CHAPTER

14

Measurement and Geometry

Circles

Squares, rectangles and triangles are all geometrical shapes that have straight edges. The circle is the first geometrical object we come across that has a curved edge.

The points on a circle each lie a fixed distance from the point O, called the **centre**. This distance is called the **radius**.

In this chapter we shall obtain formulas for the circumference of a circle and also for the area of a circle. These formulas use an amazing number called **pi** (π), which is approximately 3.14.

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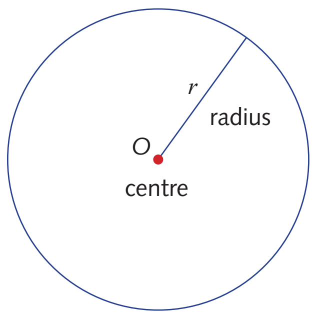
14A Features of the circle

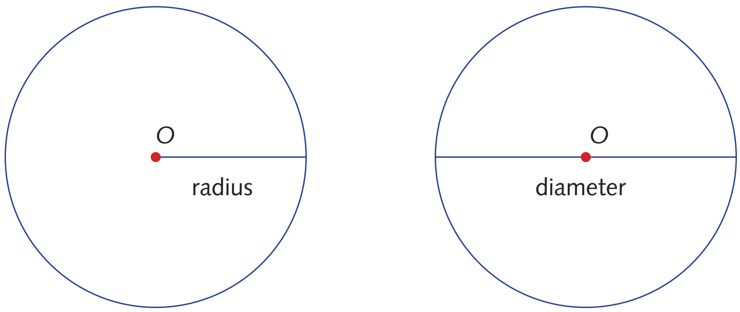
A **circle** is formed by all points that lie a fixed distance r, called the **radius**, from a fixed point O, called the **centre**.



Given a circle, any interval drawn from the centre to any point on the circle is called a **radius** of the circle. (The plural of the word 'radius' is radii.) Thus we use the word 'radius' in two senses: it means an interval joining the centre to a point on the circle, and it also means the length of such an interval.

Since the distance from the centre to any point on the circle is always the same, all radii of a given circle have the same length. This is how compasses work.

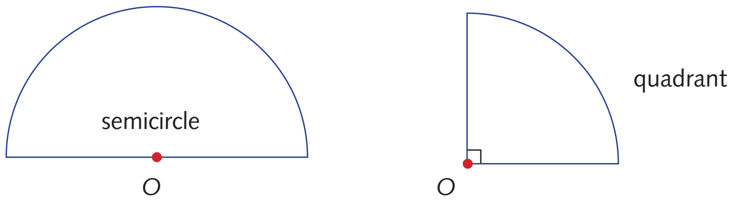
Any interval joining two points on the circle and passing through the centre is called a **diameter** of the circle. Any two diameters of a given circle have the same length.



Notice that the diameter of a circle is equal to twice its radius, and so the radius of a circle is half the diameter.

A diameter divides the circle into two equal parts. Each part is called a **semicircle**.

If we draw a radius that cuts the semicircle into two equal parts, then each part is called a **quadrant**. The Latin root *quad* means 'four'. A circle can be cut into four quadrants.



Any two radii divide the circle into two (not necessarily equal) pieces. Each piece is called a **sector**. The word 'sector' comes from the Latin word *secare*, meaning 'to cut'. When you cut up a pizza, you normally cut the pizza into sectors.

The angle between two radii is called the **angle contained in the sector**.

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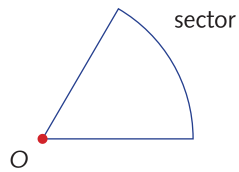

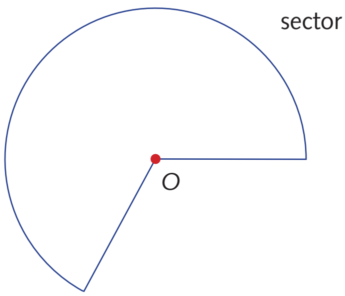

A quadrant and a semicircle are special kinds of sectors for which the angle of the sectors are 90° and 180° respectively.

