CHAPTER

5

Measurement and Geometry

Review of geometry

The word *geometry* comes from two Greek words meaning 'earth measurement'. Its name explains the origin of the subject. In ancient Egypt, the river Nile regularly overflowed its banks and washed away boundary markers between neighbouring properties. The boundaries had to be found again and this led to the study of the different shapes and sizes of triangles.

Geometry deals with points, lines, planes, angles, circles, triangles, and so on. The ancient Greeks valued its study greatly, and their work established the geometry you will meet in this book.

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5A Angles at a point

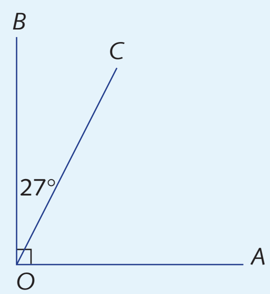
Clear, logical reasoning has always been an important part of geometry. It is very important in geometry to draw diagrams. This makes it easier to construct convincing arguments.

Angles at a point – geometric arguments

- The following can be used in arguments:
  - Adjacent angles can be added.
  - Angles in a revolution add to 360°.
  - Angles in a straight angle add to 180°.
  - Vertically opposite angles are equal.
- Two lines are called **perpendicular** if they meet at right angles.
- The Greek letters α (alpha), β (beta), γ (gamma) and θ (theta) are often used as pronumerals to represent angle size in geometry.

Example 1

Find $\angle AOC$ in the diagram opposite.



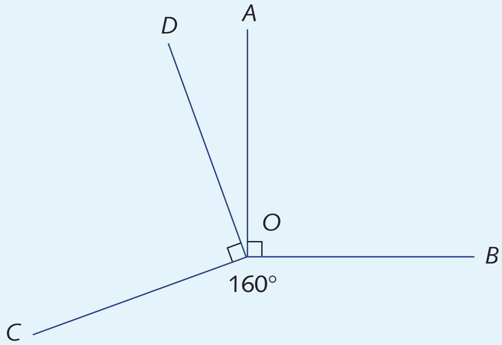
Solution

$\angle AOC + 27° = 90°$ (adjacent angles at O)

so $\angle AOC = 63°$

Example 2

Find $\angle AOD$ in the diagram opposite.



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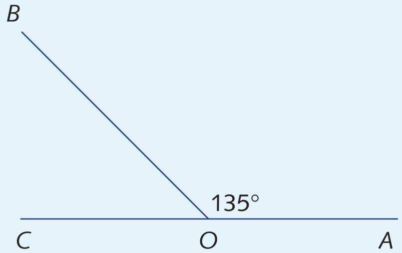
Solution

$\angle AOD + 90° + 160° + 90° = 360°$ (revolution at O)

so $\angle AOD = 20°$

Example 3

Find $\angle BOC$ in the diagram below.



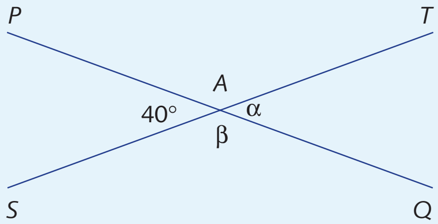
Solution

$\angle BOC + 135° = 180°$ (straight angle $\angle AOC$)

so $\angle BOC = 45°$

Example 4

Find α and β in the diagram below.



Solution

$\alpha = 40°$ (vertically opposite angles at A)

Also, $\beta + 40° = 180°$ (straight angle $\angle PAQ$),

so $\beta = 140°$

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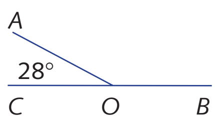
Exercise 5A

Example 1, 2, 3

1 Find the value of $\angle AOB$ in each diagram below, giving reasons for your answer. In parts **g** and **j**, find the reflex angle $\angle AOB$.

a


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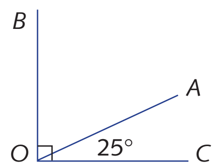

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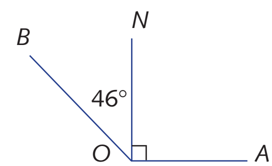

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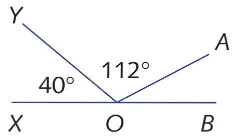

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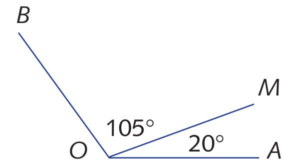
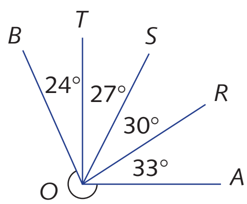


g


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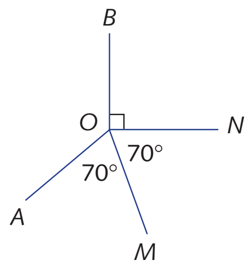

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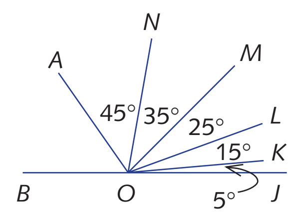

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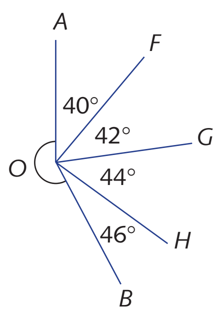

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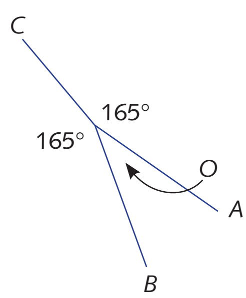

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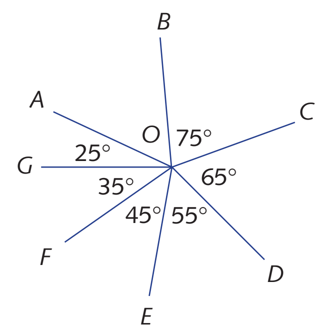
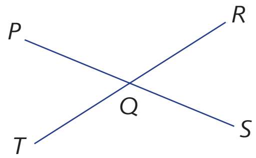


2 Name the angle that is vertically opposite to $\angle PQR$ in each diagram below.

a


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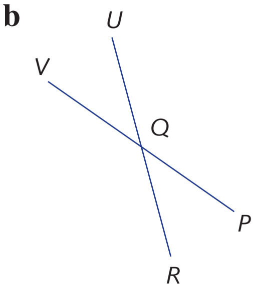

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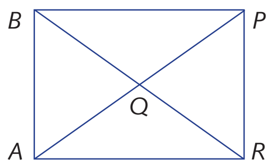

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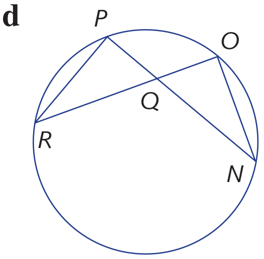

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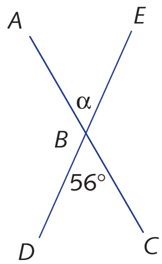
Example 4

3 Find the sizes of the angles denoted by α, β, γ and θ in each diagram below, giving reasons.

a



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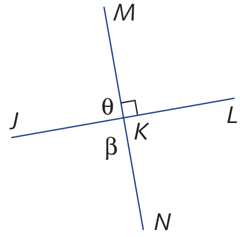

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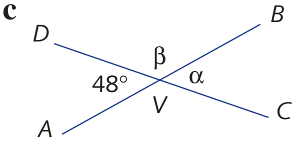

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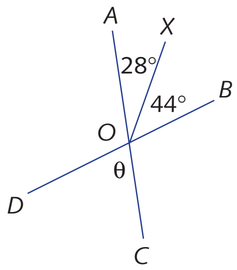

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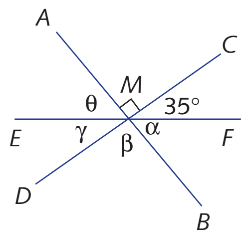

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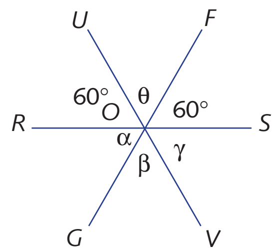

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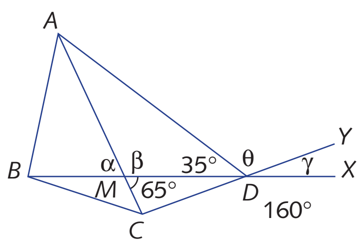

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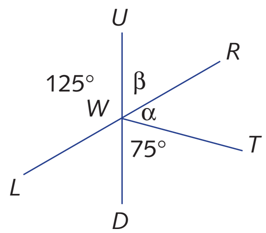
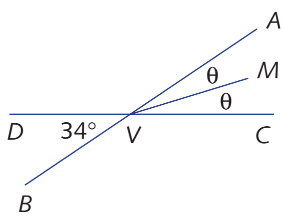


4 In each diagram below, angles marked with the same Greek letter are equal in size. Find the value of each, giving reasons.

a



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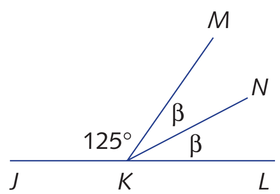

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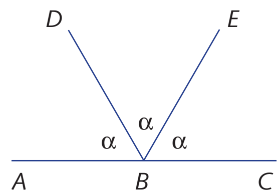

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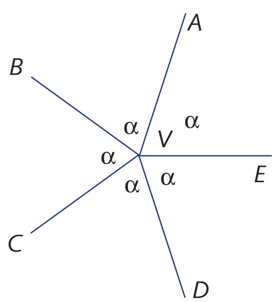

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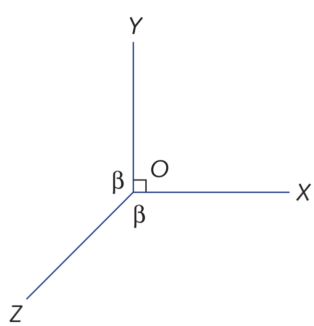

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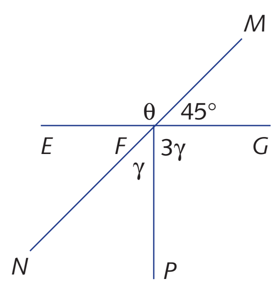

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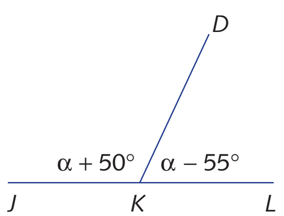

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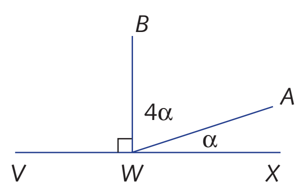

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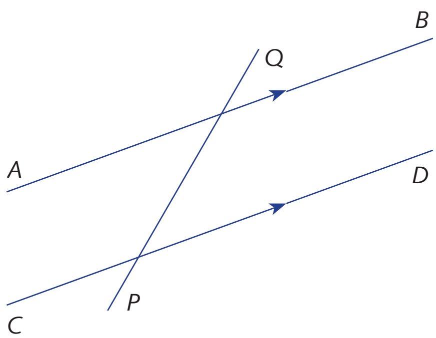
5B Angles associated with transversals

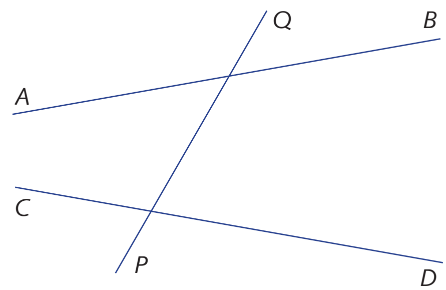
This section and the next one involve the relationships between angles and parallel lines.

As always in geometry, reasons should be as specific as possible. Make sure that you name any parallel lines that you are using in an argument.

