0,0)$ to $A(1,2)$:



Using the same language as we did for the hill:

run $= 1$ and rise $= 2$

The gradient of the interval is $\frac{2}{1} = 2$.

Now look at the interval joining $A(1,2)$ to $B(2,4)$:

run $= 1$ and rise $= 2$

So the gradient of the interval is 2.

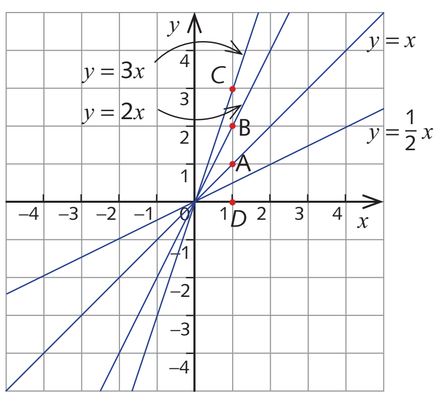
Finally, for the interval joining $A(2,4)$ to $C(3,6)$:

run $= 1$ and rise $= 2$

So the gradient of the interval is 2.

In each case the gradient is the same. We say that the **gradient of the line** is the change in y as we move 1 unit to the right. It doesn't matter which point we start from.

The diagram below shows the lines $y = x$, $y = 2x$, $y = 3x$ and $y = \frac{1}{2}x$. The steepest of these lines is $y = 3x$, while the least steep is $y = \frac{1}{2}x$.



What makes one line steeper than another? Suppose we go from the origin to the point A on the line $y = x$. To reach A from the origin, we need to move 1 unit to the right in the horizontal direction, and then 1 unit upwards in the vertical direction. So gradient $= 1$.

To reach B from the origin, we need to move 1 unit to the right in the horizontal direction, and then 2 units upwards in the vertical direction. So gradient $= 2$.

To reach C from the origin, we need to move 1 unit to the right in the horizontal direction, and then 3 units upwards in the vertical direction. So gradient $= 3$.

From this we can see:

The gradient of $y = \frac{1}{2}x$ is $\frac{1}{2}$.

The gradient of $y = x$ is 1.

The gradient of $y = 2x$ is 2.

The gradient of $y = 3x$ is 3.

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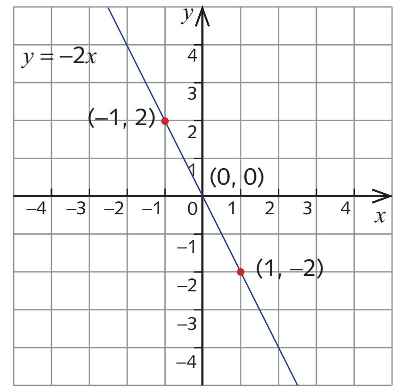
Positive gradient

Lines sloping up from left to right have positive gradient.

Lines through the origin with negative gradient

The graph of $y = -2x$ is shown.



Start at the point $(-1, 2)$. We move 1 unit to the right and 2 units down to arrive at the origin, so:

$$\text{rise} = -2$$

The run is the change in x-coordinate as we move from left to right on the graph, so:

$$\text{run} = 1$$

Hence, the gradient $= \dfrac{-2}{1} = -2$

The gradient of $y = -\dfrac{1}{2}x$ is $-\dfrac{1}{2}$.

Similarly:

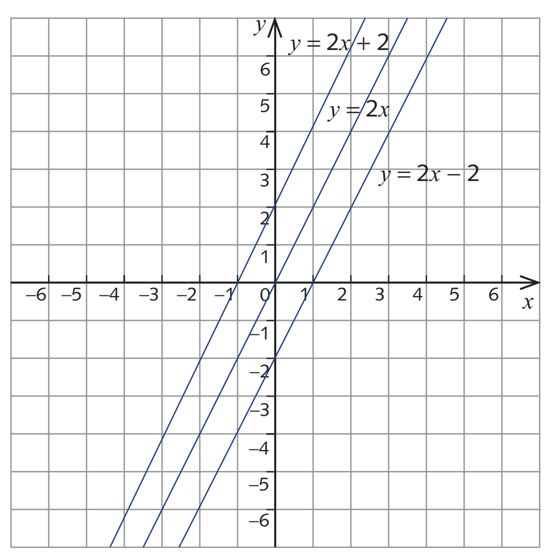
- The gradient of $y = -x$ is -1.
- The gradient of $y = -2x$ is -2.
- The gradient of $y = -3x$ is -3.

Negative gradient

Lines sloping down from left to right have negative gradient.

Straight lines not passing through the origin

The graphs of $y = 2x$, $y = 2x - 2$ and $y = 2x + 2$ are shown in the graph to the right.



Notice that they are parallel and therefore have the same gradient.

You can check this by choosing intervals on the lines $y = 2x - 2$ and $y = 2x + 2$ and calculating the gradient.

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In summary, to find the gradient of a line we follow these steps.

- Find a point on the line.
- Move along the line by moving 1 unit to the right in the horizontal direction.
- Find the change in the y-coordinate that has occurred. This gives the gradient of the line.

Example 13

Find the gradient of the line $y = 3x - 1$.

Solution

Substitute an x value to find one point on the line.

If we put $x = 1$, this gives $y = -2$. So $(1, 2)$ is a point on the line.

Substitute another x value to find a second point on the line.

If we put $x = 2$, this gives $y = 5$. So $(2, 5)$ is another point on the line.

The change in the y-coordinates is $5 - 2 = 3$ when we move 1 unit in the positive x-direction. Hence, the gradient of the line is 3.

Example 14

Find the gradient of the line with equation $y = -3x + 5$.

Solution

First, put $x = 0$. This gives $y = 5$, so $(0, 5)$ is a point on the line.

Now move 1 unit to the right in the horizontal direction.

We get $x = 1$ and $y = -3 \times 1 + 5 = 2$. So $(1, 2)$ is another point on the line.

The change in y is $2 - 5 = -3$, so the gradient is -3.

Exercise 18D

Example 11, 12

1 Find the y-intercept of each line by substituting $x = 0$.

a $y = 2x + 3$ **b** $y = 4x - 7$ **c** $y = -2x + 5$

d $y = -4x - 8$ **e** $y = 5x$

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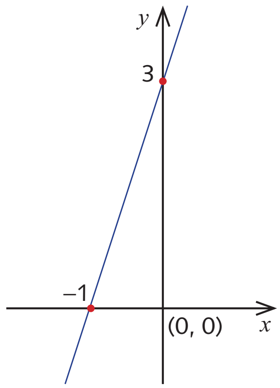
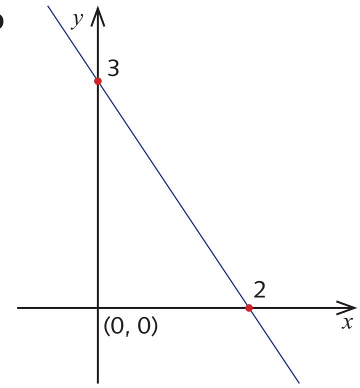
2 Read off the y-intercept from each of these graphs.

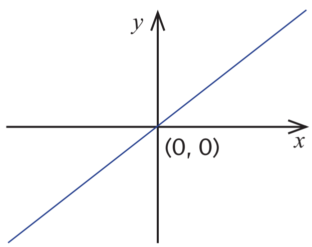
a



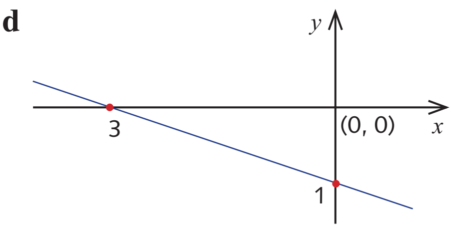
b



c



d



Example 13

3 For each of the four equations shown below:

i make up a table of values for $x = -1, 0, 1$

ii draw the graph of the line

iii find the gradient of the line

a $y = 2x$ **b** $y = 2x + 1$ **c** $y = 3x - 2$ **d** $y = x + 3$

4 For each of the four equations shown below:

i find the points on the line that have x-values of 1 and 2

ii hence find the gradient of the line

a $y = 3x$ **b** $y = 4x - 1$ **c** $y = 2x - 5$ **d** $y = x + 2$

5 **a** Find the gradient of the line $y = 3x - 1$ and the gradient of the line $y = 2x + 4$.

b Which of the lines in part **a** is steeper?

Example 14

6 For each of the four equations shown below:

i make up a table of values for $x = 0, 1, 2$

ii draw the graph of the line

iii find the gradient of the line

a $y = -2x$ **b** $y = -x + 1$ **c** $y = -3x + 5$ **d** $y = -2x + 3$

7 For each line given below, find the points on the line that have x-values of 1 and 2, and hence find the gradient of the line.

a $y = -3x$ **b** $y = -4x + 2$ **c** $y = -2x + 5$ **d** $y = -x + 2$

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18E More on gradients

In Examples 13 and 14 in the previous section, we saw that the gradient of the line $y = 3x - 1$ is 3 and the gradient of the line $y = -3x + 5$ is -3.

From these and other examples we have done, you may have guessed that the gradient of the line written in the form $y = mx + c$ is the number m in front of the x, which is called the **coefficient** of x. If so, you are correct! Well done.

Equations of the form $y = mx + c$

- The graph of each equation of the form $y = mx + c$ is a straight line.
- The number m gives the **gradient** of the line and c gives the **y-intercept**.

Example 15

Find the gradient and y-intercept of the lines:

a $y = 7x - 4$

b $y = -x + 3$

Solution

a The gradient of the line $y = 7x - 4$ is 7. The y-intercept is -4.

b The gradient of the line $y = -x + 3$ is -1. The y-intercept is 3.

We can now quickly compare the steepness of any two lines that have positive gradients.

Example 16

Which of the lines $y = 4x - 6$ and $y = 6x - 4$ is steeper?

Solution

The gradient of the line $y = 4x - 6$ is 4.

The gradient of the line $y = 6x - 4$ is 6.

Hence, the line $y = 6x - 4$ is steeper.

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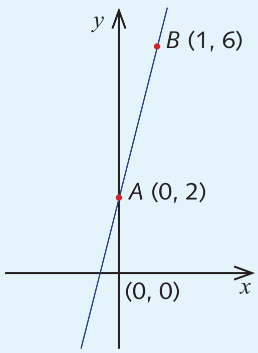
Example 17

The line shown opposite has equation $y = mx + 2$.
Find the value of m.



Solution

The points $A(0,2)$ and $B(1,6)$ lie on the line. The change in the y-coordinate, as x changes from 0 to 1, is 4, so the gradient of the line is 4. Hence, $m = 4$.

(As a check, if $y = 6$ and $x = 1$, then $6 = m + 2$)

Exercise 18E

Example 15

1 Write down the gradient of each line.

a $y = 7x$

b $y = 9x + 2$

c $y = -12x + 3$

d $y = -x + 8$

2 Write down the gradient of each line.

a $y = 5 - 7x$

b $y = -3x + 12$

c $y = 6 - 11x$

d $y = 9 - x$

Example 16

3 For each pair of lines, state which is steeper.

a $y = 5x$, $y = 4x - 2$

b $y = 7x + 2$, $y = 9x - 5$

c $y = 3 + 2x$, $y = 3x + 2$

4 **a** For each line given below, state whether it slopes upwards or downwards.

i $y = -7x$

ii $y = 4x - 9$

iii $y = -9x + 8$

iv $y = -x + 12$

b For each equation in part **a**, find the value of m.

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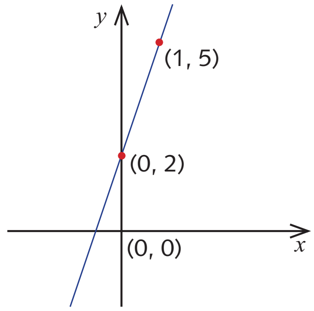
Example 17

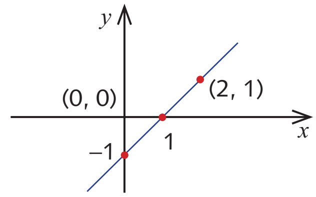
5 In each problem below, find the value of m.

a



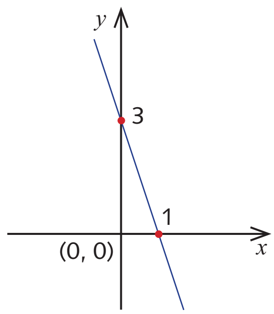
$y = mx + 2$

b



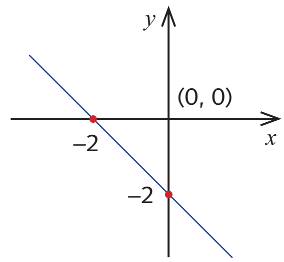
$y = mx - 1$

c



$y = mx + 3$

d



$y = mx - 2$

6 Find the value of m for the line that connects the points with coordinates $(3,-2)$ and $(0,7)$.

7 Find the value of m for the line that connects the points with coordinates $(-2,0)$ and $(5,14)$.

18F Applications to constant rate problems

The idea of a rate was introduced in Section 10B. For example, 60 km/h and 50 litres/minute are examples of constant rates. We can use straight-line graphs to help us solve problems involving constant rates. In Section 10C, we looked at the familiar example of an object moving at constant speed.

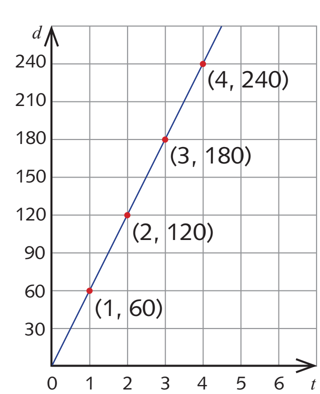
For example, suppose that a car is travelling at 60 km/h. We can plot a table of values for the distance d (in km) travelled by the car after t hours.

t (hours)	0	1	2	3	4
d (kilometres)	0	60	120	180	240

We can see from the table that the equation relating d and t is $d = 60t$.

We take the horizontal axis to be the t-axis and the vertical axis to be the d-axis. The graph opposite is drawn for t-values from 0 to 4 and d takes values from 0 to 240.

The graph has a gradient of 60 and passes through the origin. The gradient gives the speed of the car, in km/h.



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Example 18

A car is travelling at a constant speed of 100 km/h.

a What is the formula for the distance d (in km) travelled by the car in t hours?

b What is the gradient of the straight-line graph of d against t?

Solution

a In 1 hour, the car travels 100 km.

In 2 hours, the car travels 200 km.

The formula is $d = 100t$.

b The gradient of the straight line graph is 100.

Constant rate

Questions involving a constant rate give rise to straight-line graphs. The gradient of the line is the constant rate.

Example 19

A cylindrical tank can hold a maximum of 40 litres of water. It has 10 litres of water in it to start with. Water is flowing slowly in at a rate of 5 litres per minute.

a Prepare a table of values showing how much water is in the container at 1-minute intervals from 0 up to 6 minutes.

b Plot the graph of the volume V (in litres) of water in the tank against time t (in minutes) since the start.

c Give the formula for V in terms of t.

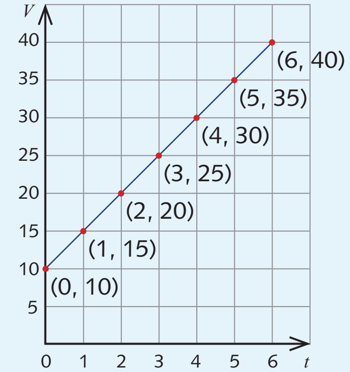
Solution

a

t (minutes)	0	1	2	3	4	5	6
V (litres)	10	15	20	25	30	35	40

b (The scale on the t-axis is '0.5 cm represents 1 minute' and the scale on the V-axis is '0.5 cm represents 5 litres'.)

c From the graph, the gradient is 5 and the V-axis intercept is 10 litres. The formula is $V = 5t + 10$, where t takes values from 0 to 6 minutes inclusive.



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Exercise 18F

Example 18

1 Maria decides to ride her bicycle to the next town, which is 60 km away. She rides at 15 km/h.

a Let t denote the number of hours that have elapsed since she set out. Prepare a table of values showing how far (d km) she is from her starting point at 1-hour intervals up to $t = 4$ hours.

b Plot the points (t, d) from your table of values and draw the graph.

Note: For travel graphs it is usual to plot distance travelled against time.

c Give the formula for d in terms of t.

d Use the formula to find how far Maria has ridden after $3\frac{1}{4}$ hours.

Example 19

2 A tank can hold a maximum of 40 litres of water. Initially it has 15 litres of water in it. Water is flowing slowly in at a rate of 5 litres per minute. Let V be the volume (in litres) of water in the tank t minutes from the start.

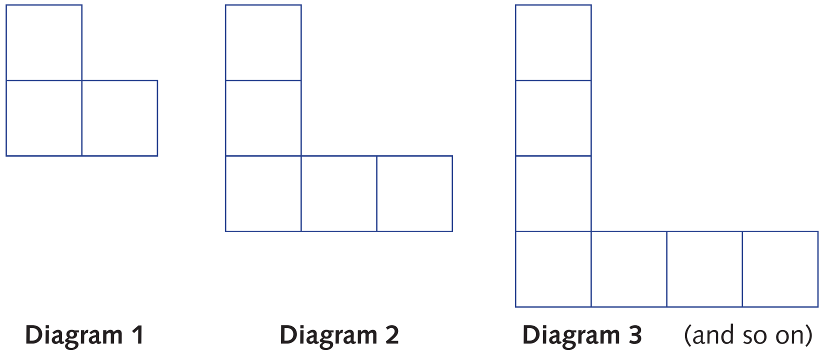
a Prepare a table showing the values of V at 1-minute intervals up to $t = 5$.

b Plot the graph of V against t from your table of values.

c Give the formula for V in terms of t.

d Use the formula to find the volume of water in the tank after $2\frac{1}{2}$ minutes.

3 We have a pile of matchsticks. We form the letter L with squares made up of matchsticks, as shown in the diagrams below.



a Copy and complete the table below, where n is the number of the diagram.

Diagram number (n)	Number of matches (M)
1	10
2	
3	
4	
5	
6	

b Find a formula for M in terms of n.

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c Plot the points (n, M) for values of n from 1 to 10, using your table of values. (Draw a line through the points you plot, although in this question doing so does not make *practical* sense – why not?)

d Find the value of M for $n = 12$.

e What is the value of n for the diagram that uses 124 matches?

18G Not all graphs are straight lines

Up to this point, we have considered only straight-line graphs. However, there are many kinds of equations that do not produce straight lines when plotted, as the following example shows. You will often encounter these kinds of equations as you progress in your study of mathematics.

Example 20

For the rule $y = x^2$:

a calculate the table of values for the x values $-3, -2, -1, 0, 1, 2, 3$

b plot the points

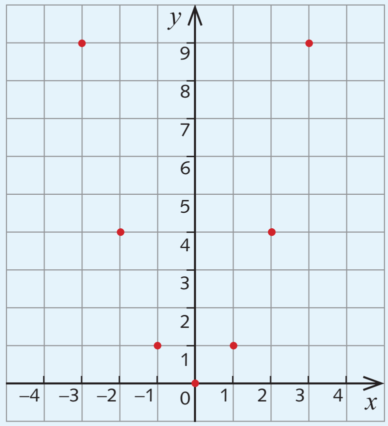
c join the points with a smooth curve

Solution

a

x	–3	–2	–1	0	1	2	3
y	9	4	1	0	1	4	9

b



You can see immediately that the points do not lie on a straight line.

c Draw a smooth freehand curve through the points. Compare your curve with your neighbour's.

The curve, $y = x^2$, is called a **parabola**.

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Exercise 18G

Example 20

1 For each given rule, complete the table, decide on the set of values for the y-axis, and plot the points. Draw a smooth freehand curve through the points.

a $y = x^2 - 4$

x	−3	−2	−1	0	1	2	3
y							

b $y = x^3$

x	−3	−2	−1	0	1	2	3
y							

c $y = x^3 + 1$

x	−3	−2	−1	0	1	2	3
y							

d $y = (x - 3)^2$

x	0	1	2	3	4	5	6
y							

e $y = (x + 3)^2$

x	−6	−5	−4	−3	−2	−1	0
y							

f $y = x^3 - 4x$

x	−3	−2	−1	0	1	2	3
y							

g $y = x^3 - 9x$

x	−4	−3	−2	−1	0	1	2	3	4
y									

h $y = x^2 - 2x$

x	−2	−1	0	1	2	3	4
y							

i $y = 9 - x^2$

x	−6	−5	−4	−3	−2	−1	0	1	2
y									

Review exercise

1 On a single set of axes, draw the graphs of:

a $y = x$

b $y = -5x$

c $y = -\frac{1}{4}x$

d $y = \frac{2}{3}x$

e $y = -\frac{5}{4}x$

2 On a single set of axes, draw the graphs of:

a $y = 3x - 6$

b $y = -x + 4$

c $y = \frac{1}{2}x + 4$

d $y = -\frac{1}{4}x - 2$

e $y = -3x + 2$

3 Find the y-intercept of each of these lines by substituting $x = 0$ into the equation.

a $y = x - 4$

b $y = 3x + 10$

c $y = 4 - 3x$

d $y = 5x - 10$

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4 For each line given below, find the points on the line with x-values of 1 and 2, and hence calculate the gradient of the line.

a $y = 3x - 2$ **b** $y = 2x - 8$ **c** $y = -5x + 6$

5 **a** For each line given below, state whether it slopes upwards or downwards.

i $y = 2x$ **ii** $y = -3x$ **iii** $y = 4x - 6$ **iv** $y = 2 - x$

b Each equation above has the form $y = mx + b$, where m and b are numbers. For each equation, find the value of b.

6 Check whether or not each of these points lies on the line with equation $y = 2x + 3$.

a $(3,9)$ **b** $(-2,-1)$ **c** $(-1,-5)$ **d** $(-4,-5)$

7 The x-coordinate of a point on the line $y = 10 - 3x$ is 3. Write down the y-coordinate.

8 The y-coordinate of a point on the line $y = 8 + 2x$ is 72. Write down the x-coordinate.

9 If the points $(-1,a)$, $(b,5)$ and $(c,-20)$ lie on the line with equation $y = 5x + 5$, find the values of a, b and c.

10 Write down the gradients and y-axis intercepts for the lines with the equations given below.

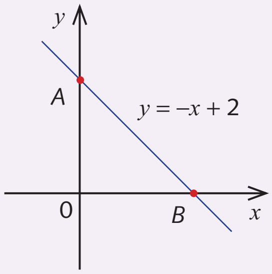
a $y = 5x - 8$

b $y = 3x + 7$

c $y = -3x + 11$

11 If the points $(-2,a)$, $(b,16)$ and $(c,28)$ lie on the line with equation $y = 3x + 7$, find the values of a, b and c.

12 The equation of the line passing through points A and B in the diagram below is $y = -x + 2$.



a Find the coordinates of points A and B.

b Find the length of the line interval AB.

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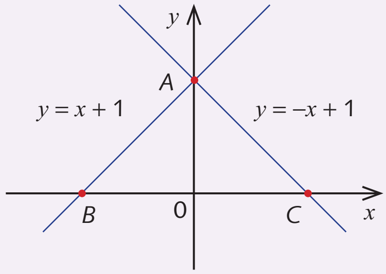
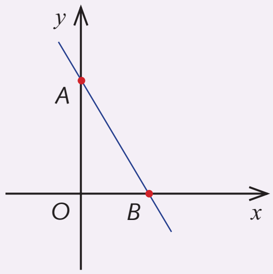
13 The graphs of $y = x + 1$ and $y = -x + 1$ are shown below.



a Find the coordinates of points A, B and C.

b Find the area of triangle ABC.

14 The line through points A and B in the diagram below has equation $y = -5x + c$, where c is a positive number. The area of triangle OAB is 10 cm^2.



a Find the value of c.

b Find the coordinates of A and B.

15 **a** The line with equation $y = mx + 3$ is parallel to the line with equation $y = -5x$. State the value of m.

b Find the y-coordinate of the point on the line with equation $y = -5x + 4$ for which the x-coordinate is 7.

c Find the x-coordinate of the point on the line with equation $y = -5x + 10$ for which the y-coordinate is 0.

d Find the x-coordinate of the point on the line with equation $y = -5x - 2$ for which the y-coordinate is 13.

Challenge exercise

1 A man is walking home at 6 km/h. He starts at a point 18 km from his home. Draw a graph representing his trip home. State the gradient and vertical axis intercept, and give a formula that describes the trip.

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2 A tank can hold up to 50 litres of water. It is full to start with. Water is flowing slowly out at a rate of 5 litres per minute.

a Prepare a table of values showing how much water is in the tank at 1-minute intervals.

b Plot the graph of the volume V (in litres) of water in the tank against time t (in minutes) since the start.

c Give the formula for V in terms of t.

3 The towns Cunadilla and Frevnelle are 110 km apart. David lives in Cunadilla, and Lilly lives in Frevnelle. They decide to meet on the road between the two towns. Lilly cycles at 15 km/h and David cycles at 10 km/h. They both leave their towns at 11 a.m.

Let D and L be the respective distances of David and Lilly from Cunadilla at time t hours after 11 a.m.

a Write down a formula for D in terms of t.

b Write down a formula for L in terms of t.

c Draw up a table of values for each formula. For David, consider values of t between 0 and 11. For Lilly, consider values of t between 0 and 7.

d On the same set of axes, draw the graphs of D against t, and L against t.

e At what time do the two friends meet?

f How far from Cunadilla do they meet?

4 In a 100-m handicap race, Priscilla is given a 5-m start over Wendy. That is, when the starter's gun is fired, Priscilla starts 5 m ahead of the starting line for the 100 m. Wendy starts from the starting line. Priscilla runs at 7.5 m/s and Wendy runs at 8 m/s.

a Write down a formula for P, the distance in metres of Priscilla from the starting line t seconds after the starter's gun is fired.

b Write down a formula for W, the distance in metres of Wendy from the starting line t seconds after the starter's gun is fired.

c Draw up a table of values for each formula.

d On the same set of axes, draw the graphs of P against t, and W against t.

e How long does it take for Wendy to overtake Priscilla?

f How far has each of the girls run when Priscilla is overtaken?

g What start should Priscilla be given if we want Wendy to catch up to her at the 100-m mark?

5 **a** Prepare a table of values for the graph of $y = 3x$.

b Plot the points from your table of values.

c Draw the graph of $y = 3x$.

d On the same set of axes, draw the graph of $y = 3x + 3$.

e **i** Describe a translation that moves the graph of $y = 3x$ to the graph of $y = 3x + 3$.

ii Find a second translation that does this.

iii How many such translations are there?

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f What is the equation of the graph obtained by reflecting the line $y = 3x$ in:

i the x-axis? **ii** the y-axis?

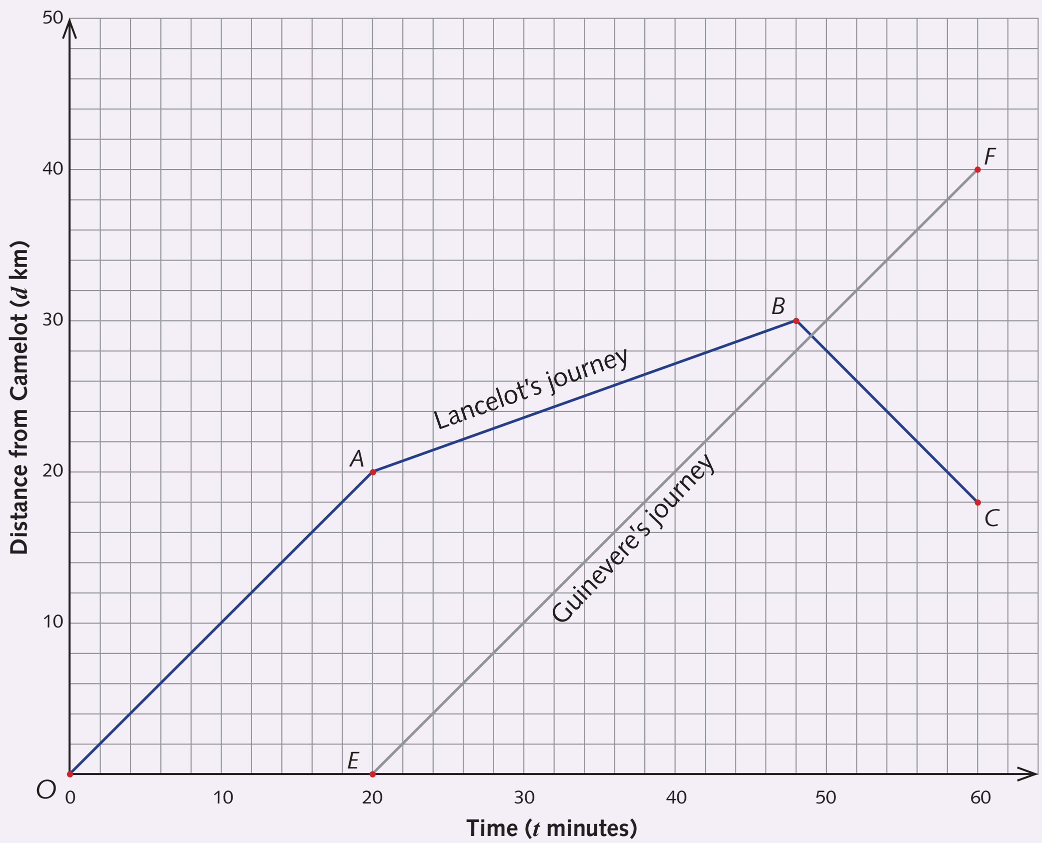
g Draw the graphs of the equations in part **f**.

h What is the equation of the graph obtained by rotating the line $y = 3x$:

i 180° about the origin in an anticlockwise direction?

ii 90° about the origin in an anticlockwise direction?

6 The line graphs below show the journeys of Lancelot and Guinevere along a road leading out of Camelot. The vertical axis shows the distance from Camelot and the horizontal axis the time taken after Lancelot leaves Camelot.



a State the gradient of each line interval.

i OA **ii** AB **iii** BC **iv** EF

b Give the speed of Lancelot for the different stages of his journey.

c Find the equation of each stage of Lancelot's journey.

d Find the equation of Guinevere's journey.

e If Lancelot continues back to Camelot at the same speed, when will he be back in Camelot?

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CHAPTER

19

Statistics and Probability

Statistics

People deal with large amounts of information every day. When we read newspapers, watch television or open our mail, we may be looking at information that has been organised so that we can understand it easily. For example, we can tell how much water we use at home by looking at a water bill.

When we collect, organise, represent and analyse information, we are using **statistics**. The information is collected and we call this information **data**. A person who does this kind of work is called a statistician.

Analysing statistical data can help us understand more about a group of people, the habits of animals or changes in the weather over time. This information helps us make decisions and predict outcomes.

One very important aspect of statistics is collecting representative data. This can be done by taking a census or a survey or recording data through observations. These ideas are discussed and developed using many of the techniques that have been introduced in earlier years.

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19A Comparing means and medians

Mean

You have already heard of the mean and know that it is commonly called the average. We recall that to calculate the mean, we find the sum of the values and divide this by the number of values.

Mean

$$\text{mean} = \frac{\text{sum of values}}{\text{number of values}}$$

Example 1

The heights of eight people measured in centimetres are shown below:

178 187 175 183 174 180 177.5 182.5

Find the average.

Solution

$$\begin{aligned}\text{Average} &= \frac{178 + 187 + 175 + 183 + 174 + 180 + 177.5 + 182.5}{8} \\ &= 179.625 \text{ cm}\end{aligned}$$

Median

The **median** is the 'middle value' when all values are arranged in order of size. Here are some numbers in order of size:

2, 2, 3, 3, 3, 4, 5, 11, 13, 18, 18, 19, 21

This data set has an odd number of values. The middle value is 5, since it has the same number of values on either side of it. Hence the median of this dataset is 5.

Here are some more numbers:

1, 3, 4, 4, 5, 6, 8, 11, 13, 13, 19, 21

This data set has an even number of values. The middle values are 6 and 8. We take the average of 6 and 8 to calculate the median.

$$\text{Median} = \frac{6 + 8}{2} = 7$$

Hence, the median of this data set is 7 even though it does not occur in the dataset.

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Median

- When the data set has an odd number of values, the median is the middle value.
- When the number of values is even, the median is the average of the two middle values.

Example 2

Students measured their heights to the nearest centimetre, and recorded the results shown below.

164 168 167 158 164 154 170 175 164 168

Calculate the median.

Solution

Arrange the data in order:

154, 158, 164, 164, 164, 167, 168, 168, 170, 175

The median lies between 164 and 167, so we need to take the average of these two values.

$$\begin{aligned}\text{Median} &= \frac{164+167}{2}\\ &= 165.5\end{aligned}$$

Example 3

Students measured their heights to the nearest centimetre, and recorded the results shown below.

164 168 167 158 164 154 170 175 164 168

Calculate the mean.

Solution

The mean is found by dividing the sum of the values by the number of values in the dataset.

$$\begin{aligned}\text{Mean} &= \frac{(164+168+167+158+164+154+170+175+164+168)}{10}\\ &= 165.2\text{ cm}\end{aligned}$$

The difference between the mean and median can make quite an impact. Consider the following examples of the mean and median of house prices. You might think that you could not afford to buy a house in the suburb in Example 4, based on the mean. This is because the mean is affected by the two extremely high prices. The median gives a clearer picture of what the 'average' house in this suburb might cost.

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Example 4

Listed below are some house prices achieved at auction last weekend.

$320 000, $299 000, $308 000, $335 000, $1 005 000, $325 000, $985 000

a Calculate the average house price.

b Calculate the mean, excluding the two extremely high prices.

c What is the median of all of the house prices?

Solution

a $\text{Mean} = \dfrac{\text{sum of values}}{\text{number of values}}$

$= \dfrac{(320\,000 + 299\,000 + 308\,000 + 335\,000 + 1\,005\,000 + 325\,000 + 985\,000)}{7}$

$= \$511\,000$

b $\text{Mean} = \dfrac{320\,000 + 299\,000 + 308\,000 + 335\,000 + 325\,000}{5}$

$= \$317\,400$

c Arrange the values in order. The median is the middle value.

$299 000, $308 000, $320 000, $325 000, $335 000, $985 000, $1 005 000

The median is $325 000.

Stem-and-leaf plots

Stem-and-leaf plots were introduced in *ICE-EM Mathematics Year 7*.

Example 5

The heights of 20 students, in centimetres, are given below.

164	158	152	167	146	149	167	171	181	154
167	158	164	172	176	180	178	165	159	153

a Represent this information on a stem-and-leaf plot.

b Find the median height.

c Find the average height.

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Solution

a Since each data value contains three digits, the first two will be the stem and the last digit will be the leaf. Also, since the smallest height is 146 cm and the largest height is 181 cm, the stems will be 14, 15, 16, 17 and 18.

This produces the following stem-and-leaf plot.

14	6 9
15	2 3 4 8 8 9
16	4 $\boxed{4\ 5}$ 7 7 7
17	1 2 6 8
18	1 0

17 | 2 means 172

b There are 20 items, so the median is the average of 164 and 165, which are the 10th and 11th items of the ranked data set.

$$\text{Median} = \frac{164 + 165}{2} = 164.5$$

c Mean

$$= \frac{164 + 158 + 152 + 167 + 146 + 149 + 167 + 171 + 181 + 154 + 167 + 158 + 164 + 172 + 176 + 180 + 178 + 165 + 159 + 153}{20}$$

$$= 164.05$$

In this case the mean and median are quite close.

Example 6

Form a stem-and-leaf plot for the following data and give the median and mean of the data below.

59, 57, 56, 47, 45, 43, 36, 33, 32, 31, 31, 30, 30, 25, 24, 23, 22, 11

Solution

The 18 data items have been ordered in descending order.

1	1
2	2 3 4 5
3	0 0 1 1 2 3 6
4	3 5 7
5	6 7 9

4 | 3 means 43

$$\text{Median} = \frac{31 + 32}{2} = 31.5$$

$$\text{Mean} = \frac{59 + 57 + 56 + 47 + 45 + 43 + 36 + 33 + 32 + 31 + 31 + 30 + 32 + 25 + 24 + 23 + 22 + 11}{18}$$

$$= 35\tfrac{5}{18}$$

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If a data set contains items that are clearly ‘very different’ to most of the items, then the median is the better choice to give you an idea of centre.

For example:

$12, 17, 19, 23, 24, 34, 38$

has a mean of 23.857 correct to three decimal places and the median is 23.

The data set:

$12, 17, 19, 23, 24, 34, 118$

has a mean of 35.286 correct to three decimal places and the median is 23.

The mean is changed by the value 78. The median does not change at all.

Exercise 19A

Example 3

1 The mean of each of the following data sets is 50. Determine the median of each of these.

a $23, 56, 37, 29, 58, 97$

b $48, 49, 50, 50, 51, 52$

c $0, 102, 58, 67, 71, 2$

Example 2, 3

2 The weights of a group of students, in kilograms, are given below.

47 46 45 46 42 41 45 41 49 46 42 43

a What is the median?

b Calculate the mean, correct to two decimal places.

3 The mean price of bags of potatoes at different supermarkets was \$7.50. The sum of the data was \$90.00. How many bags of potatoes were included in the survey?

Example 4

4 Sue spent the following amounts on her lunch each day over the course of two working weeks.

\$14 \$6 \$12 \$44.50 \$8.50 \$12.50 \$8 \$10 \$8.50 \$9.50

a Calculate the median for these data.

b Calculate the mean for these data.

c Compare the median and mean and comment on which is the better indicator of how much Sue usually spent on her daily lunch.

5 A list of data has 10 entries. Each entry is 1, 2 or 3. What could the list be if the average is:

a 1? **b** 2? **c** 3?

Example 6

6 For the stem-and-leaf plot shown, calculate the mean and find the median.

1	5 5 6 8 9
2	4 5
3	0 0 1 1 2 3 5
4	0 1 2 3 3 5

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7 The chest measurement in centimetres of 23 people is taken. The results are recorded in the stem-and-leaf plot shown.

8	8 9
9	3 3 3 4 5 6 7 9 9
10	0 1 2 3 4 6 6 7 7
11	1 1 9

a Find the median.

b Find the mean.

c Find the mean and median if the readings less than 90 and greater than 110 are not included.

8 The following list gives the area in hectares of each of the suburbs of a city.

7.5 2.2 5.2 19.2 2.4 41.3 11.3 27.6 9.0 2.3 28.4 3.2 3.6

a Find the mean and the median areas.

b Which do you think is a better measure of centre for the data set? Explain your answer.

9 The birth weights, in kilograms, of the first 20 babies born at a hospital in a selected month are as follows.

3.0	2.8	3.6	2.8	3.6	3.7	3.2	3.9	3.6	4.2
3.7	2.7	3.1	3.0	2.5	2.6	3.6	2.4	2.9	3.2

a Represent these data with a stem-and-leaf plot.

b Find the median value.

c Find the mean value.

10 **a** Find three different sets of four positive whole numbers that have a mean of 3 and a median of 2.

b Find seven different sets of five positive whole numbers that have a mean of 3 and a median of 2.

19B Sampling data

Population

An investigator usually wants to generalise about a whole class of individuals or objects. This class is called a **population**. For example, consider the following scenario. You have come up with a training strategy that you believe enables participants to increase their swimming speeds by at least 10%. You decide to set up a study to test the effectiveness of your training strategy. If your participants seem to benefit from the program, you would like to be able to say more than just that the program worked for these particular people. You would like to be able to generalise and say that it would be effective for a larger group. This larger group, the population, could be the members of a swimming club.

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Likewise, if a cure for a disease that works for one group of patients has been found, you would like to be able to conclude that it will work for all similar patients. Here the population is the class of similar patients.

An early step in your study should be to determine exactly what you want your population to be. It could be people in certain age group living in a particular state or attending your school.

You start with the population and with a question you want to answer.

Census

Sometimes it is possible to conduct your investigation for every member of the population; this is called taking a **census**.

A census of the whole population is taken in Australia every 5 years. The Australian Census aims to measure accurately the number of people in Australia on Census night wherever they are, from Australia's research hubs in Antarctica to remote Indigenous communities in northern Australia.

A census is the most comprehensive way of providing a snapshot of the people of Australia, our key characteristics and where we live. Census data support planning, decision making and funding at all levels of government, and are behind the services and facilities you use in your area every day.

It is often impossible to carry out a census. It is then necessary to look at a subset of the population.

Sampling data

The subset of the population that you choose to work with is called a **sample**. You can select a sample from a population in many different ways. Unfortunately, not all samples are equally useful if you want to be able to generalise your results.

The results of a study conducted with members of a swimming club, no matter how chosen, would not be appropriate if you wanted to make statements about the effectiveness of your swimming training program for everybody. The method of choosing a sample is very important. The best methods of sampling involve the planned introduction of chance.

Simple random sample

The simplest way of obtaining a representative sample from a population is to select a **simple random sample**. In a simple random sample, each member of the population has an equal chance of being selected; that is, no member of the population is systematically excluded from the sample nor are any particular members of the population more likely to be included. Each member of the sample is also selected **independently**; that is, the selection of a member in no way influences the selection of another member.

Uses of sampling

An example of the need for sampling is to investigate whether particular species of animals are prospering. In particular if we wanted to discover whether green tree frogs in an area of Queensland are surviving and are healthy, we might collect information about the numbers of frogs living in different areas, the weight of each frog and length of the hind legs. The type of information collected would depend on the questions we wanted to answer.

It would not be practical to collect information about all the green tree frogs in Queensland. It is not possible. Instead, we usually collect information about a smaller group within a population. The smaller group is a sample.

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Once scientists have some information about the sample, they try to make valid predictions about the entire population of frogs.

Usually, there are some numerical facts about the population that the investigators want to know, for example:

- the mean weight of a frog
- the percentage of frogs that have spots on their necks.

These cannot be determined exactly, but can only be estimated from a sample. Then an important issue is accuracy. That is, how close are the estimates going to be?

Selecting random samples

If you wanted to randomly select a sample of students in a school, you could write each student's school number on a small piece of paper, place the pieces of paper in a bucket, making sure that the slips of paper were well mixed up, and then withdraw a piece of paper. Mix up the pieces again and select the second, and so on. This could also be done with balls with numbers on them placed in a bin. Calculators and Excel have random number generators which could be used for this process.

