CHAPTER

4

Number and Algebra

Negative numbers

You have probably come across examples of negative numbers already. They are the numbers that are less than zero. Negative numbers are used, for example, in the measurement of temperature.

The temperature 0°C is the temperature at which water freezes, known as *freezing point*. The temperature that is 5 degrees colder than freezing point is written as –5°C.

In some Australian cities, the temperature drops below zero. Canberra has a lowest recorded temperature of –10°C. The lowest recorded temperature in Australia was –23°C, recorded at Charlotte's Pass in NSW. Here are some other lowest temperatures:

Alice Springs	–7°C	Paris	–24°C	London	–16°C

Negative numbers are also used to record heights below sea level.

For example, the surface of the Dead Sea in Israel is at 423 metres below sea level. This can also be written as –423 metres above sea level. This is the lowest point on land anywhere on Earth. The lowest point on land in Australia is at Lake Eyre, which is 15 metres below sea level. This can also be written as –15 metres above sea level.

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4A Negative integers

The whole numbers together with the negative whole numbers are called the **integers**.

…, –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, …

The numbers 1, 2, 3, 4, 5, … are called the **positive integers**.

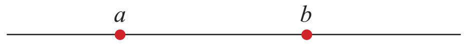
The numbers … , –5, –4, –3, –2, –1 are called the **negative integers**.

The number 0 is neither positive nor negative.

The number line

The integers can be represented by points on a number line. The line is infinite in both directions, with the positive integers to the right of zero and the negative integers to the left of zero. The numbers are equally spaced.

An integer a is **less than** another integer b if a lies to the left of b on the number line. The symbol $<$ is used for **less than**. For example, $-3 < -1$.



An integer b is **greater than** another integer a if b lies to the right of a on the number line. The symbol $>$ is used for **greater than**. For example, $1 > -5$.

A practical illustration of this is that a temperature of –10°C is colder than a temperature of –5°C, and $-10 < -5$. Similarly, $-5 < 0$.

Example 1

a List all the integers less than 5 and greater than –2.

b List all the integers less than 1 and greater than –4.

Solution

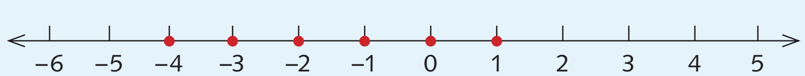
a –1, 0, 1, 2, 3, 4

b –3, –2, –1, 0

Example 2

Draw a number line and indicate on it with dots all the integers greater than –5 and less than 2.

Solution



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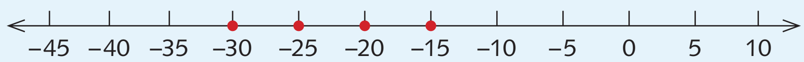
Example 3

a The sequence 10, 5, 0, –5, –10, ... is ‘going down by fives’.
Write down the next four terms.

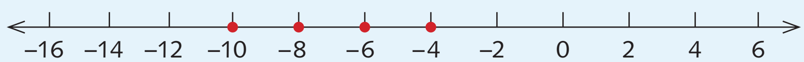
b The sequence –16, –14, –12, ... is ‘going up by twos’.
Write down the next four terms.

Solution

a The next four terms are –15, –20, –25, –30.



b The next four terms are –10, –8, –6, –4.



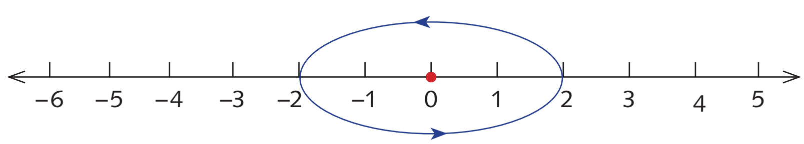
The opposite of an integer

We call –2 the number **opposite** to 2.

The integer –2 has opposite –(–2) = 2.

The operation of forming the opposite can be visualised by putting a pin in the number line at 0 and rotating the number line by 180°.

The opposite of 2 is –2.



The opposite of –2 is 2, so –(–2) = 2.

The number 0 is different from all of the other numbers. The opposite of 0 is 0.

The operation of taking the opposite can be visualised by swapping the number on one side of zero with the matching number the same distance from zero on the other side.

Exercise 4A

Example 1

1 **a** List the integers less than –16 and greater than –22.

b List the integers greater than –114 and less than –105.

c List the integers less than 3 and greater than –8.

Example 2

2 **a** Draw a number line and mark the numbers –12, –2, 2 and –8 on it.

b Draw a number line and mark the numbers –8, –3, –5 and 4 on it.

c Draw a number line and mark the integers less than 5 and greater than –6 on it.

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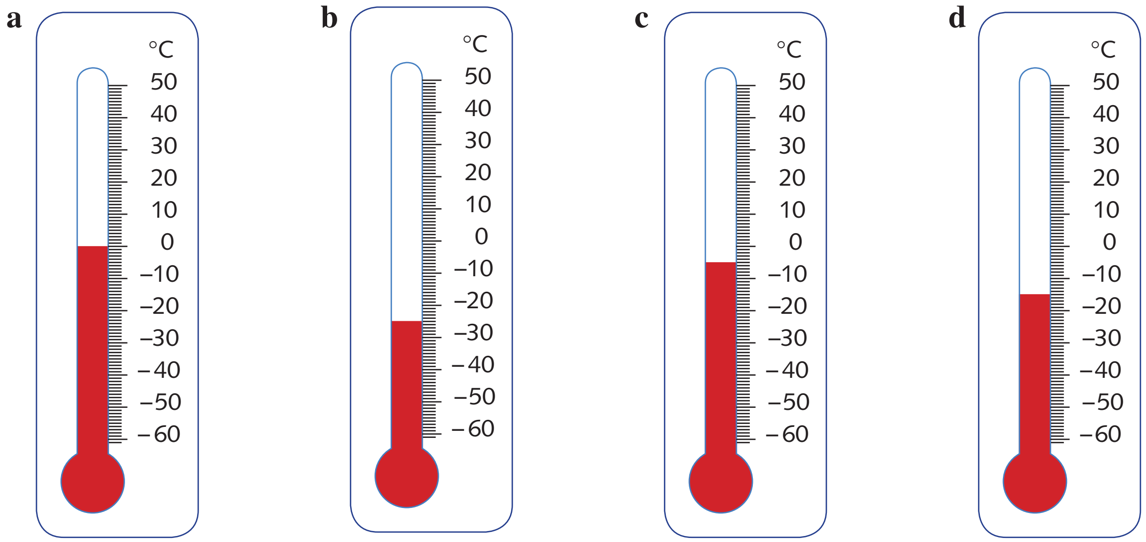
Example 3

3 The sequence –16, –13, –10, ... is 'going up by threes'. Find the next three terms.

4 The sequence 15, 11, 7, ... is 'going down by fours'. Find the next three terms.

5 The sequence –50, –43, –36, ... is 'going up by sevens'. Find the next four terms.

6 Determine the readings for each of the thermometers shown below.



7 Determine the opposites of these integers.

a 6 **b** –3 **c** 34 **d** –5

e –72 **f** 67 **g** –456 **h** 10 000

4B Addition and subtraction of a positive integer

If a submarine is at a depth of –250 m and then rises by 20 m, its final position is –230 m. This can be written as –250 + 20 = –230.

