CHAPTER

10

Number and Algebra

Rates and ratios

Paul travels at 60 kilometres per hour.

The Smith household uses 500 litres of water per day.

These are examples of rates. Rates are used here to measure speed and water consumption, and we encounter many different kinds of rates in everyday life. For example, Mary earns $15 an hour working at the supermarket.

Ratios are usually used to compare two related quantities. For example, salad dressing may be made using a ratio of one part vinegar to two parts oil.

We will see that many practical problems can be solved by working with ratios in appropriate ways.

Many problems about rates and ratios can be solved by using a familiar and simple idea – the unitary method.

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10A Review of the unitary method

The unitary method was introduced in Chapter 2, and was used again in Chapter 7 where we looked at percentages. In this section, we review the unitary method, with particular attention to the setting out of problems.

Here is an example about comparing prices. Examples like this occur whenever we go shopping

Example 1

If 12 mangoes cost \$24, how much do 7 mangoes cost?

Solution

The unit in this example is 1 mango.

	12 mangoes cost \$24.
$\div 12$	1 mango costs \$2.
$\times 7$	7 mangoes cost \$14.

Sometimes the numbers involved are nicely related. This enables us to take a shortcut, as shown in the following two examples.

Example 2

If 12 mangoes cost \$27, how much do 8 mangoes cost?

Solution

The easiest unit to choose here is 4 mangoes, because 4 is the highest common factor (HCF) of 12 and 8.

	12 mangoes cost \$27.
$\div 3$	4 mangoes cost \$9.
$\times 2$	8 mangoes cost \$18.

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Example 3

If 14 apples cost \$12, how many apples can I buy for \$30?

Solution

The easiest unit to choose is \$6, because 6 is the HCF of 12 and 30.

\$12 buys 14 apples.

$\boxed{\div 2}$ \$6 buys 7 apples.

$\boxed{\times 5}$ \$30 buys 35 apples.

Example 4

Ginger beer costs \$3.35 for a cartoon of 6 cans. If the shop is prepared to split up its cartons, how much will they charge for 11 cans? Give your answer correct to the nearest 5 cents.

Solution

6 cans cost 335 cents.

$\boxed{\div 6}$ 1 can costs $\frac{335}{6}$ cents.

$\boxed{\times 11}$ 11 cans cost $11 \times \frac{335}{6}$ cents $= \frac{3685}{6}$ cents (Leave as an improper fraction.)

$\approx$ \$6.15 (correct to the nearest 5 cents)

Exercise 10A

Example 1, 2

1 **a** If 3 kg of potatoes cost \$3.60, find the cost of 4 kg.

b If 5 pens cost \$15, find the cost of 6 pens.

c If 8 tennis balls cost \$15, find the cost of 20 tennis balls.

d If 9 billiard balls weigh 1440 g, how much do 6 billiard balls weigh?

e If the length of a row of 12 seats is 480 cm, how long will a row of 21 seats be?

f The donuts of a certain donut manufacturer contain 25 g of fat ‘per standard serve’. If one standard serve is 2 donuts, how much fat, in grams, is consumed by a person who eats 5 donuts?

Example 3, 4

2 **a** If 10 bananas cost \$4, how many can I buy for \$14?

b How many shares can I buy for \$5400 if 1000 shares cost \$14?

c At a sale, exercise books cost \$8 per dozen. How many can I buy for \$22?

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d A caterer must buy 4 lettuces to make sandwiches for 50 people. How many lettuces will he need to buy to feed 225 people?

e The first two pages of a student's essay contain 800 words. How many pages are needed if the essay is to meet the requirement of 5000 words? (Assume that each page contains the same number of words.)

3 **a** On a map, 4 cm represents 2 km. How far apart are two suburbs A and B if they are 15 cm apart on the map?

b On a map, 10 cm represents a distance of 50 km. What distance on the map would represent 60 km?

c At a certain time of day, a man 1.8 m tall casts a shadow 2 m long. What is the height of a person who casts a shadow of length 1.8 m?

d If a tree of height 12 m casts a shadow of length 16.8 m, how long a shadow will a tree of height 9 m cast?

e A chef's favourite cake recipe uses 8 eggs and 500 g of flour. If he scales the recipe up to use a dozen eggs, how many grams of flour should he use?

4 **a** If 100% of a school's population is 840, what is 75% of the population?

b If 60% of an amount is $540, what is 40% of the amount?

c I own 32 copies of the 4 cent postage stamp issued by Jaga Jaga Island in 1930, and this is 20% of the total number of these stamps in existence. How many more of these stamps must I buy in order to own 25% of the total?

5 *Note*: These problems may require some written calculations.

a If 2 kg of tomatoes cost 5, how many kilograms of tomatoes can I buy for $8?

b If a dozen apples costs $5, how much do 17 apples cost?

c If 350 g of tinned pineapple contains 210 g of fruit, how much fruit will 500 g contain?

d If 750 mL of soft drink contains 288 g of sugar, how much sugar will 1 litre contain?

6 *Hint*: These problems can be done choosing a 'unit' that is a fraction.

a Three-fifths of an amount is $243. Find two-fifths of the amount.

b A man can cut down 12 trees in three-quarters of an hour. How many trees can he cut down in half an hour, working at the same rate?

c One-third of the class can plant 28 plants during one period. How many plants could half of the class plant in the same time?

d 100 g of crispbread contains 1.2 g of saturated fat. How much crispbread contains 3 g of saturated fat?

e The label on a certain cat medicine states that 3.2 mL should be given for every kilogram of body weight. How much medicine should be given to a kitten weighing 2.5 kg?

7 If you are shopping, which is the better buy?

a 400 g of tinned peaches costing $6.40, or 550 g costing $9.00

b 500 g of lychees costing $5.50, or 300 g costing $3.40

c 140 g of toothpaste costing $1.68, or 180 g costing $2.16

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8 If a box of cereal is selling on special at \$3.60 for a 375 g packet, how much is 500 g of the cereal at this price? If there is actually a 500 g box selling for \$4.50, which is the cheaper way to buy 500 g of cereal, and by how much?

9 A ship has taken 15 days to travel $\frac{3}{7}$ of its journey. How much longer will it take to complete the journey?

10 If 2 painters can paint 3 rooms in 9 hours, how long does it take for 3 painters to paint 4 rooms, working at the same rate?

10B Solving problems using the unitary method

Many problems that reverse a previous operation can be solved quickly using the unitary method.

Example 5

The price of a shirt has been discounted by 20%, and the discounted price is \$64. What was the original price?

Solution

We know that 80% of the price is \$64, and we want to find 100% of the price.

80% is \$64.

$\boxed{\div 4}$ 20% is \$16.

$\boxed{\times 5}$ 100% is \$80.

Calculations involving rates

Quantities such as 600 L per hour, 35 km/h \$15 per m^2 and \$25 per litre are called **rates**. The unitary method usually makes rate questions quite straightforward.

Example 6

Water is flowing into a dam at a constant rate of 600 L per hour.

a How much water flows into the dam in 2 hours?

b How much water flows into the dam in 3.5 hours?

c How long does it take for 12 000 L of water to flow into the dam?

d How long does it take for 10 000 L of water to flow into the dam?

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Solution

a 600 L flows into the dam in 1 hour.

$\boxed{\times 2}$ 1200 L flows into the dam in 2 hour.

b 600 L flows into the dam in 1 hour.

$\boxed{\div 2}$ 300 L flows into the dam in half an hour.

$\boxed{\times 7}$ 2100 L flows into the dam in 3.5 hours.

c 600 L flows into the dam in 1 hour.