Transversal

A **transversal** is a line that crosses two other lines. In both diagrams below, the line *PQ* is a transversal to the lines *AB* and *CD*.



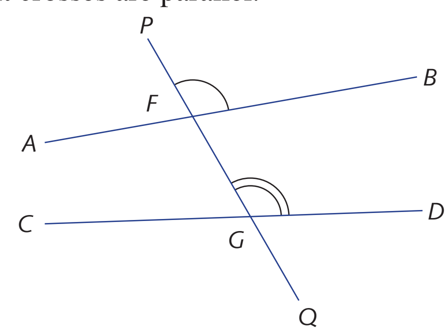


Notice that *PQ* is a transversal whether or not the two lines that it crosses are parallel.

Corresponding angles

The two marked angles in the diagram are called **corresponding angles**, because they are in *corresponding* positions around the two vertices *F* and *G*.

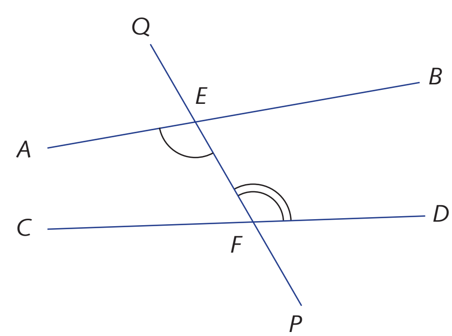
There are actually four pairs of corresponding angles in the diagram. Can you name the other three pairs?



Alternate angles

In the diagram opposite, the two marked angles are called **alternate angles**, because they are on *alternate* sides of the transversal *PQ*. The two angles must also be inside the two parallel lines.

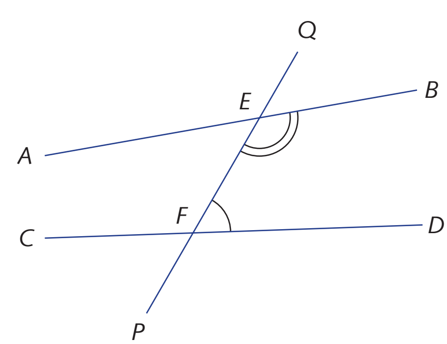
There is a second pair of alternate angles in the diagram. Can you name the angles of this pair?



Co-interior angles

In the diagram opposite, the two marked angles are called **co-interior angles**, because they are between the two lines and on the same side of the transversal *PQ*.

There is a second pair of co-interior angles in the diagram. Name the angles of this pair.



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A transversal lying across parallel lines

We learned in Chapter 6 of *ICE-EM Mathematics Year 7* that when a transversal lies across two parallel lines, the alternate angles, the corresponding angles and the co-interior angles are closely related.

Angles associated with transversals

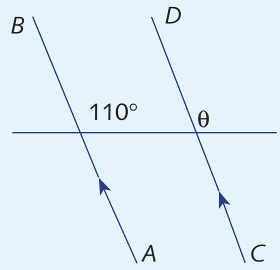
A transversal is a line that crosses two other lines. If the lines crossed by the transversal are parallel, then:

- **corresponding** angles are equal
- **alternate** angles are equal
- **co-interior** angles are supplementary.

Here are three examples that show how to set out your work with corresponding angles, alternate angles and co-interior angles. Your reason should first name the type of angles involved, and then name the relevant pair of parallel lines.

Example 5

Find θ in the diagram opposite.

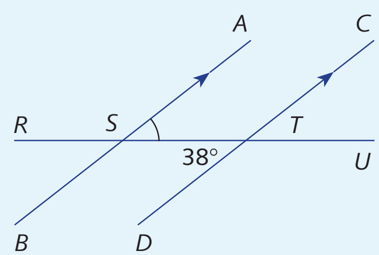


Solution

$\theta = 110°$ (corresponding angles, $AB \parallel CD$)

Example 6

Find $\angle AST$ in the diagram opposite.



Solution

$\angle AST = 38°$ (alternate angles, $AB \parallel CD$)

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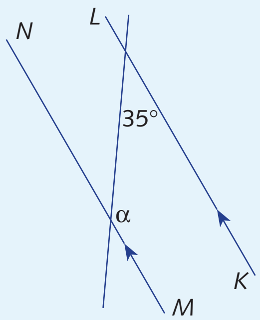
Example 7

Find α in the diagram below.



Solution

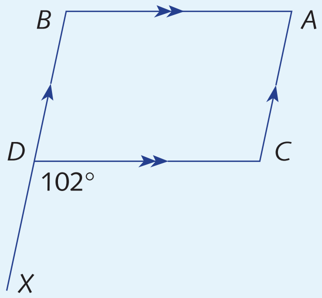
$$\alpha + 35° = 180° \text{ (co-interior angles, } KL \parallel MN)$$
$$\alpha = 145°$$

Two-step solutions

The solution of the problem below needs two steps, with a reason for each step. Notice that the pairs of parallel lines used are different in the two steps.

Example 8

Find $\angle BAC$ in the diagram below.



Solution

First, $\angle DCA = 102°$ (alternate angles, $AC \parallel BD$).

Hence, $\angle BAC = 78°$ (co-interior angles, $AB \parallel CD$).

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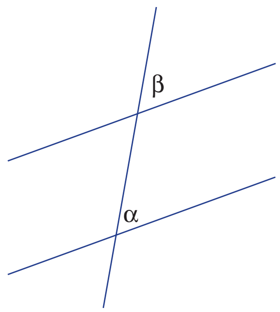
Exercise 5B

1 In each diagram below, identify each pair of angles marked with α and β as corresponding angles, alternate angles or co-interior angles.

a



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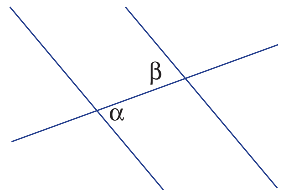

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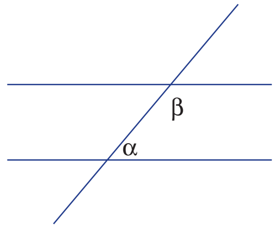

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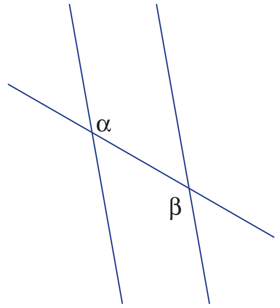

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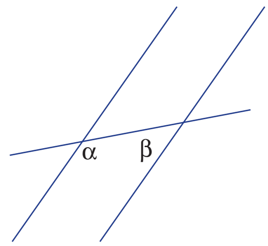

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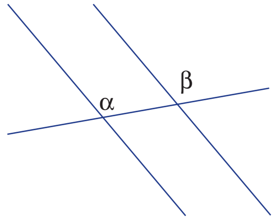

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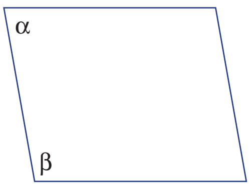

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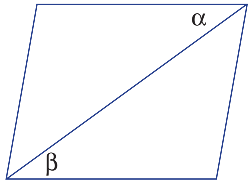

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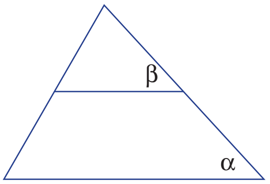

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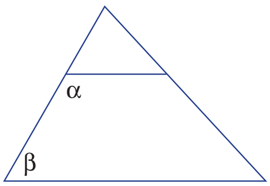

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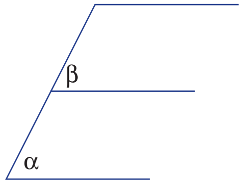

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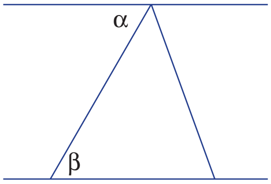
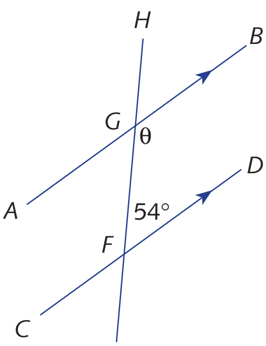


Example 5, 6, 7

2 Find the values of the pronumerals α, β, γ and θ in the diagrams below. Give careful reasons for all your statements, mentioning the relevant parallel lines.

a



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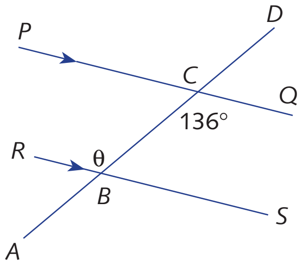

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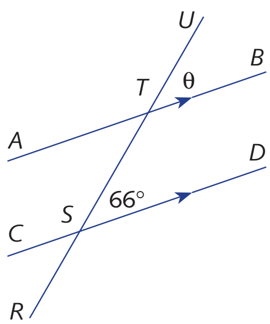

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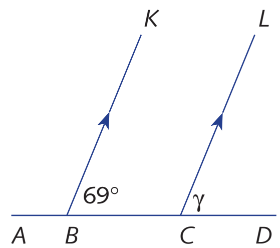
d



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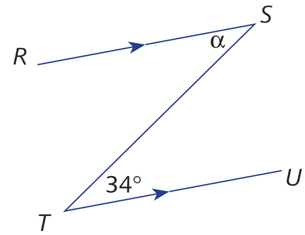

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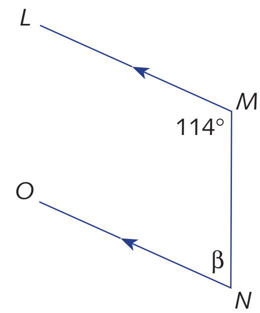

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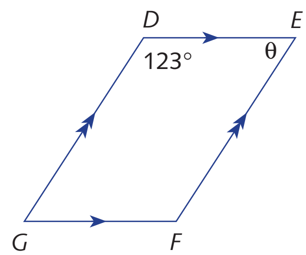

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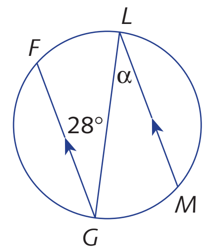

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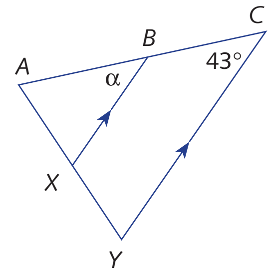

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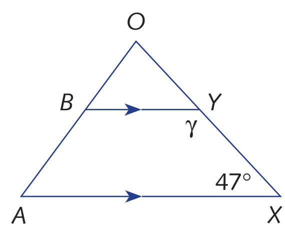

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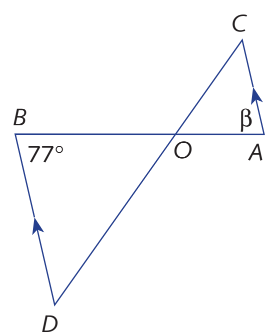

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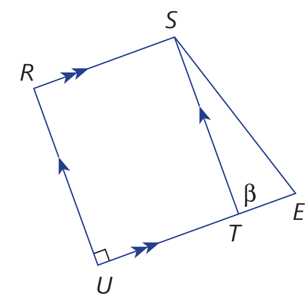
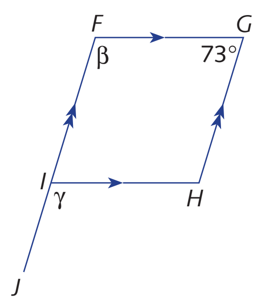


Example 8

3 Find the values of α, β, γ and θ in the diagrams below, giving reasons.

a



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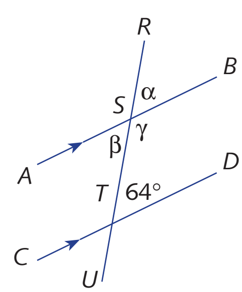

