CHAPTER

2

Number and Algebra

Fractions and decimals

In Chapter 1 we reviewed the whole numbers. This chapter is a review of fractions and decimals.

Fractions have been used since ancient times. For instance, dividing a harvest into equal parts and distributing different amounts to different families would often have involved fractions.

Fractions were used by the ancient Egyptians and Babylonians. Systematic use of decimal fractions did not appear until the sixteenth century. Decimals are now a central part of day-to-day calculations in almost every walk of life.

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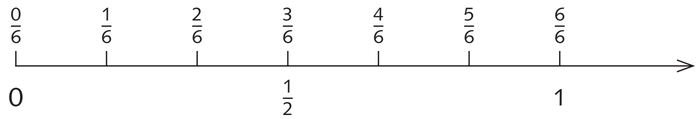
2A Equivalent fractions and simplest form

Equivalent fractions

Two fractions are **equivalent** if they are represented by the same point on the number line. We regard equivalent fractions as being equal.

The markers $\frac{3}{6}$ and $\frac{1}{2}$ are represented by the same point on the number line below, so they are equivalent.



This is simply the statement that, starting at 0, taking one step of length $\frac{1}{2}$ and taking 3 steps of length $\frac{1}{6}$ both get us to the same point.

$$\begin{aligned}\frac{1}{2} &= \frac{1\times 3}{2\times 3}\\ &= \frac{3}{6}\end{aligned}$$

To form equivalent fractions, start with a fraction and either:

- multiply its numerator and denominator by the same non-zero whole number.

  For example:

  $$\frac{3}{4} = \frac{3\times 6}{4\times 6} = \frac{18}{24}$$

- divide the numerator and denominator by the same common factor.

  For example:

  $$\frac{18}{24} = \frac{18\div 6}{24\div 6} = \frac{3}{4}$$

Simplest form and cancelling

A fraction is in **simplest form** or **lowest terms** if the numerator and denominator have no common factor other than 1.

For example, the fraction $\frac{5}{12}$ is in simplest form because the highest common factor of 5 and 12 is 1. However, $\frac{9}{12}$ is not in simplest form since 9 and 12 have 3 as a common factor.

Indeed $\frac{9}{12} = \frac{3}{4}$.

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Example 1

Simplify:

a $\dfrac{36}{45}$

b $\dfrac{98}{21}$

Solution

a $\dfrac{36}{45} = \dfrac{\cancel{36}^{4}}{\cancel{45}_{5}}$ (dividing each by 9)

$= \dfrac{4}{5}$

b $\dfrac{98}{21} = \dfrac{\cancel{98}^{14}}{\cancel{21}_{3}}$ (dividing each by 7)

$= \dfrac{14}{3}$

$= 4\frac{2}{3}$

Note: Two fractions are equivalent if they have the same simplest form.

Example 2

Write 'yes' or 'no' to indicate whether the fractions in each pair are equivalent.

a Is $\dfrac{3}{4}$ equivalent to $\dfrac{45}{60}$?

b Is $\dfrac{5}{9}$ equivalent to $\dfrac{70}{108}$?

c Is $\dfrac{99}{100}$ equivalent to $\dfrac{9}{10}$?

d Is $\dfrac{9}{27}$ equivalent to $\dfrac{33}{99}$?

Solution

a Yes. (Multiply numerator and denominator of $\dfrac{3}{4}$ by 15.)

b No. $\left(\dfrac{5}{9}\text{ is in simplest form. The simplest form of }\dfrac{70}{108}\text{ is }\dfrac{35}{54}.\right)$

c No. (Both fractions are in simplest form and they are different.)

d Yes. $\left(\text{The simplest form of both fractions is }\dfrac{1}{3}.\right)$

Equivalent fractions and simplest form

- Two fractions are said to be **equivalent** if they are represented by the same point on a number line. Equivalent fractions are equal.
- We say that a fraction is in **simplest form** if the numerator and denominator have no common factor other than 1.
- We can test when two fractions are **equivalent** by seeing if they have the same simplest form.
- Starting with a given fraction, the fractions obtained by multiplying its numerator and its denominator by the same non-zero whole number are equivalent to it.
- Starting with a given fraction, the fractions obtained by dividing its numerator and its denominator by the same common factor are equivalent to it.

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Comparison of fractions: which is larger?

If two or more fractions have the same denominator, then the one with the larger numerator is the larger fraction.

Equivalent fractions are used to compare fractions when the denominators of the fractions given are different. First find equivalent fractions with the same denominator for the given fractions, and then compare the numerators.

Example 3

Order the following fractions from smallest to largest:

$\frac{3}{10}, \frac{3}{4}, \frac{1}{2}, \frac{7}{20}$ and $\frac{3}{5}$.

Solution

The LCM of the denominators 2, 4, 5, 10 and 20 is 20.

$\frac{3}{10} = \frac{6}{20}, \frac{3}{4} = \frac{15}{20}, \frac{1}{2} = \frac{10}{20}, \frac{7}{20} = \frac{7}{20}, \frac{3}{5} = \frac{12}{20}$

Writing these in order: $\frac{6}{20}, \frac{7}{20}, \frac{10}{20}, \frac{12}{20}, \frac{15}{20}$

The order is $\frac{3}{10} < \frac{7}{20} < \frac{1}{2} < \frac{3}{5} < \frac{3}{4}$.

Exercise 2A

Example 1

1 Write each fraction in simplest form.

a $\frac{10}{15}$ **b** $\frac{17}{34}$ **c** $\frac{18}{21}$ **d** $\frac{20}{24}$

e $\frac{45}{35}$ **f** $\frac{56}{14}$ **g** $\frac{105}{147}$ **h** $\frac{84}{224}$

i $\frac{20}{25}$ **j** $\frac{24}{36}$ **k** $\frac{14}{21}$ **l** $\frac{24}{32}$

m $\frac{45}{36}$ **n** $\frac{56}{42}$ **o** $\frac{105}{84}$ **p** $\frac{112}{48}$

Example 2

2 Test whether the fractions in each pair are equivalent.

a $\frac{2}{3}, \frac{10}{15}$ **b** $\frac{2}{3}, \frac{12}{21}$ **c** $\frac{2}{3}, \frac{22}{33}$

d $\frac{7}{8}, \frac{56}{64}$ **e** $\frac{7}{8}, \frac{49}{64}$ **f** $\frac{3}{5}, \frac{18}{30}$

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Example 3

3 Write the six fractions with a common denominator, and hence order them from smallest to largest.

a $\frac{11}{10}, \frac{2}{5}, \frac{5}{4}, \frac{9}{10}, \frac{3}{4}, \frac{4}{5}$

b $\frac{5}{6}, \frac{4}{5}, \frac{7}{10}, \frac{3}{4}, \frac{11}{15}, \frac{23}{30}$

4 Rewrite these fractions with denominator 63.

$\frac{1}{3}, \frac{2}{3}, \frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{1}{9}, \frac{2}{9}, \frac{5}{9}, \frac{8}{21}$

5 Arrange the fractions from smallest to largest.

a $\frac{7}{8}, \frac{2}{3}, \frac{5}{6}$

b $\frac{11}{16}, \frac{2}{3}, \frac{3}{4}$

c $\frac{11}{12}, \frac{31}{45}, \frac{2}{3}$

d $\frac{7}{12}, \frac{5}{8}, \frac{2}{3}$

e $\frac{25}{54}, \frac{1}{2}, \frac{5}{9}$

f $\frac{31}{48}, \frac{2}{3}, \frac{11}{16}$

2B Addition and subtraction of fractions

A **proper fraction** is greater than or equal to 0 and less than 1. For example, $\frac{2}{7}$ and $\frac{3}{8}$.

A **mixed numeral** is a whole number plus a proper fraction. For example, $1\frac{1}{3}$ and $2\frac{2}{5}$ are mixed numerals.

An **improper fraction** is a fraction whose numerator is greater than or equal to the denominator, so that it is greater than or equal to 1. For example, $\frac{9}{8}$ and $\frac{13}{7}$.

An improper fraction can be written as a mixed numeral or a whole number. In general, improper fractions should be written as mixed numerals with the fractional parts in simplest form.

