CHAPTER

6

Number and Algebra

Algebra – part 1

Algebra is an important part of mathematical language. It helps us to state ideas more simply. It also enables us to make general statements about mathematics and solve problems that are difficult to do otherwise.

Algebra was introduced to the Arabs in about 830 CE by Abu Abdullah Muhammad bin Musa al-Khwarizmi. The last part of his name gives us the word algorithm. His work was also influential in the introduction of algebra into Europe in the early thirteenth century.

ICE-EM Mathematics 8 3ed ISBN 978-1-108-40429-7 Cambridge University Press

6A Notation and substitution

In algebra, letters are used to stand for numbers. For example, if a box contains x stones and you put five more in, then there are $x + 5$ stones in the box. You may or may not know what the value of x is. In algebra, we call the letter that represents a number a **pronumeral**.

Substitution involves giving a value to the letter.

For example, if $x = 20$, then $x + 5 = 25$.

We first recall some conventions and notation in algebra.

Multiplication:	$5 \times x = 5x$	$a \times b = ab$	
Division:	$x \div 5 = \frac{x}{5}$	$a \div b = \frac{a}{b}$	
Powers:	$x \times x = x^2$	$a \times a \times a = a^3$	$z \times z \times z \times z = z^6$

The following table reminds us of the meanings of some commonly occurring algebraic expressions.

$2x + 3$	The number x is multiplied by 2, and 3 is added to the result.
$5x - 3$	The number x is multiplied by 5, and 3 is subtracted from the result.
$3(x - 1)$	One is subtracted from the number x, and the result is multiplied by 3.
$x^2 + 4$	The number x is multiplied by itself, and 4 is added to the result.
$\frac{x}{5} + 6$	The number x is divided by 5, and 6 is added to the result.
$\frac{x + 5}{6}$	5 is added to the number x, and the result is divided by 6.

In algebra we write:

$x^1 = x$ $\qquad 0 + x = x$ $\qquad 1x = x$

Example 1

a For $x = 2$, find $x - 3$.

b For $x = -3$, find $2x + 3$.

Solution

a When $x = 2$:

$$\begin{aligned} x - 3 &= 2 - 3 \\ &= -1 \end{aligned}$$

b When $x = -3$:

$$\begin{aligned} 2x + 3 &= 2 \times (-3) + 3 \\ &= -6 + 3 \\ &= -3 \end{aligned}$$

ICE-EM Mathematics 8 3ed ISBN 978-1-108-40429-7

Example 2

For $m = -1$ and $n = \frac{1}{2}$, evaluate:

a $m + n$ **b** $m - n$ **c** $2m - n$ **d** $m^2 + n^2$

Solution

a $m + n = -1 + \frac{1}{2}$
$= -\frac{1}{2}$

b $m - n = -1 - \frac{1}{2}$
$= -1\frac{1}{2}$

c $2m - n = 2 \times (-1) - \frac{1}{2}$
$= -2 - \frac{1}{2}$
$= -2\frac{1}{2}$

d $m^2 + n^2 = (-1)^2 + \left(\frac{1}{2}\right)^2$
$= 1 + \frac{1}{4}$
$= 1\frac{1}{4}$

Example 3

For $m = 2$, evaluate:

a m^2 **b** $(-m)^2$ **c** $-m^2$

d m^3 **e** $(-m)^3$ **f** $(3m)^2$

Solution

a $m^2 = 2^2$
$= 4$

b $(-m^2) = (-2)^2$
$= -2 \times -2$
$= 4$

c $-m^2 = -2^2$
$= -4$

d $m^3 = 2^3$
$= 8$

e $(-m)^3 = (-2)^3$
$= (-2) \times (-2) \times (-2)$
$= -8$

f $(3m)^2 = (3 \times 2)^2$
$= 6^2$
$= 36$

ICE-EM Mathematics 8 3ed ISBN 978-1-108-40429-7 © Brown et al. 2017 Cambridge University Press

Exercise 6A

Example 1

1 Substitute 2 for x in each expression and evaluate.

a $x+5$ **b** $4x$ **c** $\frac{x}{2}$

d $\frac{7x}{2}$ **e** $3(x+4)$ **f** $2(x-1)$

g $\frac{x+8}{2}$ **h** $\frac{9-x}{4}$

2 Substitute –6 for x in each expression and evaluate.

a $x+3$ **b** $-2x$ **c** $\frac{x}{6}$

d $\frac{x}{2}$ **e** $2(x+5)$ **f** $2(x-5)$

g $\frac{x+8}{2}$ **h** $5(x-4)$

3 Evaluate each expression when $n=-3$.

a n^2 **b** $(-n)^2$ **c** $-n^2$

d n^3 **e** $(-n)^3$ **f** $(3n)^2$

g $3n^2$

Example 2, 3

4 Evaluate each expression when $a=-2$, $b=5$ and $c=-1$.

a $a+b$ **b** a^2+c **c** $a-c$

d c^5 **e** a^2+b^2 **f** $a-b$

g a^2-c^2 **h** $-c$ **i** $\frac{b}{c}$

j $\frac{b}{c^3}$ **k** $(2c)^2$ **l** $(2c)^3$

5 Evaluate each expression when $a=-2$, $b=5$ and $c=-\frac{1}{3}$.

a c^2 **b** a^2-c^2 **c** $a-b$ **d** $b-a$

6 Evaluate each expression when $x=-2$ and $y=-1$.

a $-(2x)$ **b** $-xy$ **c** $2x+3y$ **d** $\frac{x}{y}$

e $\frac{6x}{y}$ **f** x^2+y^2 **g** $x+y$ **h** $-5x+6y$

i $-3y$ **j** x^2y **k** x^3+y^3

7 Evaluate each expression when $a=3$, $b=5$ and $c=-1$.

a $a+b+c$ **b** $a-c+b$ **c** $a^2+b^2+c^2$

d $a^2-b^2-c^2$ **e** $(-a)^2+b^3+c$ **f** $c-a-b$

g $c-b+a^2$ **h** $-(2a)^2+b^3+c$ **i** $(-3a)^3+2a-c$

ICE-EM Mathematics 8 3ed ISBN 978-1-108-40429-7

6B Solving equations

Joe has a pencil case that contains a number of pencils. He has three other pencils.