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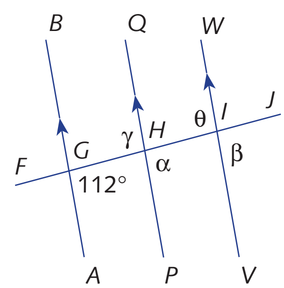

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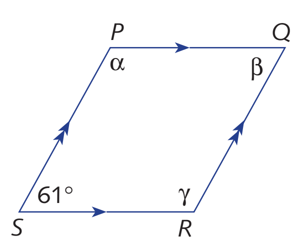

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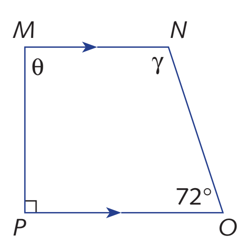

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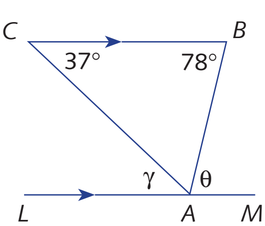

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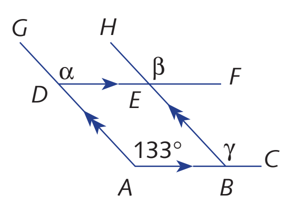

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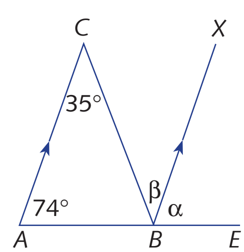

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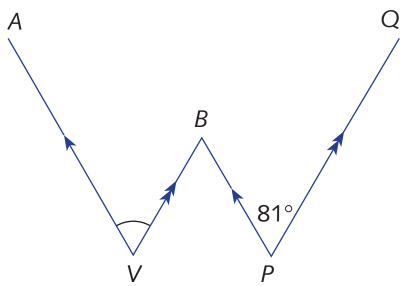
4 Find the size of the marked angle $\angle AVB$ in each diagram below. The solution to each part will require two steps, each with its own reason.

a



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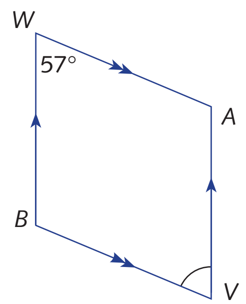

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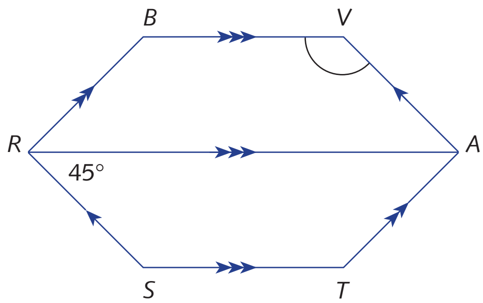

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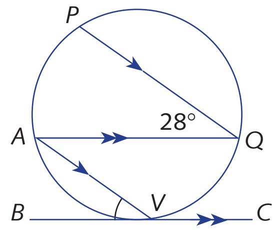


5C Further problems involving parallel lines

This section deals with more complicated problems involving parallel lines:

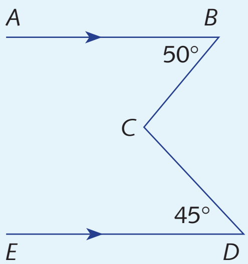
- problems where construction lines need to be added
- problems involving algebra.

Adding construction lines to solve a problem

Some problems cannot be solved until one or more extra lines, called **construction lines**, have been added to the diagram, as in the example below.

Example 9

Find $\angle BCD$ in the diagram opposite.



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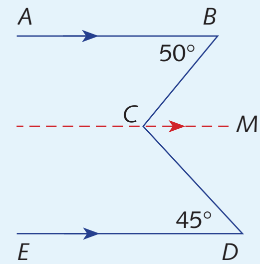
Solution

Construct the line CM through C parallel to AB and ED, as shown in the diagram opposite.

Then $\angle BCM = 50°$ (alternate angles, $AB \parallel CM$),

and $\angle DCM = 45°$ (alternate angles, $ED \parallel CM$).

Hence, $\angle BCD = 95°$ (adjacent angles at C).

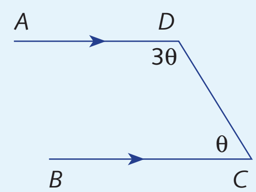


Problems involving algebra

In the example below, the value of θ is found using geometric arguments and algebra.

Example 10

Find θ in the diagram opposite.



Solution

$$\begin{aligned}\theta + 3\theta &= 180° \quad \text{(co-interior angles, } AD \parallel BC\text{)}\\ 4\theta &= 180°\\ \theta &= 45°\end{aligned}$$

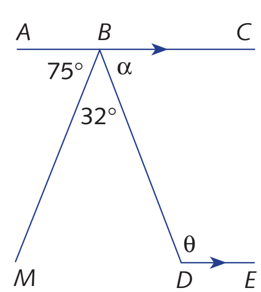
Further problems involving parallel lines

- Extra **construction lines** may need to be added to the diagram.
- Algebra may be required.

Exercise 5C

1 Find the values of α, β, γ and θ in the diagrams below, giving reasons.

a



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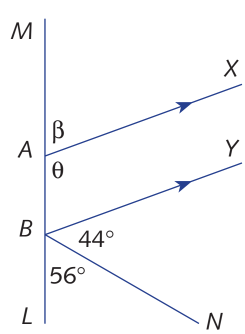
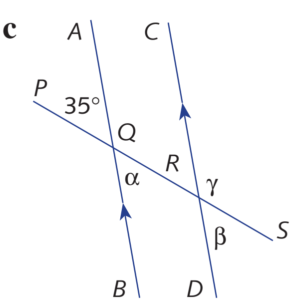


c



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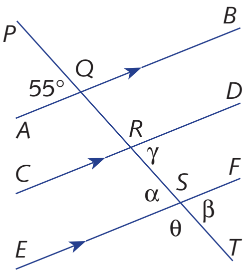
d



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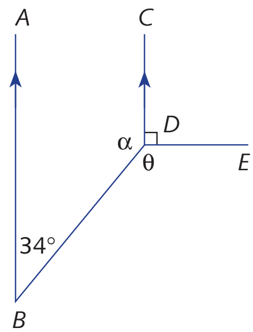

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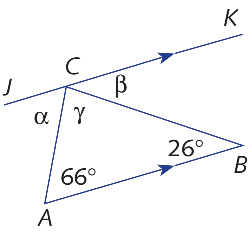

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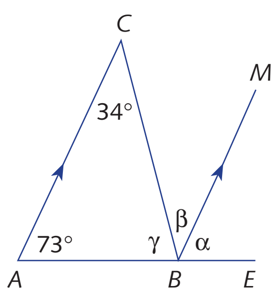

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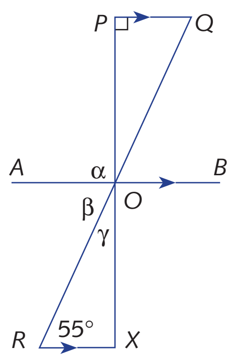
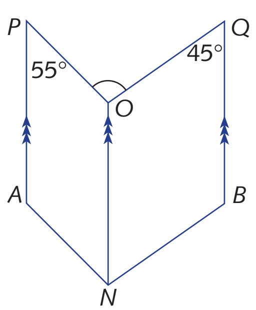


2 Find the size of the marked angle $\angle POQ$ in each diagram below. Several steps may be required.

a



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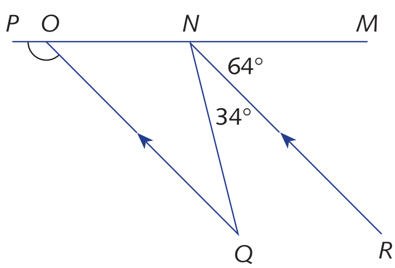

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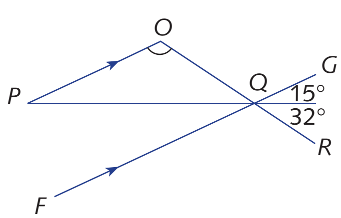

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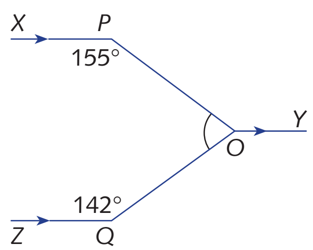
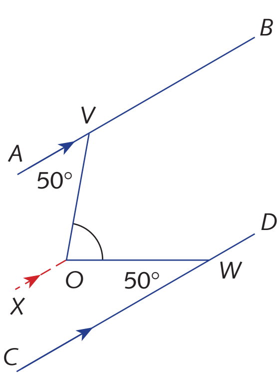


Example 9

3 Copy each diagram, then add a suitable construction line in order to find the size of the marked angle $\angle VOW$ – the first has been done for you. Give reasons.

a



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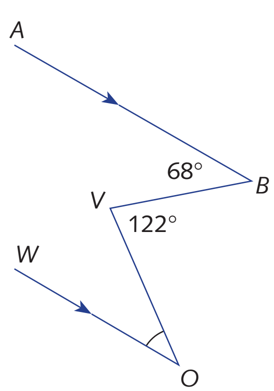

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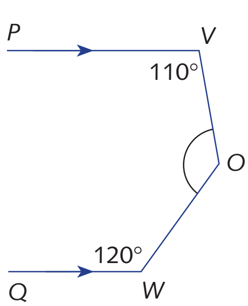

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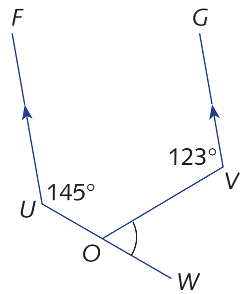
d



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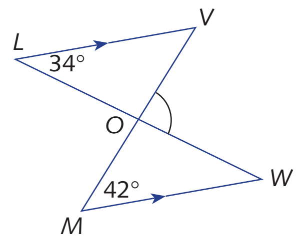

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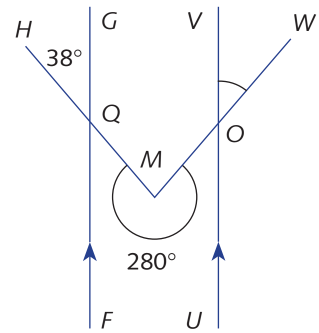

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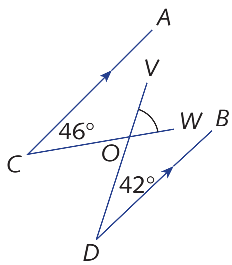

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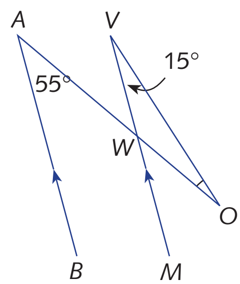
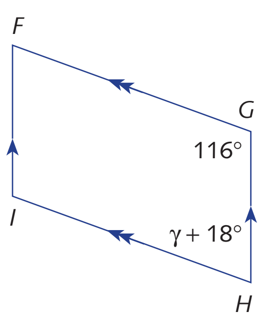


Example 10

4 Find the values of α, β, γ and θ, giving reasons.

a



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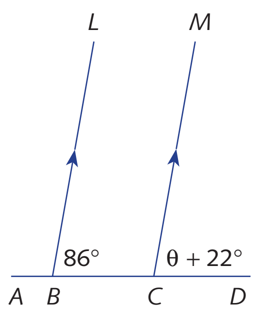

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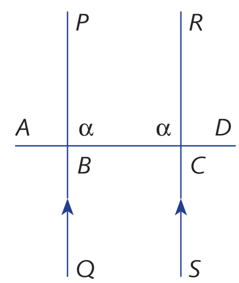

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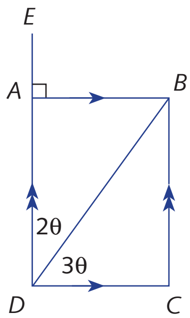

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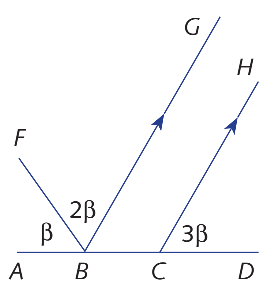

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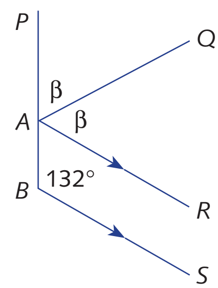

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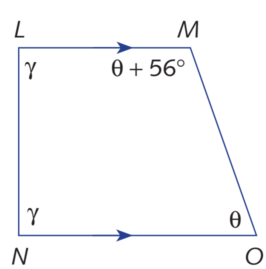

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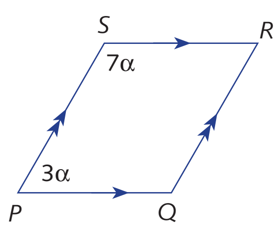

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5D Proving that two lines are parallel

Corresponding, alternate and co-interior angles can be used to prove that two lines are parallel.