Joseph has $3000 and he spends $5000. He now has a debt of $2000, so it is natural to interpret this as 3000 – 5000 = –2000.

The number line and addition

The number line provides a useful picture for addition and subtraction of integers.

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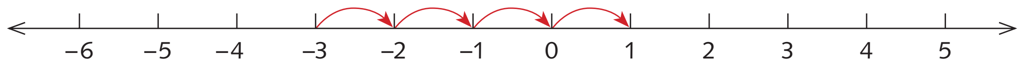
Addition of a positive integer

When you add a positive integer, move to the right along the number line.



For example, to calculate $-3+4$, start at -3 and move 4 steps to the right. We see that $-3+4=1$.

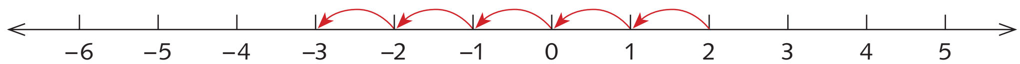
A practical example of this is: 'I started with a debt of $3 and I earned $4. I now have $1'.

Subtraction of a positive integer

We can think of subtraction as **taking away**.

When you subtract a positive integer, move to the left along the number line.

For example, to calculate $2-5$, start at 2 and move 5 steps to the left. We see that $2-5=-3$.



The same question arises in a practical way: 'I had $2 and I spent $5. I now have a debt of $3'.

Example 4

Calculate:

a $-6+9$ **b** $-1+7$ **c** $-2+9$ **d** $-9+2$

Solution

a $-6+9=3$ (Start at -6 on the number line and move 9 steps to the right.)
b $-1+7=6$ (Start at -1 on the number line and move 7 steps to the right.)
c $-2+9=7$ (Start at -2 on the number line and move 9 steps to the right.)
d $-9+2=-7$ (Start at -9 on the number line and move 2 steps to the right.)

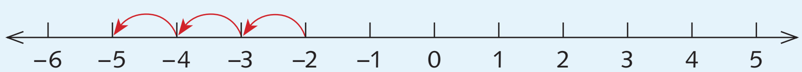
Example 5

Calculate:

a $-2-3$ **b** $2-8$

Solution

a



Start at -2 and move 3 steps to the left. We see that $-2-3=-5$.

b $2-8=-6$ (Start at 2 on the number line and move 8 steps to the left.)

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Exercise 4B

Example 4

1 Write the answers to these additions.

a $-5+4$ **b** $-7+11$ **c** $-6+2$
d $-11+15$ **e** $-4+9$ **f** $-7+11$
g $-1+10$ **h** $-3+10$ **i** $-15+7$
j $-11+11$ **k** $-17+10$ **l** $-13+4$
m $-16+16$ **n** $-70+33$ **o** $-60+99$
p $-21+15$ **q** $-75+12$ **r** $-96+6$
s $-345+600$ **t** $-23+89$

Example 5

2 Write the answers to these subtractions.

a $-5-6$ **b** $6-8$ **c** $1-9$
d $3-11$ **e** $7-12$ **f** $-2-8$
g $4-10$ **h** $2-5$ **i** $-3-4$
j $4-6$ **k** $-10-6$ **l** $-26-12$
m $-17-10$ **n** $-13-6$ **o** $14-20$
p $3-96$ **q** $54-59$ **r** $-66-30$
s $28-64$ **t** $12-21$ **u** $-100-28$
v $-136-144$

3 Write the answers to these additions and subtractions.

a $-450+50$ **b** $-500+80$
c $-100-70$ **d** $-315+430$
e $-420-45$ **f** $-300+500$
g $-500-700$ **h** $-1500+500$
i $-4000-600$ **j** $8000-10\,000$
k $4500-5000$ **l** $-5000-2500$

4 Work from left to right to calculate:

a $2-4+8$ **b** $4-10+20$
c $4-12+8$ **d** $23-30+17$
e $2+3-5$ **f** $3-7+11$
g $-3-7+12$ **h** $4-11-21$

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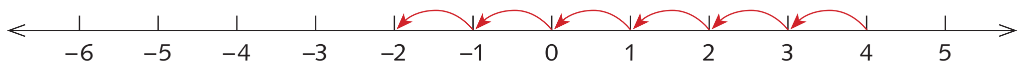
4C Addition and subtraction of a negative integer

In the previous section, we considered addition and subtraction of a positive integer. In this section, we will add and subtract negative integers.

Addition of a negative integer

Adding a negative integer to another integer means that you take a certain number of steps to the *left* on a number line.

The result of the addition $4 + (-6)$ is the number you get by moving 6 steps to the left, starting at 4.



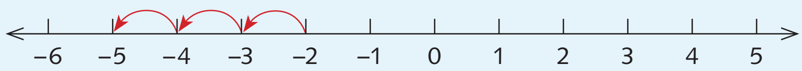
$4 + (-6) = -2$

Note that $4 + (-6)$ is the same as $4 - 6$.

Example 6

Calculate $-2 + (-3)$.

Solution



$-2 + (-3)$ is the number you get by moving 3 steps to the left, starting at -2. That is, -5.

Notice that $-2 - 3$ is also equal to -5.

All additions of this form can be completed in a similar way. For example:

$4 + (-7) = -3$ and $4 - 7 = -3$

$-11 + (-3) = -14$ and $-11 - 3 = -14$

This suggests the following rule.

To add a negative integer, subtract its opposite

For example:

$$4 + (-10) = 4 - 10 = -6$$

$$-7 + (-12) = -7 - 12 = -19$$

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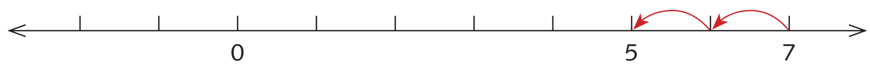
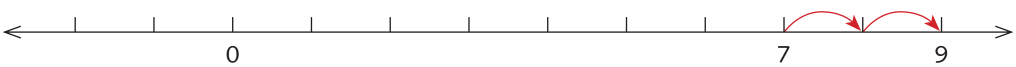
Subtraction of a negative integer

We have already seen that adding –2 means taking 2 steps to the left. For example:



$7 + (-2) = 5$

We want subtracting –2 to be the reverse process of adding –2, so to subtract –2, we will take 2 steps to the *right*. For example:



$7 - (-2) = 9$

This is a very simple way to state this rule:

To subtract a negative number, add its opposite

For example:

$$\begin{aligned} 7 - (-2) &= 7 + 2 \\ &= 9 \end{aligned}$$

Example 7

Evaluate:

a $20 + (-3)$ **b** $-3 + (-8)$ **c** $6 - (-20)$ **d** $-13 - (-8)$ **e** $-20 - (-3)$

Solution

a $\begin{aligned} 20 + (-3) &= 20 - 3 \\ &= 17 \end{aligned}$

b $\begin{aligned} -3 + (-8) &= -3 - 8 \\ &= -11 \end{aligned}$

c $\begin{aligned} 6 - (-20) &= 6 + 20 \\ &= 26 \end{aligned}$

d $\begin{aligned} -13 - (-8) &= -13 + 8 \\ &= -5 \end{aligned}$

e $\begin{aligned} -20 - (-3) &= -20 + 3 \\ &= -17 \end{aligned}$

Example 8

Calculate $6 - (-17)$.