$\boxed{\times 20}$ 12 000 L flows into the dam in 20 hours.

d 600 L flows into the dam in 1 hour.

$\boxed{\div 6}$ 100 L flows into the dam in 10 minutes.

$\boxed{\times 100}$ 10 000 L flows into the dam in 1000 minutes.

Hence, it takes 16 hours and 40 minutes for 10 000 L to flow into the dam.

Alternative method

d Number of hours for 10 000 L to flow into the dam $= 10\,000 \div 600$

$$= \frac{50}{3}$$

$$= 16\tfrac{2}{3}$$

Hence, it will take 16 hours and 40 minutes for 10 000 L to flow into the dam.

Example 7

Edward can paint one house in 2 days. James can paint one house in 3 days. Working at these rates, how long does it take to paint a house if they work together?

Solution

Edward: 1 house, 2 days

$\boxed{\times 3}$ 3 houses, 6 days

James: 1 house, 3 days

$\boxed{\times 2}$ 2 houses, 6 days

Together: 5 houses, 6 days

$\boxed{\div 5}$ 1 house, $\frac{6}{5}$ days

Hence, it will take $1\frac{1}{5}$ days to paint one house if they work together.

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Changing units

Careful use of successive one-step conversions allows the units to be changed, as in the previous example. Here is another example involving changes of two units.

Example 8

Van is earning $18 per hour making packages. How much is he earning in cents per second?

Solution

Van earns $18 per hour.

Van earns 1800 cents per hour.

Van earns $\frac{1800}{60} = 30$ cents per minute.

Van earns $\frac{30}{60}$ cents per second = half a cent per second.

Exercise 10B

Example 5, 6

1 **a** The price of a dress has been discounted by 20% and the discounted price is $96. What was the original price?

b Water is flowing out of a dam at a constant rate of 200 litres per hour. How much water has flowed out of the dam in 3 hours?

c If 10% of a mine's production in a day is 2100 tonnes, what is the total production for the day?

d If 70% of an amount is $847, what is the whole amount?

e A car travels 96 km on 12 L of petrol. How much petrol would it use in travelling 200 km?

2 **a** A car travelling at a constant speed travels 130 km in 2 hours. How far will it travel in 3 hours?

b A truck travels 55 km on 10 L of petrol. How far will it travel on 36 L?

c If 100 US dollars (US$100) is worth 180 Australian dollars (A$180), how much is US$40 worth in Australian dollars?

d A watch loses 3 minutes every 6 months. How much time will it lose in 8 months?

e A car travels 80 km on 12 L of petrol. How far could it travel on 300 L of petrol?

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3 **a** If 20% of a farmer's crop is 620 tonnes, what is the total crop?

b Nina is in hospital and has drunk 840 mL of water, which is 70% of her daily allowance. What is her daily allowance?

c If 120% of an amount is $156, what was the original amount?

d Hail destroyed 58% of a crop of plums, and only 10 500 kg was sent to market. How much would have been sent to market if there had been no hailstorm?

e A farmer has used 6000 L of petrol harvesting his crop, and has so far harvested 35%. How much more petrol, correct to the nearest 100 L, will he need to harvest the whole crop?

Example 8

4 **a** A guesthouse has used 630 L of milk over a 30-day period. At what rate, in litres per week, is it using milk?

b A firm pays its workers $1800 per 40-hour week. What rate of pay is this in dollars per minute?

c 350 megalitres of water has flowed into a dam in two and a half days. What is the average flow rate, in megalitres per day?

d A car has travelled 240 km on 48 L of petrol. At what rate, in litres per 100 km, is the car using petrol?

e A log of wood weighs 3 kg and has a volume of 2400 cm^3. What is its density, in kilograms per cm^3?

f A woman has been paid $3800 for 250 hours of work. At what weekly rate is she being paid, assuming there are 40 hours in a normal working week?

g A battered old truck has driven the 740 km from Wagga Wagga to Wee Waa in $18\frac{1}{2}$ hours. What was its average speed, in kilometres per hour?

5 **a** A large tank is leaking water at 20 mL per second. Express this rate in litres per hour.

b Every student in a school of 800 throws away an average of 200 g of rubbish a day. If there are 200 days in the school year, what is the rate at which the students throw away rubbish, in kilograms per year?

c The water stored in Warragamba Dam went from 41.6% of the dam's capacity to 40.4% in a 30-day period. At what weekly rate is the water being used?

d A car travels 100 km on 15 L of petrol. At what rate is the car using petrol, in units of kilometres per litre?

e At a certain time, one Australian dollar bought 72 US cents.

- **i** Express A$12 in US dollars.
- **ii** Express US$18 in Australian dollars.

f At a certain time, one Australian dollar bought 40 British pence. Express in Australian dollars the price of a jug that cost 55 British pounds. (There are 100 British pence in a British pound.)

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6 **a** A truck has driven the 1050 km from Adelaide to Innamincka in $13\frac{1}{2}$ hours. What was its average speed, in kilometres per hour? Give your answer correct to the nearest kilometre per hour.

b A bullet is fired at a speed of 800 metres per second. What is its speed in kilometres per hour?

c At a certain time, one Australian dollar bought 72 US cents.

i Express A$21.06 in US dollars, correct to the nearest US cent.

ii Express US$21 in Australian dollars, correct to the nearest Australian cent.

7 **a** If 20% of the marbles in a box are blue and there are 34 blue marbles, how many marbles are there in the box?

b If 110% of an amount is $220, what was the original amount?

Example 7

8 Natalie can paint one house in 4 days, and Erin can paint one house in 5 days. Working at these rates, how long does it take if they work together?

10C Speed

Speed is one of the most familiar rates of all. It is a measure of how far something goes for each given period of time.

Constant speed

If the speed of an object does not change over time, we say that it is travelling with **constant speed**. For example, a car travelling at a constant speed of 60 kilometres per hour would travel 60 km in 1 hour. It would travel 120 km in 2 hours and 150 km in $2\frac{1}{2}$ hours.

Looking at things the other way around, if I travel at a constant speed for 100 km and it takes me 1 hour to complete the journey, then my speed is 100 kilometres per hour. This is usually written as 100 km/h. If I take 2 hours to travel the 100 km (at a constant speed), then my speed is only 50 km/h.

If the distance is measured in metres and the time is measured in seconds, the speed is measured in metres per second. If an athlete runs 100 metres in 10 seconds, her speed is 10 metres per second. This is usually written as 10 m/s.

The speed of a moving object is thus the distance travelled divided by the time the object takes to travel that distance:

$$\text{speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

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Example 9

Mary travelled at a constant speed for 60 km and it took 4 hours to complete the journey. What was her speed?

Solution

$$\begin{aligned} \text{speed} &= \frac{\text{distance}}{\text{time}} \\ &= \frac{60}{4} \text{ km/h} \\ &= 15 \text{ km/h} \end{aligned}$$

or

60 km in 4 h

$\boxed{\div 4}$ 15 km in 1 h

Speed = 15 km/h

Average speed

When we drive a car or ride a bike, it is very rare for our speed to remain the same for a long period of time. Most of the time, especially in the city, we are slowing down or speeding up, so our speed is not constant. If we travel 20 km in 1 hour, then we say that our **average speed** is 20 km/h, even though we may have travelled much faster than this at some times in that hour, and come to a complete stop at others. When we calculate speed, we often mean average speed, and in all our unitary method calculations in this chapter we make the assumption of constant speed.

Example 10

Paul rides 6 km on his bike in three-quarters of an hour. What is his average speed?