We do not know how many pencils there are in the pencil case but we do know he has a total of 11 pencils. Let x be the number of pencils in the pencil case. Then the facts we are given tell us that:

$$x + 3 = 11$$

This statement is called an **equation** and we need to **solve** the equation for x. The solution is $x = 8$.

Reading equations

The equation $2x + 4 = 10$ can be read as 'two times a number x plus 4 is equal to 10'.

The instruction 'Solve the equation $2x + 4 = 10$' can also be read as 'A number x is multiplied by 2, and then 4 is added. The result is 10. Find the number.'

The number in this case is 3. This can be checked by $2 \times 3 + 4 = 10$.

Equivalent equations

Consider these equations

$$2x + 3 = 9 \quad ①$$
$$2x + 5 = 11 \quad ②$$

Equation ② is obtained from equation ① by adding 2 to each side of the equation. Equation ① is obtained by subtracting 2 from both sides of equation ②. Equations ① and ② are said to be **equivalent** equations.

Equation ③ below is obtained from equation ② by subtracting 5 from each side of the equation.

$$2x = 6 \quad ③$$
$$x = 3 \quad ④$$

Equation ④ is obtained from equation ③ by dividing each side of the equation by 2. You can obtain equation ③ from equation ④ by multiplying each side by 2. All of the above equations are **equivalent**.

Equivalent equations

- If we add the same number to, or subtract the same number from, both sides of an equation, the new equation is **equivalent** to the original equation.
- If we multiply or divide both sides of an equation by the same non-zero number, the new equation is equivalent to the original equation.
- Equivalent equations have exactly the same solutions.

ICE-EM Mathematics 8 3ed ISBN 978-1-108-40429-7

Example 4

Solve each equation for x and check that LHS = RHS.

a $x+3=5$ **b** $3x=23$ **c** $2x-3=11$ **d** $\frac{3x}{4}=10$

Solution

a

$$x+3=5$$

$\boxed{-3}$ $x+3-3=5-3$ (Subtract 3 from both sides.)

$$x=2$$

Check: Left-hand side (LHS) $= 2+3$
$= 5$
= right-hand side (RHS)

b

$$3x=23$$

$\boxed{\div 3}$ $\frac{3x}{3}=\frac{23}{3}$ (Divide both sides by 3.)

$$=7\tfrac{2}{3}$$

Check: LHS $= 3\times 7\tfrac{2}{3}$
$= 3\times\frac{23}{3}$
$= 23$
$=$ RHS

c

$$2x-3=11$$

$\boxed{+3}$ $2x-3+3=11+3$ (Add 3 to both sides.)

$$2x=14$$

$\boxed{\div 2}$ $x=7$ (Divide both sides by 2.)

Check: LHS $= 2\times 7-3$
$= 11$
$=$ RHS

d

$$\frac{3x}{4}=10$$

$\boxed{\times 4}$ $3x=40$ (Multiply both sides by 4.)

$\boxed{\div 3}$ $x=\frac{40}{3}$ (Divide both sides by 3.)

$$=13\tfrac{1}{3}$$

Check: LHS $= \frac{3\times 13\frac{1}{3}}{4}$
$= \frac{40}{4}$
$= 10$
$=$ RHS

ICE-EM Mathematics 8 3ed ISBN 978-1-108-40429-7

From now on we will not include the box to the left in the solutions.

Example 5

Solve each equations for x.

a $2x - 11 = -7$ **b** $6 - 4x = 22$ **c** $\frac{x}{5} + 7 = 22$

Solution

a $2x - 11 = -7$

$2x = -7 + 11$ (Add 11 to both sides.)

$2x = 4$

$x = 2$ (Divide both sides by 2.)

b $6 - 4x = 22$

$-4x = 16$ (Subtract 6 from both sides.)

$x = -4$ (Divide both sides by -4.)

c $\frac{x}{5} + 7 = 22$

$\frac{x}{5} = 15$ (Subtract 7 from both sides.)

$x = 15 \times 5$ (Multiply both sides by 5.)

$= 75$

Example 6

In each case below, write an equation and solve it.

a A number has 7 added to it and the result is 35.

b A number is divided by 11 and 6 is added to it. The result is 13.

Solution

a Let x be the number.

$x + 7 = 35$

$x = 28$ (Subtract 7 from both sides.)

The number is 28.

b Let x be the number.

$\frac{x}{11} + 6 = 13$

$\frac{x}{11} = 13 - 6$ (Subtract 6 from both sides.)

$\frac{x}{11} = 7$

$x = 77$ (Multiply both sides by 11.)

The number is 77.

