CHAPTER

3

Number and Algebra

Review of factors and indices

This chapter is about the whole numbers 0, 1, 2, 3, ... and multiplication. The whole number 6 divides exactly into the whole number 24. The number 1001 can be written as $7 \times 13 \times 11$, which is a product of prime numbers. Each whole number can be factorised into prime numbers. In this chapter, we will learn how to factor numbers. The numbers 72 and 81 share some common factors. The number 9 is called the **highest common factor** of 72 and 81. Factoring numbers into products of primes gives a simple way to find their highest common factor. It will also assist us in finding the square roots and cube roots of numbers. Primes and factoring have important applications in telecommunications technology.

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3A Factors, prime and composite numbers, multiples

All numbers in this chapter are whole numbers.

Factors

The numbers 1, 2, 3, 6, 9 and 18 are the **factors** of 18, because each of them divides into 18 without remainder.

Common factors and the HCF

Below, we list the factors of 18 and 24 and compare the lists. The factors that are common to both lists have been boxed.

Factors of 18: $\boxed{1}, \boxed{2}, \boxed{3}, \boxed{6}, 9, 18$

Factors of 24: $\boxed{1}, \boxed{2}, \boxed{3}, 4, \boxed{6}, 8, 12, 24$

The **common factors** of 18 and 24 are 1, 2, 3 and 6.

The largest of these common factors is 6. This is called the **HCF** – the **highest common factor** – of 18 and 24.

Example 1

Find the highest common factor of 18, 72 and 54.

Solution

The factors of 18 are 1, 2, 3, 6, 9 and 18.

The factors of 72 are 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36 and 72.

The factors of 54 are 1, 2, 3, 6, 9, 18, 27 and 54.

So the highest factor common to 18, 72 and 54 is 18.

Multiples

To write down the non-zero multiples of 4, count in 4s until you have as many as you want:

4, 8, 12, 16, 20, 24, 28, 32, …

Common multiples and the LCM

Look at the lists of the first few non-zero multiples of 18 and 24 below and compare them. The multiples that are common to both lists have been boxed.

Multiples of 18: $18, 36, 54, \boxed{72}, 90, 108, 126, \boxed{144}, 162, 180, 198, \boxed{216}, \ldots$

Multiples of 24: $24, 48, \boxed{72}, 96, 120, \boxed{144}, 168, 192, \boxed{216}, \ldots$

So the **common multiples** of 18 and 24 are 72, 144, 216, . . . The least of these common multiples is 72. This is called the **LCM** – the **lowest common multiple** – of 18 and 24.

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Example 2

Find the lowest common multiple of 6, 8 and 12.

Solution

Non-zero multiples of 6 are 6, 12, 18, 24, 30, 36, ...

Non-zero multiples of 8 are 8, 16, 24, 32, 40, ...

Non-zero multiples of 12 are 12, 24, 36, ...

So the lowest common multiple of 6, 8 and 12 is 24.

Factors and the HCF

- A **factor** of a number is a number that divides into it without remainder. For example, 6 is a factor of 24 because $24 = 6 \times 4$ or, equivalently, $24 \div 6 = 4$.
- There is natural pairing of the factors of any number. For example, the factors of 24 are 1 and 24, 2 and 12, 3 and 8, 4 and 6.
- The **highest common factor**, or **HCF**, of two or more numbers is the largest number that is a factor of all of them.

Multiples and the LCM

- The non-zero **multiples** of a number are found by counting in that number. For example, the non-zero multiples of 4 are 4, 8, 12, 16, 20, 24, 28, 32, ...
- The **lowest common multiple**, or **LCM**, of two or more numbers is the smallest number that is a multiple of all of them.

Prime numbers and composite numbers

A **prime number** is a number greater than 1 whose only factors are 1 and itself.

The list of prime numbers begins 2, 3, 5, 7, 11, ... There are infinitely many prime numbers.

The whole numbers greater than 1 that are not prime are called **composite numbers**. Such numbers have at least three factors, including 1 and itself. The list of composite numbers begins

4, 6, 8, 9, 10, ...

The numbers 0 and 1 are special, because they are neither prime nor composite.

Prime numbers and composite numbers

- A **prime number** is a number greater than 1 whose only factors are 1 and itself.
- A **composite number** is a whole number greater than 1 that is not prime. It will therefore have at least 3 factors.
- The numbers **0** and **1** are neither prime nor composite.

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Exercise 3A

Example 1

1 List all the factors of both numbers in each pair, then write down their HCF.

a 8 and 12 **b** 6 and 20 **c** 14 and 9
d 20 and 22 **e** 1 and 8 **f** 5 and 15

2 Find the HCF of:

a 30 and 24 **b** 15 and 21 **c** 12 and 72
d 25 and 16 **e** 36 and 16 **f** 26 and 65
g 12, 18 and 30 **h** 15, 6 and 14 **i** 60, 20 and 10

Example 2

3 List the first few non-zero multiples of both numbers in each pair, then write down their LCM.

a 4 and 6 **b** 10 and 12 **c** 3 and 7
d 5 and 10 **e** 1 and 5 **f** 6 and 15

4 Find the LCM of:

a 8 and 12 **b** 8 and 9 **c** 17 and 1
d 12 and 15 **e** 7 and 49 **f** 20 and 90
g 3, 4 and 6 **h** 8, 9 and 12 **i** 12, 8, 10 and 30

5 **a** **i** Count to 30 in 2s.
ii Count to 30 in 3s.
iii Count to 30 in 5s.

b Write down all the numbers from 2 to 30 that you did not list in part **a**. This list, together with 2, 3 and 5, is a list of all the 10 prime numbers less than 30.

c Write down all the composite numbers between 30 and 50.

6 Find all the prime divisors of:

a 8 **b** 12 **c** 15 **d** 30
e 44 **f** 38 **g** 125

7 **a** Write 10 and 14 as products of two prime numbers.

b Write 30 and 42 as products of three prime numbers.

c Write each of 8 and 9 as a power of a prime number.

8 Write each of these prime numbers as the sum of two squares.

a 5 **b** 13
c 17 **d** 29
e 37 **f** 41

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9 The Guavalo Palace has three long corridors, whose lengths are 24 m, 48 m and 60 m, respectively. The decorators are designing a standard piece of carpeting that will fit an exact number of times into each corridor. What is the longest piece of carpet that they can choose?