See www.cambridge.edu.au/go for information on how to do this with Excel or a calculator.

19C Variation of means and proportions

In this chapter, consideration of variation across datasets leads us to explore the variation of sample means and of sample proportions across datasets collected or obtained under the same or similar circumstances. Sample proportions are considered for categorical data, and sample means for numerical data.

Categorical data

Categorical data are where each observation falls into one of a number of distinct categories.

Such data are everywhere in everyday life, for example:

- gender
- hair colour
- place of birth.

Relative frequencies for samples

A large bin contains 200 red, blue and green balls. They are thoroughly mixed up. Twenty balls are withdrawn without looking, and the colour is noted. Here the population is the bin of balls and the random sample of each ball consists of balls that have been withdrawn.

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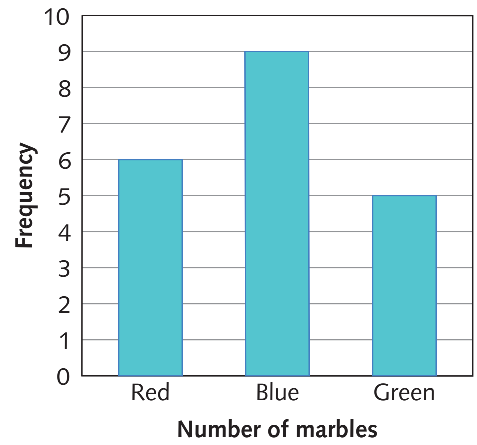
These data obtained are categorical data. In this sample, there are 6 red balls, 9 blue balls and 5 green balls. The categories are the colours.

A column graph has been constructed for this data.



One way of describing this data is through **relative frequencies** or **proportions**.

$$\text{Relative frequency (proportion)} = \frac{\text{frequency}}{\text{size of data set}}$$

The relative frequency is sometimes given as a percentage.

In this case the sample dataset has 20 items. The frequencies and relative frequencies are shown.

Category	Red	Blue	Green
Frequency	6	9	5
Relative frequency as a percentage	30%	45%	25%

Variability

Thirty samples of 20 balls are taken. The number of red balls in each sample is recorded:

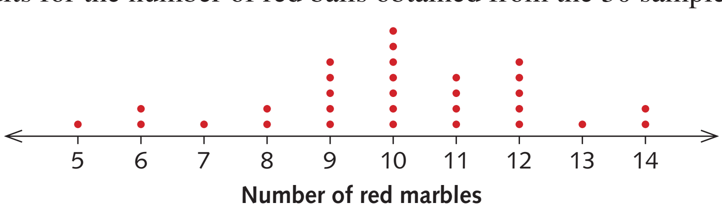
8 9 9 11 10 11 6 12 10 14 11 14 9 9 12 12
12 7 10 13 5 9 10 12 10 11 10 10 8 6

This is summarised in the following table.

Number of red balls	5	6	7	8	9	10	11	12	13	14
Frequency	1	2	1	2	5	7	4	5	1	2

This table tells us that 9 red balls were obtained from 5 of the samples, 10 red balls were obtained from 7 of the samples, and so on.

A dot plot of the results for the number of red balls obtained from the 30 samples is shown below.



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The frequency of each proportion (relative frequency) is given in the following table.

Proportion of red balls	25%	30%	35%	40%	45%	50%	55%	60%	65%	70%
Frequency	1	2	1	2	5	7	4	5	1	1

You can see that the sample proportion of red balls varies from 25% to 70%.

For categorical data, we are interested in relative frequencies or proportions of the different categories. If we have sample data that are representative of some general situation, we are interested in using the sample data to **estimate** proportions for the more general situation. In the case above, it would be estimating how many red balls there are in the bin.

Great care must be taken with the method of selecting the sample. At this stage we have no method for determining the accuracy of our estimates.

Numerical data

Some examples of numerical data are:

- time in minutes to get to work
- length in centimetres of the left hands of 13-year-old girls
- weight of boys who are in Year 9.

Sample means

Rods are known to have lengths between 50 cm and 100 cm. They are measured to the nearest centimetre. A random sample of size 20 is taken. The population is all of the rods.

The results are as shown below.

61 79 59 91 72 99 80 73 89 89 91 54 51 78 56 75 69 90 60 66

Mean = 74

The mean of the sample is 74 cm, correct to the nearest whole number. This provides us with an *estimate* of the mean lengths of the rods for the population.

Variability of sample means

Ten random samples of 20 rods are taken from the same population and the means recorded.

Sample number	1	2	3	4	5	6	7	8	9	10
Sample mean	74	77	70	73	74	66	76	70	74	81

The sample means vary from 66 through to 81.

The sampling procedure was also undertaken with samples of size 100. (There are a lot more than 100 rods.) The means are given correct to the nearest whole number.

Sample number	1	2	3	4	5	6	7	8	9	10
Sample mean	76	73	77	74	74	74	75	70	75	77

You can see that the means vary less with this larger sample size. The sample means vary from 70 to 77.

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Exercise 19C

1 A bin contains black, blue and green marbles. A sample of 20 marbles is taken from the bin. The results are as shown:

green, black, green, blue, green, black, green, blue, black, black, green, black, black, green, green, green, black, green, green, black

a Draw a column graph showing the frequency of each category.

b Find the relative frequency of each category.

c Find the relative frequency of each category as a percentage.

2 A second sample of 20 marbles is taken from the bin. The results are as shown in the table.

Category	Green	Blue	Black
Frequency	11	5	4

Calculate the relative frequency of each category.

3 The mass of 22 people is recorded in kilograms as shown.

77	85.5	63	80.5	79.5	94	66	69	65	58	69.5
73	74	68	80	66	54.5	64	84	73	89	94

a Two random samples of 10 people are chosen as shown. Find the mean of each sample.

i 66, 80, 63, 73, 84, 94, 69.5, 64, 79.5, 65

ii 65,58,94,73,77,64,89,84,66,63

b List the 10 smallest masses and calculate the average.

c List the 10 largest masses and calculate the average.

Note: The average mass of the 22 people is 73.93 kg.

4 A section of the Murray River is known to contain three types of fish. A random sample of fish is taken from the river and the results are recorded. The fish are released back into the river.

Type of fish	Murray cod	Redfin	Catfish	Murray perch
Number in sample	15	10	12	3

a Draw a column graph with this information.

b Find the relative frequency of each type of fish.

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5 A sample of red-eyed tree frogs is taken from an area surrounding a pond in central Queensland.

The lengths of the frogs in millimetres are as follows.

33 35 45 50 55 58 32 56 60 32

Find the sample mean of these data.

6 Use the data shown.

1 3 5 7 9 11 13 15 17

a Find the smallest and the largest sample mean obtainable from samples of 5.

b The average of the whole dataset is 8.5. Is there a sample of 5 which has a mean of 8.5?

7 The 1420 students at a school were asked 'Which method of transport do you use to get to school?' The responses were as follows.

Method	Train	Tram	By foot	Cycle	Car
Number	450	320	150	240	260

a Construct a table showing the relative frequency expressed as a percentage (correct to one decimal place) of each type of transport.

b Draw a column graph representing this data.

Activity 1 – Number of red counters (sample proportion)

A jar contains 100 counters. There are both red counters and black counters in the jar.

Take samples of 10 with replacement and good mixing and estimate the proportion of red counters in the jar. Compare results.

Activity 2 – Colourful Yummies (sample proportion)

Packets of Colourful Yummies come in several colours and are sold in packets that contain all of the colours.

Population: This could be the packing of Colourful Yummies by the manufacturer or a large bin of the sweet collected in the classroom. The benefit of the second is that the population proportion of a particular colour could be known.

The questions could then be:

a What is the proportion of red Colourful Yummies in a packet prepared by the manufacturer?

b What is the proportion of red Colourful Yummies in the large bin?

Sample: Samples could be prepared from the large bin or use the packages prepared by the manufacturer.

Compare the results of all students and display the results with a dot plot. Each of these results is an estimate for the proportion of red Colourful Yummies in the population.

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Activity 3 – Yes/no question (sample proportion)

a Write a *yes*/*no* question on a topic that interests you for a survey of a sample of 40 students.

b Decide what population you will sample from. This could be all girls in Year 8, or students in Year 9, or boarders.

c First, test your question on a small group of people.

d Decide on a method for random sampling.

e Work out your sample proportion of a 'yes' response.

f Compare your sample proportion of a 'yes' response with the sample proportion that others obtained.

Each result is an estimate of the proportion of 'yes' responses for the population.

Activity 4 – Even digits (sample proportion)

You know that the digits 0, 2, 4, 6, 8 are the even digits and 1, 3, 5, 7, 9 are the odd digits.

We know that 50% of digits are even.

a Randomly select 20 digits. This can be done by using a calculator or computer or by having a bucket full of pieces of paper with a digit on each. Of course, there must be the same number of occurrences of each digit.

b Compare your results with others and record all of the results on a dot plot.

c Try this with larger samples.

Activity 5 – Ages of 5-cent coins (sample means)

The Australian 5-cent coin was first minted in 1966 with an initial mintage of 75.427 million. Very few of these original coins are still in circulation.

The lowest minted year was 1972, with 8.25 million coins, and the highest minded year was 2005, with 194.3 million coins.

The average today is 80 million coins per year.

a Collect a large number of 5-cent coins.

b Take samples of 20 and record the age of each.

c Calculate the mean age of the 20 coins.

d Compare your results with others and record all of the results on a dot plot.

e Try this with larger samples.

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CHAPTER

20

Review and problem-solving

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20A Review

Chapter 10: Rates and ratios

1 Copy and complete the following equivalent ratios.

a $2:3=\square:9$ **b** $9:24=\square:8$
c $10:15=\square:4$ **d** $8:12=\square:3$

2 Express each of these ratios in simplest form.

a $5:10$ **b** $22:4$ **c** $125:750$ **d** $2:10$
e $32:72$ **f** $96:24$ **g** $225:150$ **h** $20:100$

3 In a box of toffees, the ratio of red wrappers to green wrappers is $5:6$, while the ratio of green wrappers to blue wrappers is $3:10$. Find the ratio of red wrappers to blue wrappers.

4 Divide 198 in the ratio $7:4$.

5 Divide \$1125 in the ratio $2:1$.

6 Divide \$2190 in the ratio $3:2:1$.

7 If a dozen eggs costs \$5.64, what is the cost of 30 eggs?

8 A man works for 7 hours and gets paid \$157.50.

a What is his hourly rate?
b How much does he get paid for 10 hours work?

9 A car uses 40 L of petrol to travel 320 km. How far can it travel with 50 L of petrol?

10 The cost of 1 kg of a certain type of fish is \$8.20.

a How much does 4.5 kg of this kind of fish cost?
b How many kilograms can you buy for \$49.20?

11 Express each of the following as a ratio in simplest form.

a 50 g to 200 g **b** 700 g to 1 kg

12 In a co-educational school of 1029 pupils, 504 are girls. What is the ratio of the number of girls to the number of boys?

13 The ratio of the number of birds to the number of cats owned by people in a neighbourhood is $5:3$. The number of cats is 96. How many birds are there?

14 In an apartment block, 24 apartments have two bedrooms and 32 have one bedroom. Give the ratio of the number of two-bedroom apartments to the number of one-bedroom apartments.

15 Four bricklayers can build a certain wall in 10 days. How long would it take five bricklayers to build it?

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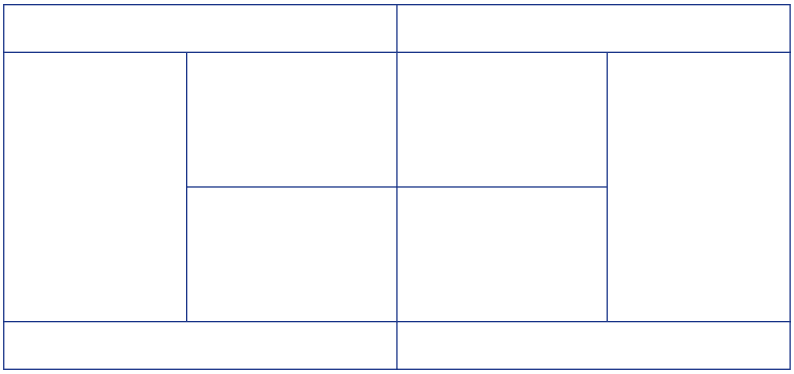
16 This plan of a tennis court is drawn to a scale of 1 cm to represent 230 cm. By taking measurements with your ruler, answer the questions below.



a What is the approximate length of the court?

b What is the approximate width of the court?

c Find the approximate distance from the base line to the service line on the same side of the net.

d What is the approximate area of the court?

e What is the approximate distance between the singles side line and doubles side line on each side of the court?

17 Convert each scale to ratios of whole numbers, reduced to lowest terms. Make sure to convert both measurements to the same unit first where necessary.

a 3 cm : 72 cm **b** 2 cm : 0.4 m **c** 5 cm : 10 mm **d** 27 cm : 3 km

18 Copy and complete each conversion of scale.

a 4 : 9 = 1 cm: _____ cm **b** 4 : 9 = 1 cm: _____ mm **c** 4 : 9 = 1 cm: _____ m

19 A car travels at 90 km/h for 5 hours. How far does it go?

20 A train takes 5 hours to complete a journey of 400 km. What is the average speed of the train?

21 A car travels a distance of 480 km at an average speed of 60 km/h. How long did the journey take?

22 Elizabeth walks 20 km in 5 hours. How far, at the same rate, will she walk in:

a 1 hour? **b** 2 hours? **c** 3 hours?

23 An aircraft travels 6500 km in 13 hours. What is its average speed?

24 A train travels for 45 minutes at 80 km/h. How far does it go?

25 Claire travels 40 km by train in $\frac{3}{4}$ of an hour and then cycles for 10 km in 48 minutes.

a How long is she travelling in total?

b What is her average speed during the train trip?

c What is her average speed during the bike trip?

d What is her average speed over the whole trip?

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Chapter 11: Algebra – part 2

1 Express each of these expressions as a single fraction.

a $\frac{x}{3}+\frac{x}{3}$

b $\frac{4x}{5}-\frac{2x}{5}$

c $\frac{z}{11}+\frac{6z}{11}$

d $\frac{3x}{5}+\frac{x}{2}$

e $\frac{x}{4}+\frac{6x}{7}$

f $\frac{2x}{9}-\frac{3x}{4}$

g $\frac{4x}{7}-\frac{x}{2}$

h $\frac{3x}{5}+\frac{3x}{7}$

i $\frac{2x}{5}+\frac{4x}{15}-\frac{x}{3}$

j $\frac{x}{2}-\frac{2x}{5}+\frac{x}{3}$

2 Expand the brackets and collect like terms.

a $4(x-3)+6$

b $-2(7x+3)-8x$

c $-(x-6)-5(x-2)$

d $2\left(\frac{2x}{3}+4\right)+\frac{x}{3}$

e $5\left(\frac{x}{2}+5\right)-\frac{x}{2}$

f $3\left(\frac{3x}{4}-2\right)+\frac{x}{2}$

g $3\left(\frac{2x}{5}+2\right)-\frac{3x}{7}$

h $4\left(\frac{3x}{5}+1\right)-\frac{2x}{3}$

i $6\left(\frac{4x}{7}-1\right)-\frac{3x}{11}$

3 Solve these equations for x.

a $2x+5x+1=22$

b $4(x+1)+2=15$

c $-3(2x+1)+5=17$

d $x+3=2$

e $2x+4=-2$

f $2-x=8$

g $\frac{x}{5}+4=11$

h $2x-5=-6x+7$

i $2(x-2)=11x$

j $\frac{3x}{5}+6=\frac{21}{5}$

k $2x-12=2(3+4x)$

4 Solve these equations.

a $\frac{3x}{7}+\frac{x}{7}=6$

b $\frac{4p}{9}+\frac{2p}{9}=1$

c $-\frac{x}{5}+\frac{2x}{5}=1$

d $\frac{2x}{7}+\frac{x}{5}=2$

e $\frac{m}{3}+\frac{2m}{5}=1$

f $\frac{5m}{9}-\frac{4m}{5}=1$

5 Given that $y=3x-5$, find the value of y when $x=5$.

6 **a** A number is multiplied by -5 and 3 is subtracted. The result is -4. Write an equation and find the number.

b Six is added to a number, and the result is multiplied by -3. The result is 28. Write an equation and find the number.

c Seven is subtracted from a number, and the result is multiplied by 4. The result is 15. Write an equation and find the number.

d When a number is multiplied by 5 and divided by 11, the result is 22.8. Find the number.

e If you add 13 to a number, you get the same result as when you subtract half the number from 4. What is the number?

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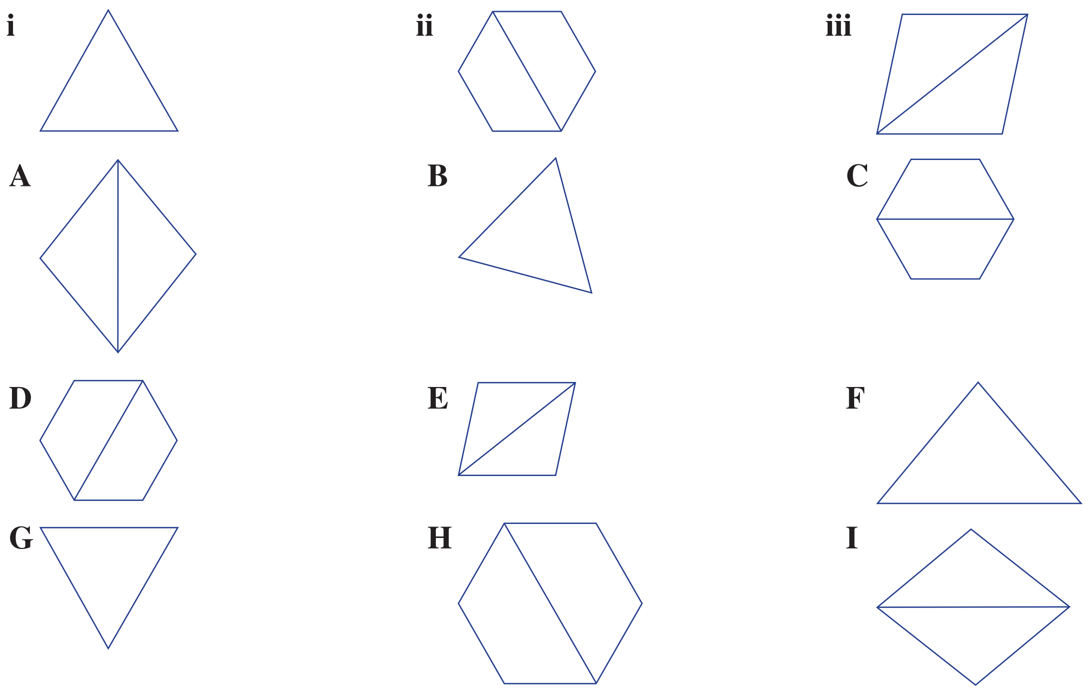
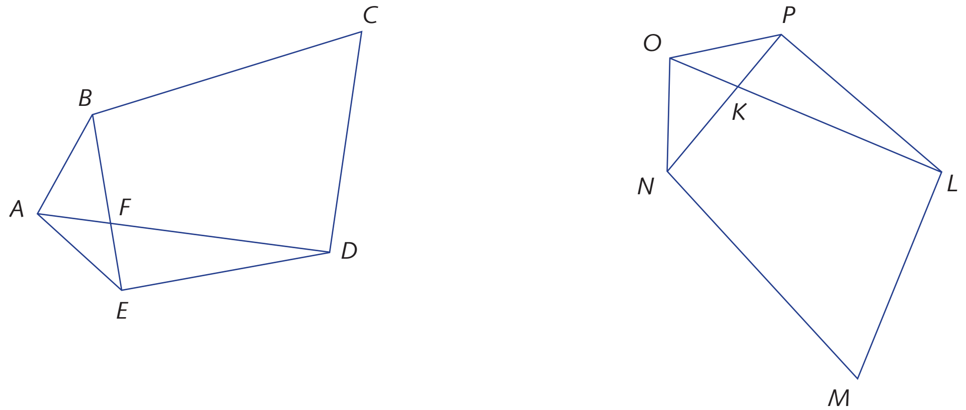
Chapter 12: Congruent triangles

1 List the figures in the collection below that are congruent to figures **i**, **ii** and **iii**.



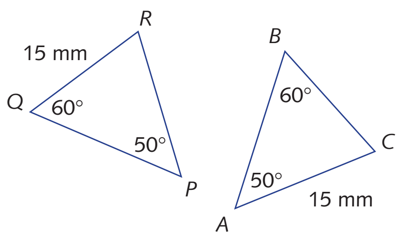
2 Complete the pairing of matching vertices, matching sides and matching angles of these two congruent figures.



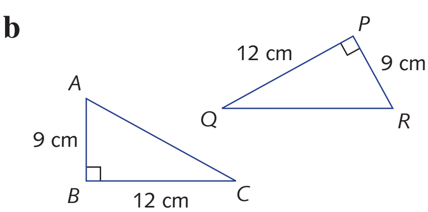
a $A \leftrightarrow$	**b** $B \leftrightarrow$	**c** $C \leftrightarrow$	**d** $D \leftrightarrow$
e $E \leftrightarrow$	**f** $F \leftrightarrow$	**g** $AD \leftrightarrow$	**h** $AB \leftrightarrow$
i $CD \leftrightarrow$	**j** $\angle ABC \leftrightarrow$	**k** $\angle BED \leftrightarrow$	**l** $\angle AFB \leftrightarrow$

3 In each part below, say whether the two triangles are congruent. If they are, write a congruence statement, including the appropriate congruence test.

a



b



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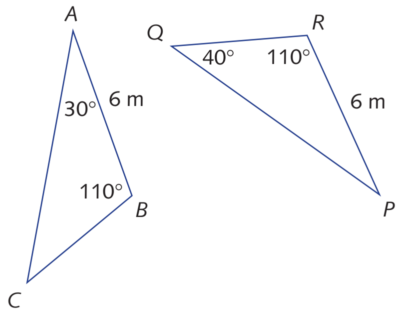
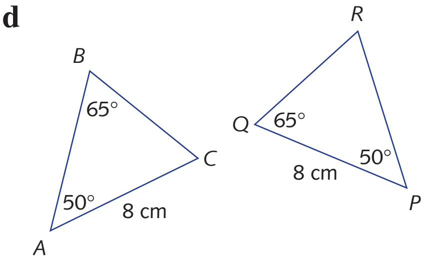
c

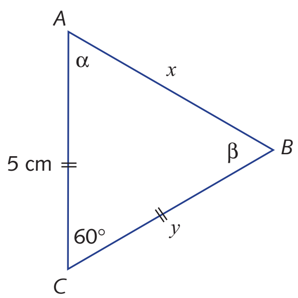


d

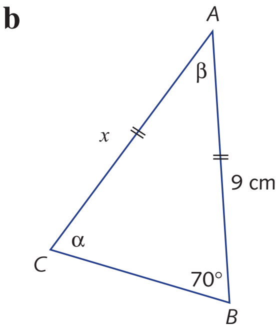


4 Find the values of x, y, α, β, and γ, giving reasons.

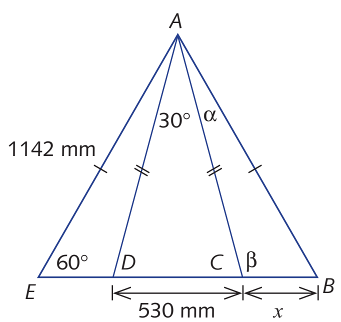
a



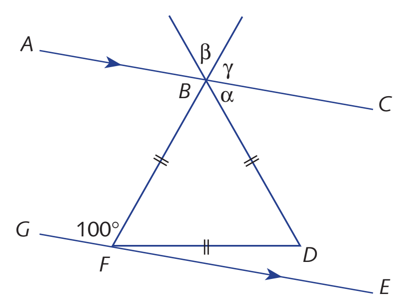
b



c



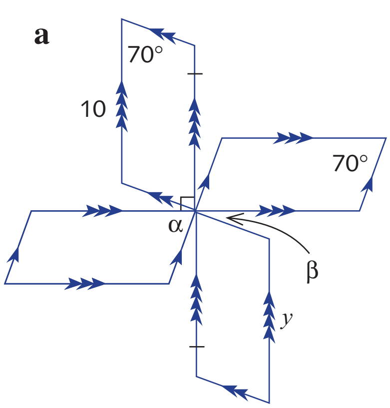
d



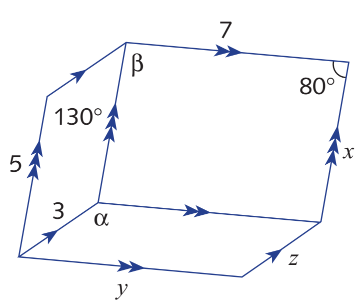
Chapter 13: Congruence and special quadrilaterals

1 Use the properties of a parallelogram to find the values of x, y, z, α, and β in the diagrams below.

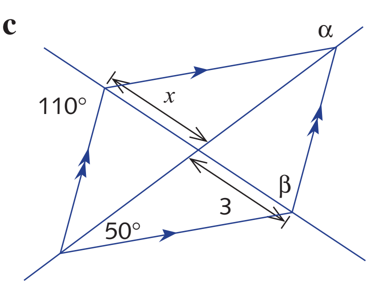
a



b



c



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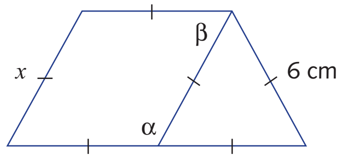
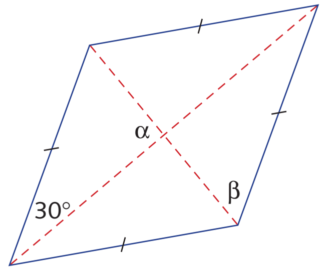
2 Use the properties of a rhombus to find the values of x, y, z, α, and β in the diagrams below.

a



b



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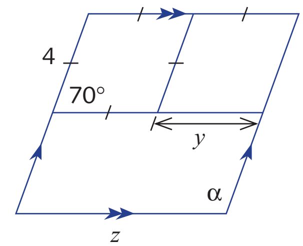
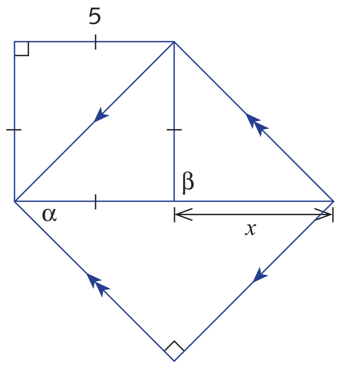
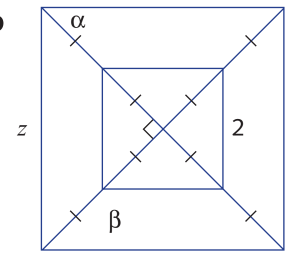


3 Find the values of x, z, α, and β in the diagrams below.

a



b



4 Each of the groups in the column on the left matches with at least one of the descriptions on the right. Can you match each group on the left with a description so that each description is used only once?

	Quadrilateral		Description
A	Parallelogram	I	Opposite sides are parallel and all four angles are right angles
B	Rhombus	II	Opposite sides are parallel
C	Rectangle	III	Opposite sides are parallel and all four angles are right angles and all four sides are equal
D	Square	IV	Opposite sides are parallel and all four sides are equal

5 **a** Complete this sentence by inserting the word 'square' or 'rhombus'.

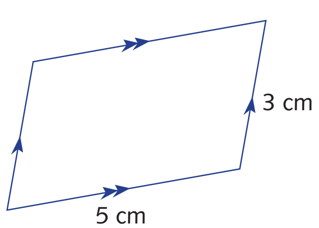
A '_______' is always a '_______' but a '_______' is not necessarily a '_______'.

b Complete this sentence by inserting the word 'square' or 'rectangle'.

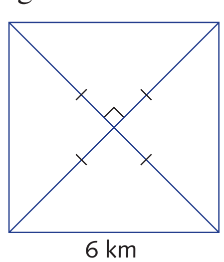
A '_______' is always a '_______' but a '_______' is not necessarily a '_______'.

6 Find the perimeter of each parallelogram.

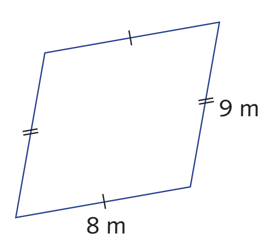
a



b



c



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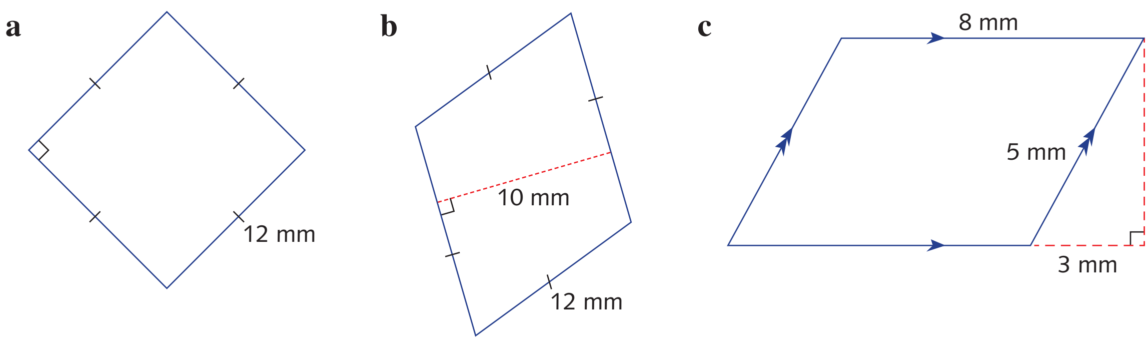
7 Find the area of each parallelogram.



Chapter 14: Circles

1 Use your compasses, ruler and protractor to draw:

a a sector of a circle with diameter 10 cm and containing an angle of 130°

b a sector of a circle with radius 4 cm and containing an angle of 60°

c a semicircle with diameter 7 cm

d a quadrant with radius 3 cm

2 Find the circumferences of the two circles specified below. In each case, give the answer:

i in terms of π

ii as an approximate value, using $\pi \approx \frac{22}{7}$

iii as an approximate value, using $\pi \approx 3.14$

a Diameter 8 mm **b** Radius 3 m

3 Find the approximate value of the area of each of the circles in Question **2**, using $\pi \approx \frac{22}{7}$.

4 Find, for a sector with radius 12 cm and containing an angle of 72°:

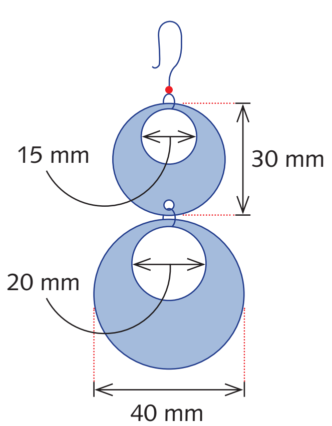
a the perimeter of the sector

b the area of the sector

Give each answer in terms of π.

5 Find the area of the annulus formed by two concentric circles, one of diameter 9 m and the other of diameter 5 m. Leave your answer in terms of π.

6 Find the approximate value of the area of plastic, represented by the shaded area in the figure opposite, needed to make the piece of jewellery shown. Use $\pi \approx 3.14$.



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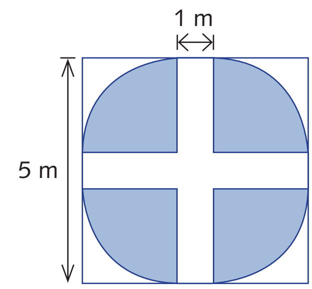
7 Clive has a square area of garden with side length 5 m. He is planning to design a vegetable garden with two straight paths, each of width 1 m, across the garden. The paths are to be at right angles to each other, as shown. The diagram is symmetric. He also plans to make four vegetable garden beds in the shape of quadrants, as shown, and to put pavers in the rest of the garden.

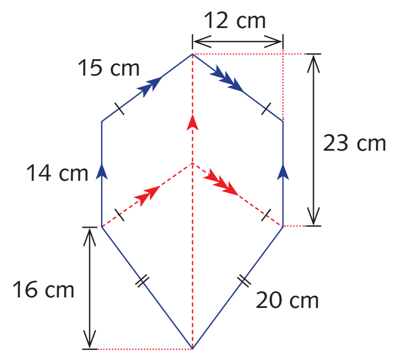


What will be the value of the area of the vegetable garden beds once the paths and pavers have been laid? Give your answer in square metres, in terms of π.

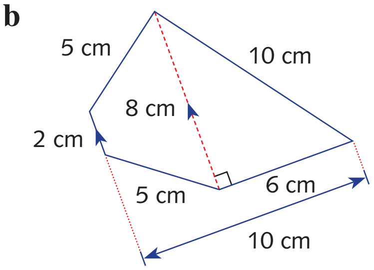
Chapter 15: Areas, volumes and time

1 Find the perimeter and area of each of these figures. Give answers to **d**, **e** and **f** in terms of π.

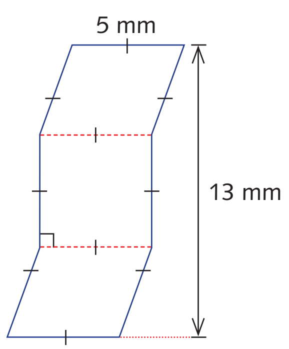
a



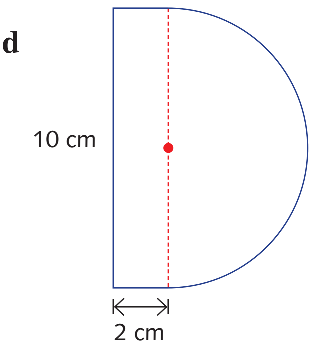
b



c

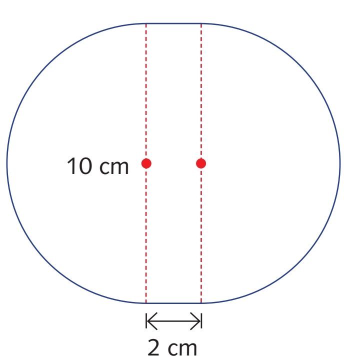


d



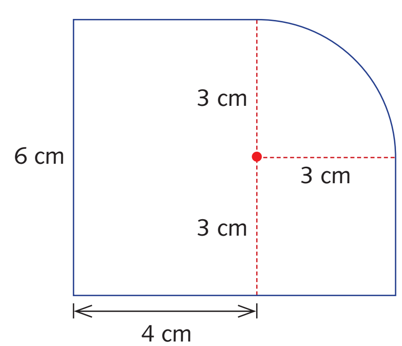
A semicircle and rectangle

e



Two semicircles and rectangle

f



Quadrant of circle and rectangles

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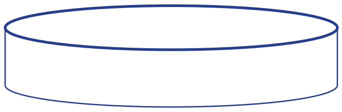
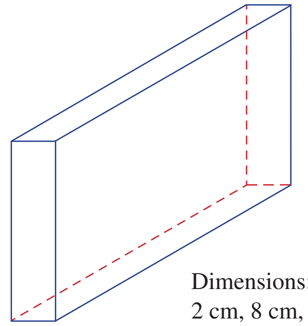
2 Find the volumes of these solids. Each solid is a prism or a cylinder.
Give your answer for **a** and **d** in terms of π.

a



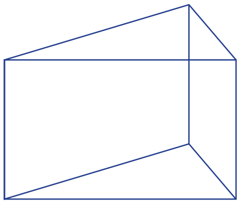
Radius 6 cm
Height 2.1 cm

b



Dimensions:
2 cm, 8 cm, 12 cm

c

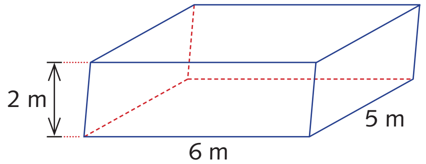


Area of cross-section 5 cm^2
Height 6.2 cm

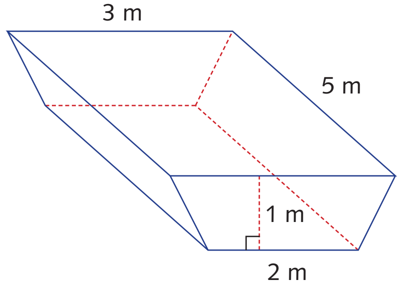
d

Radius 2 cm
Length 5 cm

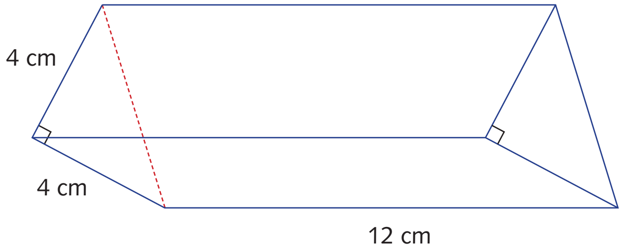
e



f



3 Find the surface area and volume of the prism shown below. The cross-section is a right-angled isosceles triangle.



Chapter 16: Probability

1 A bag contains three red beads, two blue beads and four yellow beads. If one bead is drawn at random from the bag, find the probability that the bead is:

a red

b red or blue

c red or yellow or blue

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2 A two-digit number – that is, a number from 10 to 99 – is chosen at random. Find the probability that the number:

a is greater than 72

b contains the digit 6 at least once (that is, a number such as 64, or 16 or 66)

3 A letter is chosen at random from the word RANDOM.

a List the sample space for this experiment.

b i If E is the event 'the letter chosen is a vowel', write down the outcomes that satisfy E.

ii What is the probability of E occurring?

4 The numbers 1 to 25 are written on 25 cards. If a card is selected at random, find the probability that the number on the card is:

a 14

b greater than 20

c even

d a multiple of 5

e odd and a multiple of 3

f neither even nor a multiple of 5

5 In a group of 200 students, 60 study geography, 80 study economics and 70 study neither.

a Represent this information on a Venn diagram.

b If a student is selected at random from the group, what is the probability that the student studies:

i geography?

ii geography and economics?

iii at least one of these subjects?

6 The table shows the type of accommodation and car ownership status of 150 university students.

Accommodation	Car	No car
Residential college	10	50
Apartment off campus	30	40
Living at home	15	5
Total	**55**	**95**

For a randomly selected student from this university, what is the probability that the student:

a lives in a residential college and has a car?

b has a car?

c lives at home and has a car?

d has no car?

e lives in an apartment off campus and does not have a car?

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Chapter 17: Formulas and factorisation

1 Find the value of each of these expressions for $a = -4$.

a $10 + 2a$ **b** $5 - 2a$ **c** $94 + a^2$

2 Given that $y = 3x - 5$, find the value of y when $x = 5$.

3 Given that $v = u + at$, find the value of:

a v when $u = 4$, $t = 12$ and $a = 6$

b a when $v = 24$, $u = 4$ and $t = 10$

c t when $v = 5$, $u = 12$ and $a = 14$

4 A rectangle is 4 m longer than it is wide. Let the width be w metres.

a Write an expression in terms of w for the length, ℓ, of the rectangle.

b Write an expression in terms of w for the perimeter, P, of the rectangle. Find P and ℓ when $w = 5$.

c Write an expression for the area, A, of the rectangle.

d Find A when $w = 3$.

e Find w when $P = 32$.

f Find w when $A = 45$.

5 Find the highest common factor of each pair of terms.

a $8, 4$ **b** $5x, 5y$ **c** $14x, 63x$ **d** $17y,\ 51y^2$

e $18ab^2, 9ab$ **f** $15x^2y, 25yx^2$ **g** $27a^2b^2, 9a^2b$ **h** $8a^3, 9b^2$

6 Factorise by taking the highest common factor.

a $3y + 6$ **b** $8m - 56$ **c** $-2 + 44p$

d $80y + 5$ **e** $-12z + 36$ **f** $5x^2 - x$

g $12y^2 + 4$ **h** $-42x + 36x^2$ **i** $12ab^2 + 2ab$

j $8a + 4ab$ **k** $10xy + 5xy^2$ **l** $25mnp - 5m^2p$

7 Expand:

a $2(m + 3)$ **b** $-2(3x - 4)$ **c** $-4(3x - 2)$

d $(3x + 2)(x - 5)$ **e** $a(a^2 + 3)$ **f** $(2 - x)(2 + x)$

g $(2z + 1)(2z + 4)$ **h** $(2z - 3)(2z + 3)$ **i** $(x + 2)^2$

j $(2b + 3)^2$ **k** $(2s - 6)(2s + 3)$ **l** $(2a + b)(2a - 3b)$

8 Factorise:

a $x^2 + 6x + 9$ **b** $a^2 + 4a + 4$ **c** $x^2 - 7x + 12$

d $x^2 + 4x + 3$ **e** $a^2 - 4a + 4$ **f** $x^2 + 9x + 14$

g $x^2 - 5x - 14$ **h** $a^2 - 2a - 24$ **i** $a^2 - 5a + \frac{25}{4}$

j $x^2 - 7x + 6$ **k** $x^2 - x - 30$ **l** $x^2 - 4x - 21$

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Chapter 18: Graphing straight lines

1 Complete each table of values for the given formula. List the coordinates of the points you get from the table and plot them on a number plane, drawing a line through the points.

a $y = -3x$

x	–3	–2	–1	0	1	2	3
y							

b $y = 3x - \frac{1}{2}$

x	–3	–2	–1	0	1	2	3
y							

2 **a** On a single set of axes, draw the graphs of:

i $y = 3x$ **ii** $y = 3x + 2$ **iii** $y = 3x - 1$

b State the gradient of each of the graphs in part **a**.

3 **a** On a single set of axes, draw the graphs of:

i $y = x$ **ii** $y = 4x$ **iii** $y = -2x$

iv $y = \frac{1}{2}x$ **v** $y = -\frac{3}{4}x$

b For each line in part **a**, find the points on the line with x values of 1 and 2, and hence find the gradient of the line.

c For each line in part **a**, state whether the line slopes upwards or downwards.

d Which of the lines in part **a** has:

i the steepest upwards slope? **ii** the steepest downwards slope?

4 **a** On a single set of axes, draw the graphs of:

i $y = 4x + 1$ **ii** $y = -2x + 1$ **iii** $y = \frac{2}{3}x + 1$

b State the value of the slope m for each of the lines in part **a**.