ICE-EM Mathematics 8 3ed ISBN 978-1-108-40429-7

Exercise 6B

Example 4

1 Solve these equations. Check your solutions.

a $m+4=11$ b $2a=6$ c $\frac{x}{5}=10$

d $4x+5=16$ e $5x-3=12$ f $2m+8=12$

g $\frac{2x}{3}=12$ h $\frac{3x}{4}=15$ i $2n+12=20$

Example 5

2 Solve these equations. Check your solutions.

a $2x-6=-12$ b $2x+6=-15$ c $2x-3=-8$

d $6-4x=26$ e $5-2x=12$ f $6-5x=10$

g $\frac{x}{3}+4=22$ h $\frac{x}{4}-5=10$ i $\frac{x}{5}+7=15$

3 Solve each equation for m.

a $m+3=6$ b $m-11=7$ c $m+8=-10$

d $m-5=-11$ e $6-m=10$ f $-8+m=6$

g $m+6=3$ h $8-m=-6$ i $m-11=-5$

4 Solve each equation for n.

a $2n=-6$ b $-3n=9$ c $-5n=25$

d $-3n=16$ e $12n=100$ f $18n=46$

g $5n=17$ h $-6n=-50$ i $3n=17$

5 Solve each equation for x. Check your solutions.

a $\frac{x}{3}=12$ b $\frac{x}{2}=15$

c $\frac{x}{5}=-16$ d $\frac{x}{10}=-2$

e $\frac{x}{3}=-6$ f $\frac{x}{4}=-8$

6 Solve each equation for x. Check your solutions.

a $2x+1=7$ b $5x-1=11$ c $7x+3=17$

d $4x+2=18$ e $1-5x=21$ f $5-20x=100$

g $2-10x=44$ h $5x-11=30$ i $10x+23=100$

7 Solve these equations.

a $x-4=5$ b $3a=36$

c $3z-7=17$ d $11b+4=121$

e $\frac{5x}{4}=30$ f $\frac{x}{7}-2=8$

ICE-EM Mathematics 8 3ed ISBN 978-1-108-40429-7

Example 6

8 In each case, write an equation, and solve it.

a A number a has 5 added to it, and the result is 21.

b A number x is multiplied by 7, and the result is 35.

c A number z is multiplied by 5, and the result is 37.

d A number m is multiplied by 5, and then 3 is added. The result is 50.

e A number n is divided by 6, and the result is 10.

f A number p is divided by 3, and 5 is subtracted from it. The result is 23.

9 Solve each equation for x.

a $x + 5 = 6$ **b** $5x - 2 = 3$ **c** $2x + 7 = 4$ **d** $\frac{4x}{7} = 11$ **e** $\frac{5x}{11} = 7$

10 In each case, write an equation and solve it.

a A number x is subtracted from 20, and the result is 10.

b A number m is multiplied by 2, and the result is subtracted from 6. The final result is 20.

c A number n is divided by 8, and 6 is added to the result. The final result is 20.

d A number p is multiplied by 7, and then 10 is added. The result is 60.

e A number x is subtracted from 6, and the result is –10.

f A number y is multiplied by 7, and the result is subtracted from 15. The final result is –6.

g A number k is divided by 10, and 7 is subtracted from the result. The final result is –1.

6C Expanding brackets

The **distributive law for multiplication over addition** implies:

$$\begin{aligned} 3 \times (x + 4) &= 3 \times x + 3 \times 4 \\ &= 3x + 12 \end{aligned}$$

The **distributive law for multiplication over subtraction** implies:

$$\begin{aligned} 3 \times (x - 4) &= 3 \times x - 3 \times 4 \\ &= 3x - 12 \end{aligned}$$

We can describe these results in general by using algebra.

The distributive law

- The distributive law for multiplication over addition:

  $a(b + c) = ab + ac$

- The distributive law for multiplication over subtraction:

  $a(b - c) = ab - ac$

This process of rewriting an expression to remove brackets is usually referred to as **expanding brackets**.

ICE-EM Mathematics 8 3ed ISBN 978-1-108-40429-7 © Brown et al. 2017 Cambridge University Press

Example 7

Use the distributive law to expand the brackets.

a $5(x-4)$ **b** $4(3x+2)$ **c** $-2(4+x)$ **d** $-4(5-x)$

Solution

a $5(x-4) = 5 \times x - 5 \times 4$
$= 5x - 20$

b $4(3x+2) = 4 \times 3x + 4 \times 2$
$= 12x + 8$

c $-2(4+x) = -8 - 2x$

d $-4(5-x) = -20 + 4x$

Exercise 6C

Example 7

1 Expand brackets.

a $2(3a-1)$ **b** $5(6p+7)$ **c** $4(3-x)$ **d** $3(a-2)$

e $7(6-2x)$ **f** $3(7-x)$ **g** $-4(5-x)$ **h** $-6(7-2p)$

i $-3(5x-2)$ **j** $-4(6x-3)$ **k** $-3(2-x)$ **l** $-2(3x+1)$

2 First, substitute $x = 5$ in each expression and evaluate it. Next, expand the brackets and again substitute $x = 5$.

a $2(x+3)$ **b** $5(2x+4)$ **c** $2(x-2)$ **d** $6(x-3)$

3 Substitute $a = 2$ in each expression and evaluate.

a $3 + 2(a-2)$ **b** $6(a-2)$ **c** $3 + 2(5a-2)$

4 Evaluate each expression when $z = -2$.

a $5z - 2(z-4)$ **b** $2 - 3(2-4z)$ **c** $7z + 5(2z-4)$

6D Solving equations with brackets

We will now employ the methods introduced in the last section to solve equations that contain brackets.

Example 8

Solve these equations by first expanding the brackets.

a $2(3x-1) = 15$ **b** $3(1-2x) = -13$

ICE-EM Mathematics 8 3ed ISBN 978-1-108-40429-7

Solution

a $2(3x - 1) = 15$

$6x - 2 = 15$ (Expand the brackets.)

$6x = 17$ (Add 2 to both sides of the equation.)

$x = \frac{17}{6}$ (Divide both sides of the equation by 6.)

$= 2\frac{5}{6}$

b $3(1 - 2x) = -13$

$3 - 6x = -13$ (Expand the brackets.)

$-6x = -13 - 3$ (Subtract 3 from both sides of the equation.)

$-6x = -16$

$x = \frac{-16}{-6}$ (Divide both sides of the equation by −6.)

$= 2\frac{2}{3}$ (Simplify the fraction.)

Example 9

Eight is subtracted from a number, x, and the result is multiplied by 2. The final result is 10. Write an equation and solve it.

Solution

$(x - 8) \times 2 = 10$

$2(x - 8) = 10$

$2x - 16 = 10$ (Expand the brackets.)

$2x = 26$ (Add 16 to both sides.)

$x = 13$ (Divide both sides by 2.)