10 The baker's delivery truck visits the caravan park every 4 days. The fishmonger visits every 5 days. If they both visit today, how many days will pass before they visit on the same day again?

11 Three different bugs living in a puddle take 4, 8 and 3 minutes, respectively, to swim around the edge of the puddle. If they start at the same point at 12 p.m., what time will it be when they are all together again at that point?

12 1001 is the product of which three prime numbers?

13 **Goldbach's conjecture** states that every even number greater than 2 can be expressed as the sum of two prime numbers. Show that this is true for the even numbers between 140 and 160.

14 The prime factors of 210 are 2, 3, 5 and 7. Find six other factors of 210 without using division.

15 Explain why a square number has an odd number of factors, and a non-square number has an even number of factors. (*Hint*: Think about the pairing procedure that you used to find all the factors of a number.)

16 Using Question **15**, find the smallest and largest three-digit numbers that have exactly three factors.

3B Indices and the index laws

In this section we discuss some properties of products of repeated factors. We begin with a review of powers.

Powers

Powers provide a useful way of writing a product of repeated factors. For example, 16 can be written as the product of four 2s:

$$16 = 2 \times 2 \times 2 \times 2$$
$$= 2^4 \quad \text{(We read this as '2 to the power 4'.)}$$

- The small number 4 at the top right is called the **index** or **exponent**. (*Note*: The plural of *index* is *indices*.)
- The whole expression 2^4 is called a **power**. It is the fourth power of 2. We say 'two to the fourth'.
- The number 2 in this expression is called the **base**.

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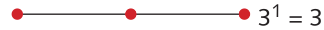
Arrays provide a useful way to visualise powers. Here are some examples with powers of 3.

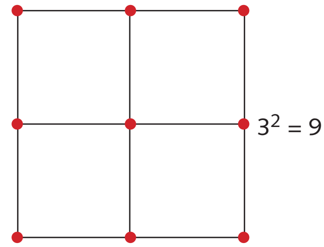
$3^1 = 3$

Read 3^1 as 'three to the power of one'.



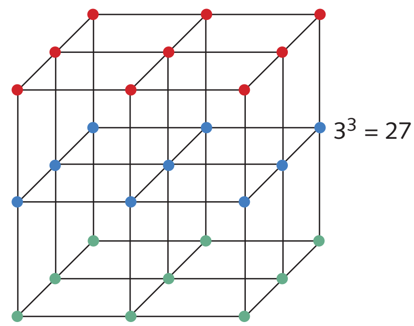
$3^2 = 3 \times 3 = 9$

Read 3^2 as 'three squared' or 'three to the power of two'.



$3^3 = 3 \times 3 \times 3 = 27$

Read 3^3 as 'three cubed' or 'three to the power of three'.



$3^4 = 3 \times 3 \times 3 \times 3 = 81$

Read 3^4 as 'three to the power of four'.

$3^n = 3 \times 3 \times 3 \times \ldots \times 3 \times 3$, with n factors of 3 in the product.

$a^m = a \times a \times a \times \ldots \times a \times a$, with m factors of a in the product.

Index notation

- A **power** is the product of a certain number of factors, all of which are the same. For example:
  $2^4 = 2 \times 2 \times 2 \times 2$
- The number 2 in the example on the previous page is called the **base** of the power.
- The number 4 in the example on the previous page is called the **index** or **exponent**.
- For any number a, $a^1 = a$.

Index laws

There are rules, called the **index laws**, for handling indices with the same base.

Index law 1: A rule for multiplying powers of the same base

There is a very simple way to multiply two or more powers of the same base. Here is an example that explains the rule.

$$7^2 \times 7^3 = (7 \times 7) \times (7 \times 7 \times 7) \quad \text{(Write out each power.)}$$
$$= 7^5 \quad \text{(There are five 7s multiplied together.)}$$

Notice that we add the two indices to get the result:

$$7^2 \times 7^3 = 7^{2+3}$$
$$= 7^5$$

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Index law 1

When multiplying numbers written using powers, if the base number is the same, add the indices.

$a^m \times a^n = a^{m+n}$

Example 3

Write $5^2 \times 5^7$ as a single power of 5.

Solution

$$\begin{aligned} 5^2 \times 5^7 &= 5^{2+7} \\ &= 5^9 \end{aligned}$$

Index law 2: A rule for dividing powers of the same base

There is also a simple rule for dividing powers of the same base. Again, an example shows what the rule must be.

$$\begin{aligned} 5^7 \div 5^3 &= \frac{5^7}{5^3} \\ &= \frac{\cancel{5}^1 \times \cancel{5}^1 \times \cancel{5}^1 \times 5 \times 5 \times 5 \times 5}{\cancel{5}^1 \times \cancel{5}^1 \times \cancel{5}^1} && \text{(Three factors cancel out.)} \\ &= 5 \times 5 \times 5 \times 5 && \text{(This leaves } 7 - 3 = 4 \text{ factors behind.)} \\ &= 5^{7-3} && \text{(Notice that we subtract the indices to get the result.)} \\ &= 5^4 \end{aligned}$$

Index law 2

When dividing a number a^m by another number a^n, subtract the indices.

$a^m \div a^n = a^{m-n}$ or $\frac{a^m}{a^n} = a^{m-n}$

Example 4

Write: **a** $3^6 \div 3^2$ as a single power of 3 **b** $3^7 \div 3^6$ as a single power of 3

Solution

a $$\begin{aligned} 3^6 \div 3^2 &= \frac{3^6}{3^2} \\ &= 3^4 \end{aligned}$$

b $$\begin{aligned} 3^7 \div 3^6 &= \frac{3^7}{3^6} \\ &= 3 \end{aligned}$$

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Index law 3: A rule for raising a power to a power

Once again, an example makes the rule for raising a power to a power clear.

$$\begin{aligned}(8^3)^4 &= 8^3 \times 8^3 \times 8^3 \times 8^3 && \text{(This is what '}8^3\text{ to the power of 4' means.)}\\ &= 8^{3+3+3+3} && \text{(index law 1)}\\ &= 8^{3\times 4}\\ &= 8^{12}\end{aligned}$$

Index law 3

When a power is raised to another power, multiply the indices.

