CHAPTER

7

Number and Algebra

Percentages

We encounter percentages often, and they can be very useful in many aspects of our lives. For example, we see discounts offered in shops everywhere – as smart shoppers, we definitely need to know how to do percentage calculations if we want the best deal.

The word 'percentage' comes from the Latin *per centum*, meaning 'per hundred'. A percentage is another way of writing a fraction with a denominator of 100. The symbol for percentage is %. For example:

$$8\% = \frac{8}{100}, \quad 25\% = \frac{25}{100}, \quad 99\% = \frac{99}{100}, \quad 150\% = \frac{150}{100}$$

In this chapter, you will learn how to perform several different types of useful calculations involving percentages.

Also, we review the previous study of percentages and consider more applications of percentages, such as discounts, profits and loss, interest and GST.

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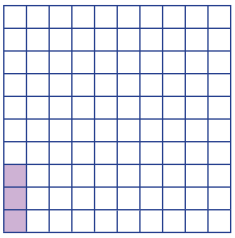
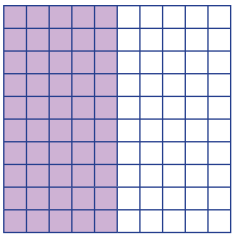
7A Review of percentages

To understand the basic idea of percentages, we need only look at the meaning of 'out of a hundred'.

A square that has been cut into 100 smaller squares can be used to model percentages. If we colour in three of them, we say that 'three out of a hundred' or 'three per cent' are coloured in.



If we colour 50 of them, we say that '50 out of a hundred' or '50 per cent' are coloured in. Half of the squares are coloured in.



Converting percentages to fractions

A percentage is a fraction with a denominator of 100. To convert a percentage to its fraction equivalent, write it as a fraction with a denominator of 100 and then simplify. For example:

$$65\% = \frac{65}{100} = \frac{13}{20} \qquad 150\% = \frac{150}{100} = 1\tfrac{1}{2}$$

For percentages such as $12\frac{1}{2}\%$ or $33\frac{1}{3}\%$, an extra step is required to convert the numerators to whole numbers:

$$12\tfrac{1}{2}\% = \frac{12\frac{1}{2}}{100} = \frac{25}{200} = \frac{1}{8}$$

(Multiply the numerator and denominator of the fraction by 2.)

$$33\tfrac{1}{3}\% = \frac{33\frac{1}{3}}{100} = \frac{100}{300} = \frac{1}{3}$$

(Multiply the numerator and denominator of the fraction by 3.)

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Converting percentages to decimals

A percentage can be converted to a decimal by writing it as a fraction with a denominator of 100 and then converting to a decimal.

$$65\% = \frac{65}{100} = 0.65 \qquad 150\% = \frac{150}{100} = 1.5$$

For a percentage such as 37.5%, divide 37.5 by 100 in one step:

$$37.5\% = \frac{37.5}{100} = 0.375$$

Converting fractions and decimals to percentages

We know from the meaning of the word 'percentage' that fractions with a denominator of 100 convert directly to percentages. For example:

$$\frac{2}{100} = 2\% \qquad \frac{37}{100} = 37\%$$

$$\frac{175}{100} = 175\% \qquad \frac{100}{100} = 1 = 100\%$$

Equivalent fractions can be used for some fractions whose denominators are not 100 to find the corresponding percentage:

$$\frac{2}{10} = \frac{20}{100} = 20\% \qquad \frac{3}{5} = \frac{60}{100} = 60\%$$

$$\frac{3}{20} = \frac{15}{100} = 15\% \qquad \frac{3}{2} = \frac{150}{100} = 150\%$$

More generally, to convert a fraction to a percentage, multiply by 100%, which is the same as multiplying by 1.

$$\frac{2}{5} = \frac{2}{5} \times \frac{100}{1}\% = 40\% \qquad \frac{2}{3} = \frac{2}{3} \times \frac{100}{1}\% = 66\tfrac{2}{3}\%$$

To convert a decimal to a percentage, the procedure is the same – multiply by 100%, which is the same as multiplying by 1.

$$0.6 = 0.6 \times 100\% = 60\% \qquad 3.2 = 3.2 \times 100\% = 320\%$$

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Here are some commonly used percentages and their fraction equivalents. It can be very useful to know these. Try to learn them in both directions, but you can quickly work them out if you forget them.

For example, $\frac{1}{5} = 20\%$ and 37.5% is $\frac{3}{8}$.

Fraction	Percentage
$\frac{1}{2}$	50%
$\frac{1}{4}$	25%
$\frac{1}{3}$	$33\frac{1}{3}\%$
$\frac{3}{4}$	75%
$\frac{2}{3}$	$66\frac{2}{3}\%$
$\frac{1}{5}$	20%
$\frac{2}{5}$	40%

Fraction	Percentage
$\frac{3}{5}$	60%
$\frac{4}{5}$	80%
$\frac{1}{8}$	$12\frac{1}{2}\%$
$\frac{3}{8}$	$37\frac{1}{2}\%$
$\frac{5}{8}$	$62\frac{1}{2}\%$
$\frac{7}{8}$	$87\frac{1}{2}\%$
$\frac{1}{1}$	100%

Percentage of a quantity

We often talk about a percentage of a particular quantity. For example:

- 20% of all suitcases are brown.
- 52% of voters in a recent election voted for a particular political party.
- 80% of the students at a particular school access the internet at least once a day.

In such cases, we may want to know the actual number of objects or people involved. The following example shows how to do calculations of this type.

Example 1

32% of people interviewed watched the Australian Open tennis final. Calculate how many of the people interviewed watched the final if:

a 300 people were interviewed

b 250 people were interviewed

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Solution

a If 300 people were interviewed, then 32 of every 100 people interviewed watched the final. That is, $32 \times 3 = 96$ people watched it. This can be set out as follows:

32% of 300 people interviewed watched the final.

$$\begin{aligned}\text{Number of people who watched} &= 32\% \text{ of } 300 \\ &= \frac{32}{100} \times \frac{300}{1} \\ &= 96\end{aligned}$$

b 32% of 250 people interviewed watched the final.

$$\begin{aligned}\text{Number of people who watched} &= 32\% \text{ of } 250 \\ &= \frac{32}{100} \times \frac{250}{1} \\ &= \frac{8}{25} \times \frac{250}{1} \\ &= 80\end{aligned}$$

Expressing one quantity as a percentage of another

Sometimes we want to describe one quantity as a percentage of another.

Example 2

There are 50 people in a swimming club and 35 of them go to squad training. Calculate the percentage of the club members who go to squad training.

Solution

35 out of the 50 or $\frac{35}{50}$ go to squad training.

$$\begin{aligned}\text{Percentage going to squad training} &= \frac{35}{50} \times 100\% \\ &= 70\%\end{aligned}$$

Thus 70% of the swimming club members go to squad training.