Solution

$$\begin{aligned} 6 - (-17) &= 6 + 17 \\ &= 23 \end{aligned}$$

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Example 9

Calculate $-17 + (-6) - (-22)$.

Solution

$$\begin{aligned} -17 + (-6) - (-22) &= -17 - 6 + 22 \\ &= -23 + 22 \\ &= -1 \end{aligned}$$

Example 10

The minimum temperature on Saturday is −12°C and the maximum temperature is −1°C. Calculate the difference (maximum temperature – minimum temperature).

Solution

$$\begin{aligned} \text{maximum temperature} - \text{minimum temperature} &= -1 - (-12) \\ &= -1 + 12 \\ &= 11^\circ\text{C} \end{aligned}$$

Example 11

Evaluate:

a $347 + (-625)$

b $456 - (-356)$

c $-234 + 568$

d $-120 - (-105)$

Solution

a $$\begin{aligned} 347 + (-625) &= 347 - 625 \\ &= -(625 - 347) \\ &= -278 \end{aligned}$$

b $$\begin{aligned} 456 - (-356) &= 456 + 356 \\ &= 812 \end{aligned}$$

c $$\begin{aligned} -234 + 568 &= 568 + (-234) \\ &= 568 - 234 \\ &= 334 \end{aligned}$$

d $$\begin{aligned} -120 - (-105) &= -120 + 105 \\ &= -(120 - 105) \\ &= -15 \end{aligned}$$

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Exercise 4C

Example 6, 7a, b

1 Write the answers to these additions.

a $7 + (-2)$ **b** $-7 + 2$ **c** $-5 + 12$ **d** $-15 + (-3)$
e $-13 + 17$ **f** $-7 + (-3)$ **g** $-7 + (-2)$ **h** $-6 + (-11)$
i $12 + (-5)$ **j** $-13 + 5$ **k** $-13 + (-4)$ **l** $-24 + 8$

Example 7c, d, e, 8

2 Write the answers to these subtractions.

a $3 - 6$ **b** $5 - 11$ **c** $-4 - 7$ **d** $13 - (-3)$
e $-12 - 8$ **f** $-5 - (-3)$ **g** $-7 - (-5)$ **h** $6 - (-11)$
i $-11 - 5$ **j** $-14 - (-5)$ **k** $-23 - (-15)$ **l** $31 - (-10)$

3 Write the answer to these additions and subtractions.

a $2 + (-3)$ **b** $4 + (-6)$ **c** $5 + (-8)$ **d** $4 + (-7)$
e $6 + (-11)$ **f** $6 - (-3)$ **g** $-4 - (-1)$ **h** $-6 - (-2)$
i $-6 - 3$

Example 9

4 Evaluate:

a $13 - 27 + (-26)$ **b** $-12 - 14 + 8$ **c** $-41 + 56 - 2$
d $32 - 42 - (-8)$ **e** $7 + 13 - 16$ **f** $-32 - (-42) - (-3)$
g $-37 - 17 + 25$ **h** $6 - (-27) + 45$ **i** $16 + (-3) - (-7)$
j $-100 + 48 - (-80)$ **k** $-50 - 70 - (-100)$ **l** $20 - (-10) - (-30)$

Example 11

5 Evaluate:

a $256 - (-100)$ **b** $-850 - (-246)$ **c** $-658 - (-790)$ **d** $-9860 - (-3755)$

Example 10

6 The temperature in Warsaw on a winter's day went from a minimum of –17°C to a maximum of –1°C. By how much did the temperature rise?

7 The temperature in Daylesford on a very cold winter's day went from –2°C to 9°C. What was the change in temperature?

8 The following table shows a list of minimum and maximum temperatures for a number of cities. Complete the table, showing the increase in each case.

Minimum	Maximum	Increase
–11°C	6°C	
–16°C	7°C	
–35°C	–4°C	
–25°C	–15°C	
–7°C	–2°C	
–13°C	2°C	
–11°C	5°C	

9 The temperature in Canberra on a very cold day went from 8°C to –2°C. What was the change in temperature?

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4D Multiplication and division with negative integers

Multiplication with negative integers

$5 \times (-3)$ means 5 lots of -3 added together. That is:

$$5 \times (-3) = (-3) + (-3) + (-3) + (-3) + (-3)$$
$$= -15$$

Just as $8 \times 6 = 6 \times 8$, we will take -3×5 to be the same as $5 \times (-3)$.

All products such as $5 \times (-3)$ and -3×5 are treated in the same way.

For example:

$$-6 \times 3 = 3 \times (-6) = -18 \qquad -15 \times 4 = 4 \times (-15) = -60$$

The question remains as to what we might mean by multiplying two negative integers together. We first investigate this by looking at a multiplication table.

In the left-hand column below, we are taking multiples of 5. The products go down by 5 each time.

In the right-hand column, we are taking multiples of -5. The products go up by 5 each time.

$3 \times 5 = 15$	$3 \times (-5) = -15$
$2 \times 5 = 10$	$2 \times (-5) = -10$
$1 \times 5 = 5$	$1 \times (-5) = -5$
$0 \times 5 = 0$	$0 \times (-5) = 0$
$-1 \times 5 = -5$	$-1 \times (-5) = ?$
$-2 \times 5 = -10$	$-2 \times (-5) = ?$

The pattern suggests that it would be natural to take $-1 \times (-5)$ equal to 5 and $-2 \times (-5)$ equal to 10 so that the pattern continues in a natural way.

All products such as $-5 \times (-2)$ and $-5 \times (-1)$ are treated in the same way.
For example:

$$-6 \times (-2) = 12$$

$$-3 \times (-8) = 24$$

$$-20 \times (-5) = 100$$

We have the following rules.

The sign of the product of two integers

- The product of a negative number and a positive number is a negative number. For example: $-4 \times 7 = -28$.
- The product of two negative numbers is a positive number. For example: $-4 \times (-7) = 28$.

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Example 12

Evaluate each product.

a $4 \times (-20)$

b -7×10

c $-25 \times (-40)$

Solution

a $4 \times (-20) = -80$

b $-7 \times 10 = -70$

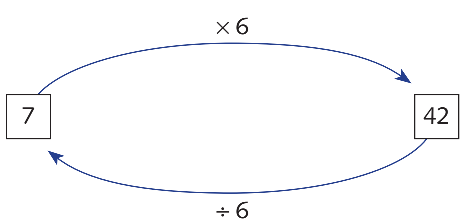
c $-25 \times (-40) = 25 \times 40$
$\qquad\qquad = 1000$

Division with negative integers

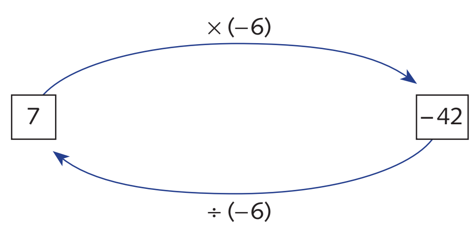
Every multiplication statement, for non-zero numbers, has an equivalent division statement. For example, $7 \times 3 = 21$ is equivalent to $21 \div 3 = 7$ and to $21 \div 7 = 3$. We will use this fact to establish the rules for division involving the integers.