$(a^m)^n = a^{m \times n}$

Example 5

Write $(5^3)^2$ as a single power of 5.

Solution

$$\begin{aligned}(5^3)^2 &= 5^{3\times 2}\\ &= 5^6\end{aligned}$$

Index law 4: Powers of products

It is often useful to expand a power of a product.
For example, we can expand:

$$\begin{aligned}(3\times 4)^5 &= (3\times 4)\times(3\times 4)\times(3\times 4)\times(3\times 4)\times(3\times 4)\\ &= (3\times 3\times 3\times 3\times 3)\times(4\times 4\times 4\times 4\times 4)\\ &= 3^5 \times 4^5\end{aligned}$$

Index law 4

A power of a product is the product of the powers.

$(a \times b)^m = a^m \times b^m$

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Example 6

a Expand the brackets in $(5 \times 4)^2$ and write the answer as a product of powers. Then calculate the answer.

b Use the above index law to evaluate $5^4 \times 2^4$.

Solution

a
$$\begin{aligned}(5 \times 4)^2 &= 5^2 \times 4^2 \\ &= 25 \times 16 \\ &= 400\end{aligned}$$

b
$$\begin{aligned}5^4 \times 2^4 &= (5 \times 2)^4 \\ &= 10^4 \\ &= 10\,000\end{aligned}$$

Index law 5: Powers of quotients

The brackets in a power of a quotient can also be expanded.

$$\begin{aligned}\left(\frac{2}{3}\right)^5 &= \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \\ &= \frac{2 \times 2 \times 2 \times 2 \times 2}{3 \times 3 \times 3 \times 3 \times 3} \\ &= \frac{2^5}{3^5}\end{aligned}$$

Index law 5

A power of a quotient is the quotient of the powers.

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

Example 7

Evaluate $\frac{20^3}{5^3}$.

Solution

$$\begin{aligned}\frac{20^3}{5^3} &= \left(\frac{20}{5}\right)^3 \\ &= 4^3 \\ &= 64\end{aligned}$$

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Indices

Index law 1

To multiply powers of the same base, add the indices.

$a^m \times a^n = a^{m+n}$

Index law 2

To divide powers of the same base, subtract the indices.

$a^m \div a^n = a^{m-n}$

Index law 3

To raise a power to a power, multiply the indices.

$(a^m)^n = a^{m \times n}$

Index law 4

A power of a product is the product of the powers.

$(a \times b)^m = a^m \times b^m$

Index law 5

A power of a quotient is the quotient of the powers.

$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Zero index

Clearly $\frac{4^3}{4^3} = 1$. If the index laws are to apply then $4^{3-3} = 4^0 = 1$. Hence, we define $a^0 = 1$, for all non-zero a.

For example:

$2^0 = 1, 5^0 = 1, 2 \times 6^0 = 2s$ and $(2 \times 6)^0 = 1$.

Exercise 3B

1 Copy and complete this sentence.

'The expression 5^3 is a __________, in which 5 is the _________ and 3 is the _________.'

2 Evaluate:

a 4^2

b 9^2

c 12^2

d 100^2

e 2000^2

f 25^2

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3 **a** Write down three powers of base 7.

b Write down three powers with index 7.

c Write 49, 144 and 8 as powers with indices greater than 1.

d Write 64 as a power with:

i index 1

ii index 2

iii index 3

iv index 6

4 **a** Evaluate all the powers of 2 up to 2^{10}.

b Evaluate all the powers of 5 up to 5^4.

5 Simplify each of the following, using the index law given in brackets.

a $2^2 \times 2^3$ (index law 1)

b $3^2 \times 4^2 \times 3^2$ (index law 1)

c $3^3 \div 3^2$ (index law 2)

Example 3

6 Write each expression as a single power.

a $7^3 \times 7^2$ **b** $11^2 \times 11^4$ **c** $11^5 \times 11$

d $3^2 \times 3^7$ **e** $13^2 \times 13^5$ **f** $10^2 \times 10^3$

Example 4

7 Write each expression as a single power.

a $7^3 \div 7^2$ **b** $11^5 \div 11^2$ **c** $5^5 \div 5^2$

d $\frac{5^7}{5^2}$ **e** $\frac{7^{11}}{7^2}$ **f** $\frac{5^{12}}{5^7}$

Example 5

8 Write each expression as a single power.

a $(2^3)^2$ **b** $(3^5)^2$ **c** $(2^5)^2$

d $(11^2)^3$ **e** $(5^2)^3$ **f** $(7^2)^4$

9 Write each expression as a single power.

a $3^6 \times 3^7$ **b** $8^5 \times 8^2$ **c** $5^3 \times 5 \times 5^4$

d $10^{12} \times 10^{13} \times 10^{20}$ **e** $6^7 \div 6^3$ **f** $2^{12} \div 2^4$

g $7^{10} \div 7$ **h** $4^6 \times 4^5 \div 4^9$ **i** $(12^4)^3$

j $(11^7)^9$ **k** $(4^5)^2 \times 4^6$ **l** $(10^4)^6 \div 10^{20}$

Example 6

10 Evaluate:

a $2^4 \times 5^4$ **b** $4^3 \times 5^3$ **c** $2^5 \times 5^3$

11 Evaluate:

a 5^0 **b** 7^0 **c** 2×6^0

d $(2 \times 6)^0$ **e** 7×11^0 **f** $6^2 \times 6^0$

g $\frac{7^{11}}{7^{10} \times 7^0}$ **h** $(6^7)^0$ **i** $(5^0)^6$

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3C Order of operations

Brackets are used to control the order in which operations are done. For example:

a $(3+4)\times 5 = 7\times 5$
$= 35$

b $(5\times 4)^3 = 20^3$
$= 8000$

c $20-(8+3) = 20-11$
$= 9$

However, when there are no brackets, we need a set of conventions, known as the **order of operations**, to control the order in which operations are done.

Order of operations

- Evaluate expressions inside brackets first.
- In the absence of brackets, and within brackets, carry out operations in the following order:
  - indices
  - multiplication and division from left to right
  - addition and subtraction from left to right.

