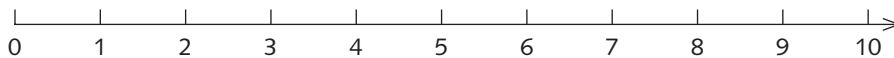# Whole numbers

This chapter is a brief review of whole numbers. Whole numbers, also known as the **counting numbers**, are the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, …

The whole numbers can be visualised as points equally spaced on the number line.

0	1	2	3	4	5	6	7	8	9	10

We can add, subtract, multiply, divide and take powers of numbers. Performing calculations using these operations can be done mentally using a variety of strategies or by using written algorithms.

We begin this chapter with a review of addition and subtraction of whole numbers.

Addition

Any two numbers can be added. When two or more whole numbers are added, the result is called the **sum**.

The order in which we add two or more whole numbers does not matter. Adding them in any order will give the same answer.

Subtraction

Subtraction gives the difference between two numbers. For example, the difference of 8 and 5 is 3, and this is written $8 - 5 = 3$.

We can think of subtraction as:

- taking away one number from another. For example: I had eight tennis balls, but I gave five away, so now I only have three tennis balls.

- finding what we have to add to one number to get the other. For example: I have eight tennis balls and she has five. I have to add 3 to 5 to get 8, so the difference is 3.

Example 1

Calculate these sums mentally by adding the numbers in a more convenient order.

a $29 + 60 + 21$ **b** $33 + 42 + 67$

Solution

a
$$
\begin{aligned}
29 + 60 + 21 &= 29 + 21 + 60 \\
&= 50 + 60 \\
&= 110
\end{aligned}
$$

b
$$
\begin{aligned}
33 + 42 + 67 &= 33 + 67 + 42 \\
&= 100 + 42 \\
&= 142
\end{aligned}
$$

Calculate these differences mentally

a $45 - 27$

b $136 - 77$

Solution

a
$$45 - 27 = 25 - 7$$
$$= 18$$

or

$$45 - 27 = 48 - 30$$
$$= 18$$

or

$$27 + 3 + 15 = 45$$
Therefore $45 - 27 = 18$

Note: Other strategies are possible

b
$$136 - 77 = 139 - 80$$
$$= 59$$

or

$$77 + 23 + 36 = 136$$
Therefore $136 - 77 = 23 + 36$
$$= 59$$

The standard algorithms

Example 3

The numbers of students and staff who attended the local swimming carnival from five schools were $1023, 896, 704, 199$ and 1189. How many people attended in total?

Solution

$$
\begin{array}{r}
1\ 0\ 2\ 3 \\
8\ 9\ 6 \\
7\ 0\ 4 \\
1\ 9\ 9 \\
+\ 1_2\ 1_3\ 8_3\ 9 \\
\hline
4\ 0\ 1\ 1
\end{array}
$$

4011 people attended the swimming carnival.

There are two standard algorithms for subtraction. It is easier to check your calculations if you use Method 1 in Example 4.

Example 4

Of the 4011 people who attended the swimming carnival, 2118 left by the end of lunchtime. How many were present for the start of the afternoon races?

Solution

Method 1

$$
\begin{array}{r}
4 \quad {}^{1}0 \; {}^{1}1 \; {}^{1}1 \\
- \; 2_{1} \; 1_{1} \; 1_{1} \; 8 \\
\hline
1 \quad 8 \quad 9 \quad 3
\end{array}
$$

Method 2

$$
\begin{array}{r}
{}^{3}\!\!\!\!/4 \; {}^{9}\!\!\!\!/1 \; {}^{10}\!\!\!\!/0 \; {}^{1}\!\!\!\!/1 \quad {}^{1}1 \\
- \; 2 \quad 1 \quad 1 \quad 8 \\
\hline
1 \quad 8 \quad 9 \quad 3
\end{array}
$$

1893 people were at the swimming carnival for the start of the afternoon races. A subtraction can always be checked by addition. In this case, $1893 + 2118 = 4011$.

Addition and subtraction

- The **any-order property for addition** states that a list of numbers can be added together in any order to give the sum of the numbers.
- The any-order property summarises the commutative and associative laws for addition.
- Numbers in addition and subtraction calculations can often be rearranged to make mental calculations simpler.
- Written algorithms for addition and subtraction are useful when the numbers are large.
- Subtraction is the reverse of addition. For example, $8 + 5 = 13$ and $13 - 5 = 8$.

