Algebra – part 2

We review and extend the use of negative numbers and fractions in algebra. In the course of this chapter, we will introduce new ways of solving problems from topics you have studied arising in the year, such as percentages, speed and ratios.

11A Expanding brackets and collecting like terms

We learned how to expand brackets in Chapter 6. We now review that work, including expressions with negative integers and negative fractions.

Example 1

Simplify:

a $-3a \times (-4)$

b $-(-3a)$

c $6a - (-a)$

d $-7a \times 6a$

Solution

a $-3a \times (-4) = 12a$

b $-(-3a) = 3a$

c $6a - (-a) = 6a + a = 7a$

d $-7a \times 6a = -42a^2$

We recall two distributive laws introduced earlier:

$$a(b + c) = ab + ac \qquad \text{and} \qquad a(b - c) = ab - ac$$

These are used in the following examples.

Example 2

Expand the brackets in each expression.

a $3(2x - 5)$

b $-4(2x - 6)$

c $-\dfrac{1}{2}(4x - 18)$

d $-(4 - 5a)$

Solution

a $3(2x - 5) = 6x - 15$

b $-4(2x - 6) = -8x + 24$

c $-\dfrac{1}{2}(4x - 18) = -2x + 9$

d $-(4 - 5a) = -4 + 5a$

Example 3

Expand the brackets and collect like terms.

a $3 - (6 - 5x)$

b $2x - (8 - 10x)$

c $3(-2x + 5) + 4(3x + 6)$

d $-4(-2x + 5) - 6(4 - 2x)$

Solution

a $3-(6-5x) = 3-6+5x$
$\qquad\qquad\quad = -3+5x$

b $2x-(8-10x) = 2x-8+10x$
$\qquad\qquad\qquad\quad = 12x-8$

c $3(-2x+5)+4(3x+6) = -6x+15+12x+24$
$\qquad\qquad\qquad\qquad\quad = -6x+12x+15+24$
$\qquad\qquad\qquad\qquad\quad = 6x+39$

d $-4(-2x+5)-6(4-2x) = 8x-20-24+12x$
$\qquad\qquad\qquad\qquad\quad\ = 8x+12x-20-24$
$\qquad\qquad\qquad\qquad\quad\ = 20x-44$

Exercise 11A

Example 1

1 Simplify:

a $-3a\times 2$ **b** $-6b\times(-5)$ **c** $-(-6m)$ **d** $2\times(-3a)$

e $-5a\times(-3b)$ **f** $-(-5h)$ **g** $-7\times(-3a)$ **h** $-m\times 3$

i $2a\times(-3a)$ **j** $-4x\times(-3x)$ **k** $-3a\times 5a$

Example 2

2 Expand the brackets in each case.

a $3(x+2)$ **b** $5(2x+3)$ **c** $6(6x+3)$

d $-5(4x+7)$ **e** $7(5a-6)$ **f** $-4(5+2b)$

g $7(3z+5)$ **h** $6(2a-b)$ **i** $-11(6a-5)$

j $-7(6-x)$ **k** $-6(-3x-4)$ **l** $-5(2a-3b)$

m $-3(-6m+5)$ **n** $-3(-m+8)$ **o** $-7(-7m-3)$

p $-4(2-3c)$ **q** $\frac{1}{3}(2x-9)$ **r** $-\left(10+\frac{1}{4}x\right)$

Example 3a, b

3 Expand the brackets and collect like terms in each case.

a $3(2x-5)+5x$ **b** $6(x-3)+10$ **c** $4(2x-3)-5$

d $6(x-7)+6x$ **e** $-4(n-6)+4n$ **f** $7(2m-5)+8$

g $-4(8k+7)+60k$ **h** $7(-8m-3)+5m$ **i** $-6(-h+3)-18$

j $-4(-2x+3)-8x$ **k** $-4(3-3k)-12k+12$ **l** $-5(-7-7m)-20m$

Example 3c, d

4 Expand the brackets and collect like terms.

a $4(2x+6)+5(3x+2)$ **b** $2(5x+3)+3(x+4)$

c $6(x-7)+6(x-5)$ **d** $5(2m-3)+6(3m+4)$

e $2(3m-4)-5(m+4)$ **f** $6(4k-2)-(k+6)$

g $7(z+6)-2(3z+4)$ **h** $7(5a+2)-(4a-3)$

i $7(2x+7)-5(6x-2)$ **j** $7(p-2)-8(3-p)$

k $8(q-5)-2(q+3)$ **l** $6(m-4)-6(3m-1)$

Recall how to add and subtract fractions. When the denominators of two fractions are the same, we use the common denominator and add the numerators. For example:

$$\frac{3}{7} + \frac{2}{7} = \frac{3+2}{7} = \frac{5}{7}$$

When the denominators of the fractions to be added are different, we first find equivalent fractions with the lowest common denominator. We then proceed as for fractions with the same denominator. For example:

$$\frac{3}{7} + \frac{2}{5} = \frac{15}{35} + \frac{14}{35} \quad \text{or} \quad \frac{3}{7} + \frac{2}{5} = \frac{15+14}{35}$$
$$= \frac{29}{35} \qquad\qquad\qquad = \frac{29}{35}$$

We will use the second notation most of the time.

Subtraction of two fractions is undertaken in a similar way.

We use the same procedures that we previously used to add and subtract algebraic fractions that are like terms.

Example 4

Express as a single fraction.

a $\dfrac{x}{7} + \dfrac{4x}{7}$ **b** $\dfrac{2x}{5} + \dfrac{x}{3}$ **c** $\dfrac{2z}{11} + \dfrac{z}{2}$ **d** $-\dfrac{m}{3} + \dfrac{2m}{5}$

Solution

a $\dfrac{x}{7} + \dfrac{4x}{7} = \dfrac{x+4x}{7}$

$\qquad\qquad = \dfrac{5x}{7}$

b $\dfrac{2x}{5} + \dfrac{x}{3} = \dfrac{6x+5x}{15}$

$\qquad\qquad = \dfrac{11x}{15}$

c $\dfrac{2z}{11} - \dfrac{z}{2} = \dfrac{4z-11z}{22}$

$\qquad\quad = \dfrac{-7z}{22}$

$\qquad\quad = -\dfrac{7z}{22}$

d $-\dfrac{m}{3} + \dfrac{2m}{5} = \dfrac{-5m+6m}{15}$

$\qquad\qquad = \dfrac{m}{15}$

(It is better to put the sign in front of the entire fraction.)

Example 5

Expand the brackets and collect like terms.