In the following examples, the brackets have been removed from the expressions **a** to **c** above.

The order of operations conventions mean that the operations are done in different order, and the answers are quite different. We can now see the effect of removing the brackets from **a**, **b** and **c**.

a $3+4\times 5 = 3+20$
$= 23$
(Do multiplication before addition.)

b $5\times 4^3 = 5\times 64$
$= 320$
(Calculate the power first.)

c $20-8+3 = 12+3$
$= 15$
(Do addition and subtraction from left to right.)

Example 8

Evaluate:

a $3\times(6+5)$ **b** $17+3\times 2$ **c** 7×3^2 **d** $(50+10)^2$

Solution

a $3\times(6+5) = 3\times 11$
$= 33$

b $17+3\times 2 = 17+6$
$= 23$

c $7\times 3^2 = 7\times 9$
$= 63$

d $(50+10)^2 = 60^2$
$= 3600$

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Example 9

Evaluate:

a $3^3 \times (2^2 + 12) \div 6 - 4^2$

b $3^3 \times 2^2 + 12 \div (6 - 4)^2$

Solution

a $3^3 \times (2^2 + 12) \div 6 - 4^2 = 27 \times (4 + 12) \div 6 - 16$
$= 27 \times 16 \div 6 - 16$
$= 432 \div 6 - 16$
$= 72 - 16$
$= 56$

b $3^3 \times 2^2 + 12 \div (6 - 4)^2 = 27 \times 4 + 12 \div 4$
$= 108 + 3$
$= 111$

Exercise 3C

Example 8

1 Use the order of operations conventions to evaluate these expressions.

a $3 \times (5 + 11)$ **b** $3 \times 5 + 11$ **c** $(20 - 8) \times 2$

d $20 - 8 \times 2$ **e** $(9 \times 2)^3$ **f** 9×2^3

g $(20 \div 2)^2$ **h** $20 \div 2^2$ **i** $(11 - 1)^4$

j $11 - 1^4$ **k** $(90 + 10)^2$ **l** $90 + 10^2$

2 Evaluate these expressions. You will need to use the 'work from left to right' convention.

a $40 - (17 - 12)$ **b** $40 - 17 - 12$ **c** $52 - (16 + 12)$

d $52 - 16 + 12$ **e** $(44 - 20) - (12 - 8)$ **f** $44 - 20 - 12 - 8$

g $(60 + 25) - (20 + 15)$ **h** $60 + 25 - 20 + 15$ **i** $48 \div (12 \times 2)$

j $48 \div 12 \times 2$ **k** $90 \div (6 \div 3)$ **l** $90 \div 6 \div 3$

Example 9

3 Evaluate:

a $6^2 + 20 \div 2^2 + 15$ **b** $4^3 + 6 \times 8 + 5^2$ **c** $2^3 + (2^2 - 2) \times 2^4$

d $2^2 \times 2^2 + 2 \div 2 - 2^2$ **e** $2^3 \times 2^3 + 2 \div 2 - 2^3$ **f** $2^3 \times (2^3 + 2) \div 2 - 2^3$

g $18 \div 6 - 5 + 3^2 \times 4^3$ **h** $7^3 \times 2^2 + 12$ **i** $\dfrac{3^3 \times 4^2}{2 \times 3^2}$

4 Copy these statements, inserting brackets if necessary to make them true.

a $4 \times 3 + 7 = 40$ **b** $70 - 20 \div 5 = 10$ **c** $2^2 + 6 \times 4 \div 2^2 = 7$

d $4 + 3^2 - 2 = 11$ **e** $3 + 2^2 \times 2^3 - 7^2 = 7$ **f** $4 + 4^4 \div 2^3 + 2 = 26$

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5 Copy these statements, inserting brackets if necessary to make them true.

a $4 \times 2 + 7 = 36$ **b** $4 \times 2 + 7 = 15$ **c** $20 - 2 + 2 - 2 + 2 = 12$

d $20 - 2 + 2 - 2 + 2 = 16$ **e** $70 - 20 \div 5 = 10$ **f** $70 - 20 \div 5 = 66$

g $2^2 + 2 \times 8 \div 2^2 = 12$ **h** $2^2 + 2 \times 8 \div 2^2 = 8$ **i** $2^2 + 2 \times 8 \div 2^2 = 144$

j $11 - 2 + 3^2 = 144$ **k** $11 - 2 + 3^2 = 18$ **l** $11 - 2 + 3^2 = 0$

3D Divisibility tests

We have already seen that a number is divisible by another number if the second number divides it exactly, with no remainder.

Sometimes it is easy to see that one number divides another exactly. For example, 2 divides 12 because $12 = 2 \times 6$.

But what about situations where it is not so easy to see if one number is divisible by another? Sometimes it is helpful to know about divisibility without having to use the division algorithm.

Summary of the divisibility tests

Number	Divisibility test	Examples	
2	Last digit is divisible by 2	12 876 yes	12 877 no
3	Sum of digits is divisible by 3	381 yes	382 no
4	The number consisting of last two digits is divisible by 4	3124 yes	3126 no
5	Last digit is 0 or 5	1245 yes	1246 no
6	Divisible by 2 and 3	3336 yes	3338 no
8	The number consisting of last three digits is divisible by 8	3016 yes	3014 no
9	Sum of digits is divisible by 9	3339 yes	9993 no
10	Last digit is 0	9990 yes	9909 no

We will now look in greater detail why the tests work.

Divisibility tests for 2, 10 and 5

A number that ends in a zero is divisible by 10. For example, 20, 450 and 23 890 are all divisible by 10. This can be seen in the counting pattern:

10, 20, 30, 40, . . . , 670, 680, 690, 700, . . .

Numbers that do not end in 0 are not divisible by 10.

The numbers that are divisible by 2 – the even numbers – can be listed by counting by 2s:

0, 2, 4, 6, 8, 10, . . . , 452, 454, 456, . . .

It is easy to see why this test for divisibility by 2 works.

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For example, we write 352 as a multiple of 10 plus its last digit: $35 \times 10 + 2$. Since we know that any multiple of 10 is divisible by 2, then we only need to look at the last digit. If this last digit is divisible by 2, then the entire number is divisible by 2. If the last digit of a number is not even, then the number is not divisible by 2.