Solution

$$\begin{aligned} \text{average speed} &= \frac{\text{distance travelled}}{\text{time taken}} \\ &= 6 \div \frac{3}{4} \\ &= \frac{{}^{2}\cancel{6}}{1} \times \frac{4}{\cancel{3}} \\ &= 8 \end{aligned}$$

Paul's average speed is 8 km/h.

Alternatively, we can use unitary method to work out speeds.

Paul rides 6 km in $\frac{3}{4}$ of an hour.

$\boxed{\div 3}$ Paul rides 2 km in $\frac{1}{4}$ of an hour.

$\boxed{\times 4}$ Paul rides 8 km in 1 hour.

Paul's average speed is 8 km/h.

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Example 11

Anthony travelled at an average speed of 50 km/h for 4 hours and 30 minutes. How far did he travel?

Solution

A speed of 50 km/h means that: in 1 hour, Anthony travelled 50 km
in 4 hours, Anthony travelled 200 km
in $\frac{1}{2}$ hour, Anthony travelled 25 km.

Hence, in $4\frac{1}{2}$ hours, Anthony travelled $(200 + 25)$ km $= 225$ km.

Alternative solution

We can take the formula:

$$\text{speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

and rearrange it as:

$$\text{distance travelled} = \text{speed} \times \text{time taken}$$

$$\begin{aligned}\text{Hence, the distance Anthony travelled} &= 50 \text{ km/h} \times 4\tfrac{1}{2} \text{ hours} \\ &= 225 \text{ km}\end{aligned}$$

Example 12

A couple go for a walk of 10 km. They walk at an average speed of 2.5 km/h. How long does it take them to complete the walk?

Solution

In each hour, they walk 2.5 km.

$$\begin{aligned}\text{The number of hours needed to walk 10 km} &= \text{the number of lots of 2.5 in 10} \\ &= 10 \div 2.5 \\ &= 4\end{aligned}$$

Hence, the couple take 4 hours to complete the walk.

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Example 13

A cyclist completes a circuit of 15 km. She cycles at an average speed of 12 km/h. How long does it take her to complete the circuit?

Solution

In each hour, she cycles 12 km.

$$\begin{aligned}\text{The number of hours needed to cycle 15 km} &= \text{the number of lots of 12 in 15}\\ &= 15 \div 12\\ &= \frac{5}{4}\\ &= 1\tfrac{1}{4}\end{aligned}$$

Hence, the cyclist takes $1\frac{1}{4}$ hours or 1 hour and 15 minutes to complete the circuit.

Alternatively, it takes 1 hour to go 12 km

$\frac{1}{4}$ hour to go 3 km

Hence, it takes $1\frac{1}{4}$ hours to go 15 km.

We see from the last two examples that in general:

$$\text{time taken} = \frac{\text{distance travelled}}{\text{speed}}$$

Be careful that you are consistent with units. For example, if speed is given in kilometres per hour but distance is given in metres, you need to convert the distance to kilometres before calculating the time taken in hours.

Speed

- The **speed** of an object moving at a constant speed is the distance travelled divided by the time the object takes to travel that distance:

  $$\text{speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

- Depending on the information we are given, we can rewrite this as:

  distance travelled = speed × time taken

  or as:

  $$\text{time taken} = \frac{\text{distance travelled}}{\text{speed}}$$

- Alternatively, we can use the unitary method.
- It is important to be consistent with units when calculating speed, distance travelled or time taken.

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Exercise 10C

Example 9, 10

1 A train takes 4 hours to complete a journey of 360 km. What is the average speed of the train?

Example 11

2 A car travelled at 100 km/h for 4 hours. How far did it go?

Example 12, 13

3 A car travels a distance of 240 km at an average speed of 60 km/h. How long does the journey take?

4 Jonathon walks 12 km in 3 hours. If he walks at the same speed, how far will he walk in:

a 1 hour? **b** 2 hours? **c** 4 hours? **d** 7 hours? **e** $5\frac{1}{2}$ hours?

5 A car travels 320 km in 5 hours. If it travels at the same speed, how far will it travel in:

a 1 hour? **b** 2 hours? **c** 10 hours? **d** 7 hours? **e** $5\frac{1}{2}$ hours?

6 Complete the table.

	Speed	Distance	Time
a		100 km	2 hours
b		30 m	6 minutes
c		15 km	$\frac{1}{2}$ hour
d	30 m/s		4 seconds
e	55 km/h		11 hours
f	60 km/h		$\frac{1}{3}$ hour

7 A boy cycles for 2 hours and 20 minutes at 18 km/h. How far does he go?

8 An aircraft travels 4230 km in $4\frac{1}{2}$ hours. What is its average speed?

9 A train travels for 48 minutes at 80 km/h. How far does it go?

10 A plane is flying at a speed of 640 km/h. How far will it travel between 10:30 a.m. and 11:15 a.m. the same day?

11 Henry travels 48 km by train in $\frac{3}{4}$ of an hour and then cycles 12 km in $\frac{1}{2}$ of an hour.

a How long is he travelling in total?

b What is his average speed during the train trip?

c What is his average speed during the bike trip?

d What is his average speed over the whole trip?

12 A standard (or Olympic) triathlon race involves 1500 m of swimming, followed by a 40-km bike ride, and finally a 10-km run. If a competitor took 20 minutes for the swim, 55 minutes for the ride and 35 minutes for the run, what was her average speed throughout the event?

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13 Michael swims 10 laps of a 50 m pool in 8 minutes. He rests for 10 minutes and then swims a further 15 laps at 68 seconds per lap. What is his average speed in m/s, correct to two decimal places, for:

a the first 10 laps? **b** the last 15 laps?

c the whole swim, excluding rest time?

14 A family are travelling in a car at a steady speed of 85 km/h, and have covered 320 km since they left home at 8 a.m. They plan to have lunch at the next town, which is 422 km from home. When, to the nearest minute, will they have lunch?

15 David and Renae stand 120 m apart. David begins running towards Renae at a speed of 4 m/s. At the same time, Renae runs towards David at a speed of 6 m/s. How long does it take for them to meet?

10D Ratios

Ratios provide a way of comparing two or more related quantities. Ratios are closely connected to fractions, but in many problems they are more convenient to use than fractions.

Suppose that I have 5 red jelly beans and 7 yellow jelly beans. The ratio of the number of red jelly beans to the number of yellow jelly beans is written as:

number of red jelly beans : number of yellow jelly beans = 5 : 7

Example 14

A bag of bread rolls contains 13 wholemeal rolls and 9 multigrain rolls. Write down:

a the ratio of the number of wholemeal rolls to the number of multigrain rolls

b the ratio of the number of multigrain rolls to the number of wholemeal rolls

c the ratio of the number of wholemeal rolls to the total number of rolls

Solution

a Number of wholemeal rolls : number of multigrain rolls = 13 : 9

b Number of multigrain rolls : number of wholemeal rolls = 9 : 13

c Number of wholemeal rolls : total number of rolls = 13 : 22

Now suppose that I mix 200 mL of cordial and 700 mL of water in a jug. The mixture then contains two parts of cordial to every seven parts of water. This is written as.

cordial : water = 2 : 7 (Read this as 'The ratio of cordial to water is 2 to 7.')

or as:

water : cordial = 7 : 2 (Read this as 'The ratio of water to cordial is 7 to 2.')