Features of the circle

- A circle is formed by all the points that lie a fixed distance r from a fixed point O.
- Any interval drawn from the centre to a point on the circle is called a **radius** of the circle.
- Any interval joining two points on the circle and passing through the centre is called a **diameter** of the circle.
- A diameter divides the circle into two equal parts. Each part is called a **semicircle**.
- If a radius is drawn cutting a semicircle into two equal parts, then each part is called a **quadrant**.
- Any two radii divide the circle into two pieces. Each piece is called a **sector**.

Exercise 14A

1 Use your compasses to draw a circle with:

a radius 5 cm

b radius 7 cm

c diameter 12 cm

2 Use your protractor and compasses to draw a sector with:

a radius 5 cm and containing an angle of 45°

b diameter 14 cm and containing an angle of 150°

3 Complete these statements.

a A quadrant is a sector containing an angle of ______°.

b A semicircle is a sector containing an angle of ______°.

c Two quadrants can be joined to form a ______.

4 **a** If you cut a circle into 3 equal sectors, what is the angle of the sector?

b Repeat for 5, 8 and 10 equal sectors.

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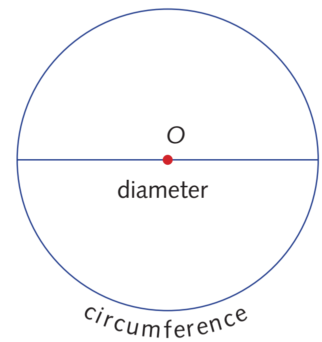
14B Circumference of a circle

Suppose we have drawn a circle of radius 2 cm. An obvious question to ask is 'What is the distance around the circle?' In other words, imagine that our circle is made of string. If we pull it tight and lay it on our ruler, what is the length of the string?



The distance around the edge of a circle (that is, the perimeter of the circle) is called its **circumference**. This word comes from the Latin words *circum*, meaning 'around', and *ferre*, which means 'to carry'.

The Greeks noticed that if you double the diameter, you double the circumference, and if you triple the diameter, you triple the circumference. In other words, the ratio of the circumference to the diameter is always the same.

Here are some (approximate) measurements of the circumferences and diameters of some circles.

Diameter (cm)	Circumference (cm)	$\frac{\text{Circumference}}{\text{Diameter}}$
2	6.3	3.15
3	9.4	3.13
4	12.6	3.15
5	15.7	3.14

You will notice from the table that, even though we measure different diameters and matching circumferences, the *ratio* of the two is always (approximately) the same. It is measurement and rounding error that makes the ratios appear slightly different each time.

This constant ratio is given the symbol π (pronounced 'pie') in mathematics. It is the Greek letter *pi*, and is equivalent to our 'p'. Thus, in any circle:

$$\frac{\text{circumference}}{\text{diameter}} = \pi$$

Using C for circumference and d for diameter, we can write the formula for the circumference of a circle as:

$$C = \pi d$$

Since the diameter is twice the radius, we can rewrite this formula, using the letter r for the radius, as:

$$C = 2\pi r$$

These formulas should be memorised.

The number π is an example of a decimal that does not terminate or repeat. We can display the first few places of the number π by writing:

$$\pi = 3.141\,592\,653\,58\ldots$$

We will normally round π to two decimal places, and take π as 3.14. When greater accuracy is required, we can take more decimal places, for example, $\pi \approx 3.1416$.

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There is also a fraction that is close (but *not equal*) to π. This is the fraction $3\frac{1}{7} = \dfrac{22}{7}$.
Note that $\dfrac{22}{7} = 3.142\,857\ldots$ while $\pi = 3.141\,592\,6\ldots$, so these numbers agree only to two decimal places.
We will write $\pi \approx 3.14$ or $\pi \approx \dfrac{22}{7}$ to express the *approximate* equality of the two sides. In fact π is slightly closer to $\dfrac{22}{7}$ than to 3.14.