If the last digit of a number is 5, then it is divisible by 5. For example, 15, 225 and 439 785 are all divisible by 5. Also, it follows that a number divisible by 10 is also divisible by 5.

So any number that ends in 0 or 5 is also divisible by 5. If the last digit of a number is not 0 or 5, then the number is not divisible by 5. The counting pattern shows the numbers that are divisible by 5:

$$5, 10, 15, 20, 25, \ldots, 985, 990, 995, 1000, \ldots$$

Divisibility tests for 3 and 9

A number is divisible by 3 if the sum of its digits is divisible by 3. Any number whose digit sum is not divisible by 3 is not itself divisible by 3. For example, 39 is divisible by 3 because the sum of its digits is equal to $3 + 9 = 12$, and 12 is divisible by 3.

A number is divisible by 9 if the sum of its digits is divisible by 9. Any number whose digit sum is not divisible by 9 is not itself divisible by 9. For example, 13 968 is divisible by 9 because the sum of its digits is equal to $1 + 3 + 9 + 6 + 8 = 27$ and 27 is divisible by 9.

The proof of why the test for divisibility by 9 works is illustrated for 684 in the example below.

In expanded form, 684 can be written as:

$$684 = 100 \times 6 + 10 \times 8 + 4$$

We know that $100 = 99 + 1$ and $10 = 9 + 1$, so we can rewrite the above expression as:

$$\begin{aligned} 684 &= (99 \times 6 + 1 \times 6) + (9 \times 8 + 1 \times 8) + 4 \\ &= 99 \times 6 + 6 + 9 \times 8 + 8 + 4 \\ &= (99 \times 6 + 9 \times 8) + 6 + 8 + 4 \end{aligned}$$

Since 9 divides 99, 9 divides $(99 \times 6 + 9 \times 8)$. We are left with $6 + 8 + 4$ (the sum of the digits), which is equal to $6 + 8 + 4 = 18$. This is divisible by 9, so the entire number is divisible by 9.

Note also that any number that is divisible by 9 is also divisible by 3, so 684 is divisible by both 9 and 3. The proof of the test for divisibility by 3 is essentially the same as that for divisibility by 9.

Divisibility test for 6

A number is divisible by 6 if it is both even and divisible by 3. This is because we know that $2 \times 3 = 6$. It follows that if a number is divisible by 3 and it is even, then it is divisible by 6.

Divisibility tests for 4 and 8

A number is divisible by 4 if the number formed by the last two digits is divisible by 4. Conversely, if the number formed by the last two digits of a number is not divisible by 4, then the number itself is not divisible by 4.

For example, 124 is divisible by 4 because the number formed by its last two digits is 24, which is divisible by 4.

The reason why the test for divisibility by 4 works is illustrated in the following example.

$$\begin{aligned} 324 &= 300 + 24 \\ &= (3 \times 100) + 24 \end{aligned}$$

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Since we know that 100 is divisible by 4, we are left with 24. Since 24 is divisible by 4, then 324 is divisible by 4.

A number is divisible by 8 if the number formed by the last three digits is divisible by 8. Conversely, if the number formed by the last 3 digits of a number is not divisible by 8, then the number itself is not divisible by 8.

For example, 5472 is divisible by 8 because 472 is divisible by 8. Since 8 divides 1000, 8 divides any multiple of 1000, so we only need to deal with the last three digits.

Divisibility test for 11

To test a number for divisibility by 11, we follow this procedure. Add up the digits in the odd positions, then add up the digits in the even positions. If the difference of these two numbers is divisible by 11, then the given number is divisible by 11.

For example, 82 786 is divisible by 11 because:

$$\begin{aligned}(8+7+6)-(2+8) &= 21-10\\ &= 11\end{aligned}$$

Example 10

Is 1848 divisible by 2, 3, 4, 5, 9 and 11?

Solution

1848 is divisible by 2 as it ends in 8.

The sum of the digits is $1+8+4+8=21$. Since 21 is divisible by 3, 1848 is divisible by 3.

Since the number formed from the last two digits (48) is divisible by 4, the given number is divisible by 4.

1848 does not end in 5 so it is not divisible by 5.

The sum of the digits is $1+8+4+8=21$. Since 21 is not divisible by 9, 1848 is not divisible by 9.

The sum of the odd-position digits minus the sum of the even-position digits is $(8+8)-(1+4)=16-5=11$. This is divisible by 11, so 1848 is also divisible by 11.

Exercise 3D

Example 10

1 Test each number for divisibility by 2, 4 and 8.

a 2896 **b** 56 374 **c** 1 858 732 **d** 280 082

2 Test each number for divisibility by 3, 9 and 11.

a 5679 **b** 7425 **c** 71 643 **d** 1727

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3 Test each number for divisibility by 6 and 12.

(A number is divisible by 12 if it is divisible by both 4 and 3, because $12 = 4 \times 3$.)

a 3798 **b** 5772 **c** 9909 **d** 48 882

4 Test each number for divisibility by 2, 3, 4, 5, 6, 8, 9, 10, 11 and 12.

a 672 **b** 49 395

c 136 290 **d** 242 010 000 437 000 361

5 Fill in the boxes to make a six-digit number 3__1__3__ divisible by:

a 3 **b** 5 **c** 3 and 5

d 2, 4 and 9 **e** 3, but not 9 **f** 11

g 4 and 11 **h** 6 and 8, but not 9 **i** 3, but not 6

6 List the other numbers by which a number is divisible if it is divisible by:

a 9 **b** 2 and 3 **c** 2 and 5 **d** 8

7 Test each number for divisibility by 12, 15 and 18.
(A number is divisible by 15 if it is divisible by both 3 and 5, because $15 = 3 \times 5$. A number is divisible by 12 if it is divisible by both 4 and 3, because $12 = 4 \times 3$. A number is divisible by 18 if it is divisible by both 2 and 9, because $18 = 2 \times 9$.)

a 36 450 **b** 21 942 **c** 2 041 200 **d** 2 007 000 000

8 Using only digits 3 and 2 write down the smallest number divisible by 6.

9 Using only digits 1 and 2, write down the smallest number divisible by 9.

3E Prime factorisation and its applications

Any whole number can be broken down, by factorisation, into a product of prime numbers. This means that prime numbers are the building blocks from which all whole numbers greater than 1 are built up by multiplication. For example:

$$\begin{aligned} 12 &= 3 \times 4 \\ &= 3 \times 2 \times 2 \\ &= 2^2 \times 3 \end{aligned} \quad \text{or} \quad \begin{aligned} 12 &= 6 \times 2 \\ &= 2 \times 3 \times 2 \\ &= 2^2 \times 3 \end{aligned}$$

We have just found the **prime factorisation** of 12 in two different ways. The prime factorisation of 12 is $2^2 \times 3$.