For example:

$$\frac{20}{6} = 3\frac{1}{3}$$

To add and subtract fractions with the same denominator, simply add or subtract the numerators.

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Example 4

Evaluate:

a $\frac{18}{19}+\frac{13}{19}$

b $1\frac{1}{6}+2\frac{5}{6}$

c $\frac{5}{16}+\frac{11}{16}+\frac{7}{16}$

Solution

a $\frac{18}{19}+\frac{13}{19}=\frac{31}{19}$

$=1\frac{12}{19}$

b $1\frac{1}{6}+2\frac{5}{6}=1+2+\frac{1}{6}+\frac{5}{6}$

$=3+\frac{6}{6}$

$=4$

c $\frac{5}{16}+\frac{11}{16}+\frac{7}{16}=\frac{23}{16}$

$=1\frac{7}{16}$

Example 5

Evaluate:

a $\frac{12}{13}-\frac{5}{13}$

b $1\frac{8}{19}-\frac{9}{19}$

Solution

a $\frac{12}{13}-\frac{5}{13}=\frac{7}{13}$

b $1\frac{8}{19}-\frac{9}{19}=\frac{27}{19}-\frac{9}{19}$

$=\frac{18}{19}$

If the denominators are different, use the lowest common multiple of the denominators to find equivalent fractions.

Example 6

Evaluate:

a $\frac{2}{3}+\frac{5}{8}$

b $\frac{5}{9}-\frac{1}{6}$

Solution

a The LCM of 3 and 8 is 24.

$\frac{2}{3}+\frac{5}{8}=\frac{16}{24}+\frac{15}{24}$

$=\frac{31}{24}$

$=1\frac{7}{24}$

b The LCM of 9 and 6 is 18.

$\frac{5}{9}-\frac{1}{6}=\frac{10}{18}-\frac{3}{18}$

$=\frac{7}{18}$

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Example 7

Evaluate:

a $2\frac{3}{8}+1\frac{5}{6}$

b $21\frac{1}{6}-19\frac{5}{8}$

Solution

a
$$\begin{aligned}2\tfrac{3}{8}+1\tfrac{5}{6} &= 2\tfrac{9}{24}+1\tfrac{20}{24}\\ &= 3+\frac{9}{24}+\frac{20}{24}\\ &= 3+\frac{29}{24}\\ &= 3+1+\frac{5}{24}\\ &= 4\tfrac{5}{24}\end{aligned}$$

b Notice that $\frac{1}{6}<\frac{5}{8}$.

$$\begin{aligned}21\tfrac{1}{6}-19\tfrac{5}{8} &= 21\tfrac{4}{24}-19\tfrac{15}{24}\\ &= 20+1\tfrac{4}{24}-\left(19+\frac{15}{24}\right)\\ &= (20-19)+\left(\frac{28}{24}-\frac{15}{24}\right)\\ &= 1\tfrac{13}{24}\end{aligned}$$

or

$$\begin{aligned}21\tfrac{1}{6}-19\tfrac{5}{8} &= 21+\frac{1}{6}-19-\frac{5}{8}\\ &= 21-19+\frac{4}{24}-\frac{15}{24}\\ &= 2-\frac{11}{24}\\ &= 1\tfrac{13}{24}\end{aligned}$$

Many applications involve addition and subtraction of fractions.

Example 8

Five identical trucks, carrying $\frac{5}{6}$ load, $\frac{3}{8}$ load, $\frac{3}{4}$ load, $\frac{1}{2}$ load and $\frac{2}{3}$ load of potatoes, arrived at the chip factory. How many truckloads of potatoes were delivered in total?

Solution

$$\begin{aligned}\frac{5}{6}+\frac{3}{8}+\frac{3}{4}+\frac{1}{2}+\frac{2}{3} &= \frac{20}{24}+\frac{9}{24}+\frac{18}{24}+\frac{12}{24}+\frac{16}{24}\\ &= \frac{75}{24}\\ &= \frac{25}{8}\\ &= 3\tfrac{1}{8}\end{aligned}$$

$3\frac{1}{8}$ truckloads of potatoes were delivered.

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Example 9

Brad has a roll of electrical cable with $3\frac{3}{10}$ m left on it. How much is left if he uses $2\frac{7}{8}$ m?

Solution

$$\begin{aligned} 3\tfrac{3}{10} - 2\tfrac{7}{8} &= \frac{33}{10} - \frac{23}{8} \\ &= \frac{132}{40} - \frac{115}{40} \quad \text{(LCM is 40.)} \\ &= \frac{17}{40} \end{aligned}$$

There is $\frac{17}{40}$ m of cable left.

Exercise 2B

Example 4

1 Find the value of:

a $\frac{3}{4}+\frac{5}{4}$ **b** $\frac{1}{5}+\frac{3}{5}+\frac{4}{5}$ **c** $\frac{3}{10}+\frac{2}{10}$

d $\frac{3}{5}+\frac{2}{5}+\frac{7}{5}$ **e** $\frac{2}{3}+\frac{5}{3}$ **f** $\frac{2}{7}+\frac{8}{7}$

Example 5

2 Evaluate:

a $\frac{3}{4}-\frac{1}{4}$ **b** $\frac{7}{12}-\frac{5}{12}$ **c** $2\frac{2}{5}-1\frac{1}{5}$

d $1\frac{7}{19}-\frac{9}{19}$ **e** $2\frac{2}{5}-1\frac{4}{5}$ **f** $3\frac{5}{12}-2\frac{7}{12}$

Example 6a

3 Evaluate:

a $\frac{2}{3}+\frac{3}{4}$ **b** $\frac{9}{10}+\frac{10}{11}$ **c** $\frac{5}{6}+\frac{3}{10}$

d $\frac{1}{9}+\frac{1}{10}$ **e** $\frac{8}{9}+\frac{7}{8}$ **f** $\frac{5}{7}+\frac{3}{5}$

g $\frac{1}{3}+\frac{5}{8}$ **h** $\frac{7}{8}+\frac{3}{7}$ **i** $\frac{5}{10}+\frac{3}{5}$

Example 6b

4 Evaluate:

a $\frac{5}{6}-\frac{2}{3}$ **b** $\frac{1}{3}-\frac{1}{4}$ **c** $\frac{20}{33}-\frac{35}{77}$

d $\frac{5}{7}-\frac{3}{5}$ **e** $\frac{7}{8}-\frac{5}{6}$ **f** $\frac{8}{9}-\frac{7}{8}$

g $\frac{1}{9}-\frac{1}{10}$ **h** $\frac{2}{7}-\frac{1}{5}$ **i** $\frac{11}{12}-\frac{5}{6}$

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Example 7a

5 Evaluate:

a $2\frac{1}{5}+3\frac{2}{3}$ **b** $1\frac{2}{3}+1\frac{1}{4}$ **c** $2\frac{4}{5}+1\frac{1}{4}$

d $3\frac{1}{2}+2\frac{2}{3}$ **e** $1\frac{1}{5}+2\frac{2}{3}$ **f** $5\frac{1}{10}+3\frac{7}{8}$

g $1\frac{1}{11}+3\frac{4}{5}$ **h** $7\frac{3}{8}+2\frac{1}{5}$ **i** $6\frac{4}{5}+7\frac{5}{8}$

Example 7b

6 Evaluate:

a $3\frac{3}{4}-1\frac{1}{2}$ **b** $13\frac{3}{4}-2\frac{7}{8}$ **c** $27\frac{5}{8}-14\frac{1}{3}$

d $52\frac{3}{4}-26\frac{7}{8}$ **e** $22\frac{1}{4}-11\frac{1}{3}$ **f** $52\frac{7}{10}-3\frac{5}{11}$

Example 8

7 A man walks $2\frac{7}{8}$ km and then runs for $1\frac{3}{4}$ km. How far has he travelled in total?

Example 9

8 The distance from Davidson to Clare is $5\frac{3}{10}$ km. A boy rides $2\frac{3}{4}$ km from Davidson along the road to Clare. How much further does he have to ride to Clare?

9 Jacinta has $\frac{2}{3}$ of a litre of water in a jug and then pours in $\frac{1}{4}$ of a litre. How much water is in the jug now?