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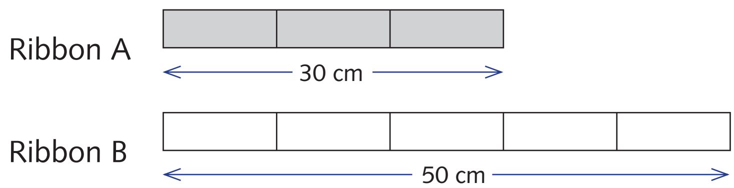
In this example, 'one part' is 100 mL. Another mixture of identical strength could be made by taking 'one part' to be 1 L, and mixing 2 L of cordial with 7 L of water. This idea of **parts** is very useful in dealing with problems involving ratios.

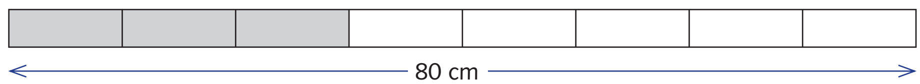
Now consider an example involving ratios of lengths. The next diagram shows two ribbons, a grey one that is 30 cm long and a white one 50 cm long.



Taking 'one part' to be 10 cm:

length of grey ribbon : length of white ribbon = 3 : 5

Now suppose that we join the two ribbons together to make a single ribbon of length 80 cm, as shown.



Then:

length of grey ribbon : total length = 3 : 8

and:

length of white ribbon : total length = 5 : 8

We can also write that the grey section is $\frac{3}{8}$ of the total length, and the white section is $\frac{5}{8}$ of the total length.

Example 15

A mixture contains 200 mL of milk and 500 mL of water. What is the ratio of milk to water?

Solution

Take 1 part to be 100 mL.

200 mL of milk = 2 parts

500 mL of water = 5 parts

Hence, the ratio of milk to water = 2 : 5

Ratios and fractions

If you cut a rope into two equal lengths, the ratio of the two parts is 1 : 1. In this case, each piece is half the total length. Can you see a connection between the ratio 1 : 1 and the fraction $\frac{1}{2}$?

In the following examples, we will explore this link between ratios and fractions.

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Example 16

An alloy of gold and silver contains 2 parts of gold to 5 parts of silver by mass.

a What fraction of the alloy is gold?

b What fraction of the alloy is silver?

c How much of each does 700 g of the alloy contain?

Solution

a There are $2 + 5 = 7$ parts in the alloy, 2 of which are gold.

Hence, the fraction of gold in the alloy $= \dfrac{2}{7}$

b There are 5 parts of silver in the alloy. Hence, the fraction of silver in the alloy $= \dfrac{5}{7}$

c There are 200 g of gold and 500 g of silver.

Example 17

One-tenth of the population has red hair. What is the ratio of the number of redheads to the rest of the population?

Solution

$$\text{Rest of the population} = 1 - \frac{1}{10}$$

$$= \frac{9}{10} \text{ of the whole population}$$

Since $\dfrac{1}{10}$ of the population has red hair, divide the population into 10 parts, each of size $\dfrac{1}{10}$.

Then 1 part = redheads

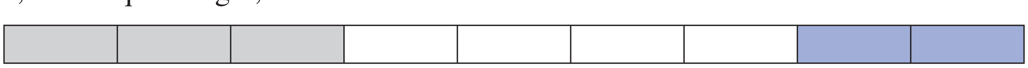
There 9 parts = rest of the population

so redheads : the rest of the population is $1 : 9$.

Ratios with three or more terms

We often have a mixture of more than two things. For example, when making a cake, we sometimes mix flour, sugar, butter and milk.

Suppose that a streamer is made by joining three grey ribbons, four white ribbons and two blue ribbons, all of equal length, as shown.



There are three parts of grey, four parts of white and two parts of blue, so we say that the ratio of grey to white to blue is:

grey : white : blue = $3 : 4 : 2$

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Example 18

A batch of concrete is made from 10 kg of sand, 2 kg of cement, 5 kg of water and 2 kg of gravel. Express the parts as a ratio and express each ingredient as a fraction of the whole.

Solution

The ratio is sand : cement : water : gravel = $10:2:5:2$

The concrete contains 10 parts of sand to 2 parts of cement to 5 parts of water to 2 parts of gravel. There are 19 parts altogether, each part being 1 kg of material, so:

- $\frac{10}{19}$ of the concrete is sand
- $\frac{2}{19}$ of the concrete is cement
- $\frac{5}{19}$ of the concrete is water
- $\frac{2}{19}$ of the concrete is gravel.

Reducing a ratio to simplest form

A ratio involving whole numbers can be reduced to simplest form – just like a fraction – by dividing all the terms in the ratio by their highest common factor (HCF). Sometimes doing this can make it easier to understand the situation we are looking at, as in the next example.

Example 19

A mixture contains 6 parts of oil, 2 parts of insecticide and 10 parts of water by volume. Express the ratio of oil : insecticide : water in simplest form.

Solution

oil : insecticide : water $= 6:2:10$

$\boxed{\div 2}$ $= 3:1:5$

Equivalent ratios behave like equivalent fractions.

$1:3 = 2:6 = 3:9 = 4:12$, and so on, just as $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12}$, and so on.

In other cases, we may need to find a common denominator for a set of fractions, so that we can easily compare them.

We first multiply the fraction by the lowest common denominator and then simplify the ratios if necessary.

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Example 20

A mixture contains $\frac{1}{10}$ sand, $\frac{1}{5}$ soil and $\frac{7}{10}$ rocks. Work out the ratio sand : soil : rocks.

Solution

$$\begin{aligned} \text{sand} : \text{soil} : \text{rocks} &= \frac{1}{10} : \frac{1}{5} : \frac{7}{10} \\ &= 1 : 2 : 7 \end{aligned}$$

Example 21

Eliminate any fractions in these ratios, then reduce them to simplest form.

a $4\frac{1}{2} : 3$

b $\frac{1}{2} : \frac{2}{3} : \frac{5}{6}$

Solution

a $4\frac{1}{2} : 3 = 9 : 6$
$= 3 : 2$

b $\frac{1}{2} : \frac{2}{3} : \frac{5}{6} = 3 : 4 : 5$

Exercise 10D

Express all ratios in simplest form.

Example 14

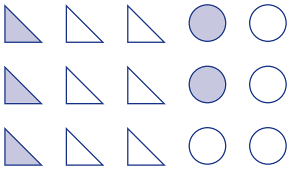
1 In a bowl of fruit, there are 6 oranges and 7 apples. Write down:

a the ratio of the number of apples to the number of oranges

b the ratio of the number of oranges to the number of apples

c the ratio of the number of oranges to the number of pieces of fruit

2 For the diagram opposite, write down:



a the ratio of the number of triangles to the number of circles

b the ratio of the number of blue circles to the number of white circles

c the ratio of the number of blue triangles to the number of white triangles

Example 15

3 A jug of orange juice is made up of 100 mL of pure orange juice and 900 mL of water. What is the ratio of pure orange juice to water?

Example 16

4 In a park, there are 23 native trees for every 40 exotic trees.

a What is the fraction of native trees in the park?

b What is the fraction of exotic trees in the park?

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Example 17

5 One-third of the flowers in a garden are blue. What is the ratio of the number of blue flowers to the number of other-coloured flowers?

Example 18

6 A rope is cut into sections so that the resulting lengths are in the ratio $1:2$.

a Express the length of the shorter piece of rope as a fraction of the total length of rope.

b Express the length of the longer piece of rope as a fraction of the total length.

7 A bowl contains green and blue marbles. One-fifth of the marbles are green. What is the ratio of the number of green marbles to the number of blue marbles?

8 In a bus, $\frac{2}{7}$ of the passengers are male. What is the ratio of the number of male passengers to the number of female passengers?