Here are some more examples:

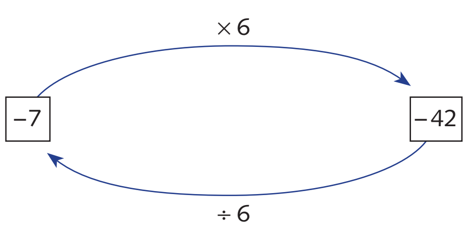
$7 \times 6 = 42$ is equivalent to $42 \div 6 = 7$.



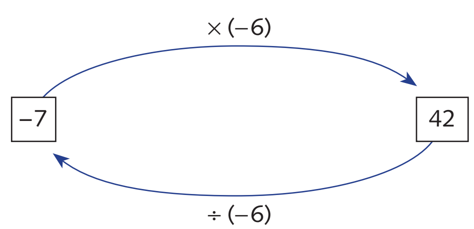
$7 \times (-6) = -42$ is equivalent to $-42 \div (-6) = 7$.



$-7 \times 6 = -42$ is equivalent to $-42 \div 6 = -7$.



$-7 \times (-6) = 42$ is equivalent to $42 \div (-6) = -7$.



The sign of the quotient of two integers

- The quotient of a positive number and a negative number is a negative number. For example: $28 \div (-7) = -4$.
- The quotient of a negative number and a positive number is a negative number. For example: $-28 \div 7 = -4$.
- The quotient of two negative numbers is a positive number. For example: $-28 \div (-7) = 4$.

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Example 13

Evaluate:

a $-54 \div 9$ **b** $-28 \div (-4)$ **c** $72 \div (-9)$

Solution

a $-54 \div 9 = -6$ **b** $-28 \div (-4) = 7$ **c** $72 \div (-9) = -8$

Example 14

Evaluate:

a $\frac{-63}{7}$ **b** $\frac{-72}{-4}$ **c** $\frac{108}{-12}$

Solution

a $\frac{-63}{7} = -9$ **b** $\frac{-72}{-4} = 18$ **c** $\frac{108}{-12} = -9$

Exercise 4D

Example 12

1 Carry out these multiplications.

a $6 \times (-2)$ **b** $7 \times (-2)$
c $7 \times (-10)$ **d** $12 \times (-4)$
e $12 \times (-18)$ **f** -5×3
g -6×4 **h** -5×12
i -11×7 **j** -12×5
k $-20 \times (-7)$ **l** $16 \times (-4)$
m $-7 \times (-20)$ **n** $-14 \times (-14)$
o -19×9 **p** $25 \times (-4)$

Example 13

2 Carry out these divisions.

a $-18 \div 3$ **b** $-28 \div 2$ **c** $-35 \div 5$ **d** $-27 \div 3$
e $-120 \div 4$ **f** $18 \div (-3)$ **g** $36 \div (-2)$ **h** $45 \div (-9)$
i $21 \div (-3)$ **j** $120 \div (-3)$ **k** $-52 \div (-4)$ **l** $-72 \div (-8)$
m $-52 \div (-13)$ **n** $-600 \div (-10)$ **o** $-168 \div 6$ **p** $-385 \div 11$

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Example 14

3 Evaluate:

a $\frac{4}{-1}$ **b** $\frac{-6}{-1}$ **c** $\frac{8}{-2}$ **d** $\frac{12}{-4}$

4 Evaluate:

a $\frac{-60}{-12}$ **b** $\frac{-65}{13}$ **c** $\frac{-72}{12}$ **d** $\frac{-120}{-8}$

e $\frac{136}{-4}$ **f** $\frac{-610}{5}$ **g** $\frac{-240}{15}$ **h** $\frac{396}{-4}$

i $\frac{-70}{10}$ **j** $\frac{720}{-24}$ **k** $\frac{-360}{-90}$ **l** $\frac{-4323}{3}$

5 Evaluate:

a $3 \times (-4) \times (-7)$ **b** $5 \times (-7) \times (-8)$

c $60 \times (-3) \times (-12)$ **d** $-45 \times (-8) \times 10$

e $48 \times (-10) \div 3$ **f** $8 \times (-10) \div 5$

6 Copy and complete these statements.

a $2 \times ... = -50$ **b** $5 \times ... = -75$

c $-7 \times ... = 63$ **d** $... \times (-8) = 72$

e $80 \div ... = -10$ **f** $-45 \div ... = 5$

g $-321 \div ... = 3$

h $5664 \div ... = -708$

4E Indices and order of operations

You need to be particularly careful with the order of operations conventions when working with indices.

For example, $-4^2 = -16$ and $(-4)^2 = 16$. In the first case, 4 is first squared and then the opposite is taken. In the second case, -4 is squared. Notice how different the two answers are.

Also:

$$-3 \times 5 + 2 = -15 + 2 = -13 \quad \text{and} \quad -3 \times (5+2) = -3 \times 7 = -21$$

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Order of operations

- Evaluate expressions inside brackets first.
- In the absence of brackets, carry out operations in the following order:
  - powers
  - multiplication and division from left to right
  - addition and subtraction from left to right.

Example 15

Evaluate:

a $(-5)^2$ **b** -5^2 **c** $-6-5+4$ **d** $6\times(-3)+8$

Solution

a $(-5)^2 = -5\times(-5)$
$= 25$

b $-5^2 = -(5\times 5)$
$= -25$

c $-6-5+4 = -11+4$
$= -7$

d $6\times(-3)+8 = -18+8$
$= -10$

Example 16

Evaluate:

a $7\times(-6+8)$

b $-3+8\times(7-12)$

c $6-(5+3)$

d $4\times(-6)+3\times 8$

e $3\times(-6)^2+3\times 21$

Solution

a $7\times(-6+8) = 7\times 2$
$= 14$

b $-3+8\times(7-12) = -3+8\times(-5)$
$= -3+(-40)$
$= -43$

c $6-(5+3) = 6-8$
$= -2$

d $4\times(-6)+3\times 8 = -24+24$
$= 0$

e $3\times(-6)^2+3\times 21 = 3\times 36+63$
$= 108+63$
$= 171$

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Exercise 4E

Example 15 a, b

1 Evaluate:

a $(-9)^2$ **b** $-(7)^2$ **c** $3\times(-5)^2$ **d** $-9\times(-4)^2$

e $(-20)^2\times-(4)^2$ **f** $(-13)^2$ **g** $(-6)^3$ **h** $(-3)^4$

i $(-5)^5$ **j** $(-7)^6$ **k** $(-1)^7$

Example 15c

2 Evaluate:

a $-6+30-15$ **b** $-5-(-10)+30$ **c** $-8+12-18$

d $-6+12-(-15)$ **e** $-15+7-10$ **f** $75-(-40)+60$

Example 16

3 Evaluate:

a $-3\times(-16+8)$ **b** $-4+6\times(11-14)$ **c** $-6-18+4$

d $-6\times(-3+12)$ **e** $-2\times(-6+16)-25$ **f** $-15+5\times(-3+12)$

g $(-11+5)\times12+(-15)$ **h** $(-18-4)\times26-(-12)$

4 Evaluate:

a $-(3-27)$ **b** $-(37-64)$ **c** $15+(8-16)$

d $-53+(5-11)$ **e** $16-21+5\times(-3)$ **f** $-3\times(86-97)$

g $-15\times(3-11)$ **h** $6\times(14-41)$ **i** $-8\times(11-28)$

5 Evaluate:

a $-5\times(-26+8)$ **b** $-4+6\times(15-12)$ **c** $-6-(25+4)$

d $-6\times(-4+12)$ **e** $-2\times(-8+16)-40$ **f** $-99+5\times(-62+12)$

g $-81+5\times(51+(-45))$ **h** $-28-4\times(26-(-14))$

6 Evaluate:

a $60\div(-5)\div12$ **b** $90\times(-4)\div10$

c $60\div10\times3$ **d** $80\times(-5)\div25$

7 Evaluate:

a $(-20)^2+3\times(-10)$ **b** $(-20)^2\times(-10)^3$

c $3\times(-10)^3+10^2$ **d** $(-4)\times(-10)^2\times(-10)$

8 A shop manager buys 200 shirts at \$26 each and sells them for a total of \$4000. Calculate the total purchase price, and subtract this from the total amount gained from sales.

9 A man puts \$2000 into a bank account every month for 12 months.

a What is his bank balance at the end of 12 months, given that he does not withdraw any money?

b He writes a cheque for \$40 000. What is his new bank balance?

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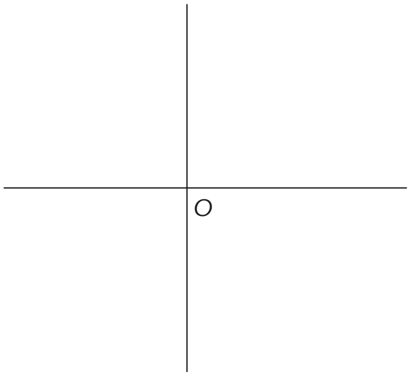
4F The Cartesian plane

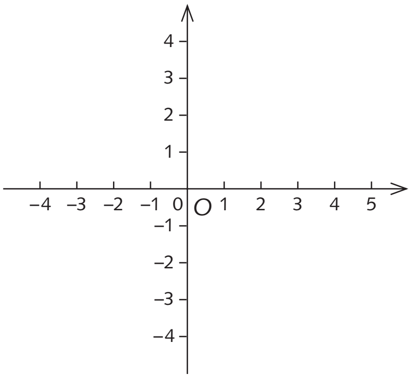
We have previously represented numbers as points on the number line. This idea can be extended by taking pairs of numbers to represent points in a plane. This is called the **Cartesian plane**.

We start with two perpendicular straight lines. They intersect at a point O called the **origin**. We leave the right angle sign out for clarity.



Each of the lines is called an **axis**. The plural of *axis* is *axes*.

Next we mark off intervals of unit length along each axis, and mark each axis as a number line with 0 at the point O. The arrows are drawn to show that the axes extend infinitely in both directions and to indicate which way is the positive direction.



The axes are called the **coordinate axes** or sometimes the **Cartesian coordinate axes**. They are named after the French mathematician and philosopher René Descartes 1596–1650. He introduced coordinate axes to show how algebra could be used to solve geometric problems. Although the idea is simple, it revolutionised mathematics.

Now we imagine adding vertical and horizontal lines to the diagram through the integer points on the axes. We can describe each point where the lines meet by a pair of integers. This pair of integers is called the **coordinates** of the point. The first number is the **horizontal coordinate** and the second number is the **vertical coordinate**.

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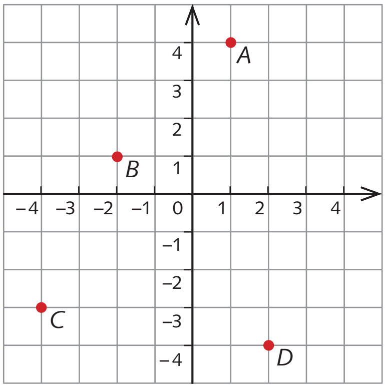
For example, the coordinates of the point labelled A below are (1, 4). This is where the line through the point 1 on the horizontal axis and the line through the point 4 on the vertical axis meet. We move 1 unit to the right of the origin and 4 units up to reach A.



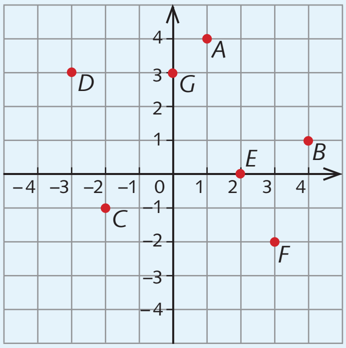
The point D has coordinates (2, –4).
We move 2 units to the right of the origin and 4 units down to get D.

The point B has coordinates (–2, 1).
We move 2 units to the left of the origin and 1 unit up to get B.

The point C has coordinates (–4, –3).
We move 4 units to the left of the origin and 3 units down.

Example 17

Give the coordinates of each of the points marked on the Cartesian plane below.

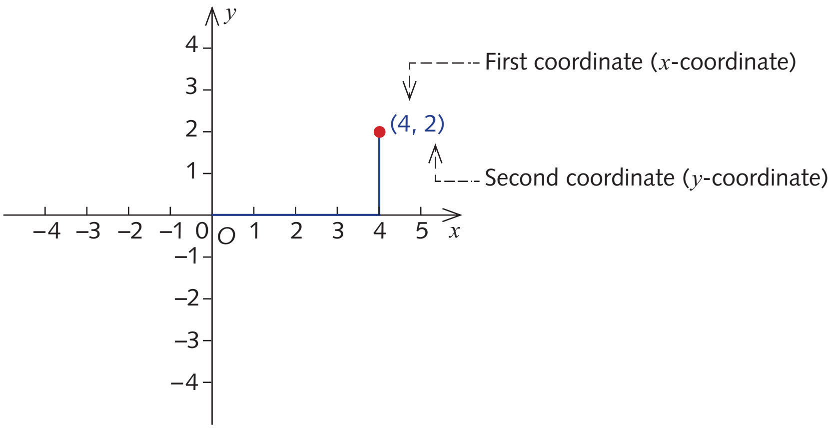


Solution

$A(1,4)$	$B(4,1)$	$C(-2,-1)$	$D(-3,3)$
$E(2,0)$	$F(3,-2)$	$G(0,3)$	

Remember, the first coordinate tells us where to go from the origin in the horizontal direction. If it is negative, we go to the left of the origin; if it is positive, we go to the right of the origin.

The second coordinate tells us where to go from the origin in the vertical direction. If it is negative, we go below the origin; if it is positive, we go above the origin.



The first coordinate is usually called the x-coordinate and the second coordinate is usually called the y-coordinate.