a $2\left(\dfrac{x}{3}+4\right)+\dfrac{x}{3}$

b $5\left(\dfrac{4x}{7}+6\right)+\dfrac{2x}{3}$

Solution

a $2\left(\dfrac{x}{3}+4\right)+\dfrac{x}{3}=\dfrac{2x}{3}+8+\dfrac{x}{3}$

$\qquad\qquad\qquad = \dfrac{3x}{3}+8$

$\qquad\qquad\qquad = x+8$

b $5\left(\dfrac{4x}{7}+6\right)+\dfrac{2x}{3}=\dfrac{20x}{7}+30+\dfrac{2x}{3}$

$\qquad\qquad\qquad = \dfrac{60x+14x}{21}+30$

$\qquad\qquad\qquad = \dfrac{74x}{21}+30$

Exercise 11B

Example 4

1 Simplify.

a $\dfrac{2x}{5}+\dfrac{x}{5}$

b $\dfrac{2x}{7}+\dfrac{3x}{7}$

c $\dfrac{5z}{11}-\dfrac{2z}{11}$

d $\dfrac{3m}{11}-\dfrac{2m}{11}$

e $\dfrac{2x}{3}+\dfrac{x}{2}$

f $\dfrac{4x}{5}+\dfrac{x}{2}$

g $\dfrac{5x}{7}-\dfrac{3x}{4}$

h $\dfrac{3a}{7}+\dfrac{a}{2}$

i $\dfrac{5x}{6}-\dfrac{4x}{7}$

j $\dfrac{5x}{7}+\dfrac{3x}{2}-\dfrac{5x}{4}$

k $\dfrac{6b}{11}+\dfrac{2b}{3}-\dfrac{b}{2}$

l $\dfrac{5x}{7}-\dfrac{x}{3}$

m $\dfrac{3x}{4}+\dfrac{3x}{2}-\dfrac{5x}{4}$

n $\dfrac{7x}{24}+\dfrac{2x}{3}-\dfrac{x}{4}$

o $\dfrac{5c}{3}-\dfrac{c}{5}$

p $\dfrac{7x}{24}+\dfrac{5x}{6}-\dfrac{x}{3}$

q $\dfrac{x}{6}-\dfrac{2x}{3}-\dfrac{x}{4}$

r $\dfrac{x}{24}+\dfrac{2x}{3}-\dfrac{3x}{4}$

s $\dfrac{7x}{12}+\dfrac{5x}{6}-\dfrac{2x}{3}$

t $-\dfrac{7x}{24}-\dfrac{5x}{6}-\dfrac{5x}{4}$

2 Collect like terms.

a $\dfrac{x}{3}+\dfrac{x}{4}+\dfrac{y}{3}+\dfrac{y}{4}$

b $\dfrac{x}{3}-\dfrac{x}{4}+\dfrac{y}{3}-\dfrac{y}{4}$

c $\dfrac{x}{3}+\dfrac{y}{4}+\dfrac{y}{3}+\dfrac{y}{5}$

d $\dfrac{y}{3}-\dfrac{x}{4}+\dfrac{y}{3}+\dfrac{x}{2}$

e $\dfrac{2x}{3}-\dfrac{x}{5}+\dfrac{2y}{3}-\dfrac{3y}{4}$

f $-\dfrac{2x}{3}+\dfrac{2y}{3}+\dfrac{4x}{5}-\dfrac{5y}{8}$

g $\dfrac{2x}{3}-\dfrac{3x}{7}+\dfrac{3y}{4}-\dfrac{5y}{8}$

h $-\dfrac{5x}{3}+\dfrac{11x}{6}-\dfrac{7y}{4}-\dfrac{5y}{8}$

i $\dfrac{2x}{5}-\dfrac{3x}{10}+\dfrac{y}{4}-\dfrac{11y}{8}$

Example 5

3 Expand the brackets and collect like terms.

a $4\left(\dfrac{x}{5}+6\right)+\dfrac{x}{5}$

b $5\left(\dfrac{4x}{7}+6\right)+\dfrac{2x}{7}$

c $5\left(\dfrac{2x}{3}+\dfrac{1}{2}\right)+\dfrac{4x}{7}$

d $-3\left(\dfrac{3x}{5}+2\right)-\dfrac{4x}{11}$

e $12\left(\dfrac{4x}{3}+\dfrac{1}{2}\right)+\dfrac{3x}{7}$

f $-7\left(\dfrac{7x}{12}+2\right)-\dfrac{4x}{3}$

g $-4\left(-\dfrac{x}{5}+8\right)+\dfrac{x}{5}+12$

h $-5\left(\dfrac{3x}{7}+12\right)-\dfrac{4x}{7}+30$

4 Expand the brackets and collect like terms.

a $2\left(3x+\dfrac{1}{2}\right)+x+\dfrac{1}{2}$

b $3\left(5x+\dfrac{3}{2}\right)+2x+\dfrac{1}{2}$

c $5\left(3x-\dfrac{3}{5}\right)-4x+\dfrac{1}{10}$

d $-2\left(3x+\dfrac{3}{2}\right)+6x+\dfrac{11}{2}$

e $-2\left(3x-\dfrac{11}{2}\right)+7x+4\tfrac{1}{2}$

f $6\left(-3x+\dfrac{3}{4}\right)-4x+\dfrac{11}{2}$

11C Solving equations

Setting up and solving equations is central to the study and application of mathematics. Solving equations was introduced in Chapter 6. We first review these methods, and then consider equations that involve adding or subtracting algebraic fractions.

Example 6

Solve each equations.

a $x+3=-2$

b $-2x=10$

c $\dfrac{x}{3}=-5$

d $2x+5=-6$

e $5-11x=-7$

Solution

a $\qquad x+3=-2$

$\boxed{-3}\qquad x=-2-3$ (Subtract 3 from both sides of the equation.)

$\qquad\qquad =-5$

(continued over page)

b
$$-2x = 10$$
$\boxed{\div(-2)}$ $x = \dfrac{10}{-2}$ (Divide both sides of the equation by −2.)
$$= -5$$

or

$\boxed{\times(-1)}$ $2x = -10$ (Multiply both sides of the equation by −1.)