5 Calculate the slope of the line that passes through the points shown in the table below.

x	–3	–2	–1	0	1	2	3
y	6	3	0	–3	–6	–9	–12

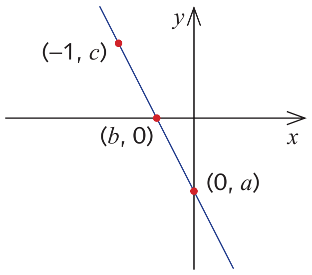
6 For the straight-line graph of $y = 3x - 2$, find the y-coordinate of the point on the line with x-coordinate:

a $x = -2$ **b** $x = 0$ **c** $x = 12$

7 Check whether or not each of these points lies on the line with equation $y = -5x + 2$.

a $(2, 8)$ **b** $(-2, 8)$ **c** $(2, -8)$ **d** $(-2, -8)$

8 The graph of $y = -4x - 2$ is shown opposite. Find the values of a, b and c.



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9 **a** The x -coordinate of a particular point on the line $y = 7 - 2x$ is 8. Write down the y-coordinate.

b The y-coordinate of a particular point on the line $y = 5 + 3x$ is 17. Write down the x-coordinate.

10 If the points $(2, a)$, $(-1, b)$ and $(c, -6)$ lie on the line with equation $y = -\frac{3}{4}x$, find the values of a, b and c.

11 Find the y-intercept of each of these lines by substituting $x = 0$.

a $y = 8x + 1$ **b** $y = -4x - 3$ **c** $y = -\frac{1}{2}x + 5$

12 Complete the table of values and plot the graph for each of the given formulas, drawing a smooth curve through the points.

a $y = 4 - x^2$

x	–3	–2	–1	0	1	2	3
y							

b $y = (x + 1)^2$

x	–3	–2	–1	0	1	2	3
y							

c $y - x^3 - 1$

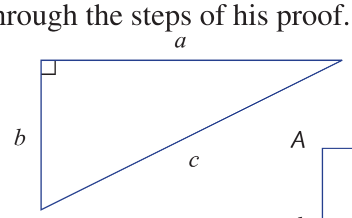
x	–2	–1	0	1	2
y					

20B Problem-solving

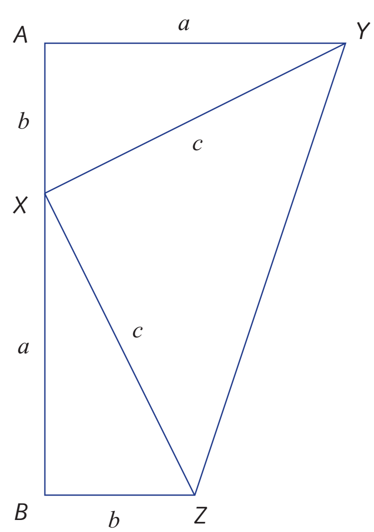
Pythagoras' theorem

The American president James Garfield (1831–1881) gave a simple proof of Pythagoras' theorem. In this problem, you will be working through the steps of his proof.

Begin by drawing a right-angled triangle with side lengths a, b and c, as shown opposite.



Now draw the triangle again to form a trapezium, as shown opposite.



1 Explain why this shape is a trapezium.

2 Explain why $\angle YXZ$ is a right angle.

3 Write down the area of the trapezium, in terms of a and b, using the formula for the area of a trapezium.

4 Find the area of the trapezium, in terms of a, b and c, by adding up the areas of the three triangles.

5 Explain why $\frac{1}{2}(a + b)(a + b) = \frac{1}{2}(2ab + c^2)$.

6 Use the result in step **5** to prove Pythagoras' theorem.

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Scones

A chef has a large ball of dough. Its volume is 5000π cm^3. The chef intends to roll the dough out into a circular slab of thickness 2 cm, and then cut it into as many scones as she can, using a round cutter of diameter 5 cm.

1 Find the diameter of the rolled-out slab of dough.

2 Find how many scones the chef will be able to make using the dough from the dough-ball, assuming that when she finishes cutting out the first lot of scones, she collects the scraps of dough, rolls them out again and keeps cutting, and that she continues this process until there is either no dough left or not enough to make another scone. (The thickness is always 2 cm.)

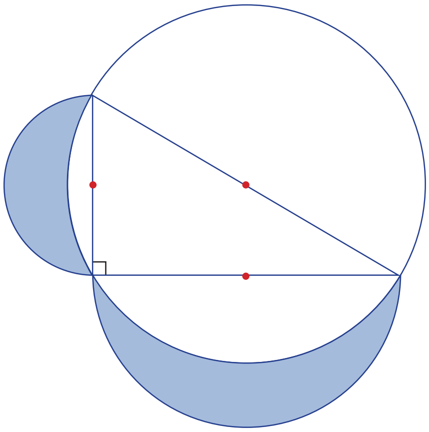
A litre

We wish to build a rectangular prism of height 10 cm and volume 1000 cm^3 (= 1 L). All dimensions are to be integer number of centimetres. One such container is a cube with edges 10 cm long. Find all the others.

Circles

Start with a right-angled triangle. Construct three circles, using the three sides as diameters so that each circle has the midpoint of a side as its centre, as shown opposite. Assume that the area of the triangle is 36 cm^2. Find the combined area of the shaded regions of the two smaller circles that lie outside the largest circle.

What is surprising about the result?



A quadrilateral

We are given a quadrilateral and are told that three of its sides are of equal length and the fourth side is the same length as each of the diagonals. Find the interior angles of the quadrilateral.

Unusual numbers

1 What number gives the same answer when it is subtracted from 10 as it does when it is multiplied by 10?

2 What number gives the same answer when it is divided by 10 as it does when it is added to 10?

3 What numbers give the same result when they are divided into 10 as they do when they are multiplied by 10? (There are two such numbers.)

4 What numbers give the same result when they are divided into 10 as they do when they are subtracted from 10? (Again, two numbers are possible. Give your answers correct to three decimal places. Some trial and error may be needed here.)

Which fits better?

Which fits better – a square peg in a round hole or a round peg in a square hole? Explain your assumptions and answer mathematically.

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Racing and chasing

A runaway dog, called Fido, is running at 3 m/s and is being chased by his owner, who is riding her bicycle at 5 m/s. How long will it be before she catches up with the dog?

The answer to this question is: 'It all depends on how far she is behind Fido to start with'. Obviously, if the distance between them is 100 m she will take a lot longer to catch up than if she starts only 20 m behind. Five times as long, in fact.

This question is one involving the concept of **closing speed**. By finding the difference between the front-runner's speed and the chaser's speed, the question can be solved very easily. It makes use of the fact that, since speed = distance ÷ time, then it follows that time = distance ÷ speed.

The owner is cycling at $(5 - 3)$ m/s = 2 m/s faster than the dog. This is the speed at which she is closing the distance between herself and Fido. You can imagine that the distance between them is being 'eaten up' at the rate of 2 m/s. If you do this, it is obvious that, if Fido is 100 metres ahead when the chase begins, he will be 98 m ahead after 1 second, 96 m ahead after 2 seconds, and after (100 m) ÷ (2 m/s) = 50 s, he will be 0 m ahead. With a 20-m head start, the chase will only last 10 seconds.

We can also use a similar idea to find out where the chase ends. Since the chase lasts for 50 seconds, the owner will travel (50 s) × (5 m/s) = 250 m until she catches the dog. (Fido will have run (50 s) × (3 m/s) = 150 m, which, when added to his 100 m start, will also be 250 m from where his owner started the chase.)

Try these questions, using similar ideas.

1 In a cross-country race, Manus sees a friend 70 m ahead of him. He knows that his friend runs at a steady speed of 2.8 m/s in races of this length. Manus wishes to catch up with him, so he speeds up to 3.2 m/s.

a How long will it take Manus to catch up with his friend?

b If Manus is a kilometre from the finish when he sets out after his friend, how far from the finish line will he overtake him?

2 The leading driver in an Indy car race is having engine trouble. He has had to cut his speed to 140 km/h but is 500 m ahead of the car in second place. The driver in second place maintains a speed of 150 km/h. How long does it take for the second driver to catch up with the leader?

3 At 7:30 a.m. a moving van (van 1) leaves Melbourne and heads for Mildura, 550 km away. At 10 a.m., another van (van 2) leaves Bendigo, 150 km from Melbourne on the Melbourne–Bendigo–Mildura road. It averages 90 km/h. Van 1 travels at an average speed of 80 km/h, but stops for lunch for an hour at Sea Lake, 360 km from Melbourne. Van 2 fills up at Ouyen, 100 km before Mildura. This takes 40 min.

Draw up a time/place table showing how far from Melbourne each van is at the times when they start, stop and pass each other. Then answer these questions.

a How far ahead is van 1 when van 2 leaves Bendigo?

b When and where does van 2 pass van 1?

c When does van 2 leave Ouyen, and when it does, where is van 1?

d At what time does each van reach Mildura, to the nearest minute?

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Closing the gap

When objects are travelling towards each other, the distance between them gets 'eaten up' quickly. If a car is travelling at 100 km/h towards a truck that is travelling at 60 km/h, the car and truck have a closing speed of 160 km/h. In other words, the vehicles will be 160 km closer to each other after 1 hour, provided they are at least 160 km apart in the first place. When two objects are travelling towards each other, their closing speed is found by adding their individual speeds.

1 A car and a truck are approaching each other on a straight road. They are 20 km apart and the truck is travelling at 60 km/h. Find how long it takes for the car to pass the truck if the car maintains a steady speed of:

a 90 km/h

b 100 km/h

c 80 km/h.

2 Jill and Sophie both leave home at 8:25 a.m. and walk to school. Jill walks at a speed of 75 m/min and Sophie takes her time and walks at 60 m/min. They meet at school at 8:40 a.m.

a How far does each girl live from the school?

b Suppose Jill and Sophie start at two points on the same road and walk towards each other at the above speed and from the same times. How far apart are the points?

3 A meteor is heading straight for Earth at a speed of 90 000 km/h. A rocket is launched towards the meteor at a speed through space of 20 000 km/h. If it takes the rocket two days to collide with and destroy the meteor, how far from Earth is it when the rocket is launched, and how far from Earth does the collision take place?

20C Fibonacci sequences

A Fibonacci sequence is a sequence F_1, F_2, F_3, … of numbers in which each term from the third one onwards is the sum of the two terms that immediately precede it. You have to have two numbers to start with, F_1 and F_2. These are called the **seeds**. Then:

$$F_3 = F_2 + F_1,$$

$$F_4 = F_3 + F_2,$$

and so on. The classic Fibonacci sequence has 1 and 1 as its seeds. Its first 10 terms are:

$$1, 1, 2, 3, 5, 8, 13, 21, 34 \text{ and } 55$$

Use a calculator where appropriate in the following.

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Activity 1

Write out the classic Fibonacci sequence as far as its 25th term, F_{25}. Before you calculate F_{11}, make a rough guess of what the value of F_{25} will be. See how good your guess turns out to be.

Activity 2

Pick any two numbers as seeds and work out the first 20 terms for that Fibonacci sequence. Pick entirely different seed numbers from the person beside you, and keep your list at least reasonably neat, as we will be coming back to it in a little while.

Activity 3

Swap your two seed numbers from Activity 2 around and figure out the first 20 terms in the new Fibonacci sequence. (If, for example, your sequence in Activity 2 started $6, 11, 17, 28, 45, \ldots$, your new sequence will start $11, 6, 17, 23, 40, \ldots$) Yes, you do get quite different numbers from the ones in Activity 2.

Activity 4

It is now time to make a few observations about your Fibonacci sequences.

- The classic sequence (in Activity 1) has two odd seeds. This gives a certain pattern of odd and even terms throughout the sequence. What happens if you start with two even seeds or an odd and an even seed? Explain.
- Compare the 10th terms in each of the sequences you generated in Activities 2 and 3. Which one is larger? Compare the 20th terms as well. Can you explain what is happening?
- Use a calculator to divide the term F_{10} in the first sequence by the term F_9 immediately before it. Write your answer down. Then do the same with the second sequence. Now repeat the calculations but with F_{20} and F_{19} for both sequences. Do you notice anything interesting? Did any of the other students who are doing this activity get the same number? They should all have found the same answer, although there may be very small differences in the sixth decimal places.
- For the classic Fibonacci sequence, the first two terms larger than 1000000 are $F_{32} = 1\,346\,269$ and $F_{33} = 2\,178\,309$. Use these two values to see if what you noticed in the previous ratio calculations also holds for high-order terms in the classic Fibonacci sequence.

The number you obtained (to a good approximation) in the ratio calculations is famous and interesting enough to deserve its own Greek letter. It is called Φ (*phi*, pronounced to rhyme with 'spy') and is known as the **golden mean** or **golden ratio**. It is a very interesting number with a long history.

Search Google and you will discover some amazing facts about Φ. It appears in many different ways in geometry and architecture.

Now try calculating these values and see what you notice about them.

a Φ^2 **b** $\dfrac{1}{\Phi}$ **c** $(2\Phi - 1)^2$

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Answers to exercises

Chapter 1

Exercise 1A

1 **a** 78 **b** 103 **c** 157
d 138 **e** 100 **f** 140

2 **a** 61 **b** 126 **c** 81
d 223 **e** 65 **f** 96

3 **a** 18 **b** 26 **c** 48
d 68 **e** 58 **f** 75

4 **a** 1810 **b** 9330 **c** 10 125
d 1580 **e** 1699 **f** 789

5 **a** 375 **b** 1493 **c** 1019
d 1075 **e** 339 **f** 898

6 2675 people

7 14 000 people

8 9904 cats

9 821 chairs, 1055 chairs and tables altogether

10 775 409 900 L, 218 008 100 L difference

11 **a** 69 mm **b** 197 mm **c** 128 mm

12 10 071

13 16 229 290 Roman Catholics

14 2 002 263

Exercise 1B

1 **a** 630 **b** 13 200 **c** 540 **d** 770

2 **a** 693 **b** 504 **c** 672 **d** 1666

3 **a** 107 **b** 88 **c** 106 **d** 494

4 **a** 23 remainder 4 **b** 49
c 66 remainder 8 **d** 113 remainder 1

5 **a** 4942 **b** 16 588 **c** 14 694 **d** 29 988
e 38 016 **f** 31 325 **g** 4860 **h** 22 464

6 **a** 278 **b** 135 **c** 1078 **d** 83
e 675 **f** 136 **g** 8677 **h** 4360

7 **a** 63 remainder 9 **b** 65 remainder 10
c 146 remainder 8 **d** 503 remainder 3
e 122 **f** 212 remainder 14
g 83 remainder 4 **h** 247 remainder 15

8 840 seats

9 792 apartments

10 1428 cars

11 103 200 tomatoes

12 3315 lights

13 Plan A costs $33.00, Plan B costs $35.76.

14 561 858 990 km

15 $520

16 17 bags

17 **a** 1553 bags with 5 left over
b 345 mega-bags with 9 left over

18 **a** **i** 384 weeks **ii** 288 weeks **iii** 192 weeks
b 24 weeks, 18 weeks, 12 weeks

19 53 times

Review exercise

1 **a** 60 **b** 57 **c** 62 **d** 61 **e** 30
f 97 **g** 90 **h** 75 **i** 90 **j** 41
k 53 **l** 55 **m** 94 **n** 82 **o** 36
p 45 **q** 140 **r** 140 **s** 80 **t** 130
u 150

2 **a** 22 **b** 4 **c** 28 **d** 8 **e** 93
f 19 **g** 75 **h** 109 **i** 194 **j** 5
k 260 **l** 75

3 **a** 900 **b** 7800 **c** 700 **d** 420 **e** 760
f 1600 **g** 600 **h** 4700 **i** 0

4 12 979 km^2

5 **a** $231 + 26\underline{4} + 1\underline{3}6 = \underline{6}31$
b $\underline{843} + 637 + 567 = \underline{2}047$
c $\underline{3}7\underline{6} \times 9 = 33\underline{8}4$

6 5930 dogs

7 551 girls

8 24 chocolates

9 45 boys

10 54 boxes

11 74 people

12 16 380 kg

13 35 trees

Challenge exercise

1 2.5 km

2 There are at least 384 solutions. One is $98\,765 + 1234 = 99\,999$.

3 22 birds, 14 beasts

4 $n \to 2n + 4 \to 10n + 32 \to 100n + 320 - 320 \to n$. Hence you get the number you started with.

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5 **a** 5 **b** 21 779 **c** 402, 529

6 **a** 108 **b** 21 **c** 35 **d** 42

7 **a** 7307 **b** 243

8 $69\,237 \div 231 = 299$ remainder 168

9 5 a.m.

Chapter 2

Exercise 2A

1 **a** $\frac{2}{3}$ **b** $\frac{1}{2}$ **c** $\frac{6}{7}$ **d** $\frac{5}{6}$ **e** $1\frac{2}{7}$

f 4 **g** $\frac{5}{7}$ **h** $\frac{3}{8}$ **i** $\frac{4}{5}$ **j** $\frac{2}{3}$

k $\frac{2}{3}$ **l** $\frac{3}{4}$ **m** $1\frac{1}{4}$ **n** $1\frac{1}{3}$ **o** $1\frac{1}{4}$

p $2\frac{1}{3}$

2 **a** yes **b** no **c** yes **d** yes

e no **f** yes

3 **a** $\frac{2}{5} < \frac{3}{4} < \frac{4}{5} < \frac{9}{10} < \frac{11}{10} < \frac{5}{4}$

b $\frac{7}{10} < \frac{11}{15} < \frac{3}{4} < \frac{23}{30} < \frac{4}{5} < \frac{5}{6}$

4 $\frac{21}{63}, \frac{42}{63}, \frac{9}{63}, \frac{18}{63}, \frac{27}{63}, \frac{7}{63}, \frac{14}{63}, \frac{35}{63}, \frac{24}{63}$

5 **a** $\frac{2}{3} < \frac{5}{6} < \frac{7}{8}$ **b** $\frac{2}{3} < \frac{11}{16} < \frac{3}{4}$ **c** $\frac{2}{3} < \frac{31}{45} < \frac{11}{12}$

d $\frac{7}{12} < \frac{5}{8} < \frac{2}{3}$ **e** $\frac{25}{54} < \frac{1}{2} < \frac{5}{9}$ **f** $\frac{31}{48} < \frac{2}{3} < \frac{11}{16}$

Exercise 2B

1 **a** 2 **b** $1\frac{3}{5}$ **c** $\frac{1}{2}$ **d** $2\frac{2}{5}$ **e** $2\frac{1}{3}$ **f** $1\frac{3}{7}$

2 **a** $\frac{1}{2}$ **b** $\frac{1}{6}$ **c** $1\frac{1}{5}$ **d** $\frac{17}{19}$ **e** $\frac{3}{5}$ **f** $\frac{5}{6}$

3 **a** $1\frac{5}{12}$ **b** $1\frac{89}{110}$ **c** $1\frac{2}{15}$ **d** $\frac{19}{90}$ **e** $1\frac{55}{72}$

f $1\frac{11}{35}$ **g** $\frac{23}{24}$ **h** $\frac{73}{56}$ **i** $\frac{11}{10}$

4 **a** $\frac{1}{6}$ **b** $\frac{1}{12}$ **c** $\frac{5}{33}$ **d** $\frac{4}{35}$ **e** $\frac{1}{24}$

f $\frac{1}{72}$ **g** $\frac{1}{90}$ **h** $\frac{3}{35}$ **i** $\frac{1}{12}$

5 **a** $5\frac{13}{15}$ **b** $2\frac{11}{12}$ **c** $4\frac{1}{20}$ **d** $6\frac{1}{6}$ **e** $3\frac{13}{15}$

f $8\frac{39}{40}$ **g** $4\frac{49}{55}$ **h** $9\frac{23}{40}$ **i** $14\frac{17}{40}$

6 **a** $2\frac{1}{4}$ **b** $10\frac{7}{8}$ **c** $13\frac{7}{24}$ **d** $25\frac{7}{8}$ **e** $10\frac{11}{12}$

f $49\frac{27}{110}$

7 $4\frac{5}{8}$ km **8** $2\frac{11}{20}$ km

9 $\frac{11}{12}$ of a litre of water **10** $\frac{7}{8}$

11 $\frac{4}{7}$ **12** $\frac{7}{30}$ **13** $\frac{7}{12}$

14 **a** $\frac{11}{24}$ **b** $\frac{7}{12}$ **c** $\frac{17}{24}$ **d** $\frac{7}{24}$

Exercise 2C

1 **a** $\frac{5}{24}$ **b** $\frac{33}{64}$ **c** 10 **d** $\frac{33}{100}$

e $13\frac{1}{3}$ **f** $13\frac{7}{11}$

2 **a** $\frac{1}{6}$ **b** $\frac{1}{77}$ **c** $\frac{1}{4}$ **d** $\frac{7}{10}$

e $\frac{1}{14}$ **f** $\frac{1}{6}$ **g** $\frac{2}{3}$ **h** $\frac{49}{60}$

i 2 **j** $\frac{2}{5}$ **k** $\frac{7}{13}$ **l** $\frac{2}{5}$

m $\frac{35}{96}$ **n** $\frac{1}{4}$ **o** $\frac{3}{5}$

3 **a** $2\frac{4}{5}$ **b** $8\frac{8}{15}$ **c** $22\frac{7}{50}$ **d** $3\frac{5}{33}$ **e** $2\frac{2}{3}$

f $2\frac{7}{10}$ **g** $\frac{3}{5}$ **h** $2\frac{17}{32}$ **i** $5\frac{5}{9}$

4 847 students

5 **a** 12 L **b** $6\frac{1}{4}$ km **c** 6250 m **d** $37\frac{1}{2}$ m

e 48 kg **f** 480 mm **g** 60 m **h** $87\frac{1}{2}$ km

6 **a** $1\frac{23}{32}$ **b** $2\frac{17}{30}$ **c** $1\frac{1}{6}$ **d** $4\frac{1}{6}$

e $\frac{3}{40}$ **f** $\frac{5}{77}$ **g** $\frac{15}{88}$ **h** $\frac{1}{36}$

i $1\frac{1}{15}$ **j** 24 **k** 25 **l** $2\frac{1}{2}$

7 **a** $5\frac{5}{8}$ **b** $1\frac{11}{15}$ **c** $3\frac{1}{2}$ **d** $8\frac{1}{3}$

e $\frac{8}{15}$ **f** $1\frac{13}{20}$ **g** $\frac{4}{5}$ **h** $\frac{44}{75}$

i $\frac{64}{75}$

8 28 table tops

9 $10\frac{10}{19}$ seconds

10 **a** $\frac{1}{2}$ **b** 3

11 **a** $\frac{3}{7}$ **b** 270

12 360

13 **a** 4500 **b** $\frac{1}{4}$

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14 \$63 **15** 780 ha

16 2025 L **17** 42 km **18** $\frac{87}{40}$ m

Exercise 2D

1 **a** 360 **b** 605 **c** 600

2 **a** \$128 **b** 90 kg **c** \$2205 **d** 2875 m^3

3 **a** \$1448 **b** 968 students

4 28 students

5 **a** 250 litres **b** \$540
c \$175 **d** $2\frac{1}{2}$ hours = 150 minutes

6 96 goals

7 $28\frac{4}{5}$ hectare

8 $\frac{1}{6}$ litre of milk

Exercise 2E

1 **a** $2\frac{1}{10}$ **b** $5\frac{23}{1000}$ **c** $6\frac{71}{100}$ **d** $2\frac{3}{500}$
e $\frac{19}{250}$ **f** $5\frac{17}{25}$ **g** $\frac{17}{2000}$ **h** $2\frac{1}{125}$
i $16\frac{7}{8}$ **j** $23\frac{5}{8}$

2 **a** 51.75 **b** 0.875 **c** 36.12 **d** 112.34
e 87.195

3 **a** $2.35 < 2.435 < 2.5 < 2.5834 < 2.83$
b $18.009\,957\,3 < 18.02 < 18.1 < 18.1002 < 18.21$
c $6.6 < 6.66 < 60.006 < 60.66 < 66.06$
d $47.682 < 55.16 < 55.2 < 55.24 < 56.001$

4 $4.31\left(4\frac{31}{100}\right) < 4.6\left(4\frac{3}{5}\right) < 4.625\left(4\frac{5}{8}\right) < 4.75\left(4\frac{3}{4}\right)$

Exercise 2F

1 **a** 47.57 **b** 129.454 **c** 519.42 **d** 217.28

2 **a** 23 **b** 0.3 **c** 0.26 **d** 0.26
e 0.026 **f** 0.75 **g** 0.561 **h** 26 300
i 250

3 **a** 39.41 **b** 114.335 **c** 3841.2 **d** 3193.61
e 1349.52 **f** 2225.92

4 **a** 0.21 **b** 0.048 **c** 0.063 **d** 0.0042
e 0.000 24 **f** 0.004 **g** 37.8048 **h** 0.000 122

5 **a** 1.188 **b** 1.84 **c** 1.37 **d** 8.54
e 4.85 **f** 8.64

6 **a** 40 **b** 4 000 000 **c** 12.6 **d** 902.1
e 35 **f** 70 000 **g** 80 000 **h** 12
i 5632

7 0.3 million = 300 000, 60 000

8 4.97 m

9 1.5666 million = 1 566 600

10 **a** 1.756 kg **b** 3.073 kg **c** 0.7024 kg
d 1.4926 kg **e** 0.371 394 kg **f** 0.3512 kg

11 64 190 000 kg

12 219 576 335 kg

13 perimeter 87.42 m, area 440.7354 m^2

14 **a** 0.04 **b** 76.93 **c** 4
d 510.92 **e** 28.405 26 **f** 56.5

15 **a** 68 955.351 g **b** 71 039.252 g **c** 93 309 g

Exercise 2G

1 **a** 0.4 **b** 0.8 **c** 0.12 **d** 0.875
e 4.15 **f** 7.375 **g** 45.75 **h** 7.85
i 3.4 **j** 1.12

2 **a** $0.\dot{2}85\,71\dot{4}$ **b** $0.\dot{4}28\,57\dot{1}$ **c** $0.\dot{6}$
d $0.\dot{2}$ **e** $0.\dot{2}\dot{7}$ **f** $0.\dot{4}$
g $0.\dot{6}\dot{3}$ **h** $0.\dot{8}$ **i** $0.\dot{1}53\,84\dot{6}$

3 **a** $0.1\dot{6}$ **b** $0.8\dot{3}$ **c** $0.41\dot{6}$ **d** $0.58\dot{3}$
e $0.91\dot{6}$ **f** $0.\dot{4}$

4 **a** $64.\dot{5}$ **b** $78.8\dot{3}$ **c** $45.\dot{1}\dot{8}$ **d** $0.91\dot{6}$
e $3.\dot{2}85\,71\dot{4}$

5 **a** 463.15 **b** 7.3 **c** 79.50 **d** 0.065
e 8.0 **f** 86

6 **a** 0.29 **b** 0.83 **c** 0.57 **d** 0.58
e 0.44

7 **a** 0.75 **b** 0.7467 **c** 0.746 746 75

Review exercise

1 **a** $\frac{22}{33}, \frac{10}{15}$ **b** $\frac{35}{42}, \frac{20}{24}$ **c** $\frac{18}{21}, \frac{6}{7}$ **d** $\frac{17}{34}, \frac{112}{224}$

2 **a** $\frac{3}{2} > \frac{8}{6} > \frac{5}{4} > \frac{10}{12} > \frac{2}{3} > \frac{2}{4}$
b $3\frac{1}{8} > 3\frac{1}{9} > 2\frac{6}{7} > 2\frac{5}{6} > 2\frac{3}{7} > 2\frac{2}{9}$

3 **a** $2\frac{1}{2}$ **b** $\frac{5}{6}$ **c** $1\frac{1}{4}$ **d** $\frac{3}{40}$

4 **a** $11\frac{1}{4}$ **b** $3\frac{1}{13}$ **c** 11 **d** $7\frac{1}{32}$
e 3 **f** $\frac{5}{56}$ **g** $\frac{5}{9}$ **h** $15\frac{1}{2}$
i $2\frac{21}{32}$

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5 **a** $\frac{3}{4}$ **b** $1\frac{1}{12}$ **c** $\frac{1}{12}$ **d** $3\frac{8}{15}$
e $5\frac{17}{20}$ **f** $5\frac{19}{20}$ **g** $2\frac{1}{8}$ **h** $1\frac{3}{10}$

6 **a** $3\frac{5}{12}$ **b** $4\frac{1}{8}$ **c** 3 **d** $7\frac{3}{10}$
e 14 **f** $3\frac{1}{4}$

7 $\frac{2}{15}$

8 \$4.50 for transport, \$1.80 for telephone calls and \$10.80 on clothes; $\frac{1}{20}$ of his pocket money is saved.

9 $\frac{4}{15}$ **10** 2060 **11** 21

12 **a** 0.0897 **b** 654.079 **c** 3.141 59
d 3.142 **e** 2.718 281 8285 **f** 2.7183

13 **a** 45.6 **b** 0.28 **c** 10 305.2 **d** 0.043
e 2.487 **f** 9.712 **g** 0.000 003 **h** 0.034
i 1008.36

14 **a** 38.025 **b** 34.8495 **c** 460.912 **d** 5.94
e 48.781 **f** 94.43 **g** 203.7 **h** 0.246
i 0.000 690 **j** 200 **k** 1675 **l** $0.1\dot{3}$

15 **a** \$5.63 **b** \$9.80 **c** \$32.63

16 6, \$86.69

17 **a** 9 cents **b** 60 cents **c** 2 cents

18 **a** 15 **b** $\frac{1}{5}, 12$ **c** $\frac{1}{20}$

19 \$8.50 **20** 1 hour **21** 150 cm

22 **a** $26\frac{2}{3}$ kg **b** $1386\frac{2}{3}$ kg

23 **a** $\frac{1}{8}$ **b** $\frac{3}{11}$ **c** $\frac{5}{18}$ **d** $\frac{6}{11}$
e $\frac{7}{23}$ **f** $\frac{49}{69}$ **g** 1 **h** 1

Challenge exercise

1 15 **2** 3.36 metres **3** $\frac{1}{8}$

4 **a i** $\frac{1}{6}$ **ii** $\frac{1}{20}$ **iii** $\frac{1}{56}$ **iv** $\frac{1}{90}$
b When subtracting consecutive unit fractions, the answer is a unit fraction whose denominator is the product of the denominators of the two fractions.
c $\frac{1}{3} - \frac{1}{4}$

5 **a** $\frac{5}{8}$ **b** $\frac{1}{4}$ **c** $\frac{1}{2}$ **d** $\frac{1}{2}$
e $\frac{1}{2}$ **f** $\frac{1}{4}$ **g** $1\frac{4}{5}$ **h** $\frac{5}{8}$

6 **e** $0.\dot{0}3\dot{7}$, $0.\dot{3}7\dot{0}$, $0.\dot{7}0\dot{3}$. Then $0.\dot{0}7\dot{4}$, $0.\dot{1}$, $0.\dot{1}4\dot{8}$.

7 **b** $0.\dot{0}09\dot{9}$
c $0.\dot{0}99\dot{0}$, $0.\dot{9}90\dot{0}$, $9.\dot{9}00\dot{9}$ Hence $\frac{99}{100} = 0.\dot{9}00\dot{9}$
d $0.\dot{0}19\dot{8}$, $0.\dot{0}29\dot{7}$, $0.\dot{0}39\dot{6}$

8 210 seconds

9 **a** $\frac{1}{5} + \frac{1}{10}$ **b** $\frac{1}{2} + \frac{1}{4} + \frac{1}{5}$
c $\frac{1}{5} + \frac{1}{10} + \frac{1}{15} + \frac{1}{30}$

Chapter 3

Exercise 3A

1 **a** factors of 8: 1, 8, 2, 4; factors of 12: 1, 12, 2, 6, 3, 4; HCF: 4
b factors of 6: 1, 6, 2, 3; factors of 20: 1, 20, 2, 10, 4, 5; HCF: 2
c factors of 14: 1, 14, 2, 7; factors of 9: 1, 9, 3; HCF: 1
d factors of 20: 1, 20, 2, 10, 4, 5; factors of 22: 1, 22, 2, 11; HCF: 2
e factors of 1: 1; factors of 8: 1, 8, 2, 4; HCF: 1
f factors of 5: 1, 5; factors of 15: 1, 15, 3, 5; HCF: 5

2 **a** 6 **b** 3 **c** 12 **d** 1
e 4 **f** 13 **g** 6 **h** 1
i 10

3 **a** multiples of 4: 4, 8, 12, 16, 20; multiples of 6: 6, 12, 18, 24, 30; LCM: 12
b multiples of 10: 10, 20, 30, 40, 50, 60; multiples of 12: 12, 24, 36, 48, 60; LCM: 60
c multiples of 3: 3, 6, 9, 12, 15, 18, 21; multiples of 7: 7, 14, 21; LCM: 21
d multiples of 5: 5, 10, 15, 20; multiples of 10: 10, 20, 30, 40; LCM: 10
e multiples of 1: 1, 2, 3, 4, 5; multiples of 5: 5, 10, 15, 20, 25; LCM: 5
f multiples of 6: 6, 12, 18, 24, 30; multiples of 15: 15, 30, 45, 60: LCM: 30

4 **a** 24 **b** 72 **c** 17 **d** 60
e 49 **f** 180 **g** 12 **h** 72
i 120

5 **a i** 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30
ii 3, 6, 9, 12, 15, 18, 21, 24, 27, 30
iii 5, 10, 15, 20, 25, 30
b 7, 11, 13, 17, 19, 23, 29
c 30, 32, 33, 34, 35, 36, 38, 39, 40, 42, 44, 45, 46, 48, 49, 50

6 **a** 2 **b** 2, 3 **c** 3, 5 **d** 2, 3, 5
e 2, 11 **f** 2, 19 **g** 5

7 **a** $10 = 2 \times 5$, $14 = 2 \times 7$
b $30 = 2 \times 3 \times 5$, $42 = 2 \times 3 \times 7$
c $8 = 2^3$, $9 = 3^2$

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8 **a** 1^2+2^2 **b** 2^2+3^2 **c** 1^2+4^2 **d** 2^2+5^2
e 1^2+6^2 **f** 4^2+5^2

9 12 m **10** 20 days **11** 12:24 p.m.

12 7, 11, 13

13 $140=3+137, 142=3+139, 144=5+139, 146=7+139,$
$148=11+137, 150=11+139, 152=13+139,$
$154=17+137, 156=17+139, 158=19+139, 160=11+149$
(*Note*: There may be other possibilities.)

14 6, 10, 14, 15, 21, 35 (*Note*: There may be other possibilities.)

15 The factor that is squared to produce a square appears only once in the list of factors; all other factors appear in pairs.

16 121, 961

Exercise 3B

1 power; base; index *or* exponent

2 **a** 16 **b** 81 **c** 144 **d** 10 000
e 4 000 000 **f** 625

3 **a** $7^2, 7^3, 7^4$ **b** $2^7, 3^7, 4^7$
c $49=7^2, 144=12^2, 8=2^3$
(*Note*: There may be other possibilities.)
d **i** 64^1 **ii** 8^2 **iii** 4^3
iv 2^6

4 **a** 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024
b 5, 25, 125, 625

5 **a** 2^5 **b** $3^4\times4^2$ **c** 3^1 or 3

6 **a** 7^5 **b** 11^6 **c** 11^6 **d** 3^9
e 13^7 **f** 10^5

7 **a** $7^1=7$ **b** 11^3 **c** 5^3 **d** 5^5
e 7^9 **f** 5^5

8 **a** 2^6 **b** 3^{10} **c** 2^{10} **d** 11^6
e 5^6 **f** 7^8

9 **a** 3^{13} **b** 8^7 **c** 5^8 **d** 10^{45}
e 6^4 **f** 2^8 **g** 7^9 **h** 4^2
i 12^{12} **j** 11^{63} **k** 4^{16} **l** 10^4

10 **a** $10^4=10\,000$ **b** $20^3=8000$ **c** $10^3\times2^2=4000$

11 **a** 1 **b** 1 **c** 2 **d** 0 **e** 7
f 36 **g** 7 **h** 1 **i** 1

Exercise 3C

1 **a** 48 **b** 26 **c** 24 **d** 4 **e** 5832
f 72 **g** 100 **h** 5 **i** 10 000 **j** 10
k 10000 **l** 190

2 **a** 35 **b** 11 **c** 24 **d** 48 **e** 20
f 4 **g** 50 **h** 80 **i** 2 **j** 8
k 45 **l** 5

3 **a** 56 **b** 137 **c** 40 **d** 13 **e** 57
f 32 **g** 574 **h** 1384 **i** 24

4 **a** $4\times(3+7)=40$ **b** $(70-20)\div5=10$
c $(2^2+6\times4)\div2^2=7$ **d** $4+3^2-2=11$
e $(3+2^2)\times2^3-7^2=7$ **f** $(4+4^4)\div(2^3+2)=26$

5 **a** $4\times(2+7)=36$ **b** $4\times2+7=15$
c $20-(2+2)-(2+2)=12$ **d** $20-(2+2-2+2)=16$
e $(70-20)\div5=10$ **f** $70-(20\div5)=66$
g $(2^2+2)\times8\div2^2=12$ **h** $2^2+2\times(8\div2^2)=8$
i $(2^2+2\times8\div2)^2=144$ **j** $(11-2+3)^2=144$
k $(11-2)+3^2=18$ **l** $11-(2+3^2)=0$

Exercise 3D

1 **a** 2896 is divisible by 2, 4 and 8.
b 56 374 is divisible by 2.
c 1 858 732 is divisible by 2 and 4.
d 280 082 is divisible by 2.

2 **a** 5679 is divisible by 3 and 9.
b 7425 is divisible by 3, 9 and 11.
c 71 643 is divisible by 3 and 11.
d 1727 is divisible by 11.

3 **a** 3798 is divisible by 6.
b 5772 is divisible by 6 and 12.
c 9909 is divisible by neither 6 nor 12.
d 48 882 is divisible by 6.

4 **a** 672 is divisible by 2, 3, 4, 6, 8 and 12.
b 49 395 is divisible by 3 and 5.
c 136 290 is divisible by 2, 3, 5, 6, 10 and 11.
d 242 010 000 437 000 361 is divisible by 3.

5 **a** 301 032 **b** 301 030 **c** 321 030
d 301 032 **e** 321 330 **f** 391 930
g 391 732 **h** 321 432 **i** 321 231
(*Note*: There are many other possibilities.)

6 **a** 3 **b** 6 **c** 10 **d** 2 and 4

7 **a** 36 450 is divisible by 15 and 18.
b 21 942 is divisible by 18.
c 2 041 200 is divisible by 12, 15 and 18.
d 2 007 000 000 is divisible by 12, 15 and 18.

8 2232 **9** 12 222

Exercise 3E

1 **a** 2^3 **b** $2^3\times3$ **c** $3^2\times5$ **d** $3^4\times11$
e $3\times5\times7^2$ **f** $2^3\times3\times11^2$

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2 **a** 432, 36 **b** 1620, 27 **c** 6480, 9

3 **a** 9 **b** 180 **c** 7 **d** 336 **e** 120
f 216 **g** 18 **h** 4 **i** 35

4 **a** 15 (cube root)
b 27 (square root), 9 (cube root)
c 66 (square root)

5 **a** $3^2 \times 5^2$, 15 (square root)
b $2^2 \times 3^4$, 18 (square root)
c 2^9, 8 (cube root)
d $2^6 \times 3^3$, 12 (cube root)
e $2^3 \times 3^2 \times 5$
f $2^4 \times 11^2$, 44 (square root)
g $2^3 \times 3^3$, 6 (cube root)
h $2^3 \times 11^3$, 22 (cube root)

6 30

7 $360 = 5 \times 3^2 \times 2^3$, 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, 360

Review exercise

1 C 2 D

3 **a** $8+88$ **b** $32+8$ **c** $176+8$
d $200+280$ **e** $8+288$
(*Note*: There are other possibilities.)

4 **a** 24 **b** 90 **c** 48 **d** 360 **e** 30
f 180 **g** 540 **h** 1020 **i** 600

5 **a** 3 **b** 15 **c** 1 **d** 10 **e** 7
f 4 **g** 6 **h** 6 **i** 21

6 **a** 36 **b** 121 **c** 2500 **d** 961
e 7569 **f** 20 449

7 **a** 8^6 **b** 2^4 **c** 3^{15} **d** 4^{28} **e** 10^{12}
f 7^{17} **g** 5^6 **h** 8^4 **i** 12^{14}

8 **a** B **b** C

9 **a** 36 seconds **b** 9 times

10 $28 = 2 \times 2 \times 7$; 29 is prime; $30 = 2 \times 3 \times 5$; 31 is prime; $32 = 2 \times 2 \times 2 \times 2 \times 2$; $33 = 3 \times 11$; $34 = 2 \times 17$; $35 = 5 \times 7$; $36 = 2 \times 2 \times 3 \times 3$; 37 is prime

11 **a** 2^8, 16 **b** $3^2 \times 7^2$, 21 **c** $2^2 \times 17^2$, 34
d $3^2 \times 29^2$, 87 **e** $5^2 \times 3^6$, 135 **f** $2^8 \times 3^4$, 144

12 **a** $3^2 + 8^2$ **b** $4^2 + 11^2$ **c** $2^2 + 15^2$

13 **a** 64 is divisible by 4.
b 1336 is divisible by 4.
c 3972 is divisible by both 3 and 4, and so also by 12.

14 **a** 21 212 **b** 22 210 **c** 10 800 **d** 11 132
e 24 000 **f** 10 010
(*Note*: There are many other possibilities!)

15 **a** 103 **b** 6553 **c** 39 529 **d** 53 353
e 616 137 **f** 1

16 10:24 a.m.

17 **a** 4 **b** 4 **c** 4 **d** 4

18 **a** 6 **b** 6 **c** 6 **d** 6

19 **a** 1, 3, 37, 111
b 1, 7, 11, 13, 77, 91, 143, 1001
c 1, 11, 101, 1111

20 **a** 324 **b** 120

Challenge exercise

1 2340

2 328 020; there are 16 such numbers.