10 A box of chocolates has 24 chocolates in it. Marie ate $\frac{1}{2}$ of the box of chocolates and then ate $\frac{3}{8}$ of the same box of chocolates the next day. What fraction of the box of chocolates did she eat in total?

11 I have a length of wire and cut off $\frac{3}{7}$ of it. What fraction of the wire do I have left?

12 Dimitri devotes $\frac{1}{3}$ of the day to schoolwork and he spends $\frac{1}{10}$ of the day watching television. As a fraction of the day, how much more time is spent on schoolwork than on television?

13 A family travelling to Albury cover one-third of the journey before 1 p.m. and a further one-quarter of the journey between 1 p.m. and 2 p.m. What fraction of the journey have they travelled by 2 p.m.?

14 In a packet of jelly beans, $\frac{1}{3}$ of the jelly beans are purple, $\frac{1}{8}$ are black and $\frac{1}{4}$ are red.

a What fraction of the jelly beans are either purple or black?

b What fraction of the jelly beans are either purple or red?

c What fraction of the jelly beans are purple or red or black?

d What fraction of the jelly beans are not purple or red or black?

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2C Multiplication and division of fractions

Multiplication of fractions

When multiplying fractions, we first multiply the two numerators, then multiply the two denominators, and then simplify if we can.

Always look out for cancelling. In most cases, cancel common factors first, then multiply.

Example 10

Evaluate $\frac{10}{21} \times \frac{9}{16}$.

Solution

$$\frac{10}{21} \times \frac{9}{16} = \frac{\cancel{10}^{5}}{\cancel{21}_{7}} \times \frac{\cancel{9}^{3}}{\cancel{16}_{8}}$$ (Divide 10 and 16 by 2; divide 9 and 21 by 3.)

$$= \frac{5}{7} \times \frac{3}{8}$$

$$= \frac{15}{56}$$

When multiplying mixed numerals, first convert to improper fractions, cancel common factors if possible and then multiply.

Example 11

Evaluate $5\frac{1}{4} \times 2\frac{1}{3}$.

Solution

$$5\tfrac{1}{4} \times 2\tfrac{1}{3} = \frac{21}{4} \times \frac{7}{3}$$ (Convert to improper fractions.)

$$= \frac{\cancel{21}^{7}}{4} \times \frac{7}{\cancel{3}_{1}}$$ (Divide 21 and 3 by 3.)

$$= \frac{7}{4} \times \frac{7}{1}$$

$$= \frac{49}{4}$$

$$= 12\tfrac{1}{4}$$

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Remember that multiplication of fractions is required when the word 'of' is used.

An everyday use of multiplication of fractions arises when we take part of a quantity or measurement. The key word here is 'of'.

Example 12

Approval has been given for $\frac{4}{5}$ of a class of 30 students to be immunised. How many injections will the nurse give to students in that class?

Solution

$$\begin{aligned}\frac{4}{5} \text{ of } 30 &= \frac{4}{5} \times 30 \\ &= \frac{4}{5} \times \frac{30}{1} \\ &= \frac{4}{\cancel{5}^{1}} \times \frac{\cancel{30}^{6}}{1} \\ &= \frac{4}{1} \times \frac{6}{1} \\ &= 24\end{aligned}$$

Hence, 24 injections will be given.

Example 13

There is $\frac{5}{8}$ of a packet of cereal at the start of the week, and Jack eats $\frac{2}{3}$ of that amount by the end of the week.

a What fraction of a whole packet of cereal has Jack eaten by the end of the week?

b Assuming Jack will eat the same amount of cereal each week, how much does he eat in 5 weeks?

Solution

a We need to work out $\frac{2}{3}$ of $\frac{5}{8}$.

$$\begin{aligned}\frac{2}{3} \times \frac{5}{8} &= \frac{2 \times 5}{3 \times 8} \\ &= \frac{10}{24} \\ &= \frac{5}{12}\end{aligned}$$

b Jack has eaten $\frac{5}{12}$ of a packet of cereal over the week.

For 5 weeks, Jack needs

$$\begin{aligned}\frac{5}{12} \times 5 &= \frac{5}{12} \times \frac{5}{1} \\ &= \frac{25}{12} \\ &= 2\tfrac{1}{12} \text{ packets.}\end{aligned}$$

Jack will eat $2\frac{1}{12}$ packets of cereal over 5 weeks.

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Division of fractions

The **reciprocal** of a non-zero fraction is obtained by interchanging the numerator and the denominator. For example, the reciprocal of $\frac{3}{7}$ is $\frac{7}{3}$ and the reciprocal of 5 is $\frac{1}{5}$.

The product of a fraction and its reciprocal is always 1.

$$\frac{3}{7} \times \frac{7}{3} = 1 \quad \text{and} \quad 5 \times \frac{1}{5} = 1$$

Division of fractions is the reverse process of multiplication, so to divide by a non-zero fraction we multiply it by its reciprocal.

Example 14

Evaluate:

a $\frac{2}{3} \div \frac{4}{27}$

b $16 \div \frac{2}{3}$

c $\frac{2}{3} \div 16$

Solution

a The reciprocal of $\frac{4}{27}$ is $\frac{27}{4}$.

$$\begin{aligned}\frac{2}{3} \div \frac{4}{27} &= \frac{2}{3} \times \frac{27}{4} \\ &= \frac{\cancel{2}^{1}}{\cancel{3}_{1}} \times \frac{\cancel{27}^{9}}{\cancel{4}_{2}} \\ &= \frac{9}{2} \\ &= 4\tfrac{1}{2}\end{aligned}$$

b The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

$$\begin{aligned}16 \div \frac{2}{3} &= \frac{16}{1} \times \frac{3}{2} \\ &= \frac{\cancel{16}^{8}}{1} \times \frac{3}{\cancel{2}_{1}} \\ &= \frac{24}{1} \\ &= 24\end{aligned}$$

c The reciprocal of 16 is $\frac{1}{16}$.

$$\begin{aligned}\frac{2}{3} \div 16 &= \frac{2}{3} \times \frac{1}{16} \\ &= \frac{\cancel{2}^{1}}{3} \times \frac{1}{\cancel{16}_{8}} \\ &= \frac{1}{24}\end{aligned}$$

As with multiplication, mixed numerals must first be converted to improper fractions before dividing.

Example 15

Robin has $6\frac{3}{4}$ m of timber and wants to cut from it pieces that are $2\frac{1}{4}$ m long.

How many such lengths can he cut?

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Solution

$$6\tfrac{3}{4} \div 2\tfrac{1}{4} = \frac{27}{4} \div \frac{9}{4}$$ (Convert each mixed numeral to an improper fraction.)

$$= \frac{27}{4} \times \frac{4}{9}$$ $\left(\text{Multiply by the reciprocal of } \frac{9}{4}\text{, that is } \frac{4}{9}.\right)$

$$= \frac{\cancel{27}^{3}}{\cancel{4}_{1}} \times \frac{\cancel{4}^{1}}{\cancel{9}_{1}}$$ (Cancel and then multiply.)

$$= 3$$

Thus cutting $6\frac{3}{4}$ m of timber into pieces of length $2\frac{1}{4}$ m gives exactly 3 pieces.

Multiplying and dividing fractions

- To multiply fractions:
  - first cancel factors common to a numerator and a denominator
  - then multiply the numerators and multiply the denominators.
- The **reciprocal** of a non-zero fraction is formed by interchanging the numerator and the denominator.
- To divide by a non-zero fraction, multiply by its reciprocal.
- To multiply or divide mixed numerals, first change them to improper fractions.