Factorisation into primes always gives the same answer, apart from the order of the factors. This fact is called the **fundamental theorem of arithmetic**. The final answer is usually written using index notation, with the primes placed in increasing order.

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Here are two of the many ways to carry out the factorisation of 420 into primes.

$$\begin{aligned} 420 &= 2 \times 210 \\ &= 2 \times 21 \times 10 \\ &= 2 \times 3 \times 7 \times 2 \times 5 \\ &= 2^2 \times 3 \times 5 \times 7 \end{aligned} \qquad \text{or} \qquad \begin{aligned} 420 &= 60 \times 7 \\ &= 6 \times 10 \times 7 \\ &= 2 \times 3 \times 2 \times 5 \times 7 \\ &= 2^2 \times 3 \times 5 \times 7 \end{aligned}$$

The example below shows another standard way to set out these calculations.

Example 11

Express 1260 as the product of its prime factors.

Solution

2	1260
2	630
3	315
3	105
5	35
7	7
	1

$$\begin{aligned} \text{Hence, } 1260 &= 2 \times 3 \times 5 \times 7 \times 3 \times 2 \\ &= 2 \times 2 \times 3 \times 3 \times 5 \times 7 \\ &= 2^2 \times 3^2 \times 5 \times 7 \end{aligned}$$

Using prime factorisation to find the LCM

At the beginning of this chapter, we found the lowest common multiple (LCM) of two numbers by writing down lists of the multiples of each number. For larger numbers this is not very practical – prime factorisation provides a better method.

To find the LCM of 24 and 18, we first factorise each number as a product of primes:

$$\begin{aligned} 24 &= 2 \times 2 \times 2 \times 3 \\ &= 2^3 \times 3 \end{aligned} \qquad \begin{aligned} 18 &= 2 \times 3 \times 3 \\ &= 2 \times 3^2 \end{aligned}$$

Both numbers have powers of 2 as a factor. The larger, 2^3, must be taken as a factor of the LCM, otherwise the LCM could not be a multiple of 24. Similarly, the larger power of 3 is 3^2. It must be taken as a factor of the LCM because the LCM must be a multiple of 18. So the LCM of 24 and 18 is:

$$\begin{aligned} 2^3 \times 3^2 &= 8 \times 9 \\ &= 72 \end{aligned}$$

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Example 12

What is the LCM of 180 and 144?

Solution

Factorise each number into a product of primes.

$$180 = 2 \times 2 \times 3 \times 3 \times 5 = 2^2 \times 3^2 \times 5 \qquad 144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^4 \times 3^2$$

Take the greatest power of each prime.

The LCM of 180 and 144 is:

$$2^4 \times 3^2 \times 5^1 = 16 \times 9 \times 5 = 720$$

Using prime factorisation to find the HCF

Prime factorisation provides an effective method for finding the highest common factor (HCF) of two or more numbers.

To find the HCF of 24 and 18, we again factorise each number into primes:

$$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3^1 \qquad 18 = 2 \times 3 \times 3 = 2^1 \times 3^2$$

Each number contains a power of 2 and a power of 3 in its prime factorisation.

To get the highest common factor of 24 and 18, we take the lower power of 2, which is $2 = 2^1$, and multiply it by the lower power of 3, which is $3 = 3^1$. No higher power than 3^1 can divide the HCF. No higher power than 2^1 can divide the HCF. The HCF of 24 and 18 is therefore $2 \times 3 = 6$.

The reason this works is that we cannot take any higher power of 2 and still have a factor of 18, nor can we take a higher power of 3 and still have a factor of 24.

Example 13

What is the HCF of 180 and 144?

Solution

Factorise both numbers into primes.

$$180 = 2 \times 2 \times 3 \times 3 \times 5 = 2^2 \times 3^2 \times 5^1 \qquad 144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^4 \times 3^2$$

Take the smallest index of each prime common factor.

The HCF of 180 and 144 is $2^2 \times 3^2 = 36$.

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Factors and divisibility

- The **fundamental theorem of arithmetic** states that every whole number greater than 1 can be expressed as a product of prime numbers. This product for each number is unique apart from the order of the prime factors.
- Prime factorisation can be used to find the lowest common multiple. Take the *greatest* power for each prime.
- Prime factorisation can be used to find the highest common factor. Take the *smallest* power for each prime.

Squares and square roots

The **square** of a number is the product of that number with itself. For example, 8 squared is the number $8 \times 8 = 64$. The number 64 is called a **perfect square**, because it is the square of a whole number.

The **square root** of a number is the number that when multiplied by itself gives the first number. For example, $8^2 = 64$, so 8 is the square root of 64.

We write $\sqrt{64} = 8$.

Cubes and cube roots

The **cube** of a number is defined similarly. So the cube of 4 is $4 \times 4 \times 4 = 4^3 = 64$.

The **cube root** of 64 is 4 because $4^3 = 64$, and we write $\sqrt[3]{64} = 4$.

The cube root of 125 is 5, because the cube of 5 is 125. We write $\sqrt[3]{125} = 5$, because $5^3 = 125$.

We can see that taking the cube root is the reverse operation to cubing.

Note that most numbers do not have a square or cube root that is a whole number. For example:

- Neither $\sqrt{2}$ nor $\sqrt[3]{2}$ is a whole number.
- $\sqrt{9} = 3$ because $3^2 = 9$, but $\sqrt[3]{9}$ is not a whole number.
- $\sqrt{8}$ is not a whole number, but $\sqrt[3]{8} = 2$ because $2^3 = 8$.