$\boxed{\div(-2)}$ $x = -5$

c
$$\dfrac{x}{3} = -5$$
$\boxed{\times 3}$ $x = -15$ (Multiply both sides by 3.)

d
$$2x + 5 = -6$$
$\boxed{-5}$ $2x = -11$ (Subtract 5 from both sides of the equation.)

(Divide both sides by 2.)

$\boxed{\div 2}$ $x = \dfrac{-11}{2}$

$$= -5\tfrac{1}{2}$$

e
$$5 - 11x = -7$$
$\boxed{-5}$ $-11x = -12$ (Subtract 5 from both sides of the equation.)

$\boxed{\div -11}$ $x = \dfrac{-12}{-11}$ (Divide both sides by −11.)

$$= 1\tfrac{1}{11}$$

or

$$11x = 12$$ (Multiply both sides of the equation by −1.)
$$x = 1\tfrac{1}{11}$$ (Divide both sides by 11.)

Example 7

Collect like terms and solve.

a $5x + 3x + 4 = 36$ **b** $2(3z + 4) + 5 = 20$

c $5x + 5 = 2x - 4$ **d** $-5x + 3 = 2x - 11$

Solution

a
$$5x + 3x + 4 = 36$$
$$8x + 4 = 36$$ (Collect like terms.)
$\boxed{-4}$ $8x = 32$ (Subtract 4 from both sides.)
$\boxed{\div 8}$ $x = 4$ (Divide both sides by 8.)

b
$$2(3z + 4) + 5 = 20$$
$$6z + 8 + 5 = 20 \quad \text{(Expand brackets.)}$$
$$6z + 13 = 20 \quad \text{(Collect like terms.)}$$
$$\boxed{-13} \quad 6z = 7 \quad \text{(Subtract 13 from both sides.)}$$
$$\boxed{\div 6} \quad z = 1\tfrac{1}{6} \quad \text{(Divide both sides by 6.)}$$

c
$$5x + 5 = 2x - 4$$
$$\boxed{-2x} \quad 3x + 5 = -4 \quad \text{(Subtract } 2x \text{ from both sides of the equation.)}$$
$$3x = -9 \quad \text{(Subtract 5 from both sides of the equation.)}$$
$$\boxed{\div 3} \quad x = -3 \quad \text{(Divide both sides by 3.)}$$

d
$$-5x + 3 = 2x - 11$$
$$\boxed{+5x} \quad 3 = 7x - 11 \quad \text{(Add } 5x \text{ to both sides.)}$$
$$\boxed{+11} \quad 14 = 7x \quad \text{(Add 11 to both sides.)}$$
$$\boxed{\div 7} \quad 2 = x \quad \text{(Divide both sides by 7.)}$$

We can also use the methods from the previous section to solve equations involving fractions. We consider two ways of solving the equation $\dfrac{m}{5} + \dfrac{m}{3} = 1$.

Method 1

Simplify the left-hand side.

$$\frac{m}{5} + \frac{m}{3} = 1$$
$$\frac{3m + 5m}{15} = 1 \quad \text{(Obtain a common denominator.)}$$
$$\frac{8m}{15} = 1$$
$$\boxed{\times 15} \quad 8m = 15 \quad \text{(Multiply both sides of the equation by 15.)}$$
$$\boxed{\div 8} \quad m = 1\tfrac{7}{8} \quad \text{(Divide both sides of the equation by 8.)}$$

Method 2

Multiply through by the lowest common multiple.

$$\frac{m}{5} + \frac{m}{3} = 1$$
$$3m + 5m = 15 \quad \text{(Multiply both sides of the equation by 15.)}$$
$$8m = 15 \quad \left(\frac{m}{5} \times 15 = 3m \text{ and } \frac{m}{3} \times 15 = 5m \right)$$
$$m = 1\tfrac{7}{8} \quad \text{(Divide both sides of the equation by 8.)}$$

Example 8

Solve each equation for m.

a $\dfrac{m}{6} - \dfrac{m}{7} = 10$

b $\dfrac{2m}{5} - \dfrac{m}{4} = 21$

Solution

a We shall use method 2.