The commutative and associative laws of addition say, for example, that $3 + 4 = 4 + 3$ and $(3 + 4) + 5 = 3 + (4 + 5)$.

Exercise 1A

Example 1

1 Calculate these sums mentally by adding the numbers in a more convenient order.

   a $19 + 48 + 11$ **b** $28 + 53 + 22$

   c $15 + 57 + 85$ **d** $37 + 88 + 13$

   e $22 + 24 + 26 + 28$ **f** $31 + 33 + 37 + 39$

2 Calculate these sums mentally by adding the numbers in stages.

   a $25 + 36$ **b** $37 + 89$

   c $54 + 27$ **d** $189 + 34$

   e $12 + 16 + 15 + 22$ **f** $23 + 24 + 28 + 21$

Example 2

3 Calculate these differences mentally, either by subtracting in stages or by adding in stages.

a $35 - 17$ **b** $84 - 36$

c $134 - 76$ **d** $53 - 27$

e $96 - 28$ **f** $252 - 177$

4 Calculate:

a $36 + 130 + 1644$ **b** $910 + 49 + 8371$

c $9221 + 839 + 65$ **d** $324 + 538 + 718$

e $229 + 1038 + 432$ **f** $251 + 489 + 12 + 37$

5 Carry out these subtractions.

a $762 - 387$ **b** $1921 - 428$

c $1405 - 386$ **d** $2764 - 1689$

e $918 - 579$ **f** $1297 - 399$

Example 3

6 The numbers of people living in five apartment blocks in the city are $1098, 956, 423, 156$ and 42. How many people live in these apartment blocks in total?

Example 4

7 In 1975, the population of Hobart was $217\,135$. In 1985, the population was $231\,135$. What was the population increase from 1975 to 1985?

8 There are 3047 cats living in Brownville and 6857 in Wugtown. What is the combined cat population of the two towns?

9 A furniture store ordered 234 tables and 587 more chairs than tables. How many chairs were ordered? How many items were ordered in total?

10 The Mitchell Dam holds $496\,709\,000$ litres of water. The Gaudry Weir holds $278\,700\,900$ litres. If both water storage facilities are filled to capacity, how much water do they hold in total? What is the difference, in litres, between the capacities of the two facilities?

11 Doug is 1708 mm tall, Colin is 1639 mm tall and Peter is 1836 mm tall. How much taller is:

a Doug than Colin?

b Peter than Colin?

c Peter than Doug?

12 In 2004, there were 5938 students in state primary schools and 4133 students in state secondary schools in Toowoomba. What was the total number of students in state primary and secondary schools in that year?

13 In 1898, the population of the German Empire was $45\,234\,061$. If $28\,318\,592$ were Lutherans, $124\,567$ belonged to other protestant denominations, $561\,612$ were Jewish and the remainder Roman Catholic, how many Roman Catholics were there?

14 In 1891, the population of London was $4\,211\,743$. The counties of Durham, Gloucestershire and Devonshire had populations of $1\,024\,369$, $548\,886$ and $636\,225$, respectively. By how many did the population of London exceed the total population of these three counties?

Multiplication and division are useful arithmetic operations. We use them all the time in everyday life. Repeated addition can be completed as a multiplication.

For example:

$$3 \times 5 = 5 + 5 + 5$$

Multiplication

Any two whole numbers can be multiplied together. The result is called the **product** of the numbers.

For example, the product of 15 and 13 is $15 \times 13 = 195$.

A list of whole numbers can be multiplied, two at a time, in any order, and the result will always be the same.

The following strategies are useful when doing multiplication mentally.

- Grouping numbers together in ways that will make the calculations easier.

Example 5

Find the product $8 \times 5 \times 7 \times 2$.

Solution

$$\begin{aligned} 8 \times 5 \times 7 \times 2 &= 8 \times 7 \times 10 \\ &= 560 \end{aligned}$$ (Multiply $5 \times 2 = 10$ first.)