3 **a i** 187 979 **ii** 24 453
iii 18 204 978 **iv** 183 704 778
b The long multiplication algorithm makes the process clear.

$$\begin{array}{rccccc} & & 5 & 3 & 8 & 9 \\ \times & & & & 1 & 1 \\ \hline & & 5 & 3 & 8 & 9 \\ & 5 & 3_1 & 8_1 & 9 & 0 \\ \hline & 5 & 9 & 2 & 7 & 9 \\ \hline \end{array}$$

4 301 or 721

5 19 employees, each works 7 hours

6 34 and 35

7 24 000 000 000 hours later it will be 7 p.m., so 4 hours before that it will be 3 p.m.

8 25 201 9 31 10 128 and 78 125

11 **a** $2^9 \times 3^5 \times 5^1 \times 7^3 \times 13^1 \times 17^2 \times 19^1$
b 11 students. For ages that are not prime numbers, we use the factors and multiply. Ensure you have used each prime factor only as often as it occurs in the prime factorisation.

Age	Prime factors	Number of students
13	13	1
14	$(2\times7)(2\times7)(2\times7)$	3
15	(3×5)	1
16	$(2\times2\times2\times2)$	1
17	$(17)(17)$	2
18	$(2\times3\times3)(2\times3\times3)$	2
19	19	1

12 360 13 **a** 501 **b** 5541

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14 **a** $m = 6, n = 4$
b $a = 1, b = 2, c = 4, d = 5, e = 3$
c $a = 3, b = 2, c = 4, d = 5, e = 6, f = 1$

15 66 or 318 or 402
16 $729 = 3^6$

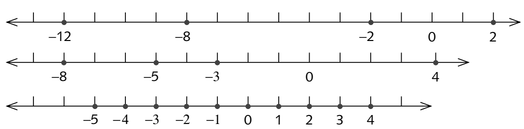
17 15
18 1111 011 111 000

Chapter 4

Exercise 4A

1 **a** −21, −20, −19, −18, −17
b −113, −112, −111, −110, −109, −108, −107, −106
c −7, −6, −5, −4, −3, −2, −1, 0, 1, 2

2 **a** **b** **c**



3 −7, −4, −1
4 3, −1, −5
5 −29, −22, −15, −8

6 **a** 0°C **b** −25°C **c** −5°C **d** −15°C

7 **a** −6 **b** 3 **c** −34 **d** 5
e 72 **f** −67 **g** 456 **h** −10 000

Exercise 4B

1 **a** −1 **b** 4 **c** −4 **d** 4
e 5 **f** 4 **g** 9 **h** 7
i −8 **j** 0 **k** −7 **l** −9
m 0 **n** −37 **o** 39 **p** −6
q −63 **r** −90 **s** 255 **t** 66

2 **a** −11 **b** −2 **c** −8 **d** −8
e −5 **f** −10 **g** −6 **h** −3
i −7 **j** −2 **k** −16 **l** −38
m −27 **n** −19 **o** −6 **p** −93
q −5 **r** −96 **s** −36 **t** −9
u −128 **v** −280

3 **a** −400 **b** −420 **c** −170 **d** 115
e −465 **f** 200 **g** −1200 **h** −1000
i −4600 **j** −2000 **k** −500 **l** −7500

4 **a** 6 **b** 14 **c** 0 **d** 10
e 0 **f** 7 **g** 2 **h** −28

Exercise 4C

1 **a** 5 **b** −5 **c** 7 **d** −18
e 4 **f** −10 **g** −9 **h** −17
i 7 **j** −8 **k** −17 **l** −16

2 **a** −3 **b** −6 **c** −11 **d** 16
e −20 **f** −2 **g** −2 **h** 17
i −16 **j** −9 **k** −8 **l** 41

3 **a** −1 **b** −2 **c** −3 **d** −3
e −5 **f** 9 **g** −3 **h** −4
i −9

4 **a** −40 **b** −18 **c** 13 **d** −2
e 4 **f** 13 **g** −29 **h** 78
i 20 **j** 28 **k** −20 **l** 60

5 **a** 356 **b** −604 **c** 132 **d** −6105

6 16°C
7 An increase of 11°C

8

Minimum	Maximum	Increase
−11°C	6°C	17°C
−16°C	7°C	23°C
−35°C	−4°C	31°C
−25°C	−15°C	10°C
−7°C	−2°C	5°C
−13°C	2°C	15°C
−11°C	5°C	16°C

9 −10°C

Exercise 4D

1 **a** −12 **b** −14 **c** −70 **d** −48
e −216 **f** −15 **g** −24 **h** −60
i −77 **j** −60 **k** 140 **l** −64
m 140 **n** 196 **o** −171 **p** −100

2 **a** −6 **b** −14 **c** −7 **d** −9
e −30 **f** −6 **g** −18 **h** −5
i −7 **j** −40 **k** 13 **l** 9
m 4 **n** 60 **o** −28 **p** −35

3 **a** −4 **b** 6 **c** −4 **d** −3

4 **a** 5 **b** −5 **c** −6 **d** 15
e −34 **f** −122 **g** −16 **h** −99
i −7 **j** −30 **k** 4 **l** −1441

5 **a** 84 **b** 280 **c** 2160 **d** 3600
e −160 **f** −16

6 **a** $2 \times (-25) = -50$ **b** $5 \times (-15) = -75$
c $-7 \times (-9) = 63$ **d** $-9 \times (-8) = 72$
e $80 \div (-8) = -10$ **f** $-45 \div (-9) = 5$
g $-321 \div (-107) = 3$ **h** $5664 \div (-8) = -708$

Exercise 4E

1 **a** 81 **b** −49 **c** 75 **d** −144
e −6400 **f** 169 **g** −216 **h** 81
i −3125 **j** 117 649 **k** −1

2 **a** 9 **b** 35 **c** −14 **d** 21
e −18 **f** 175

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3 **a** 24 **b** –22 **c** –20 **d** –54
e –45 **f** 30 **g** –87 **h** –560

4 **a** 24 **b** 27 **c** 7
d –59 **e** –20 **f** 33
g 120 **h** –162 **i** 136

5 **a** 90 **b** 14 **c** –35 **d** –48
e –56 **f** –349 **g** –51 **h** –188

6 **a** –1 **b** –36 **c** 18 **d** –16

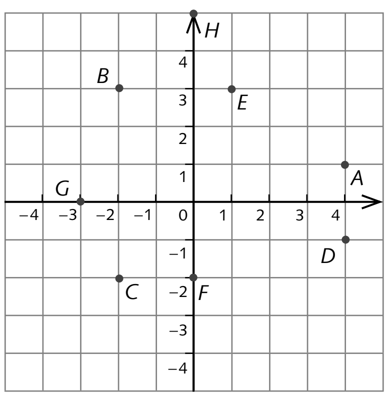
7 **a** 370 **b** –400 000 **c** –2900 **d** 4000

8 –$1200

9 **a** $24 000 **b** –$16 000

Exercise 4F

1

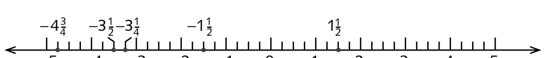


2 **a** $A(1, 1)$, $B(2,–3)$, $C(0, 6)$, $D(–4, 0)$, $E(–4,–2)$, $F(–4, 5)$, $G(4,–4)$
b **i** 3
ii 3

3 **a** square, 9 cm^2
b triangle, 18 cm^2
c trapezium, 71.5 cm^2
d right-angled triangle, 7.5 cm^2
e AO is perpendicular to BC.
f parallelogram, 8 cm^2

Exercise 4G

1



2 **a** $-2<-\frac{7}{5}<-1<\frac{1}{4}<\frac{1}{2}<1<\frac{-5}{-3}$
b $-\frac{11}{5}<-\frac{7}{4}<\frac{14}{5}<\frac{7}{2}<\frac{-11}{-3}$
c $-\frac{11}{12}<-\frac{4}{5}<-\frac{2}{3}<\frac{12}{13}$
d $-2<-1\frac{11}{13}<-\frac{15}{13}<-1$

3 **a** $2\frac{1}{2}$ **b** $2\frac{1}{2}$ **c** –1 **d** $-\frac{1}{3}$
e $-1\frac{1}{2}$ **f** $-\frac{1}{15}$ **g** $-\frac{1}{2}$ **h** $-\frac{3}{5}$
i $2\frac{1}{4}$ **j** $-3\frac{3}{5}$ **k** $-2\frac{5}{18}$ **l** $-2\frac{1}{14}$
m $-\frac{1}{2}$ **n** $1\frac{1}{2}$ **o** $-\frac{11}{15}$ **p** $-\frac{5}{12}$
q $2\frac{17}{36}$ **r** $-3\frac{31}{35}$ **s** $-4\frac{9}{91}$ **t** $\frac{37}{56}$

4 **a** $-1367\frac{1}{2}$ **b** $-2107\frac{1}{3}$ **c** $789\frac{1}{7}$

5 **a** $-\frac{3}{4}$ **b** $\frac{5}{24}$ **c** $\frac{55}{108}$ **d** $-\frac{11}{12}$
e $-\frac{5}{6}$ **f** $-1\frac{1}{2}$ **g** $-\frac{5}{9}$ **h** $-\frac{7}{12}$

6 **a** –6 **b** $-1\frac{1}{8}$ **c** $-\frac{3}{5}$ **d** $-\frac{9}{11}$
e $1\frac{1}{2}$ **f** $-\frac{9}{10}$ **g** 4 **h** $\frac{1}{14}$

7 **a** $-\frac{19}{30}$ **b** $-3\frac{23}{72}$ **c** $\frac{7}{8}$
d $-1\frac{7}{12}$ **e** $-\frac{1}{4}$ **f** $-2\frac{1}{4}$
g $5\frac{1}{2}$ **h** 0 **i** –5

8 **a** $2\frac{1}{2}$ **b** 0 **c** 2
d $-6\frac{1}{2}$ **e** 0 **f** $-1\frac{9}{20}$

9 **a** $\frac{1}{4}$ **b** $-\frac{1}{8}$ **c** $\frac{1}{16}$ **d** $-\frac{1}{32}$
e $\frac{1}{2}$ **f** –2 **g** –3 **h** $\frac{8}{27}$

Review exercise

1 **a** 23 **b** –14 **c** 15 **d** –95
e –16 **f** –68 **g** –180 **h** –60
i 70 **j** –80 **k** –160 **l** 85

2 –10

3 **a** 25°C **b** –25°C

4 **a** 27°C **b** –27°C

5 **a** –250 **b** –396 **c** –1750 **d** 2040
e –48 **f** 1150 **g** 3200 **h** 500
i –440 **j** –2400 **k** 128 **l** –200

6 **a** –25 **b** –4 **c** –7 **d** 17
e 4 **f** 5 **g** 8 **h** 150
i –3 **j** –6 **k** 8 **l** –104

7 **a** 4 **b** –63 **c** –60
d 60 **e** –168 **f** –16
g 16 **h** 49 **i** –90

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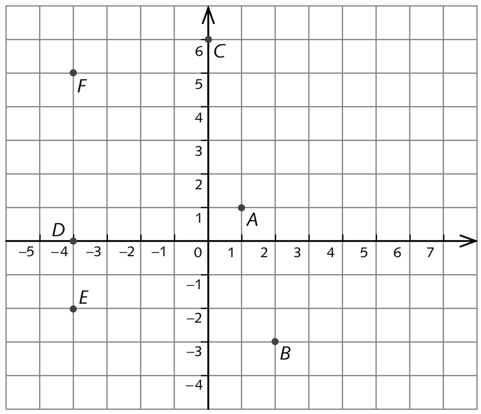
8 −56 **9** −6

10



11 $A(4,4)$, $B(-4,3)$, $C(-4,3)$, $D(3,3)$, $E(1,1)$, $F(0,-4)$, $G(-2,0)$

12 **a** $2\frac{1}{2}$ **b** $4\frac{1}{2}$ **c** $-\frac{10}{13}$

d $-\frac{1}{4}$ **e** $-1\frac{1}{4}$ **f** $5\frac{6}{11}$

13 **a** $-\frac{1}{6}$ **b** −30 **c** $-1\frac{1}{7}$

d 1 **e** $-\frac{3}{10}$ **f** $\frac{1}{3}$

g $-\frac{1}{10}$ **h** $-\frac{5}{24}$ **i** $\frac{1}{36}$

14 644 V

15 **a** 1 **b** −1 **c** 1 **d** −1 **e** 1

Clearly, −1 to an even power is 1, whereas −1 to an odd power is −1.

16 220 m

17 **a** $12\frac{1}{2}$ **b** −3 **c** −500 **d** 0

18 **a** $\frac{5}{8}$ **b** $\frac{7}{8}$ **c** $-\frac{1}{8}$

19 **a** $2\frac{1}{4}$ **b** $-4\frac{3}{4}$ **c** $4\frac{3}{4}$ **d** $-4\frac{3}{8}$

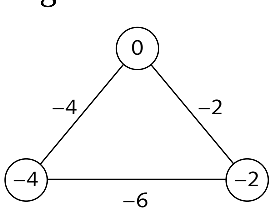
e $-\frac{5}{14}$ **f** $-2\frac{4}{5}$ **g** $\frac{3}{14}$ **h** $1\frac{5}{14}$

20 **a** 11 **b** −11 **c** 10 **d** $\frac{1}{8}$

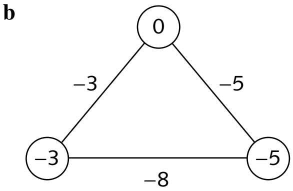
21 **a** 2 **b** −2 **c** $-\frac{2}{3}$ **d** $\frac{4}{9}$

Challenge exercise

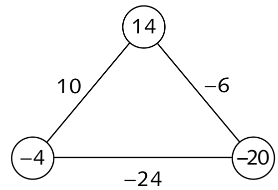
1 **a**



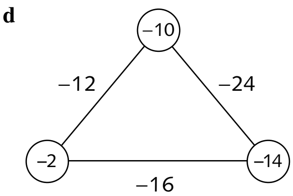
b



c



d



2 **a** $4+(-5)-(-2)=1$ **b** $4+(-2)-(-5)=7$

c $-2+(-5)-(4)=-11$

3 **a** $-3+3-(-4)=4$ **b** $-3+(-4)-3=-10$

c $-4+3-(-3)=2$

4 −2

5 **a** $(2+-3)\times(3+4)=-7$

b $2+(-3\times 3)+(4\times 2)=1$

c $((2-5)\times 6+7)\times 6-5=-71$

6

8	−6	4
−2	2	6
0	10	−4

7 **a** **i** −3 **ii** −3 **b** −50

8 100 **9** −6 **10** **a** −10 **b** −50

11 −21

12 **a** $\frac{1}{3}$ **b** $\frac{1}{10}$ **c** $-\frac{1}{10}$ **d** $-\frac{4}{45}$

Chapter 5

Note: In the answers for this chapter, reasons have only been given for selected questions.

Note that in geometry there are often two or more different valid arguments leading to the same conclusion.

Exercise 5A

1 **a** 152° (straight angle $\angle COB$)

b 65° (adjacent angles at O)

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c 136° (adjacent angles at O)
d 28° (straight angle $\angle XOB$)
e 180° (straight angle $\angle AOB$)
f 125° (adjacent angles at O)
g 246° (revolution at O)
h 130° (revolution at O)
i 55° (straight angle $\angle BOJ$)
j 188° (revolution at O)
k 30° (revolution at O)
l 60° (revolution at O)

2 **a** $\angle SQT$ **b** $\angle VQU$ **c** $\angle AQB$ **d** $\angle NQO$

3 **a** $\alpha = 56°$ (vertically opposite angles at B)
b $\beta = 90°$ (vertically opposite angles at K), $\theta = 90°$ (straight angle)
c $\alpha = 48°, \beta = 132°$
d $\theta = 72°$
e $\alpha = 55°, \beta = 90°, \gamma = 35°, \theta = 55°$
f $\alpha = \beta = \gamma = \theta = 60°$
g $\alpha = 65°, \beta = 115°, \gamma = 20°, \theta = 125°$
h $\alpha = 50°, \beta = 55°$

4 **a** $\theta = 17°$ (vertically opposite angles at V)
b $\beta = 27\frac{1}{2}°$ (straight angle $\angle JKL$)
c $\alpha = 60°$
d $\alpha = 72°$
e $\beta = 135°$
f $\gamma = 33\frac{3}{4}°, \theta = 135°$
g $\alpha = 92\frac{1}{2}°$
h $\alpha = 18°$

Exercise 5B

1 **a** corresponding **b** alternate
c co-interior **d** alternate
e co-interior **f** corresponding
g co-interior **h** alternate
i corresponding **j** co-interior
k corresponding **l** alternate

2 **a** $\theta = 126°$ (co-interior angles, $AB \parallel CD$)
b $\theta = 136°$ (alternate angles, $PQ \parallel RS$)
c $\theta = 66°$ (corresponding angles, $AB \parallel CD$)
d $\gamma = 69°$ (corresponding angles, $BK \parallel CL$)
e $\alpha = 34°$ (alternate angles, $RS \parallel TU$)
f $\beta = 66°$ (co-interior angles, $LM \parallel ON$)
g $\theta = 57°$ (co-interior angles, $DG \parallel EF$)
h $\alpha = 28°$ (alternate angles, $FG \parallel LM$)
i $\alpha = 43°$ (corresponding angles, $BX \parallel CY$)
j $\gamma = 133°$ (co-interior angles, $AX \parallel BY$)
k $\beta = 77°$ (alternate angles, $CA \parallel BD$)
l $\beta = 90°$ (corresponding angles, $RU \parallel ST$)

3 **a** $\beta = 107°$ (co-interior angles, $FI \parallel GH$), $\gamma = 107°$ (corresponding angles, $FG \parallel IH$)
b $\alpha = \beta = 64°, \gamma = 116°$ (various arguments)
c $\alpha = \beta = \gamma = \theta = 112°$ (various arguments)
d $\alpha = 119°$ (co-interior angles, $PQ \parallel SR$), $\beta = 61°$ (co-interior angles, $PS \parallel QR$), $\gamma = 119°$
e $\theta = 90°$ (co-interior angles, $MN \parallel PO$), $\gamma = 108°$ (co-interior angles, $MN \parallel PO$)
f $\gamma = 37°$ (alternate angles, $BC \parallel ML$), $\theta = 78°$ (alternate angles, $BC \parallel ML$)
g $\alpha = \beta = \gamma = 133°$ (various arguments)
h $\alpha = 74°$ (corresponding angles, $AC \parallel BX$), $\beta = 35°$ (alternate angles, $AC \parallel BX$)

4 **a** $\angle PBV = 81°$ (alternate angles, $PQ \parallel VB$), $\angle AVB = 81°$ (alternate angles, $AV \parallel BP$)
b $\angle WAV = 123°$ (co-interior angles, $WB \parallel AV$), $\angle AVB = 57°$ (co-interior angles, $AW \parallel BV$)
c $\angle RAV = 45°$ (alternate angles, $RS \parallel VA$), $\angle AVB = 135°$ (co-interior angles, $RA \parallel BV$)
d $\angle QAV = 28°$ (alternate angles, $PQ \parallel AV$), $\angle AVB = 28°$ (alternate angles, $AQ \parallel BC$)

Exercise 5C

1 **a** $\alpha = 73°$ (straight angle $\angle ABC$), $\theta = 107°$ (alternate angles, $AC \parallel DE$)
b $\theta = 100°$ (corresponding angles, $AX \parallel BY$), $\beta = 80°$ (straight angle $\angle MAB$)
c $\alpha = 35°$ (vertically opposite angles at Q), $\beta = 35°$ (corresponding angles, $AB \parallel CD$), $\gamma = 145°$ (straight angle $\angle CRD$)
d $\alpha = \beta = \gamma = 55°, \theta = 125°$
e $\alpha = 146°$ (co-interior angles, $AB \parallel CD$), $\theta = 124°$ (revolution at D)
f $\alpha = 66°, \beta = 26°, \gamma = 88°$
g $\alpha = 73°, \beta = 34°, \gamma = 73°$
h $\alpha = 90°, \beta = 55°, \gamma = 35°$

2 **a** $\angle PON = 125°$ (co-interior angles, $PA \parallel ON$), $\angle QON = 135°$ (co-interior angles, $QB \parallel ON$), $\angle POQ = 100°$ (revolution at O)
b 116° **c** 133° **d** 63°

3 **a** 100° **b** 54° **c** 130° **d** 88°
e 76° **f** 42° **g** 88° **h** 40°

4 **a** $\gamma = 46°$ **b** $\theta = 64°$ **c** $\alpha = 90°$
d $\theta = 18°$ **e** $\beta = 30°$ **f** $\beta = 66°$
g $\gamma = 90°, \theta = 62°$ **h** $\alpha = 18°$

Exercise 5D

1 **a** $HJ \parallel KM$ (co-interior angles are supplementary)
b $AB \parallel CZ$ (alternate angles are equal)
c $FP \parallel GQ$ (corresponding angles are equal)

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d $AD \parallel EH$ (corresponding angles are equal)

e $RS \parallel UT$ (co-interior angles are supplementary), $RU \parallel ST$ (co-interior angles are supplementary)

f $DE \parallel GF$ (alternate angles are equal), $DG \parallel EF$ (alternate angles are equal)

g $DF \parallel AC$ (corresponding angles are equal), $AV \parallel BF$ (alternate angles are equal)

h $KL \parallel NM$ (co-interior angles are supplementary)

2 a $AB \parallel CD$ (corresponding angles are equal), $\beta = 54°$ (corresponding angles, $AB \parallel CD$)

b $AB \parallel CD$ (alternate angles are equal), $\alpha = 100°$ (alternate angles, $AB \parallel CD$)

c $AB \parallel CD$ (alternate angles are equal), $\gamma = 28°$ (alternate angles, $AB \parallel CD$)

d $AB \parallel CD$ (co-interior angles are supplementary), $\theta = 109°$ (co-interior angles, $AB \parallel CD$)

3 a $\alpha = 66°$ b $\theta = 53°$

c $\gamma = 13°$ d $\beta = 37°$

Exercise 5E

1 a 90° b 30° c 58° d 28° e 94°

2 a $\theta = 38°$ (angle sum of ΔABJ)

b $\gamma = 57°$ (angle sum of ΔCTU)

c $\alpha = 65°$ (angle sum of ΔABC)

d $\beta = 136°$ (angle sum of ΔIBT)

e $\angle BIR = 110°$ (vertically opposite angles at I), $\beta = 27°$ (angle sum of ΔBIR)

f $\angle POQ = 69°$ (angle sum of ΔOPQ), $\alpha = 69°$ (vertically opposite angles at O)

g $\theta = 90°$

h $\gamma = 60°$

3 a $\alpha = 120°$ (exterior angle of ΔABC)

b $\gamma = 25°$ (exterior angle of ΔEGL)

c $\beta = 45°$ (exterior angle of ΔETX)

d $\angle AOT = 58°$ (exterior angle of ΔAOT), $\alpha = 58°$ (vertically opposite angles at O)

e $\theta = 23°$ (exterior angle of ΔADE)

f $\angle LKC = 94°$ (straight angle $\angle JKL$), $\theta = 118°$ (exterior angle of ΔCKL)

g $\beta = 159°$

h $\gamma = 115°$

4 a $\alpha = 45°$ (straight angle $\angle LMN$), $\beta = 135°$ (exterior angle of ΔMNT)

b $\theta = 108°, \gamma = 107°$

c $\alpha = 75°, \beta = 51°$

d $\theta = 95°$

e $\gamma = 100°$

f $\alpha = 120°, \theta = 84°$

g $\alpha = 98°, \beta = 115°$

h $\alpha = 81°, \beta = 116°$

5 a $\alpha = 15°$ b $\alpha = 30°$ c $\beta = 26\frac{1}{2}°$

d $\alpha = 40°$ e $\theta = 62°$ f $\theta = 18°$

g $\beta = 67°$ h $\gamma = 14°$

6 a $\alpha = 58°$ (alternate angles, $AR \parallel CD$), $\beta = 28°$ (alternate angles, $CA \parallel DR$), $\gamma = 94°$ (angle sum of ΔADR)

b $\alpha = 50°$ (co-interior angles, $FI \parallel GH$), $\beta = 50°$ (alternate angles, $FI \parallel GH$), $\gamma = 10°$ (angle sum of ΔFIH)

c $\alpha = 75°, \theta = 30°$ d $\alpha = 55°, \beta = \gamma = 60°$

e $\beta = \theta = 55°$ f $\alpha = \beta = 58°, \gamma = 42°$

g $\alpha = 36°, \beta = 54°, \gamma = 90°$ h $\alpha = 152°, \beta = 68°, \gamma = 84°$

7 a $\angle APQ = 97°$ (angle sum of ΔAPQ), $\angle AVB = 97°$ (corresponding angles, $PQ \parallel VB$)

b 25° c 107° d 70° e 36°

f 119° g 8° h 72°

Exercise 5F

1 a $\alpha = 65°$ (base angles of isosceles ΔABC) $\beta = 50°$ (angle sum of ΔABC)

b $\angle A = \gamma$ (base angles of isosceles ΔABC), $\gamma = 45°$ (angle sum of ΔABC)

c $\alpha = \beta$ (base angles of isosceles ΔFGH), $\alpha = \beta = 66°$ (exterior angle of ΔFGH)

d $\alpha = 36°$ (angle sum of isosceles ΔPQR)

e $\alpha = \beta = 60°$ (equilateral ΔFGH)

f $\alpha = 42°$ (angle sum of ΔABM), $\angle C = 48°$ (base angles of isosceles ΔABC), $\beta = 42°$ (angle sum of ΔACM)

2 a isosceles: $AB = AC$ (opposite angles of ΔABC are equal)

b not isosceles: $\angle A = 52°$ (angles are all different)

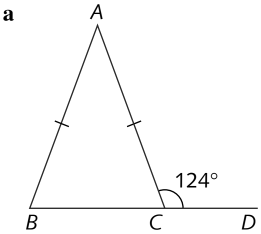
c isosceles: $\angle C = 64°$ (angle sum of ΔABC), $AB = AC$ (opposite angles of ΔABC are equal)

d isosceles: $\angle A = 36°$ (exterior angle of ΔABC), $AC = BC$ (opposite angles of ΔABC are equal)

e not isosceles: $\angle C = 76°$ and $\angle BAC = 38°$

f not isosceles

3 a



$\angle ABC = 56°$ (base angles of isosceles ΔABC)

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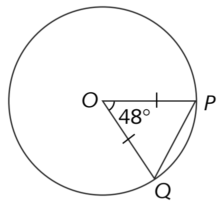
b



$\angle OPQ = 66°$ (base angles of isosceles ΔOPQ)

4 **a** $\alpha = 39°$ **b** $\alpha = 72°, \beta = 90°$
c $\alpha = 25°, x = 7$ **d** $\alpha = 45°, x = 17$
e $\beta = 45°, y = 11$ **f** $\alpha = \gamma = 60°, \beta = 120°, x = y = 15$

5 **a** $\alpha = 110°$ **b** $\alpha = 66°$ **c** $\beta = 36°$
d $\alpha = 56°, \beta = 62°$ **e** $\beta = 31°$ **f** $\alpha = 90°$

Exercise 5G

1 **a** $\alpha = 68°$ (angle sum of quadrilateral $PQRS$)
b $\alpha = 90°$ (angle sum of quadrilateral $ABCD$)
c $\alpha = 110°$ (angle sum of quadrilateral $ABCD$)
d $\beta = 65°$ (angle sum of quadrilateral $PQRS$)
e $\alpha = 62°$ (angle sum of quadrilateral $ABCD$)

2 **a** 30° **b** 90° **c** 110° **d** 240°

3 **a** $\alpha = \gamma = 104°, \beta = 76°$ **b** $\alpha = 90°, \beta = 126°$
c $\alpha = 124°$ **d** $\alpha = 90°$

4 **a** $\alpha = 136°, \beta = \gamma = 44°$
b $\alpha = 163°$
c $\alpha = 50°, \beta = 130°, \gamma = 101°, \theta = 40°$
d $\alpha = 99°$
e $\alpha = 50°, \beta = 57°$
f $\beta = 49°$

5 **a** $\beta = 67°$ **b** $\alpha = 72°$ **c** $\alpha = 135°$
d $\alpha = 42°$ **e** $\alpha = 17°$ **f** $\alpha = 79°, \beta = 138°$

6 **a** 90°, 45°, 45° **b** 135°, 22.5°, 22.5°

Review exercise

1 **a** $\theta = 75°$ (corresponding angles, $AB \parallel SD$)
b $\beta = 90°$ (co-interior angles, $QP \parallel SR$)
c $\alpha = 55°$ (corresponding angles, $AB \parallel CD$)
d $\alpha = 20°$ (vertically opposite angles),
$\beta = 20°$ (corresponding angles, $AB \parallel CD$),
$\gamma = 160°$ (straight angle $\angle CRD$)
e $\beta = 90°$ (co-interior angles, $LM \parallel ON$)
f $\alpha = 71°$ (co-interior angles, $AB \parallel CD$),
$\beta = 109°$ (co-interior angles, $AD \parallel BC$)

2 **a** 60° **b** 27° **c** 16° **d** 6°

3 **a** 53° **b** 104° **c** 46° **d** 165°

4 **a** $\theta = 110°$ **b** $\alpha = 33°$
c $\alpha = 100°$ **d** $\alpha = 26°$
e $\alpha = 126°, \beta = 37°$ **f** $\alpha = 106°, \beta = 20°$

g $\alpha = 80°, \beta = 40°, \gamma = 60°$ **h** $\alpha = 111°, \beta = 69°$
i $\alpha = 32°$

5 **a** $\alpha = 112°$ **b** $\alpha = 78°, \beta = 58°$
c $\alpha = 20°$ **d** $\alpha = 76°, \beta = 104°, \gamma = 52°$
e $\alpha = 36°$ **f** $\alpha = 130°, \beta = 50°, \gamma = 130°$

6 **a** 82° **b** 73° **c** 76°

7 If any angle is reflex, then the angle sum is greater than 180°.

8 **a** $\alpha = 60°$ **b** $\beta = 80°, \gamma = 170°$ **c** $\beta = 85°, \gamma = 70°$

Challenge exercise

1 $\alpha = 95°$

2 **a** $\alpha = 40°$ **b** $\alpha = \theta + (\gamma - \beta)$
c $\alpha = (180 - \theta) + (\beta - \gamma)$

3 $\alpha = 2\beta$

4 **a** 360° **b** 540° **c** 720° **d** 900°
e 1080° **f** 1800°
sum of internal angles = (number of sides − 2) × 180°

5 $\Delta AFG, \Delta ABC, \Delta BDK, \Delta CEL, \Delta JBC, \Delta JBH, \Delta JCI$

6 $\angle ABC = 71°$ **7** 30° or 150° **8** 360°

Chapter 6

Exercise 6A

1 **a** 7 **b** 8 **c** 1 **d** 7
e 18 **f** 2 **g** 5 **h** $1\frac{3}{4}$

2 **a** −3 **b** 12 **c** −1 **d** −3
e −2 **f** −22 **g** 1 **h** −50

3 **a** 9 **b** 9 **c** −9 **d** −27
e 27 **f** 81 **g** 27

4 **a** 3 **b** 3 **c** −1 **d** −1
e 29 **f** −7 **g** 3 **h** 1
i −5 **j** −5 **k** 4 **l** −8

5 **a** $\frac{1}{9}$ **b** $3\frac{8}{9}$ **c** −7 **d** 7

6 **a** 4 **b** −2 **c** −7 **d** 2
e 12 **f** 5 **g** −3 **h** 4
i 3 **j** −4 **k** −9

7 **a** 7 **b** 9 **c** 35 **d** −17
e 133 **f** −9 **g** 3 **h** 88
i −722

Exercise 6B

1 **a** $m = 7$ **b** $a = 3$ **c** $x = 50$ **d** $x = 2\frac{3}{4}$
e $x = 3$ **f** $m = 2$ **g** $x = 18$ **h** $x = 20$
i $n = 4$

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2 **a** $x=-3$ **b** $x=-10\frac{1}{2}$ **c** $x=-2\frac{1}{2}$ **d** $x=-5$
e $x=-3\frac{1}{2}$ **f** $x=-\frac{4}{5}$ **g** $x=54$ **h** $x=60$
i $x=40$

3 **a** 3 **b** 18 **c** −18 **d** −6
e −4 **f** 14 **g** −3 **h** 14
i 6

4 **a** −3 **b** −3 **c** −5 **d** $-5\frac{1}{3}$
e $8\frac{1}{3}$ **f** $2\frac{5}{9}$ **g** $3\frac{2}{5}$ **h** $8\frac{1}{3}$
i $5\frac{2}{3}$

5 **a** 36 **b** 30 **c** −80 **d** −20
e −18 **f** −32

6 **a** 3 **b** $2\frac{2}{5}$ **c** 2 **d** 4
e −4 **f** $-4\frac{3}{4}$ **g** $-4\frac{1}{5}$ **h** $8\frac{1}{5}$
i $7\frac{7}{10}$

7 **a** $x=9$ **b** $a=12$ **c** $z=8$ **d** $b=10\frac{7}{11}$
e $x=24$ **f** $x=70$

8 **a** $a+5=21,\ a=16$ **b** $7x=35,\ x=5$
c $5z=37,\ z=7\frac{2}{5}$ **d** $5m+3=50,\ m=9\frac{2}{5}$
e $\frac{n}{6}=10,\ n=60$ **f** $\frac{p}{3}-5=23,\ p=84$

9 **a** 1 **b** 1 **c** $-1\frac{1}{2}$ **d** $19\frac{1}{4}$ **e** $15\frac{2}{5}$

10 **a** $20-x=10,\ x=10$ **b** $6-2m=20,\ m=-7$
c $\frac{n}{8}+6=20,\ n=112$ **d** $7p+10=60,\ p=7\frac{1}{7}$
e $6-x=-10,\ x=16$ **f** $15-7y=-6,\ y=3$
g $\frac{k}{10}-7=-1,\ k=60$

Exercise 6C

1 **a** $6a-2$ **b** $30p+35$ **c** $12-4x$
d $3a-6$ **e** $42-14x$ **f** $21-3x$
g $-20+4x$ **h** $-42+12p$ **i** $-15x+6$
j $-24x+12$ **k** $-6+3x$ **l** $-6x-2$

2 **a** $2x+6$, 16 **b** $10x+20$, 70
c $2x-4$, 6 **d** $6x-18$, 12

3 **a** 3 **b** 0 **c** 19

4 **a** 2 **b** −28 **c** −54

Exercise 6D

1 **a** $3\frac{1}{2}$ **b** $5\frac{2}{5}$ **c** $\frac{1}{3}$ **d** $1\frac{5}{6}$
e $\frac{2}{5}$ **f** $1\frac{2}{5}$ **g** $4\frac{1}{3}$ **h** $-\frac{7}{12}$
i $1\frac{1}{15}$ **j** $-1\frac{3}{5}$ **k** $\frac{5}{12}$ **l** $1\frac{1}{9}$

2 **a** $3(6+x)=-10,\ x=-9\frac{1}{3}$
b $3(m-6)=5,\ m=7\frac{2}{3}$
c $2(10+p)=4,\ p=-8$
d $3(n-5)=14,\ n=9\frac{2}{3}$
e $2(x-3)=-10,\ x=-2$
f $-2(3+x)=-2,\ x=-2$
g $-4(x-6)=-10,\ x=8\frac{1}{2}$

Exercise 6E

1 like: **a, b, c, d, g, h, k, l, o, p, s, t**
unlike: **e, f, i, j, m, n, q, r**

2 **a** $7x$ **b** $-x$ **c** $3x$ **d** $5x$
e $-x$ **f** $-4x$ **g** $12x$ **h** $51m$
i $-9n$ **j** $97x$ **k** $9n$ **l** $-45p$
m $-20x$ **n** 0 **o** $13m+11n$ **p** $7m+4n$
q $7m+8n$ **r** $x+5y$ **s** $2x+y$ **t** $11m-9n$
u $7q-2p-5r$ **v** $7a-18b$

3 **a** $93xy$ **b** $21xy$ **c** $16xy^2$
d $7x+10y$ **e** $-5v-7z$ **f** $16y+4x$
g $7x^3+4y^3+5x^2$ **h** $-4x^2+13y^2$ **i** $3x^2+2xy$
j $-14x^2-9y^2$

4 **a** $7x+40$ **b** $5x+9$ **c** $7x+16$
d $14x+38$ **e** $-2x+55$ **f** $8x+11$
g $8x-42$ **h** $-2x+3$ **i** $6x+4$
j $26x+22$ **k** $11x+21$ **l** $18ax+16x$

5 **a** −3 **b** 4 **c** $1\frac{1}{2}$ **d** 5
e $\frac{4}{5}$ **f** −2

6 **a** 3 **b** 4 **c** 4 **d** $6\frac{2}{3}$

7 **a** $4\frac{1}{2}$ **b** −15 **c** −47 **d** $14\frac{1}{2}$
e $2\frac{9}{14}$ **f** $2\frac{16}{23}$

Exercise 6F

1 **a** $x+6=24,\ x=18$ **b** $5x=35,\ x=7$
c $3x=37,\ x=12\frac{1}{3}$ **d** $6x+4=48,\ x=7\frac{1}{3}$
e $\frac{x}{6}=20,\ x=120$ **f** $\frac{x}{7}-5=33,\ x=266$

2 −16 **3** $-1\frac{3}{4}$ **4** \$2.40

5 $4\frac{1}{5}$ **6** 7 and 35 **7** $2\frac{1}{2}$ and $7\frac{1}{2}$

8 4 **9** 88 **10** 92

11 \$3.50

12 Let $\$x$ be the cost of the apple. Then $\$(x+0.60)$ is the cost of the milkshake, and $\$(x+0.60+0.30)$ is the cost of the sausage roll, so $x+(x+0.60)+(x+0.60+0.30)=3.00$, and $x=0.50$. Hence, the apple costs \$0.50, the milkshake costs \$1.10 and the sausage roll costs \$1.40.

Exercise 6G

1 **a** $12ab$ **b** $12x^2$ **c** $50xy$ **d** $77ab$
e $10a^2$ **f** $66c^2$ **g** $24\ mn$ **h** $77\ m^2n^2$
i $12n^2$ **j** $42m^3$ **k** $22m^4$ **l** $35a^2b$

2 **a** $25n^2$ **b** $16z^2$ **c** $256z^2$ **d** $9c^4$

3 **a** $8x^2$ **b** $21x^2$ **c** $6x^2$ **d** $6x^2y$
e $8x^2y^2$ **f** $12x^3y^2$

4 **a** $2x$ **b** $8x$ **c** $24y$ **d** $\frac{np}{5}$
e $3x$ **f** $18x$ **g** $18x$ **h** $\frac{9}{2}ab$

5 **a** $\frac{4}{y}$ **b** a **c** a **d** 4
e $3ab$ **f** a^2 **g** $\frac{1}{a}$ **h** $\frac{1}{a}$
i 1 **j** a **k** ab **l** b

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6 a $\frac{4}{3}pq$ b $20ab$ c $20a^2b^2$

7 a $\frac{x^2}{3}$ cm^2 b 27 cm^2

8 a $\frac{x^2}{12}$ cm^2 b 48 cm^2

9 $\frac{6x}{5}$ cm

10 a $\frac{10x^2}{3}$ cm^2 b 2430 cm^2

11 $\frac{64x^2}{y}$ cm^2

12 840 tiles

Review exercise

1 a 2 b 2 c 7 d −4
e 14 f −5 g 18 h 28
i −2 j 14 k 4 l $-\frac{5}{9}$

2 a −3 b 7 c −9 d −2
e 9 f −27 g −7 h 13
i −28 j −35 k −1 l 125

3 a 125 b 5 c 14 d −14
e −66 f 4356 g −20 h 128

4 a $3a+12$ b $18-6x$ c $2x+xy$ d $2x-22$
e $4x-12$ f $7x+10$ g $6a+ax+21x$ h $x-42$
i $18-6x$ j $-19x+48$

5 a $x=-4$ b $x=-18$ c $x=-1$ d $a=-3$
e $m=-1$ f $m=7$

6 a $x=4$ b $a=-1$ c $x=13$ d $x=\frac{4}{3}$
e $x=-1$ f $x=7\frac{1}{3}$

7 a $8x^5y^5$ b $4x^5y^4$ c xy d mn
e $16x^5y^3$ f $4ab^2$

8 $12 9 10 10 $x=2$

11 $x=4$ 12 $x=2\frac{3}{4}$

Challenge exercise

1 a 8, 72, 70, 75, 50, 5 c x

2 64 smaller cubes; 0 faces: 8; 1 face: 24; 2 faces: 24; 3 faces: 8
In general, 0 faces: $(n-2)^3$; 1 face: $6(n-2)^2$; 2 faces: $12(n-2)$; 3 faces: 8

3 6 left: 33; 12 left: 51. In general, $3x+15$

4 child ticket: $2.14; adult ticket: $4.28

5 50 laps 6 14, 41, 23, 32, 50 7 54

8 No. Let the 2-digit integer be $10a+b$, where a and b are the digits of the tens and units places.
Then
$11(a+b)=10a+b$
$11a+11b=10a+b$,
so $a+10b=0$, which is impossible because $0<a<10$ and $0<b<10$.