$$\frac{m}{6} - \frac{m}{7} = 10$$
$$7m - 6m = 420 \qquad \text{(Multiply both sides of the equation by 42.)}$$
$$m = 420$$

$$\begin{aligned}
Check : \text{LHS} &= \frac{m}{6} - \frac{m}{7} \\
&= \frac{240}{6} - \frac{420}{7} \\
&= 70 - 60 \\
&= 10 \\
&= \text{RHS}
\end{aligned}$$

b We shall use method 1.

$$\frac{2m}{5} - \frac{m}{4} = 21$$
$$\frac{8m - 5m}{20} = 21 \qquad \text{(Obtain a common denominator.)}$$
$$\frac{3m}{20} = 21$$
$$\boxed{\times 20} \quad 3m = 420 \qquad \text{(Multiply both sides of the equation by 20.)}$$
$$\boxed{\div 3} \qquad m = 140 \qquad \text{(Divide both sides of the equation by 3.)}$$

Exercise 11C

Example 6

1 Solve these equations.

a $x + 7 = -12$

b $x - 11 = -14$

c $x + 7 = 7$

d $5x = -12$

e $-9x = -27$

f $-8x = 8$

g $\dfrac{x}{5} = -7$

h $\dfrac{-5x}{2} = -6$

i $3x + 1 = -8$

j $5x - 7 = -14$

k $-2x + 11 = -12$

l $4x - 6 = -8$

Example 7

2 Solve these equations.

a $2x + 7x + 5 = 41$ **b** $6x - 3x + 5 = 3$ **c** $z - 5z + 10 = 13$

d $5(x + 3) + 4 = 39$ **e** $3(2z + 5) + 5 = 28$ **f** $-4(3x + 1) + 7 = 22$

g $6x + 4 = 7x + 5$ **h** $-3y + 11 = y + 13$ **i** $7x + 2 = 3x + 1$

j $5x - 2 = 8x + 1$ **k** $x + 14 = 4x + 10$ **l** $13x + 4 = 2x + 7$

m $3(2x - 4) = x$ **n** $5(2x - 4) = 11x$ **o** $\dfrac{p}{3} - 5 = p$

p $\dfrac{1}{2}(6 - 2x) = x - 3$

3 Solve these equations.

a $\dfrac{x}{3} - 4 = 11$ **b** $\dfrac{2x}{5} + 5 = 25$ **c** $\dfrac{5x}{7} + 2 = 11$

d $5 - \dfrac{4x}{5} = 7$ **e** $\dfrac{5x}{7} - 2 = 30$ **f** $\dfrac{7x}{11} - 1 = 6$

Example 8

4 Solve these equations.

a $\dfrac{x}{5} + \dfrac{2x}{5} = 1$ **b** $\dfrac{4m}{7} - \dfrac{2m}{7} = 10$ **c** $\dfrac{3m}{4} - \dfrac{m}{4} = 20$ **d** $\dfrac{5m}{2} + \dfrac{2m}{3} = 6$

e $\dfrac{3x}{4} + \dfrac{2x}{5} = 6$ **f** $\dfrac{3x}{5} + \dfrac{x}{7} = 5$ **g** $\dfrac{3x}{11} + \dfrac{3x}{4} = 8$ **h** $\dfrac{5x}{8} - \dfrac{x}{4} = 72$

i $\dfrac{5x}{6} - \dfrac{3x}{4} = 40$ **j** $\dfrac{x}{7} - \dfrac{x}{8} = 1$ **k** $3\left(\dfrac{m}{6} - \dfrac{m}{9}\right) = 21$ **l** $\dfrac{3x}{8} + \dfrac{3x}{4} + 12 = 8$

5 Is there a solution to the equation $x + 3 = x + 5$? Explain your answer.

6 Trang can run up a hill at 3 m/s and down the hill at 7 m/s. It takes her 60 seconds in total to run up and then down the hill. What distance does she cover going from the bottom to the top of the hill?

11D Problem-solving with equations

The next three examples demonstrate the usefulness of equations in solving different kinds of problems.

Example 9

I think of a number, multiply it by 3 and add 4 to the result. The number I obtain is 10 more than the number I first thought of. What was my original number?

Solution

Let x be the number.

$$3x + 4 = x + 10$$
$$2x + 4 = 10 \qquad \text{(Subtract } x \text{ from both sides of the equation.)}$$
$$2x = 6 \qquad \text{(Subtract 4 from both sides of the equation.)}$$
$$x = 3 \qquad \text{(Divide both sides of the equation by 2.)}$$

The number I first thought of was 3.