Exercise 2C

1 Evaluate:

a $\frac{2}{3}$ of $\frac{5}{16}$ **b** $\frac{3}{4}$ of $\frac{11}{16}$ **c** $\frac{5}{11}$ of 22

d $\frac{3}{5}$ of $\frac{11}{20}$ **e** $\frac{5}{12}$ of 32 **f** $\frac{6}{11}$ of 25

Example 10

2 Evaluate:

a $\frac{1}{2} \times \frac{1}{3}$ **b** $\frac{1}{7} \times \frac{1}{11}$ **c** $\frac{3}{4} \times \frac{1}{3}$ **d** $\frac{7}{8} \times \frac{4}{5}$ **e** $\frac{3}{28} \times \frac{2}{3}$

f $\frac{5}{18} \times \frac{3}{5}$ **g** $\frac{16}{21} \times \frac{7}{8}$ **h** $\frac{14}{15} \times \frac{7}{8}$ **i** $\frac{14}{3} \times \frac{3}{7}$ **j** $\frac{7}{11} \times \frac{22}{35}$

k $\frac{7}{5} \times \frac{5}{13}$ **l** $\frac{3}{4} \times \frac{8}{15}$ **m** $\frac{5}{8} \times \frac{7}{12}$ **n** $\frac{15}{22} \times \frac{11}{30}$ **o** $\frac{7}{4} \times \frac{12}{35}$

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Example 11

3 Evaluate:

a $1\frac{1}{5}\times2\frac{1}{3}$ **b** $2\frac{2}{3}\times3\frac{1}{5}$ **c** $4\frac{1}{10}\times5\frac{2}{5}$

d $2\frac{4}{11}\times1\frac{1}{3}$ **e** $1\frac{3}{5}\times1\frac{2}{3}$ **f** $2\frac{1}{4}\times1\frac{1}{5}$

g $2\frac{1}{5}\times\frac{3}{11}$ **h** $2\frac{1}{4}\times1\frac{1}{8}$ **i** $3\frac{1}{3}\times1\frac{2}{3}$

Example 12

4 $\frac{7}{8}$ of a group of 968 students have brown eyes. How many students is this?

5 Find the value of:

a $\frac{3}{5}$ of 20 L **b** $\frac{5}{8}$ of 10 km **c** $\frac{5}{8}$ of 10 000 m **d** $\frac{3}{4}$ of 50 m

e $\frac{4}{5}$ of 60 kg **f** $\frac{3}{5}$ of 800 mm **g** $\frac{3}{7}$ of 140 m **h** $\frac{7}{8}$ of 100 km

Example 14

6 Evaluate:

a $\frac{5}{8}\div\frac{4}{11}$ **b** $\frac{7}{15}\div\frac{2}{11}$ **c** $\frac{2}{3}\div\frac{4}{7}$ **d** $15\div\frac{18}{5}$

e $\frac{3}{4}\div10$ **f** $\frac{5}{11}\div7$ **g** $\frac{5}{8}\div\frac{11}{3}$ **h** $\frac{5}{12}\div15$

i $\frac{2}{3}\div\frac{5}{8}$ **j** $16\div\frac{2}{3}$ **k** $10\div\frac{2}{5}$ **l** $\frac{15}{8}\div\frac{3}{4}$

7 Evaluate:

a $6\frac{3}{4}\div1\frac{1}{5}$ **b** $2\frac{3}{5}\div1\frac{1}{2}$ **c** $5\frac{1}{4}\div1\frac{1}{2}$ **d** $10\div1\frac{1}{5}$

e $5\frac{1}{3}\div10$ **f** $2\frac{1}{5}\div1\frac{1}{3}$ **g** $7\frac{1}{5}\div9$ **h** $2\frac{1}{5}\div3\frac{3}{4}$

i $5\frac{1}{3}\div6\frac{1}{4}$

Example 13

8 It takes a cabinet maker $2\frac{1}{4}$ days to make a polished table top. How many can he make in 9 weeks?

Example 15

9 How many seconds does it take for a man running at $9\frac{1}{2}$ m/s to run 100 m?

10 **a** What number multiplied by $2\frac{1}{4}$ gives $1\frac{1}{8}$?

b What number divided by $1\frac{1}{3}$ gives $2\frac{1}{4}$?

11 At Castlefield College, $\frac{4}{7}$ of the students are boys.

a What fraction of students are girls?

b If there are 360 boys, how many girls are there?

12 Four-fifths of the jelly beans in a jar are black. There are 288 black jelly beans in the jar. How many jelly beans are there in total?

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13 Three-quarters of the trees in a forest are acacias. It is known that there are 6000 trees in the forest.

a How many acacias are there in the forest?

b What fraction of the trees are not acacias?

14 Three-sevenths of a sum of money is \$36. How much is $\frac{4}{7}$ of the sum of money?

15 Five-twelfths of a farm covers 325 hectares (ha). What is the area of the whole farm?

16 A tank that is $\frac{2}{3}$ full contains 1350 L of water. How many litres does it hold when it is full?

17 After cycling $\frac{1}{3}$ of a journey, a cyclist has 28 km further to go. What is the length of the journey?

18 A rope $10\frac{7}{8}$ m is cut into five equal length pieces of rope. How long is each piece?

2D The unitary method

The idea of the **unitary method** is to base calculations on one part of the whole. Often, the 'part' is a fraction of the whole. Here we take another look at 'of' and multiplication from this point of view.

Fraction of a quantity: the unitary method

In the **unitary method** of solving problems, the most important step is the very first line, where the 'parts' and the 'whole' are identified.

Example 16

The class was told that each pupil must read $\frac{3}{8}$ of a 496-page book before the first day of term. How many pages must each pupil read?

Solution

The 'whole' is the 496-page book.

The 'part' is $\frac{1}{8}$ of the pages in the book,

so 8 parts is 496 pages. (This line is the key step.)

$\boxed{\div 8}$ 1 part is 62 pages. (The operation is in the box to the left.)

$\boxed{\times 3}$ 3 parts is 186 pages. (We have now got to the '3' in $\frac{3}{8}$.)

Hence, each pupil must read 186 pages.

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Going from a fraction to the whole

The unitary method can be used to solve problems where we are given a fraction of the whole.

Example 17

There are 51 people at Sacha's barbecue. This is $\frac{3}{5}$ of those invited. How many people were invited?

Solution

The 'part' to use is $\frac{1}{5}$ of those invited,

so 3 parts is 51 people. (This line is the key step.)

$\boxed{\div 3}$ 1 part is 17 people.

$\boxed{\times 5}$ 5 parts is 85 people. (This is the 'whole'.)

Hence, 85 people were invited to the party.

Exercise 2D

Example 16

1 Use the unitary method to solve these problems.

a $\frac{2}{3}$ of 540 **b** $\frac{5}{8}$ of 968 **c** $\frac{5}{12}$ of 1440

2 Use the unitary method to solve these problems.

a Find $\frac{4}{5}$ of \$160.

b Find $\frac{3}{7}$ of 210 kg.

c Amina's father agrees to pay $\frac{7}{12}$ of the price of a new saxophone. If the saxophone costs \$3780, how much will Amina's father pay?

d Binh's farm has produced 4600 cubic metres of hay. If he keeps $\frac{3}{8}$ of this to feed his stock, how much will he have left to sell at the market?

Example 17

3 **a** $\frac{3}{8}$ of an amount of money is \$543. What is the amount of money?

b $\frac{3}{4}$ of the students in a school are boys. There are 726 boys in the school. How many students are there in the school?

4 Twenty-one students have signed up for the school music camp. This is $\frac{3}{7}$ of the maximum number of people allowed to attend the camp. How many more students can still sign up?

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5 Use the unitary method in reverse to solve these problems.

a If $\frac{3}{5}$ of a container used for storing milk is 150 litres, what is the capacity of the container?

b Massima has saved \$495, which is $\frac{11}{12}$ of the cost of a stereo. What is the price of the stereo?

c Jehan saves \$105 every week, which is $\frac{3}{5}$ of his weekly wage. What is his weekly wage?

d Conrad is recovering from an operation and has been told to walk 10 000 steps per day. In 45 minutes, he has walked 3000 steps. If he walks at the same rate, how long will it take him to walk the required number of steps?

6 In the basketball grand final, Simon scored 16 goals. If Simon scored $\frac{1}{6}$ of his team's goals, what was their final number of goals?

7 One goat needs two-fifths of a hectare if it is to produce high-quality milk. How many hectares are needed for a herd of 72 goats?

8 Sam wants to make five banana pancakes. He has a recipe for 20 pancakes, and the recipe requires two-thirds of a cup of milk. How much milk is needed for 5 pancakes?