Exercise 6D

Example 8

1 Solve each equation for x.

a $2(x + 1) = 9$ **b** $5(x - 3) = 12$ **c** $2(2x + 4) = 14$

d $3(2x - 1) = 8$ **e** $5(3 + 2x) = 19$ **f** $5(5 - 3x) = 4$

g $6(x - 2) = 14$ **h** $4(2 - 3x) = 15$ **i** $5(3x - 2) = 6$

j $-5(2x + 2) = 6$ **k** $-3(4x - 5) = 10$ **l** $-3(2 - 3x) = 4$

Example 9

2 In each case, write an equation and solve it.

a Six is added to a number, x, and the result is multiplied by 3. The final result is −10.

b Six is subtracted from a number, m, and the result is multiplied by 3. The final result is 5.

ICE-EM Mathematics 8 3ed ISBN 978-1-108-40429-7 © Brown et al. 2017 Cambridge University Press

c Ten is added to a number, p, and the result is multiplied by 2. The final result is 4.

d Five is subtracted from a number, n, and the result is multiplied by 3. The final result is 14.

e Three is subtracted from a number, x, and the result is multiplied by 2. The final result is -10.

f Three is added to a number, x, and the result is multiplied by -2. The final result is -2.

g Six is subtracted from a number, x, and the result is multiplied by -4. The final result is -10.

6E Collecting like terms and solving equations

Like terms

If a boy has 3 pencil cases with the same number, x, of pencils in each, he has $3x$ pencils in total.

If he is given 2 more pencil cases with x pencils in each of the 5 cases, then he has $3x + 2x = 5x$ pencils in total. This is correct as the number of pencils in each of the five cases is x. $3x$ and $2x$ are said to be **like terms**.

If Janna has x packets of chocolates, each containing y chocolates, then she has $x \times y = xy$ chocolates. If David has twice as many chocolates as Janna, he has $2 \times xy = 2xy$ chocolates. Together they have $2xy + xy = 3xy$ chocolates.

The terms $2xy$ and xy are **like terms**. The pronumerals are the same and have the same exponents in each term.

The distributive law can be used to explain the addition and subtraction of like terms.

$$\begin{aligned} 2xy + xy &= 2 \times xy + 1 \times xy \\ &= (2+1)xy \\ &= 3xy \end{aligned}$$

Example 10

Which of these pairs consist of like terms?

a $3x, 5x$ **b** $4x^2, 8x$ **c** $4x^2y, 12x^2y$

d $ab, 2ba$ **e** $3mn^2, 5nm^2$

ICE-EM Mathematics 8 3ed ISBN 978-1-108-40429-7

Solution

a $5x$ and $3x$ are like terms.

b These are not like terms because the powers of x differ.

c These are like terms because each is a number times x^2 times y.

d These are like terms because $ab = ba$.

e These are unlike terms because the powers of n and m are different.

Adding and subtracting like terms

Like terms can be added and subtracted, as shown in the example below.

Example 11

Simplify each expression by collecting like terms.

a $4x + 3x - 2x$

b $11m - 2m + 8n - 6n$

c $4m + 2n + 5m + 6n$

d $6m + 2n - 3m - n$

Solution

a $4x + 3x - 2x = 5x$

b $11m - 2m + 8n - 6n = 9m + 2n$

c $$\begin{aligned} 4m + 2n + 5m + 6n &= 4m + 5m + 2n + 6n \\ &= 9m + 8n \end{aligned}$$

d $$\begin{aligned} 6m + 2n - 3m - n &= 6m - 3m + 2n - n \\ &= 3m + n \end{aligned}$$

Example 12

Simplify each expression by collecting like terms together.

a $2x^2 + 3x^2 + 5x^2$

b $3xy + 2xy$

c $4x^2 - 3x^2$

d $2x^2 + 3x + 4x$

e $4x^2y - 3x^2y + 3xy^2$

Solution

a $2x^2 + 3x^2 + 5x^2 = 10x^2$

b $3xy + 2xy = 5xy$

c $4x^2 - 3x^2 = 1x^2 = x^2$

d $2x^2 + 3x + 4x = 2x^2 + 7x$

e $4x^2y - 3x^2y + 3xy^2 = x^2y + 3xy^2$

ICE-EM Mathematics 8 3ed ISBN 978-1-108-40429-7 © Brown et al. 2017 Cambridge University Press

Example 13

Expand the brackets and collect like terms.

a $2(x-6)+5x$

b $3+3(x-1)$

c $5+x-2(3x-4)$

Solution

a $2(x-6)+5x = 2x-12+5x$
$= 7x-12$

b $3+3(x-1) = 3+3x-3$
$= 3x$

c $5+x-2(3x-4) = 5+x-6x+8$
$= 13-5x$

Example 14

Solve these equations by first collecting like terms.

a $2x+3=5x+1$

b $2(3x-2)-4(x+1)=2$

c $5(3x-2)=6x+3$

Solution

a $2x+3=5x+1$

$2x+3-2x=5x+1-2x$ (Subtract $2x$ from both sides.)

$3=3x+1$ (Collect like terms.)

$2=3x$ (Subtract 1 from both sides.)

$x=\frac{2}{3}$ (Divide both sides by 3.)

b $2(3x-2)-4(x+1)=2$

$6x-4-4x-4=2$ (Expand the brackets.)

$2x-8=2$ (Collect like terms.)

$2x=10$ (Add 8 to both sides.)

$x=5$ (Divide both sides by 2.)

c $5(3x-2)=6x+3$

$15x-10=6x+3$ (Expand the brackets.)

$9x=13$ (Collect like terms.)

$x=1\frac{4}{9}$ (Divide both sides by 9.)