Proving that two lines are parallel

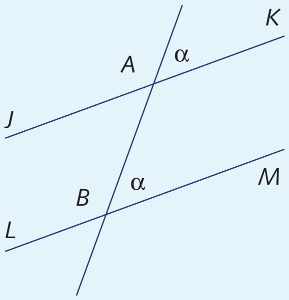
Suppose that a transversal crosses two other lines.

- If the corresponding angles are equal, then the lines are parallel.
- If the alternate angles are equal, then the lines are parallel.
- If the co-interior angles are supplementary, then the lines are parallel.

These three results are called the **converses** of the previous three results, because the logic works in reverse. Here are three examples that show how to set out arguments using them.

Example 11

Find any parallel lines in the diagram opposite.

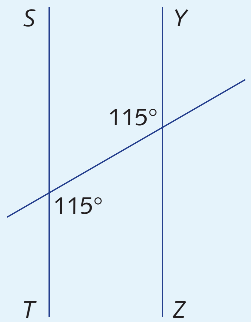


Solution

$JK \parallel LM$ (corresponding angles are equal)

Example 12

Find any parallel lines in the diagram opposite.



Solution

$ST \parallel YZ$ (alternate angles are equal)

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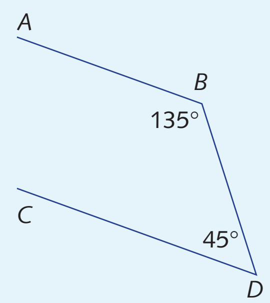
Example 13

Find any parallel lines in the diagram opposite.



Solution

$135° + 45° = 180°$ (co-interior angles are supplementary)

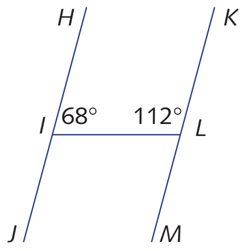
Hence, $AB \parallel CD$

Exercise 5D

Example 11, 12, 13

1 In each diagram below, name all pairs of parallel lines, giving reasons.

a



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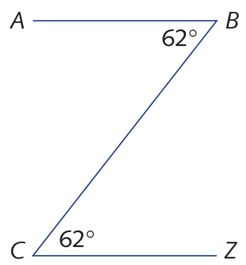

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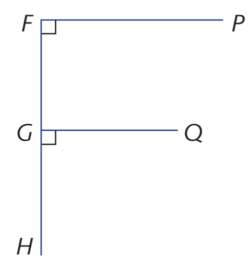

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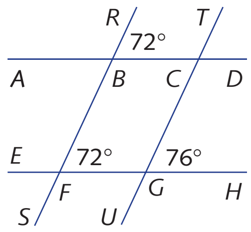

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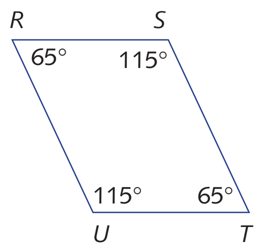

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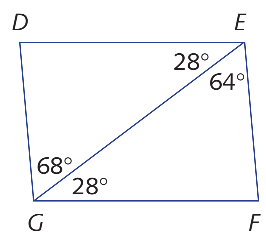

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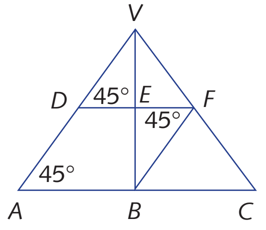

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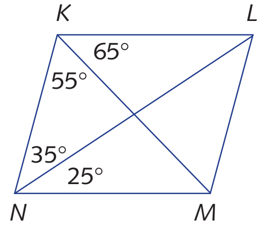

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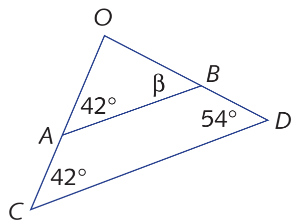
2 In each diagram below, give a reason why $AB \parallel CD$. Hence, find the values of α, β, γ and θ.

a



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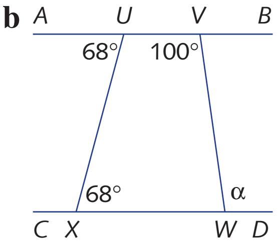

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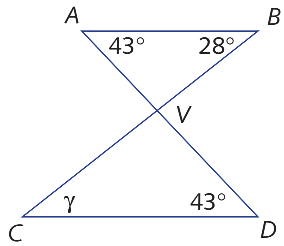
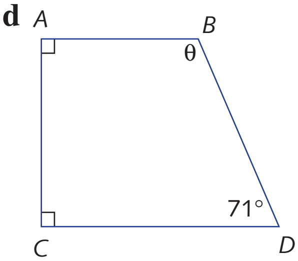
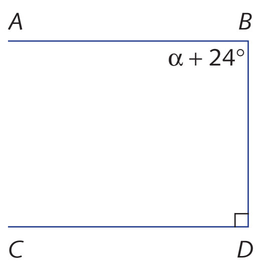


d



3 In each diagram below, write down the values of α, β, γ and θ that will make AB parallel to CD.

a



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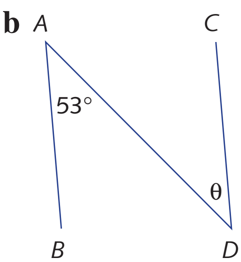

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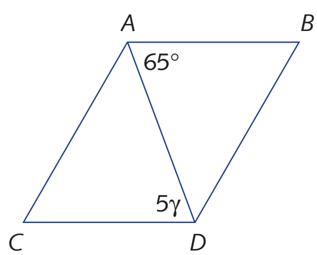

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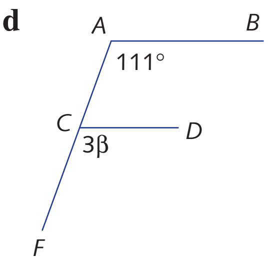
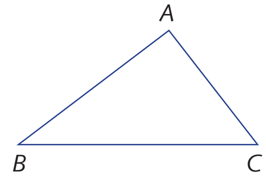


5E Angles in triangles

This section proves and applies two theorems about the angles of any triangle. In mathematics, a true statement and its proof are called a *theorem* – a Greek word meaning 'a thing to be gazed upon' or 'a thing contemplated by the mind'. You may have seen these theorems already, but proving them will probably be new to you.

Triangles

A **triangle** is formed by taking three non-collinear points A, B and C and joining the three intervals AB, BC and CA.



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These intervals are called the **sides** of the triangle, and the three points are called its **vertices**. (The singular of *vertices* is *vertex*.)

Proving the theorem about the angle sum of a triangle

We proved last year that the sum of the interior angles of a triangle is 180°. As we discussed then, checking a theorem many times doesn't prove it – however many triangles you check, there are always more to check. Here is a totally convincing proof that applies to every triangle, whatever its shape.

The theorem and its proof are set out very formally in the manner traditional for geometry. The proof uses only the methods we have already developed about angles at a point and across transversals.

Theorem: The sum of the interior angles of a triangle is 180°.

Proof: Let ABC be a triangle.

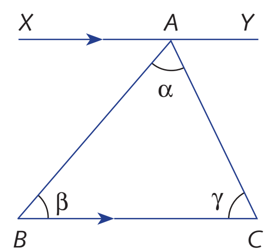
Let $\angle BAC = \alpha$, $\angle ABC = \beta$ and $\angle ACB = \gamma$.

We must prove that $\alpha + \beta + \gamma = 180°$.

Draw the line XY parallel to BC and passing through the vertex A.

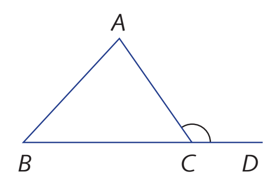
Then $\angle XAB = \beta$ (alternate angles, $XY \parallel BC$), and $\angle YAC = \gamma$ (alternate angles, $XY \parallel BC$).

Hence, $\alpha + \beta + \gamma = 180°$ (straight angle $\angle XAY$).



The exterior angles of a triangle

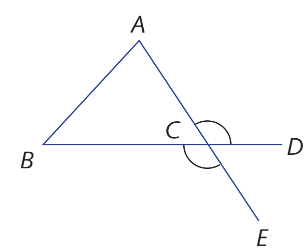
Let ABC be a triangle, with the side BC produced to D. (The word 'produced' means 'extended'.) Then the angle $\angle ACD$ between the side AC and the extension CD is called an **exterior angle** of the triangle.



The angles $\angle CAB$ and $\angle ABC$ are called the **opposite interior angles**, because they are *opposite* the exterior angle at the vertex C.

There are two exterior angles at each vertex, as shown in the lower diagram opposite, and because they are vertically opposite, they are equal in size:

$\angle ACD = \angle BCE$ (vertically opposite at C).



Proving the exterior angle theorem

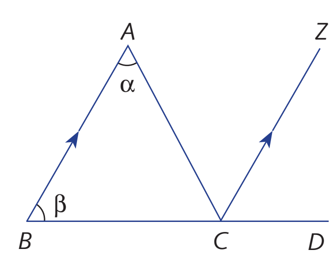
We proved last year that an exterior angle of a triangle is the sum of the opposite interior angles. The following is valid in every situation, whatever shape the triangle may have.

Theorem: An exterior angle of a triangle equals the sum of the two interior opposite angles.

Proof: Let ABC be a triangle, with BC produced to D.

Let $\angle BAC = \alpha$ and $\angle CBA = \beta$.

We must prove that $\angle ACD = \alpha + \beta$.



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Draw the line CZ through C parallel to BA.

$\angle ZCD = \beta$ (corresponding angles, $BA \parallel CZ$), and $\angle ACZ = \alpha$ (alternate angles, $BA \parallel CZ$).

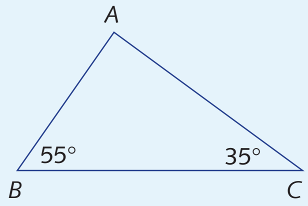
Hence, $\angle ACD = \alpha + \beta$ (adjacent angles at C).

Angles in triangles

- The sum of the interior angles of a triangle is 180°.
- An exterior angle of a triangle equals the sum of the opposite interior angles.

Example 14

Find $\angle A$ in the triangle below.



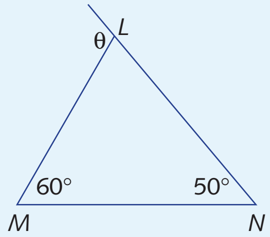
Solution

$\angle A + 55° + 35° = 180°$ (angle sum of ΔABC)

so $\angle A = 90°$

Example 15

Find θ in the triangle below.