Chapter 7

Exercise 7A

1 a 100% b 66% c 40% d 50%
e 37.5% f 37.5%

2 a $\frac{1}{10}$ b $\frac{1}{4}$ c $\frac{17}{20}$ d $\frac{1}{2}$
e $\frac{2}{5}$ f $\frac{1}{5}$ g $1\frac{1}{4}$ h $6\frac{3}{10}$
i $2\frac{1}{20}$ j $1\frac{19}{50}$ k $4\frac{24}{25}$ l $431\frac{1}{4}$

3 a 0.55 b 0.37 c 0.19 d 0.8
e 0.3 f 0.1 g 2.5 h 1.48
i 39.12 j 0.03 k 0.07 l 0.01

4 a 23% b 99% c 11% d 150%
e 230% f 999% g 420% h 670%
i 140% j 90% k 50% l 10%
m 67% n 81% o 33% p 99.9%
q 12.3% r 70.2% s 999.9% t 123.4%

5 a 50% b 98% c 16% d 5%
e 60% f 76% g 125% h 2.5%
i 368% j 120% k 250% l 125%
m 7.5% n 240% o 42.5%

6 a 65% b 23% c 75% d 50%
e 30% f 60% g 150% h 110%
i 180% j 100% k 7% l 2%
m 105% n 409% o 703%

7 a $\frac{1}{40}$ b $\frac{3}{40}$ c $\frac{33}{400}$
d $\frac{3}{8}$ e $\frac{1}{8}$ f $\frac{5}{8}$
g $\frac{7}{8}$ h $\frac{2}{3}$

8

Decimal	Fraction	Percentage
0.5	$\frac{1}{2}$	50%
0.25	$\frac{1}{4}$	25%
0.75	$\frac{3}{4}$	75%
0.4	$\frac{2}{5}$	40%
1	$\frac{1}{1}=\frac{2}{2}=\cdots$	100%
0.367	$\frac{367}{1000}$	36.7%
0.29	$\frac{29}{100}$	29%
0.403	$\frac{403}{1000}$	40.3%
0.375	$\frac{3}{8}$	37.5%
1.25	$\frac{5}{4}$	125%
2.75	$\frac{11}{4}$	275%
2.0	2	200%

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9 a 30% b 25% c 80% d $66\frac{2}{3}\%$
e 5% f 40%

10 a 234 b 36 c 170 d 2910 e 100
f 182 g 72 h 840 i 350

11 a 20 b 40 c 200 d 150 e 1.5
f 45 g 63 h 390 i 17.1 j 200
k 236 l 7

12 a 34 b 102 c 238 d 17 e 8.5
f 25.5

13 a 80 b 240 c 40

14 a 45.6 b 136.8 c 91.2 d 45.6

15 a 9.6 m b 60 lollies c 81 children

16 a 7 songs b 112 stories c 24 students

17 36% 18 80% 19 62.5%

Exercise 7B

1 a $66\frac{2}{3}\%$ b $16\frac{2}{3}\%$ c $83\frac{1}{3}\%$
d $77\frac{7}{9}\%$ e $91\frac{2}{3}\%$ f $71\frac{3}{7}\%$

2 a $\frac{61}{500}$ b $\frac{27}{400}$ c $\frac{21}{400}$ d $\frac{91}{600}$
e $\frac{1}{15}$ f $\frac{61}{1200}$ g $\frac{73}{600}$ h $\frac{87}{400}$
i $\frac{74}{175}$ j $\frac{11}{175}$ k $\frac{67}{150}$ l $\frac{89}{1200}$

3 a 2 b $\frac{3}{200}$ c $1\frac{6}{25}$ d $2\frac{1}{10}$
e $\frac{1}{200}$ f $1\frac{51}{200}$

4 a $0.1225 = \frac{49}{400}$ b $0.0675 = \frac{27}{400}$
c $0.0575 = \frac{23}{400}$ d $0.0806 = \frac{403}{5000}$
e $0.1205 = \frac{241}{2000}$ f $2.27 = 2\frac{27}{100}$
g $117.9 = 117\frac{9}{10}$ h $57.11 = 57\frac{11}{100}$

5 a $33\frac{1}{3}\%$ b 28% c $33\frac{1}{3}\%$ d $22\frac{4}{5}\%$
e $25\frac{25}{67}\%$ f 14%

6 2nd test $86\frac{2}{3}\%$, 3rd test 85%, 1st test 84%

7 a 60% b 40%

8 $39\frac{1}{16}\%$ 9 95%

10 a 1182 b The first drug is more effective (98.5 > 97.5).

11 a $\frac{1}{3}\%$ b $3\frac{1}{3}\%$

12 school C (highest), school A, school B

13 white: 37.5%; wholemeal: $23\frac{1}{3}\%$; multigrain: 32.5%; raisin: $6\frac{2}{3}\%$

14 a 0.25% b $15\frac{5}{8}\%$

15 a 17.48 kg b 12.24 kg

Exercise 7C

1 a $\frac{11}{20}$ b $\frac{9}{10}$
c $\frac{31}{50}$ d $\frac{149}{400}$

2 a 10% b $4\frac{1}{2}\%$ c 48%
d $30\frac{3}{5}\%$ e 10% f $8\frac{1}{3}\%$
g 14% h 12% i $33\frac{1}{3}\%$

3 a 6% b 6%

4 a 2795 b 35%

5 a i 90 ii 240 iii 150 b 120

6 a 170 b 130 c 200

7 0.9%

8 a 65% b 5.25%

9 18% 10 12% 11 9.2% 12 22%

13 They are not percentages of the same whole.

14 The 40% of females come from all of the supporters, not just from those who wear red shirts.

Exercise 7D

1 a 856 b 1060 c 810 d 297
e 4905 f 6390

2 a 100% increase b 300% increase
c 400% increase d 10% increase
e 25% increase f $9\frac{1}{11}\%$ decrease
g $33\frac{1}{3}\%$ increase h 20% decrease
i 25% decrease

3 a \$54 000 b \$51 000
c \$36 000 d 52 080 hectares

4 $16\frac{2}{3}\%$ 5 55 800 hectares

6 a 450 b 5750 c 5170 d 6545

7 a \$2889 b \$4928 c \$64.60
d \$51.52 e \$60.45

8 50 600 9 $23\frac{17}{21}\%$ 10 2 689 600

11 a 1 025 000 b 1 075 000

12 $46\frac{2}{13}\%$ 13 $33\frac{1}{3}\%$

14 a 80% b 75% c machine A

15 \$74 400 16 a \$572 000 b 11%

17 a 16% profit b 9% profit
c 10% loss d 5% loss

18 a \$7600 loss b \$6400 profit
c \$500 000 profit d \$105 000 loss

19 \$72 20 \$207 21 \$20

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Exercise 7E

1 40

2 **a** \$187 560 **b** \$168 000 **c** \$518 400 **d** \$81 000

3 \$25 000

4 **a** 275 000 m^2 **b** 220 000 m^2

5 \$120 000 6 1500

7 **a** 541 820 **b** 456 700 **c** 121 000 **d** 12 050

8 \$800 9 68% 10 \$160 11 \$352

12 **a** \$80 **b** \$62 **c** \$230 **d** \$85

Review exercise

1 **a** 92% **b** 75% **c** $37\frac{1}{2}$% **d** $87\frac{1}{2}$% **e** 60% **f** $66\frac{2}{3}$% **g** 75% **h** 110% **i** $66\frac{2}{3}$% **j** $12\frac{1}{2}$% **k** 124% **l** 200%

2 **a** $\frac{1}{2}$ **b** $\frac{17}{20}$ **c** $\frac{43}{100}$ **d** 2 **e** $3\frac{2}{5}$ **f** $1\frac{17}{20}$ **g** $\frac{1}{8}$ **h** $6\frac{6}{25}$

3 **a** 8 **b** 4.5 **c** 46 **d** 27 **e** 194 **f** 18 **g** 3468 **h** 374 000 **i** 45

4 utes: 9801; four-wheel drives: 5346; sedans: 8019; station wagons: 6534

5 5% 6 6 g

7 **a** 375 **b** 210 **c** 1400 **d** 504 **e** 2625 **f** 40 000 **g** 11 **h** 24 **i** 0.078 **j** 21 **k** 29 985 **l** 3.3

8 **a** 37.5 mL **b** 19c

9 110 10 848 11 352.5 g 12 9924

13 3312 g 14 233.6 g 15 33.6 g

16 6 carbon, 1 nitrogen and 7 hydrogen atoms

17 \$100 18 \$25 19 \$55

20 **a** 112.5 mL **b** 28c

21 2400 22 5200 23 51% 24 18 25 \$4875

Challenge exercise

1 \$13.75 2 103.5 cm^2 3 100 000 4 50%

5 4960 6 250 mL 7 87.5 mL

8 **a i** 16 cm^2 **ii** 112 cm^2 **b** 25.6 cm^2 **c** 35.2 cm^2 **d** 27.5%

9 20 10 12, 24, 36, 48 11 1%

Chapter 8

Exercise 8A

1 **a** 81 **b** 121 **c** 49 **d** 225

2 **a** 8 **b** 12 **c** 14 **d** 30

3 **a** 625 **b** 225 **c** 144 **d** 64

4 **a** 10 **b** 20 **c** 25 **d** 13 **e** 25 **f** 26

5

Row	a	b	a^2	b^2	$c^2=a^2+b^2$	c
1	3	4	9	16	25	5
2	6	8	36	64	100	10
3	9	12	81	144	225	15
4	12	16	144	256	400	20
5	15	20	225	400	625	25

Pattern: a increases by 3, b by 4 and c by 5

6

Row	a	b	a^2	b^2	$c^2=a^2+b^2$	c
1	3	4	9	16	25	5
2	5	12	25	144	169	13
3	7	24	49	576	625	25
4	9	40	81	1600	1681	41
5	11	60	121	3600	3721	61

Pattern: a increases by 2; the difference between b and c is 1.

7 **a** $x = 20$ **b** $x = 13, \quad y = 15$ **c** $x = 26$ **d** $x = 30, \quad y = 26$ **e** $x = 25, \quad y = 26$ **f** $x = 15, \quad y = 25, \quad z = 20$

Exercise 8B

1 **a** 24 **b** 21 **c** 16

2 15 cm

3 **a** yes, $\angle ACB$ **b** no **c** yes, $\angle ABC$

4 **a** $x = 18$ **b** $x = 7$ **c** $x = z = 10, y = 6$ **d** $x = 12$ **e** $x = 13$ **f** $x = 25$ **g** $x = 5, \quad y = 13, \quad z = 84$ **h** $x = z = 12, w = y = 16$

5 390 cm

6 90 cm

7 31.2 m

8 **a** no **b** yes **c** yes

9 40 km

10 **a** 2 cm **b i** 8 cm **ii** 18 cm **iii** 50 cm **iv** $2n^2$

Exercise 8C

1 **a** 2.24 m **b** 4.90 m **c** 4.36 m **d** 1.73 m **e** 5.20 m **f** 3.46 m

2 4 m 3 5.39 cm 4 16.97 cm

5 5.20 cm 6 $\sqrt{3}$ cm $\approx$ 1.73 cm 7 $3\sqrt{2}$ cm $\approx$ 4.24 cm

8 5.39 km 9 2 m 10 5.39 km

11 3.61 km 12 4.24 cm

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Review exercise

1 **a** 34 cm **b** 30 cm **c** 25 cm **d** 20 m **e** 32 cm **f** 48 cm

2 **a** no **b** yes, $\angle ABC$ **c** yes, $\angle ACB$ **d** yes, $\angle ABC$ **e** no **f** yes, $\angle CAB$

3 20 km **4** 16.5 m **5** 2.65 m

6 **a** 4.24 cm **b** 2.82 cm

7 4 m

Challenge exercise

1 $\sqrt{a^2+a^2}=\sqrt{2a^2}=a\sqrt{2}$ **2** $\sqrt{3}x$ m

3 $\frac{\sqrt{3}}{4}x^2$ m^2

4 **a** $x=\sqrt{360}\approx 18.96$ cm
b $x=4$ cm, $y=\sqrt{84}\approx 9.16$ cm
c $x=\sqrt{75}\approx 8.65$ cm
d $x=\sqrt{160}\approx 12.64$ cm
e $x=\sqrt{1305}\approx 36.12$ cm

5 180 cm^2

6 height = 3.46 cm, area = 6.93 cm^2

7 $AE=\sqrt{21}\approx 4.58$, $DE=\sqrt{12}\approx 3.64$, $DB=4$

8 7 m

Chapter 9

9A Review

Chapter 1: Whole numbers

1 588 ants **2** 4440 pies

3 Buy 2 lots of 20 packets for \$19.00, \$7 saved

4 1560 **5** 628

6 Bashir has 241 marbles, Andrew has 241 marbles.

7 629 **8** 59 **9** 52 437

Chapter 2: Fractions and decimal

1 **a** $3\frac{4}{7}$ **b** $\frac{1}{5}$ **c** $2\frac{1}{8}$ **d** $\frac{2}{9}$ **e** $2\frac{2}{3}$

2 **a** $7\frac{9}{10}$ **b** $2\frac{5}{12}$ **c** $9\frac{11}{24}$ **d** $8\frac{11}{20}$ **e** $9\frac{1}{36}$ **f** $1\frac{5}{9}$

3 **a** 90 **b** 160 **c** $3\frac{1}{2}$ **d** $7\frac{7}{8}$ **e** $20\frac{5}{6}$ **f** $\frac{10}{27}$ **g** $4\frac{2}{3}$ **h** $13\frac{5}{7}$

4 $41\frac{2}{3}$ metres **5** $13\frac{1}{8}$ **6** 33 **7** \$700

8 **a** $\frac{1}{2}<\frac{2}{3}<\frac{3}{4}<\frac{4}{5}<\frac{5}{6}$ **b** $\frac{1}{4}<\frac{11}{25}<\frac{6}{10}<\frac{13}{20}<\frac{4}{5}$

9 **a** $\frac{1}{5}$ **b** $\frac{1}{5}$

10 1 minute 4 seconds

11 **a** 4.56 **b** 2.8 **c** 10.1 **d** 5.75 **e** 37.8758 **f** 8083.39

Chapter 3: Review of factors and indices

1 **a** 1, 2, 3, 4, 6, 12 **b** 1, 17
c 1, 2, 3, 5, 6, 9, 10, 15, 18, 27, 30, 45, 54, 90, 135, 270
d 1, 3, 5, 9, 15, 27, 45, 135 **e** 1, 2, 29, 58
f 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144

2 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157

3 **a** 35 **b** 144 **c** 5610 **d** 300

4 **a** 1 **b** 3 **c** 6 **d** 10

5 **a** product of 24 and 76 = 1824, HCF × LCM = 4 × 456 = 1824
b product of 102 and 54 = 5508, HCF × LCM = 6 × 918 = 5508

6 **a** 3 **b** 5 **c** 10 **d** 2 **e** 81 **f** 6 **g** 1 **h** 8 **i** 3

7 **a** 64 **b** 11 **c** 56 **d** 25 **e** 4096 **f** 47 **g** 15 625

8 For example:
a 25 671, 19 914, 456 **b** 73 656, 1008, 2226
c 1020, 121 980, 59 220
(*Note*: There are more possibilities.)

9 There are eight possibilities; two of them are 84 920 and 94 820.

10 **a** $2\times3\times5\times7$ **b** $2^4\times3^2\times5\times7$
c $2^2\times109$ **d** $2\times3^2\times7\times11$

11 **a** **i** 1372 **ii** 1859 **iii** 7560
b $2^3\times3^3\times5\times7$

12 **a** HCF = 720, LCM = 64 800
b HCF = 14, LCM = 147 000

13 **a** 400 **b** 4000 **c** 3000 **d** 4370

Chapter 4: Negative numbers

1 **a** 8 **b** 86 **c** −30 **d** −11 **e** 997 **f** 21

2 **a** 6 **b** 15 **c** −25 **d** 98 **e** $-14\frac{3}{10}$ **f** $60\frac{4}{5}$ **g** $\frac{17}{104}$ **h** $\frac{11}{12}$ **i** $-3\frac{13}{20}$ **j** $19\frac{13}{15}$

3 16.3°C

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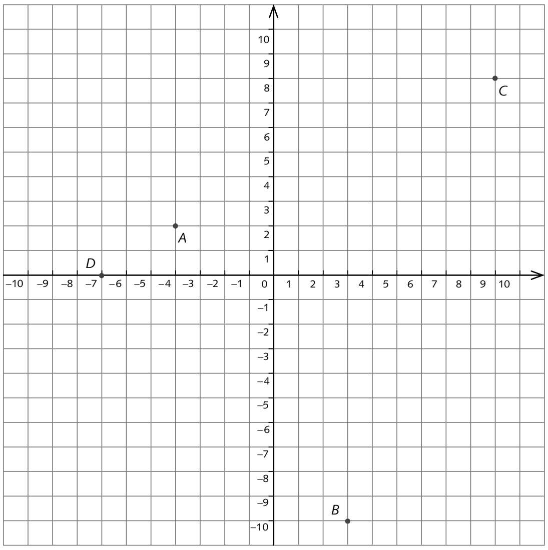
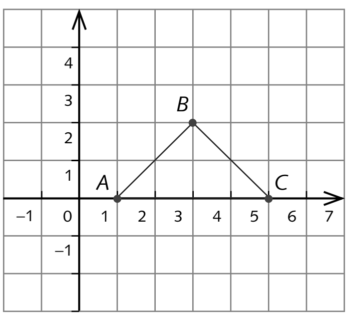
4



5



Area of $\Delta ABC = \frac{1}{2}bh$

$= \frac{1}{2} \times 4 \times 2$

$= 4$ unit squares

6 $-\frac{9}{8} < -\frac{7}{9} < -0.02 < 0.2 < \frac{12}{8} < \frac{1}{0.4}$

7 \$2359.70 **8** 9691 metres **9** 12

Chapter 5: Review of geometry

1 **a** 138° **b** 64° **c** 59°

2 **a** $\alpha=150°, \beta=30°, \gamma=150$ **b** $\alpha=80°$
c $\alpha=130°, \beta=135°$ **d** $\alpha=60°, \beta=120°$
e $\alpha=45°, \beta=135°$ **f** $\alpha=36°, \beta=144°$

3 **a** $\gamma=135°$ **b** $\alpha=75°$
c $\alpha=66\frac{2}{3}°$ **d** $\alpha=90°$

Chapter 6: Algebra – part 1

1 **a** 6 **b** 6 **c** 1 **d** $1\frac{2}{3}$

2 **a** -1 **b** -1 **c** 4 **d** -8
e 1 **f** 4 **g** 6 **h** 1

3 **a** $m=5$ **b** $m=14$ **c** $m=4$
d $m=24$ **e** $x=7\frac{1}{2}$ **f** $x=60$
g $x=-6$ **h** $x=-1$ **i** $x=-75$

4 **a** $2x-6$ **b** $-3x-12$ **c** $5x-10$
d $ax+3a$ **e** $-3x+12$ **f** $-12+4x$

5 **a** $7x-y$ **b** $3x+3y$ **c** $4x$
d 0 **e** 0 **f** $5x^2y$

6 **a** $-12\frac{3}{5}$ **b** -12 **c** $1\frac{3}{25}$ **d** $108\frac{1}{5}$
e 3 **f** $\frac{3}{8}$ **g** $-1\frac{4}{25}$ **h** $97\frac{1}{2}$

7 **a** $x=-13$ **b** $b=3$ **c** $a=11$
d $b=-1\frac{1}{13}$ **e** $x=100$ **f** $x=4$

8 **a** $30x+7y$ **b** $13y+\frac{9}{4}x$
c $20xy+6y+24x$ **d** $4x^2+x+27$
e $15x+\frac{17x^2}{3}$ **f** $2xy+13x+5y$
g $ay-ax-10a$ **h** $4x^2y-15xy^2-6xy$

9 $\frac{2(x+14)-8}{2}-x=x+14-4-x=10$

10 4 rides

11 5 pages on a week night, 15 pages on Saturday night

12 $3\frac{5}{6}$ cm

Chapter 7: Percentages

1 **a** 40% **b** $83\frac{1}{3}$% **c** 87.5%
d $77\frac{7}{9}$% **e** $71\frac{3}{7}$% **f** 7.5%

2 **a** $\frac{17}{200}$ **b** $\frac{1}{6}$ **c** $\frac{23}{150}$
d $\frac{13}{150}$ **e** $\frac{3}{70}$ **f** $\frac{11}{200}$

3 **a** $1\frac{1}{2}$ **b** $\frac{1}{40}$ **c** $1\frac{9}{25}$
d $2\frac{1}{25}$ **e** $\frac{1}{400}$ **f** $\frac{21}{400}$

4 **a** $\frac{27}{50}$ **b** $\frac{18}{25}$ **5** 24

6 **a** 1250 **b** 2000 **c** 2800
d 594 **e** 3675 **f** 22 400

7 **a** \$186 000 **b** \$57 000
c \$36 800 **d** 133 920 hectares

8 **a** \$4068 **b** \$22.95 **c** \$81.70
d \$210

9 **a** 15% **b** 6% **c** 40% **d** 18%

10 **a** 5100 **b** 2400 **c** 2200

11 \$33

Chapter 8: Pythagoras' theorem

1 **a** 26 **b** 41 **c** 61

2 **a** $x=25$ cm, $y=51$ cm **b** $x=\sqrt{128}$ cm

3 40 cm **4** 12 m **5** 4.23 m **6** 6 m

7 **a** no **b** yes, $\angle BCA$ **c** yes, $\angle BAC$

9B Problem solving

Kaprekar's routine

1 Eventually a subtraction produces either 495 or 594. Since $954-459=495$, repeating the process will keep on producing 495.

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2 The numbers that are produced by this process all have '9' in the middle and two other digits with a sum of nine (for example, 297, 693, 198 and so on). Their digit sum is obviously always 18. As the process is repeated, the outside numbers approach 4 and 5; for example, $891 \rightarrow 792 \rightarrow 693$ and so on.

3 $6174 (= 7641 - 1467)$ This number is known as **Kaprekar's constant**.

4 There is no five-digit self-producing integer. However, there are three different cyclic patterns that numbers with five digits can fall into. Two have four members ($63\,954 \rightarrow 61\,974 \rightarrow 82\,962 \rightarrow 75\,933$ and so on, and $62\,964 \rightarrow 71\,973 \rightarrow 83\,952 \rightarrow 74\,943$ and so on), the other has two members ($59\,994 \rightarrow 53\,955$ and so on).

5 *Challenge:* 6 333 176 664, 9 753 086 421, 9 975 084 201

Angle chasing

1 **a** 305° **b** 126° **c** 107° **d** 325°

2 $\theta = 342°, \alpha = 72°$

Multiples

1 **a** $481 = 13 \times 37, 148 = 4 \times 37, 814 = 22 \times 37$

b $185 = 5 \times 37, 851 = 23 \times 37, 518 = 14 \times 37$
$259 = 7 \times 37, 592 = 16 \times 37, 925 = 25 \times 37$
$296 = 8 \times 37, 962 = 26 \times 37, 629 = 17 \times 37$
$740 = 20 \times 37, 74 = 2 \times 37, 407 = 11 \times 37$
$814 = 22 \times 37, 481 = 13 \times 37, 148 = 4 \times 37$
$925 = 25 \times 37, 259 = 7 \times 37, 592 = 16 \times 37$

2 **a** $68\,839 = 1679 \times 41, 88\,396 = 2156 \times 41, 83\,968 = 2048 \times 41, 39\,688 = 968 \times 41, 96\,883 = 2363 \times 41$

b They are all multiples of 41.

3 **a i** 555 **ii** 222 **iii** 777

The product turns out to be a three-digit number with all digits the same as the middle multiplicand.

b $999, 37 \times 3 = 111$

c 1443, 1887. The product turns out to be a four-digit number, with the first and last digits the same as the middle multiplicand and each of the two digits in the middle is the sum of the outer digits.

d $41 \times 5 \times 271 = 55\,555, 41 \times 2 \times 271 = 22\,222,$
$41 \times 7 \times 271 = 77\,777, 41 \times 9 \times 271 = 99\,999,$
$41 \times 13 \times 271 = 144\,443, 41 \times 17 \times 271 = 188\,887$

Geometry challenge

1 **a** $\alpha = 30°, \beta = 86°$ **b** $\alpha = 83°, \beta = 120°, \gamma = 83°$

c $\alpha = 66°, \beta = 38°$ **d** $\alpha = 107°, \beta = 140°$

e $\alpha = 51°, \beta = 129°$ **f** $\alpha = 30°, \beta = 147°$

2 **a** $\angle BAC = 70°, \angle ACB = 55°$

b $\angle FIC = 55°$, ΔFIC is isosceles triangle because base angles are equal.

c $\angle DIB = 55°$, ΔDIB is isosceles triangle because base angles are equal thus $DB = DI$.

d $\angle GIF = 53°, \Delta EFG = 17°$

3 **a** $\angle AOC = 2\alpha$ **b** $\angle AOB = 2\beta$

c $2\alpha + 2\beta = 180°$ (straight angle BOC) so $\alpha + \beta = 90°$. Hence, $\angle BAC = 90°$.

ΔBDC and ΔCDE are right-angled triangles.

d 115°

Pythagoras' theorem

a $(a+b)^2 = (a+b)(a+b) = a(a+b) + b(a+b)$
$= a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$

b Each of the 4 triangles has area $\frac{1}{2}ab$

c $c^2 + 2ab = a^2 + 2ab + b^2$. Hence, $c^2 = a^2 + b^2$

Pythagorean triples

The two methods always produce primitive triples.

Sports percentages

Activity 1

140%, 108.3%, 125%, 93.8%, 75%, 68%

Activity 2

	Team	Wins	Losses	Goals For	Goals Against	Percentage (%)	Points
1	Cassowaries	5	1	157	117	134.2	20
2	Pelicans	5	1	142	111	127.9	20
3	Hawks	3	3	160	132	121.2	12
4	Brolgas	3	3	132	128	103.1	12
5	Jabirus	3	3	120	123	97.6	12
6	Emus	2	4	97	120	80.8	8
7	Kookaburras	2	4	105	140	75.0	8
8	Eagles	1	5	113	155	72.9	4

Activity 3

1 The Kookaburras' percentage did not change, because their old percentage happened to be equal to the ratio of goals in the latest game.

2 The Hawks' and Cassowaries' percentages went down even though they won.

3 Yes; the Eagles are an example.

4 They both sum to 1026, because each goal scored contributes 1 to both the Goals For and Goals Against columns.

Exercise 9C

1 **a** $27 = 7 \times 3 + 6$, hence $27 \equiv 6 \pmod 7$

b $359\,412\,294 = 10 \times 35\,941\,229 + 4$, hence $359\,412\,294 \equiv 4 \pmod{10}$

c $53 = 12 \times 4 + 5$, hence $53 \equiv 5 \pmod{12}$

d $3 = 8 \times 0 + 3$, hence $3 \equiv 3 \pmod 8$

2 **a** 0 **b** 4 **c** 6 **d** 4

3 **a** 51 **b** 57 **c** 51 **d** 52
e 58 **f** 61 **g** 61 **h** 67

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4 Yes, because $47 \equiv 1 \pmod 2$.

5 Bessie, because $80 \equiv 2 \pmod 6$.

6 6 a.m.

7 4, 11, 18, 25; Friday

8 1891, 1895, 1903, 1907, 1911, 1915, 1919
(1900 is not a leap year)

9 **a** 1 (look at the last two digits)
b 81 (look at the last two digits)
c 31 (look at the last two digits)
d 6 (look at the last two digits)

10 **a** 2 p.m. **b** Thursday **c** East

11 **a** $-13 = 5 \times (-3) + 2$ **b** $-13 \equiv 2 \pmod 5$

13 **a** $-20 = 7 \times (-3) + 1$, hence $-20 \equiv 1 \pmod 7$.

b $-30 = 8 \times (-4) + 2$, hence $-30 \equiv 2 \pmod 8$.
c 1 **d** 2 **e** 10 **f** 0

14 **a** $-16 \equiv 0$ 4; $-1 \equiv 15$ **b** $-30 \equiv 18$; $5 \equiv 53$

Exercise 9D

1 **a** 3 **b** 5 **c** 0 **d** 2 **e** 4
f 3 **g** 0 **h** 76 **i** 4 **j** 24

2 **a** 1 **b** 4 **c** 1 **d** 0 **e** 4 **f** 2

3 **a** 1 **b** 0 **c** 0 **d** 1

4 **a** 1 **b** 4 **c** 4 **d** 1
e 5 **f** 4 **g** 4 **h** 1

5 **a** 1 **b** 5 **c** 9 **d** 0

6 They are all zero. When you add an integer to itself a number of times equal to the modulus, then the sum is zero.

7 4 times, because $74 \equiv 2 \pmod 6$.

8 10 a.m., because $7 \times 247 \equiv 1 \pmod{24}$.

9 Wednesday, because
$280 + 25 + 211 + 143 \equiv 0 + 4 + 1 + 3 \equiv 1 \pmod 7$.

10 3rd finger, because $37 \times 19 \equiv 3 \pmod 5$.

11 30°, because $60 \times 9 + 40 \times 12 + 90 \times 13 \equiv 30 \pmod{360}$.

12 **a** $10^n \equiv 1^n \equiv 1 \pmod 9$.
b An example will demonstrate the result better than a general argument:

$2467 = 2 \times 10^3 + 4 \times 10^2 + 6 \times 10 + 7 \equiv$
$2 \times 1 + 4 \times 1 + 6 \times 1 + 7 \equiv 2 + 4 + 6 + 7 \pmod 9$.

c If two numbers are congruent modulo 9, then they are congruent modulo 3.

13 **a** $10 \equiv -1 \pmod{11}$, so $100 \equiv 1 \pmod{11}$, so $1000 \equiv -1 \pmod{11} \ldots$

b $2467 \equiv 2 \times 10^3 + 4 \times 10^2 + 6 \times 10 + 7 \equiv 2 \times (-1) + 4 \times 1 + 6 \times (-1) + 7 \equiv -2 + 4 - 6 + 7 \pmod{11}$.

14 **a** 1, 1, 2, 0, 2, 2, 1, 0, 1, 1… length = 8

b 1, 1, 0, 1, 1… length = 3

c 1, 1, 2, 3, 1, 0, 1, 1… length = 6

d 1, 1, 2, 3, 0, 3, 3, 1, 4, 0, 4, 4, 3, 2, 0, 2, 2, 4, 1, 0, 1, 1 …length = 20

e 1, 1, 2, 3, 5, 2, 1, 3, 4, 1, 5, 0, 5, 5, 4, 3, 1, 4, 5, 3, 2, 5, 1, 0, 1, 1…length = 24

f 1, 1, 2, 3, 5, 1, 6, 0, 6, 6, 5, 4, 2, 6, 1, 0, 1, 1…length = 16

g 1, 1, 2, 3, 5, 8, 4, 3, 7, 1, 8, 0, 8, 8, 7, 6, 4, 1, 5, 6, 2, 8, 1, 0, 1, 1…length = 24

h 1, 1, 2, 3, 5, 8, 2, 10, 1, 0, 1, 1… length = 10

i 1, 1, 2, 3, 5, 8, 3, 1, 4, 5, 9, 4, 3, 7, 0, 7, 7, 4, 1, 5, 6, 1, 7, 8, 5, 3, 8, 1, 9, 0, 9, 9, 8, 7, 5, 2, 7, 9, 6, 5, 1, 6, 7, 3, 0, 3, 3, 6, 9, 5, 4, 9, 3, 2, 5, 7, 2, 9, 1, 0, 1, 1…length = 60

In each sequence, the zeros cycle regularly, and the cycle length of the sequence is a multiple of the cycle length of the zeros.

Chapter 10

Exercise 10A

1 **a** \$4.80 **b** \$18 **c** \$37.50
d 960 g **e** 840 cm **f** 62.5 g

2 **a** 35 **b** 4500 **c** 33 **d** 18 **e** 12.5

3 **a** 7.5 km **b** 12 cm **c** 1.62 m
d 12.6 m **e** 750 g

4 **a** 630 **b** \$360 **c** 8

5 **a** 3.2 kg **b** \$7.08 **c** 300 g **d** 384 g

6 **a** \$162 **b** 8 **c** 42
d 250 g **e** 8 mL

7 **a** 400 g **b** 500 g
c They are the same.

8 \$4.80; the 500 g box is cheaper by 30c

9 A further 20 days **10** 8 hours

Exercise 10B

1 **a** \$120 **b** 600 litres **c** 21 000 tonnes
d \$1210 **e** 25 litres

2 **a** 195 km **b** 198 km **c** A\$72
d 4 min **e** 2000 km

3 **a** 3100 t **b** 1200 mL **c** \$130
d 25 000 kg **e** 11 100 L

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4 **a** 147 L/week **b** \$0.75/min **c** 140 ML/day **d** 20 L/(100 km) **e** $\frac{1}{800}$ kg/cm^3 **f** \$608/week **g** 40 km/h

5 **a** 72 L/h **b** 32 000 kg/year **c** 0.28% / week **d** $6\frac{2}{3}$ km/L **e** **i** US\$8.64 **ii** A\$25 **f** A\$137.50

6 **a** 78 km/h **b** 2880 km/h **c** **i** US\$15.16 **ii** A\$29.17

7 **a** 170 marbles **b** \$200 **8** $2\frac{2}{9}$ days

Exercise 10C

1 90 km/h **2** 400 km **3** 4 h

4 **a** 4 km **b** 8 km **c** 16 km **d** 28 km **e** 22 km

5 **a** 64 km **b** 128 km **c** 640 km **d** 448 km **e** 352 km

6 **a** 50 km/h **b** 5 m/min **c** 30 km/h **d** 120 m **e** 605 km **f** 20 km

7 42 km **8** 940 km/h **9** 64 km **10** 480 km

11 **a** $1\frac{1}{4}$ h **b** 64 km/h **c** 24 km/h **d** 48 km/h

12 $28\frac{1}{11}$ km/h

13 **a** 1.04 m/s **b** 0.74 m/s **c** 0.83 m/s

14 12:58 p.m. **15** 12 sec

Exercise 10D

1 **a** 7:6 **b** 6:7 **c** 6:13

2 **a** 3:2 **b** 1:2 **c** 1:2 **3** 1:9

4 **a** $\frac{23}{63}$ **b** $\frac{40}{63}$ **5** 1:2

6 **a** $\frac{1}{3}$ **b** $\frac{2}{3}$

7 1:4 **8** 2:5

9 **a** 1:2 **b** 2:3 **c** 2:3 **d** 2:3 **e** 3:1 **f** 3:2 **g** 16:1 **h** 9:1

10 **a** 25:23 **b** $\frac{25}{48}$

11 9:20 **12** 3:4:1

13 **a** 5:14 **b** 5:12 **c** 5:9 **d** 124:85 **e** 6:5 **f** 3:2 **g** 3:2 **h** 3:1

14 24:25

15 **a** 2:3 **b** 2:3 **c** 2:3 **d** 4:9

16 **a** 2:3 **b** 3:1 **c** 1:1 **d** 2:1

17 **a** 32 **b** 48 **c** $5\frac{1}{3}$

18 59:22 **19** 6 cm, 14.4 cm and 15.6 cm

20 **a** 1:1 **b** 2:1 **c** 4:1 **d** 8:1

21 \$120 000, \$150 000 and \$180 000, respectively

22 **a** 11:3:4 **b** 20:13:7 **c** 2:4:1 **d** 16:30:39 **e** 42:27:1 **f** 46:82:89

23 5:3:7 **24** 18 yellow, 12 red and 15 blue

Exercise 10E

1 12 **2** 12 **3** 39 cm

4 shirt: \$128; tie: \$80 **5** \$104

6 **a** \$20 and \$25 **b** 450 kg and 270 kg

7 **a** 54 m and 42 m **b** 120 cm and 24 cm

8 **a** \$9, \$18 and \$45 **b** 25 kg, 30 kg, 40 kg

9 \$2000, \$4000 and \$6000 **10** 3 cm, 3 cm

11 Jane: \$45 500; Anthony: \$36 400

12 9 boys, 15 girls **13** $398\frac{7}{16}$ mm

14 60°, 50° and 70° **15** 46 cm, 69 cm, 138 cm

16 **a** 270 **b** 90 **17** 1:1 **18** 30 cm

Exercise 10F

1 **a** 1:8 **b** 3:11 **c** 1:200 **d** 1:45 **e** 1:300 000 **f** 1:4 000 000 **g** 5:1 **h** 40:1 **i** 25:1

2 **a** 1 cm:7.5 cm **b** 12 cm:40 cm **c** 1 cm:7 m **d** 1 cm:46 m **e** 1 cm:3 km **f** 12 cm:192 km **g** 1 cm:1 mm **h** 1 cm:0.2 mm **i** 1 cm:0.05 mm **j** 1 cm:0.002 mm

3 **a** 125 km **b** 1000 km **c** 150 km **d** 12.5 km

4 **a** 6 km **b** 22 km **c** 15 km **d** 9.6 km **e** 1 km **f** 0.2 km **g** 0.3 km **h** 0.04 km

5 **a** 80 m, 150 m **b** 12 000 m^2

6 **a** 4 m, 1 m **b** 4 m^2

7 **a** 3 cm **b** $1\frac{1}{2}$ cm **c** 1.4 cm **d** 5.6 cm **e** 0.6 cm

8 **a** 1:30 000 000 **b** 1:2000 **c** 50:1 **d** 2000:3 **e** 1:25 **f** $1:1.2\times10^{19}$

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9 **a** ≈3200 km **b** ≈1000 km **c** ≈500 km
d ≈960 km **e** ≈340 km **f** ≈1000 km
g ≈4160 km **h** ≈3760 km **i** ≈680 km
j ≈220 km

10 **b** $30\text{ mm}\times 20\text{ mm}$, approximately $980\,000\text{ km}^2$

11 **a** 94 mm; 9.4 mm **b** 63 mm; 6.3 mm

12 **a** 1:470
b Victoria St frontage 32 m, James St frontage 28 m
c $6\text{ m}\times 6\text{ m}$, area $\approx 36\text{ m}^2$

Review exercise

1 **a** 441, 294 **b** 196, 245, 294
c 171, 114, 57 **d** 300, 360
e 180, 480 **f** 1200, 1800, 3600

2 **a** 2:1 **b** 3:7 **c** 1:6
d 16:27 **e** 147:90:100 **f** 13:26:37
g 10:13 **h** 4:3

3 **a** \$75 **b** \$55 4 $6\frac{2}{3}$ weeks

5 **a** 120 **b** 75

6 **a** $1\frac{1}{2}$ stitches/second **b** $\frac{1}{2}$ row/minute
c 30 stitches/minute

7 **a** 100 minutes **b** $5\frac{1}{4}$ km/h **c** $11\frac{2}{3}$ km

8 \$1.50, \$6 and \$7.50 9 16:25

10 **a** 60 m **b** 6 m 11 \$11 900 12 40

13 first: \$9800; second: \$7000; third: \$2800

14 **a** 2:3 **b** 2:3 **c** 2:3 **d** 4:9

15 12 losses, 15 wins 16 1:2

17 **a** 5:1 **b** 1:3 **c** 1:2 **d** 1:2

18 48, 72 19 2:1

20 **a** 10.5 km **b** 12.25 km

Challenge exercise

1 24 km/h 2 \$185 3 3 4 5:11

5 37.5 cm 6 169:87 7 3.75 h

8 **a** 19:161 **b** 8:37 **c** 4:11

Chapter 11

Exercise 11A

1 **a** $-6a$ **b** $30b$ **c** $6m$
d $-6a$ **e** $15ab$ **f** $5h$
g $21a$ **h** $-3m$ **i** $-6a^2$
j $12x^2$ **k** $-15a^2$

2 **a** $3x+6$ **b** $10x+15$ **c** $36x+18$
d $-20x-35$ **e** $35a-42$ **f** $-20-8b$
g $21z+35$ **h** $12a-6b$ **i** $-66a+55$
j $-42+7x$ **k** $18x+24$ **l** $-10a+15b$
m $18m-15$ **n** $3m-24$ **o** $49m+21$
p $-8+12c$ **q** $\frac{2}{3}x-3$ **r** $-10-\frac{1}{4}x$

3 **a** $11x-15$ **b** $6x-8$ **c** $8x-17$
d $12x-42$ **e** 24 **f** $14m-27$
g $28k-28$ **h** $-51m-21$ **i** $6h-36$
j -12 **k** 0 **l** $35+15m$

4 **a** $23x+34$ **b** $13x+18$ **c** $12x-72$
d $28m+9$ **e** $m-28$ **f** $23k-18$
g $z+34$ **h** $31a+17$ **i** $-16x+59$
j $15p-38$ **k** $6q-46$ **l** $-12m-18$

Exercise 11B

1 **a** $\frac{3x}{5}$ **b** $\frac{5x}{7}$ **c** $\frac{3z}{11}$
d $\frac{m}{11}$ **e** $\frac{7x}{6}$ **f** $\frac{13x}{10}$
g $-\frac{x}{28}$ **h** $\frac{13a}{14}$ **i** $\frac{11x}{42}$
j $\frac{27x}{28}$ **k** $\frac{47b}{66}$ **l** $\frac{8x}{21}$
m x **n** $\frac{17x}{24}$ **o** $\frac{22c}{15}$
p $\frac{19x}{24}$ **q** $-\frac{3x}{4}$ **r** $-\frac{x}{24}$
s $\frac{3x}{4}$ **t** $-\frac{19x}{8}$

2 **a** $\frac{7x}{12}+\frac{7y}{12}$ **b** $\frac{x}{12}+\frac{y}{12}$ **c** $\frac{x}{3}+\frac{47y}{60}$
d $\frac{x}{4}+\frac{2y}{3}$ **e** $\frac{7x}{15}-\frac{y}{12}$ **f** $\frac{2x}{15}+\frac{y}{24}$
g $\frac{5x}{21}+\frac{y}{8}$ **h** $\frac{x}{6}-\frac{19y}{8}$ **i** $\frac{x}{10}-\frac{9y}{8}$

3 **a** $x+24$ **b** $\frac{22x}{7}+30$ **c** $\frac{82x}{21}+\frac{5}{2}$
d $-\frac{119x}{55}-6$ **e** $\frac{115x}{7}+6$ **f** $-\frac{65x}{12}-14$
g $x-20$ **h** $-\frac{19x}{7}-30$

4 **a** $7x+\frac{3}{2}$ **b** $17x+5$ **c** $11x-\frac{29}{10}$
d $\frac{5}{2}$ **e** $x+\frac{31}{2}$ **f** $-22x+10$

Exercise 11C

1 **a** -19 **b** -3 **c** 0
d $-2\frac{2}{5}$ **e** 3 **f** -1
g -35 **h** $2\frac{2}{5}$ **i** -3
j $-1\frac{2}{5}$ **k** $11\frac{1}{2}$ **l** $-\frac{1}{2}$

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2 **a** $x = 4$ **b** $x = -\frac{2}{3}$ **c** $z = -\frac{3}{4}$
d $x = 4$ **e** $z = 1\frac{1}{3}$ **f** $x = -1\frac{7}{12}$
g $x = -1$ **h** $y = -\frac{1}{2}$ **i** $x = -\frac{1}{4}$
j $x = -1$ **k** $x = 1\frac{1}{3}$ **l** $x = \frac{3}{11}$
m $x = 2\frac{2}{5}$ **n** $x = -20$ **o** $p = -7\frac{1}{2}$
p $x = 3$

3 **a** 45 **b** 50 **c** $12\frac{3}{5}$
d $-2\frac{1}{2}$ **e** $44\frac{4}{5}$ **f** 11

4 **a** $x = 1\frac{2}{3}$ **b** $m = 35$ **c** $m = 40$
d $m = 1\frac{17}{19}$ **e** $x = 5\frac{5}{23}$ **f** $x = 6\frac{19}{26}$
g $x = 7\frac{37}{45}$ **h** $x = 192$ **i** $x = 480$
j $x = 56$ **k** $m = 126$ **l** $x = -3\frac{5}{9}$

5 No, there is no solution to this equation. There is no value of x that will make the left-hand side equal the right-hand side.