Comparing fractions using percentages

One useful way to compare your results in a French test, in which you scored $\frac{16}{20}$, and a German test, in which you scored $\frac{42}{50}$, is to change both scores to percentages.

In this case, $\frac{16}{20} = 80\%$ and $\frac{42}{50} = 84\%$, so you did better in the German test.

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Percentages

- A **percentage** is a fraction with denominator of 100. For example:
  $5\% = \frac{5}{100}$
- To convert a percentage to a fraction, write the percentage as a fraction with a denominator of 100 and then simplify.
- To convert a percentage to a decimal, write the percentage as a fraction with a denominator of 100 and then convert to a decimal.
- To convert a fraction or a decimal to a percentage, multiply by 100%.
- To express one quantity as a percentage of another, write the first quantity as a fraction of the second, and then convert to a percentage by multiplying by 100%.
- A useful way to compare two fractions is to convert them both to percentages.

Example 3

a Convert 45% to a fraction.

b Convert 135% to a fraction.

c Convert 45% to a decimal.

d Convert 135% to a decimal.

Solution

a $45\% = \frac{45}{100}$
$= \frac{9}{20}$

b $135\% = \frac{135}{100}$
$= 1\frac{7}{20}$

c $45\% = \frac{45}{100}$
$= 0.45$

d $135\% = \frac{135}{100}$
$= 1.35$

Example 4

Express each fraction as a percentage.

a $\frac{27}{100}$

b $\frac{127}{1000}$

Solution

a $\frac{27}{100} = 27\%$

b $\frac{127}{1000} = \frac{127}{1000} \times \frac{100}{1}\%$
$= \frac{127}{10}\%$
$= 12.7\%$

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Example 5

Express each fraction as a percentage.

a $\frac{4}{5}$ **b** $\frac{37}{50}$ **c** $2\frac{4}{5}$

Solution

a $\frac{4}{5} = \frac{4}{5} \times \frac{100}{1}\%$

$= \frac{4}{\cancel{5}^{1}} \times \frac{\cancel{100}^{20}}{1}\%$

$= 80\%$

b $\frac{37}{50} = \frac{37}{50} \times \frac{100}{1}\%$

$= \frac{37}{\cancel{50}^{1}} \times \frac{\cancel{100}^{2}}{1}\%$

$= 74\%$

c $2\frac{4}{5} = \frac{14}{5} \times \frac{100}{1}\%$

$= \frac{14}{\cancel{5}^{1}} \times \frac{\cancel{100}^{20}}{1}\%$

$= 280\%$

Example 6

Write $7\frac{1}{4}\%$ as a fraction.

Solution

$7\frac{1}{4}\% = \frac{7\frac{1}{4} \times 4}{100 \times 4}$

$= \frac{29}{400}$

Example 7

a What is 20% of 415?

b What is 7% of 197?

Solution

a 20% of $415 = \frac{20}{100} \times \frac{415}{1}$

$= \frac{1}{5} \times \frac{415}{1}$

$= 83$

b 7% of $197 = \frac{7}{100} \times \frac{197}{1}$

$= \frac{1379}{100}$

$= 13.79$

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Example 8

There are 200 Year 8 students in a school. Of these, 112 are girls. What percentage of the students are girls?

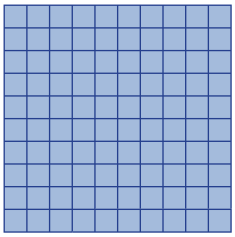
Solution

$$\text{Percentage of girls in the school} = \frac{112}{200} \times 100\%$$
$$= \frac{112}{2}\%$$
$$= 56\%$$

Exercise 7A

1 What percentage of each square is shaded?

a



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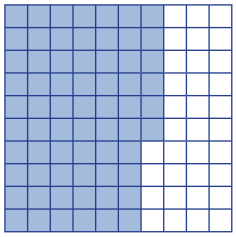

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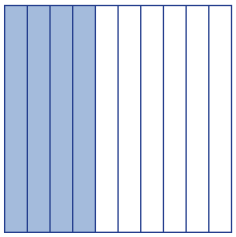

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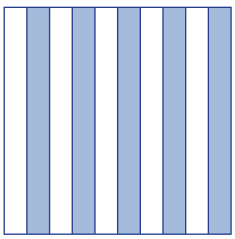
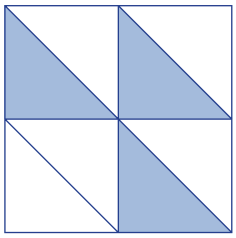
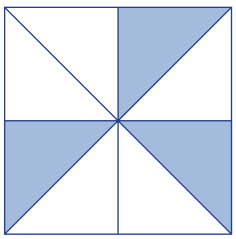


e



f



Example 3a, b

2 Convert each percentage to a fraction.

a 10% **b** 25% **c** 85%

d 50% **e** 40% **f** 20%

g 125% **h** 630% **i** 205%

j 138% **k** 496% **l** 43125%

Example 3c, d

3 Convert each percentage to a decimal.

a 55% **b** 37% **c** 19%

d 80% **e** 30% **f** 10%

g 250% **h** 148% **i** 3912%

j 3% **k** 7% **l** 1%

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Example 4

4 Express each fraction as a percentage.

a $\frac{23}{100}$ **b** $\frac{99}{100}$ **c** $\frac{11}{100}$ **d** $\frac{150}{100}$ **e** $\frac{230}{100}$

f $\frac{999}{100}$ **g** $\frac{42}{10}$ **h** $\frac{67}{10}$ **i** $\frac{14}{10}$ **j** $\frac{9}{10}$

k $\frac{5}{10}$ **l** $\frac{1}{10}$ **m** $\frac{670}{1000}$ **n** $\frac{810}{1000}$ **o** $\frac{330}{1000}$

p $\frac{999}{1000}$ **q** $\frac{123}{1000}$ **r** $\frac{702}{1000}$ **s** $\frac{9999}{1000}$ **t** $\frac{1234}{1000}$

Example 5

5 Express each fraction as a percentage.

a $\frac{2}{4}$ **b** $\frac{49}{50}$ **c** $\frac{4}{25}$ **d** $\frac{1}{20}$ **e** $\frac{3}{5}$

f $\frac{19}{25}$ **g** $1\frac{1}{4}$ **h** $\frac{1}{40}$ **i** $3\frac{17}{25}$ **j** $\frac{6}{5}$

k $\frac{10}{4}$ **l** $\frac{25}{20}$ **m** $\frac{3}{40}$ **n** $2\frac{2}{5}$ **o** $\frac{17}{40}$