Numbers such as π that are neither terminating nor recurring decimals are called **irrational numbers**. The number $\sqrt{2}$, which you met in Chapter 8 on Pythagoras' theorem, is similar to this. You will learn more about irrational numbers later in your study of mathematics.

The number π, in particular, is one that you will learn more and more about as you progress in mathematics. It is a truly amazing number!

Example 1

Find the circumference of a circle:

a with diameter 14 cm

b with radius 21 cm

Give each answer:

i in terms of π

ii as an approximate value, using $\pi \approx \dfrac{22}{7}$

Solution

a i $C = \pi d$
$= 14\pi$ cm

ii $C = 14\pi$
$\approx 14 \times \dfrac{22}{7}$
$= 44$ cm

b i $C = 2\pi r$
$= 2 \times \pi \times 21$
$= 42\pi$ cm

ii $C = 42\pi$
$\approx 42 \times \dfrac{22}{7}$
$= 132$ cm

Note that the fractional answers in part **ii** are only approximate, because we have used an approximation for π.

Example 2

Find the circumference of a circle:

a with diameter 5 cm

b with radius 10 cm

Give each answer:

i in terms of π

ii as an approximate value, using $\pi \approx 3.14$

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Solution

a i $C = \pi d$
$= 5\pi$ cm

ii $C = 5\pi$
$\approx 5 \times 3.14$
$= 15.70$ cm

b i $C = 2\pi r$
$= 2 \times \pi \times 10$
$= 20\pi$ cm

ii $C = 20\pi$
$\approx 20 \times 3.14$
$= 62.80$ cm

Circumference of the circle

In any circle:

- the ratio of the circumference to the diameter is
  $\frac{\text{circumference}}{\text{diameter}} = \pi$
- circumference = $\pi \times$ diameter, or $C = \pi d$
- circumference = $2\pi \times$ radius, or $C = 2\pi r$.

Exercise 14B

1 Take a large round object and try to measure (approximately) its circumference by rolling the object along a ruler or measuring tape. Measure the diameter of the object as carefully as you can and see how close the ratio of the circumference to the diameter is to π.

2 Use your ruler to measure (approximately) the diameter of a 20c coin in millimetres. Write down the radius and find the circumference of the coin. Repeat the exercise with a 10c coin and a 5c coin.

Example 1i

3 Find the circumference of a circle with the given radius. Leave your answers in terms of π.

a 14 cm **b** 7 cm **c** $3\frac{1}{2}$ mm **d** 42 m

Example 1ii

4 Use $\pi \approx \frac{22}{7}$ to find the approximate value of the circumference of a circle with radius:

a 14 cm **b** 7 cm **c** $3\frac{1}{2}$ mm **d** 42 m

5 Find the circumference of a circle with the given diameter. Leave your answers in terms of π.

a 14 cm **b** 7 cm **c** $3\frac{1}{2}$ mm **d** 42 m

6 Use $\pi \approx \frac{22}{7}$ to find the approximate value of the circumference of a circle with diameter:

a 14 cm **b** 7 cm **c** $3\frac{1}{2}$ mm **d** 42 m

Example 2i

7 Find the circumference of a circle with the given radius. Leave your answers in terms of π.

a 10 cm **b** 5 cm **c** 20 mm **d** 15 m

Example 2ii

8 Use $\pi \approx 3.14$ to find the approximate value of the circumference of a circle with radius:

a 10 cm **b** 5 cm **c** 20 mm **d** 15 m

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9 Find the circumference of a circle with the given diameter. Leave your answers in terms of π.

a 10 cm **b** 5 cm **c** 20 mm **d** 15 m

10 Use $\pi \approx 3.14$ to find the approximate value of the circumference of a circle with diameter:

a 10 cm **b** 5 cm **c** 20 mm **d** 15 m

11 If the radius of a circle is doubled, what happens to the circumference?

12 A circle has circumference 66 cm. Using $\pi = \frac{22}{7}$, find the approximate value of the diameter of the circle.