Solution

$\theta = 60° + 50°$ (exterior angle of ΔLMN)

so $\theta = 110°$

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Exercise 5E

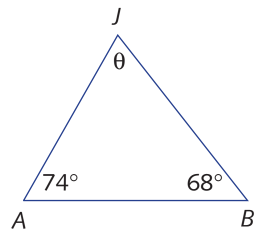
1 Find the third angle of a triangle, two of whose angles are given.

a 60°, 30° **b** 120°, 30° **c** 111°, 11° **d** 62°, 90° **e** 28°, 58°

Example 14, 15

2 Find the values of α, β, γ and θ in the diagrams below, giving reasons.

a



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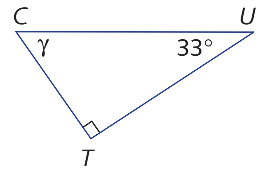

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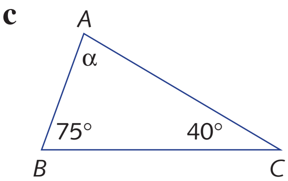

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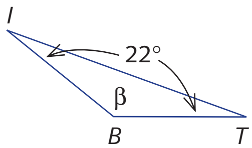

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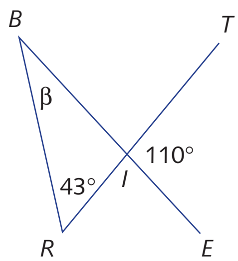

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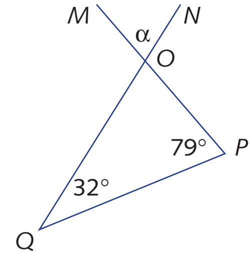

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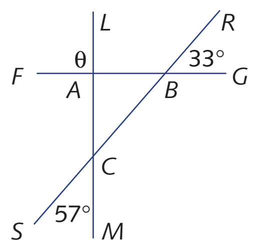

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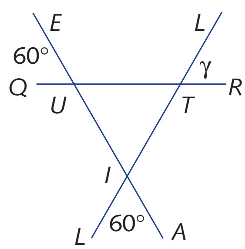
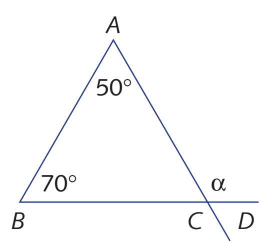


3 Use exterior angles – *not* interior angles – to find α, β, γ and θ in each diagram below. Give reasons.

a



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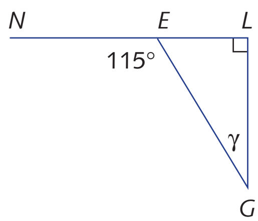

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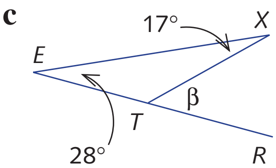

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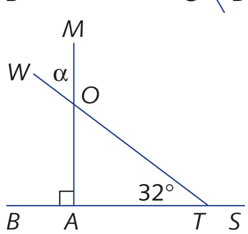

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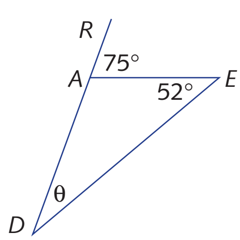

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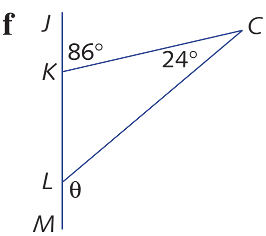

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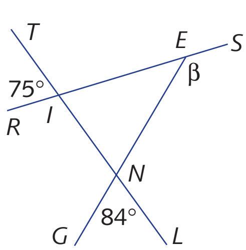

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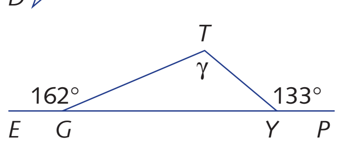

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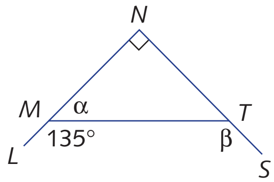
4 Find the values of the pronumerals in each diagram, giving reasons.

a



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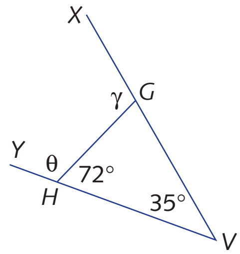

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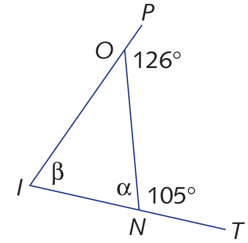

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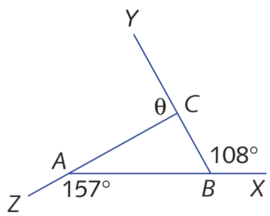

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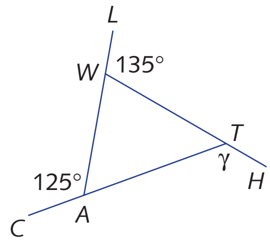

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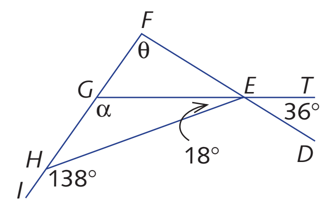

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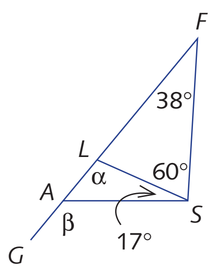

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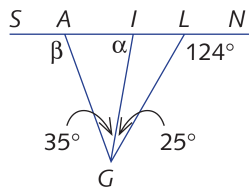
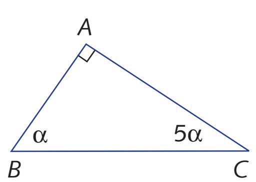


5 Find α, β, γ and θ in these diagrams, giving reasons.

a



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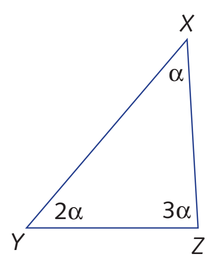

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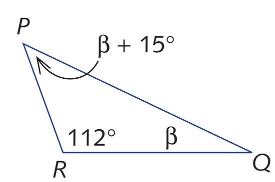

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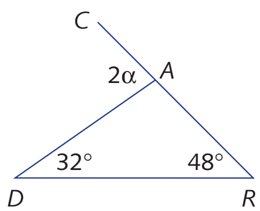

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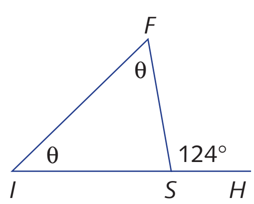

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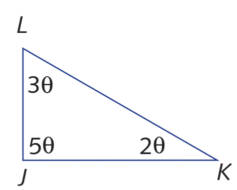

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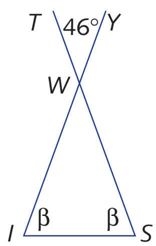

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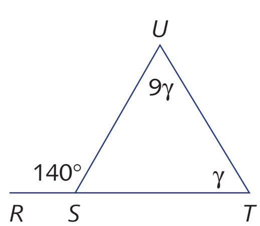

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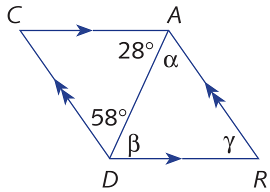
6 Find α, β, γ and θ in these diagrams, giving reasons.

a



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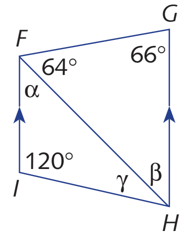

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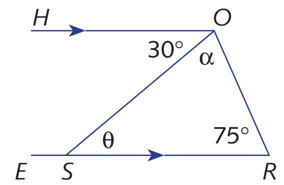

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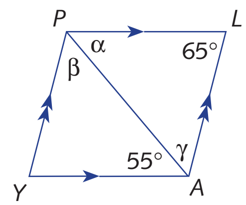

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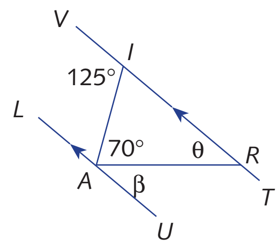

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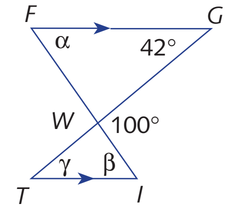

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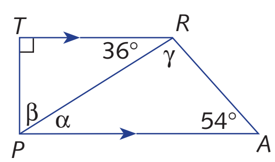

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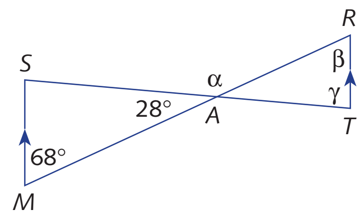
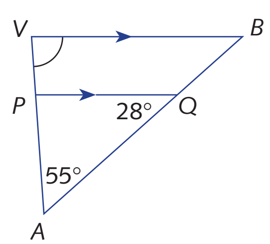


7 Find the size of the marked angle $\angle AVB$ in each diagram, giving reasons.

a



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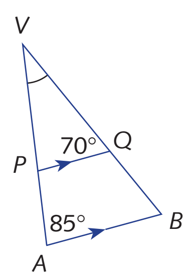

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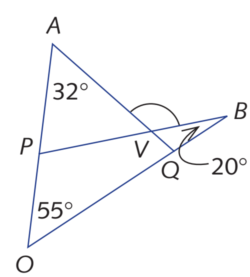

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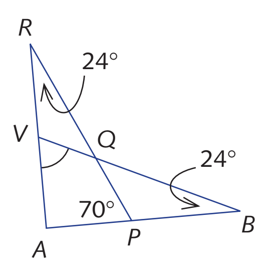

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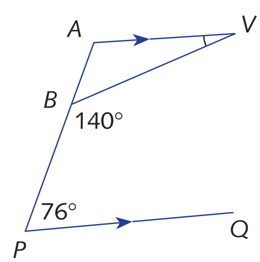

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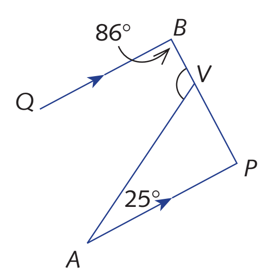

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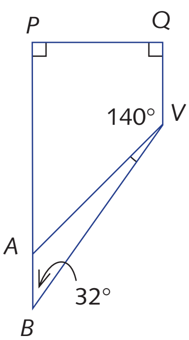

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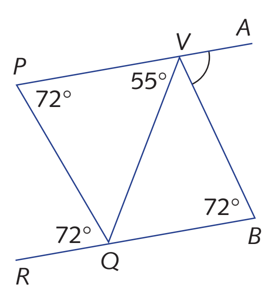

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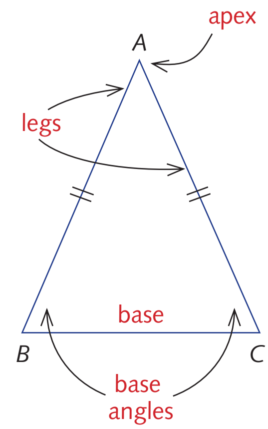
5F Isosceles and equilateral triangles

Isosceles triangles



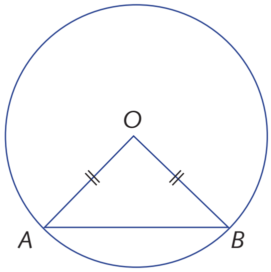
An **isosceles triangle** is a triangle in which two (or more) sides have equal length.