Example 19

9 Reduce each ratio to simplest form.

a $2:4$ **b** $12:18$ **c** $320:480$ **d** $96:144$

e $12:4$ **f** $36:24$ **g** $512:32$ **h** $243:27$

10 The number of students enrolled at a school is 1200. Of these, 625 are male.

a What is the ratio of the number of male students to the number of female students?

b What fraction of the school population is male?

11 Jonathon has 27 CDs and 60 DVDs in his collection. Write down the ratio of the number of CDs to the number of DVDs in his collection.

Example 20

12 A salad dressing consists of $\frac{3}{8}$ vinegar, $\frac{1}{2}$ oil and $\frac{1}{8}$ lemon juice by volume. Find the ratio vinegar : oil : lemon juice.

Example 21

13 Eliminate any fractions in these ratios, then reduce them to simplest form.

a $2\frac{1}{2}:7$ **b** $3\frac{1}{3}:8$ **c** $2.5:4.5$ **d** $6\frac{1}{5}:4\frac{1}{4}$

e $\frac{3}{4}:\frac{5}{8}$ **f** $\frac{1}{3}:\frac{2}{9}$ **g** $5\frac{1}{4}:3\frac{1}{2}$ **h** $\frac{7}{8}:\frac{7}{24}$

14 In a school of 1029 students, 504 are boys. What is the ratio of the number of boys to the number of girls?

15 A rectangle has a length of 6 cm and a width of 4 cm. A second rectangle has length 9 cm and width 6 cm. Find the ratio of:

a the lengths of the rectangles

b the widths of the rectangles

c the perimeters of the rectangles

d the areas of the rectangles

16 A rectangle has a length of 6 cm and a width of 4.5 cm. A second rectangle has length 9 cm and width 1.5 cm. Find the ratio of:

a the lengths of the rectangles

b the widths of the rectangles

c the perimeters of the rectangles

d the areas of the rectangles

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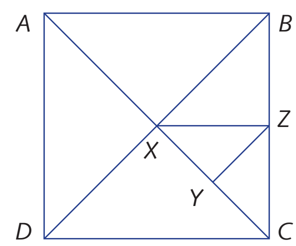
17 The ratio of the number of times the pedals are turned on a bicycle to the number of times the rear wheel revolves is called the *gear ratio*. If the gear ratio is $9:4$, how many times will the rear wheel turn if I pedal:

a 72 times? **b** 108 times? **c** 12 times?

18 A 250 mL packet of milk contains 295 mg calcium and 110 mg of phosphorus. Give the ratio of calcium to phosphorus in simplest form.

19 The lengths of a triangle are in the ratio $5:12:13$. If the perimeter of the triangle is 36 cm, what is the length of each side?

20 In the diagram opposite, $ABCD$ is a square with diagonals AC and BD meeting at X, Z is the midpoint of BC and Y is the midpoint of XC. Find the following ratios of areas.



a area $(\Delta\, AXD)$: area $(\Delta\, AXB)$

b area $(\Delta\, AXD)$: area $(\Delta\, ZXB)$

c area $(\Delta\, AXD)$: area $(\Delta\, ZYC)$

d area $(\Delta\, DBC)$: area $(\Delta\, XYZ)$

21 The cost of a new office building is to be shared between three people in the ratio $4:5:6$. If the office building costs \$450 000, how much does each person have to pay?

22 Express each ratio in simplest form.

a $33:9:12$ **b** $100:65:35$ **c** $2\frac{1}{2}:5:1\frac{1}{4}$

d $1\frac{1}{3}:2\frac{1}{2}:3\frac{1}{4}$ **e** $7:4\frac{1}{2}:\frac{1}{6}$ **f** $11\frac{1}{2}:20\frac{1}{2}:22\frac{1}{4}$

23 An alloy is made by combining 6.25 kg of metal A, 3.75 kg of metal B and 8.75 kg of metal C. Find the ratio of metal A to metal B to metal C.

24 At an agricultural show in the 1950s, freshly hatched chickens were dyed yellow, red and blue. The ratio of the numbers of the different-coloured chickens was yellow : red : blue = $6:4:5$. If there were 45 chickens in total, how many were there of each colour?

10E Using ratios in problems

Ratio problems are usually best solved using the language of parts. The rest of the working is just another form of the unitary method.

For example in a bookshop, the ratio of the number of novels to the number of textbooks is $4:7$. There are 480 novels. How many textbooks are there?

We can use the language of parts to solve this question.

There are 480 novels, so:

$$4 \text{ parts} = 480$$
$$1 \text{ part} = 120$$

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Thus:

7 parts = 840

Hence, there are 840 textbooks.

The ratio 4 : 7 = 480 : 840

Example 22

The ratio of boys to girls in a school is 2 : 3. If there are 264 boys:

a how many girls are there?

b how many students are there altogether?

Solution

a

2 parts = 264 students

÷ 2

1 part = 132 students

× 3

3 parts = 396 students

There are 396 girls in the school.

b 5 parts = 660 students (Multiply the second line above by 5.)

Thus there are 660 students altogether.

Dividing a quantity in a given ratio

A very common application of ratio is the division of a quantity in a given ratio. The key step here is to add the parts.

Example 23

A man divides his estate of \$360 000 in the ratio 4 : 3 : 3 amongst his daughter and his two sons. How much does each receive?

Solution

There are 4 + 3 + 3 = 10 parts in total.

10 parts = \$360 000

÷ 10

1 part = \$36 000

× 3

3 parts = \$108 000

4 parts = \$144 000 (Multiply through the second line above by 4.)

Hence, the daughter receives \$144 000 and each son receives \$108 000.

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Exercise 10E

Example 22

1 The ratio of the number of boys to the number of girls in a class is 5 : 4. If there are 15 boys, how many girls are there?

2 A bowl contains green and red glass balls. The ratio of the number of green balls to the number of red balls is 2 : 3. If there are 18 red balls, how many green balls are there?

3 A ribbon is cut so that the lengths of the parts are in the ratio 8 : 5. If the shorter piece is 15 cm in length, what was the length of the original ribbon?

4 The ratio of the cost of a shirt to the cost of a tie is 8 : 5. If the shirt costs $48 more than the tie, find the cost of the shirt and the cost of the tie.

5 Two sums of money are in the ratio 5 : 8. The smaller amount is $65. Find the larger amount.

6 **a** Divide $45 in the ratio 4 : 5. **b** Divide 720 kg in the ratio 5 : 3.

7 **a** Divide 96 m in the ratio 9 : 7. **b** Divide 144 cm in the ratio 5 : 1.

8 **a** Divide $72 in the ratio 1 : 2 : 5. **b** Divide 95 kg in the ratio 5 : 6 : 8.

Example 23

9 Three friends decide to divide $12 000 amongst them in the ratio 1 : 2 : 3. How much does each receive?

10 An interval AB is 6 cm in length. A point C is a point on AB such that the ratio of the length AB to the length CB is 2 : 1. Find the lengths of AC and CB.

11 Jane and Anthony run a business. They have decided that all profits will be divided between them in the ratio 5 : 4, with Jane receiving the larger share. In 2005, the business made $81 900. How much does each person receive?

12 There are 24 children in a class. The ratio of the number of boys to the number of girls is 3 : 5. How many boys and how many girls are there?

13 160 mm of snow contains as much water as 15 mm of rain. A town within the Arctic Circle receives about 4250 mm of snow a year. If the snow had fallen as rain, what would the equivalent rainfall for the year have been?

14 The angles of a triangle are in the ratio 6 : 5 : 7. Find the size of each of the angles.

15 A piece of string 253 cm long is to be divided in the ratio 2 : 3 : 6. How long is each part?

16 Students in a school are told to choose one out of three sports options: tennis, basketball or swimming. Given that the pupils choose the options in the ratio 4 : 2 : 3 and that 120 choose tennis, find:

a the number of pupils in the school

b the number of students who choose swimming

17 In a class of 30 students, the ratio of boys to girls is 2 : 3. If 6 boys join the class, find the new ratio of boys to girls in the class.