- Using the distributive law. For example, $3 \times 23 = 3 \times (20 + 3) = 60 + 9 = 69$

Example 6

Calculate:

a 21×36 **b** 28×36

Solution

a
$$\begin{aligned} 21 \times 36 &= 20 \times 36 + 1 \times 36 \\ &= 720 + 36 \\ &= 756 \end{aligned}$$

b
$$\begin{aligned} 28 \times 36 &= 30 \times 36 - 2 \times 36 \\ &= 1080 - 72 \\ &= 1008 \end{aligned}$$

There are other possible methods.

The **multiplication algorithm** is used for larger numbers.

Example 7

A brick wall is to have 37 rows, with 128 bricks in each row.

How many bricks are needed?

Solution

$$
\begin{array}{r}
1\ 2\ 8 \\
\times\quad 3\ 7 \\
\hline
8\ 9\ 6 \\
3\ 8\ 4\ 0 \\
\hline
4\ 7\ 3\ 6 \\
\hline
\end{array}
$$

(Multiply 128 by 7.)

(Multiply 128 by 30.)

4736 bricks are required.

Multiplication of whole numbers

- The **any-order property for multiplication** states that a list of numbers can be multiplied together, in any order, to give the product of the numbers.
- The any-order property summarises the commutative and associative laws for multiplication.
- Multiplication is **distributive** over addition and subtraction:

  $(20 + 1) \times 36 = 20 \times 36 + 1 \times 36 = 720 + 36 = 756$

  $(30 - 2) \times 36 = 30 \times 36 - 2 \times 36 = 1080 - 72 = 1008$

- When multiplying mentally, it is often effective to:
  - take the factors in a different order
  - use the distributive law.
- Larger numbers can be multiplied using the multiplication algorithm.
- It is important to know your multiplication tables.

The commutative and associative laws of multiplication say, for example, that $3 \times 4 = 4 \times 3$ and $(3 \times 4) \times 5 = 3 \times (4 \times 5)$.

Division

Any whole number can be divided by any non-zero whole number, called the **divisor**, to give a **quotient** and a **remainder**.

Division without remainder is the reverse process of multiplication. For example, $30 \div 5 = 6$ is the reverse of $30 = 6 \times 5$. Six is the quotient and the remainder is 0.

Often, we do not have exact division. For example, 5 does not divide exactly into 32. We write $32 \div 5 = 6$ remainder 2. This is another way of writing $32 = 6 \times 5 + 2$.

There are two quite different ways to interpret a division such as '$32 \div 5 = 6$ remainder 2'.

- 'Divide 32 people into groups of 5. There will be 6 groups, with 2 people left over.'

- 'Divide 32 people into 5 equal groups. Each group will have 6 people, and there will be 2 people left over.'

Example 8

53 chocolates are put into boxes of 12. How many boxes are filled, and how many chocolates are left over?

Solution

$53 \div 12 = 4$ remainder 5

That is, $53 = 4 \times 12 + 5$

We can fill 4 boxes with 5 chocolates left over.

There are several useful strategies for mental division.

Example 9

a Calculate $864 \div 8$ mentally.   **b** Calculate $752 \div 8$ mentally.

Solution

a Split 864 into $800 + 64$.   Divide successively by 2.

$$\begin{aligned} 864 \div 8 &= 800 \div 8 + 64 \div 8 \\ &= 100 + 8 \\ &= 108 \end{aligned} \quad \textbf{or} \quad \begin{aligned} 864 \div 8 &= 432 \div 4 \\ &= 216 \div 2 \\ &= 108 \end{aligned}$$

b Split 752 into $800 - 48$.

$$\begin{aligned} 752 \div 8 &= 800 \div 8 - 48 \div 8 \\ &= 100 - 6 \\ &= 94 \end{aligned} \quad \textbf{or} \quad \begin{aligned} 752 \div 8 &= 376 \div 4 \\ &= 188 \div 2 \\ &= 94 \end{aligned}$$

The short division algorithm

The short division algorithm is effective when the divisor is small.

Example 10

The humbug-making machine produced 2592 humbugs in an afternoon, packaged into bags of 8. How many bags were produced?

Solution

In this case the divisor is 8, so the short division algorithm can be used.

$$8\overline{)2\,5\,^{1}9\,^{3}2} = 324$$

324 bags of humbugs were produced that afternoon.