ICE-EM Mathematics 8 3ed ISBN 978-1-108-40429-7 © Brown et al. 2017 Cambridge University Press

Exercise 6E

Example 10

1 Which of these pairs contain like terms and which do not?

a $11a$ and $4a$
b $6b$ and $-2b$
c $12m$ and $5m$
d $14p$ and $5p$
e $7p$ and $-3q$
f $-6a$ and $7b$
g $4mn$ and $7mn$
h $3pq$ and $-2pq$
i $6ab$ and $-7b$
j $4ab$ and $5a$
k $11a^2$ and $5a^2$
l $6x^2$ and $-7x^2$
m $4b^2$ and $-3b$
n $2y^2$ and $2y$
o $-5a^2b$ and $9a^2b$
p $6mn^2$ and $11mn^2$
q $6\ell^2s$ and $17\ell s^2$
r $12d^2e$ and $14de^2$
s $-4x^2y^2$ and $12x^2y^2$
t $-5a^2bc^2$ and $8a^2bc^2$

Example 11

2 Simplify these expressions; that is, collect like terms.

a $3x+4x$
b $5x-6x$
c $4x+7x-8x$
d $4x-7x+8x$
e $-11x+10x$
f $3x+13x-20x$
g $6x-11x+17x$
h $-12m+13m+50m$
i $5n+6n-20n$
j $67x-70x+100x$
k $4n-5n+10n$
l $-10p-30p-5p$
m $5x-10x-15x$
n $80p-100p+20p$
o $10m+3m+4n+7n$
p $10m-3m+7n-3n$
q $3m+2n+4m+6n$
r $3x+4y-2x+y$
s $5x+2y-3x-y$
t $5m-2n+6m-7n$
u $5q-2p+2q-5r$
v $5a-7b+2a-11b$

Example 12

3 Collect like terms.

a $45xy+32xy+16xy$
b $19xy+6xy-4xy$
c $8xy^2+9xy^2-y^2x$
d $4x+3x+3y+7y$
e $6v-11v+7z-14z$
f $6y-11x+10y+15x$
g $7x^3+6x^2+4y^3-x^2$
h $8x^2-12x^2+y^2+12y^2$
i $2x^2+x^2-5xy+7yx$
j $11x^2-20x^2-5x^2+y^2-10y^2$

Example 13

4 Expand the brackets in each expression, and collect like terms.

a $12+7(x+4)$
b $3(3+x)+2x$
c $5(x+2)+2(x+3)$
d $2(7+5x)+4(x+6)$
e $9(3-x)+7(x+4)$
f $3(2x+7)+2(x-5)$
g $10(x-4)-2(x+1)$
h $4(3+x)-3(2x+3)$
i $11(x-1)-5(x-3)$
j $7(2x+4)-3(2-4x)$
k $5x+3(2x+7)$
l $2x(3a-4)+6(2ax+4x)$

ICE-EM Mathematics 8 3ed ISBN 978-1-108-40429-7 © Brown et al. 2017 Cambridge University Press

Example 14

5 Solve each equation for x.

a $3x+2=x-4$

b $5x-4=2x+8$

c $6x+2=2x+8$

d $3x-4=x+6$

e $11x+2=x+10$

f $3x+1=2x-1$

6 Solve each equation for x.

a $3(x-2)+x+4=10$

b $2(x-5)+3x=10$

c $x+5+2(x-4)=9$

d $2x+6+x-1=25$

7 Solve these equations by first expanding the brackets.

a $4(x-2)=10$

b $2(x+3)=x-9$

c $5(x-3)-4(x+8)=2x$

d $3(x+8)+5-3x=2x$

e $13=6(x-4)+(5+3)x$

f $2.5(x+2^2)=6(3x-2)-4(x+1.5)-3$

6F Problem-solving using algebra

The algebra that has been introduced in the previous sections of this chapter can be used to solve problems that are difficult to solve otherwise. In many cases, algebra simplifies the problem into a form to which the methods for solving equations can be applied.

Example 15

Multiplying a number by 4 and adding 6 gives the same result as multiplying the number by 3 and subtracting 4. Find the number.

Solution

Let x be the number. The resulting equation is:

$$4x+6=3x-4$$

$$4x+6-3x=3x-4-3x \quad \text{(Subtract } 3x \text{ from both sides.)}$$

$$x+6=-4$$

$$x=-10 \quad \text{(Subtract 6 from both sides.)}$$

Hence, the number is -10.

Check: LHS $=4\times(-10)+6$

$=-40+6$

$=-34$

RHS $=3\times(-10)-4$

$=-34$

ICE-EM Mathematics 8 3ed ISBN 978-1-108-40429-7

Example 16

Five less than a number, multiplied by 3, is equal to the sum of 2 times the number and 12. Find the number.

Solution

Let x be the number. Then from the information given:

$$\begin{aligned} 3(x-5) &= 2x+12 && \\ 3x-15 &= 2x+12 && \text{(Expand the brackets.)} \\ 3x &= 2x+27 && \text{(Add 15 to both sides.)} \\ x &= 27 && \text{(Subtract } 2x \text{ from both sides.)} \end{aligned}$$

Hence, the number is 27.

Example 17

John is thinking of getting a mobile phone, but is having trouble deciding between two plans. One costs $30 a month, includes 10 free minutes, and charges 50 cents a minute after that. The other costs $10 a month and charges $1 a minute. After how many minutes do the two plans cost the same?

Solution

Let x be the number of minutes per month at which the cost of the two plans is the same. Then from the information given:

Cost of the first plan per month is $30 + 0.5(x - 10)$

Cost of the second plan per month is $10 + x$

$$\begin{aligned} 30 + 0.5(x-10) &= 10 + x && \\ 30 + 0.5x - 5 &= 10 + x && \text{(Expand the brackets.)} \\ 25 + 0.5x &= 10 + x && \text{(Collect like terms.)} \\ 15 + 0.5x &= x && \text{(Subtract 10 from both sides.)} \\ 15 &= 0.5x && \text{(Subtract } 0.5x \text{ from both sides.)} \\ 30 &= x && \text{(Multiply both sides by 2.)} \end{aligned}$$

The two plans cost the same amount when John has used exactly 30 minutes.