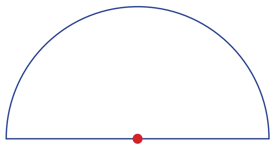
13 A circle has circumference 62.8 cm. Using $\pi \approx 3.14$, find the approximate value of the radius of the circle.

14 If the circumference of a circle is halved, what happens to the diameter?

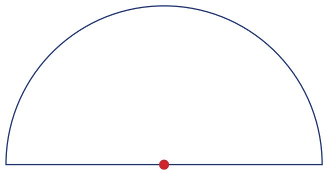
15 The perimeter of a semicircle is the distance around the semicircle (which includes the diameter).

a What is the perimeter of a semicircle with radius 35 cm?

Leave your answer in terms of π. (The dot indicates the centre of the full circle.)



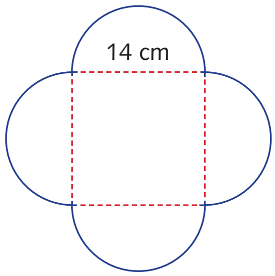
b What is the approximate value of the perimeter of a semicircle with radius 42 cm if we use $\pi \approx \frac{22}{7}$?



16 Four semicircles are drawn along the edges of a square with side length 14 cm.

a Find the perimeter of the region, giving your answer in terms of π.

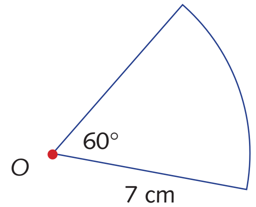
b Find the approximate value of the perimeter, using $\pi \approx \frac{22}{7}$.



17 The perimeter of a sector is the length of the arc plus twice the radius.

a Find the perimeter of the sector with radius 7 cm in which the angle at the centre is 60°. Leave your answer in terms of π.

b Now use $\pi \approx \frac{22}{7}$ to find the approximate value of the perimeter in part **a**.

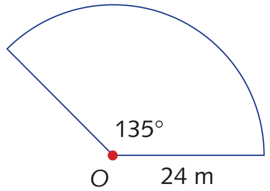


18 Use $\pi \approx 3.14$ to find the approximate value of the perimeter of the sector with:

a radius 10 m, containing an angle of 30°

b radius 9 cm, containing an angle of 10°

c radius 24 m, containing an angle of 135°



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14C Area of a circle

We know that the area of a rectangle is the length times the width. In earlier chapters, we used this idea to find simple formulas for the areas of squares, rectangles and triangles. We will now describe a way to find the **area of a circle**. By this we mean the area *inside* the circle. This problem is much more difficult than finding the area of a polygon (a straight-edged figure) such as a triangle or a quadrilateral.

The ancient Greek mathematician Archimedes came up with a number of clever ways to find a formula for the area of a circle in terms of the radius. The Greeks discovered that if you double the radius of a circle, then the area increases by a factor of 4. If you triple the radius, then the area increases by a factor of 9. The area of a circle appears to be related to the square of the radius.

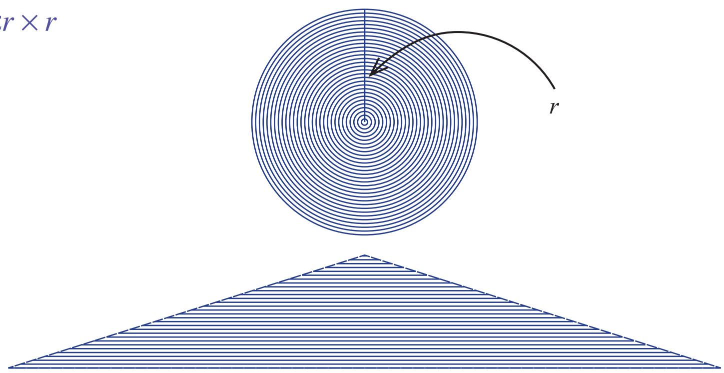
Here are two ways to dissect a circle to find a formula for its area.