Example 14

What is the cube root of 1000?

Solution

$$\begin{aligned} 1000 &= 10 \times 10 \times 10 \\ &= 10^3 \end{aligned}$$

$$\sqrt[3]{1000} = 10$$

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Finding square roots and cube roots by prime factorisation

Prime factorisation can help us find the square root of a perfect square. For larger numbers, prime factorisation is the best way of finding their square roots and cube roots, if these are whole numbers.

For example, here is the prime factorisation of 324.

$$\begin{aligned}324 &= 4\times 81\\ &= 2\times 2\times 9\times 9\\ &= 2\times 2\times 3\times 3\times 3\times 3\\ &= 2^2\times 3^4\end{aligned}$$

Since both indices are even, we can use the index laws to write 324 as the square of a whole number.

$$\begin{aligned}324 &= 2^2\times 3^4\\ &= (2^1\times 3^2)^2 \quad \text{(index laws 3 and 4)}\end{aligned}$$

Since the square of $2^1\times 3^2$ is 324, $\sqrt{324} = 2\times 3^2 = 18$.

However, the indices are not multiples of 3, so $\sqrt[3]{324}$ is not a whole number.

Example 15

Find the square root of 7056, using the prime factorisation of 7056.

Solution

7	7056
7	1008
3	144
3	48
2	16
2	8
2	4
2	2
	1

Hence, $7056 = 2^4\times 3^2\times 7^2$

$= (2^2\times 3\times 7)^2$

Therefore, 7056 is a square number and $\sqrt{7056} = 2^2\times 3\times 7 = 84$.

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It is not necessary to factor one prime at a time, as the following example shows.

Example 16

Find $\sqrt[3]{1728}$.

Solution

The prime factorisation of 1728 is:

$$\begin{aligned} 1728 &= 4 \times 432 \\ &= 4 \times 4 \times 108 \\ &= 4 \times 4 \times 4 \times 27 \\ &= (2^2)^3 \times 3^3 \\ &= (2^2 \times 3)^3 \end{aligned}$$

so $\sqrt[3]{1728} = 2^2 \times 3 = 12$

Note that we could not calculate $\sqrt{1728}$ in this way because not all the indices are even, so $\sqrt{1728}$ is not a whole number.

Perfect squares and square roots

- A number multiplied by itself gives the **square** of the original number.
- The **square root** of a square number is the number that when multiplied by itself gives the original number.
- The square of a whole number is called a **perfect square**.
- We can use prime factorisation to determine whether a whole number is a square and to find the square root of a number.

Cubes and cube roots

- The **cube** of a number a is the product $a \times a \times a = a^3$.
- Taking the **cube root** is the inverse operation of cubing.
- We can use prime factorisation to determine whether a whole number is a cube and to find the cube root of a number.

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Exercise 3E

Example 11

1 Find the prime factorisation of:

a 8 **b** 24 **c** 45

d 891 **e** 735 **f** 2904

Example 12, 13

2 Given the prime factorisations $144 = 2^4 \times 3^2$, $108 = 2^2 \times 3^3$ and $405 = 3^4 \times 5$. Find the LCM and HCF of:

a 144 and 108 **b** 108 and 405 **c** 108, 144 and 405

3 Use prime factorisation to find:

a the HCF of 36 and 45

b the LCM of 36 and 45

c the HCF of 112 and 21

d the LCM of 112 and 21

e the LCM of 12, 15 and 40

f the LCM of 24, 72 and 108

g the HCF of 72 and 126

h the HCF of 24, 60 and 112

i the HCF of 70, 105 and 280

Example 14, 15

4 Given the prime factorisation, find the square root and the cube root of each number below, if they are whole numbers.

a $3375 = 3^3 \times 5^3$ **b** $729 = 3^6$ **c** $4356 = 2^2 \times 3^2 \times 11^2$

Example 15, 16

5 Find the prime factorisation of each number below. Then find its square root and cube root, if they are whole numbers.

a 225 **b** 324

c 512 **d** 1728

e 360 **f** 1936

g 216 **h** 10 648

6 Mr and Mrs Cinco have quintuplets, three boys and two girls. When buying toys, to avoid squabbles, they always make sure that the toy collection can be shared equally whether the five children play together, the girls play alone or the boys play alone.

At the beginning of the year, the number of toys in the collection is 180. What is the smallest number of toys they can buy for the children's birthday so that the family can continue sharing the toys in the same way as before?

7 Find the prime factorisation of 360, and use it to write down all 24 factors of 360.

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Review exercise

1 The product of the factors of 12 is:

A 12 **B** 28 **C** 1728 **D** 144 **E** 3556

2 The sum of the factors of 18 is:

A 9 **B** 36 **C** 5832 **D** 39 **E** 2916

3 Write each of these numbers as the sum of two multiples of 8.

a 96 **b** 40 **c** 184 **d** 480 **e** 296

4 Find the LCMs of the following sets of numbers.

a 8 and 6 **b** 10 and 18 **c** 16 and 24

d 60 and 72 **e** 6, 10 and 15 **f** 10, 12 and 18

g 36, 45 and 27 **h** 12, 60 and 102 **i** 30, 40, 50 and 75

5 Find the HCFs of the following sets of numbers.

a 6 and 9 **b** 45 and 75 **c** 12 and 35

d 60 and 70 **e** 21 and 35 **f** 132 and 680

g 12, 18 and 30 **h** 18, 30 and 90 **i** 42, 189 and 231

6 Evaluate:

a 6^2 **b** 11^2 **c** 50^2

d 31^2 **e** 87^2 **f** 143^2

7 Write each expression as a single power.

a $8^2 \times 8^4$ **b** $2^{13} \div 2^9$ **c** $3^5 \times 3^5 \times 3^5$

d $4^9 \times 4^{19}$ **e** $10^3 \times 10^2 \times 10^3 \times 10^4$ **f** $7^{11} \div 7^2 \times 7^8$

g $(5^2)^3$ **h** $(8^2)^3 \div 8^2$ **i** $12^4 \times (12^5)^2$

8 **a** The LCM of $2^2 \times 3^1 \times 5^4$ and $2^3 \times 3^2 \times 7^2$ is:

A $2 \times 5^4 \times 7^2$ **B** $2^3 \times 3^2 \times 5^4 \times 7^2$ **C** $2^2 \times 3^1 \times 5^1 \times 7^1$

b The HCF of $2^2 \times 3^1 \times 5^4$ and $2^3 \times 3^2 \times 7^2$ is:

A 75 **B** 3528 **C** 12 **D** 210 **E** $2^3 \times 3^2$

9 Standing on the seashore, Stephanie can see two ships' lights flashing. One light flashes at intervals of 12 seconds and the other at intervals of 18 seconds.

a If they flash together once, how long is it before they flash together again?

b If they flash together at the start of a 5-minute interval, how many times will they flash together in those 5 minutes?