Example 10

Josephine bought a desktop computer and a printer at a total cost of $1560. The desktop computer cost $5\frac{1}{2}$ times as much as the printer. Write an equation and find the cost of the computer and the cost of the printer.

Solution

Let $\$x$ be the cost of the printer.

The computer cost $\$\left(5\frac{1}{2} \times x\right) = \$\dfrac{11x}{2}$

The total cost $= \$1560$

So $\dfrac{11x}{2} + x = 1560$

$\quad\quad \dfrac{13x}{2} = 1560$

$\quad\quad\quad x = \dfrac{3120}{13}$

$\quad\quad\quad\quad = 240$

Thus the printer cost $240 and the computer cost $1320.

Example 11

Grant runs half the distance to school and walks for the remainder of the journey. He runs at 3 m/s but slows to 2 m/s for the second half of his trip. He takes 50 minutes to complete the trip. Find the distance Grant has to travel to school.

Solution

We will use metres and seconds as our units. Let x m be half the distance to the school in metres. Recall that:

$\quad$ distance $=$ speed $\times$ time

and therefore:

$\quad$ time $= \dfrac{\text{distance}}{\text{speed}}$
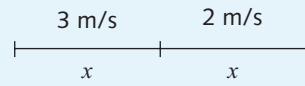
Then time spent running at 3 m/s $= \dfrac{x}{3}$ seconds

and time spent walking at 2 m/s $= \dfrac{x}{2}$ seconds

Also, time taken for the entire trip $= 50$ minutes

$= 3000$ seconds

Hence:

$$\frac{x}{3} + \frac{x}{2} = 3000$$

$$2x + 3x = 18\,000 \qquad \text{(Multiply both sides of the equation by 6.)}$$

$$5x = 18\,000 \qquad \text{(Divide both sides of the equation by 5.)}$$

$\boxed{\div 5} \qquad x = 3600$

Thus half the distance Grant travels from home to school is 3600 m, so the full distance he travels is 7200 m or 7.2 km.

Exercise 11D

Example 9

1 For each problem, write an equation and solve it to find the unknown number.

  a Six is added to a number, x, and the result is multiplied by -3. The result of this is -72.

  b One-third of a number, x, and one-quarter of the same number are added, and the result is 25.

  c Three-fifths of a number, z, and one-seventh of the same number are added and the result is 30.

  d Two-thirds of a number, m, is subtracted from three-fifths of the same number, and the result is 1.

  e The result of multiplying a number, a, by -2 and adding 6 is the same as the result of multiplying a by 3 and subtracting 4.

2 For each problem, write an equation and find the unknown number.

  a When half of a number is subtracted from two-thirds of the same number, the result is 10.

  b Ten is subtracted from a number, and the result is multiplied by -5. The result of this is 30.

  c Six is added to two-thirds of a number, and the result is -10.

  d Two-thirds is subtracted from half of a number, and the result is -1.

Example 10

3 Chloe buys x kg of bananas for \$3 a kilogram, and x kg of apples for \$2 a kilogram. The total cost is \$15.50. Find the value of x.

4 The length of a rectangle is twice its width. The perimeter is 13 cm. Let x cm be the width. Write an equation for x and find the width and length.

5 The average of three numbers is −56. Two of the numbers are 20 and −10. The third number is x. Write an equation for x and solve it.

6 The sum of eight numbers is x, and the sum of a different set of nine numbers is $2x$. The average of the smaller set of numbers is one more than the average of the larger set of numbers. Write an equation for x and solve it.

7 Five hundred dollars more than 10% of an amount of money $\$m$ is \$1280. Find m.

8 Trevor invests \$10 000 for a year. He obtains 8% per annum for a part of the money and 10% per annum for the remainder. At the end of the year, he will receive \$950 in interest. How much does he invest at 10%?

9 Anthony is twice as old as Julian. Five years ago, he was three times as old. Let x be Julian's present age. Write an equation for x and find the present ages of both Anthony and Julian.

10 Giorgia cycles to school every day. She manages to go half the distance at 4 m/s but slows to 3 m/s for the second half of her trip. She takes 35 minutes to complete the trip. Find the distance she travels, to the nearest metre.