2E Decimal notation

Decimals are an extension of the base-ten number system. The whole-number place-value notation is extended to include tenths, hundredths, thousandths, and so on. The **decimal point** separates the whole-number part from the fractional part.

Here is a reminder of what a decimal means. The decimal:

$$123.456 = 100 + 20 + 3 + \frac{4}{10} + \frac{5}{100} + \frac{6}{1000}$$

This process is called writing 123.456 in **expanded form**. We can also write $123.456 = 123\frac{456}{1000}$.

Using expanded form, we can write a decimal as a fraction or mixed numeral.

Example 18

Write 2.1255 as a mixed numeral with the fractional part in simplest form.

Solution

$$\begin{aligned} 2.1255 &= 2\tfrac{1255}{10\,000} \\ &= 2\tfrac{251}{2000} \end{aligned}$$

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Converting fractions to decimals

Converting fractions with denominators that are powers of 10 to decimals is straightforward.

Example 19

Convert $8\frac{123}{1000}$ to a decimal.

Solution

$8\frac{123}{1000} = 8.123$

Some fractions do not have denominators that are powers of 10 but they can also be converted into decimals.

Example 20

Write each fraction as a decimal.

a $\frac{3}{25}$ **b** $\frac{7}{40}$ **c** $\frac{19}{125}$

Solution

a $\frac{3}{25} = \frac{3 \times 4}{25 \times 4} = \frac{12}{100} = 0.12$

b $\frac{7}{40} = \frac{7 \times 25}{40 \times 25} = \frac{175}{1000} = 0.175$

c $\frac{19}{125} = \frac{19 \times 8}{125 \times 8} = \frac{152}{1000} = 0.152$

Comparing and ordering decimals

Selecting the larger of two decimals is done by comparing the leftmost digit and then comparing from left to right until the digits being compared are different. You can then decide which is larger.

Example 21

Five children measure their arm spans. Their results are 1.64 m, 1.5 m, 1.595 m, 1.328 m and 1.593 m.

Put these measurements in order from smallest to largest.

Solution

First compare the units. They are all the same, so compare the tenths digit. The largest tenths digit is the 6 in 1.64 and the smallest is the 3 in 1.328. Now compare the hundredths digits and finally the thousandths to see that the order is:

$1.328 < 1.5 < 1.593 < 1.595 < 1.64$

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Example 22

Order these fractions from smallest to largest by first converting them to decimals:

$\frac{3}{4}$, $7\tfrac{5}{8}$, $\frac{6265}{700}$ and $\frac{228}{400}$.

Solution

$$\begin{aligned}\frac{3}{4} &= \frac{75}{100}\\ &= 0.75\end{aligned}$$

$$\begin{aligned}7\tfrac{5}{8} &= 7\tfrac{625}{1000}\\ &= 7.625\end{aligned}$$ (Multiply numerator and denominator by 125.)

$$\begin{aligned}\frac{6265}{700} &= \frac{895}{100}\\ &= 8.95\end{aligned}$$ (Divide numerator and denominator by 7.)

$$\begin{aligned}\frac{228}{400} &= \frac{57}{100}\\ &= 0.57\end{aligned}$$ (Divide numerator and denominator by 4.)

So the order is $\frac{280}{400} < \frac{3}{4} < 7\tfrac{5}{8} < \frac{6265}{700}$ or $0.57 < 0.75 < 7.625 < 8.95$.

Exercise 2E

Example 18

1 Convert these numbers to a proper fraction or mixed numeral.

a 2.1 **b** 5.023 **c** 6.71 **d** 2.006 **e** 0.076

f 5.68 **g** 0.0085 **h** 2.008 **i** 16.875 **j** 23.625

Example 19, 20

2 Convert these fractions and mixed numerals to decimals.

a $51\tfrac{3}{4}$ **b** $\frac{7}{8}$ **c** $36\tfrac{3}{25}$ **d** $112\tfrac{17}{50}$ **e** $87\tfrac{39}{200}$

Example 21

3 Arrange these numbers in order from smallest to largest.

a 2.5834, 2.35, 2.83, 2.435, 2.5

b 18.009 9573, 18.1, 18.02, 18.1002, 18.21

c 6.6, 6.66, 66.06, 60.66, 60.006

d 55.2, 47.682, 55.24, 55.16, 56.001

Example 22

4 Convert these mixed numerals to decimals, then put them in increasing order.

$4\tfrac{3}{4}$, $4\tfrac{3}{5}$, $4\tfrac{31}{100}$, $4\tfrac{5}{8}$

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2F Operations on decimals

Addition and subtraction

Addition and subtraction of decimals follow the same ideas as for whole numbers and use the standard algorithms. It is important that we only add like to like: we can only add tens to tens, ones to ones, tenths to tenths, and so on. For this reason, we must line up the decimal points of the numbers when we use the algorithms for addition and subtraction.

Example 23

The wiring plan for an electronic robot calls for 0.82 m, 1.5 m and 13.285 m of cable. How much cable is needed in total?

Solution

$$\begin{array}{rr}
 & 0\,.\,8\;2\;0 \\
 & 1\,.\,5\;0\;0 \\
+ & 13_{1}\,.\,2_{1}\;8\;5 \\
\hline
 & 15\,.\,6\;0\;5 \\
\hline
\end{array}$$

15.605 m of cable is needed in total.

Example 24

Coffee comes in bags that weigh 4.2 kg. In one week, 1.83 kg of coffee was used. How much was left?

Solution

$$\begin{array}{rr}
 & 4\,.\,{}^{1}2\;{}^{1}0 \\
- & 1_{1}\,.\,8_{1}\;3 \\
\hline
 & 2\,.\,3\;7 \\
\hline
\end{array}$$

2.37 kg of coffee was left.

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Multiplication and division by powers of 10

Multiplying and dividing decimals by 10, 100, 1000, 10 000 and so on is easy.

Multiplication and division of decimals by powers of 10

- When any number is multiplied by 10, each digit is multiplied by 10. This corresponds to moving the decimal point one place to the right and inserting a zero if necessary.
- Multiplying by $100 = 10^2$ corresponds to moving the decimal point 2 places to the right and inserting zeros if necessary.

  Multiplying by $1000 = 10^3$ corresponds to moving the decimal point 3 places to the right and inserting zeros if necessary.
- When any number is divided by 10, each digit is divided by 10. This corresponds to moving the decimal point one place to the left and inserting a zero if necessary.
- Dividing by $100 = 10^2$ corresponds to moving the decimal point 2 places to the left and inserting zeros if necessary.

  Dividing by $1000 = 10^3$ corresponds to moving the decimal point 3 places to the left and inserting zeros if necessary.

Multiplication of decimals

Example 25

Evaluate 2.451×100.

Solution

$2.451 \times 100 = 245.1$ (The decimal point is moved two places to the right.)

Example 26

Evaluate $2.451 \div 1000$.

Solution

$2.451 \div 1000 = 0.002\,451$ (The decimal point is moved 3 places to the left and the 2 zeros are inserted.)

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Multiplying a decimal by a whole number

The following example presents a suitable setting out.

Example 27

Six children collected lengths of timber for recycling. They collected 3.87 m of timber each. How much timber did they collect in total?

Solution

Multiply the length collected by one child by 6.

$$\begin{array}{r} 3.87 \\ \times\quad 6 \\ \hline 23.22 \\ \hline \end{array}$$

The children collected 23.22 m of timber in total.

Multiplying one decimal by another

We can multiply decimals by converting each decimal to a fraction, multiplying the fractions (without cancelling) and then converting the result back to a decimal.

Example 28

Calculate 0.03×0.18.

Solution

$$\begin{aligned} 0.03 \times 0.18 &= \frac{3}{100} \times \frac{18}{100} \\ &= \frac{54}{10\,000} \\ &= 0.0054 \end{aligned}$$

or

$0.03 \times 0.18 = 0.0054$.
Multiply 18 by 3 and place the decimal point so that the total number of places after the decimal point is the same on both sides of the equation.

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Division of decimals

Dividing a decimal by a whole number

We can use the division algorithm to do this.

$$4\overline{)3.6}\quad = 0.9$$

The procedure is the same as for whole numbers. The decimal point in the quotient is aligned directly above the decimal point in the dividend.