6 126 m

Exercise 11D

1 **a** $-3(x+6) = -72, x = 18$ **b** $\frac{x}{3} + \frac{x}{4} = 25, x = 42\frac{6}{7}$
c $\frac{3z}{5} + \frac{z}{7} = 30, z = 40\frac{5}{13}$ **d** $\frac{3m}{5} - \frac{2m}{3} = 1, m = -15$
e $-2a + 6 = 3a - 4, a = 2$

2 **a** $\frac{2x}{3} - \frac{x}{2} = 10, x = 60$ **b** $-5(x-10) = 30, x = 4$
c $\frac{2x}{3} + 6 = -10, x = -24$ **d** $\frac{x}{2} - \frac{2}{3} = -1, x = -\frac{2}{3}$

3 3.1 **4** $13 = 6x$, width $= 2\frac{1}{6}$ cm, length $= 4\frac{1}{3}$ cm

5 $\frac{20 + (-10) + x}{3} = -56, x = -178$

6 $\frac{x}{8} - \frac{2x}{9} = 1, x = -10\frac{2}{7}$ **7** $m = 7800$

8 \$7500

9 $2x - 5 = 3(x - 5)$; Anthony is 20 years old, Julian is 10 years old.

10 7200 m **11** 20 km **12** 120 **13** 33°, 57°

Review exercise

1 **a** $10x + 14$ **b** $8b + 12$ **c** $42 - 18x$
d $-20x - 45$ **e** $-9a + 6$ **f** $-20 + 32x$

2 **a** $5x - 3$ **b** $7x + 3$ **c** $15y + 13$
d $8x - 200$ **e** $6x + 19$ **f** $x + 15$
g $-6 - 20x$ **h** $-2y + 21$

3 **a** $7x + 18$ **b** $13x + 9$ **c** $-x - 14$
d $22y + 4$ **e** $2x - 11$ **f** $29 + 5x$
g $-13a - 18$ **h** $-5x + 28$

4 **a** $\frac{x}{2}$ **b** $\frac{2x}{5}$ **c** $-\frac{3x}{7}$
d $\frac{13x}{10}$ **e** $\frac{19x}{20}$ **f** $-\frac{8x}{21}$
g $\frac{5x}{12}$ **h** $\frac{13x}{30}$

5 **a** $x + 10$ **b** $9 - \frac{x}{2}$ **c** $-\frac{3x}{5} - 6$
d $\frac{29x}{20} + 2$ **e** $-4 - \frac{31x}{35}$ **f** $\frac{21x}{20} - 6$

6 **a** 2 **b** -9 **c** 5
d 2 **e** -14 **f** -4
g 6 **h** -4 **i** -4
j 5 **k** 2 **l** 3
m 4 **n** -1 **o** 4
p -100 **q** -24 **r** -30
s $-\frac{1}{3}$ **t** $-3\frac{1}{4}$

7 **a** 3 **b** 2 **c** $-\frac{5}{6}$
d 2 **e** -6 **f** $1\frac{1}{7}$
g -5 **h** $2\frac{2}{3}$ **i** $-1\frac{2}{3}$
j $-\frac{1}{2}$ **k** $1\frac{5}{6}$ **l** $\frac{3}{4}$

8 **a** $x = 14$ **b** $m = 3$ **c** $x = 1\frac{2}{3}$
d $x = 3\frac{2}{11}$ **e** $m = -15$ **f** $x = -2\frac{13}{16}$

9 \$500

10 **a** $\frac{1}{2}(x + 10) = -32, x = -74$
b $\frac{x}{4} + \frac{x}{5} = 60, x = 133\frac{1}{3}$
c $-6a + 3 = 10a - 20, a = 1\frac{7}{16}$
d $\frac{3z}{5} - \frac{2z}{7} = 10, z = 31\frac{9}{11}$
e $\frac{7z}{10} + 6 = z - 4, z = 33\frac{1}{3}$

11 $m = 79\,000$

12 $x = 42$; sides are 42 cm, 6 cm, 24 cm and 28 cm

Challenge exercise

1 **a** $8\frac{11}{16}$ **b** $16\frac{1}{5}$

2 $x \to x + 30 \to 5x + 150 \to 10x + 150 \to 10x + 100 \to x + 10 \to x$

3 **a** $33\frac{1}{3}$ minutes **b** $27\frac{7}{9}$ km **4** 80 km

5 -12 **6** $18\frac{6}{13}$ km **7** \$105

8 $2\frac{12}{19}$ kg of peaches, $7\frac{7}{19}$ kg of grapes

9 $x = \frac{2}{3}$

10 $625 - x = \frac{3}{4}\left(400 - \frac{x}{4}\right), x = 400$; 500 boys.

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11 250 mL of solution A and 750 mL of solution B

12 $28\frac{4}{7}$ minutes

Chapter 12

Exercise 12A

1 congruent to i: H, K; to ii: C, I, L; to iii: A, G, J

2 **a** K **b** L **c** M
d N **e** O **f** KL
g KM **h** LN **i** ON
j $\angle KLM$ **k** $\angle OKL$ **l** $\angle NKL$

3 **a** yes **b** no **c** no
d yes (but you may like to argue about this)
e no **f** yes
g d, p and perhaps q (depending on font)
h 6

Exercise 12B

1 **a** $\Delta ABC \equiv \Delta XZY$ **b** $\Delta ABC \equiv \Delta DEF \equiv \Delta EDF$
c $\Delta ABC \equiv \Delta EFG$ **d** $\Delta MNO \equiv \Delta XYZ$

Exercise 12C

1 **a** SSS **b** yes, AAS

2 **b i** You can't.
ii The longest side of a triangle is shorter than the sum of the other two sides. (This is known as the triangle inequality.)
c It is false, by the triangle inequality.

3 **a** ≈ 10.8 cm, AAS
b impossible: $105° + 80° > 180°$

4 **a** $\Delta ABC \equiv \Delta PQR$ (SSS) **b** $\Delta ABC \equiv \Delta RPQ$ (SAS)
c $\Delta RPI \equiv \Delta CTU$ (AAS) **d** $\Delta TSP \equiv \Delta GFM$ (SSS)
e not congruent **f** $\Delta OFQ \equiv \Delta PAH$ (AAS)

5 **a** $\Delta SRT \equiv \Delta KLJ$ (AAS), $x = 10$ cm (matching sides of congruent triangles)
b $\Delta APQ \equiv \Delta BPQ$ (SSS), $\theta = 80°$ (matching angles of congruent triangles)
c $\Delta FBC \equiv \Delta MUS$ (AAS), $x = 6$ m (matching sides of congruent triangles)

6 **a** $\Delta ABC \equiv \Delta VTP$ (SSS); $\angle ACB = \angle VPT$ (matching angles of congruent triangles)
b $\Delta DFH \equiv \Delta TEM$ (AAS); $FH = EM$ (matching sides of congruent triangles)

Exercise 12D

1 **a** SAS **b** RHS
c error in measurement

2 **a** ≈ 6.1 cm
b 48°, to the nearest degree.

3 **a** $\Delta KJL \equiv \Delta YXZ$ (SAS) **b** $\Delta RST \equiv \Delta VWU$ (RHS)
c not necessarily congruent **d** $\Delta AJZ \equiv \Delta KOR$ (SAS)
e $\Delta HSI \equiv \Delta BYT$ (SAS) **f** $\Delta ABC \equiv \Delta KLW$ (AAS)

4 **a** $\Delta ACB \equiv \Delta PRQ$ (SAS), $x = 5$ m (matching sides of congruent triangles)
b $\Delta ABC \equiv \Delta BAD$ (AAS), $\theta = 70°$ (matching angles of congruent triangles)
$x = 7$ cm (matching sides of congruent triangles)
c not necessarily congruent
d $\Delta ABC \equiv \Delta CDA$ (RHS), $x = 10$ (matching sides of congruent triangles)

5 **a** $\Delta MKN \equiv \Delta TPQ$ (SAS); $\angle M = \angle T$ (matching angles of congruent triangles)
b $\Delta LDW \equiv \Delta LDM$ (SAS); $WL = ML$ (matching sides of congruent triangles)
c $\Delta JIL \equiv \Delta LKJ$ (RHS); $IJ = KL$ (matching sides of congruent triangles)
d $\Delta HIT \equiv \Delta GIW$ (SAS); $HT = GW$ (matching sides of congruent triangles)

Exercise 12E

1 **a** RHS, $x = 20$ cm **b** SSS, $\theta = 40°$
c AAS, $x = 5$ m **d** RHS, $\theta = 20°$
e AAS, $x = 8$ cm **f** SAS, $x = 10$ cm

2 **a i** RHS
ii $LS = MS$ (matching sides of congruent triangles)
b i SSS
ii $\angle OCZ = \angle ODZ$ (matching angles of congruent triangles)
c $\Delta APB \equiv \Delta AQB$ (SSS); $\angle P = \angle Q$ (matching angles of congruent triangles)
d $\Delta OSN \equiv \Delta OTN$ (RHS); $SN = TN$ (matching sides of congruent triangles)
e $\Delta FOH \equiv \Delta IOG$ (SAS); $\angle OFH = \angle OIG$ (matching angles of congruent triangles)
f $\Delta VAW \equiv \Delta GAF$ (AAS); $VW = GF$ (matching sides of congruent triangles)

3 **a** $\Delta AOB \equiv \Delta QOP$ (SAS), $\angle B = \angle P$ (matching angles of congruent triangles), $AB \parallel PQ$ (alternate angles equal)
b $\Delta AOB \equiv \Delta QOP$ (SAS), $\angle B = \angle P$ (matching angles of congruent triangles), $AB \parallel PQ$ (alternate angles equal)
c $\Delta ABQ \equiv \Delta QPA$ (SSS), $\angle A = \angle Q$ (matching angles of congruent triangles), $AB \parallel PQ$ (alternate angles equal)

4 **a** $\Delta BAP \equiv \Delta BAQ$ (SAS), $\angle ABP = \angle ABQ$ (matching angles of congruent triangles), $\angle ABQ = 90°$
b $\Delta OBP \equiv \Delta OBQ$ (SSS), $\angle ABP = \angle ABQ$ (matching angles of congruent triangles), $\angle ABQ = 90°$
c $\Delta APB \equiv \Delta QVB$ (SAS), $\angle ABP = \angle QBV$ (matching angles of congruent triangles), $\angle ABQ = 90°$

5 **a i** SSS
ii $\angle DMG = \angle DME$ (matching angles of congruent triangles), $\angle DMG = 90°$
iii $\angle GMF = \angle EMF$, $\Delta GMF \equiv \Delta EMF$ (SAS)
iv $GF = EF$ (matching sides of congruent triangles)

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b **i** SSS

ii matching angles of congruent triangles

iii SAS

iv $\angle OWA = \angle OWB$ (matching angles of congruent triangles),
$\angle OWA = 90°$, and $AW = BW$ (matching sides of congruent triangles)

Exercise 12F

1 **a** $\theta = 75°$ $(AC = BC)$, $\alpha = 30°$ (angle sum of triangle)

b $\alpha = \beta = \gamma = 60°$ (equilateral triangle)

c $\alpha = 60°$ (angle sum of triangle),
$x = y = 6$ m (equilateral triangle)

d $x = 12$ cm (base angles equal),
$\theta = 20°$ (angle sum of triangle)

e $\angle PQR = 60°$ $(PR = QR)$, $\theta = 60°$ (angle sum of triangle),
$y = 7$ cm (equilateral triangle)

f $\alpha = 60°$ (angle sum of triangle),
$x = 3$ cm (equilateral triangle)

g $\alpha = 60°$ (equilateral triangle),
$\angle QLM = \beta$ (isosceles triangles),
$\alpha = 2\beta$ (external angle of triangle),
$\beta = 30°$, $y = 5$ (isosceles)

h $\theta = 70°$ ($BG = CG$, vertically opposite angles),
$\alpha = 40°$ (angle sum of triangle),
$\beta = 140°$ (straight angle),
$\angle FGB = 70°$ (alternate angles, $AD \parallel FG$),
$\gamma = 110°$ (straight angle at G)

i $\angle ABD = 55°$ (straight angle at B),
$\angle BAD = 55°$ (isosceles triangle),
$\angle ADB = 70°$ (angle sum of ΔABD),
$\theta = 70°$ (vertically opposite),
$\angle AGH = 125°$ (co-interior angles $AC \parallel EH$),
$\alpha = 125°$ (vertically opposite)

j $\angle DCG = 55°$ (vertically opposite),
$\angle CDG = 55°$ (isosceles ΔDGC),
$\alpha = 70°$ (angle sum of triangle),
$\angle CGE = 55°$ (corresponding angles, $BD \parallel EH$),
$\beta = 125°$ (vertically opposite)

k $\angle ACE = 90°$ (co-interior angles $AB \parallel CE$),
$\alpha = 50°$ (angle sum in ΔDCA),
$\angle FDC = 50°$ $(\Delta FDC \equiv \Delta ADC)$,
$\theta = 130°$ (straight angle at D)

l $\alpha = 60°$ (equilateral triangle),
$\beta = 120°$(straight angle at K)

m $\alpha = 60°$ (vertically opposite angles, equilateral triangle),
$\gamma = \beta = 120°$ (straight angles)

n $\alpha = 60°$ $(OB = OC)$, $\beta = 25°$ (angle sum of triangle, $OB = OA$), $y = 2$ m

2 **a** angle sum of triangle

b all 3 pairs of base angles are equal

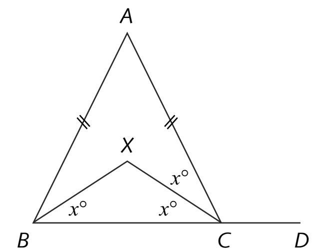
3 **a** SSS ($AC = BC, AM = BM, CM$ is common)

b matching angles of congruent triangles

4 **a** AAS ($\angle A = \angle B, \angle ACN = \angle BCN, CN$ is common)

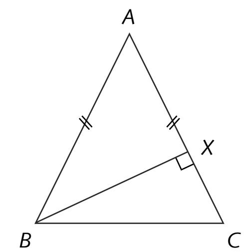
b matching sides of congruent triangles

5



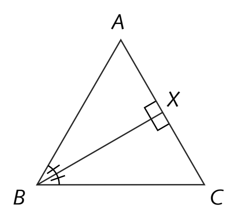
$\angle ABC = \angle ACB$ (ΔABC is isosceles).
Let $\angle XBC = x°$, then $\angle XCB = x°$ (angle bisectors of equal angles)
$\angle BXC = (180 - 2x)°$ (angle sum of ΔBXC)
$\angle ACD = (180 - 2x)°$ ($\angle ACB = 2x°$ and straight angle)

6



$\angle ACB = 90° - \frac{1}{2}\angle BAC$ (angle sum of isosceles ΔABC)
$\angle XBC = 90° - (90° - \frac{1}{2}\angle BAC)$ (right-angled ΔBXC)
$= \frac{1}{2}\angle BAC$ (angle sum of right-angled ΔBXC)

7



$\Delta AXB \equiv \Delta CXB$ (AAS)
Therefore $AB = AC$ (matching sides)
Hence $\angle BAX = \angle BCX$ (isosceles triangle)

Review exercise

1 **a** $\Delta NOC \equiv \Delta RGU$ (SSS) **b** $\Delta ETN \equiv \Delta BAP$ (SAS)

c $\Delta LNM \equiv \Delta STP$ (AAS)

2 **a** $\Delta ABC \equiv \Delta DEC$ (AAS); $ED = BA$ (both 6 cm in length);
$\angle BAC = \angle EDC$ (alternate angles);
$\angle ABC = \angle DEC$ (alternate angles)

b 4 cm

3 **a** $\Delta ABC \equiv \Delta FED$ (SSS) **b** $\Delta ABC \equiv \Delta DEF$ (RHS)

c $\Delta ABC \equiv \Delta EDF$ (SAS) **d** $\Delta ABC \equiv \Delta EFD$ (AAS)

4 **a** $\Delta ABC \equiv \Delta GHI$ (SSS)

b $\Delta ABC \equiv \Delta XYZ$ (SAS)

c $\Delta ABC \equiv \Delta LDP$ (AAS)

5 **a** $\Delta ACD \cong \Delta BDC$ (SSS) and $\Delta ACB \cong \Delta BDA$ (SSS)

b $\Delta NOK \equiv \Delta LOK$ (RHS); $\Delta NOM \equiv \Delta LOM$ (RHS);
$\Delta KNM \equiv \Delta KLM$ (SSS)

6 **a** $\alpha = 30°, \beta = 100°, \gamma = 50°$

b $\alpha = 40°, \beta = 80°$

c $a = 12, b = 12$ **d** $x = 6, y = 4$ **e** $b = 3, a = 6$

Challenge exercise

1 **b** $OP = OQ$ (radii of circle), $PM = QM$ (radii of circle), $OM = OM$ (common side); $\Delta PMO \equiv \Delta QMO$ (SSS)

c $\angle QOM = \angle POM$. Therefore OM bisects $\angle AOB$ (matching angles of congruent triangles).

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2 **b** $AP = BP$ (radii of circle), $AQ = BQ$ (radii of circle), $PQ = PQ$ (common side); $\Delta APQ \equiv \Delta BPQ$ (SSS)

c $\angle APQ = \angle BPQ$ (matching angles of congruent triangles)

d $AP = BP$ (radii of circle), $\angle APM = \angle BPM$ (proved in **c**), $PM = PM$ (common); $\Delta APM \equiv \Delta BPM$ (SAS)

e $AM = BM$ (matching sides of congruent triangles), $\angle AMP = \angle BMP$ (matching angles of congruent triangles)

Also, $\angle AMP + \angle BMP = 180°$ (straight line), so $\angle AMP = \angle BMP = 90°$

3 **b** $FA = FB$ (radii of circle), $AG = BG$ (radii of circle), $FG = FG$ (common side); $\Delta AFG \equiv \Delta BFG$ (SSS)

c $\angle AFG = \angle BFG$ (matching angles of congruent triangles)

d $FA = FB$ (radii of circle), $\angle AFM = \angle BFM$ (proved in **c**), $FM = FM$ (common); $\Delta AFM \equiv \Delta BFM$ (SAS)

e $\angle AMF = \angle BMF$ (proved in **d**), $\angle AMF + \angle BMF = 180°$ (straight line), $\angle AMF = \angle BMF = 90°$

4 **a** SAS ($AC = BC$, $\angle C$ is common)

b matching angles of congruent triangles

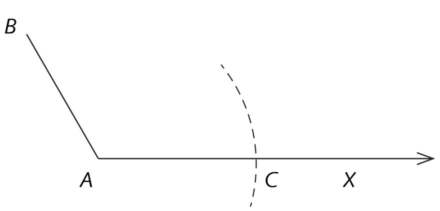
5 **a** AAS ($\angle A = \angle B$, AB is common)

b matching sides of congruent triangles

6 The angle is not included, so we may not use SAS (or any other tests) to show congruence.

7 The angle is not included, so we may not use SAS.

8



$\angle BAC$ is given. AB is given. Length BC is given. The circle with centre B and radius BC can cut the ray AX at only one point.

9 **b** $\angle APO = \alpha$ and $\angle BPO = \beta$ (isosceles triangles), so $2\alpha + 2\beta = 180°$ (angle sum of triangle), hence $\alpha + \beta = \angle APB = 90°$ (adjacent angles).

10 **b** This is quite obvious if you draw the diagrams correctly.

11 **b** SAS

c $MA = MB = MC$ (matching sides of congruent triangles)

d $\Delta MBP \equiv \Delta MCP$ (SSS), hence $\angle MPB = 90°$ (angle sum of triangle, matching angles of congruent triangles).

e When the triangle is obtuse, the circumcentre lies outside the triangle.

Chapter 13

Exercise 13A

2 $WZ = 5$ cm, $YZ = 4$ cm (opposite sides of parallelogram)

5 **a** $\alpha = 110°$ (opposite angles of parallelogram $ABCD$), $\beta = 70°$ (straight angle at B)

b $y = 6$ cm, $z = 8$ cm (diagonals of parallelogram $PQRS$ bisect each other),

$x = 12$ cm (opposite sides of parallelogram $PQRS$)

c $\alpha = 80°$ (opposite angles of parallelogram $KLQP$, straight angle at Q),

$\beta = 80°$ (opposite angles of parallelogram $LMRQ$, vertically opposite angles at M)

d $\beta = 360 - 50 - 30 = 260°$ (opposite angles of parallelogram $RSVW$ and $STUV$)

e $\alpha + 3\alpha = 180°$, $\alpha = 45°$ (co-interior angles, $GD \parallel FE$)

f $y = 7 - y$, $y = 3.5$; $x = 2x - 5$, $x = 5$ (diagonals of parallelogram $JKLM$ bisect each other); $18 - 2z = 4$, $z = 7$ (opposite sides of parallelogram $JKLM$)

6 **a** 68 cm **b** 38 km **c** 25 m **d** 72 cm

7 **b, c** Yes, diagonals of parallelogram bisect each other.

8 $AC = 4x = 2 \times 2x = 2BD$ (diagonals of parallelogram bisect each other)

9 **a** $2\alpha + 2\beta = 360°$, $\alpha + \beta = 180°$

b $RS \parallel UT$ (co-interior angles are supplementary), $RU \parallel ST$ (co-interior angles are supplementary)

10 **a** SSS

b $\angle ABD = \angle CDB$ (matching angles of congruent triangles), so $AB \parallel DC$ (alternate angles are equal); $\angle CBD = \angle ADB$ (matching angles of congruent triangles), so $AD \parallel BC$ (alternate angles are equal)

12 Let $\angle BAD = \angle BCD = 2\alpha$ (opposite angles of parallelogram)
$\angle CYD = \alpha$ (alternate angles, $BC \parallel AD$), $AX \parallel YC$ (corresponding angles equal)

13 $\Delta PAB \equiv \Delta QCD$ (SAS). Therefore $PB = DQ$, similarly $PD = BQ$
Quadrilateral with opposite sides equal is a parallelogram. (See question 10 of this exercise.)

Exercise 13B

2 **a** $x = y = z = 6$ cm (sides of rhombus $ABCD$)

b $\beta = 20°$ (diagonal of rhombus $PQRS$ bisects angles), $\alpha = 90°$ (diagonals meet at right angles), $\gamma = 70°$ (angle sum of ΔMRS)

c $x = 20$ cm (sides of rhombus $JKLM$), $\alpha = \gamma = 90°$ (diagonals meet at right angles), $\beta = 30° = \theta$ (angle sum of triangle, alternate angles, $JM \parallel KL$)

d $\alpha = \angle RTU = 20°$ (alternate angles, $RS \parallel UT$; straight angle at T; diagonal of rhombus $RSTU$ bisects angles);

$\beta = 70°$ (angle sum of ΔRUV; diagonals meet at right angles)

e $\beta = 25°$ (diagonal of rhombus $ABFE$ bisects angles), $\alpha = 50°$ (opposite angles in rhombus; vertically opposite angles at F), $\theta = 50°$ (corresponding angles, $AB \parallel EF$)

f $x = 8$ cm (sides of rhombus $APBO$), $\alpha = 60°$ (equilateral ΔAPB), $\theta = 30°$ (diagonal of rhombus $APBD$ bisects angle)

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6 **e** One circle passes through A and C; the other through B and D. Diagonals bisect each other.

Exercise 13C

1 **a** $x = y = z = 7$ m (diagonals of rectangle are of equal length and bisect each other)

b $\beta = 30°$ (diagonals of rectangle $PQRS$; base angles of isosceles ΔPTS),
$\alpha = 30°$ (alternate angles, $SP \parallel RQ$),
$\theta = 120°$ (angle sum of ΔPTS),
$\gamma = 60°$ (straight angle at T)

c $\alpha = 35°$ (diagonals of rectangle $FGQP$; base angles of isosceles ΔFMG),
$\beta = 55°$ (complementary angles),
$\theta = \gamma = 35°$ (base angles of isosceles ΔPMQ)

d $x = y = z = 5$ (diagonals of square $RSTU$),
$\theta = 90°, \alpha = \beta = 45°$ (diagonals of square)

e $\angle FGI = 45°$ (right-angled isosceles ΔFGI)
$\alpha = \beta = 180° - 45° - 45° = 90°$(straight angle at G)

f $\alpha = 20°$ (alternate angles, $RS \parallel UV$),
$\beta = 160°$ (straight angle at S)

2 **c** The diagonals are equal and bisect each other.

3 **a** All four sides are equal and one angle is a right angle.

d The diagonals are equal and bisect each other.

4 Use co-interior angles four times to show that all angles are right angles.

5 **a** $AD = BC$, $\angle ADC = \angle BCD$, and CD is common, so congruence by SAS.

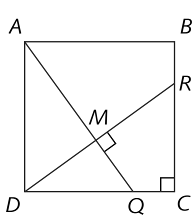
b matching sides of congruent triangles

6 **a** 5 **b** $\frac{7\sqrt{2}}{2}$

7 Show that there are four congruent triangles and thus that all vertex angles are equal.

8 **a** $DC = DA$ (sides of a square)
Let $\angle DAQ = \alpha°$
$\angle AQD = (90 - \alpha)°$ (angle sum of ΔADQ)
$\angle MDQ = 90° - (90 - \alpha)°$ (angle sum of ΔDMQ)
$\Delta DAQ \equiv \Delta CDR$ (AAS)
$AQ = DR$ (matching sides of congruent triangles)

b Consider suitable translations.



Review exercise

1 **a** Diagonals bisect each other.

b Diagonals bisect each other at right angles, bisect interior angles.

c Diagonals equal and bisect each other.

d Diagonals equal, bisect each other at right angles, and meet each side at 45°.

2 **a** $\alpha = \beta = 110°$ (opposite angles of parallelogram $BCED$; vertically opposite angles at B and E)

b $3x + 5 = 15 - 2x$, $x = 2$ (opposite sides of parallelogram $PQRS$),
$\alpha + 10 = 2\alpha - 60$, $\alpha = 70°$ (opposite angles of parallelogram);
$\beta = 110°$ (co-interior angles, $PQ \parallel SR$)

c $y = 7$ cm, $z = 5$ cm (diagonals of parallelogram $ABCD$ bisect each other),
$x = 10$ cm (opposite sides of parallelogram)

3 **a** $\alpha = 90°$ (diagonals of rhombus $MNOP$ meet at right angles),
$\beta = \theta = 25°$ (angle sum of ΔPMT, alternate angles, $PM \parallel ON$),
$\gamma = 65°$ (diagonals bisect interior angles)

b $\alpha = 55°$ (straight angle at C; diagonals of rhombus $ABCD$ bisect interior angles),
$\beta = 35°$ (alternate angles, $AB \parallel CD$; diagonals bisect interior angles)

c $OA = OC$(radii of circle)
Therefore, $AOBC$ is a rhombus.
Therefore, $x = 10, \theta = 140°$

4 **a** $x = y = z = 10$ cm (diagonals of rectangle $ABCD$)

b $\alpha = 90°, \beta = 45°$ (diagonals of square $PQRS$)

c $\theta = 60°$ (angle sum of ΔFGH); $\gamma = 30°$,
$\beta = 120°$(diagonals of rectangle $FGHI$, isosceles ΔFTI),
$\alpha = 60°$(straight angle at T)

d $\alpha = 40°$ (alternate angles, $AE \parallel DF$),
$\beta = 140°$ (straight angle at E)

5 **a** Let $\angle A = \alpha$, $\angle D = 180° - \alpha$ (co-interior angles, $AB \parallel DC$),
$\angle C = 180° - (180° - \alpha) = \alpha$ (co-interior angles, $AD \parallel BC$)

b $\angle BAC = \angle DCA$, $\angle BCA = \angle CAD$ (alternate angles, $AB \parallel DC$, $AD \parallel BC$); AC is common.
Hence $\Delta ABC \equiv \Delta CDA$ (AAS), and $AB = CD$ (matching sides of congruent triangles).

c Let M be the point of intersection of AC and BD.
$\angle BAM = \angle DCM$, $\angle ABM = \angle CDM$ (alternate angles, $AB \parallel DC$); $AB = CD$ (opposite sides of parallelogram).
Hence $\Delta ABM \equiv \Delta CDM$ (AAS), and $AM = CM$ (matching sides of congruent triangles).

6 **a** Let $\angle MAB = \alpha$ and M be the point of intersection of AC and BD.
$\angle DCA = \alpha$ (alternate angles, $AB \parallel DC$),
so $\angle DAC = \alpha$ (base angles of isosceles ΔADC).

b $\angle MBA = \beta$ (alternate angles, $AD \parallel BC$, and diagonal bisects ΔABC);
$\angle MAD = \alpha$ (diagonal bisects $\angle DAB$).
Hence $2\alpha + 2\beta = 180°$ (angle sum of ΔABD),
$\alpha + \beta = 90° = \angle AMB$ (angle sum of ΔABM).

7 Let $AB = x$. Then $CD = x$. Let $AD = y$. Then $BC = y$.
$AC^2 = AB^2 + BC^2 = x^2 + y^2$
$BD^2 = BC^2 + DC^2 = x^2 + y^2$
Hence $AC = BD$.

Challenge exercise

1 **a** SAS (vertically opposite angles at M)

b $\angle WXM = \angle YZM$ (matching angles of congruent triangles), so $WX \parallel ZY$ (alternate angles are equal).

c SAS (vertically opposite angles at M)

d $\angle WZM = \angle YXM$ (matching angles of congruent triangles), so $WZ \parallel XY$ (alternate angles are equal).

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2 **a** SAS (diagonals of parallelogram $PQRS$ bisect each other)
b matching sides of congruent triangles

3 **a** SSS
b $\angle ABD = \angle CDB$ (matching angles of congruent triangles), so $AB \parallel DC$ (alternate angles are equal); $\angle CBD = \angle ADB$ (matching angles of congruent triangles), so $AD \parallel BC$ (alternate angles are equal).

4 **a** $\alpha + x + y = \gamma + x + y$, so $\alpha = \gamma$,
b Using the angle sums of ΔABC and ΔADC.
$\beta + \frac{\alpha}{2} + \frac{\gamma}{2} = \theta + \frac{\alpha}{2} + \frac{\gamma}{2}$, so $\beta = \theta$,
c $\angle CAD = \frac{\alpha}{2} + \frac{\gamma}{2} = \angle ACD$. Therefore ΔADC is isosceles.
Therefore $AD = DC$.

5 **a** SSS (opposite sides of parallelogram are equal)
b $\angle A = \angle B$ (matching angles of congruent triangles), and $\angle A + \angle B = 180°$ (co-interior angles, $AD \parallel BC$), so $\angle A = \angle B = 90°$, hence all angles are 90°.

6 $\angle ABX = \angle BXC$ (alternate angles, $AB \parallel DC$), ΔXBC is isosceles (equal base angles)
Therefore $XC = BC$. Similarly $XD = AD$.
$DC = DX + XC = BC + AD = 2BC$. $AB = DC$ (opposite sides of parallelogram)
Hence $AB = 2BC$.

7 Let $\angle DPC = \alpha$ and $\angle PCD = \beta$, $\angle QPB = \alpha$ (vertically opposite)
$\angle PBQ = \beta$ (alternate angles, $BQ \parallel DC$), $BP = PC$ (given)
Hence $\Delta PBQ \equiv \Delta PCD$ (AAS)
$AB = CD$ (opposite sides of parallelogram)
$CD = BQ$ (matching sides of congruent triangle)
Therefore $AQ = AB + BQ = 2AB$

Chapter 14

Exercise 14A

3 **a** 90 **b** 180 **c** semicircle
4 **a** 120° **b** 72°, 45°, 36°

Exercise 14B

2 20c: $r \approx 14$ mm, $C \approx 88.0$ mm; 10c: $r \approx 11.8$ mm, $C \approx 74.1$ mm; 5c: $r \approx 9.5$ mm, $C \approx 59.7$ mm

3 **a** 28π cm **b** 14π cm
c 7π mm **d** 84π m

4 **a** 88 cm **b** 44 cm
c 22 mm **d** 264 m

5 **a** 14π cm **b** 7π cm
c $\frac{7\pi}{2}$ mm **d** 42π m

6 **a** 44 cm **b** 22 cm
c 11 mm **d** 132 m

7 **a** 20π cm **b** 10π cm
c 40π mm **d** 30π m

8 **a** 62.8 cm **b** 31.4 cm
c 125.6 mm **d** 94.2 m

9 **a** 10π cm **b** 5π cm
c 20π mm **d** 15π m

10 **a** 31.4 cm **b** 15.7 cm
c 62.8 mm **d** 47.1 m

11 It doubles.

12 21 cm

13 10 cm

14 It halves.

15 **a** $(70 + 35\pi)$ cm **b** 216 cm

16 **a** 28π cm **b** 88 cm

17 **a** $(14 + 2\frac{1}{3}\pi)$ cm **b** $21\frac{1}{3}$ cm

18 **a** 25.23 m **b** 19.57 cm **c** 104.52 m

Exercise 14C

1 20c: $A \approx 615$ mm^2; 10c: $A \approx 437$ mm^2; 5c: $A \approx 283$ mm^2

2 **a** 196π cm^2 **b** 49π cm^2
c $12\frac{1}{4}\pi$ mm^2 **d** 1764π m^2

3 **a** 616 cm^2 **b** 154 cm^2
c $38\frac{1}{2}$ mm^2 **d** 5544 m^2

4 **a** 49π cm^2 **b** $12\frac{1}{4}\pi$ cm^2
c $3\frac{1}{16}\pi$ mm^2 **d** 441π m^2

5 **a** 154 cm^2 **b** $38\frac{1}{2}$ cm^2
c $9\frac{5}{8}$ mm^2 **d** 1386 m^2

6 **a** 100π cm^2 **b** 25π cm^2
c 400π mm^2 **d** 225π m^2

7 **a** 314 cm^2 **b** 78.5 cm^2
c 1256 mm^2 **d** 706.5 m^2

8 **a** 25π cm^2 **b** $6\frac{1}{4}\pi$ cm^2
c 100π mm^2 **d** $56\frac{1}{4}\pi$ m^2

9 **a** 78.5 cm^2 **b** 19.625 cm^2
c 314 mm^2 **d** 176.625 m^2

10 It quadruples (increases by a factor of 4).

11 21 cm

12 10 cm

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13 Both decrease by a factor of 3.

14 Both decrease by a factor of 4.

15 **a** $612\frac{1}{2}\pi$ cm^2 **b** 1925 cm^2

16 2772 cm^2

17 **a** $(196+98\pi)$ cm^2 **b** 504 cm^2

18 **a** $8\frac{1}{6}\pi$ cm^2 **b** $25\frac{2}{3}$ cm^2

19 **a** 26.17 m^2 **b** 7.065 cm^2

20 216π cm^2

21 Let the radius of the circle be r. The area of the outside square is $4r^2$. The area of the circle is πr^2. The side length of the inside square is $\sqrt{2}r$. The area of the inside square is $2r^2$. Therefore, $2 < \pi < 4$.

Exercise 14D

1 176 cm^2

2 $(50-4\pi)$ cm^2; $37\frac{3}{7}$ cm^2

3 $(32+6\pi)$ cm^2

4 813.42 cm^2

5 **a** circle **b** circle **c** circle

6 118 mm^2

7 **a** 3 cm **b** $(36-9\pi)$ cm^2

8 **a** 10 cm **b** 21.5 cm^2

9 $25\frac{1}{7}$ cm^2

10 **a** 8 cm **b** 47.1 cm^2

11 $398\frac{6}{7}$ m

12 **a** $\left(15+\frac{9\pi}{2}\right)$cm^2 **b** $\left(30+\frac{169\pi}{6}\right)$cm^2

Review exercise

2 **a** 4π m **b** 86π mm
c 24π mm **d** 374π cm

3 **a** 12.56 m **b** 270.04 mm
c 75.36 mm **d** 1174.36 cm

4 **a** 88 mm **b** 110 cm **c** 264 m

5 **a** 23.13 cm **b** 7.14 cm **c** 17.85 cm
d 41.12 mm **e** $57\frac{23}{75}$ cm

6 **a** 441π mm^2 **b** 49π m^2
c 3969π cm^2 **d** 1225π m^2

7 **a** 1386 mm^2 **b** 154 m^2
c 12 474 cm^2 **d** 3850 m^2

8 **a** 50.24 m^2 **b** 200.96 mm^2 **c** 314 cm^2
d 7.065 cm^2 **e** 56.52 cm^2 **f** 628 mm^2

9 264 cm^2

10 $\left(99+20\frac{1}{4}\pi\right)$ cm^2

11 **a** 123.84 mm^2 **b** 6.88 m^2
c 526.75 mm^2 **d** 252.77 cm^2

12 52π cm

13 13 266.5 cm^2

14 107.765 cm^2 **15** $\left(\frac{25\pi}{8}-6\right)$cm^2 **16** $4:\pi$

17 $(4-\pi)$ cm^2 **18** 36π cm^2 **19** 25π cm^2

20 60π cm **21** 21.048 m **22** 708 m^2

Challenge exercise

1 **a** 91.7 m **b** 270.75 m^2

2 70π cm^2 **3** about 298 500

4 **a** 4 times **b** 4 times

5 34 cm **6** π^2 **7** 9 : 16

8 **a** 1 **b** 2

9 $\frac{1}{\sqrt{2}}$ cm^2 **10** $\pi : 2$

11 1 : 1 **12** $(50\pi-80)$ cm^2

Chapter 15

Exercise 15A

1 **a** 39 m^2 **b** 51 cm^2 **c** 42 cm^2
d 60 cm^2 **e** 62 cm^2 **f** 108 cm^2

2 **a** 13 m^2 **b** 20 cm^2
c 80 mm^2 **d** 76 cm^2

3 15.6 ha

4 1, 24; 2, 12; 3, 8; 4, 6; 6, 4; 8, 3; 2, 12; 24, 1

5 **a** $8ab$ **b** $9x^2$
c $42ab$ **d** $6y^2$

6 $685

7 8 cm; 72 cm^2

8 12 m; 60 m^2

9 13 m^2

10 **a** 20 m **b** 10 m **c** 8 m, 5.6 m
d 23.6 m^2 **e** 10 m **f** 32.8 m^2

Exercise 15B

1 **a** 42 cm^2 **b** 3 cm **c** 5 m
d 12 km **e** 60 km **f** 120 m
g 100 m **h** 16 cm

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2 a 104 cm^2 b 168 m^2 c 39 cm^2
d 63 m^2 e 640 mm^2 f 12.75 cm^2
g 20 cm^2 h 28 cm^2

3 a 24 cm^2 b 14 cm^2
c 81 m^2 d 300 cm^2

4 a 55 cm^2 b 94.5 cm^2
c 132 cm^2 d 330 cm^2

5 a 96 b 156
c 96 d 80

6 a $14x^2$ b $56x^2$
c $40x^2$ d $5x^2$

7 Join them together to form a parallelogram of base $a+b$ and height h.

8 16; 360

9 24 and 21; 252

10 a $x = 11$ cm b 66 cm^2 c 66 cm^2

11 a $L = 11$
b 92 cm^2
c yes, one obtuse angle at the top and one at the bottom

12 a 38 cm^2 b 37 cm^2 c 39 cm^2
d 40 cm^2

13 28 cm^2

Exercise 15C

1 840 mm^3

2 a 20 b 27 c 12
d 16 e 18 f 35

3 a 126 cm^3 b 180 cm^3 c 3 m
d 4 m e $156\,800\text{ m}^3$ f 256 mm^3
g 8 m h 4 cm

4 a 48 cm^3 b 54 cm^3
c 1280 cm^3 d 80 cm^3

5 600 m^3 6 4.76 m^3 7 343 cm^3 8 900 cm^3

9 9.66 m^3

Exercise 15D

1 a 60 cm^3 b 336 cm^3
c 400 cm^3 d 3300 cm^3

2 a 404 cm^2 b $12\,120\text{ cm}^3$

3 a 31.5 m^2 b 94.5 m^3

4 116 m^3 5 42 m^3

6 2430 m^3 7 576 cm^3

8 a 1920 cm^3 b 17.6 m^3

9 a 44 cm^2 b 18 cm

10 a 12 cm b 60 cm^2 c 80 cm

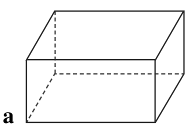
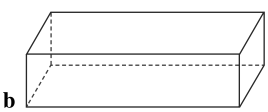
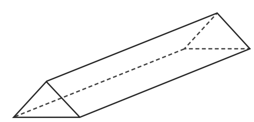
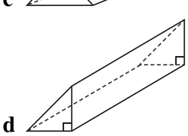
11 26 cm

Exercise 15E

1 a i $3920\pi\text{ m}^3$ ii $12\,320\text{ m}^3$
b i $196\pi\text{ cm}^3$ ii 616 cm^3
c i $5292\pi\text{ mm}^3$ ii $16\,632\text{ mm}^3$

2 $577\tfrac{1}{2}\text{ cm}^3$ 3 $56\,520\text{ mm}^3$

Exercise 15F

1 a

b

c

d


2 a 150 m^2 b 162 cm^2
c 114 mm^2 d 636 cm^2

3 a 216 cm^2 b 204 cm^2

4 a $h = 41$ cm; 1440 cm^2
b $a = 34$ mm, $b = 20$ mm; 1632 mm^2

5 42 m^2

6 a 756 m^2 b 786

Exercise 15G

1 a 3 cm^2 b 7000 cm^2
c 0.9 cm^2 d $31\,000\text{ cm}^2$

2 a 200 mm^2 b $500\,000\text{ mm}^2$
c 60 mm^2 d $2\,300\,000\text{ mm}^2$

3 a $2\,400\,000\text{ m}^2$ b 3.6 m^2
c $360\,000\text{ m}^2$ d 0.28 m^2

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4 **a** $1035000\ mm^2$ **b** $10350\ cm^2$ **c** $90\ cm \times 115\ cm$ **d** $10350\ cm^2$ **e** Yes **f** $1.035\ m^2$ **g** $0.9\ m \times 1.15\ m$ **h** $1.035\ m^2$ **i** Yes, they do.