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10 Show that each of the whole numbers between 28 and 37 can be written as the product of prime numbers.

11 Express each of these numbers as a product of its prime factors and so find its square root.

a 256 **b** 441 **c** 1156

d 7569 **e** 18 225 **f** 20 736

12 Write each prime number as the sum of two square numbers.

a 73 **b** 137 **c** 229

13 Test each of the following numbers for divisibility by 3, by 4 and by 12.

a 64 **b** 1336 **c** 3972

14 Write a five-digit number that has one of its digits repeated three times and is:

a divisible by 2 **b** divisible by 5 and 2

c divisible by 3 and 4 **d** divisible by 11

e divisible by 8 and 3 **f** divisible by 2 and 11 but not 4

15 Evaluate:

a $3\times5^2+6^2-2^3$ **b** $(3\times5^2+6)^2-2^3$ **c** $(3\times5)^2+(6^2-2)^3$

d $[(3\times5)^2+6]^2-2^3$ **e** $3\times(5^2+6^2-2)^3$ **f** $(3\times5^2-2\times6^2-2)^3$

16 A yellow bulb flashes every 4 minutes and a blue bulb flashes every 7 minutes. If both bulbs flash together at 9 a.m., what is the first time after 10 a.m. that both bulbs will flash together?

17 How many factors do each of the following have?

a 2^3 **b** 5^3 **c** 7^3 **d** 11^3

18 How many factors do each of the following have?

a 2^5 **b** 3^5 **c** 7^5 **d** 11^5

19 Write down the factors of:

a 111 **b** 1001 **c** 1111

20 Find the highest common factor of each of the following pairs.

a $2^3\times3^4, 2^2\times3^6\times5$ **b** $2^4\times3\times5, 2^3\times3^2\times5^2$

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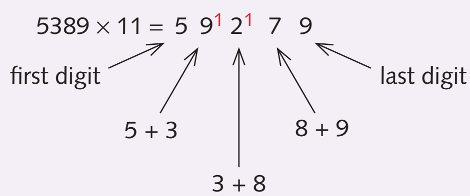
Challenge exercise

1 A number is greater than 48 squared and less than 49 squared. If 6 squared is one of its factors, and it is a multiple of 13, what is the number?

2 Given a six-digit number $328xyz$, find the values of x, y and z such that the number is divisible by 3, 4 and 5, and is the smallest six-digit number starting with 328 that has this property. How many other numbers have this property?

3 When multiplying a number by 11, we can use the following technique. First, write down the last digit of the number to be multiplied. Then, *working from right to left*, write down the sum of every pair of adjacent numbers. Remember to carry correctly. Finally, write down the first digit of the original number. For example:



a Write down, using the above method:

i $17\,089 \times 11$ **ii** 2223×11

iii $1\,654\,998 \times 11$ **iv** $1\,654\,998 \times 111$

b Briefly explain why this method works.

4 What three-digit number leaves a remainder of 1 when divided by 2, 3, 4, 5 or 6, and no remainder when divided by 7?

5 The local takeaway chicken shop has all of its employees working the same number of hours each day. The total number of hours for Thursday is 133. How many employees work on Thursdays?

6 Hoc knew that before he closed his novel, the product of the page numbers was 1190. What were the two open pages?

7 It is 7 p.m. now. What will the time be 23 999 999 996 hours later?

8 What is the smallest whole number that is divisible by 11 and leaves a remainder of 1 when divided by any of the numbers from 2 to 10?

9 What is the largest prime factor of 93 093?

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10 The product of two whole numbers is 10 000 000. Neither number is a multiple of 10. What are the two numbers?

11 A teacher multiplied the ages of all of the students (all teenagers) in her class and came up with the number 15 231 236 267 520.

a What is the prime factorisation of this number?

b Use the prime factorisation to help you work out the number of students in the class, their ages and the number of students of each age.

12 Twelve is the smallest whole number that has exactly 6 different factors. What is the smallest whole number that has exactly 24 different factors?

13 **a** What is the smallest whole number that satisfies all of the following conditions?

- Dividing by 7 gives a remainder of 4.
- Dividing by 8 gives a remainder of 5.
- Dividing by 9 gives a remainder of 6.

b What is the smallest whole number that satisfies the conditions in part **a** and also the following ones?

- Dividing by 6 gives a remainder of 3.
- Dividing by 11 gives a remainder of 8.

14 **a** If the four-digit number $8mn9$ is a perfect square, find the values of m and n.

b The digits 1, 2, 3, 4 and 5 are each used once in the five-digit number $abcde$ such that the three-digit number abc is divisible by 4, bcd is divisible by 5, and cde is divisible by 3. Find the values of a, b, c, d and e.

c The digits 1, 2, 3, 4, 5 and 6 are each used in the six-digit number $abcdef$. The three-digit number abc is divisible by 4, bcd is divisible by 5, cde is divisible by 3, and def is divisible by 11. Find the values of a, b, c, d, e and f.

15 Helena has a basket of chocolates. There are fewer than 500 chocolates in the basket. Helena knows that she can divide all of the chocolates into bags holding 2 each, 3 each or 6 each. She also knows that it is not possible to put them into bags of 4, 5, 9 or 10. If she tries to put 7 in each bag, there are 3 left over. How many chocolates are in Helena's basket?

16 What is the next number after 64 whose square roots and cube roots are both whole numbers?

17 Find the smallest positive whole number m such that $60m$ is a perfect square.

18 Find the smallest positive whole number whose digits are all 0s and 1s that is divisible by 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

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