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Exercise 4F

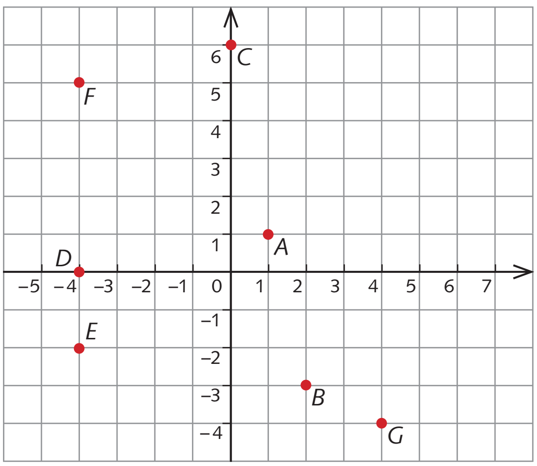
1 On a Cartesian plane, plot each of the following points. The coordinates of the points are given.

a $A(4,1)$ b $B(-2,3)$ c $C(-2,-2)$ d $D(4,-1)$

e $E(1,3)$ f $F(0,-2)$ g $G(-3,0)$ h $H(0,5)$

Example 17

2 a Give the coordinates of each of the points A to G below.



b How many of these coordinates have

i a positive x-coordinate?

ii a negative y-coordinate?

3 Do this exercise on graph paper. Set out the coordinate axes by marking intervals of length 1 unit along the horizontal and vertical axes.

a Plot the points $O(0,0), A(3,0), C(3,3)$ and $D(0,3)$, and join the points to form the intervals OA, AC, CD and DO. Describe the shape formed and evaluate its area.

b Plot the points $A(-3,0), B(6,0)$ and $C(0,4)$, and join the points to form the intervals AB, BC and CA. Describe the shape formed and evaluate its area.

c Plot the points $A(-4,-3), B(7,-3), C(7,2)$ and $D(-4,5)$, and join the points to form the intervals AB, BC, CD and DA. Describe the shape formed and evaluate its area.

d Plot the points $A(-4,4), B(1,4)$, and $C(-4,1)$, and join the points to form the intervals AC, CB, BA. Describe the shape formed and evaluate its area.

e Plot the points $O(0,0)$ and $A(1,2)$, and draw the line passing through these points. Plot the points $B(-2,3)$ and $C(0,2)$, and draw the line passing through these points. Describe the relationship between the lines.

f Plot the points $A(-1,1), B(3,3), C(7,3)$ and $D(5,1)$, and join the points to form the intervals AB, BC, CD and DA. Describe the shape formed and evaluate its area.

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4G Negative fractions

Fractions were reviewed in Chapter 2.

We can now introduce negative fractions as well. They lie to the left of 0 on the number line.

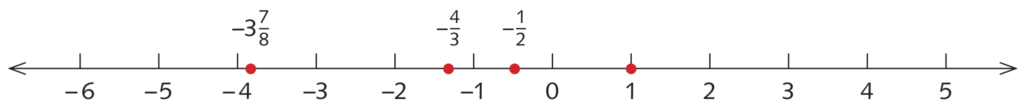
Each positive fraction has an opposite fraction. For example:

$\frac{5}{4}$ has the opposite $-\frac{5}{4} = -1\frac{1}{4} = -1.25.$

A situation where we see negative fractions is in temperatures.

A temperature of $-4.5°\text{C}$ $\left(-\frac{9}{2}°\text{C}\right)$ is 4.5°C below zero.

Several negative fractions are marked on the number line below.



The arithmetic for integers, which we have considered in the previous sections, also extends to fractions.

Addition and subtraction of negative fractions

Example 18

Write the answer to each of these additions and subtractions.

a $-\frac{1}{2} + 2$ **b** $-1\frac{1}{2} + 2$ **c** $-\frac{1}{3} - \frac{1}{2}$ **d** $-\frac{2}{3} + \frac{1}{5}$

Solution

a $-\frac{1}{2} + 2 = 1\frac{1}{2}$

b $-1\frac{1}{2} + 2 = \frac{1}{2}$

c $-\frac{1}{3} - \frac{1}{2} = -\frac{2}{6} - \frac{3}{6}$

$= -\frac{5}{6}$

d $-\frac{2}{3} + \frac{1}{5} = -\frac{10}{15} + \frac{3}{15}$

$= -\frac{7}{15}$

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Division by negative integers giving fractions

Example 19

Divide:

a 265 by -2

b -3765 by -7

Solution

a First divide 265 by 2. The result is $132\frac{1}{2}$.

For $265 \div (-2)$, the signs are different and the result is $-132\frac{1}{2}$.

b First divide 3765 by 7. The result is $537\frac{6}{7}$.

For $-3765 \div (-7)$, the signs are the same and $-3765 \div (-7) = 537\frac{6}{7}$.

Multiplication and division of negative fractions

The methods we have been using with the positive fractions also work with the negative fractions.

Example 20

Give the answer to each of these multiplications and divisions as a fraction.

a $-\frac{1}{2} \times 2$

b $-1\frac{1}{2} \div 2$

c $-\frac{1}{3} \div \frac{1}{2}$

d $-\frac{2}{3} \times \left(-\frac{1}{5}\right)$

Solution

a $-\frac{1}{2} \times 2 = -1$

b $-1\frac{1}{2} \div 2 = -\frac{3}{2} \times \frac{1}{2}$

$= -\frac{3}{4}$

c $-\frac{1}{3} \div \frac{1}{2} = -\frac{1}{3} \times \frac{2}{1}$

$= -\frac{2}{3}$

d $-\frac{2}{3} \times \left(-\frac{1}{5}\right) = \frac{2}{3} \times \frac{1}{5}$

$= \frac{2}{15}$

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Exercise 4G

1 Draw a number line from –5 to 5 and mark on it the positions of

$-3\frac{1}{2}, -4\frac{3}{4}, -3\frac{1}{4}, -1\frac{1}{2}, 1\frac{1}{2}$.