18 The ratio of length to width of a rectangle is 2 : 3. If its area is 54 cm^2, find its perimeter.

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10F Scale drawings

A **scale drawing** is used where an object being illustrated is too large to be shown at full size on the page. For example, a scale drawing might be used to show:

- the plan of a building
- a map of a suburb or a country
- a photograph of a distant galaxy.

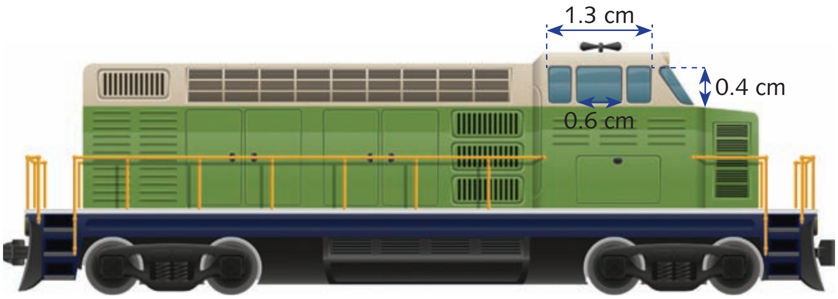
Scale drawings are also used when very small objects are to be shown, such as:

- a diagram of a cross-section of skin or hair
- an enlargement of an image on a computer screen
- a magnified photograph of an insect.

A scale drawing has the same shape as the original object, but a different size. All the lengths of the original object are reduced or magnified in the drawing in exactly the same ratio. This ratio is called the **scale** of the drawing.

Scale = length on the drawing : length on the actual object

The scale of a drawing can be given as the ratio of two numbers. For example, in the image of the side of an engine, the scale is 1 : 200.



scale = 1 : 200

This means that a length of 1 cm on the image corresponds to a length of 200 cm, or 2 m, on the actual engine. Thus, the scale can also be written as:

scale = 1 cm : 2 m

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Example 24

a Measure the overall length of the engine on the previous page (including the couplings). Then use the scale to find the approximate length (in metres) of the actual engine.

b Find the approximate width and height of the window panel (in metres), and the area of the panel.

Solution

a

Engine length in photograph = 11 cm

$\boxed{\times 200}$ Actual engine length = 2200 cm

= 22 m

b

Width of window panel in photograph = 0.6 cm

$\boxed{\times 200}$ Actual width of window panel = 120 cm

= 1.2 m

Height of window panel in photograph = 0.4 cm

$\boxed{\times 200}$ Actual height of window panel = 80 cm

= 0.8 cm

Area of window panel = 1.2×0.8

= $0.96\ \text{m}^2$

In general we express a scale in simplest form. For example, a scale of 3 : 9 is written as 1 : 3.

Scale drawings

- A **scale drawing** of an object has the same shape as the object, but a different size.
- The **scale** of the drawing is the ratio:

  length on the drawing : length on the actual object.
- A scale can be written as the ratio of two numbers, or as the ratio of two lengths. For example:

  scale = 1 : 500 or scale = 1 cm : 5 m

Example 25

Convert the two measurements in each scale to the same unit. Hence, convert the scale to a ratio of two numbers.

a 1 cm : 4 m

b 3 cm : 1 mm

Solution

a 1 cm : 4 m = 1 cm : 400 cm

= 1 : 400

b 3 cm : 1 mm = 30 mm : 1 mm

= 30 : 1

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Example 26

Convert each scale to a ratio of lengths in the units indicated.

a $1:250\,000 = 1\text{ cm} : \_\_\_\_ \text{ km}$

b $200:1 = 1\text{ cm} : \_\_\_\_ \text{ mm}$

Solution

a $$\begin{aligned}1:250\,000 &= 1\text{ cm} : 250\,000\text{ cm}\\ &= 1\text{ cm} : 2500\text{ m}\\ &= 1\text{ cm} : 2.5\text{ km}\end{aligned}$$

b $$\begin{aligned}200:1 &= 200\text{ mm} : 1\text{ mm}\\ &= 20\text{ cm} : 1\text{ mm}\\ &= 1\text{ cm} : 0.05\text{ mm}\end{aligned}$$

Problems involving scale drawing

Problems involving scale drawings can be solved in exactly the same way as the ratio problems discussed earlier.

Example 27

On a map with a scale of $1:500\,000$ two towns are 12 cm apart. How far apart are the actual towns?

Solution

1 cm represents 500 000 cm, which is 5000 m, which is 5 km

so 1 cm represents 5 km

$\boxed{\times 12}$ 12 cm represents 60 km

Thus the towns are 60 km apart.

Example 28

Eleni has photographed a tiny bug at a scale of $80:1$. If a leg on the photograph measures 4 mm, how long is the actual leg?

Solution

80 mm represents 1 mm, so:

$\boxed{\div 10}$ 8 mm represents 0.1 mm

$\boxed{\div 2}$ 4 mm represents 0.05 mm

Thus the bug's leg is 0.05 mm long.

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Exercise 10F

Use appropriate units in your answers to Questions **3–14**.

Example 25

1 Convert the two measurements in each scale to the same unit. Hence, convert the scale to a ratio of two numbers.

a 4 cm : 32 cm

b 15 cm : 55 cm

c 1 cm : 2 m

d 10 cm : 4.5 m

e 1 cm = 3 km

f 5 cm = 200 km

g 3 cm : 6 mm

h 20 cm : 5 mm

i 1 cm = 0.4 mm

Example 26

2 Copy and complete each scale conversion.

a 2 : 15 = 1 cm : ___ cm

b 3 : 10 = 12 cm : ___ cm

c 1 : 700 = 1 cm : ___ m

d 1 : 4600 = 1 cm : ___ m

e 1 : 300 000 = 1 cm : ___ km

f 1 : 1600 000 = 12 cm : ___ km

g 10 : 1 = 1 cm : ___ mm

h 50 : 1 = 1 cm : ___ mm

i 200 : 1 = 1 cm : ___ mm

j 5000 : 1 = 1 cm : ___ mm

Example 27

3 A map of a country is drawn to a scale of 1 : 12 500 000. Find the actual distance between two points whose separation on the map is:

a 1 cm

b 8 cm

c 1.2 cm

d 1 mm

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4 A map of a city is drawn to a scale of 1 cm : 2 km. Find the actual distance between two points whose separation on the map is:

a 3 cm
b 11 cm
c 7.5 cm
d 4.8 cm
e 5 mm
f 1 mm
g 1.5 mm
h 0.2 mm

5 A plan of a rectangular block of land is drawn to a scale of 1 : 1000. The side lengths of the block on the plan are 8 cm and 15 cm.

a Calculate the side lengths of the actual block.

b Calculate the area of the actual block.

6 A plan of a large ornamental doorway is drawn to a scale of 1 : 50. The doorway is 8 cm wide on the plan. Above the door is a rectangular panel, whose base equals the width of the door, and whose height is 2 cm on the plan.

a Calculate the width of the actual door and the height of the actual panel.

b Calculate the area of the actual panel.