- The equal sides AB and AC in the isosceles triangle ABC to the right are called the **legs**. They have been marked with double dashes to indicate that they are equal in length.
- The vertex A where the legs meet is called the **apex**.
- The third side BC is called the **base**.
- The angles $\angle ABC$ and $\angle ACB$ at the base are called **base angles**.

The word *isosceles* is a Greek word meaning 'equal legs' – *iso* means 'equal', and *sceles* means 'legs'.

Constructing an isosceles triangle using a circle

In the diagram below, the equal radii AO and BO of the circle form the equal legs of the isosceles triangle OAB. The interval AB is the base and the centre O is the apex.



Using two radii of a circle is usually the simplest way to construct an isosceles triangle.

Isosceles triangles and their base angles

Last year, in Chapter 13 of *ICE-EM Mathematics Year 7* we developed in an informal way two results about isosceles triangles. The first is a property that all isosceles triangles have.

- The base angles of an isosceles triangle are equal.
  (This result can also be stated as 'If two sides of a triangle are equal, then the angles opposite those sides are equal.')

The second is a test for a triangle to be isosceles, and is the converse of the first result.

- If two angles of a triangle are equal, then the sides opposite those angles are equal.

In Chapter 12 we will introduce *congruence*, which will allow us to prove these theorems properly.

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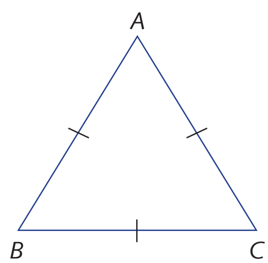
Isosceles triangles

- An **isosceles triangle** is a triangle with two (or more) sides equal.
  - The equal sides are called the **legs**; the legs meet at the **apex**; the third side is the **base**; and the angles opposite the legs are called **base angles**.
  - Two radii of a circle and the interval joining them form an isosceles triangle.
- The base angles of an isosceles triangle are equal.
- Conversely, if two angles of a triangle are equal, then the sides opposite those angles are equal.

Equilateral triangles

An **equilateral triangle** is a triangle in which all three sides have equal length. (The name comes from Latin – *equi* means 'equal', and *latus* means 'side'.)

The diagram below shows an equilateral triangle ABC. Notice how it is an isosceles triangle in three different ways, because the base could be taken as AB, BC or CA.



The interior angles of an equilateral triangle are all 60°

It is not hard to prove that all the angles of an equilateral triangle are 60° provided that we use the earlier theorem that the base angles of an isosceles triangle are equal.

Theorem: The interior angles of an equilateral triangle are all 60°.

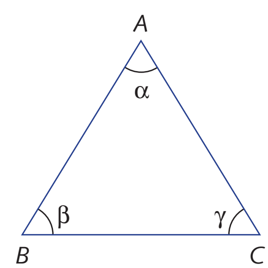
Proof: Let ABC be an equilateral triangle.

Let $\angle BAC = \alpha$, $\angle ABC = \beta$ and $\angle ACB = \gamma$.

We must prove that $\alpha = \beta = \gamma = 60°$.

$\beta = \gamma$ (opposite sides AC and AB are equal), and $\beta = \alpha$ (opposite sides AC and BC are equal), so $\alpha = \beta = \gamma$.

But $\alpha + \beta + \gamma = 180°$ (angle sum of $\triangle ABC$), so $\alpha = \beta = \gamma = 60°$.



Equilateral triangles

- An equilateral triangle is a triangle with all three sides equal.
- The angles in an equilateral triangle are all 60°.

Here are examples of arguments using each of the three theorems in this section, and the fact that all the radii of any circle are equal.

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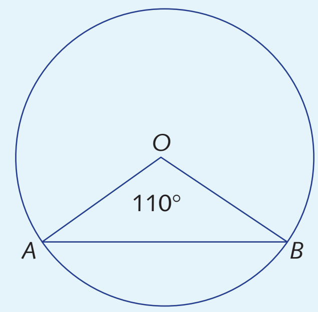
Example 16

Find $\angle OBA$ in the diagram opposite.



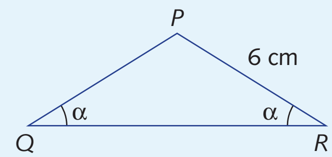
Solution

$AO = BO$ (radii)

Hence, $\angle OBA = 35°$ (base angles of isosceles ΔABO)

Example 17

Find the length of PQ in the diagram opposite.

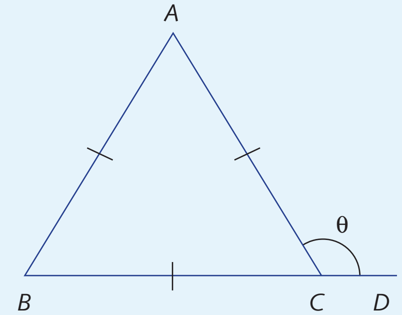


Solution

$PQ = 6$ cm (opposite angles $\angle PQR$ and $\angle PRQ$ are equal)

Example 18

Find θ in the diagram opposite.



Solution

$\angle ABC = 60°$ (equilateral ΔABC)

Hence, $\theta + 60° = 180°$ (straight angle $\angle BCD$)

so $\qquad \theta = 120°$

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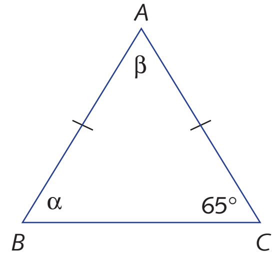
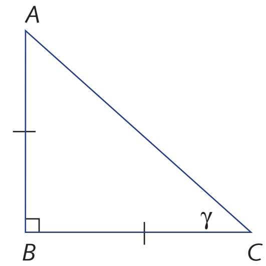
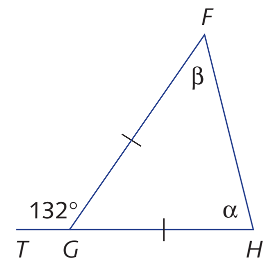
Exercise 5F

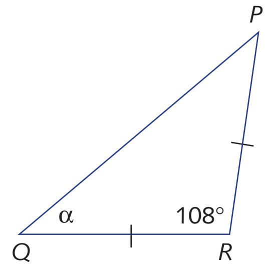
Example 18

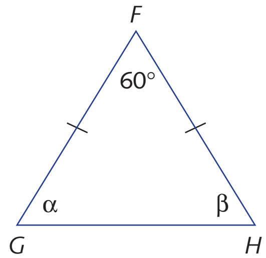
1 Find the value of each pronumeral, giving reasons.

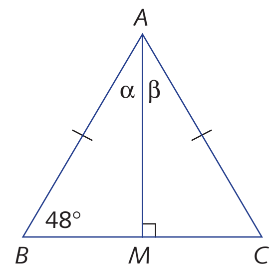
a


b


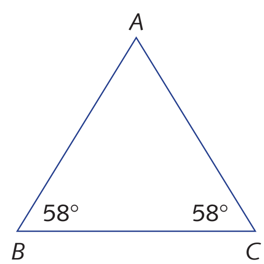
c


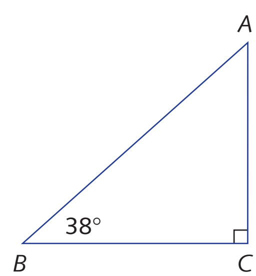
d


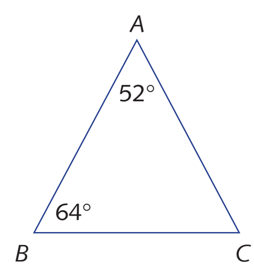
e


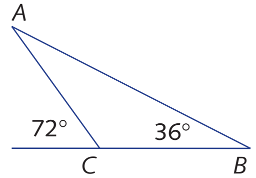
f


2 Determine whether ΔABC in each part is an isosceles triangle, naming the equal sides. Give reasons.

a


b


c


d


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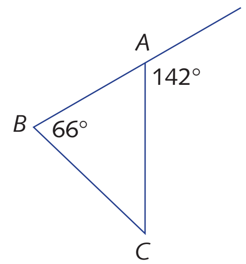
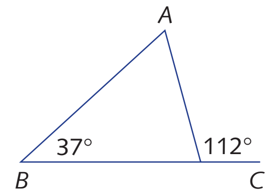
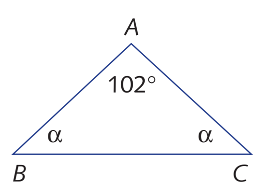


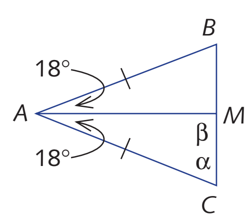
f


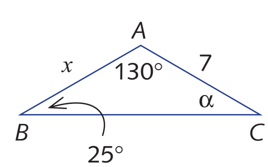
3 **a** Let ΔABC be isosceles, with $AB = AC$. Produce BC to D, and let $\angle ACD = 124°$. Draw a diagram and find $\angle ABC$, giving reasons.

b P and Q are points on a circle with centre O such that $\angle POQ = 48°$. Draw a diagram and calculate the size of $\angle OPQ$.

4 Find the values of the pronumerals in these diagrams, giving reasons.

a


b


c


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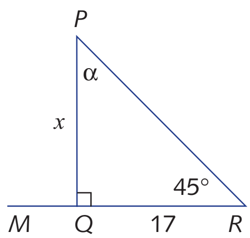
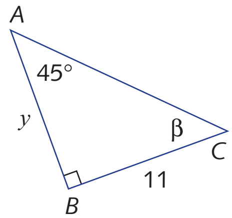
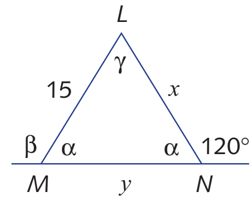
d



e

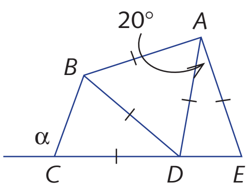


f

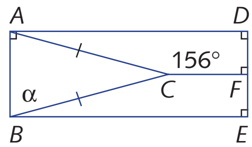


5 Find the values of the pronumerals in these diagrams, giving reasons. In part **d**, O is the centre of the circle.

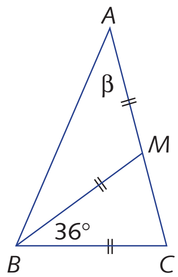
a



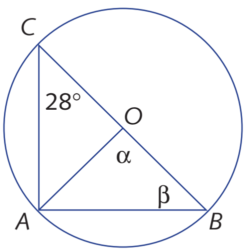
b



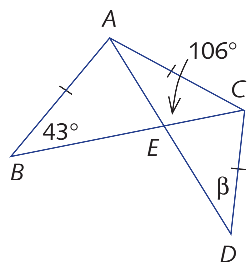
c



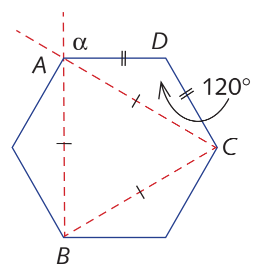
d



e

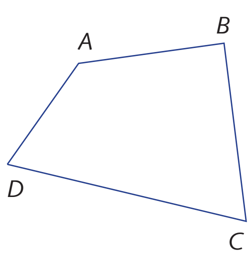


f

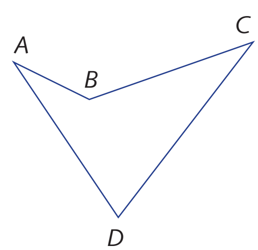


5G Quadrilaterals

Take any four points A, B, C and D in the plane, no three of which are collinear, and join the intervals AB, BC, CD and DA. Provided that no two of these intervals cross over each other, the figure $ABCD$ is called a *quadrilateral* (from Latin – *quadri* means 'four', and *latus* means 'side'). The four points are its **vertices** and the four intervals are its **sides**.