6 Write each decimal as a percentage.

a 0.65 **b** 0.23 **c** 0.75 **d** 0.5 **e** 0.3

f 0.6 **g** 1.5 **h** 1.1 **i** 1.8 **j** 1

k 0.07 **l** 0.02 **m** 1.05 **n** 4.09 **o** 7.03

Example 6

7 Write each percentage as a fraction.

a $2\frac{1}{2}\%$ **b** $7\frac{1}{2}\%$ **c** $8\frac{1}{4}\%$ **d** $37\frac{1}{2}\%$

e $12\frac{1}{2}\%$ **f** $67\frac{1}{2}\%$ **g** $87\frac{1}{2}\%$ **h** $66\frac{2}{3}\%$

8 Complete the table below, showing the decimal, fraction and percentage equivalents in each row.

Decimal	Fraction	Percentage
0.5		50%
	$\frac{1}{4}$	
		75%
0.4		
		100%
0.367		
	$\frac{29}{100}$	
0.403		
	$\frac{3}{8}$	
	$\frac{5}{4}$	
2.75		
		200%

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9 Express each quantity as a percentage of the total.

a 30 cents out of a dollar

b 250 m out of 1 km

c 160 women in a train carrying 200 people

d 16 teeth filled out of 24

e \$10 discount on a \$200 jumper

f 18 out of 45 people in a restaurant ordering pizza

10 Calculate these amounts.

a 50% of 468

b 25% of 144

c $12\frac{1}{2}\%$ of 1360

d $33\frac{1}{3}\%$ of 8730

e 20% of 500

f 40% of 455

g $37\frac{1}{2}\%$ of 192

h $87\frac{1}{2}\%$ of 960

i $66\frac{2}{3}\%$ of 525

Example 7

11 Calculate:

a 20% of 100

b 20% of 200

c 20% of 1000

d 50% of 300

e 5% of 30

f 15% of 300

g 63% of 100

h 78% of 500

i 57% of 30

j 200% of 100

k 200% of 118

l 200% of 3.5

12 **a** Find 10% of 340.

b Find 30% of 340.

c Find 70% of 340.

d Find 5% of 340.

e Find $2\frac{1}{2}\%$ of 340.

f Find $7\frac{1}{2}\%$ of 340.

13 **a** Find 25% of 320.

b Find 75% of 320.

c Find $12\frac{1}{2}\%$ of 320.

14 **a** Find 10% of 456.

b Find 30% of 456.

c Find 10% of 912.

d Find 5% of 912.

15 Use fractions to calculate:

a 20% of 48 m

b 75% of 80 lollies

c 30% of 270 children

16 Use decimals to find:

a 25% of 28 songs

b 40% of 280 stories

c 15% of 160 students

Example 8

17 Express \$1.80 as a percentage of \$5.00.

18 Express \$24.40 as a percentage of \$30.50.

19 Mike bought 72 stamps. He bought 27 more stamps than Carla. Express the number of stamps Carla bought as a percentage of Mike's.

7B Changing fractions to percentages and percentages to fractions (revisited)

In this section, we look at some harder examples of changing percentages to fractions and fractions to percentages. The emphasis here is on converting a fraction with a denominator that does not divide into a power of 10 to a percentage.

Example 9

Write $\frac{2}{7}$ as a percentage.

Solution

$$\begin{aligned}\frac{2}{7} &= \frac{2}{7} \times 100\% \\ &= \frac{200}{7}\% \\ &= 28\tfrac{4}{7}\%\end{aligned}$$

Changing percentages to fractions

We now look at how to change a percentage to a fraction. For example:

$$\begin{aligned}45\tfrac{1}{2}\% &= \frac{45\frac{1}{2}}{100} \\ &= \frac{45\frac{1}{2} \times 2}{100 \times 2} \quad \text{(Multiply numerator and denominator by 2.)} \\ &= \frac{91}{200}\end{aligned}$$

Example 10

Write each percentage as a fraction.

a $23\frac{1}{3}\%$ **b** $32\frac{4}{5}\%$ **c** $5\frac{1}{7}\%$

Solution

a
$$\begin{aligned}23\tfrac{1}{3}\% &= \frac{23\frac{1}{3}}{100} \\ &= \frac{23\frac{1}{3} \times 3}{100 \times 3} \\ &= \frac{70}{300} \\ &= \frac{7}{30}\end{aligned}$$

b
$$\begin{aligned}32\tfrac{4}{5}\% &= \frac{32\frac{4}{5}}{100} \\ &= \frac{32\frac{4}{5} \times 5}{100 \times 5} \\ &= \frac{164}{500} \\ &= \frac{41}{125}\end{aligned}$$

c
$$\begin{aligned}5\tfrac{1}{7}\% &= \frac{5\frac{1}{7}}{100} \\ &= \frac{36}{700} \\ &= \frac{9}{175}\end{aligned}$$

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Expressing one quantity as a percentage of another

We have already looked at how to express one quantity as a percentage of another in the previous section. Here are some examples in which the resulting percentages involve fractions.

Example 11

Express 55 as a percentage of 120.

Solution

$$\begin{aligned}\frac{55}{120} &= \frac{55}{120} \times \frac{100}{1}\% \\ &= \frac{55}{6} \times \frac{5}{1}\% \\ &= \frac{275}{6}\% \\ &= 45\tfrac{5}{6}\%\end{aligned}$$

Example 12

Two bakeries, Brownie and Best Bake, both bake bread. On a particular day, 27 out of 40 loaves at the Brownie Bakery were baked the previous day. At the Best Bake Bakery, 57 out of 90 loaves were baked the previous day. Which bakery was selling the greater percentage of loaves baked the previous day?

Solution

The percentage of loaves baked the previous day at the Brownie Bakery is:

$$\begin{aligned}\frac{27}{40} \times 100\% &= \frac{2700}{40}\% \\ &= \frac{135}{2}\% \\ &= 67\tfrac{1}{2}\%\end{aligned}$$

The percentage of loaves baked the previous day at the Best Bake Bakery is:

$$\begin{aligned}\frac{57}{90} \times 100\% &= \frac{570}{9}\% \\ &= 63\tfrac{1}{3}\%\end{aligned}$$

Hence, the Brownie Bakery had the greater percentage of loaves baked a day earlier.

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Example 13

Write 0.32 out of 4.8 as a percentage.