7 A plan of a house is drawn to a scale of 1 : 200. Find the measurement on the plan of:

a the hall, which has length 6 m

b the bedroom, which has width 3 m

c the walls, which have height 2.8 m

d the side of the house, which has length 11.2 m

e the kitchen table, which has width 1.2 m

8 Find the scale factor of each diagram, map or photograph described below. Give each answer as a ratio of two numbers.

a Two towns 3600 km apart are 12 cm apart on a map.

b A tree that is 80 m tall measures 4 cm on a photograph.

c A flea that is 3 mm long measures 15 cm on a microscope photograph.

d A hair that is 0.03 mm thick is 2 cm thick on a microscope photograph.

e A model aeroplane has a wingspan of 3.2 m, and the actual aeroplane has a wingspan of 80 m.

f A galaxy that is 120 light years across measures 10 cm on a telescope photograph. (You may take 1 light year to be 10^{16} m.)

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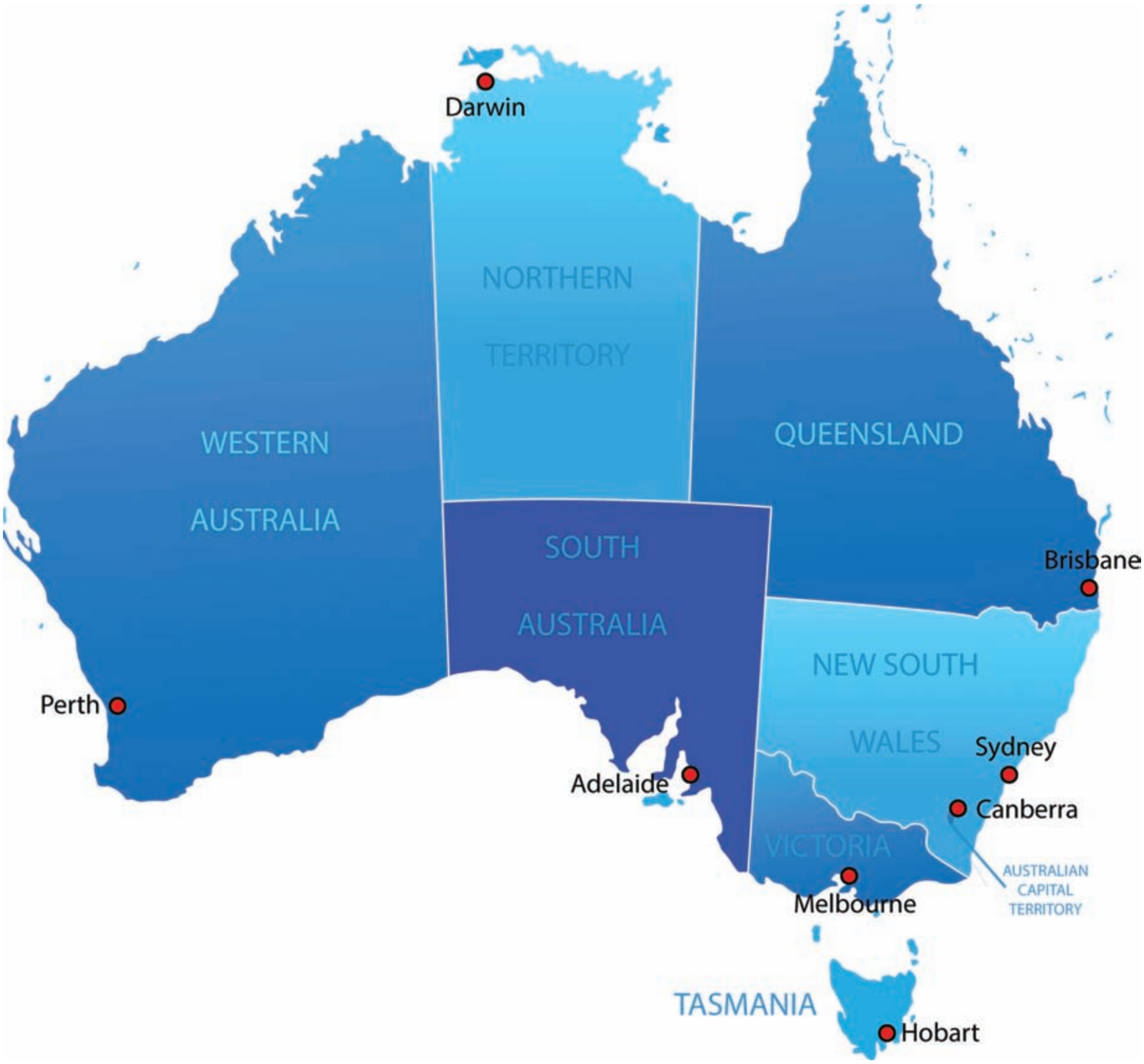
Example 24

9 The map of Australia shown below has a scale of 1: 28 000 000. Use your ruler to measure each of the following straight-line distances on the map, then calculate the actual distance.



scale = 1: 28 000 000

a Perth to Canberra

b Adelaide to Canberra

c Melbourne to Canberra

d Hobart to Canberra

e Sydney to Canberra

f Brisbane to Canberra

g The longest east–west distance lying entirely within the country

h The longest north–south distance lying entirely within the country (including Tasmania)

i The length of the border between South Australia and Western Australia

j The shortest distance across Bass Strait

10 Referring to the map in Question **9**, the shape of South Australia can be approximated by a rectangle.

a Draw a rectangle whose area is a reasonable approximation of the area of South Australia.

b Measure its side lengths, and hence calculate the approximate area of the state.

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11 The illustration of a fly shown below has a scale of 10 : 1.

scale = 10 : 1

a Measure the length of the fly in the illustration, correct to the nearest millimetre, then calculate its actual length.

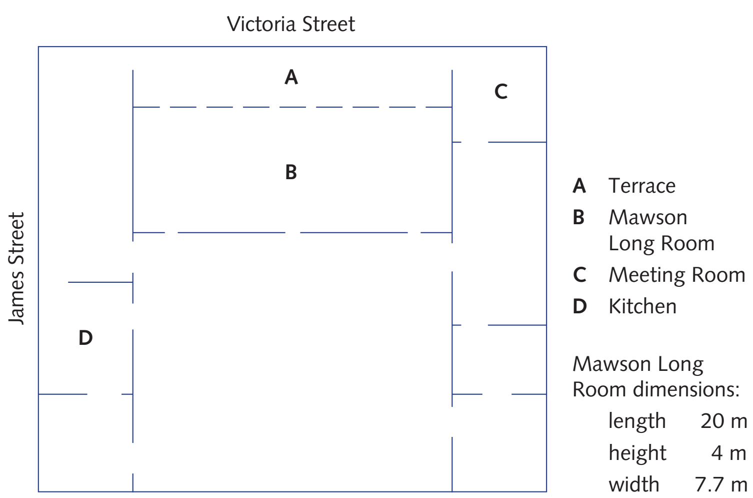
b Measure the length of the wing in the illustration, correct to the nearest millimetre, then calculate its approximate length.

12 The diagram below shows the basic floor plan of the first floor of a public building. Note that the dimensions of the Mawson Long Room are given.

a Use these dimensions, together with your own measurements of the plan, to work out, approximately, the scale of the drawing.

b Hence, find the lengths of the frontages to James Street and Victoria Street, correct to the nearest metre.

c Find the dimensions of the meeting room, correct to the nearest metre, and hence calculate its area, correct to the nearest square metre.



13 **Investigation activity:** Draw a plan of your bedroom (looking down from above) showing the positions of any doors and windows, and all items of furniture.

a First measure the dimensions of your room, and work out a suitable scale that will allow you to draw the plan on a page of your exercise book.

b Then measure the dimensions and position of each item in the room, reduce them to scale, and draw each item on the plan.

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Review exercise

Note: Some of these questions can be done mentally. Do as many as you can in your head.