Fraction notation for division

We can also write a division as a fraction.

For example, $864 \div 8 = 108$ can be written as:

$$\frac{864}{8} = 108$$

Other examples using this notation are:

$$\frac{0}{3} = 0 \quad \text{and} \quad \frac{108}{12} = 9$$

When there is a remainder, we can write the answer using a mixed numeral:

$$\frac{27}{8} = 3\frac{3}{8}$$

In the next chapter we will study calculations such as:

$$8 \times 3\frac{3}{8} = 27$$

The long division algorithm

When the divisor is large, we use the **long division algorithm**.

Example 11

On Fridays, the humbugs produced by the humbug-making machine are packed into mega-bags of 36 humbugs. One Friday, 8318 humbugs were produced. How many mega-bags were produced?

Solution

$$
\begin{array}{r}
2\ 3\ 1 \quad \text{remainder } 2 \\
36\overline{)8\ 3\ 1\ 8} \\
7\ 2 \\
\hline
1\ 1\ 1 \quad \text{(Bring down 1.)} \\
1\ 0\ 8 \\
\hline
3\ 8 \quad \text{(Bring down 8.)} \\
3\ 6 \\
\hline
2
\end{array}
$$

We use: $36 \times 1 = 36$
$36 \times 2 = 72$
$36 \times 3 = 108$

To start, consider $88 \div 36$:

$36 \times 2 = 72$ and $36 \times 3 = 108$

The 2 is placed above the line, and multiplied by 36 to give 72, which is subtracted from 83 to give the correct 'carry'.

Thus $8318 \div 36 = 231$ remainder 2,

so 231 mega-bags were produced, with 2 humbugs left over.

Example 12

Find $9617 \div 27$.

Solution

It is useful to begin by writing down at least some of the multiples of 27 to the right of the calculation.

$$
\begin{array}{r}
3\ 5\ 6 \\
27\overline{)9\ 6\ 1\ 7} \\
8\ 1 \\
\overline{1\ 5\ 1} \\
1\ 3\ 5 \\
\overline{1\ 6\ 7} \\
1\ 6\ 2 \\
\overline{5}
\end{array}
$$

$27 \times 1 = 27$

$27 \times 2 = 54$

$27 \times 3 = 81$

$27 \times 4 = 108$

$27 \times 5 = 135$

$27 \times 6 = 162$

$27 \times 7 = 189$

$27 \times 8 = 216$

$27 \times 9 = 243$

$9617 \div 27 = 356$ remainder 5.

Division of whole numbers

- Any whole number can be divided by any non-zero whole number to give a quotient and remainder.
- A division such as $30 \div 5 = 6$ can be interpreted in two ways:
    - dividing 30 people into 5 equal groups
    - dividing 300 people into groups of 5
- Division is **distributive** over addition and subtraction. For example:

    $(800 + 64) \div 8 = 800 \div 8 + 64 \div 8 = 100 + 8 = 108$

    $(800 - 48) \div 8 = 800 \div 8 - 48 \div 8 = 100 - 6 = 94$
- A division statement such as $30 \div 7 = 4$ remainder 2 is equivalent to the multiplication and addition statement $30 = 7 \times 4 + 2$.
- Every multiplication statement is equivalent to a division statement.
- To perform divisions, you must know your multiplication tables.
- When performing division mentally, the distributive law is often useful. For example:

    $$
    \begin{aligned}
    330 \div 15 &= (300 + 30) \div 15 \\
    &= 20 + 2 \\
    &= 22
    \end{aligned}
    $$
- Larger numbers can be divided using the long division algorithm.