Solution

$$\text{First } \frac{0.32}{4.8} = \frac{32}{480}$$
$$= \frac{32}{480} \times 100\%$$
$$= \frac{2}{30} \times 100\%$$
$$= \frac{2}{3} \times 10\%$$
$$= 6\tfrac{2}{3}\%$$

Exercise 7B

Example 9

1 Express each fraction as a percentage. Your answers will contain fractions.

a $\frac{2}{3}$ **b** $\frac{1}{6}$ **c** $\frac{5}{6}$

d $\frac{7}{9}$ **e** $\frac{11}{12}$ **f** $\frac{5}{7}$

Example 10

2 Express each percentage as a fraction.

a $12\frac{1}{5}\%$ **b** $6\frac{3}{4}\%$ **c** $5\frac{1}{4}\%$ **d** $15\frac{1}{6}\%$

e $6\frac{2}{3}\%$ **f** $5\frac{1}{12}\%$ **g** $12\frac{1}{6}\%$ **h** $21\frac{3}{4}\%$

i $42\frac{2}{7}\%$ **j** $6\frac{2}{7}\%$ **k** $44\frac{2}{3}\%$ **l** $7\frac{5}{12}\%$

3 Express each percentage as a fraction in simplest form.

a 200% **b** 1.5% **c** 124% **d** 210% **e** 0.5% **f** $125\frac{1}{2}\%$

4 Express each percentage first as a decimal and then as a fraction in simplest form.

a 12.25% **b** 6.75% **c** 5.75% **d** 8.06%

e 12.05% **f** 227% **g** 11 790% **h** 5711%

Example 11

5 Express the first number in each pair as a percentage of the second.

a 23, 69 **b** 35, 125 **c** 45, 135

d 456, 2000 **e** 34, 134 **f** 56, 400

6 In his vocabulary tests last term, Mick scored 21 out of 25, 13 out of 15, and 17 out of 20. By expressing his scores as percentages, rank Mick's efforts on the tests from best to worst.

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Example 12

7 The total population of a state is 2 500 000. The population of the capital city is 1 500 000.

a What percentage of the population of the state lives in the capital city?

b What percentage of the population of the state lives outside the capital city?

8 The number of people who enter Luna Park on a particular Sunday is 3200. Of these, 1250 purchase a ride on the rollercoaster. What percentage of the 3200 people purchase a ride on the rollercoaster?

9 In medieval Europe, the Black Death plague had an extremely high fatality rate. Calculate the percentage of those who died from the disease if the number of survivors in a village of 300 people infected with the disease was 15.

10 A drug for treating cholera is 98.5% effective.

a If 1200 infected people are treated with the drug, how many would you expect to recover?

b A different drug was administered, and 78 out of 80 people recovered. Using percentages, work out which drug is more effective.

11 **a** How much larger is $\frac{1}{3}$ than 33%, as a percentage?

b How much larger is $\frac{1}{3}$ than 0.3, as a percentage?

12 A survey was conducted at three different schools to find how many students travel to school by bus.

- School A reported that $\frac{2}{7}$ of its students are bus travellers.
- School B reported that 110 of its 400 students are bus travellers.
- School C reported that 28.9% of its students travel by bus.

Rank the three schools in terms of the percentage of students who travel by bus.

13 The bread sales on Saturday in a supermarket were:

- white bread, 135 loaves
- multigrain, 117 loaves
- wholemeal, 84 loaves
- raisin bread, 24 loaves

Express the number of loaves of each type sold as a percentage of total sales.

14 Express each of these situations as a percentage.

a In a population of 24 million people, 0.06 million work in the automobile industry.

b In a chess game, 5 out of 32 pieces were captured and removed from the board.

15 The average percentage, by weight, of body fat is 17% for men and 23% for women. Calculate the amount of fat, in kilograms, for:

a a woman weighing 76 kg

b a man weighing 72 kg

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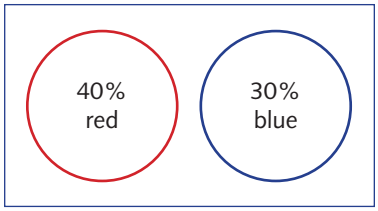
7C Sums, differences and products of percentages

Sums and differences of percentages

In some situations, but not all, percentages can be added and subtracted. There are two conditions that must be satisfied before addition or subtraction of percentages can be undertaken.

- They are percentages of the same whole.
- There is no overlap among the groups being considered.

For example, 200 people are asked their favourite colour (only one colour is allowed). 40% reply blue and 30% reply red, so $40\% + 30\% = 70\%$ have red or blue as their favourite colour.



Also, $40\% - 30\% = 10\%$ more of the people surveyed have blue rather than red as their favourite colour. In this situation both percentages are percentages of the same 200 people and there is no overlap between the groups.

Example 14

30% of the students in a school of 1200 have blond hair and 25% have black hair. What percentage of the students have either blond or black hair?

Solution

The percentage is $30\% + 25\% = 55\%$.

We could also solve the problem by working out the actual numbers involved.

30% of 1200 = 360

25% of 1200 = 300

Total = 660

$$\text{Percentage} = \frac{660}{1200} \times 100\%$$
$$= \frac{660}{12}\%$$
$$= 55\%$$

However, there is no need to do it in this way because the two conditions above are satisfied. Both the percentages given in the question are percentages of the number of students in the school, and we can safely assume that no one has both black and blond hair.

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Example 15

60% of people on a large railway platform are reading a newspaper and 24% are reading a novel. The other people on the platform are not reading. What is the percentage of people on the platform who are:

a reading? **b** not reading?

Solution

a $60\% + 24\% = 84\%$

84% of people on the platform are reading.

b Since all of the people (either reading or not reading) on the platform = 100%,
$100\% - 84\% = 16\%$.

Hence, 16% of people on the platform are not reading.

Note that in the last example there is once again no overlap between the people on the platform who are reading and those who are not reading.

In the next example, we cannot add the percentages because they are percentages of different wholes. The first, wrong, solution shows what happens if you try to do this.

Example 16

Two football teams, the Sharks and the Raiders, play a match. The Sharks club has 10 000 members and the Raiders club has 12 000 members. 15% of the Sharks club members and 20% of the Raiders club members attend the match. What percentage of the combined membership of the clubs attend the match?

Solution (Wrong!)

$15\% + 20\% = 35\%$

35% of the combined membership of the clubs attend the match.

Solution (Correct)

Number of Sharks attending = 15% of 10 000 = 1500

Number of Raiders attending = 20% of 12 000 = 2400

Number of supporters attending the match $= 1500 + 2400$
$= 3900$

Combined membership of the clubs $= 10\,000 + 12\,000$
$= 22\,000$

Percentage of combined membership at match $= \dfrac{3900}{22\,000} \times 100\%$

$= \dfrac{195}{11}\%$

$= 17\frac{8}{11}\%$

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Multiplication of percentages

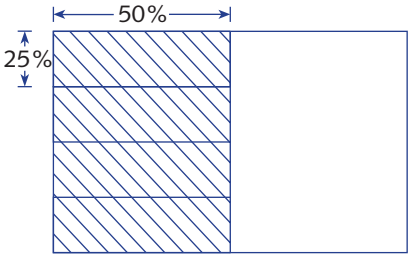
Multiplication of percentages arises in situations such as the following.

50% of a class are boys, and 25% of them have blue eyes. Hence, the percentage of students in the class who are boys with blue eyes is 25% of 50%.

We can draw a diagram to illustrate this. The large cross-hatched rectangle represents the boys in the class, and the smaller blue rectangle represents the 25% of the boys who have blue eyes.