1 **a** Divide 735 in the ratio 3 : 2. **b** Divide 735 in the ratio 4 : 5 : 6.

c Divide 342 in the ratio 3 : 2 : 1. **d** Divide 600 in the ratio 5 : 6.

e Divide 660 in the ratio 3 : 8. **f** Divide 6600 in the ratio 2 : 3 : 6.

2 Reduce each ratio to simplest form:

a 6 : 3 **b** 45 : 105 **c** 16 : 96 **d** 64 : 108

e 441 : 270 : 300 **f** 78 : 156 : 222 **g** $2\frac{1}{2} : 3\frac{1}{4}$ **h** $1\frac{1}{2} : 1\frac{1}{8}$

3 Find the cost of:

a 6 magazines, if 8 magazines cost $100 (assuming that all the magazines have the same price)

b 20 packets of rice, if 30 packets cost $82.50

4 A shirt manufacturer decides that he can supply 280 shirts in 4 weeks using 7 machinists. How long would it take for 15 machinists to produce 1000 shirts?

5 If apples are sold at 10 for $3, find the number of apples that can be bought with:

a $36 **b** $22.50

6 Sally is knitting jumpers for newly born babies.

a If she knits at a rate of 2 rows per minute, and each row of knitting contains 45 stitches, how many stitches per second does she knit?

b If it takes Sally 5 hours to knit a jumper, and there are 150 rows of knitting in each jumper, how many rows per minute does she knit?

c If it takes Sally 4 hours to knit a jumper, and there are 144 rows of knitting in each jumper with, on average, 50 stitches per row, how many stitches per minute does she knit?

7 Joe walks every morning to keep fit.

a If he walks a distance of 10 km at an average speed of 6 km/h, how long, in minutes, does his morning walk take him?

b If he walks a distance of 7 km and it takes him 80 minutes, what is his average speed, in km/h?

c If he walks at an average speed of 5 km/h for 140 minutes, how far does he walk?

8 Divide $15 in the ratio 1 : 4 : 5.

9 A 250 mL packet of milk contains 8 g of protein and 12.5 g of carbohydrate. Give the ratio of protein to carbohydrate in simplest form.

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10 A rope is cut into three pieces whose lengths are in the ratio 1 : 3 : 6. Given that the length of the longest piece is 36 m, find:

a the length of the original rope

b the length of the shortest piece of rope

11 The costs of material, labour and administration for an advertising campaign are in the ratio 8 : 5 : 2. If the total cost of the campaign is $35 700, find the cost of the labour.

12 A machine makes 720 bottles in 12 hours. How many bottles does it make in 40 minutes?

13 The total amount of prize money in a photography competition is $19 600. Given that the prize money is divided among the first, second and third prizes in the ratio 7 : 5 : 2, find the amount each prize winner receives.

14 A rectangle has length 8 cm and width 4 cm. Another rectangle has length 12 cm and width 6 cm. Find:

a the ratio of the lengths

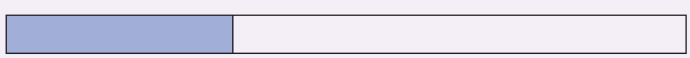
b the ratio of the widths

c the ratio of the perimeters

d the ratio of the areas

15 A hockey team played 27 matches in a season. The ratio of losses to wins was 4 : 5. How many games did the team win and how many did it lose?

16 The diagram below shows a ribbon. One-third of the ribbon is blue. Give the ratio (length of blue section):(length of white section).



17 In the diagram below, the ratio $XA : AB : BY = 2 : 3 : 1$.

Find the following ratios.

a $XB : BY$

b $XA : XY$

c $AB : XY$

d $XA : AY$

18 Three numbers are in the ratio 2 : 5 : 3. If the largest number is 120, find the other two numbers.

19 The sides of a right-angled triangle are 5 cm, 12 cm and 13 cm. A rectangle has sides 3 cm and 5 cm. Find the ratio of the area of the triangle to the area of the rectangle.

20 The scale of a map reads 1 : 350 000. Find the distance apart of two towns that are:

a 3 cm apart on the map

b 35 mm apart on the map

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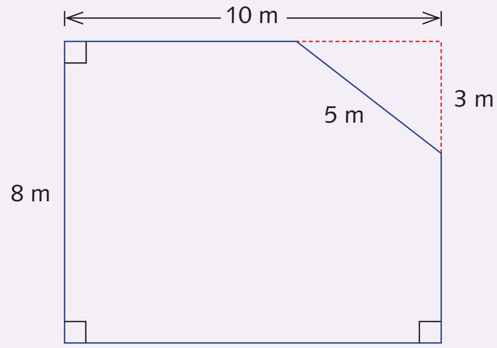
Challenge exercise

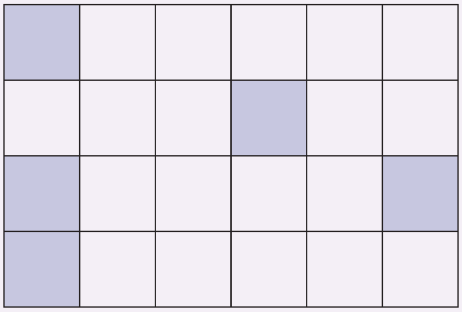
1 It took David an hour to ride 20 km from his house to the nearest town. He then spent 40 minutes on the return journey. What was his average speed?

2 The diagram below shows a metal plate that is to be painted.

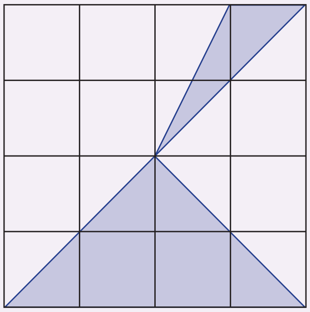


How much will it cost to paint the plate if paint costs $2.50 per square metre?

3 Some of the squares in the diagram below are shaded. More squares need to be shaded if the ratio (number of unshaded squares) : (number of shaded squares) is to be 2 : 1. How many more squares need to be shaded?



4 In the diagram below, find the ratio of the shaded area to the unshaded area.



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5 Four points A, B, C and D are marked in order on a line such that $AB : AC = 3 : 5$ and $BD : CD = 7 : 2$. If CD is 10 cm long, find the length of AB.

6 A container is filled with 64 L of orange juice, then 12 L of juice are removed and the container is topped up with lemon juice. The juice is thoroughly mixed before 12 L of the mixture is removed and the container is again topped up with lemon juice. What is the ratio of orange juice to lemon juice in the final mixture?

7 Two equal-sized vats, A and B, each containing 1000 L of oil, are being drained at a constant rate. It takes 4 hours to drain vat A completely and 5 hours to drain vat B completely. Find the time at which the amount of oil in vat B is four times the amount of oil in vat A.

8 **a** Two containers each contain 1 L of orange cordial. The first container contains cordial concentrate and water in the ratio $1 : 9$, and the second container contains cordial concentrate and water in the ratio $1 : 8$. The contents of both containers are tipped into a 2-litre bottle. What is the ratio of cordial concentrate to water in the 2-litre bottle?

b A bottle contains 1 L of cordial. The ratio of cordial concentrate to water is $1 : 5$. Another bottle contains 0.5 L of cordial. The ratio of cordial concentrate to water in this bottle is $1 : 4$. The contents of both bottles are tipped into a third container. What is the ratio of cordial concentrate to water in this third container?

c A bottle contains one-third of a litre of cordial, which consists of water and cordial concentrate in the ratio $1 : 4$. Two-thirds of a litre of water are then poured in. What is the ratio of concentrate to water now?

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