5 **a** $111\,800\ m^2$ **b** 11.18 ha

6 200 m

7 26000 ha

8 **a** $4800000\ m^2$ **b** $4.8\ km^2$

9 **a** 0.75 ha **b** 1.875 acres

10 **a** $49.92\ m^2$ **b** \$2246.40

11 **a** $22.32\ m^2$ **b** 1.86 L

12 **a** $1000000\ cm^2$ **b** 1000 L

13 **a** $5.76\ cm^2$ **b** $560000\ cm^2$ **c** $0.756\ m^2$ **d** $590\ mm^2$

14 **a** 620 L **b** 2.6 L **c** 52 mL **d** $2700\ cm^2$ **e** $0.96\ m^2$ **f** $26000\ mm^2$

15 9000 L

16 **a** $14.1372\ m^3$ **b** 14 137.2 L

17 52 days; water runs out during the 53rd day.

18 300 minutes

19 10 dosages

20 **a** **i** 9600 **ii** 2400 **iii** 4800 **b** **i** 36 **ii** 144 **iii** 576 **c** 168 hours

Exercise 15H

1 **a** 1630 **b** 1100 **c** 1849 **d** 1835 **e** 0435 **f** 1605 **g** 2357

2 **a** 2 p.m. **b** 8 a.m. **c** 8:35 a.m. **d** 10:30 p.m.

3 74 days 3 hours

4 10 days 13 hours

5 **a** 4 hours 45 min **b** 5 hours 30 min **c** 7 hours 30 minutes **d** 9 hours 20 minutes **e** 26 hours 45 min **f** 3 days 13 hours 18 minutes **g** 4 days 17 hours 25 minutes

6 15 hours 55 min

7 1 min 42 seconds; 1 min 45 seconds

8 4 hours 29 minutes

9 0.91 seconds

10 5 minutes 30 seconds

Review exercise

1 $30\ cm^2$

2 **a** $48\ m^2$ **b** $28\ cm^2$ **c** $96\ mm^2$ **d** $16\ m^2$

3 $243000\ cm^3$

4 **a** $175\ mm^3$ **b** $144\ m^3$ **c** $285\ cm^3$ **d** $36\ m^3$ **e** $3750\ cm^3$

5 **a** $100\pi\ m^3$ **b** $63\pi\ cm^3$

6 **a** 900 **b** 152 **c** 228 **d** 360

7 **a** $35000\ cm^2$ **b** $72\ cm^2$ **c** $40000\ cm^2$ **d** $670\ cm^2$

8 **a** $4500000000\ m^2$ **b** $4500\ km^2$

9 40π litres

10 80 litres

11 **a** 7 hours 45 minutes **b** 10 hours 30 minutes **c** 7 hours 5 minutes **d** 10 hours 10 minutes **e** 1 day and 5 hours **f** 2 days 12 hours 15 minutes **g** 4 days 18 hours and 36 minutes

Challenge exercise

1 $70\ cm^3$

2 16

3 $74\ cm^2$

4 $25\ cm^3$

5 $840\ cm^3$

6 3 and 4 cm respectively

7 60

8 $560\ m^2$

9 $64\ cm^2$

10 250000

Chapter 16

Exercise 16A

1 $\frac{1}{4}$

2 $\frac{1}{3}$

3 $\frac{1}{6}$

4 $\frac{1}{4}$

5 $\frac{1}{200}$

6 $\frac{1}{52}$

7 $\frac{1}{15}$

8 **a** $\frac{1}{13}$ **b** $\frac{1}{2}$ **c** $\frac{1}{4}$ **d** $\frac{3}{13}$

9 $\frac{2}{15}$

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10 **a** $\frac{3}{5}$ **b** $\frac{1}{5}$ **c** $\frac{2}{5}$

11 $\frac{1}{40}$

12 **a** $\frac{19}{37}$ **b** $\frac{18}{37}$ **c** $\frac{9}{37}$

13 $\frac{3}{5}$ **14** $\frac{12}{13}$ **15** $\frac{21}{26}$

16 $\frac{5}{6}$ **17** $\frac{7}{10}$ **18** $\frac{5}{8}$

19 $\frac{24}{25}$ **20** $\frac{2}{3}$ **21** $\frac{39}{40}$

22 $\frac{10}{13}$

23 **a** $\frac{1}{10}$ **b** $\frac{3}{10}$ **c** $\frac{2}{5}$ **d** $\frac{7}{10}$

24 **a** $\frac{1}{13}$ **b** $\frac{1}{4}$ **c** $\frac{3}{4}$ **d** $\frac{11}{13}$

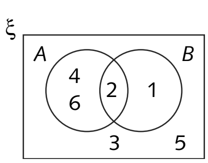
25 **a** $\frac{15}{22}$ **b** $\frac{7}{22}$ **c** $\frac{1}{22}$ **d** $\frac{3}{11}$

26 **a** $\frac{7}{15}$ **b** $\frac{19}{30}$ **c** $\frac{1}{6}$ **d** 0

27 **a** $\frac{1}{5}$ **b** $\frac{1}{15}$ **c** $\frac{2}{15}$ **d** $\frac{1}{10}$

Exercise 16B

1



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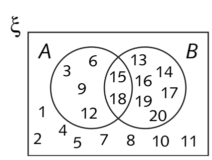
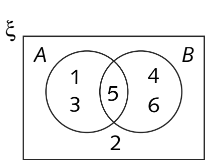


3



4 **a** $\frac{1}{2}$ **b** $\frac{2}{5}$ **c** $\frac{3}{10}$ **d** $\frac{7}{10}$

e $\frac{7}{10}$ **f** $\frac{1}{5}$ **g** $\frac{1}{5}$ **h** 0

5 **a** $\frac{1}{2}$ **b** $\frac{1}{6}$ **c** $\frac{1}{2}$ **d** $\frac{1}{6}$ **e** $\frac{1}{2}$

f 0 **g** $\frac{2}{3}$ **h** $\frac{1}{6}$ **i** $\frac{5}{6}$

6 0.7 **7** 0.5 **8** 0.62

Exercise 16C

1 **a** $\frac{39}{92}$ **b** $\frac{53}{92}$ **c** $\frac{19}{46}$ **d** $\frac{15}{92}$ **e** $\frac{23}{92}$

2 **a** $\frac{63}{100}$ **b** $\frac{13}{100}$ **c** $\frac{1}{5}$

3 **a** $\frac{2}{5}$ **b** $\frac{2}{5}$ **c** $\frac{1}{30}$ **d** $\frac{1}{3}$

e $\frac{11}{15}$ **f** $\frac{29}{30}$

4 **a** $\frac{51}{144} = \frac{17}{48}$ **b** $\frac{11}{24}$ **c** $\frac{13}{48}$

5 **a** $\frac{29}{100}$ **b** $\frac{23}{100}$ **c** $\frac{49}{100}$ **d** $\frac{13}{20}$

e $\frac{11}{100}$ **f** $\frac{13}{20}$ **g** $\frac{77}{100}$ **h** $\frac{1}{8}$

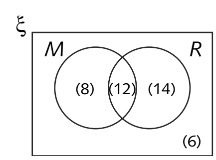
i $\frac{7}{8}$

6 **a** $\frac{4}{15}$ **b** $\frac{1}{15}$ **c** $\frac{133}{165}$

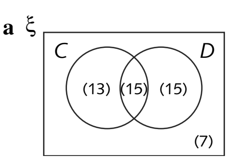
7 **a** $\frac{1}{25}$ **b** $\frac{16}{25}$ **c** $\frac{1}{10}$ **d** $\frac{23}{75}$ **e** $\frac{8}{25}$

Exercise 16D

1

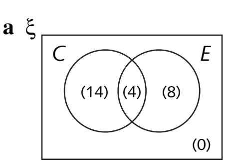


2 **a**



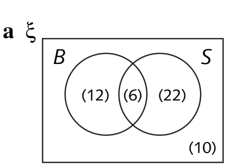
b **i** $\frac{13}{50}$ **ii** $\frac{3}{10}$ **iii** $\frac{7}{50}$

3 **a**



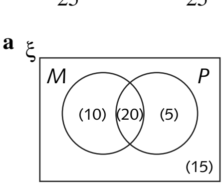
b **i** $\frac{2}{13}$ **ii** $\frac{4}{13}$ **iii** $\frac{7}{13}$

4 **a**



b **i** $\frac{3}{25}$ **ii** $\frac{11}{25}$ **iii** $\frac{6}{25}$

5 **a**



b **i** $\frac{1}{5}$ **ii** $\frac{1}{10}$ **iii** $\frac{3}{10}$

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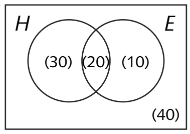
6 **a** ξ



b **i** $\frac{3}{5}$ **ii** $\frac{3}{10}$ **iii** $\frac{2}{3}$

Review exercise

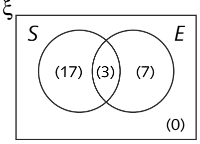
1 $\frac{2}{7}$

2 **a** $\frac{15}{23}$ **b** $\frac{8}{23}$

3 $\frac{1}{4}$ 4 $\frac{2}{3}$ 5 $\frac{2}{5}$

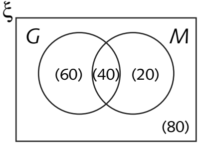
6 $\frac{7}{10}$ 7 $\frac{1}{5}$ 8 $\frac{2}{3}$

9 **a** ξ



b **i** $\frac{10}{27}$ **ii** $\frac{1}{9}$ **iii** $\frac{7}{27}$

10 **a** ξ



b **i** $\frac{3}{5}$ **ii** $\frac{3}{10}$

11 **a** $\frac{1}{4}$ **b** $\frac{3}{13}$ **c** $\frac{7}{13}$ **d** $\frac{11}{26}$

e $\frac{3}{52}$ **f** 0 **g** $\frac{10}{13}$

12 **a** $\frac{2}{5}$ **b** $\frac{2}{5}$ **c** $\frac{4}{15}$ **d** $\frac{1}{3}$

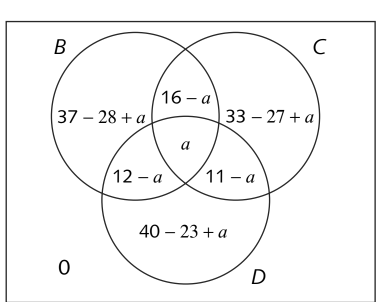
e $\frac{11}{15}$ **f** $\frac{9}{10}$

13 **a** $\frac{37}{100}$ **b** $\frac{13}{25}$ **c** $\frac{33}{100}$ **d** $\frac{39}{50}$

e $\frac{11}{50}$ **f** $\frac{67}{100}$

Challenge exercise

1 **a** ξ



b 4

c **i** $\frac{7}{75}$ **ii** $\frac{8}{75}$ **iii** $\frac{4}{75}$

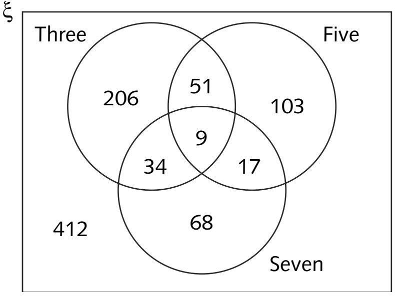
iv $\frac{7}{25}$ **v** $\frac{28}{75}$

2 **a** 350 **b** 610 **c** 200 **d** 220

3 **a** $\frac{11}{20}$ **b** $\frac{9}{20}$

4 **a** $\xi = \{100, 101, ..., 999\}$ and $|\xi| = 900$

b $\frac{1}{5}$ ξ



c $\frac{32}{225}$

d $\frac{1}{3}$

e $\frac{13}{450}$

f $\frac{47}{150}$ **g** $\frac{1}{100}$ **h** $\frac{17}{300}$

i $\frac{17}{900}$ **j** $\frac{122}{225}$

5 **a** 16 **b** 104 **c** 88

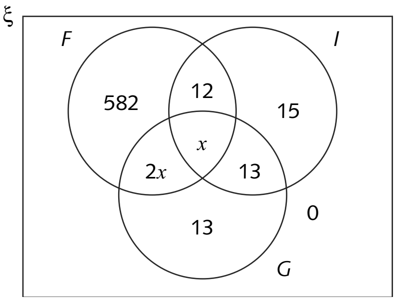
6 **a** $\frac{1}{5}$ **b** $\frac{1}{3}$ **c** $\frac{1}{15}$ **d** $\frac{7}{15}$

7 **a** 62 **b** 33

8 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{1}{4}$

9 $512

10 ξ



$$3x + 26 = 41$$
$$x = 5$$

Number who study French only = 582

$P(\text{French only}) = \frac{582}{650} = \frac{291}{325}$

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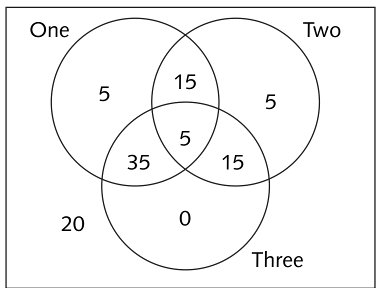
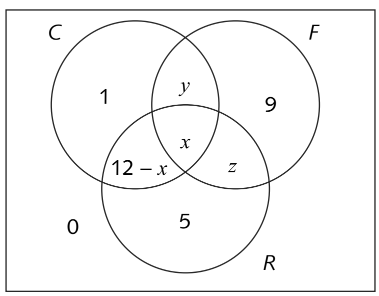
11 ξ



a $\frac{13}{20}$ b $\frac{1}{5}$

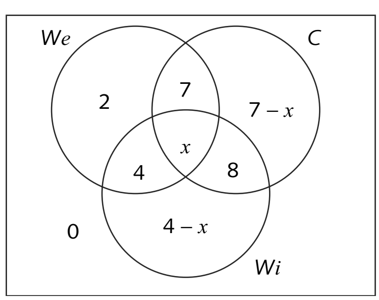
12 ξ



$$\begin{aligned}1+12-x+5&=x\\x&=9\\z&>y\\15+12+y+z&=28\\y+z&=1\\y&=0, z=1\end{aligned}$$

$P(\text{classical and folk}) = \frac{9}{28}$

13 ξ



$31 = 32 - x$
$x = 1$

a $P(\text{cold, wet and windy}) = \frac{1}{31}$

b $P(\text{cold but calm and dry}) = \frac{6}{31}$

14 **a** A **b** $A^c \cap B^c$ **c** $A^c \cap B^c$
d $\varnothing$ **e** B

Chapter 17

Exercise 17A

1 **a** 21 cm^2 **b** 18 cm

2 **a** 8 **b** 1 **c** $1\frac{7}{8}$ **d** −2

3 **a** $37\frac{7}{9}$ °C **b** 100°C
c $-17\frac{7}{9}$ °C **d** −40°C

4 **a** 24 **b** −12 **c** 292 **d** 14
e 6 **f** −6

5 **a** 5 **b** −2 **c** 8 **d** −4 **e** 9

6 **a i** 108° **ii** 120° **iii** 150°
b i 8 **ii** 10

7 **a** 14 **b** 11 **c** −5

8 **a** 48 **b** 4000 **c** $23\frac{1}{3}$

9 **a** $\frac{x}{4}$ **b** $P = \frac{x}{2} + 8$
c i 13 **ii** 20
d i 24 **ii** 64

10 $4\frac{2}{7}$

11 **a** 5 **b** 25 **c** 169

12 **a** 13 **b** 5 **c** 25

Exercise 17B

1 **a** $18x + 45$ **b** $10 + 6x$ **c** $14x - 77$
d $10x + 50$ **e** $-44 - 8x$ **f** $-18x + 30$
g $-5x + 5$ **h** $-21x - 28$ **i** $5x^2 + 10x$
j $3x^2 - 18x$ **k** $-14x^2 + 77x$ **l** $50x - 5x^2$
m $2a^2b + ab^2$ **n** $3x^3 + 9x^2$ **o** $-12x^2 + 8x^3$
p $-10x^2 - 15x$ **q** $a^2b^2 + 3ab^2$ **r** $p^2q + pq^2$
s $-3x^2y - 2xy^2$ **t** $2x^2y^2 - 2xy^3$

2 **a** 4 **b** $9x$ **c** $6x$ **d** $3b$ **e** 3 **f** 14
g 10 **h** 28 **i** 1 **j** x^2 **k** y

3 **a** $2(a + 2)$ **b** $-2(a - 2)$ **c** $-2(x + 2)$
d $10(2a + 3)$ **e** $-8(7x - 12)$ **f** $5(1 - 3x)$
g $5(4 - 5x)$ **h** $25(4 - x)$ **i** $-8(7x + 10)$
j $9(2 - 10z)$

4 **a** $5(x + 12)$ **b** $3m(2m + 1)$ **c** $-3(x - 2)$
d $3x(2x + 1)$ **e** $4(2 - x)$ **f** $8x(1 - 2x)$
g $-5x(1 - 2x)$ **h** $10(7 + a)$ **i** $3(x + 10)$
j $5m(m + 3)$ **k** $-3x(x - 2)$ **l** $4x(x + 3)$
m $5(5 - x^2)$ **n** $4x(1 - 4x)$ **o** $-m(1 - 10m)$
p $-10n(7 - n)$ **q** $4x(x^2 - 15)$ **r** $3m^2(6m + 1)$
s $-3x(1 - 2x)$ **t** $-3x(2x + 1)$ **u** $4x(2x - 1)$
v $3x(1 - 5x)$ **w** $5x(-3 + 2x)$ **x** $10a(7a + 1)$

5 **a** $3(x + 3y)$ **b** $x(y + 4)$ **c** $2(x - 4y)$
d $2x(3 + 4y)$ **e** $3a(1 - 4b)$ **f** $2m(5 + 4n)$
g $2xy(6 + 5x)$ **h** $q(p + r)$

6 **a** $3xyz(1 - 4y)$ **b** $4abc(4a + b^2)$
c $4m^2np(2 + 5n^2p^3)$ **d** $3pq(3p - 1 + 4q)$
e $5ab(a + 4c + 3bc)$

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Exercise 17C

1 a x^2-6x b $2x^2+x$ c $2x^2-8x$
d x^2-x-6 e x^2+x-56 f $x^2+5x-50$
g m^2-m-6 h z^2-36 i $a^2+12a+20$
j $n^2-3n-40$ k $z^2+8z+12$ l $10c^2+27c+18$
m $2a^2+15a-50$ n $2y^2+13y-70$ o $2s^2+10s-48$
p $y^2-18y+81$ q $x^2+10x+25$ r $4x^2-12x+9$

2 a x^2+4x+4 b $x^2-8x+16$
c $4x^2-28x+49$ d $16a^2-16a+4$
e $9m^2-30m+25$ f $16x^2-48x+36$
g $9m^2-36m+36$ h $16-8h+h^2$
i $9m^2-24m+16$

3 a a^2-36 b x^2-25 c $4x^2-16$
d $25s^2-9$ e $36m^2-4$ f $9n^2-49$
g $36c^2-49$ h $49v^2-9$ i $25-m^2$

4 a m^2-49 b $28b^2+45b+18$ c a^2-4b^2
d $m^2-3mn+2n^2$ e m^2-n^2 f $a^2+2ab+b^2$
g $2a^2+5ab-3b^2$ h g^2-9h^2 i $a^2-2ab+b^2$

Exercise 17D

1 a $(x+1)(x+3)$ b $(x+1)(x+4)$ c $(x+1)(x+6)$
d $(x+2)(x+3)$ e $(x+3)(x+8)$ f $(x+4)(x+9)$
g $(x+2)(x+8)$ h $(x+5)(x+6)$ i $(x+2)(x+10)$
j $(x+6)(x+8)$ k $(x+7)(x+9)$ l $(x+8)(x+10)$
m $(x+4)(x+7)$ n $(x+5)(x+11)$ o $(x+4)(x+8)$

2 a $(x-1)(x-5)$ b $(x-3)(x-4)$ c $(x-3)(x-5)$
d $(x-2)(x-7)$ e $(x-4)(x-6)$ f $(x-5)(x-7)$
g $(x-4)(x-8)$ h $(x-3)(x-6)$ i $(x-1)(x-12)$
j $(x-1)(x-14)$ k $(x-6)(x-7)$ l $(x-8)(x-9)$
m $(x-7)(x-8)$ n $(x-9)(x-10)$ o $(x-4)(x-13)$

3 a $(x+2)(x-3)$ b $(x-3)(x+5)$
c $(x-2)(x+5)$ d $(x+2)(x-8)$
e $(x+2)(x-6)$ f $(x-4)(x+6)$
g $(x-5)(x+6)$ h $(x+3)(x-6)$
i $(x+2)(x-9)$ j $(x-4)(x+5)$
k $(x-5)(x+8)$ l $(x+1)(x-5)$
m $(x+5)(x-9)$ n $(x-6)(x+7)$
o $(x+4)(x-5)$

4 a $(x-2)(x+2)$ b $(y-4)(y+4)$
c $(x-1)(x+1)$ d $(m-5)(m+5)$
e $(p-3)(p+3)$ f $(x-12)(x+12)$

5 a $(x+7)^2$ b $(a+5)^2$ c $(x-3)^2$
d $(x-6)^2$ e $(a-5)^2$ f $(x+3)^2$

6 a $(x+4)(x+9)$ b $(x+3)(x+12)$
c $(x+2)(x+18)$ d $(x+6)^2$
e $(x-3)(x-12)$ f $(x-4)(x-9)$
g $(x+4)(x-6)$ h $(x-3)(x+8)$
i $(x-2)(x+12)$ j $(x+1)(x-24)$
k $(x+6)(x-7)$ l $(x+5)(x-6)$
m $(x-12)(x+11)$ n $(x-5)(x+6)$
o $(x+6)(x-8)$ p $(x-4)(x+12)$
q $(x-2)(x+24)$ r $(x+1)(x-48)$
s $(x-6)(x-8)$ t $(x+4)(x+12)$
u $(x+4)(x+10)$ v $(x-2)(x-20)$
w $(x+5)(x-8)$ x $(x-2)(x+20)$
y $(x+4)(x-7)$ z $(x-2)(x+14)$

Review exercise

1 a i $-2\frac{7}{9}$ ii 104
b i $10\frac{1}{2}$ ii 4
2 a 5 b 7 c $4x$ d $21y$
3 a $2(x+3)$ b $7(x-7)$ c $-3(1-14x)$
d $5(20x+1)$ e $-14(x-2)$ f $x(8x-1)$
g $6(5x^2+2)$ h $-5y(4-5y)$
4 a $4(x+3y)$ b $x(y+6)$ c $5(3x-2y)$
d $7x(1+2y)$ e $4a(3-b)$ f $3m(2+3n)$
g $2xy(7+2x)$ h $2q(5p+4r)$
5 a $2xyz(2-3y)$ b $5abc(5a+2b^2)$
c $9m^2np(1+3n^2p^3)$ d $2pq(11p-1+2q)$
e $2ab(3a+4c+2bc)$ f $4xyz(x-2y-3z)$
6 a x^2+15x b $6x^2+7x$ c $10x^2-5x$
d $x^2+10x+24$ e x^2+2x-8 f $2x^2+9x-5$
g $4x^2-15x-4$ h $15x^2-7x-2$ i $2x^2-15x+18$
7 a x^2+2x+1 b x^2+4x+4 c $x^2+18x+81$
d x^2-2x+1 e $x^2-10x+25$ f $x^2-8x+16$
g $4x^2+4x+1$ h $9x^2-12x+4$ i $81-18x+x^2$
8 $\frac{5}{8}$
9 a 39 b 10 c $2\frac{4}{5}$
10 a $(x+2)(x+6)$ b $(x+3)(x+6)$
c $(x+5)(x+6)$ d $(x+4)(x+7)$
e $(x-3)(x-8)$ f $(x-4)(x-6)$
g $(x-2)(x-12)$ h $(x-1)(x-24)$
i $(x-4)(x+5)$ j $(x+6)(x-8)$
k $(x+2)(x-6)$ l $(x-5)(x+8)$
m $(x+1)(x-8)$ n $(x+11)(x-12)$
o $(x-5)(x+20)$
11 a $(x-8)(x+8)$ b $(y-9)(y+9)$
c $(x-11)(x+11)$ d $(m-13)(m+13)$
e $(p-15)(p+15)$ f $(x-14)(x+14)$
12 a $(x+5)^2$ b $(a+20)^2$ c $(x-8)^2$
d $(x-1)^2$ e $(a-20)^2$ f $(x+8)^2$

Challenge exercise

1 a $e^2+8e+16$
b i $e^3+12e^2+48e+64$
ii $e^4+16e^3+96e^2+256e+256$

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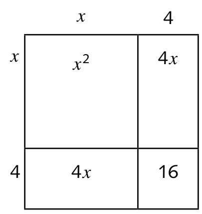
2 **a** $a^2+b^2+2ab-1$ **b** x^2-a^2-6x+9

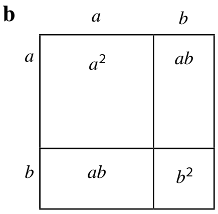
c x^4+x^2+1 **d** $x^4-2x^3+2x^2-2x+1$

e x^3-1 **f** x^3+1

3 **a**

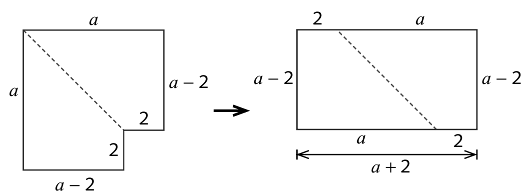


b

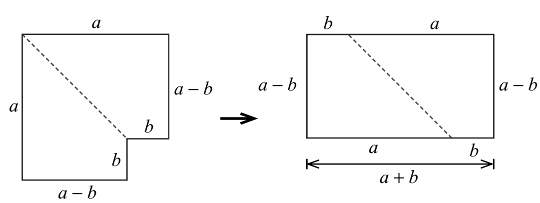


4 **a** 6 **b** 84 **c** 16 296

5 **a**



b



6 **a** 399 **b** 899 **c** 2496

d 4891 **e** 9999 **f** 39 996

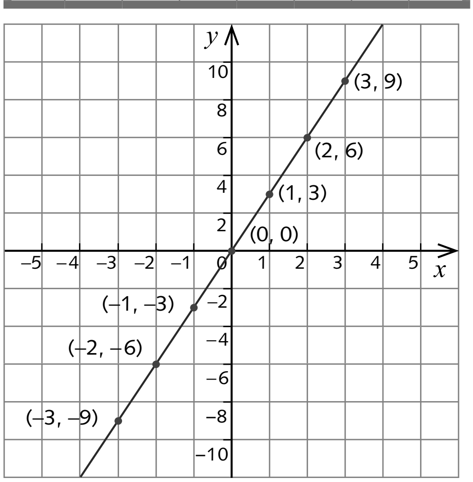
7 **a** $2n+1$ **b** 26^2-25^2

Chapter 18

Exercise 18A

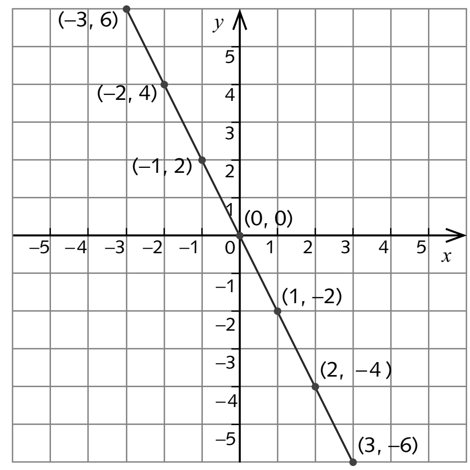
1 **a** $y=3x$

x	–3	–2	–1	0	1	2	3
y	–9	–6	–3	0	3	6	9



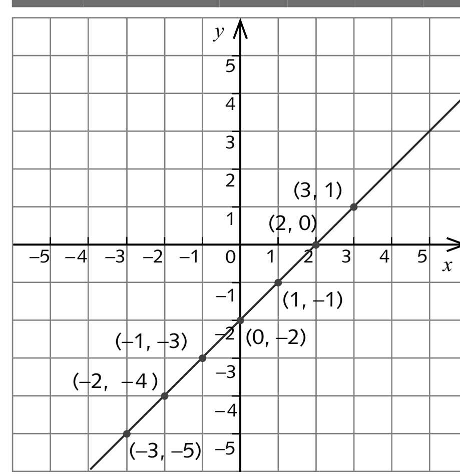
b $y=-2x$

x	–3	–2	–1	0	1	2	3
y	6	4	2	0	–2	–4	–6



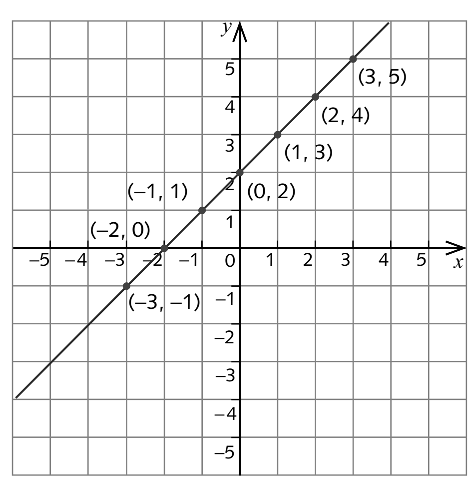
c $y=x-2$

x	–3	–2	–1	0	1	2	3
y	–5	–4	–3	–2	–1	0	1



d $y=x+2$

x	–3	–2	–1	0	1	2	3
y	–1	0	1	2	3	4	5



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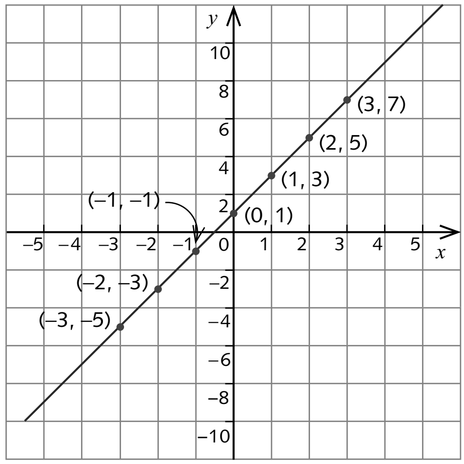
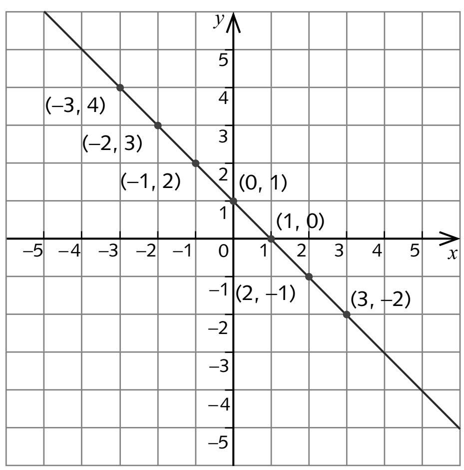
e $y = 2x + 1$

x	−3	−2	−1	0	1	2	3
y	−5	−3	−1	1	3	5	7



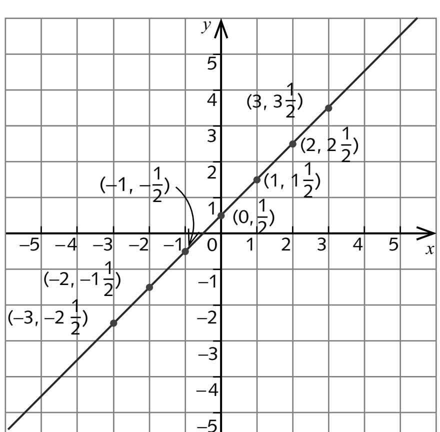
f $y = 1 - x$

x	−3	−2	−1	0	1	2	3
y	4	3	2	1	0	−1	−2



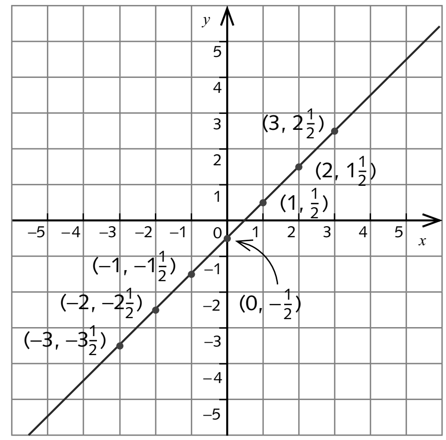
2 a $y = x + \frac{1}{2}$

x	−3	−2	−1	0	1	2	3
y	$-2\frac{1}{2}$	$-1\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$1\frac{1}{2}$	$2\frac{1}{2}$	$3\frac{1}{2}$



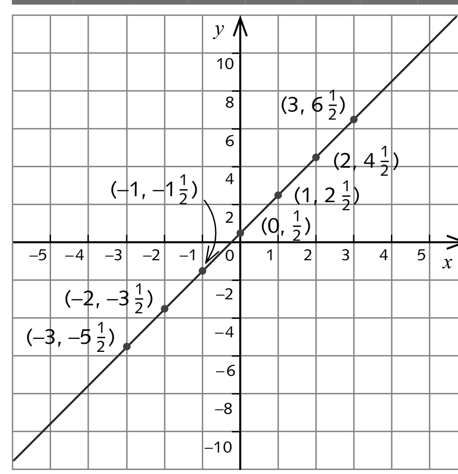
b $y = x - \frac{1}{2}$

x	−3	−2	−1	0	1	2	3
y	$-3\frac{1}{2}$	$-2\frac{1}{2}$	$-1\frac{1}{2}$	$-\frac{1}{2}$	$1\frac{1}{2}$	$1\frac{1}{2}$	$2\frac{1}{2}$



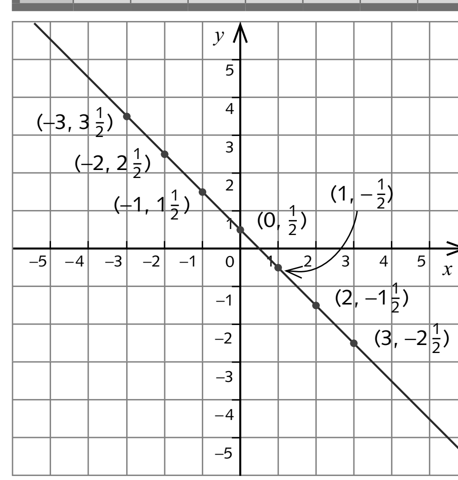
c $y = 2x + \frac{1}{2}$

x	−3	−2	−1	0	1	2	3
y	$-5\frac{1}{2}$	$-3\frac{1}{2}$	$-1\frac{1}{2}$	$\frac{1}{2}$	$2\frac{1}{2}$	$4\frac{1}{2}$	$6\frac{1}{2}$



d $y = -x + \frac{1}{2}$

x	−3	−2	−1	0	1	2	3
y	$3\frac{1}{2}$	$2\frac{1}{2}$	$1\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-1\frac{1}{2}$	$-2\frac{1}{2}$



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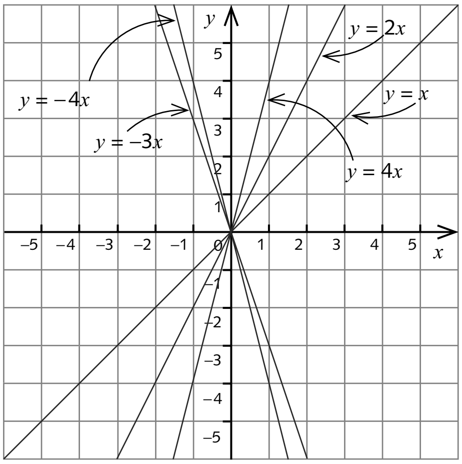
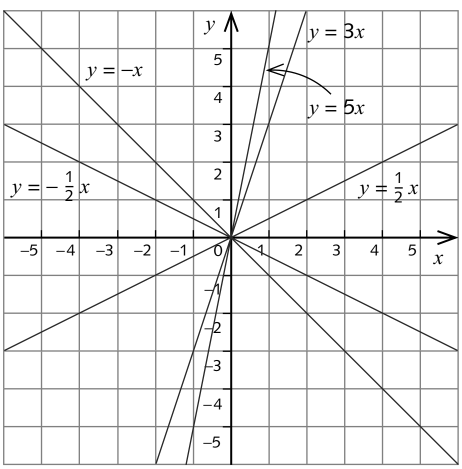
Exercise 18B

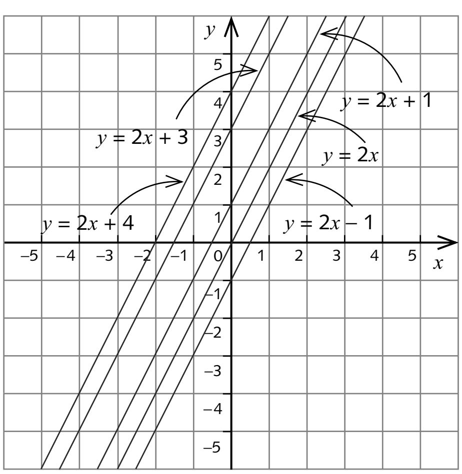
1 **a–e**



2 **a–e**

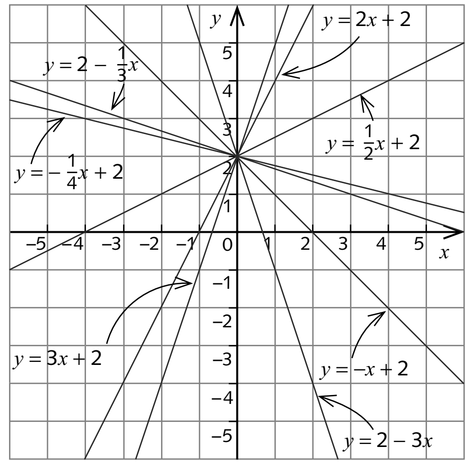


3 **a**



b They are all parallel.

4 **a**



b They are all concurrent. (All pass through (0, 2).)

Exercise 18C

1 **a** $(-1,-5)$ **b** $(0,-3)$ **c** $\left(20,37\right)$

2 **a** $(1,1)$ **b** $(0,5)$ **c** $(2,-3)$

3 **a** $(-1,-6)$ **b** $(0,-5)$ **c** $(20,15)$

4 **a** $(-1,-1)$ **b** $(0,-3)$ **c** $(15,-33)$

5 **a** $(2,4)$ **b** $\left(\frac{2}{3},0\right)$ **c** $(7,19)$

d $\left(10\frac{2}{3},-34\right)$ **e** $(-16,-50)$

6 **a** $x=-1\frac{1}{3}$ **b** $x=\frac{2}{3}$ **c** $x=-8$

d $x=12$ **e** $x=17\frac{1}{3}$

7 **a** no **b** yes **c** yes **d** no

8 **a** no **b** yes **c** yes **d** yes

9 **a** yes **b** no **c** yes **d** no

10 $a=3, b=-1\frac{1}{2}, c=5$

11 $a=6, b=2, c=9$

12 $y=-25$

13 $x=31$

14 $a=5, b=-3, c=4$

15 $a=-2, b=8, c=18$

Exercise 18D

1 **a** 3 **b** −7 **c** 5 **d** −8 **e** 0

2 **a** 3 **b** 3 **c** 0 **d** −1

3 **a** **i**

x	−1	0	1
y	−2	0	2

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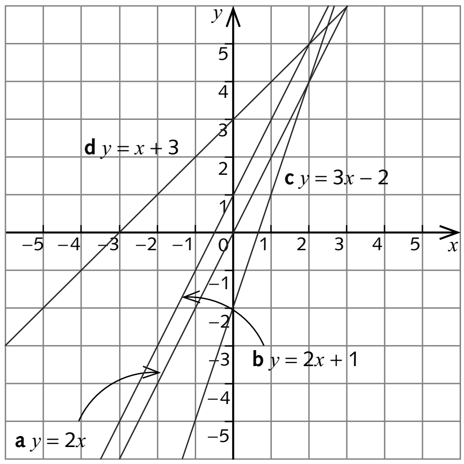
ii



iii 2

b **i**

x	–1	0	1
y	–1	1	3

iii 2

c **i**

x	–1	0	1
y	–1	1	3

iii 3

d **i**

x	–1	0	1
y	2	3	4

iii 1

4 **a** **i** (1, 3), (2, 6) **ii** 3

b **i** (1, 3), (2, 7) **ii** 4

c **i** (1,–3), (2,–1) **ii** 2

d **i** (1, 3), (2, 4) **ii** 1

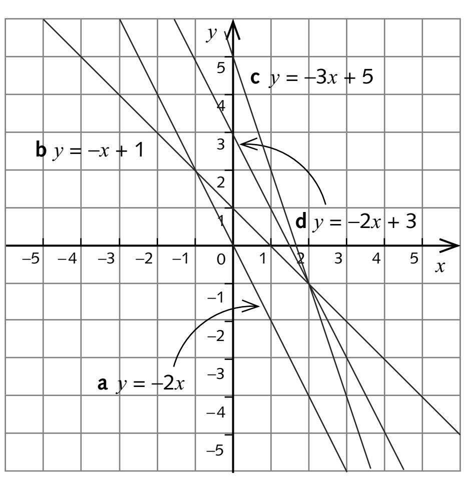
5 **a** 3 and 2, respectively

b $y = 3x - 1$ is steeper

6 **a** **i**

x	0	1	2
y	0	–2	–4

ii



iii –2

b **i**

x	0	1	2
y	1	0	–1

iii –1

c **i**

x	0	1	2
y	5	2	–1

iii –3

d **i**

x	0	1	2
y	3	1	–1

iii –2

7 **a** (1,–3), (2,–6), gradient –3

b (1, –2), (2, –6), gradient –4

c (1, 3), (2, 1), gradient –2

d (1, 1), (2, 0), gradient –1

Exercise 18E

1 **a** 7 **b** 9 **c** –12 **d** –1

2 **a** –7 **b** –3 **c** –11 **d** –1

3 **a** $y = 5x$ **b** $y = 9x - 5$ **c** $y = 3x + 2$

4 **a** **i** downwards
ii upwards
iii downwards
iv downwards

b **i** $m = -7$
ii $m = 4$
iii $m = -9$
iv $m = -1$

5 **a** $m = 3$ **b** $m = 1$ **c** $m = -3$ **d** $m = -1$

6 $m = -3$

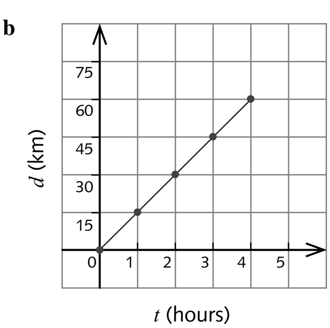
7 $m = 2$

Exercise 18F

1 **a**

t (hours)	0	1	2	3	4
d (km)	0	15	30	45	60

b



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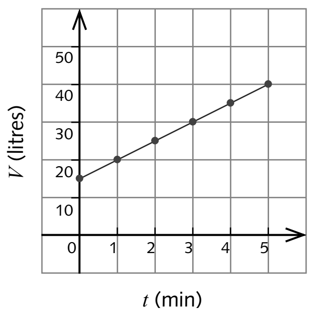
c $d = 15t$

d $48\frac{3}{4}$ km

2 a

t (mins)	0	1	2	3	4	5
d (litres)	15	20	25	30	35	40

b



c $V = 5t + 15$

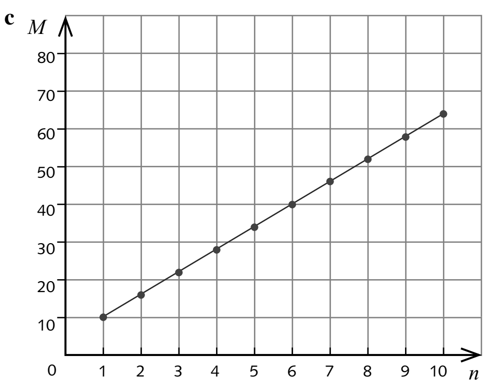
d $27\frac{1}{2}$ L

3 a

Diagram number (n)	Number of matches (M)
1	10
2	16
3	22
4	28
5	34
6	40

b $M = 6n + 4$

c



n can only have integer values.