The percentage of students who are boys and have blue eyes is:

$$\begin{aligned} 25\% \text{ of } 50\% &= \frac{1}{4} \text{ of } \frac{1}{2} \\ &= \frac{1}{8} \\ &= 12\tfrac{1}{2}\% \end{aligned}$$



Example 17

Find 20% of 70%.

Solution

$$\begin{aligned} 20\% \text{ of } 70\% &= \frac{20}{100} \times \frac{70}{100} \\ &= \frac{1}{5} \times \frac{7}{10} \\ &= \frac{7}{50} \\ &= 14\% \end{aligned}$$

Example 18

There are 65 000 spectators in a football stadium of whom 55% barrack for the Reds. Of those who barrack for the Reds, 80% are male. Find the percentage of spectators who are male and barrack for the Reds.

Solution

Percentage of spectators who are male and barrack for the Reds

$$\begin{aligned} &= 80\% \text{ of } 55\% \\ &= \frac{80}{100} \times 55\% \\ &= \frac{4}{5} \times \frac{11}{20} \\ &= \frac{44}{100} \\ &= 44\% \end{aligned}$$

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Exercise 7C

1 In each part, add the percentages and express the result as a fraction.

a $5\% + 20\% + 30\%$

b $0.5\% + 24\% + 65.5\%$

c $2\% + 10\% + 50\%$

d $12.5\% + 22.5\% + 2.25\%$

Example 17

2 Calculate:

a 20% of 50%

b 15% of 30%

c 60% of 80%

d 34% of 90%

e $12\frac{1}{2}\%$ of 80%

f $33\frac{1}{3}\%$ of 25%

g 40% of 35%

h 80% of 15%

i $66\frac{2}{3}\%$ of 50%

3 **a** Find 30% of 20%

b Find 20% of 30%.

Example 15

4 65% of a group of 4300 teenagers said they like healthy food.

a How many of the teenagers said they like healthy food?

b What is the percentage of the teenagers who said they do not like healthy food?

5 In a particular group of 600 people:

- 15% watch only the Channel Nine news every night
- 40% watch only the Channel Seven news every night
- 25% watch only the Channel Two news every night.

a How many people watch:

i the Channel Nine news?

ii the Channel Seven news?

iii the Channel Two news?

b How many people watch none of these news broadcasts?

6 When a group of 500 people were asked to name their favourite sport 34% responded 'only football' and 26% responded 'only cricket'.

a How many people responded 'only football'?

b How many people responded 'only cricket'?

c How many people did not respond 'only cricket' or 'only football'?

Example 18

7 15% of a herd of goats are male and 6% of the male goats are older than 4 years of age. What percentage of the total number of goats are male and older than 4 years of age?

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8 A jar contains dark chocolate balls and milk chocolate balls. 35% of the chocolate balls are dark chocolate and 15% of the dark chocolate balls are wrapped in gold foil. What percentage of the chocolate balls are:

a milk chocolate?

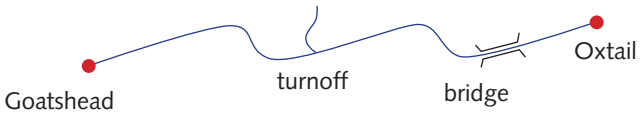
b dark chocolate in gold foil?

9 60% of a group of students are boys, and 30% of the boys have blue eyes. What percentage of the class are boys with blue eyes?

10 80% of a herd of cattle are Herefords, and 15% of the Herefords are under one year of age. What percentage of the cattle are Herefords under one year of age?

11 40% of a group of students are going to a concert, and 23% of the students going to the concert are also going to a party afterwards. What percentage of students go to both the concert and the party afterwards?

12 If you travel from Goatshead to Oxtail, you have covered 45% of the total distance between the two towns when you reach the turnoff. The bridge is 60% of the way from the turnoff to Oxtail. What percentage of the total trip still remains when you get to the bridge?



13 Explain why the following reasoning is incorrect. 20% of the students going to one school have blond hair. 30% of the students going to a nearby school have blond hair. Therefore, $20\% + 30\% = 50\%$ of the students in the two schools have blond hair.

14 Explain why the following reasoning is incorrect. 60% of the Queensland Reds' supporters wear red shirts. 40% of the Queensland Reds' supporters are female. Therefore, 40% of $60\% = 24\%$ of Queensland Reds' supporters are female and wear red shirts.

7D Percentage increase and decrease

Percentages provide a convenient way to express an increase or decrease in a quantity.

In the newspapers or on television, you will often hear such news as:

- A company's profits for the year have increased by 20%.
- The population of a city has increased by 7% over the past two years.
- The grain production in a certain area is down by 25%.

Percentage increase

Example 19

a Increase 600 by 5%.

b Increase 850 by $6\frac{1}{2}\%$.

Solution

a 5% of $600 = \dfrac{5}{100} \times 600$

$= 30$

Therefore, 600 increased by $5\% = 600 + 30$

$= 630$

b $6\frac{1}{2}\%$ of $850 = \dfrac{6\frac{1}{2}}{100} \times \dfrac{850}{1}$

$= \dfrac{13}{\cancel{200}^{4}} \times \dfrac{\cancel{850}^{17}}{1}$

$= \dfrac{13 \times 17}{4}$

$= \dfrac{221}{4}$

$= 55\frac{1}{4}$

Therefore, 850 increased by $6\frac{1}{2}\% = 850 + 55\frac{1}{4}$

$= 905\frac{1}{4}$

Example 20

A house was valued at \$200 000 in January 2004 and at \$324 000 in January 2011.

Find the percentage increase in the value of the house.

Solution

Increase in value of the house $= 324\,000 - 200\,000$

$= 124\,000$

Percentage increase in value $= \dfrac{124\,000}{200\,000} \times 100\%$

$= 62\%$

We often use percentage increase to describe such things as population increases. For example, we may read in the newspaper that the population of Queensland increased by 8% over the past five years.

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Example 21

The population of a particular mob of kangaroos in a given area is increasing by 5% every year. If the population on 1 January 2005 was 660, what was the population of kangaroos on 1 January 2006?

Solution

If the population at 1 January 2005 was 660, then the population on 1 January 2006 would be 660 + (5% of 660).

$$\begin{aligned} 5\% \text{ of } 600 &= \frac{1}{20} \times 660 \\ &= \frac{66}{2} \\ &= 33 \end{aligned}$$

The total population of kangaroos on 1 January 2006 was $660 + 33 = 693$.

Example 22

A shop finds its sales for February have increased by 24% from its January figures. In January, the sales figure was \$32 000. Find the sales for February.

Solution

$$\begin{aligned} \text{Increase in sales} &= 24\% \text{ of } 32\,000 \\ &= \frac{24}{100} \times 32\,000 \\ &= \$7680 \\ \text{February sales} &= 7680 + 32\,000 \\ &= \$39\,680 \end{aligned}$$

Percentage decrease

Everyday examples of percentage decrease are also common. For example:

- The number of pelicans in a particular breeding colony has decreased by 20% from the previous year.
- The numbers of supporters at the matches of the Gunarrong Football Club are down by 30% from the previous season.
- The number of tourists visiting the Big Turnip tourist site has decreased by 15% from the previous year.