Exercise 1B

Example 5

1 Calculate these products mentally by multiplying the numbers in a more convenient order.

a $7 \times 5 \times 9 \times 2$ **b** $12 \times 25 \times 11 \times 4$

c $15 \times 9 \times 4$ **d** $35 \times 11 \times 2$

Example 6

2 Calculate these products mentally by using the fact that one factor is close to a multiple of 10.

a 21×33 **b** 18×28 **c** 42×16 **d** 49×34

Example 9

3 Calculate these divisions mentally.

a $749 \div 7$ **b** $352 \div 4$ **c** $1272 \div 12$ **d** $2964 \div 6$

Example 8

4 Calculate the quotient and remainder in each division.

a $165 \div 7$ **b** $441 \div 9$ **c** $800 \div 12$ **d** $1244 \div 11$

Example 7

5 Carry out each calculation using the multiplication algorithm.

a 353×14 **b** 572×29 **c** 79×186 **d** 612×49

e 432×88 **f** 895×35 **g** 135×36 **h** 936×24

Example 10

6 Use short division to calculate:

a $556 \div 2$ **b** $540 \div 4$ **c** $8624 \div 8$ **d** $498 \div 6$

e $4050 \div 6$ **f** $1496 \div 11$ **g** $78\,093 \div 9$ **h** $39\,240 \div 9$

Example 12

7 Use the long division algorithm to calculate:

a $828 \div 13$ **b** $1050 \div 16$ **c** $2344 \div 16$ **d** $8554 \div 17$

e $2806 \div 23$ **f** $4042 \div 19$ **g** $1498 \div 18$ **h** $4708 \div 19$

Example 7

8 A school hall has 30 rows of seats. Each row has 28 seats. How many seats are there?

9 An apartment block has 44 floors. Each floor has 18 apartments. How many apartments are there in the block?

10 A car park has 34 rows and each row has 42 parking spaces. How many cars can be parked?

11 Pete planted 80 rows of 30 tomato plants. If each plant produced 43 tomatoes, what was the total crop?

12 An office building has 15 floors, with 23 rooms and 4 corridors on each floor. If each room has 7 lights and each corridor has 15 lights, what is the total number of lights in the building?

13 Tara is comparing mobile phone plans. She uses her phone for SMS messages only. Plan A costs $33.00 per month and includes a maximum of 150 SMS messages. Plan B has no monthly charge but charges 24c for each SMS. Which plan would have been cheaper last month, when she sent 149 SMSs?

14 The average distance between Mercury and Venus is 51 078 090 km. What is the average distance between Mars and Jupiter if it is 11 times the average distance between Mercury and Venus?

15 Tony is offered two options for buying a particular car. He can pay the marked price of $52 800 in cash. Alternatively, he can pay a $10 000 deposit and then pay instalments of $1805 a month for 24 months. How much more would he pay for the car on the instalment plan?

16 A greengrocer bought a sack of potatoes weighing 51 kg. He divided the potatoes into bags so that each bag held 3 kg of potatoes. How many bags of potatoes did he get from his sack?

17 A humbug-making machine produced 12 429 humbugs in three days.

   a How many bags of 8 is this? How many are left over?

   b How many mega-bags of 36 is this? How many are left over?

18 **a** A calendar is to be invented for a newly settled planet, which has 2304 days in its year. The designer has to split the days up into weeks. How many weeks will there be if he uses weeks of:

   i 6 days?   **ii** 8 days?   **iii** 12 days?

   b If he makes each month have 144 days, how many weeks will there be in a month for each of part **a**?

19 A family were stuck in their house after a serious storm. During the storm, the clock was damaged so that only the hour hand worked. The family stayed inside the house for 636 hours after the storm. How many times did the hour hand go around the clock before they left?

Review exercise

1 Calculate these sums mentally.

a $38 + 22$	**b** $32 + 25$	**c** $35 + 27$	**d** $42 + 19$
e $13 + 17$	**f** $8 + 89$	**g** $14 + 76$	**h** $29 + 46$
i $84 + 6$	**j** $32 + 9$	**k** $11 + 9 + 33$	**l** $2 + 15 + 38$
m $61 + 24 + 9$	**n** $4 + 42 + 36$	**o** $27 + 6 + 3$	**p** $16 + 24 + 5$
q $16 + 55 + 24 + 45$	**r** $72 + 19 + 28 + 21$	**s** $22 + 17 + 18 + 23$	**t** $23 + 37 + 64 + 6$
u $15 + 64 + 26 + 45$			

2 Carry out these subtractions mentally.

a $31 - 9$	**b** $32 - 28$	**c** $94 - 66$	**d** $56 - 48$
e $521 - 428$	**f** $405 - 386$	**g** $764 - 689$	**h** $618 - 509$
i $813 - 619$	**j** $754 - 749$	**k** $328 - 68$	**l** $462 - 387$