Dissection using concentric circles

First, suppose that we have a circle of radius r, and imagine that the inside of the circle is made up of a large number of concentric circular pieces of very thin string. The circles are then cut along a radius, straightened and laid one on top of the other, as shown in the diagram below.

These strings will now form a figure that is approximately a triangle. (The thinner the pieces of string, the closer we come to a triangle.) The base of the triangle is the circumference of the original circle, which is πd or $2\pi r$, and the height of the triangle is the radius r of the original circle, so the area inside the circle is approximately:

$$\begin{aligned}\frac{1}{2} \times \text{base} \times \text{height} &= \frac{1}{2} \times 2\pi r \times r \\ &= \pi r^2\end{aligned}$$



Dissection by sectors

The second way of finding the area is to dissect the circle into sectors, just as you would a pizza. These sectors can then be arranged alternately, as in the diagram on the next page, to form a shape that is approximately a rectangle with a width that is r and with a length that is half the circumference – that is, πr. The smaller the sectors, the closer the shape is to being a rectangle. Thus the area is approximately:

$$\begin{aligned}\text{length} \times \text{width} &= \pi r \times r \\ &= \pi r^2\end{aligned}$$

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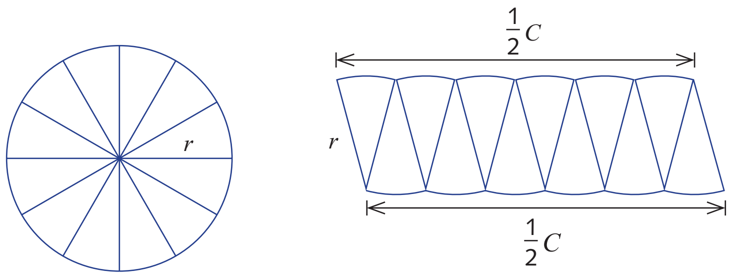

The formula for the area of a circle

The dissections shown above strongly suggest that the correct formula for the area of a circle with radius r is:

$$A = \pi r^2$$

A formal proof of this result requires an understanding of the concept of limits, which you will study in senior mathematics.

This formula should be memorised.

Example 3

Find the area of a circle:

a with radius 7 cm

b with diameter 7 cm

Give each answer:

i in terms of π

ii as an approximate value, using $\pi \approx \frac{22}{7}$

Solution

a i $A = \pi r^2$
$= 49\pi \text{ cm}^2$

ii $A = 49\pi$
$\approx 49 \times \frac{22}{7}$
$= 154 \text{ cm}^2$

b i Since the diameter is 7 cm, the radius is $\frac{7}{2}$ cm.

$A = \pi r^2$
$= \frac{49}{4}\pi \text{ cm}^2$

ii $A = \frac{49}{4}\pi$
$\approx \frac{49}{4} \times \frac{22}{7}$
$= 38\frac{1}{2} \text{ cm}^2$

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Remember that these answers are only approximate, since we have used an approximation for π.

Example 4

Using $\pi \approx 3.14$, find the approximate value of the area of a circle:

a with radius 10 cm

b with diameter 30 cm

Solution

a

$$\begin{aligned} A &= \pi r^2 \\ &\approx 3.14 \times 10 \times 10 \\ &= 314\ \text{cm}^2 \end{aligned}$$

b Since the diameter is 30 cm, the radius is 15 cm.

$$\begin{aligned} A &= \pi r^2 \\ &\approx 3.14 \times 15 \times 15 \\ &= 706.5\ \text{cm}^2 \end{aligned}$$

Area of a circle

The area of a circle is given by the formula

area = $\pi \times (\text{radius})^2$, or $A = \pi r^2$

Exercise 14C

1 Use the measurements you made in question **2** of Exercise 14B to find (approximately) the area of a 20c coin, in square millimetres. Repeat the exercise with a 10c coin and a 5c coin.