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Example 23

a Decrease 600 by 8%.

b Decrease 800 by $12\frac{1}{4}\%$.

Solution

a 8% of $600 = \dfrac{8}{100} \times \dfrac{600}{1}$

$= 48$

Decreased amount $= 600 - 48$

$= 552$

b $12\frac{1}{4}\%$ of $800 = \dfrac{12\frac{1}{4}}{100} \times \dfrac{800}{1}$

$= \dfrac{49}{\cancel{400}^{1}} \times \dfrac{\cancel{800}^{2}}{1}$

$= 98$

Decreased amount $= 800 - 98$

$= 702$

Example 24

The population of Gopedope was 80 000 on 1 February 1845. By 1 February 1901, the population had fallen by 45%. Find the population on 1 February 1901.

Solution

Drop in population of Gopedope $= 45\%$ of $80\,000$

$= \dfrac{45}{100} \times 80\,000$

$= 45 \times 800$

$= 36\,000$

Population of Gopedope on 1 February 1901 $= 80\,000 - 36\,000$

$= 44\,000$

Example 25

Rabbit Island is home to two types of rabbits.

On 1 January 2005, there were 600 grey rabbits and 400 pink-nosed rabbits.

On 1 January 2006, there were 1260 grey and 550 pink-nosed rabbits.

a For each type of rabbit, find the percentage increase in population.

b Find the percentage increase in the total rabbit population.

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Solution

a Grey rabbits increased by $1260 - 600 = 660$.

$$\text{Percentage increase} = \frac{660}{600} \times 100\%$$
$$= \frac{660}{6}\%$$
$$= 110\%$$

Pink-nosed rabbits increased by $550 - 400 = 150$.

$$\text{Percentage increase} = \frac{150}{400} \times 100\%$$
$$= 37.5\%$$

b The total number of rabbits, increased from $600 + 400 = 1000$ by $660 + 150 = 810$.

$$\text{Percentage increase} = \frac{810}{1000} \times 100\%$$
$$= 81\%$$

Discount

Discount is an important example of the use of percentages. Shops offer discounts as incentives to buy. You will see signs such as these written on shop windows such as '15% off all track pants'.

The method for calculating a discounted price is shown in the following example.

Example 26

Gloria goes to the Mighty Good Emporium to buy a bicycle. The shop is offering a 20% discount on all bicycles. The bicycle Gloria wants is marked at $460. How much does she pay for it?

Solution

$$20\% \text{ of } \$460 = \frac{20}{100} \times 460$$
$$= \frac{1}{5} \times 460$$
$$= 92$$

Gloria obtains a discount of $92. She pays $(460 - 92) = \$368$.

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Exercise 7D

Example 19

1 Find the value when:

a 800 is increased by 7%

b 1000 is increased by 6%

c 750 is increased by 8%

d 270 is increased by 10%

e 4500 is increased by 9%

f 6000 is increased by $6\frac{1}{2}\%$

2 Calculate the percentage change (increase or decrease) that occurs when:

a 20 becomes 40

b 25 becomes 100

c 2 becomes 10

d 10 becomes 11

e 8 becomes 10

f 110 becomes 100

g 90 becomes 120

h 100 becomes 80

i 48 becomes 36

3 Find the new amount after each percentage change.

a \$45 000 is increased by 20%

b \$60 000 is decreased by 15%

c \$32 000 is increased by $12\frac{1}{2}\%$

d \$42 000 hectares is increased by 24%

Example 20

4 The circulation of a newspaper on 1 January 2009 was 720 000. On 1 January 2010, the circulation was 800 000. Find the percentage increase in circulation.

Example 22

5 The farmers in a particular region of China planted 45 000 hectares of farmland with sorghum in 2005. By 2010, the area had increased by 24%. Find the area planted with sorghum in 2010.

Example 23

6 Find the value when:

a 500 is decreased by 10%

b 6250 is decreased by 8%

c 5500 is decreased by 6%

d 7000 is decreased by $6\frac{1}{2}\%$

Example 26

7 Calculate the discounted price in each case.

a A discount of 10% on a purchase of \$3210

b A discount of 12% on a purchase of \$5600

c A discount of 5% on a purchase of \$68

d A discount of 8% on a purchase of \$56

e A discount of 23% on a purchase of \$78.50

Example 24

8 The population of a city was 92 000 on 1 February 1920. By 1 February 1950, the population had fallen by 45%. Find the population at this second date.

9 The circulation of a newspaper on 1 January 2008 was 840 000. On 1 January 2009, the circulation was 640 000. Find the percentage decrease in circulation.

10 If the annual increase in the population of a country is 2.5% and the present number of inhabitants is 2 624 000, what will the population be in a year's time?

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11 The population of a city was 1 000 000 and it increased by $2\frac{1}{2}\%$ of this value in each of 3 succesive years.

a What was the population at the end of the first year?

b What was the population at the end of 3 years?

Example 20

12 A house was bought for \$300 000 in 2000. It was sold again in 2005 for \$475 000. What was the percentage increase in the value of the house?

13 A man buys a car for \$7500 and sells it two years later for \$5000. What is his percentage loss?

14 Two different machines are used to produce chocolates in a chocolate shop.

Machine A produces 520 g of chocolate from 650 g of raw materials. Machine B produces 450 g of chocolates from 600 g of raw materials. The rest of the raw materials stick to the machines.

a What percentage, by weight, of the raw materials is turned into chocolates by machine A?

b What percentage, by weight, of the raw materials is turned into chocolates by machine B?

c Which of the machines is the more efficient?

15 A floor covering shop made a profit of 6% on its total costs last year. If the total costs were \$1 240 000, what was the profit?

16 A shoe shop had total costs of \$5 200 000 on salaries, shoes etc., and their total sales was \$4 628 000.

a What was the loss?

b What was the loss expressed as a percentage of the total costs?

17 Find the profit or loss for these situations.

a 10% loss on \$76 000

b 8% profit on \$80 000

c 50% profit on \$1 000 000

d 25% loss on \$420 000

18 Find the percentage profit or loss on costs in these situations.

a Costs \$12 000 and sales \$13 920

b Costs \$52 000 and sales \$56 680

c Costs \$11 200 and sales \$100 800

d Costs \$22 000 and sales \$200 900

19 A shop has a discount sale that marks down all prices by 20%. If a customer buys a jacket originally marked at \$90, how much does he pay?

20 Gaye buys three novels and two textbooks at a shop that is advertising 25% discount on all books. The novels are marked at \$36 each and the textbooks are marked at \$72 and \$96. What is the total price after the discount has been given?