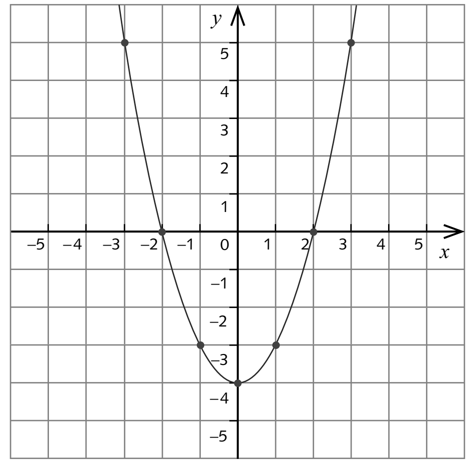
d 76

e 20

Exercise 18G

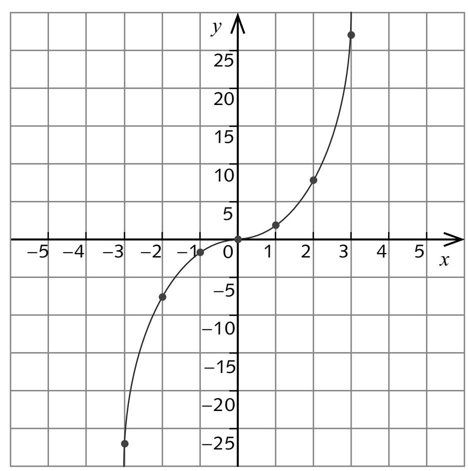
1 a

x	−3	−2	−1	0	1	2	3
y	5	0	−3	−4	−3	0	5



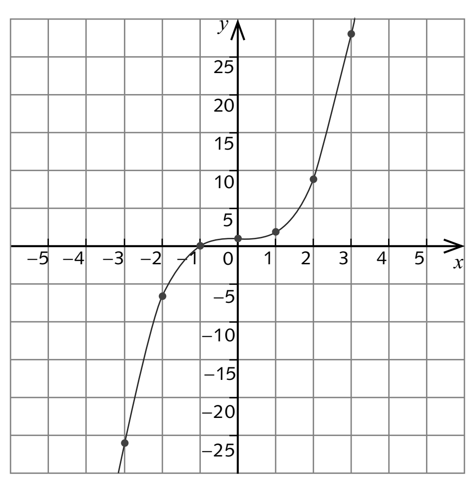
b

x	−3	−2	−1	0	1	2	3
y	−27	−8	−1	0	1	8	27



c

x	−3	−2	−1	0	1	2	3
y	−26	−7	0	1	2	9	28



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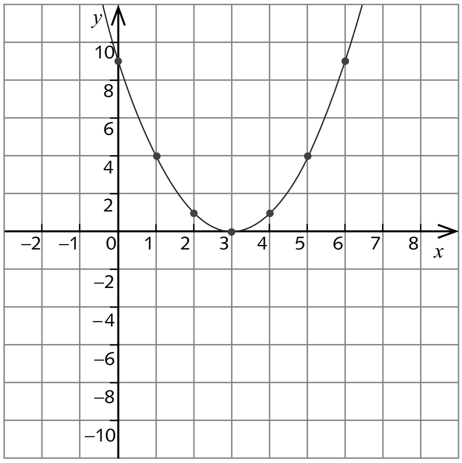
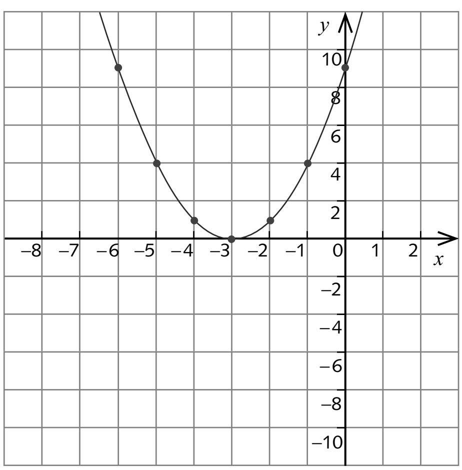
d

x	0	1	2	3	4	5	6
y	9	4	1	0	1	4	9



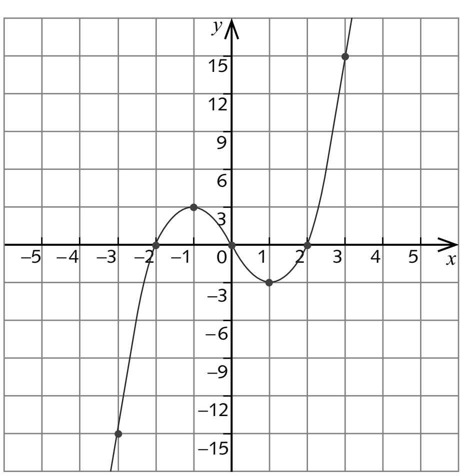
e

x	–6	–5	–4	–3	–2	–1	0
y	9	4	1	0	1	4	9



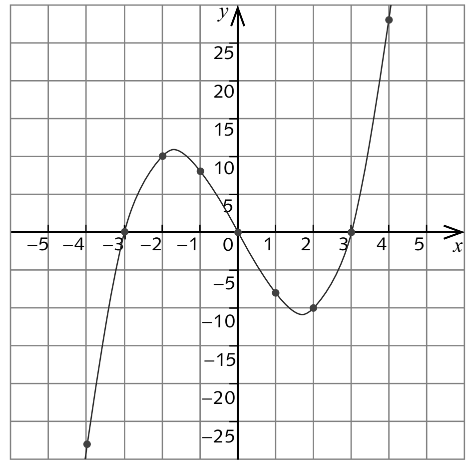
f

x	–3	–2	–1	0	1	2	3
y	–15	0	3	0	–3	0	15



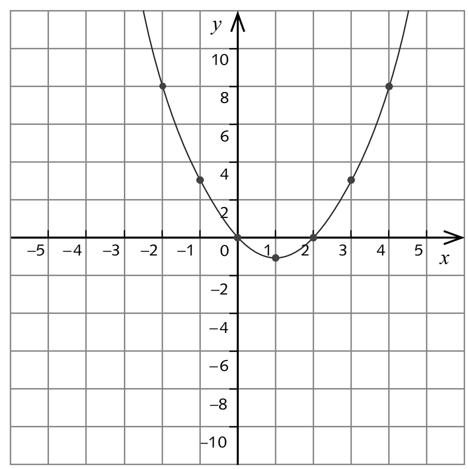
g

x	–4	–3	–2	–1	0	1	2	3	4
y	–28	0	10	8	0	–8	–10	0	28



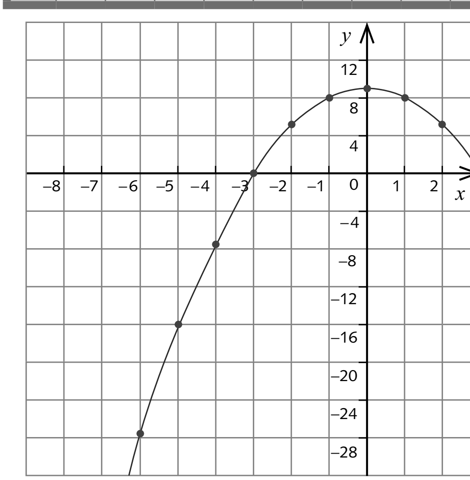
h

x	–2	–1	0	1	2	3	4
y	8	3	0	–1	0	3	8



i

x	–6	–5	–4	–3	–2	–1	0	1	2
y	–27	–16	–7	0	5	8	9	8	5



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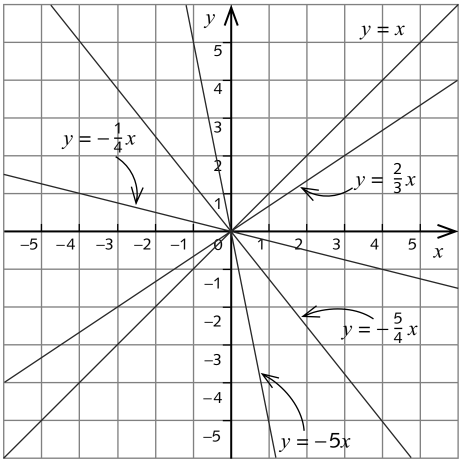
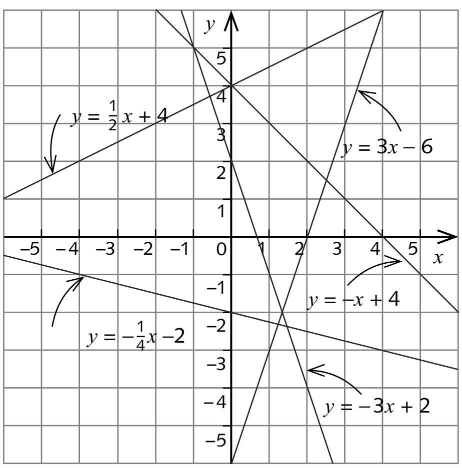
Review exercise

1 **a–e**



2 **a–e**



3 **a** -4 **b** 10 **c** 4 **d** -10

4 **a** $(1, 1), (2, 4)$, gradient 3

b $(1, -6), (2, -4)$, gradient 2

c $(1, 1), (2, -4)$, gradient -5

5 **a** **i** upwards
ii downwards
iii upwards
iv downwards
b **i** $b = 0$
ii $b = 0$
iii $b = -6$
iv $b = 2$

6 **a** yes **b** yes **c** no **d** yes

7 $y = 1$

8 $x = 32$

9 $a = 0, b = 0, c = -5$

10 **a** gradient 5, y-intercept -8
b gradient 3, y-intercept 7
c gradient -3, y-intercept 11

11 $a = 1, b = 3, c = 7$

12 **a** $A(0, 2), B(2, 0)$ **b** $2\sqrt{2}$ cm

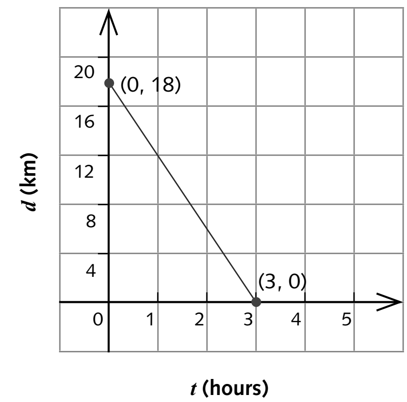
13 **a** $A(0, 1), B(-1, 0), C(1, 0)$ **b** 1

14 **a** $c = 10$ **b** $A(0, 10), B(2, 0)$

15 **a** $m = -5$ **b** $y = -31$ **c** $x = 2$ **d** $x = -3$

Challenge exercise

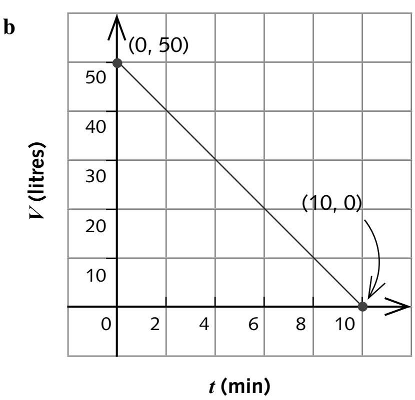
1 gradient -6, d-intercept 18, $d = 18 - 6t$



2 **a**

t (min)	0	1	2	3	4	5	6	7	8	9	10
V (litres)	50	45	40	35	30	25	20	15	10	5	0

b



c $V = 50 - 5t, 0 \le t \le 10$

3 **a** $D = 10t$
b $L = 110 - 15t$
c

t (hours)	0	1	2	3	4	5	6	7	8	9	10	11
D (km)	0	10	20	30	40	50	60	70	80	90	100	110
L (km)	110	95	80	65	50	35	20	5				

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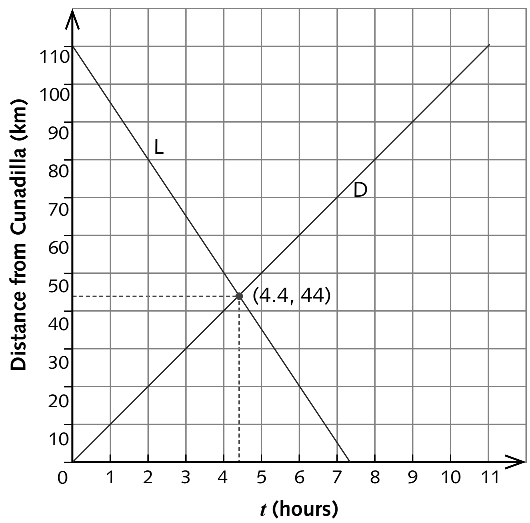
d



e 3:24 p.m.

f 44 km

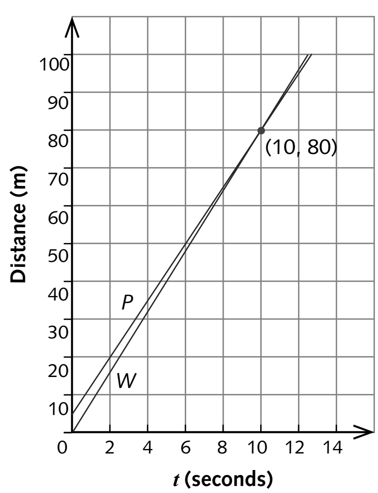
4 **a** $P = 5 + 7.5t$

b $W = 8t$

c

t (seconds)	0	1	2	3	4	5	6	7	8	9	10	11	12
P (m)	5	12.5	20	27.5	35	42.5	50	57.5	65	72.5	80	87.5	95
W (m)	0	8	16	24	32	40	48	56	64	72	80	88	95

d



e 10 seconds

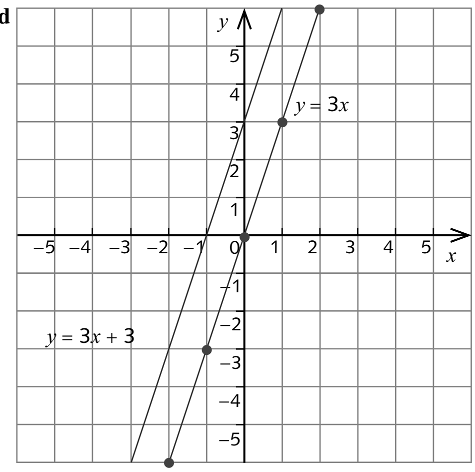
f Wendy: 80 m, Priscilla: 75 m

g 6.25 m

5 **a**

x	−2	−1	0	1	2
y	−6	−3	0	3	6

b–d



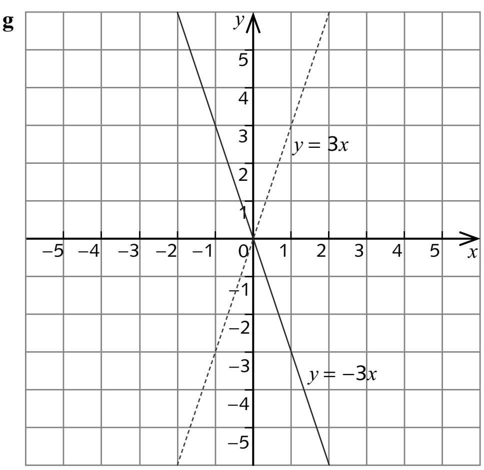
e **i** 3 up

ii 1 left

iii infinitely many

f **i** $y = -3x$ **ii** $y = -3x$

g



h **i** $y = 3x$

ii $y = -\frac{1}{3}x$

6 **a** **i** 1 **ii** $\frac{5}{14}$ **iii** −1 **iv** 1

b 1 km/min; $\frac{5}{14}$ km/min; 1 km/min

c $d = t$, $d = \frac{5}{14}t + \frac{90}{7}$, $d = -t + 78$

d $d = t - 20$

e 78 minutes after leaving Camelot

Chapter 19

Exercise 19A

1 **a** 46.5 **b** 50 **c** 62.5

2 **a** 45 **b** 44.42

3 12

4 **a** $9.75 **b** $13.35

c Sue had one very expensive lunch.

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5 **a** 1, 1, 1, 1, 1, 1, 1, 1, 1, 1

b 2, 2, 2, 2, 2 or 1, 2, 2, 2, 3 or 1, 1, 2, 3, 3

c 3, 3, 3, 3, 3, 3, 3, 3, 3, 3

6 mean = 30.4, median = 31

7 **a** median = 100.5 cm

b mean = 100.64 cm

c median = 100, mean = 99.76 cm

8 **a** mean = 12.55, median = 7.5

b The median is a better measure of centre. The suburb of 41.3 hectares is much larger than the other suburbs.

9 **a**

2	4 5 6 7 8 8 9
3	0 0 1 2 2 6 6 6 6 7 7 9
4	2

b median = 3.15 kilograms

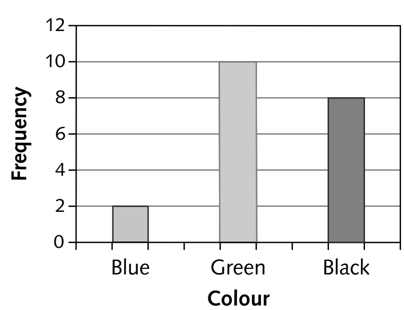
c mean = 3.205 kilograms

10 **a** 1, 1, 3, 7
1, 2, 2, 7
2, 2, 2, 6

b 1, 1, 2, 2, 9
1, 1, 2, 3, 8
1, 1, 2, 4, 7
1, 1, 2, 5, 6
1, 2, 2, 2, 8
1, 2, 2, 3, 7
2, 2, 2, 2, 7
2, 2, 2, 3, 6

Exercise 19C

1 **a** number of blue = 2, number of green = 10, number of black = 8



b, c

Category	Black	Green	Blue
Relative frequency	$\frac{8}{10}$	$\frac{10}{20}$	$\frac{2}{20}$
Relative frequency as a percentage	40%	50%	10%

2

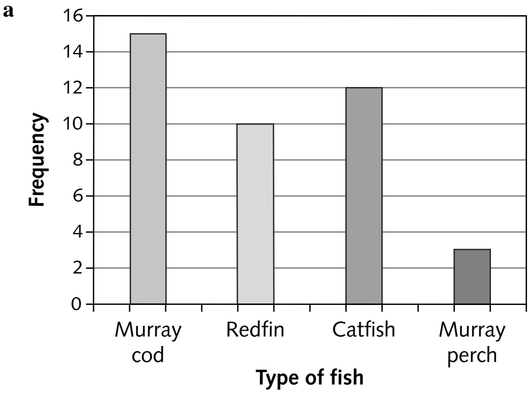
Category	Green	Blue	Black
Relative frequency	$\frac{11}{20}$	$\frac{5}{20}$	$\frac{4}{20}$
Relative frequency as a percentage	55%	25%	20%

3 **a** **i** 73.8 **ii** 73.3

b 54.5, 58, 63, 64, 65, 66, 66, 68, 69, 69.5; average = 64.3

c 94, 94, 89, 85.5, 84, 80.5, 80, 79.5, 77, 74; average = 83.75

4 **a**



b

Type of fish	Murray cod	Redfin	Catfish	Murray perch
Number in sample	15	10	12	3
Relative frequency	$\frac{15}{40}$	$\frac{10}{40}$	$\frac{12}{40}$	$\frac{3}{40}$
Relative frequency as a percentage	37.5%	25%	30%	7.5%

5 45.6 mm

6 **a** The smallest possible mean = 5

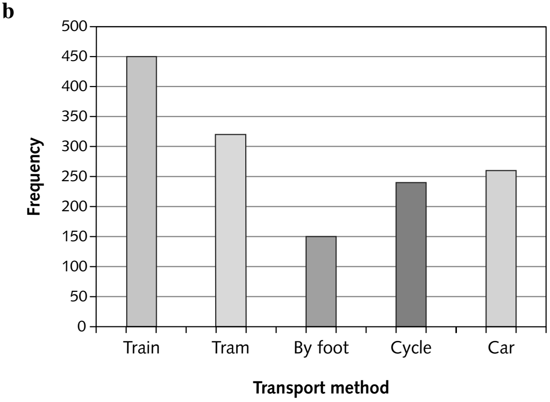
The largest possible mean = 13

b No, as $8.5 \times 5 = 42.5$

7 **a**

Method	Train	Tram	By foot	Cycle	Car
Number	450	320	150	240	260
Relative frequency	$\frac{450}{1420}$	$\frac{320}{1420}$	$\frac{150}{1420}$	$\frac{240}{1420}$	$\frac{260}{1420}$
Relative frequency as a percentage	31.7%	22.5%	10.6%	16.9%	18.3%

b



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Chapter 20

20A Review

Chapter 10: Rates and ratios

1 a 6 b 3 c 4 d 2

2 a 1 : 2 b 11 : 2 c 1 : 6 d 1 : 5
e 4 : 9 f 4 : 1 g 3 : 2 h 1 : 5

3 1 : 4 4 126 : 72 5 \$750 : \$375

6 \$1095 : \$730.00 : \$365 7 \$14.10

8 a \$22.50/h b \$225

9 400 km

10 a \$36.90 b 6 kg

11 a 1 : 4 b 7 : 10

12 24 : 25 13 160 14 3 : 4 15 8 days

16 a 23.92 m b 11.04 m c 5.52 m
d 264.08 m^2 e 1.42 m

17 a 1 : 24 b 1 : 20
c 5 : 1 d 9 : 100 000

18 a 2.25 b 22.5 c 0.0225

19 450 km 20 80 km/h 21 8 h

22 a 4 km b 8 km c 12 km

23 500 km 24 60 km

25 a 1 h 33 min b $55\frac{1}{3}$ km/h c $12\frac{1}{2}$ km/h
d 32.26 km/h (correct to two decimal places)

Chapter 11: Algebra – part 2

1 a $\frac{2x}{3}$ b $\frac{2x}{5}$ c $\frac{7z}{11}$ d $\frac{11x}{10}$
e $\frac{31x}{28}$ f $-\frac{19x}{36}$ g $\frac{x}{14}$ h $\frac{36x}{35}$
i $\frac{x}{3}$ j $\frac{13x}{30}$

2 a $4x-6$ b $-6-22x$ c $16-6x$ d $8+\frac{5x}{3}$
e $2x+25$ f $\frac{11x}{4}-6$ g $\frac{27x}{35}+6$ h $\frac{26x}{15}+4$
i $\frac{243x}{77}-6$

3 a $x=3$ b $x=2\frac{1}{4}$ c $x=-2\frac{1}{2}$ d $x=-1$
e $x=-3$ f $x=-6$ g $x=35$ h $x=1\frac{1}{2}$
i $x=-\frac{4}{9}$ j $x=-3$ k $x=-3$

4 a $x=10\frac{1}{2}$ b $p=4\frac{1}{2}$ c $x=5$
d $x=4\frac{2}{17}$ e $m=1\frac{4}{11}$ f $m=-4\frac{1}{11}$

5 10

6 a $-5x-3=-4, x=\frac{1}{5}$ b $-3(x+6)=28, x=-15\frac{1}{3}$
c $4(x-7)=15, x=10\frac{3}{4}$ d $\frac{5x}{11}=22.8, x=50.16$
e $x+13=4-\frac{x}{2}, x=-6$

Chapter 12: Congruent triangles

1 i B, G ii C, D iii A, I

2 a $A \leftrightarrow O$ b $B \leftrightarrow N$
c $C \leftrightarrow M$ d $D \leftrightarrow L$
e $E \leftrightarrow P$ f $F \leftrightarrow K$
g $AD \leftrightarrow OL$ h $AB \leftrightarrow ON$
i $CD \leftrightarrow ML$ j $\angle ABC \leftrightarrow \angle ONM$
k $\angle BED \leftrightarrow \angle NPL$ l $\angle AFB \leftrightarrow \angle OKN$

3 a not congruent b $\Delta ABC \equiv \Delta RPQ$ (SAS)
c $\Delta ABC \equiv \Delta PRQ$ (AAS) d $\Delta ABC \equiv \Delta PQR$ (AAS)

4 a $y = 5$ cm (given), $\alpha = \beta = 60°$ (isosceles ΔABC), $x = 5$ cm (equilateral ΔABC)
b $x = 9$ cm (given), $\alpha = 70°$ (isosceles ΔABC), $\beta = 40°$ (angle sum of ΔABC)
c ΔABE is equilateral, so $\alpha = 15°, \beta = 105°, x = 306$ mm.
d $\angle ABF = 80°$ (co-interior), $\gamma = 80°$ (vertically opposite at B), $\beta = 60°$ (vertically opposite at B), $\alpha = 40°$ (straight angle at B)

Chapter 13: Congruence and special quadrilaterals

1 a $y=10, \alpha=70°, \beta=20°$
b $z=3, y=7, x=5; \beta=100°; \alpha=150°$
c $x=3, \alpha=130°, \beta=70°$

2 a $x=6$ cm, $\alpha=120°, \beta=60°$
b $\alpha=90°$ (diagonals of rhombus meet at right angles), $\beta=60°$
c $y=4, z=8, \alpha=110°$

3 a $\beta=90°, \alpha=45°, x=5$ b $\alpha=45°, \beta=135°, z=2$

4 **A:** II **B:** IV **C:** I **D:** III

5 a A square is always a rhombus, but a rhombus is not necessarily a square.
b A square is always a rectangle, but a rectangle is not necessarily a square.

6 a 16 cm b 24 km c 34 m

7 a 144 mm^2 b 120 mm^2 c 32 mm^2

Chapter 14: Circles

2 a i 8π mm ii $25\frac{1}{7}$ mm iii 25.12 mm
b i 6π m ii $18\frac{6}{7}$ m iii 18.84 m

3 a $50\frac{2}{7}$ mm^2 b $28\frac{2}{7}$ m^2

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4 **a** $(24+4.8\pi)$ cm **b** 28.8π cm^2

5 2.75π m^2 **6** 1471.875 mm^2 **7** 4π m^2

Chapter 15: Areas, volumes and time

1 **a** 98 cm; 636 cm^2 **b** 28 cm; 44 cm^2
c 40 mm; 65 mm^2
d $(14+5\pi)$ cm; $(20+12.5\pi)$ cm^2
e $(4+10\pi)$ cm; $(20+25\pi)$ cm^2
f $(20+1.5\pi)$ cm; $(33+2.25\pi)$ cm^2

2 **a** 75.6π cm^3 **b** 192 cm^3 **c** 31 cm^3
d 20π cm^3 **e** 60 m^3 **f** 12.5 m^3

3 $16(7+3\sqrt{2})$ cm^2; 96 cm^3

Chapter 16: Probability

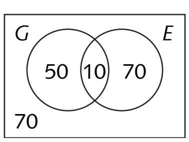
1 **a** $\frac{1}{3}$ **b** $\frac{5}{9}$ **c** 1

2 **a** $\frac{3}{10}$ **b** $\frac{1}{5}$

3 **a** $\xi=\{R, A, N, D, O, M\}$
b i $\{A, O\}$ **ii** $P(E)=\frac{1}{3}$

4 **a** $\frac{1}{25}$ **b** $\frac{1}{5}$ **c** $\frac{12}{25}$ **d** $\frac{1}{5}$
e $\frac{4}{25}$ **f** $\frac{2}{5}$

5 **a** ξ



b i $\frac{3}{10}$ **ii** $\frac{1}{20}$ **iii** $\frac{13}{20}$

6 **a** $\frac{1}{15}$ **b** $\frac{11}{30}$ **c** $\frac{1}{10}$
d $\frac{19}{30}$ **e** $\frac{4}{15}$

Chapter 17: Formulas and factorisation

1 **a** 2 **b** 13 **c** 110

2 10

3 **a** 76 **b** 2 **c** $-\frac{1}{2}$

4 **a** $\ell=w+4$ **b** $P=4(w+2)$; 28, 9 **c** $w(w+4)$
d 21 **e** 6 **f** 5

5 **a** 4 **b** 5 **c** $7x$
d $17y$ **e** $9ab$ **f** $5x^2y$
g $9a^2b$ **h** 1

6 **a** $3(y+2)$ **b** $8(m-7)$ **c** $2(22p-1)$
d $5(16y+1)$ **e** $12(3-z)$ **f** $x(5x-1)$
g $4(3y^2+1)$ **h** $6x(6x-7)$ **i** $2ab(6b+1)$
j $4a(2+b)$ **k** $5xy(2+y)$ **l** $5mp(5n-m)$

7 **a** $2m+6$ **b** $8-6x$ **c** $-12x+8$
d $3x^2-13x-10$ **e** a^3+3a **f** $4-x^2$
g $4z^2+10z+4$ **h** $4z^2-9$ **i** x^2+4x+4
j $4b^2+12b+9$ **k** $4s^2-6s-18$ **l** $4a^2-4ab-3b^2$

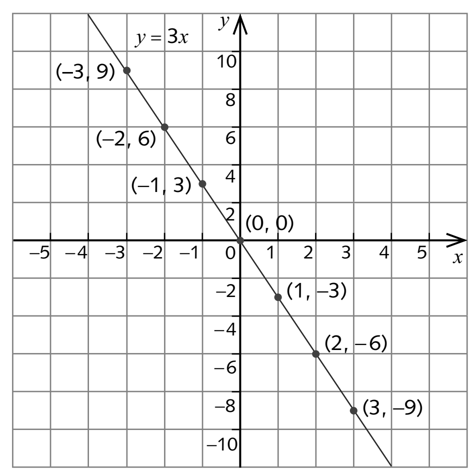
8 **a** $(x+3)^2$ **b** $(a+2)^2$ **c** $(x-3)(x-4)$
d $(x+1)(x+3)$ **e** $(a-2)^2$ **f** $(x+2)(x+7)$
g $(x+2)(x-7)$ **h** $(a+4)(a-6)$ **i** $\left(a-\frac{5}{2}\right)^2$
j $(x-1)(x-6)$ **k** $(x+5)(x-6)$ **l** $(x+3)(x-7)$

Chapter 18: Graphing straight lines

1 **a**

x	–3	–2	–1	0	1	2	3
y	9	6	3	0	–3	–6	–9

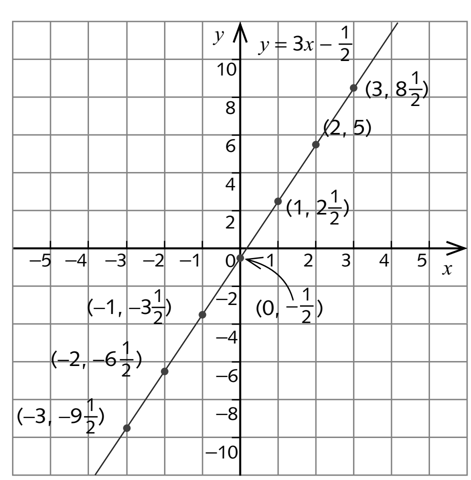
$(-3, 9), (-2, 6), (-1, 3), (0, 0), (1, -3), (2, -6), (3, -9)$



b

x	–3	–2	–1	0	1	2	3
y	$-9\frac{1}{2}$	$-6\frac{1}{2}$	$-3\frac{1}{2}$	$-\frac{1}{2}$	$2\frac{1}{2}$	$5\frac{1}{2}$	$8\frac{1}{2}$

$\left(-3,-9\frac{1}{2}\right), \left(-2,-6\frac{1}{2}\right), \left(-1,-3\frac{1}{2}\right), \left(0,-\frac{1}{2}\right), \left(1, 2\frac{1}{2}\right),$
$\left(2, 5\frac{1}{2}\right), \left(3, 8\frac{1}{2}\right)$



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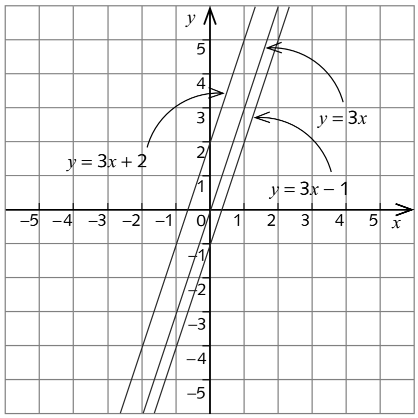
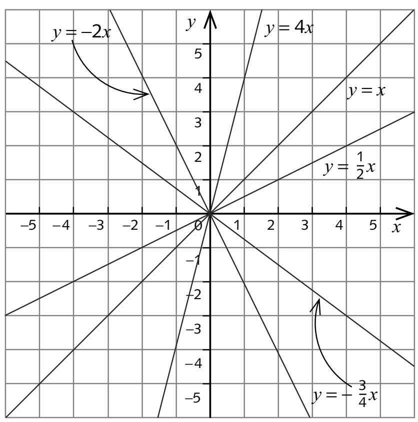
2 a 

b Each has gradient 3.

3 a 

b i (1, 1), (2, 2), gradient 1

ii (1, 4), (2, 8), gradient 4

iii (1, −2), (2, −4), gradient −2

iv $(1, \frac{1}{2})$, (2, 1), gradient $\frac{1}{2}$

v $\left(1, -\frac{3}{4}\right), \left(2, -\frac{3}{2}\right)$, gradient $-\frac{3}{4}$

c i Upwards

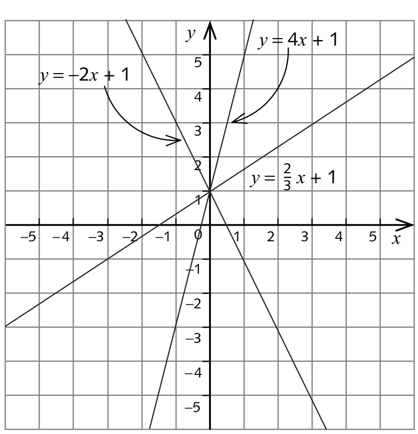
ii Upwards

iii Downwards

iv Upwards

v Downwards

d i $y = 4x$ **ii** $y = -2x$

4 a 

b i 4 **ii** −2 **iii** $\frac{2}{3}$

5 slope = −3

6 **a** $y = -8$ **b** $y = -2$ **c** $y = 34$

7 **a** no **b** no **c** yes **d** no

8 $a = -2, b = \frac{1}{2}, c = 2$

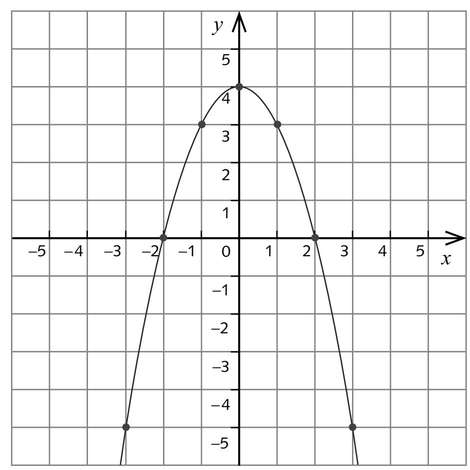
9 **a** $y = -9$ **b** $x = 4$

10 $a = -\frac{3}{2}, b = \frac{3}{4}, c = 8$

11 **a** $y = 1$ **b** $y = -3$ **c** $y = 5$

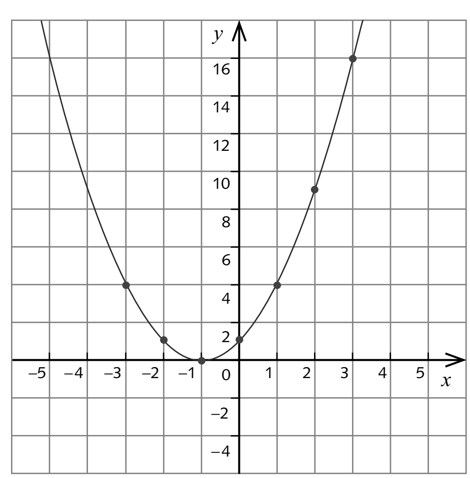
12 **a** $y = 4 - x^2$

x	−3	−2	−1	0	1	2	3
y	−5	0	3	4	3	0	−5



b $y = (x + 1)^2$

x	−3	−2	−1	0	1	2	3
y	4	1	0	1	4	9	16



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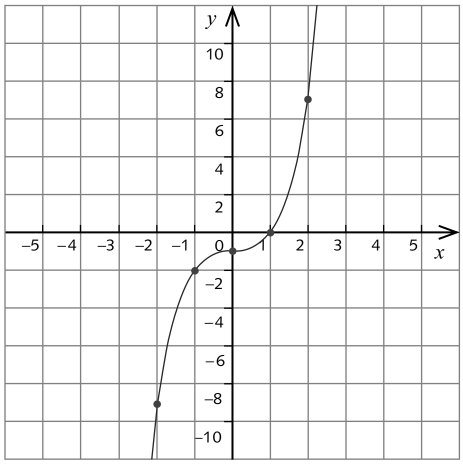
c $y = x^3 - 1$

x	–2	–1	0	1	2
y	–9	–2	–1	0	7



20B Problem-solving

Pythagoras' theorem

1 The co-interior angles at A and B sum to 180°, so $AY \parallel BZ$.

2 $\angle BXZ + \angle AXY + 90° = 180°$ (angle sum of triangle and straight angle at X)

3 $\frac{1}{2}(a+b)h = \frac{1}{2}(a+b)^2$

4 $2 \times \frac{1}{2}ab + \frac{1}{2}c^2 = ab + \frac{1}{2}c^2$

5 $\frac{1}{2}(a+b)^2 = ab + \frac{1}{2}c^2$ (from **3** and **4**)

6 $\frac{1}{2}(a^2 + 2ab + b^2) = ab + \frac{1}{2}c^2$ (from **5**), so $a^2 + b^2 = c^2$

Scones

1 100 cm 2 400 scones

A litre

The other two side lengths can be (in cm; order can be swapped): 1, 100; 2, 50; 4, 25; 5, 20.

Circles

36 cm^2. If the sides of the triangle are a, b and c then the area = $\frac{\pi a^2}{8} + \frac{\pi b^2}{8} - \frac{\pi c^2}{8} + \frac{ab}{2} = \frac{ab}{2} = 36$

A quadrilateral

108°, 108°, 72° and 72°.

Unusual numbers

1 $\frac{10}{11}$ 2 $-11\frac{1}{9}$ 3 $1, -1$

4 8.873 and 1.127 (The exact answers are $5+\sqrt{15}$ and $5-\sqrt{15}$.)

Which fits better?

We can express the wasted space as a percentage of the area of the hole. Then the round hole gives wastage of about 36%, while the square hole gives 21%. Hence a round peg in a square hole fits better. The ratios are $4 : \pi$ and $\pi : 2$.

Racing and chasing

1 **a** 175 s **b** 440 m **2** 3 minutes

3 **a** 50 km **b** 12 : 20 at Sea Lake
c 14 : 00; van 1 is 440 km from Melbourne.
d van 1: 15 : 23; van 2: 15 : 07

Closing the gap

1 **a** 8 min **b** 7.5 min **c** $8\frac{4}{7}$ min

2 **a** Jill: 1125 m, Sophie: 900 m
b 2025 m

3 5 280 000 km; collision took place 960 000 km from Earth.

20C Fibonacci sequences

Activity 1

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10 946, 17 711, 28 657, 46 368, 75 025
(How good was your guess?)

Activities 2 and 3

These will vary but the one that starts with the seeds in ascending order of size will have larger values for F_n from $n = 4$ onwards.

Activity 4

- If both seeds are even, all terms will be even. If either or both seeds are odd, the pattern followed will be … EOOEOOEOO… because the sum of two odds is even and the sum of an odd and an even is odd.
- The sequence whose seeds are in ascending order will have larger values than the corresponding terms in the other sequence. F_3 will be the same for both but F_4 will be larger by the difference between the seeds. The difference continues to grow steadily from then on.
- When F_n is divided by F_{n-1}, the answer is a good approximation to $\Phi \approx 1.6180339...$, which improves for higher values of n.

a $\Phi^2 = \Phi + 1$ **b** $\frac{1}{\Phi} = \Phi - 1$ **c** $(2\Phi - 1)^2 = 5$

These all follow from the fact that $\Phi = \frac{\sqrt{5}+1}{2}$.

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