11 Frances travels between two places, Akville and Bracktown. She walks half the distance at 5 km/h and runs the other half at 10 km/h. If the total time for her journey is 3 hours, what is the distance from Akville to Bracktown?

12 When one-third of a number is added to three-quarters of a number, the result is 10 more than the number. Find the number.

13 If $\dfrac{3}{4}$ of one of the acute angles of a right-angled triangle is $15\frac{1}{4}°$ larger than $\dfrac{1}{6}$ of the other, find the acute angles.

Review exercise

1 Expand the brackets.

   a $2(5x+7)$ **b** $4(2b+3)$ **c** $6(7-3x)$

   d $-5(4x+9)$ **e** $-3(3a-2)$ **f** $-4(5-8x)$

2 Expand the brackets and collect like terms.

   a $5(x-2)+7$ **b** $3(4x+1)-5x$

   c $3(5y+3)+4$ **d** $5(x-40)+3x$

   e $6(x+2)+7$ **f** $-5(2x-3)+11x$

   g $-4(1+5x)-2$ **h** $-3(4y-7)+10y$

3 Expand the brackets and collect like terms.

a $2(2x+3)+3(x+4)$ **b** $6(x+5)+7(x-3)$

c $3(x-2)-4(x+2)$ **d** $2(3y+4)+4(4y-1)$

e $5(x+2)-3(7+x)$ **f** $7(5+x)-2(3+x)$

g $-5(a+2)-4(2a+2)$ **h** $-2(x-5)-3(x-6)$

4 Express each expression as a single fraction.

a $\dfrac{3x}{8}+\dfrac{x}{8}$ **b** $\dfrac{4x}{5}-\dfrac{2x}{5}$ **c** $\dfrac{2x}{7}-\dfrac{5x}{7}$ **d** $\dfrac{4x}{5}+\dfrac{x}{2}$

e $\dfrac{6x}{5}-\dfrac{x}{4}$ **f** $\dfrac{2x}{7}-\dfrac{2x}{3}$ **g** $\dfrac{2x}{3}+\dfrac{x}{2}-\dfrac{3x}{4}$ **h** $\dfrac{3x}{5}-\dfrac{2x}{3}+\dfrac{x}{2}$

5 Expand the brackets and collect like terms.

a $2\left(\dfrac{x}{3}+5\right)+\dfrac{x}{3}$ **b** $3\left(3-\dfrac{x}{4}\right)+\dfrac{x}{4}$ **c** $-2\left(\dfrac{x}{5}+3\right)-\dfrac{x}{5}$

d $2\left(\dfrac{3x}{5}+1\right)+\dfrac{x}{4}$ **e** $-\left(4+\dfrac{2x}{7}\right)-\dfrac{3x}{5}$ **f** $2\left(\dfrac{2x}{5}-3\right)+\dfrac{x}{4}$

6 Solve each equation for x.

a $x+3=5$ **b** $x+2=-7$ **c** $x-2=3$ **d** $x-4=-2$

e $x+5=-9$ **f** $x-1=-5$ **g** $6x=36$ **h** $5x=-20$

i $-7x=28$ **j** $-6x=-30$ **k** $2x+5=9$ **l** $7x+3=24$

m $2x-3=5$ **n** $4x-8=-12$ **o** $8x-2=30$ **p** $\dfrac{x}{5}=-20$

q $\dfrac{x}{2}+3=-9$ **r** $2-\dfrac{x}{5}=8$ **s** $3x+5=4$ **t** $5+4x=-8$

7 Solve each equation for x.

a $3x+2x+6=21$ **b** $3(x+2)+3=15$ **c** $-2(3x+2)+4=5$

d $3x=x+4$ **e** $2x=x-6$ **f** $3x=8-4x$

g $8x+2=7x-3$ **h** $5x-7=-x+9$ **i** $2x-4=5x+1$

j $15x+4=3x-2$ **k** $2x-5=-4x+6$ **l** $x-21=5x-24$

8 Solve these equations.

a $\dfrac{2x}{7}+\dfrac{x}{7}=6$ **b** $\dfrac{5m}{9}-\dfrac{2m}{9}=1$ **c** $\dfrac{x}{5}+\dfrac{2x}{5}=1$

d $\dfrac{3x}{7}+\dfrac{x}{5}=2$ **e** $\dfrac{m}{3}-\dfrac{2m}{5}=1$ **f** $\dfrac{4x}{9}-\dfrac{4x}{5}=1$

9 Four people each paid $2 towards the cost of a raffle ticket, which won a prize of \$x. The prize was shared equally amongst the four, and each person made a profit of \$123. What was the amount of the prize?