21 A shopkeeper paid \$200 each for two handbags. He sold the first handbag for a profit of 25% of the cost price. He sold the second at a loss of 15% of the cost price. How much profit did the shopkeeper make on the two bags?

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7E The unitary method

The unitary method was introduced in Section 2D. Here is a reminder of how it works.
The unitary method is very useful for doing the problems of the previous section in reverse.

Example 27

If 8 oranges cost \$5.60, how much do 3 oranges cost?

Solution

$\boxed{\div 8}$ 8 oranges cost \$5.60

1 orange costs $\frac{5.60}{8} = 0.7$

$= 70\text{c}$

$\boxed{\times 3}$ 3 oranges cost $3 \times 70\text{c} = 210\text{c}$

$= \$2.10$

Notice that we first worked out the cost of one orange. The word 'unit' is another word for 'one' (of some object). This is why the method is called the *unitary* method.

The same idea can be used to find 100% of an amount if you are given a percentage of that amount.

Example 28

7% of an amount of money is \$84. What is the sum of money?

Solution

7% of the amount is 84.

$\boxed{\div 7}$ 1% of the amount is 12.

$\boxed{\times 100}$ 100% of the amount is \$1200.

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Example 29

8% of an amount of money is \$6000. How much is 3%?

Solution

8% of the amount is 6000.

$\boxed{\div 8}$ 1% of the amount is 750.

$\boxed{\times 3}$ 3% of the amount is \$2250.

Example 30

A 15% discount is offered on all goods at a clothing shop. A suit has a discounted price of \$425. What was the original price of the suit?

Solution

Discounted price = (100 − 15)% of the original price
= 85% of the original price

85% of the original price = 425

$\boxed{\div 85}$ 1% of the original price $= \frac{425}{85}$
$= 5$

$\boxed{\times 100}$ 100% of the original price = 500

The original price of the suit was \$500.

Note: We could have used 5% as the unit here.

Alternative method

Let x be the original price.

$$85\% \text{ of } x = 425$$
$$\frac{85}{100} \times x = 425$$
$$x = 425 \times \frac{100}{85}$$
$$= 500$$

The original price is \$500.

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Example 31

A population of ants increases by 20% in a week and the new population is 125 400. What was the population at the beginning of the week?

Solution

We can use 1% as the unit in this problem.

120% of the original population is 125 400.

$\boxed{\div 120}$ 1% of the original population is 1045.

$\boxed{\times 100}$ 100% of the original population is 104 500.

The population at the beginning of the week was 104 500.

Sometimes the method can be modified to make our calculations simpler.

Alternative method using 20% as the unit

120% of the original population is 125 400.

$\boxed{\div 6}$ 20% of the original population is 20 900.

$\boxed{\times 5}$ 100% of the original population is 104 500.

The population at the beginning of the week was 104 500.

Example 32

John spends 24% of his weekly wage on rent and he spends 25% of the remainder in repayments on a motorbike. He still has $228 left. How much does he earn in one week?

Solution

$$\begin{aligned}\text{Percentage of wage left after payment of rent} &= 100\% - 24\% \\ &= 76\%\end{aligned}$$

$$\begin{aligned}\text{Percentage of wage spent in repayments on motorbike} &= 25\% \text{ of } 76\% \\ &= \frac{1}{4} \text{ of } 76\% \\ &= 19\%\end{aligned}$$

$$\begin{aligned}\text{Percentage of wage remaining} &= 76\% - 19\% \\ &= 57\%\end{aligned}$$

$$57\% \text{ of John's wage} = 228$$

$$\begin{aligned}1\% \text{ of of John's wage} &= \frac{228}{57} \\ &= 4\end{aligned}$$

Hence, 100% of John's wage is $400.

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GST (goods and services tax)

The GST on an item is calculated as 10% of the pre-GST price of that item and is included in the price. Therefore, the price including GST is 110% of the pre-GST price.

Example 33

The price of a suit before GST is added is $440. What is the price after it is added?

Solution

100% of the pre-GST price is $440.

$\boxed{\div 10}$ 10% of the price is $44.

$\boxed{\times 11}$ 110% of the price is $484.

Example 34

The price of a shirt, including GST, is $61.60. What was the original price of the shirt before GST was added?

Solution

110% of the original price is 61.60.

$\boxed{\div 11}$ 10% of the original price is $5.60.

$\boxed{\times 10}$ 100% of the original price is $56.

Exercise 7E

Example 28

1 20% of the children in a swimming pool are boys. There are 8 boys. How many children are there in the swimming pool?

Example 29

2 **a** 20% of an amount of money is $46 890. What is 80% of the amount of money?

b 5% of an amount of money is $42 000. What is 20% of the amount of money?

c $12\frac{1}{2}\%$ of an amount of money is $64 800. What is the total amount of money?

d $33\frac{1}{3}\%$ of an amount of money is $45 000. What is 60% of the amount of money?

Example 30

3 The price of a car has been discounted by 10%. The discounted price is $22 500. What was the price before the discount?

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4 A park contains paved areas, lawns and garden beds. 8% of the area of the park is garden bed. There are 22 000 m^2 of garden bed.

a What is the total area of the park?

b The lawns are 80% of the park. How many square metres of lawn are there?

5 Mr Ho invests an amount of money for one year at 5% per annum interest. The bank tells him that at the end of the year he will have $126 000. How much does Mr Ho invest?

Example 31 **6** The number of students at a school increased by 12% from 2010 to 2011. In 2011, there are 1680 students. How many students were there in 2010?

7 **a** 105% of x is 568 911. Find x.

b 10% of x is 45 670. Find x.

c 7% of w is 8470. Find w.

d 120% of a is 14 460. Find a.

Example 32 **8** Raylee gives 60% of her weekly wage to her mother and 25% of the remainder to her brother. She still has $240 left. How much does she earn in one week?

9 Felicity spent 20% of her savings on a bicycle and 15% of the remainder on a book. What percentage of her savings did she have left?

10 A discount of 15% is offered on the price of a book. The discounted price is $136. What was the original price?

Example 33 **11** The price of a bike before GST is added is $320. What is the price after GST is added?

Example 34 **12** For each item, the cost including GST is given. What was the cost of each item before GST was added?

a A shirt priced at $88

b A book priced at $68.20

c A model aeroplane priced at $253

d A chess set priced at $93.50

Review exercise

1 Write these fractions as percentages. Use fractions in your answers if necessary.

a $\frac{23}{25}$ **b** $\frac{3}{4}$ **c** $\frac{3}{8}$ **d** $\frac{7}{8}$ **e** $\frac{3}{5}$ **f** $\frac{2}{3}$

g $\frac{45}{60}$ **h** $\frac{110}{100}$ **i** $\frac{48}{72}$ **j** $\frac{12}{96}$ **k** $\frac{31}{25}$ **l** $\frac{300}{150}$

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2 Write each percentage as a mixed numeral with the fraction in simplest form.

a 50% **b** 85% **c** 43% **d** 200%

e 340% **f** 185% **g** 12.5% **h** 624%

3 Calculate:

a 40% of 20 **b** 25% of 18 **c** 23% of 200

d 18% of 150 **e** 97% of 200 **f** 3% of 600

g 85% of 4080 **h** 110% of 340 000 **i** 125% of 36

4 In a city, 33% of vehicles are utes, 18% are four-wheel drives, 27% are sedans and 22% are station wagons. If there are 29 700 vehicles in the city, how many of each type of vehicle are there?