The quadrilateral above is called a **convex quadrilateral** because none of its four interior angles is a reflex angle.



The quadrilateral above is called a **non-convex quadrilateral** because one of its interior angles ($\angle ABC$) is a reflex angle.

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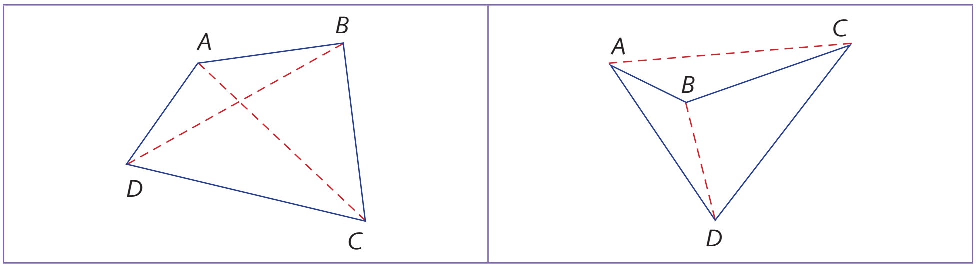
(*Note*: No interior angle can be exactly 180° because the definition requires that no three of the vertices be collinear.)



The intervals AC and BD joining non-adjacent vertices are called the **diagonals** of the quadrilateral. The left-hand quadrilateral above is convex and so both diagonals are inside the quadrilateral. The right-hand quadrilateral above is non-convex and one diagonal is inside the quadrilateral and the other is outside.

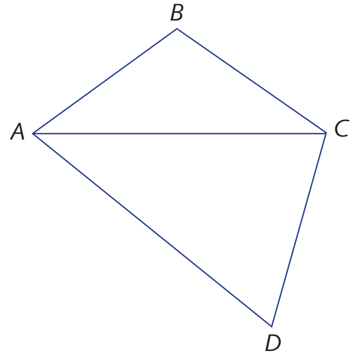
The interior angles of a quadrilateral add to 360°

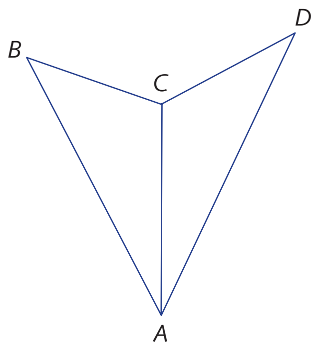
The interior angles of a quadrilateral always add to 360°.

The general proof divides the quadrilateral into two triangles and then applies the earlier theorem about the angle sum of a triangle.

Theorem: The sum of the interior angles of a quadrilateral is 360°.

Proof: Let $ABCD$ be a quadrilateral, labelled so that the diagonal AC is inside the quadrilateral.
(At least one diagonal lies inside the quadrilateral.)





We must prove that $\angle ABC + \angle BCD + \angle CDA + \angle DAB = 360°$.
Join the diagonal AC.
The interior angles of ΔABC add to 180°.
Similarly, the interior angles of ΔADC add to 180°.
But the sum of the interior angles of $ABCD$ is the sum of the angles of ΔABC and ΔADC.
Hence, the interior angles of $ABCD$ add to 360°.

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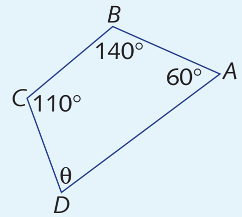
Example 19

Find θ in the diagram opposite.



Solution

$$\begin{aligned}\theta + 60° + 110° + 140° &= 360° \quad \text{(angle sum of quadrilateral } ABCD)\\ \theta + 310° &= 360°\\ \theta &= 50°\end{aligned}$$

Quadrilaterals

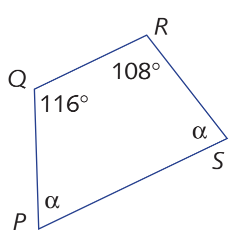
- A **quadrilateral** ABCD has four **vertices**, no three of which are collinear, and four **sides**, no two crossing each other.
- A **convex quadrilateral** has no reflex interior angle. Both **diagonals** of a convex quadrilateral lie inside the figure.
- A **non-convex quadrilateral** has one reflex interior angle. One diagonal of a non-convex quadrilateral lies outside the figure, the other inside.
- The sum of the interior angles of a quadrilateral is 360°.

Exercise 5G

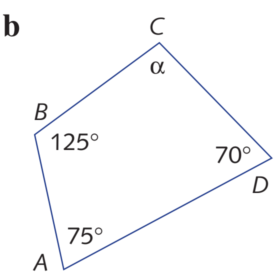
Example 19

1 Find the values of the pronumerals in these diagrams, giving reasons.

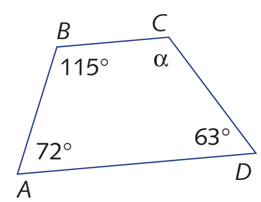
a



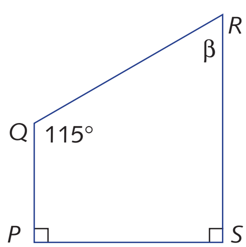
b



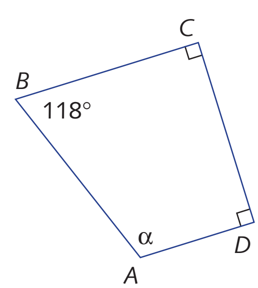
c



d



e



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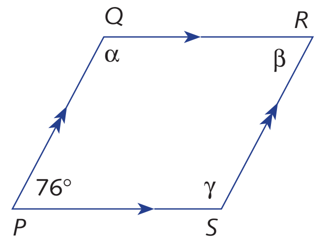
2 In each part, three angles of a quadrilateral are given. Calculate the size of the fourth angle of the quadrilateral. Sketch a quadrilateral with these angle sizes.

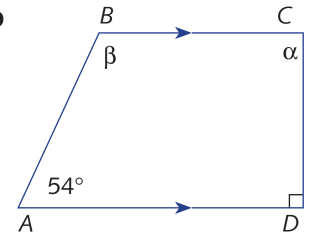
a $90°, 90°, 150°$ **b** $45°, 90°, 135°$ **c** $70°, 160°, 20°$ **d** $40°, 40°, 40°$

3 Find the values of the pronumerals, giving reasons.

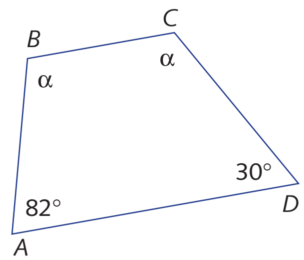
a



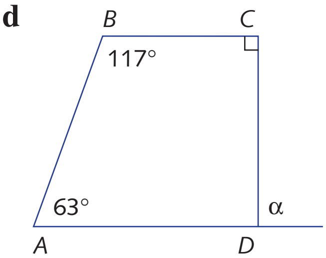
b



c

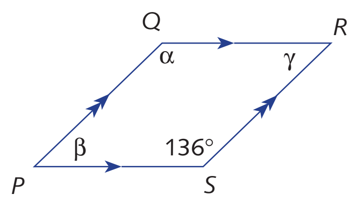


d

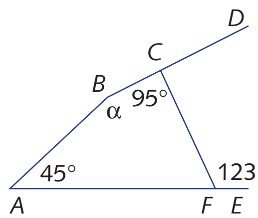


4 Find the values of the pronumerals, giving reasons. In part **f**, *O* is the centre of the circle.

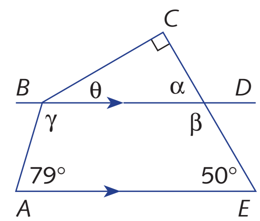
a



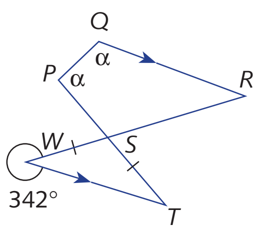
b



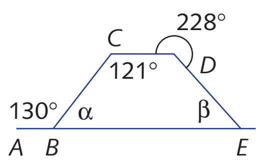
c



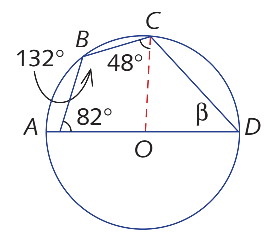
d



e

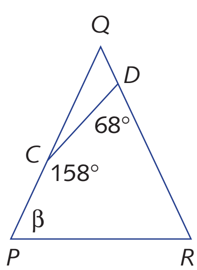


f



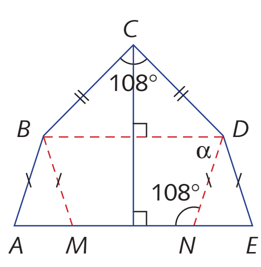
5 Find the values of the pronumerals, giving reasons.

a

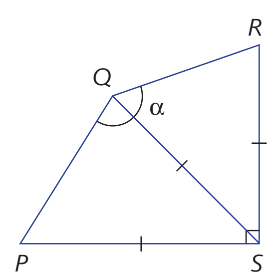


QP = QR

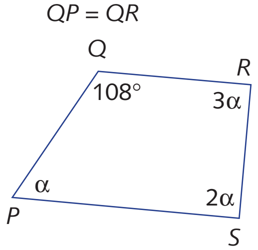
b



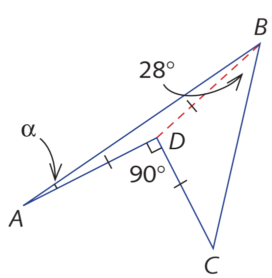
c



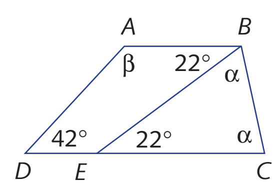
d



e



f



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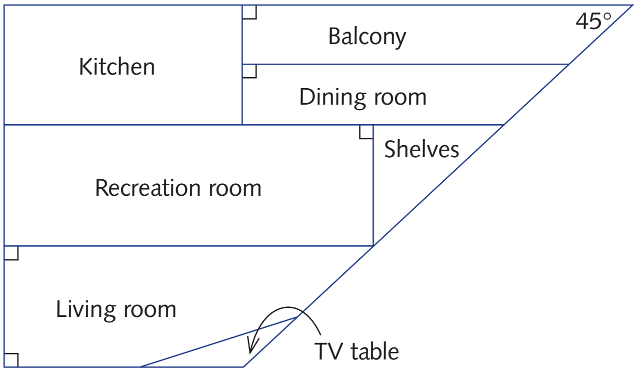
6 The following diagram is a map of the first floor of Tony's house.

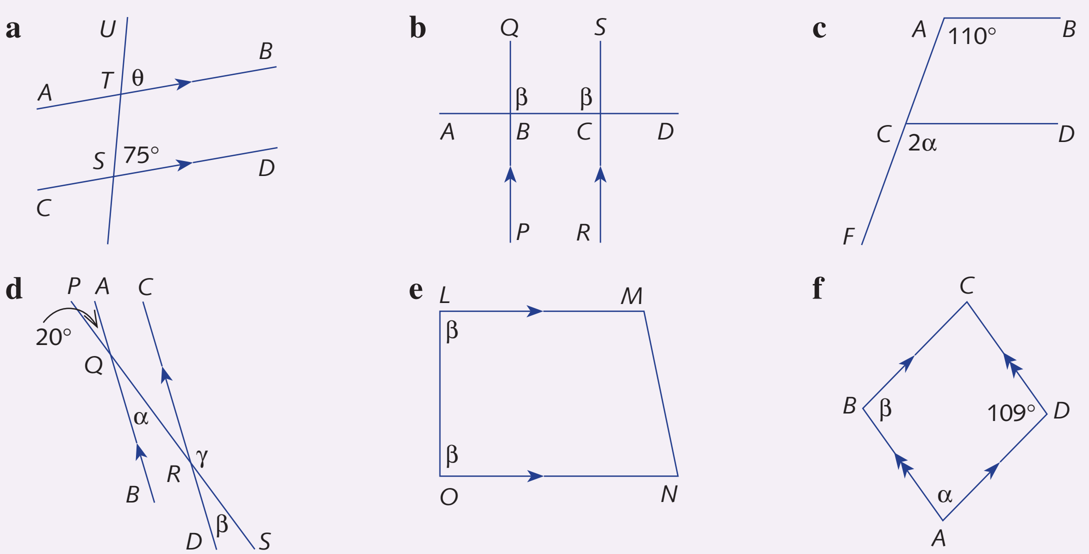


a Tony would like to build some wooden shelves at the end of the recreation room. Each shelf would be an isosceles triangle. Evaluate the internal angles of these shelves.

b Tony would also like to construct a TV table in the shape of an isosceles triangle. Evaluate the internal angles of the TV table.