Example 29

Divide 3.6 by 5.

Solution

$$5\overline{)3.6^{1}0}\quad = 0.72$$

Dividing one decimal by another

There are two methods for dividing one decimal by another.

Example 30

Evaluate $3.6 \div 0.05$.

Solution

Method 1

Multiply numerator and denominator by the same factor to obtain a whole-number divisor.

$$\begin{aligned} 3.6 \div 0.05 &= \frac{3.6 \times 100}{0.05 \times 100} \\ &= \frac{360}{5} \\ &= 72 \end{aligned}$$

Method 2

Convert each number to a fraction.

$$\begin{aligned} 3.6 \div 0.05 &= \frac{36}{10} \div \frac{5}{100} \\ &= \frac{36}{10} \times \frac{100}{5} \\ &= \frac{36}{\cancel{10}^{1}} \times \frac{\cancel{100}^{10}}{5} \\ &= \frac{360}{5} \\ &= 72 \end{aligned}$$

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Exercise 2F

Example 23, 24

1 Calculate these sums and differences using the standard algorithms.

a $9.77 + 37.8$ **b** $78.9 + 0.89 + 45 + 4.664$
c $562.6 - 43.18$ **d** $307.05 - 89.77$

Example 25, 26

2 Evaluate:

a 2.3×10 **b** 0.003×100 **c** $2.6 \div 10$
d $260 \div 1000$ **e** $260 \div 10\,000$ **f** 0.0075×100
g $56.1 \div 100$ **h** $2.63 \times 10\,000$ **i** 2.5×100

Example 27

3 Evaluate:

a 5.63×7 **b** 22.867×5 **c** 426.8×9
d 456.23×7 **e** 56.23×24 **f** 69.56×32

Example 28

4 Calculate:

a 0.3×0.7 **b** 0.8×0.06 **c** 0.9×0.07 **d** 0.006×0.7
e 0.06×0.004 **f** 0.05×0.08 **g** 31.504×1.2 **h** 0.061×0.002

Example 29

5 Evaluate:

a $2.376 \div 2$ **b** $9.2 \div 5$ **c** $5.48 \div 4$
d $42.7 \div 5$ **e** $9.7 \div 2$ **f** $43.2 \div 5$

Example 30

6 Calculate:

a $36 \div 0.9$ **b** $2800 \div 0.0007$ **c** $0.378 \div 0.03$
d $63.147 \div 0.07$ **e** $84 \div 2.4$ **f** $560 \div 0.008$
g $7200 \div 0.09$ **h** $0.144 \div 0.012$ **i** $450.56 \div 0.08$

7 The population of Sydney was 3.9 million in 1996 and increased to 4.2 million in 2001. By what number of people did the population increase over the 5 years? What was the average annual increase?

8 What is the perimeter of a table with side lengths 1.8 m, 0.87 m, 0.87 m and 1.43 m?

9 Between June 1996 and June 2001, the population of Brisbane increased by 133 400 people to reach 1.7 million. What was the population in June 1996?

10 What is the mass, in kilograms, of each of the following amounts of olive oil if one litre of olive oil has a mass of 0.878 kg?

a 2 L **b** 3.5 L **c** 0.8 L
d 1.7 L **e** 423 mL **f** 400 mL

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11 On average, Melburnians ate 18.34 kg of cheese each in 2001. If the population of Melbourne in 2001 was 3.5 million, what was the total amount of cheese eaten in Melbourne in that year?

12 If the 321 700 people living in Darwin produce an average of 1.87 kg of garbage per person per day, how much garbage is produced by Darwin residents in one non-leap year?

13 A rectangular block of land measures 15.78 m by 27.93 m. Calculate the perimeter and area of the block.

14 Find, as a decimal, the missing number in each statement.

a $10 \times \square = 0.4$ **b** $109.9 = \square \div 0.7$

c $\square \times 2.3 = 9.2$ **d** $\square = 5.3 \times 96.4$

e $\square \div 1.098 = 25.87$ **f** $4.52 \div 0.8 = \square$

15 Gold is measured in troy ounces. One troy ounce is equal to 31.103 g. What is the weight, in grams, of each of the following gold nuggets?

a The Welcome Nugget, found at Ballarat in 1858, weighing 2217 ounces

b The Welcome Stranger Nugget, found at Moliagul in 1869, weighing 2284 ounces

c The Beyers and Holterman specimen, which was reported to have contained 3000 ounces of gold

2G Fractions, decimals and rounding

We have already seen how to convert certain fractions to decimals.

For example:

$$\frac{3}{10} = 0.3, \frac{3}{4} = 0.75$$

The method used was to multiply the denominator by some number to produce a power of 10 and multiply the numerator by that same number. Thus,

$$\frac{3}{8} = \frac{3 \times 125}{8 \times 125} = \frac{375}{1000} = 0.375$$

This method will only work if the only prime factors of the denominator are 2 or 5, and will produce a decimal with a finite number of digits. Such decimals are called **terminating decimals**. We will see shortly that a fraction whose denominator has a prime factor other than 2 or 5 does not produce a terminating decimal.

A better method for converting fractions to decimals is to use the division algorithm.

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Example 31

Evaluate $\frac{3}{8}$ as a decimal by using the division algorithm.

Solution

$$\begin{array}{r} 0.3\ 7\ 5 \\ 8\overline{)3.0^{6}0^{4}0} \end{array}$$

So $\frac{3}{8} = 0.375$.

Recurring decimals

If we apply the division algorithm to a fraction whose denominator has a prime factor other than 2 or 5, we see that the process does not terminate.

Example 32

Use the division algorithm to write each fraction as a repeating decimal.

a $\frac{4}{11}$

b $\frac{5}{12}$

Solution

a

$$\begin{array}{r} 0.\ 3\ 6\ 3\ 6\ 3\ 6\ 3\ldots \\ 11\overline{)4.{}^{4}0{}^{7}0{}^{4}0{}^{7}0{}^{4}0{}^{7}0{}^{4}0\ldots} \end{array}$$

Hence, $\frac{4}{11} = 0.363636363636\ldots$

We write $\frac{4}{11} = 0.\dot{3}\dot{6}$

b

$$\begin{array}{r} 0.\ 4\ 1\ 6\ 6\ 6\ 6\ 6\ldots \\ 12\overline{)5.{}^{5}0{}^{2}0{}^{8}0{}^{8}0{}^{8}0{}^{8}0{}^{8}0\ldots} \end{array}$$

Hence, $\frac{5}{12} = 0.416666666666\ldots$

We write $\frac{5}{12} = 0.41\dot{6}$

We write 0.363636… as $0.\dot{3}\dot{6}$ to indicate that the digits 36 repeat indefinitely. This is an example of a recurring decimal. We place dots on the first and last digit of the recurring cycle.

We write $0.4166666666\ldots$ as $0.41\dot{6}$ to show that only the digit 6 repeats indefinitely. This is an example of an eventually recurring decimal.

Note the conventions for writing the repeating cycles in the final answers:

- In $0.41\dot{6} = 0.416666666666\ldots$ the dot goes over the repeating digit 6.
- In $0.\dot{3}\dot{6} = 0.363636363636\ldots$, the dots go over the first and last digits of the repeating cycle.

Other fractions give decimals with longer repeating strings of digits.

For example, $\frac{2}{7} = 0.285714285714\ldots$ This can be written as $0.\dot{2}8571\dot{4}$.

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The infinite string of digits in the statement $\frac{5}{12} = 0.416666666666\ldots$ means that the sequence 0.4, 0.41, 0.416, 0.4166, 0.41666, 0.416666, 0.4166666, 0.41666666 . . . gets closer and closer to $\frac{5}{12}$. No term can ever quite get to $\frac{5}{12}$, but we can find terms 'as close as we like' to $\frac{5}{12}$ by taking more and more digits in the string.

Similarly, the statement $\frac{4}{11} = 0.363636363636\ldots$ means that the sequence 0.3, 0.36, 0.363, 0.3636, 0.36363, 0.363636, 0.3636363, 0.36363636 gets 'as close as we like' to $\frac{4}{11}$.