Example 18

The average of eight numbers is 56. A ninth number is added and the average is then 60. What is the ninth number?

ICE-EM Mathematics 8 3ed ISBN 978-1-108-40429-7 © Brown et al. 2017 Cambridge University Press

Solution

If the average of eight numbers is 56, then the sum of these numbers is 448.

Let n be the ninth number. Then $\frac{n+448}{9} = 60$.

Solving for n:

$$\frac{n+448}{9} = 60$$

$$n + 448 = 540 \quad \text{(Multiply both sides of the equation by 9.)}$$

$$n = 92 \quad \text{(Subtract 448 from both sides of the equation.)}$$

The ninth number is 92.

Exercise 6F

In each of the following problems introduce a pronumeral, write an equation and solve it.

1 In each case, write an equation in x and solve it.

a A number has 6 added to it, and the result is 24.

b A number is multiplied by 5, and the result is 35.

c A number is multiplied by 3, and the result is 37.

d A number is multiplied by 6, and then 4 is added. The result is 48.

e A number is divided by 6, and the result is 20.

f A number is divided by 7, and 5 is subtracted from it. The result is 33.

Example 15

2 Multiplying a number by 3 and adding 8 gives the same result as multiplying the number by 2 and subtracting 8. Find the number.

Example 16

3 Multiplying a number by 2 and subtracting 6 gives the same result as multiplying the number by 10 and adding 8. Find the number.

Example 17

4 Anthony goes to the fair with his family. They ride together on the Ferris Wheel. They buy one adult and three child tickets for $12. If the child tickets are half the cost of the adult tickets, how much does one child ticket cost?

5 I start with a number, multiply it by 5 and add 7, and I end up with 28. What was the number I started with?

6 The sum of two numbers is 42, and one of the numbers is five times the other. Find the two numbers.

7 The sum of two numbers is twice the difference, and one number is five more than the other. Find the two numbers.

8 Five times a number minus two times the number is equal to six times the same number minus 12. What is the number?

ICE-EM Mathematics 8 3ed ISBN 978-1-108-40429-7

Example 18

9 In class tests this year, Juan has so far scored 75, 69, 81 and 87. If he wants to get an average of 80 after his next test, what score does Juan need to get?

10 If Amy has an average score of 67 after four tests, and after doing a fifth test her average increases to 72, what score did she get on the fifth test?

11 The local movie theatre sells popcorn in small, medium and large containers. The small size costs $1 less than the medium size, and the large size costs twice as much as the medium size. A group of friends go to the movies and decide to buy one container of each size, and they pay $13. How much does the medium-size popcorn cost?

12 Licia bought her lunch from the school canteen for $3.00. She had a sausage roll, a caramel milkshake and an apple. She paid 60 cents more for the milkshake than for the fruit, and 30 cents more for the sausage roll than for the milkshake. Write an expression for how much she spent, and use it to find the cost of each item.

6G Multiplying and dividing algebraic fractions

The **any-order property of multiplication** was discussed in Chapter 1. In Chapter 3, index notation was used to simplify expressions. For example, $x \times x \times x = x^3$ and $y \times y = y^2$.

Here is another example using index notation.

$$\begin{aligned} 3x \times 2y \times 2xy &= 3 \times x \times 2 \times y \times 2 \times x \times y \\ &= 3 \times 2 \times 2 \times x \times x \times y \times y \\ &= 12x^2y^2 \end{aligned}$$

Example 19

Simplify each expression.

a $5 \times 2a$ **b** $3a \times 2a$ **c** $5xy \times 2xy$ **d** $6x^2y \times 3x$

Solution

a $5 \times 2a = 10a$

b $\begin{aligned} 3a \times 2a &= 3 \times 2 \times a \times a \\ &= 6a^2 \end{aligned}$

c $\begin{aligned} 5xy \times 2xy &= 5 \times 2 \times x \times x \times y \times y \\ &= 10x^2y^2 \end{aligned}$

d $\begin{aligned} 6x^2y \times 3x &= 6 \times 3 \times x \times x \times x \times y \\ &= 18x^3y \end{aligned}$

ICE-EM Mathematics 8 3ed ISBN 978-1-108-40429-7

In Section 2A we learned to simplify fractions by cancelling common factors. We can do the same dealing with algebraic fractions, provided that the pronumeral we cancel is not equal to zero.

Example 20

Simplify:

a $\dfrac{8x}{2}$ **b** $\dfrac{7xy}{x}$ **c** $\dfrac{56abc}{35c}$

Solution

a $\dfrac{8x}{2} = \dfrac{\cancel{8}^{4}x}{\cancel{2}^{1}}$
$= 4x$

b $\dfrac{7xy}{x} = \dfrac{7\cancel{x}^{1}y}{\cancel{x}^{1}}$
$= 7y$

c $\dfrac{56abc}{35c} = \dfrac{\cancel{56}^{8}ab\cancel{c}^{1}}{\cancel{35}^{5}\cancel{c}^{1}}$
$= \dfrac{8ab}{5}$

Example 21

Simplify:

a $\dfrac{x}{x}$ **b** $\dfrac{x^2}{x}$ **c** $\dfrac{x^3}{x^2}$ **d** $\dfrac{x^2}{x^3}$

Solution

a $\dfrac{x}{x} = \dfrac{\cancel{x}^{1}}{\cancel{x}^{1}}$
$= 1$

b $\dfrac{x^2}{x} = \dfrac{x \times x}{x}$
$= \dfrac{x \times \cancel{x}^{1}}{\cancel{x}^{1}}$
$= x$

c $\dfrac{x^3}{x^2} = \dfrac{x \times x \times x}{x \times x}$
$= \dfrac{x \times \cancel{x}^{1} \times \cancel{x}^{1}}{\cancel{x}^{1} \times \cancel{x}^{1}}$
$= x$

d $\dfrac{x^2}{x^3} = \dfrac{\cancel{x}^{1} \times \cancel{x}^{1}}{x \times \cancel{x}^{1} \times \cancel{x}^{1}}$
$= \dfrac{1}{x}$