Review exercise

1 Find the value of the pronumeral in each diagram. Give reasons in each case.



2 State the complement of each angle.

a 30° b 63° c 74° d 84°

3 State the supplement of each angle.

a 127° b 76° c 134° d 15°

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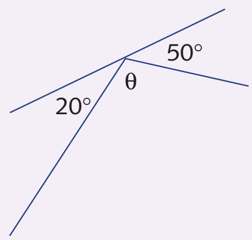
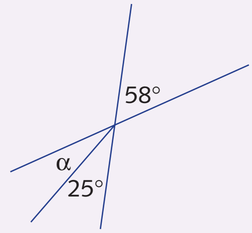
4 Find the value of each pronumeral.

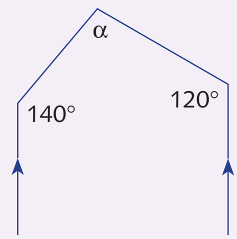
a



b



c



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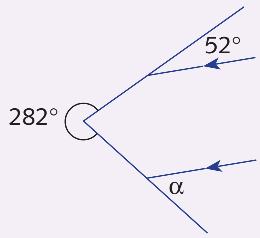
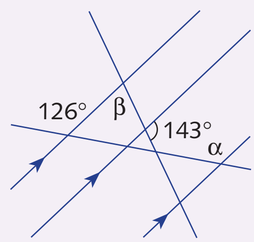


e



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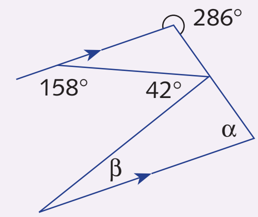

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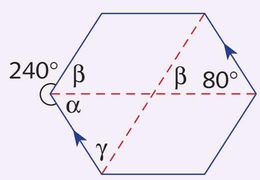

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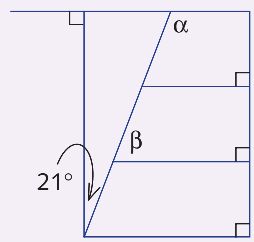

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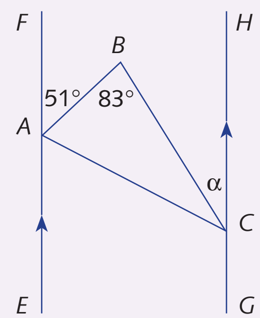
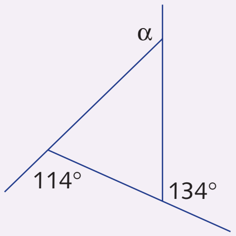


5 Find the values of the pronumerals in these diagrams.

a



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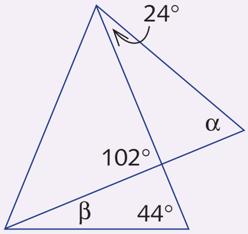

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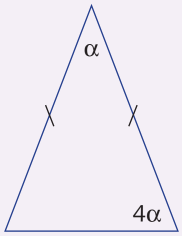

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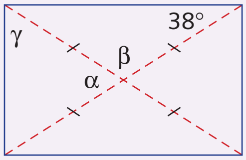

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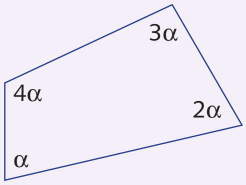

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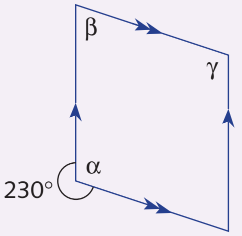

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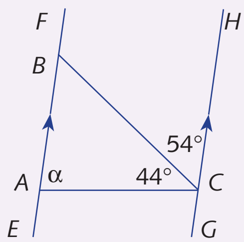
6 Find the value of α in each of these diagrams.

a



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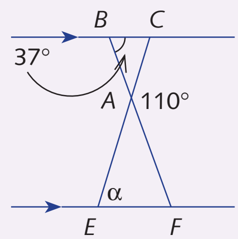

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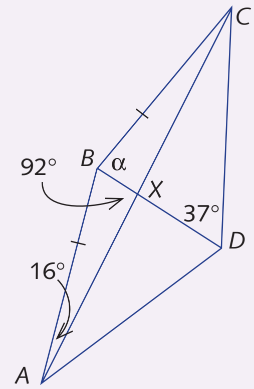
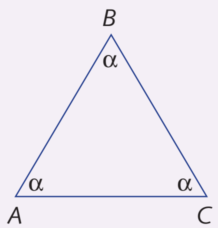


7 Explain why a triangle cannot have a reflex angle as one of its internal angles.

8 Find the values of α, β and γ in the diagrams below, and explain your reasoning.

a



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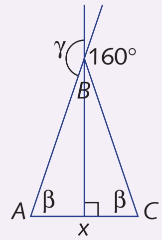

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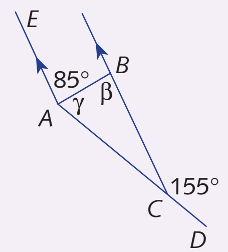
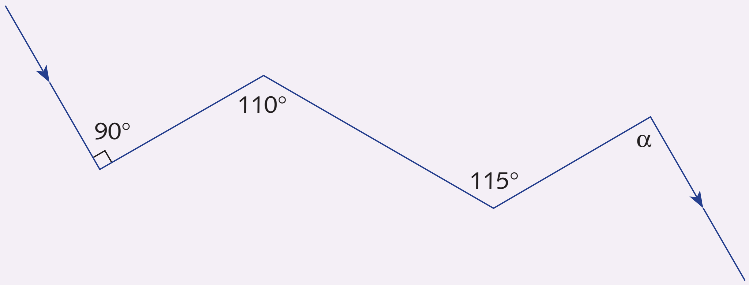
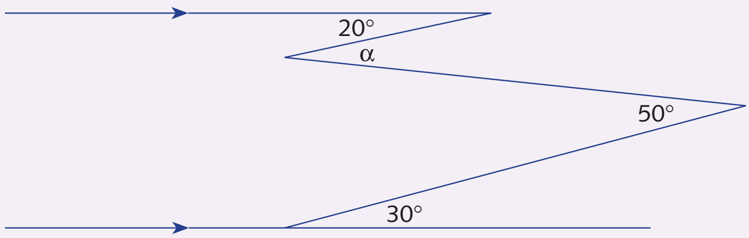


Challenge exercise

1 Calculate the unknown angle.



2 **a** Find the value of α.



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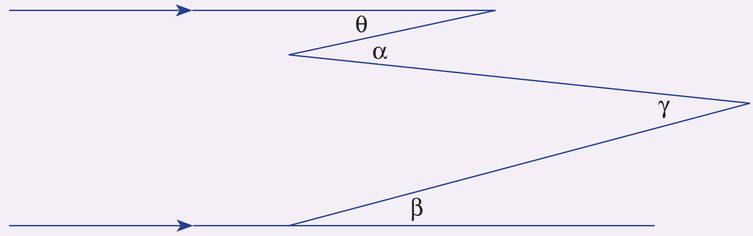
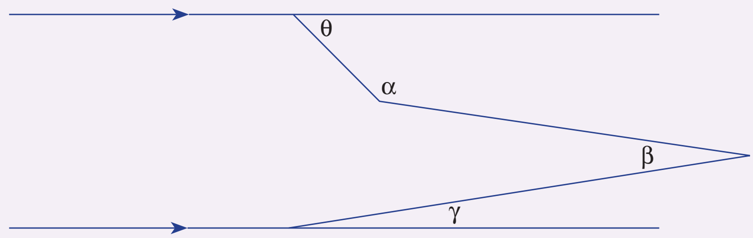
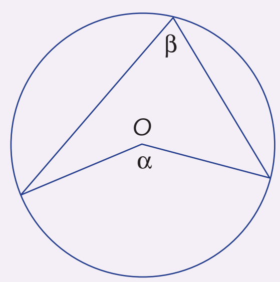
b Find α in terms of β, γ and θ.



c Find α in terms of β, γ and θ.

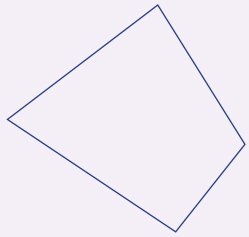


3 The point O is the centre of the circle. Find α in terms of β.

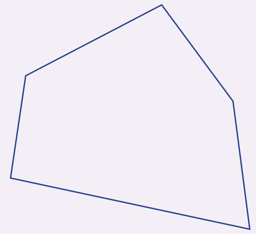


4 The sum of the internal angles of a triangle is 180°. Use this property of a triangle to find the sum of the internal angles in each of the diagrams below. What relationship do you notice?

a



b



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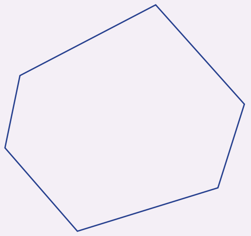
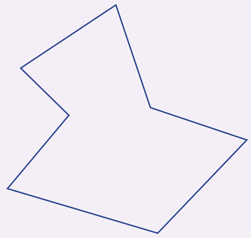
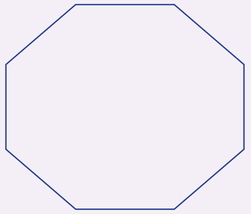


d



e



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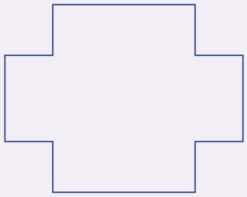

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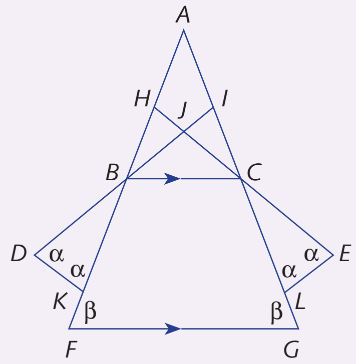
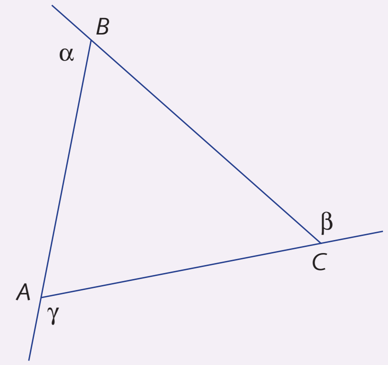
5 Name all the isosceles triangles in the diagram below.



6 Consider the isosceles triangle ABC, with $AB = AC$. Suppose D is on AB, and E is on AC, so that $AD = DE$. If $\angle CED = 142°$, calculate $\angle ABC$.

7 $ABCD$ is a square and AEB is an equilateral triangle. Calculate the two possible values of the size of $\angle DEC$.

8 Find the value of $\alpha + \beta + \gamma$ in the diagram below.



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