3 Carry out these multiplications mentally.

a $25 \times 4 \times 9$ **b** $50 \times 78 \times 2$ **c** $1 \times 35 \times 20$ **d** $3 \times 7 \times 4 \times 5$

e $5 \times 76 \times 2$ **f** $8 \times 5 \times 40$ **g** $6 \times 5 \times 10 \times 2$ **h** $5 \times 47 \times 20$

i $13 \times 6 \times 0$

4 Lake Superior, the largest freshwater lake in the world, has an area of 82 732 km^2. The second largest, Lake Victoria, has an area of 69 753 km^2. How much larger is Lake Superior?

5 Copy and complete the calculations by finding a digit for each *.

a
```
    2 3 1
    2 6 *
  + 1 * 6
  ---------
    * 3 1
```

b
```
    * * *
    6 3 7
  + 5 6 7
  ---------
  * 0 4 7
```

c
```
    * 7 *
  ×     9
  ---------
  3 3 * 4
```

6 There are 3257 dogs in Snugsville and 2673 in Cudstown. What is the combined dog population of the two towns?

7 A school has 2137 students. The number of boys at the school is 1586. How many girls are there at the school?

8 There are 576 chocolates to be divided among 24 people. How many chocolates will each person receive?

9 A youth club has 80 members. There are 10 more boys than girls. How many boys are there?

10 There are 4000 apples packed into boxes, each box holding 75 apples. How many boxes are required?

11 A club started the year with 82 members. During the year, 36 people left and 28 people joined. How many people belonged to the club at the end of the year?

12 A fruit grower sold 780 boxes of oranges. On average, the boxes weigh 21 kg each. What was the total weight (in kg) of the boxes of oranges?

13 An orchard of 60 hectares (ha) has 2100 trees. How many trees are there on each hectare (on average)?

Challenge exercise

1 The towns Thomas, Callaghan, Barker, Dixon and Evans are located on a straight highway in that order. The distance from Thomas to Evans is 20 km. The distance from Thomas to Dixon is 15 km. The distance from Callaghan to Evans is 10 km. Barker is halfway between Callaghan and Dixon. What is the distance from Callaghan to Barker?

2   Using each of the digits 1 to 9 only once, write them in such a way that you produce two numbers that add up to 99 999.

3   A family had a collection of animals. When the council asked them how many birds and how many beasts they had, they answered, 'Well, we have 36 heads and 100 feet in total.' How many birds and how many beasts did the family have?

4   Take a number that does not contain zero and in which all the digits are different.

- Double the number and then add 4.

- Multiply by 5 and then add 12.

- Multiply by 10 and then subtract 320.

- Cross out the zeroes in your answer.

What number do you get? Explain your answer.

5   **a** What is the smallest number that must be subtracted from 5762, so that the result is exactly divisible by 19?

  b What number divided by 367 will give 59 as the quotient and 126 as the remainder?

  c Divide 931 into two parts such that one part is greater than the other by 127.

6   **a** The average of 7 numbers is 36. An eighth number is chosen so that the average is now 45. What is the eighth number?

  b The average of 7 numbers is 17 and the average of four of them is 14. What is the average of the other three numbers?

  c The average of 6 numbers is 29 and the average of four other numbers is 44. What is the average of the 10 numbers?

  d A batsman had an average of 38 runs for 5 innings. If he scored 62 in his next innings, what was then his average?

Division by factors

7   **a** Divide 263 052 by 36 by first dividing by 9 and then by 4.

  b Divide 56 133 by 231 by first dividing by 3, then by 7 and finally by 11 ($231 = 3 \times 7 \times 11$).

8   Consider the following division using factors: $69\,237 \div 45$:

$$
\begin{array}{r|l}
5 & 6\,9\,2\,3\,7 \\
\hline
9 & 1\,3\,8\,4\,7 \quad \text{remainder 2} \\
\hline
& 1\,5\,3\,8 \quad \text{remainder 5}
\end{array}
$$

Quotient $= 1538$

Remainder $= 5 \times 5 + 2$
          $= 27$

Use this method to divide 69 237 by 231.

9   If the current time is 1 p.m., what will the time be in 1000 hours?