2 Arrange each of these sets of numbers from smallest to largest.

a $-2, \frac{1}{4}, -1, \frac{-5}{-3}, -\frac{7}{5}, \frac{1}{2}, 1$

b $-\frac{7}{4}, \frac{-11}{-3}, -\frac{11}{5}, \frac{14}{5}, \frac{7}{2}$

c $-\frac{2}{3}, -\frac{4}{5}, -\frac{11}{12}, \frac{12}{13}$

d $-1\frac{11}{13}, -\frac{15}{13}, -2, -1$

Example 18

3 Evaluate:

a $-\frac{1}{2}+3$

b $-1\frac{1}{2}+4$

c $-\frac{2}{3}-\frac{1}{3}$

d $-\frac{2}{3}+\frac{1}{3}$

e $-\frac{3}{4}-\frac{3}{4}$

f $-\frac{2}{3}+\frac{3}{5}$

g $-\frac{3}{4}+\frac{1}{4}$

h $-\frac{1}{5}-\frac{2}{5}$

i $-\frac{3}{4}+3$

j $-\frac{3}{5}-3$

k $-1\frac{1}{2}+\frac{-7}{9}$

l $\frac{3}{7}-2\frac{1}{2}$

m $-3\frac{1}{2}+3$

n $-2\frac{1}{2}+4$

o $-\frac{2}{5}-\frac{1}{3}$

p $-\frac{3}{4}+\frac{1}{3}$

q $-\frac{3}{4}+3\frac{2}{9}$

r $-\frac{3}{5}-3\frac{2}{7}$

s $-\frac{5}{13}+\left(-3\frac{5}{7}\right)$

t $\frac{3}{8}-\left(-\frac{2}{7}\right)$

Example 19

4 Divide:

a 2735 by –2

b –6322 by 3

c –5524 by –7

Example 20 a, d

5 Evaluate:

a $2\times\left(-\frac{3}{8}\right)$

b $-\frac{3}{8}\times\left(-\frac{5}{9}\right)$

c $-\frac{11}{12}\times\left(-\frac{5}{9}\right)$

d $-\frac{11}{5}\times\frac{5}{12}$

e $4\times\left(-\frac{3}{8}\right)\times\frac{5}{9}$

f $5\times\left(-\frac{3}{10}\right)$

g $-\frac{2}{3}\times\frac{5}{6}$

h $-\frac{5}{12}\times\frac{7}{5}$

Example 20 b, c

6 Evaluate:

a $2\div\left(-\frac{1}{3}\right)$

b $\frac{3}{4}\div\left(-\frac{2}{3}\right)$

c $-3\div 5$

d $\frac{3}{4}\div\left(-\frac{11}{12}\right)$

e $-\frac{2}{3}\div\left(-\frac{4}{9}\right)$

f $-\frac{3}{8}\div\frac{5}{12}$

g $-\frac{2}{3}\div\left(-\frac{1}{6}\right)$

h $\frac{5}{7}\div 10$

7 Rewrite each expression in simplest form.

a $\frac{1}{5}-\left(\frac{1}{3}+\frac{1}{2}\right)$

b $-2\frac{7}{8}+\left(-\frac{1}{3}\right)\times\frac{4}{3}$

c $\left(-1\frac{1}{2}+1\frac{3}{4}\right)\times 3\frac{1}{2}$

d $-\frac{1}{4}-\frac{1}{3}\times 4$

e $\frac{1}{2}-2\times\frac{3}{8}$

f $-\frac{3}{4}-\left(2-\frac{1}{2}\right)$

g $4-\left(-3\frac{1}{2}\right)-2$

h $\frac{1}{6}-\frac{1}{3}+\frac{1}{6}$

i $2\times\left(-\frac{1}{3}\right)-4\frac{1}{3}$

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8 Evaluate:

a $-\frac{1}{2}+2\tfrac{1}{2}-\frac{1}{4}+\frac{3}{4}$

b $-3\tfrac{1}{4}-\frac{1}{4}+6-2\tfrac{1}{2}$

c $2\times\left(-\frac{3}{2}\right)\times\left(-\frac{4}{7}\right)+\frac{2}{7}$

d $3\tfrac{1}{3}\times(-3)+10\tfrac{1}{2}\div 3$

e $5\tfrac{1}{4}\div(-7)+\frac{3}{4}$

f $-\frac{1}{2}-\frac{1}{3}-\frac{1}{4}-\frac{1}{5}-\frac{1}{6}$

9 Evaluate:

a $\left(-\frac{1}{2}\right)^2$

b $\left(-\frac{1}{2}\right)^3$

c $\left(-\frac{1}{2}\right)^4$

d $\left(-\frac{1}{2}\right)^5$

e $-16\times\left(-\frac{1}{2}\right)^5$

f $-32\times\left(-\frac{1}{4}\right)^2$

g $24\times\left(-\frac{1}{2}\right)^3$

h $\left(-\frac{1}{3}\right)^3\times -8$

Review exercise

1 Evaluate:

a $25+(-2)$

b $-36+22$

c $-35+50$

d $-51+(-44)$

e $-32+16$

f $-45+(-23)$

g $-160+(-20)$

h $-50+(-10)$

i $110+(-40)$

j $-120+40$

k $-135+(-25)$

l $60-(-25)$

2 In an indoor cricket match, a team has made 25 runs and lost 7 wickets. What is the score of the team? (A run adds 1 to the score and a wicket subtracts 5.)

3 The temperature in June at a base in Antarctica varied from a minimum of –60°C to a maximum of –35°C. What was the value of:

a maximum temperature – minimum temperature?

b minimum temperature – maximum temperature?

4 The temperature in Canberra had gone down to –5°C. The temperature in a heated house was a cosy 22°C. What was the value of:

a inside temperature – outside temperature?

b outside temperature – inside temperature?

5 Evaluate:

a $125\times(-2)$

b -36×11

c -35×50

d $-51\times(-40)$

e -3×16

f $-50\times(-23)$

g $-160\times(-20)$

h $-50\times(-10)$

i $11\times(-40)$

j -120×20

k $-32\times(-4)$

l -25×8

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6 Evaluate:

a $125 \div (-5)$ **b** $-36 \div 9$

c $-35 \div 5$ **d** $-51 \div (-3)$

e $-16 \div (-4)$ **f** $-50 \div (-10)$

g $-160 \div (-20)$ **h** $-1500 \div (-10)$

i $120 \div (-40)$ **j** $-120 \div 20$

k $-40 \div (-5)$ **l** $624 \div (-6)$

7 Evaluate:

a $-4 \times (6 - 7)$ **b** $7 \times (11 - 20)$

c $-3 \times (5 + 15)$ **d** $-6 \times (-4 - 6)$

e $-12 \times (-6 + 20)$ **f** $-(-4)^2$

g $(3 - 7) \times (11 - 15)$ **h** $(10 - 3) \times (-3 + 10)$

i $(-5 - 10) \times (10 - 4)$

8 Start with the number -5, add 11 and then subtract 20. Multiply the last number you obtained by 4. What is the final result?

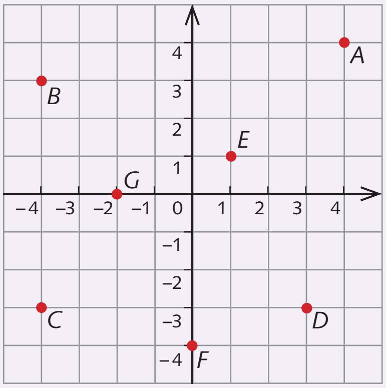
9 Start with -100, subtract 200 and then add -300. Divide the result by 100. What is the final result?