10 For each problem, write an equation and solve it to find the unknown number.

   a Ten is added to a number, x, and the result is multiplied by $\frac{1}{2}$. The result of this is -32.

   b One-quarter of a number, x, and one-fifth of the same number are added. The result is 60.

   c The result of multiplying a number, a, by -6 and adding 3 is the same as the result of multiplying a by 10 and subtracting 20.

   d Two-sevenths of a number, z, is subtracted from three-fifths of the same number, and the result is 10.

   e Six is added to seven-tenths of a number, z, and the result is the same as subtracting 4 from z.

11 One thousand dollars more than 20% of an amount of money $\$m$ is $\$16\,800$. Find m.

12 The perimeter of this quadrilateral is 100 cm. Find the value of x and the lengths of the four sides.
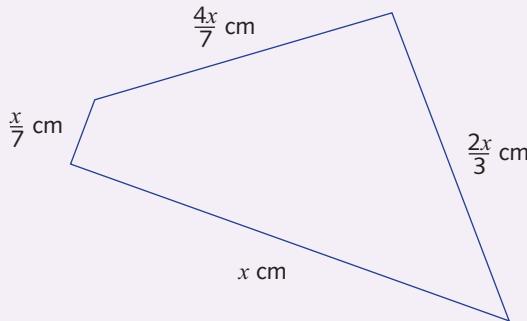
Challenge exercise

1 Solve each equation for x.

   a $\dfrac{x-4}{5} + \dfrac{2x-5}{6} = 3$

   b $\dfrac{x-5}{4} + \dfrac{5x-3}{10} = -5$

2 Think of a number, add 30 to it, multiply the result by 5, add 5 times the number you first thought of, subtract 50, divide the result by 10 and subtract 10. Use algebra to show that you always get the number you first thought of.

3 Two towns are 50 km apart. Tom starts from town A and travels at 50 km/h towards town B. Clarrie starts from town B at the same time that Tom started, and travels at 40 km/h towards town A.

  a After how many minutes do they meet?

  b How far from A are they when they meet?

4 Elizabeth travels from Crocville to Barratown. For the first d km she can travel at 80 km/h, but after that she has to reduce her speed to 60 km/h. It takes Elizabeth 3 hours to travel the 200 km from Crocville to Barratown. How far does Elizabeth drive at 80 km/h?

5 I think of a number. I add one-third of that number and one-quarter of the same number to get 5 more than the number I first thought of. What is this number?

6 David travels from town A to town B. He walks half the distance at 3 km/h and runs the other half at 10 km/h. The total time for the journey is 4 hours. What is the distance from A to B?

7 A sum of money is divided equally between three friends, Harry, Larry and Carrie. An equal amount is divided between Anne, Leslie and Bronwyn in the ratio $2:3:5$. If Harry receives $28 more than Anne, how much does Bronwyn receive?

8 David buys $2x$ worth of grapes at $5 a kilogram, and x worth of peaches at $7 a kilogram. He buys a total of 10 kg of peaches and grapes. How much of each fruit does he buy?

9 If x is subtracted from both the numerator and denominator of $\dfrac{3}{4}$, the result is $\dfrac{7}{10}$. Find x.

10 A school has 1025 students. A total of 400 students cannot swim. This consists of $\dfrac{1}{5}$ of the boys and $\dfrac{4}{7}$ of the girls. If x boys can swim, write an equation for x and solve it. How many boys are there in the school?

11 Solution A contains 40% of concentrated acid and solution B contains 60% of the same concentrated acid. How many cubic centimetres of each type are needed to produce 1000 cm^3 of mixture containing 55% concentrated acid?

12 Water flows from tank A to tank B at a rate of 2 litres per minute. Initially tank A has 200 litres in it and tank B has 100 litres in it. Water drains from tank B at 0.5 litres per minute. After how many minutes are there equal volumes of water in the two tanks?