Rounding decimals

The last few decimal places of a decimal such as 3.141 5927 may have no practical value. For example, if 3.141 5927 represents the number of kilometres between two farmyard gates, then the final digit 7 represents $\frac{7}{10}$ of a millimetre, which is completely irrelevant. When we round numbers, we write them correct to a certain number of decimal places.

Writing a decimal correct to a number of decimal places

The rules for rounding are as follows.

Suppose that we want to round 10.125 89 correct to two decimal places.

- Identify the rounding digit in the second decimal place – in this case it is 2.
- Look at the next digit to the right of the rounding digit – in this case it is 5.
  - If this next digit is 0, 1, 2, 3 or 4, leave the rounding digit alone.
  - If this next digit is 5, 6, 7, 8 or 9, increase the rounding digit by 1.

  In this case, the next digit is 5, so the rounding digit increases from 2 to 3.
- Now discard all the digits after the rounding digit.

Hence 10.125 89 is approximately 10.13, correct to two decimal places. We write this as 10.125 89 ≈ 10.13 correct to two decimal places.

Example 33

Write 3.164 93 correct to:

a 2 decimal places **b** 3 decimal places **c** 4 decimal places

Solution

a 3.164 93 ≈ 3.16 to 2 decimal places: the rounding digit is 6, and the digit to the right is smaller than 5.

b 3.164 93 ≈ 3.165 to 3 decimal places: the rounding digit is 4, and the digit to the right is larger than 5.

c 3.164 93 ≈ 3.1649 to 4 decimal places: the rounding digit is 9, and the digit to the right is smaller than 5.

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The rounding procedure can also be used with repeating decimals.

Example 34

Write $0.\dot{3}\dot{6}$ correct to:

a 2 decimal places **b** 3 decimal places **c** 4 decimal places

Solution

$0.\dot{3}\dot{6} = 0.363636\ldots$

a $0.\dot{3}\dot{6} = 0.36$ (correct to 2 decimal places)
b $0.\dot{3}\dot{6} = 0.364$ (correct to 3 decimal places)
c $0.\dot{3}\dot{6} = 0.3636$ (correct to 4 decimal places)

Exercise 2G

Example 31

1 Convert these fractions and mixed numerals to decimals.

a $\frac{2}{5}$ **b** $\frac{4}{5}$ **c** $\frac{3}{25}$ **d** $\frac{7}{8}$ **e** $4\frac{3}{20}$

f $7\frac{3}{8}$ **g** $45\frac{3}{4}$ **h** $7\frac{17}{20}$ **i** $\frac{17}{5}$ **j** $\frac{28}{25}$

Example 32

2 Convert these fractions to decimals.

a $\frac{2}{7}$ **b** $\frac{3}{7}$ **c** $\frac{2}{3}$ **d** $\frac{2}{9}$ **e** $\frac{3}{11}$

f $\frac{4}{9}$ **g** $\frac{7}{11}$ **h** $\frac{8}{9}$ **i** $\frac{2}{13}$

3 Convert these fractions to decimals.

a $\frac{1}{6}$ **b** $\frac{5}{6}$ **c** $\frac{5}{12}$ **d** $\frac{7}{12}$ **e** $\frac{11}{12}$ **f** $\frac{4}{9}$

4 Convert these fractions and mixed numerals to decimals. (They are all recurring decimals.)

a $64\frac{5}{9}$ **b** $78\frac{5}{6}$ **c** $45\frac{2}{11}$ **d** $\frac{11}{12}$ **e** $3\frac{2}{7}$

Example 33

5 Round these decimals correct to the given numbers of decimal places.

a 463.1529 (2 places) **b** 7.2811 (1 place) **c** 79.497 (2 places)

d 0.0649 (3places) **e** 7.99 (1 place) **f** 85.6 (nearest whole number)

6 Express each fraction as a decimal correct to 2 decimal places.

a $\frac{2}{7}$ **b** $\frac{5}{6}$ **c** $\frac{4}{7}$ **d** $\frac{7}{12}$ **e** $\frac{4}{9}$

Example 34

7 Write $0.7\dot{4}\dot{6}$ correct to:

a 2 decimal places **b** 4 decimal places **c** 8 decimal places

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Review exercise

1 From this list of fractions: $\frac{35}{42}, \frac{22}{33}, \frac{10}{15}, \frac{17}{34}, \frac{112}{224}, \frac{20}{24}, \frac{18}{21}, \frac{6}{7}$ write two fractions that are equivalent to each of the following.

a $\frac{12}{18}$ **b** $\frac{5}{6}$ **c** $\frac{42}{49}$ **d** $\frac{1}{2}$

2 Order each set of fractions from largest to smallest.

a $\frac{3}{2}, \frac{2}{3}, \frac{10}{12}, \frac{8}{6}, \frac{5}{4}, \frac{2}{4}$ **b** $2\frac{3}{7}, 3\frac{1}{8}, 2\frac{2}{9}, 2\frac{5}{9}, 3\frac{1}{9}, 2\frac{6}{7}$

3 Evaluate:

a $\frac{5}{12} \div \frac{1}{6}$ **b** $\frac{7}{15} \div \frac{14}{25}$ **c** $\frac{19}{13} \times \frac{52}{81} \div \frac{16}{45}$ **d** $\frac{5}{16} \times \frac{15}{16} \div \frac{125}{32}$

4 Convert mixed numerals to fractions before carrying out these calculations.

a $5\frac{1}{4} \times 2\frac{1}{7}$ **b** $3\frac{1}{5} \div 1\frac{1}{25}$ **c** $5\frac{1}{2} \times 6\frac{1}{2} \div 3\frac{1}{4}$

d $7\frac{6}{7} \times 3\frac{15}{16} \div 4\frac{2}{5}$ **e** $1\frac{1}{3} \times 2\frac{1}{4}$ **f** $\frac{5}{12} \times \frac{3}{14}$

g $\frac{5}{12} \div \frac{3}{4}$ **h** $3\frac{7}{8} \div \frac{1}{4}$ **i** $1\frac{1}{4} \times 2\frac{1}{8}$

5 Calculate:

a $\frac{5}{12} + \frac{1}{3}$ **b** $\frac{2}{3} + \frac{5}{12}$ **c** $\frac{5}{6} - \frac{3}{4}$ **d** $1\frac{1}{3} - 2\frac{1}{5}$

e $3\frac{1}{4} + 2\frac{3}{5}$ **f** $3\frac{3}{4} + 2\frac{1}{5}$ **g** $2\frac{1}{2} - \frac{3}{8}$ **h** $1\frac{1}{2} - \frac{1}{5}$

6 Evaluate:

a $1\frac{3}{4} - \frac{5}{6} + 2\frac{1}{2}$ **b** $5 - 1\frac{1}{2} + \frac{5}{8}$ **c** $3\frac{1}{4} - 2\frac{3}{4} + 2\frac{1}{2}$

d $6\frac{1}{2} + 2\frac{3}{5} - 1\frac{4}{5}$ **e** $2\frac{3}{4} + 9\frac{1}{2} + 1\frac{3}{4}$ **f** $2\frac{1}{3} - \frac{3}{4} + 1\frac{2}{3}$

7 Nina is training hard to become a better swimmer. She spends 17 hours per week doing freestyle, 9 hours per week doing backstroke, and 4 hours per week in the gym. What fraction of her training time is spent out of the pool?

8 Alejandro allocates $\frac{1}{4}$ of his pocket money to transport costs, $\frac{1}{10}$ to telephone calls and $\frac{3}{5}$ to clothes, and he saves the rest. If he receives \$18 per week, what are his expenses for transport, telephone calls and clothes? What fraction of his pocket money does Alejandro save each week?

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9 In a garden, $\frac{1}{3}$ of the area is used for vegetables and $\frac{2}{5}$ for flowers and plants. If the rest of the garden consists of lawns, what fraction of the garden is occupied by lawns?

10 There are 1236 boys at a school. This is $\frac{3}{5}$ of the total number of students at the school. How many students are there at the school?

11 A football squad consists of 24 players. Only $\frac{1}{8}$ of the squad are injured. How many players are fit to play?