10 On a Cartesian plane, plot each of the following points. The coordinates of the points are given.

a $A(1,1)$ **b** $B(2,-3)$ **c** $C(0,6)$

d $D(-4,0)$ **e** $E(-4,-2)$ **f** $F(-4,5)$

11 Find the coordinates of each of the points A to G below.



12 Evaluate:

a $-\frac{1}{2} + 3$ **b** $-1\frac{1}{2} + 6$ **c** $-\frac{9}{13} - \frac{1}{13}$

d $-\frac{7}{8} + \frac{5}{8}$ **e** $-\frac{5}{12} + \left(-\frac{5}{6}\right)$ **f** $-\frac{5}{11} + 6$

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13 Evaluate:

a $\frac{2}{3}\times\left(-\frac{1}{4}\right)$ **b** $3\frac{1}{3}\times(-9)$ **c** $-\frac{4}{3}\times\frac{3}{4}\div\frac{7}{8}$

d $-\frac{2}{3}\times\frac{3}{4}\times(-2)$ **e** $\frac{3}{4}\times\left(-\frac{4}{5}\right)\times\frac{1}{2}$ **f** $\frac{1}{2}-\frac{1}{3}\times\frac{1}{2}$

g $-\frac{2}{3}\times\frac{1}{4}\times\frac{3}{5}$ **h** $-\frac{5}{8}\times\frac{1}{2}\times\frac{2}{3}$ **i** $\frac{1}{4}-\frac{2}{3}\div 3$

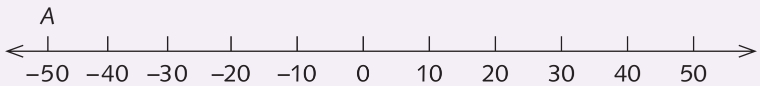
14 AC electricity, which is what you get from a power point, alternates (hence the name alternating current) between positive and negative voltages. The highest positive voltage it reaches is 1.4 times the rated voltage, and the lowest voltage is –1.4 times the rated voltage. If the power point is rated as 230 V AC, what is the difference between the highest and lowest voltages? (‘V’ stands for the unit of measure ‘volts’)

15 Evaluate:

a $(-1)^2$ **b** $(-1)^3$ **c** $(-1)^4$ **d** $(-1)^5$ **e** $(-1)^6$

Comment on the results.

16 Johann is asked to walk along a straight pathway that is marked out as a number line, as shown in the diagram. Distances are in metres.



Johann starts at the point A, which is at –50 on the number line.

He is asked to walk along the path and follow these directions:

1 Walk from A to the point –10.
2 Turn through 180°.
3 Walk to the point –20.
4 Turn through 180°.
5 Walk to the point 50.
6 Turn through 180°.
7 Walk back to A.

How far has he walked?

17 The average of a list of numbers is obtained by adding the numbers, and then dividing by the number of numbers. For example, the average of 20, 30, 50 and 60 is given by

$$(20+30+50+60)\div 4 = 160\div 4 = 40 \quad \text{or} \quad \frac{20+30+50+60}{4}=\frac{160}{4}=40$$

Find the average of each of these lists of numbers.

a –20, –30, 40, 60 **b** –6, –1, –10, –20, 22

c –200, –300, –500, –1000 **d** –2, –3, –4, 0, 8, 1

18 Find the average of each of these lists of numbers.

a $-1\frac{1}{4}, 2\frac{1}{2}$ **b** $-1, -1\frac{1}{2}, 0, 6$ **c** $-\frac{3}{4}, -\frac{1}{2}, -\frac{1}{8}, \frac{1}{2}, \frac{1}{4}$

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19 If $a = -1\frac{1}{4}$ and $b = 3\frac{1}{2}$, evaluate:

a $a + b$ **b** $a - b$ **c** $b - a$ **d** ab

e $\dfrac{a}{b}$ **f** $\dfrac{b}{a}$ **g** $\dfrac{a+2}{b}$ **h** $\dfrac{b-a}{b}$

20 Evaluate:

a $(-3)^2 + 2 \times (-3) + 8$ **b** $(-3)^3 - 2 \times (-3) + 10$

c $6 - (-2)^2 - (-2)^3$ **d** $\left(-\frac{1}{2}\right)^3 - \left(-\frac{1}{2}\right)^2 + \frac{1}{2}$

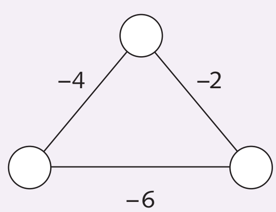
21 Evaluate:

a $1 \div \frac{1}{2}$ **b** $1 \div \left(-\frac{1}{2}\right)$ **c** $1 \div \left(-\frac{3}{2}\right)$ **d** $1 \div \left(-\frac{3}{2}\right)^2$

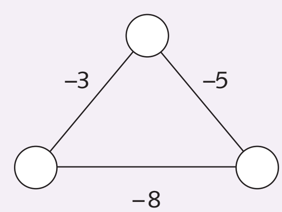
Challenge exercise

1 The integers on the edges of each triangle below are the sums of the integers in the adjoining two circles. Find the numbers in the circles.

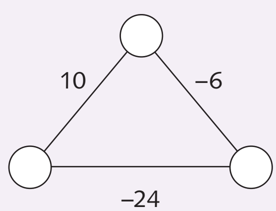
a



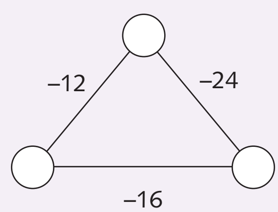
b



c



d



2 Put the three numbers 4, – 2 and –5 into the boxes below

□ + □ − □ = ____

so that the answer is:

a 1 **b** 7 **c** –11

3 Put the three numbers 3, – 3 and – 4 into the boxes below

□ + □ − □ = ____

so that the answer is:

a 4

b –10

c 2

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4 Which number needs to be placed in the box below to make the following a true statement?

$3 - \square + (-5) = 0$

5 Place brackets in each statement to make it true.

a $2 + -3 \times 3 + 4 = -7$

b $2 + -3 \times 3 + 4 \times 2 = 1$

c $2 - 5 \times 6 + 7 \times 6 - 5 = -71$

6 This is a magic square. All rows, columns and diagonals have the same sum. Complete the magic square.

8	−6	4
		−4

7 **a** Find the value of $1 - 2 + 3 - 4 + 5 - 6$ by:

i working from left to right

ii pairing $(1 - 2) + (3 - 4) + (5 - 6)$

b Evaluate $1 - 2 + 3 - 4 + 5 - 6 + 7 - 8 + \ldots + 99 - 100$.

8 Evaluate $100 + 99 - 98 - 97 + 96 + 95 - 94 - 93 + \ldots + 4 + 3 - 2 - 1$.

9 The average of five given numbers is 2. If the least of these is deleted, the average is 4. What is the smallest number?

10 Find the value of:

a $(2 - 4) + (6 - 8) + (10 - 12) + (14 - 16) + (18 - 20)$

b $2 - 4 + 6 - 8 + 10 - 12 + \ldots + 98 - 100$

11 What is the least product you could obtain by multiplying any two of the following numbers: $-7, -5, -1,$ 1 and 3?

12 Evaluate:

a $\frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} + \frac{1}{5 \times 6}$

b $\frac{1}{2 \times 3} - \frac{1}{3 \times 4} + \frac{1}{4 \times 5} - \frac{1}{5 \times 6}$

c $-\frac{1}{2 \times 3} + \frac{1}{3 \times 4} - \frac{1}{4 \times 5} + \frac{1}{5 \times 6}$

d $-\frac{1}{5 \times 6} - \frac{1}{6 \times 7} - \frac{1}{7 \times 8} - \frac{1}{8 \times 9}$

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