Example 22

Simplify:

a $\dfrac{60p^2q}{12p}$ **b** $\dfrac{50x^2y^2}{20xa}$

Solution

a $\dfrac{60p^{\cancel{2}^{1}}q}{12\cancel{p}} = 5pq$

b $\dfrac{50x^{\cancel{2}^{1}}y^2}{20\cancel{x}a} = \dfrac{5xy^2}{2a}$

ICE-EM Mathematics 8 3ed ISBN 978-1-108-40429-7

Exercise 6G

Example 19

1 Simplify, using index notation:

a $2a \times 6b$ **b** $3x \times 4x$ **c** $5xy \times 10$ **d** $7ab \times 11$

e $5a \times 2a$ **f** $6c \times 11c$ **g** $4m \times 6n$ **h** $7mn \times 11mn$

i $4n^2 \times 3$ **j** $7m^2 \times 6m$ **k** $11m^2 \times 2m^2$ **l** $7a^2b \times 5$

2 Rewrite each expression without brackets.

a $(5n)^2$ **b** $(4z)^2$ **c** $(16z)^2$ **d** $(3c^2)^2$

3 Simplify:

a $2x \times 4x$ **b** $7x \times 3x$ **c** $3x \times 2x$

d $2xy \times 3x$ **e** $4xy \times 2xy$ **f** $6xy \times 2x^2y$

Example 20

4 Simplify:

a $\frac{6x}{3}$ **b** $\frac{8xy}{y}$ **c** $\frac{24xyz}{xz}$ **d** $\frac{mnp}{5m}$

e $\frac{9zx}{3z}$ **f** $\frac{18xy}{y}$ **g** $\frac{18xyz}{yz}$ **h** $\frac{72abc}{16c}$

Example 21

5 Simplify:

a $\frac{4x}{xy}$ **b** $\frac{a^2}{a}$ **c** $\frac{a^3}{a^2}$ **d** $\frac{4ab}{ab}$ **e** $\frac{3a^2b}{a}$ **f** $\frac{a^3}{a}$

g $\frac{a}{a^2}$ **h** $\frac{a^2}{a^3}$ **i** $\frac{a}{a}$ **j** $\frac{a^2b}{ab}$ **k** $\frac{a^2b^2}{ab}$ **l** $\frac{ab^2}{ab}$

Example 22

6 Simplify:

a $\frac{48p^2q}{36p}$ **b** $\frac{100a^2b^2}{5ba}$ **c** $\frac{200a^3b^2}{10a}$

7 A rectangle has length x cm and width $\frac{x}{3}$ cm.

a What is the area of the rectangle? **b** Find the area if $x = 9$.

8 A triangle has base $\frac{2x}{3}$ cm and height $\frac{x}{4}$ cm.

a What is the area of the triangle in terms of x? **b** Find the area if $x = 24$.

9 A rectangle has length x cm and area $\frac{6x^2}{5}$ cm^2. Find its width.

10 A rectangle has length $2x$ cm and width $\frac{5x}{3}$ cm.

a What is the area of the rectangle? **b** Find the area if $x = 27$.

11 A square that has side length $8x$ cm is divided into y rectangles of equal area. What is the area of each of these rectangles?

12 A floor of a room is rectangular in shape, has length $10x$ metres and width $7x$ metres. The floor is to be tiled with rectangular tiles each of length $\frac{x}{3}$ metres and width $\frac{x}{4}$ metres. How many tiles are needed to tile this floor area?

ICE-EM Mathematics 8 3ed ISBN 978-1-108-40429-7

Review exercise

1 Evaluate each expression for $a = 4$, $b = -2$, $c = -3$ and $d = 5$.

a $d - 3$ **b** $c + 5$ **c** $3d - 8$ **d** $5b + 6$

e $10 - 2b$ **f** $7 - 3a$ **g** $3(a + 2)$ **h** $7(1 - c)$

i $\frac{a}{b}$ **j** $ad - bc$ **k** b^2 **l** $\frac{ac + 7}{5 - 2b}$

2 Substitute $x = -3$ in each expression and evaluate it.

a $2x + 3$ **b** $1 - 2x$ **c** $4x + 3$ **d** $1 + x$

e x^2 **f** x^3 **g** $2 - x^2$ **h** $4 + x^2$

i $-1 + x^3$ **j** $x^3 - 8$ **k** $(2 + x)^3$ **l** $(2 - x)^3$

3 Substitute $m = 2$, $n = -3$ and $p = 11$ in each expression and evaluate it.

a $m^2 + p^2$ **b** $m - n$ **c** $p - n$ **d** $n - p$

e mnp **f** $m^2n^2p^2$ **g** $m - 2p$ **h** $p^2 + m^2 - n$

4 Expand the brackets in each expression.

a $3(a + 4)$ **b** $6(3 - x)$

c $x(2 + y)$ **d** $-2(11 - x)$

e $14(x - 3) - 10(x - 3)$ **f** $3(x + 10) - 4(5 - x)$

g $2a(3 - x) + 3x(a + 7)$ **h** $-5(x + 6) + 3(2x - 4)$

i $-2(x - 3) + 4(3 - x)$ **j** $-7(2x - 4) + 5(4 - x)$

5 Solve each equation. Check your solutions.

a $2x - 3 = -11$ **b** $x + 6 = -12$ **c** $4x - 6 = -10$

d $3 - a = 6$ **e** $2m - 6 = -8$ **f** $10 - 2m = -4$

6 Solve each equation. Check each answer.

a $x + 3x - 5 = 11$ **b** $3(a + 4) = 9$ **c** $2x + 1 = 3(x - 4)$

d $x + 3 = 4x - 1$ **e** $2x + 6 = 3 - x$ **f** $x + 2 = 4(x - 5)$

7 Expand the brackets where necessary and simplify each of these expressions.

a $(x^2y^2) \times (2xy)^3$ **b** $(-2xy)^2 \times x^3y^2$ **c** $\frac{x^2y^2}{xy}$