5 When the kindergarten children arrived yesterday, $\frac{1}{4}$ of them were wearing skirts, $\frac{2}{5}$ were wearing shorts, $\frac{3}{10}$ were wearing dresses and the rest of the children were wearing jeans. What percentage of the children were wearing jeans?

6 What is the weight of fat, in grams, in one 200 g container of yoghurt, if 3% of the yoghurt, by weight, is fat?

7 Find:

a 12.5% of 3000 **b** 3.5% of 6000 **c** $33\frac{1}{3}$% of 4200

d 5.6% of 9000 **e** 62.5% of 4200 **f** 12.5% of 320 000

g 5.5% of 200 **h** 2.4% of 1000 **i** 0.1% of 78

j 15% of 140 **k** 99.95% of 30 000 **l** 1.375% of 240

8 The label on a 750 mL bottle of soft drink states that it contains 5% pure lemon juice.

a How many millilitres of pure lemon juice does it contain?

b If pure lemon juice costs 50 cents per 100 mL, what is the value of the lemon juice in the bottle?

9 In a group of 500 people, 22% are left-handed. How many of the group are left-handed?

10 In a flock of 2650 sheep, 32% have black wool. How many black sheep are there?

11 A breakfast cereal contains $23\frac{1}{2}$% oats, by weight. What is the weight of oats in a package of cereal containing 1500 g of the cereal?

12 There are 16 540 high-school students in Sunview, and 60% of them attend state high schools. What is the number of state high-school students in Sunview?

13 It is known that watermelons are 92% water. A particular watermelon weighs 3600 g. How much does the water in this watermelon weigh?

14 A meat pie weighs 320 g. It is found that 73% of the pie is meat. What is the weight of the meat in the pie?

15 A packet of shortbread biscuits weighs 280 g. It is known that the biscuits are 12% butter, by weight. Find the weight of butter in one packet of biscuits.

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16 In total, 14 carbon, hydrogen and nitrogen atoms make up one toxic molecule. If the number of carbon atoms is 5 more than the number of nitrogen atoms, and $42\frac{6}{7}\%$ of the atoms are carbon, find the number of each type of atom.

17 The price of a shirt including GST is \$110. What is the pre-GST price of the shirt?

18 A hotel charges a fee of 2% for a guest using a credit card. If the hotel bill is \$1250 before the fee is applied, how much is the fee?

19 A shop is offering a discount of 10% on all goods. If a garden table is priced at \$550, what is the discount?

20 The label on a 750 mL bottle of soft drink states that it contains 15% pure mango juice.

a How many millilitres of mango juice does it contain?

b If pure mango juice costs 25 cents per 100 mL, what is the value of the mango juice in the bottle?

21 In a clothing factory, 28% of the employees are female. There are 672 female employees. How many employees are there in total?

22 A sheep station has 1872 merino sheep. This is 36% of all the sheep on the station. How many sheep are there on the station?

23 A farm grows both barley and wheat. 30% of the barley is grown organically and 60% of the wheat is grown organically. 70% of the grain produced by the farm is wheat and the remainder is barley. What percentage of the farm's grain is organically grown?

24 On a test containing 30 true/false questions, David gave 50% more right answers than wrong ones. How many questions did he answer correctly?

25 A car salesyard is offering 15% off all new cars. If a car is priced at \$32 500, what is the discount being offered?

Challenge exercise

1 Fred Callum earns a salary of \$450 per week for a 40-hour week. His weekly salary is increased by 10% and his number of hours worked is decreased by 10%. Calculate his new hourly salary.

2 The length of a rectangle is increased by 15% and the width decreased by 10%. If the area of the original rectangle is 100 cm^2, find the area of the new rectangle.

3 In a lottery, only 0.002% of tickets won prizes. If there were two prizes, how many tickets were sold?

4 An illness has struck the town of Abelanne. On 1 January, 20% of the residents have the illness. On 1 February, 10% of the people who had the disease have recovered but 40% of those who did not have the disease now have it. What percentage of the population has the disease on 1 February?

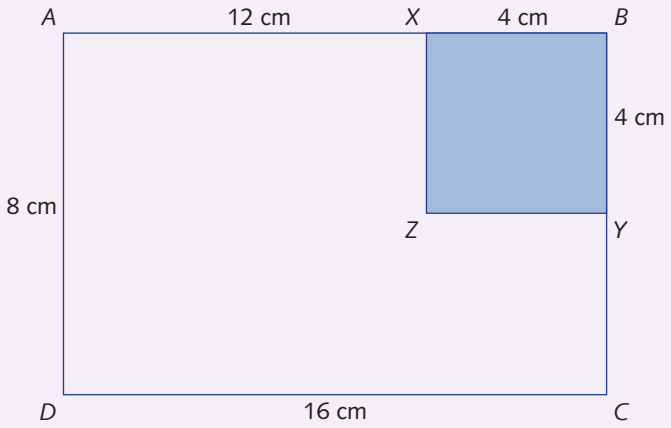
5 In an election in a small island nation of 10 000 people, 80% of the population voted. Of those who voted, 63% voted for the New Liberation Party. How many people did not vote for the New Liberation Party?

6 A dairy farmer found that the milk supplied by his cows was 5% cream and 95% skimmed milk. He wanted to know how much skimmed milk he should add to a litre of milk to reduce the percentage of cream to 4%. (*Recall*: 1 L = 1000 mL). What is the answer?

7 A girl makes up a cordial drink so that 10% of the drink is cordial concentrate. The rest is water. The girl has 350 mL of the drink in a jug. She now decides that she wants the cordial concentrate content to be only 8%. How much extra water does she need to put in the jug?

8 **a** **i** Find the area of the shaded region *XBYZ*.

ii Find the area of the unshaded region *AXZYCD*.



b Find 20% of the area of rectangle *ABCD*.

c Find 80% of the shaded area + 20% of the area of *AXZYCD*.

d Express the answer to part **c** as a percentage of the area of rectangle *ABCD*.

9 35% of students in a class have brown hair. Five brown-haired students join the class so that 48% now have brown hair. How many students were previously in the class?

10 What two-digit numbers increase by 75% when their digits are reversed?

11 The number 12 321 is a **palindrome** since it is unchanged when its digits are reversed. What percentage of five-digit numbers are palindromes?

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