12 Round these decimals correct to the given number of decimal places.

a	0.089 734	4 dec. places	**b**	654.0786	3 dec. places
c	3.141 592 65	5 dec. places	**d**	3.141 592 65	3 dec. places
e	2.718 281 828 46	10 dec. places	**f**	2.718 281 828 46	4 dec. places

13 Calculate:

a 4.56×10 **b** 0.0028×100 **c** $1.030\,52 \times 10\,000$

d $0.000\,043 \times 1000$ **e** $24.87 \div 10$ **f** $971.2 \div 100$

g $0.003 \div 1000$ **h** $340 \div 10\,000$ **i** $1000 \times 1.008\,36$

14 Calculate:

a $3.045 + 34.98$ **b** $34.7605 + 0.089$ **c** $452.906 + 8.006$

d $45 - 39.06$ **e** $56.089 - 7.308$ **f** $451.23 - 356.8$

g 67.9×3 **h** 0.41×0.6 **i** 0.345×0.002

j $4 \div 0.02$ **k** $67 \div 0.04$ **l** $0.08 \div 0.6$

15 Calculate the cost of:

a 3.75 kilograms of butter at $1.50 a kilogram

b $3\frac{1}{2}$ dozen eggs at $2.80 per dozen

c 4.35 kilograms of cheese at $7.50 a kilogram

16 Inouk wanted to edge her garden beds with timber. She measured the lengths of the edges as 1.48 metres, 9.2 metres, 0.93 metres, 4.02 metres and 5.45 metres. How many 4.2-metre lengths of timber garden edging did she have to buy? If the timber garden edging costs $3.44 per metre, how much did Inouk's garden edging project cost her?

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17 Calculate how much you save on each item when you buy in bulk.

a 3 cans of beetroot for \$2.85 or 98 cents each

b 6 jars of tomato paste for \$8.40 or \$1.50 each

c 4 bottles of soft drink for \$3.70 or 93 cents each

18 One-quarter of a class arrived at 7 p.m. for the school concert, half of them arrived at 7.15 p.m., three children told the teacher that they were not able to attend, and the rest are running late. If there are 60 children in the class:

a how many children arrived at 7 p.m.?

b what fraction of the class are running late? How many children is this?

c what fraction told the teacher they were not able to attend the concert?

19 Belgian cheese costs \$1.70 for $\frac{1}{2}$ a kilogram. How much will $2\frac{1}{2}$ kilograms of Belgian cheese cost?

20 Linda's doctor tells her she should get 10 hours of sleep each night. She knows that she sleeps $\frac{3}{8}$ of a day already. How much more sleep should she be getting each day?

21 Catherine is $\frac{5}{6}$ of her father's height and her brother Peter is $\frac{7}{9}$ of their father's height. If Peter is 140 cm tall, how tall is Catherine?

22 Zubin's large dog eats $3\frac{1}{3}$ tins of dog food every day. Each tin contains $1\frac{1}{7}$ kg of food.

a How much dog food does the dog eat in a seven-day week?

b How much dog food does the dog eat in a 52-week year?

23 Evaluate:

a $\dfrac{\frac{3}{5}-\frac{1}{2}}{\frac{4}{5}}$ **b** $\dfrac{\frac{6}{11}-\frac{1}{3}}{\frac{7}{9}}$ **c** $\dfrac{\frac{5}{8}}{3\frac{1}{2}-1\frac{1}{4}}$ **d** $\dfrac{\frac{7}{11}}{2\frac{2}{3}-1\frac{1}{2}}$

e $\dfrac{\frac{5}{8}-\frac{1}{3}}{\frac{5}{8}+\frac{1}{3}}$ **f** $\dfrac{5\frac{4}{9}}{2\frac{1}{3}+5\frac{1}{3}}$ **g** $\dfrac{\frac{1}{2}+\frac{3}{4}+\frac{5}{6}}{\frac{2}{3}+\frac{1}{4}+\frac{7}{6}}$ **h** $\dfrac{3\frac{1}{4}-2\frac{3}{5}}{3\frac{1}{4}-2\frac{3}{5}}$

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Challenge exercise

1 Rebecca has a collection of 45 books. The collection consists of novels and textbooks written in either Chinese or English. $\frac{4}{5}$ of the novels are in English and $\frac{3}{4}$ of the textbooks are in English. The total number of books written in English is 35. How many of her books are textbooks written in English?

2 Ava thought she had bought $3\frac{1}{2}$ m of cloth from the market. When she got home, she found that the stall holder had used a ruler that was 4 cm short of 1 m. What was the real length, in metres, of Ava's cloth?

3 Angel spent $\frac{7}{24}$ of her weekly salary on Monday, $\frac{1}{4}$ on Tuesday and $\frac{1}{3}$ of it on Wednesday. What fraction of her salary was left on Thursday?

4 The fractions $\frac{1}{2}$ and $\frac{1}{3}$ are said to be **consecutive unit fractions**.

a Complete each of these subtractions.

i $\frac{1}{2} - \frac{1}{3}$

ii $\frac{1}{4} - \frac{1}{5}$

iii $\frac{1}{7} - \frac{1}{8}$

iv $\frac{1}{9} - \frac{1}{10}$

b State a rule for subtracting consecutive unit fractions.

c Write $\frac{1}{12}$ as the difference of two consecutive unit fractions.

5 Which of the fractions $\frac{1}{4}, \frac{1}{2}, \frac{5}{8}$ and $1\frac{4}{5}$ is the closest to each of the fractions below?

a $\frac{4}{6}$ **b** $\frac{4}{12}$ **c** $\frac{4}{9}$ **d** $\frac{9}{17}$

e $\frac{47}{96}$ **f** $\frac{31}{120}$ **g** $\frac{199}{100}$ **h** $\frac{299}{402}$

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6 **a** Show that $27 \times 37 = 999$. Then use the fact that $0.\dot{9} = 1$ to show that $0.\dot{0}2\dot{7} \times 37 = 1$.

b Hence, show that $\frac{1}{37} = 0.\dot{0}2\dot{7}$.

c By multiplying $\frac{1}{37} = 0.\dot{0}2\dot{7}$ by 10 and 100, show that $\frac{10}{37} = 0.\dot{2}7\dot{0}$ and $\frac{100}{37} = 2.\dot{7}0\dot{2}$, and hence show that $\frac{26}{37} = 0.\dot{7}0\dot{2}$.

d By multiplying $\frac{1}{37} = 0.\dot{0}2\dot{7}$ by 2, 3, 4 and 26, show that

$\frac{2}{37} = 0.\dot{0}5\dot{4}$, $\frac{3}{37} = 0.\dot{0}8\dot{1}$, $\frac{4}{37} = 0.\dot{1}0\dot{8}$ and $\frac{26}{37} = 0.\dot{7}0\dot{2}$.

e Using similar methods, find $\frac{1}{27}, \frac{10}{27}, \frac{19}{27}$, and then $\frac{2}{27}, \frac{3}{27}$ and $\frac{4}{27}$ as recurring decimals.

7 **a** Show that $101 \times 99 = 9999$.

b Hence, find $\frac{1}{101}$ as a recurring decimal.

c Use the methods of the previous question to find $\frac{1}{101}, \frac{100}{101}$ and $\frac{1000}{101}$ as recurring decimals, and hence find $\frac{91}{101}$ as a recurring decimal.

d Find $\frac{2}{101}, \frac{3}{101}$ and $\frac{4}{101}$ as recurring decimals.

8 Four bells commence tolling together and ring every $1, 1\frac{1}{4}, 1\frac{1}{5}$ and $1\frac{1}{6}$ seconds, respectively. After what time will they first ring together again?

9 A unit fraction is of the form $\frac{1}{n}$ where n is a whole number. Remarkably, every fraction can be written as the sum of different unit fractions. For example, $\frac{2}{3} = \frac{1}{2} + \frac{1}{6}$

a Write the number $\frac{3}{10}$ as the sum of two different unit fractions.

b Write $\frac{19}{20}$ as the sum of three different unit fractions.

c Write $\frac{2}{5}$ as the sum of four different unit fractions.

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