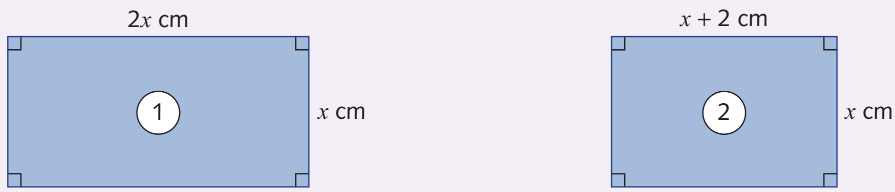
d $\frac{m^3n^2}{m^2n}$ **e** $(-4x^2y)^2 \times xy$ **f** $\frac{20a^2b^2}{5a}$

ICE-EM Mathematics 8 3ed ISBN 978-1-108-40429-7

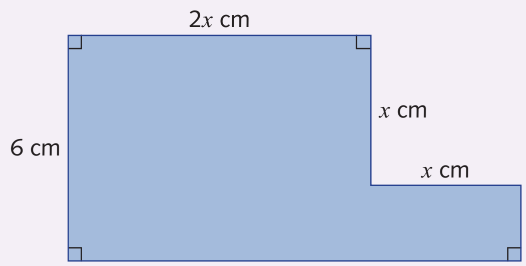
8 In a theme park, the cost of an adult ticket is 1.5 times that of a child ticket. If a family of 2 adults and 3 children paid $48, how much does an adult ticket cost?

9 In a strict monastery there are 100 monks, either senior or junior. If a senior monk is allowed to have 3 slices of bread for each meal, and a junior monk is only allowed $\frac{1}{3}$ of a slice, and altogether they consume 60 slices of bread during lunch, how many senior monks are there?

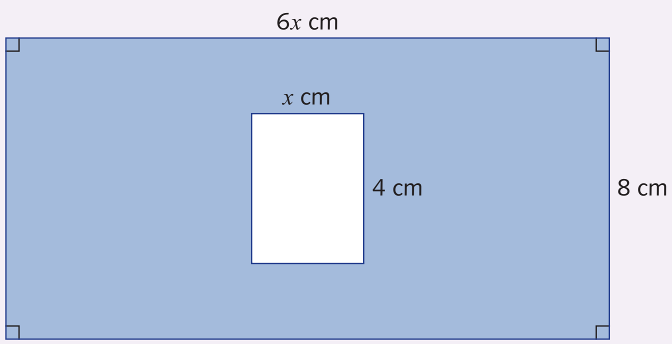
10 The perimeter of rectangle ① is numerically equal to the perimeter of rectangle ②. Find the value of x.



11 The perimeter of the shaded region is 36 cm.
Find the value of x.



12 The area of the shaded region is 121 cm^2.
Find the value of x.



ICE-EM Mathematics 8 3ed ISBN 978-1-108-40429-7

Challenge exercise

1 **a** Carry out the following operations on the number 5.

- Add 3.
- Multiply the result by 9.
- Subtract 2.
- Add the number you started with (in this case 5).
- Subtract 25.
- Divide by 10.

b Repeat the procedure with another number.

c Let the number be x and apply same the operations. State your result. What is the final number?

2 If a painted wooden cube is cut in half along each edge (making 8 smaller cubes), then each of these cubes is painted on three faces. If a painted wooden cube is cut into thirds along each edge (making 27 smaller cubes), 8 of these cubes are painted on three faces, 12 of these cubes are painted on two faces, 6 are painted on one face only, and one is not painted at all. If a painted wooden cube is cut into four parts along each edge, how many smaller cubes are there, and how many are painted on three, two, one and no faces, respectively? Find an expression for how many cubes are painted on three, two, one and no faces if the large cube is cut into n parts along each edge.

3 Paul, Kate and Sarah's mother left a bowl of jelly beans on their kitchen counter. Paul took one-third of the jelly beans, but then threw back 4 because he didn't like the black ones. Sarah ate 6 jelly beans and then took one-quarter of what remained. Kate took one-third of what remained, and then picked out and ate the last 4 black ones because she really liked them. If after all this there were 6 jelly beans left, how many were there to begin with? If there were 12 jelly beans left, how many were there to begin with? If there were x jelly beans left, how many were there to begin with?

4 A primary school is planning to hold a fete, but needs help in figuring out how much to charge for entry. They intend to have an 'all you can eat' sausage sizzle, and they think each adult will eat about \$3 worth of sausages, and each child \$2 worth. The cost of cleaning after the event will be \$300. They want an adult ticket to cost twice as much as a child ticket, and they expect that, on average, 1.7 parents and 2.3 children will come from each family. How much will they need to charge for adult and child tickets if they are expecting 120 families to attend and they don't want to make a profit or a loss? (Round to the nearest cent.)

ICE-EM Mathematics 8 3ed ISBN 978-1-108-40429-7

5 Andrew and a group of his friends have decided they want to race go-karts on the weekend, and Andrew wants to work out which of the two local go-kart places will be cheaper. Kart-Kingdom charges $15 per person to do safety training, and then 50 cents per lap that you do. Go-Mobile charges 80 cents per lap, and the safety training is free. After how many laps does Go-Mobile become more expensive than Kart-Kingdom?

6 Find all the two-digit integers such that when you reverse the digits and add that number to the original number, you get 55.

7 A two-digit number has the property that the sum of the number and its digits is 63. What is the original number?

8 Is there a two-digit integer with the property that 11 times the sum of its digits is equal to itself?

ICE-EM Mathematics 8 3ed ISBN 978-1-108-40429-7