Example 3i

2 Find the area of a circle with the given radius. Give each area in terms of π.

a 14 cm **b** 7 cm **c** $3\frac{1}{2}$ mm **d** 42 m

Example 3ii

3 Use $\pi = \frac{22}{7}$ to find the approximate value of the area of a circle with radius:

a 14 cm **b** 7 cm **c** $3\frac{1}{2}$ mm **d** 42 m

4 Find the area of a circle with the given diameter. Give each area in terms of π.

a 14 cm **b** 7 cm **c** $3\frac{1}{2}$ mm **d** 42 m

5 Use $\pi \approx \frac{22}{7}$ to find the approximate value of the area of a circle with diameter:

a 14 cm **b** 7 cm **c** $3\frac{1}{2}$ mm **d** 42 m

6 Find the area of a circle with the given radius. Give each area in terms of π.

a 10 cm **b** 5 cm **c** 20 mm **d** 15 m

Example 4

7 Use $\pi \approx 3.14$ to find the approximate value of the area of a circle with radius:

a 10 cm **b** 5 cm **c** 20 mm **d** 15 m

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8 Find the area of a circle with the given diameter. Give each area in terms of π.

a 10 cm **b** 5 cm **c** 20 mm **d** 15 m

9 Use $\pi \approx 3.14$ to find the approximate value of the area of a circle with diameter:

a 10 cm **b** 5 cm **c** 20 mm **d** 15 m

10 If the diameter of a circle is doubled, what happens to its area?

11 A circle has area 1386 cm^2. Using $\pi \approx \frac{22}{7}$, find the approximate value of the radius of the circle.

12 A circle has an area of 314 cm^2. Using $\pi \approx 3.14$, find the approximate value of the radius of the circle.

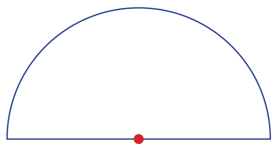
13 If the area of a circle is divided by 9, what happens to the radius? What happens to the circumference?

14 If the area of a circle is divided by 16, what happens to the diameter? What happens to the circumference?

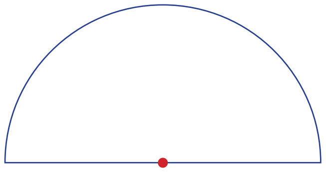
15 **a** What is the area inside a semicircle if the radius of the semicircle is 35 cm? Give your answer in terms of π.

b Find an approximate answer, using $\pi \approx \frac{22}{7}$.

(Note that the dot indicates the centre of the full circle.)



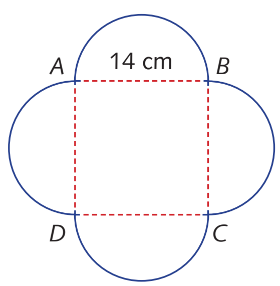
16 What is the approximate value of the area of a semicircle with radius 42 cm if we use $\pi \approx \frac{22}{7}$?



17 Four semicircles are drawn along the edges of a square with side length 14 cm.

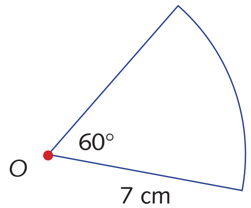
a Find the area of the entire region, giving your answer in terms of π.

b Find the approximate value of the area, using $\pi \approx \frac{22}{7}$.



18 **a** Find the area of the sector with radius 7 cm that contains an angle of 60°. Leave your answer in terms of π.

b Now use $\pi \approx \frac{22}{7}$ to find the approximate value of the area in part **a**.



19 Use $\pi \approx 3.14$ to find the approximate value of the area of a sector with:

a radius 10 m, containing an angle of 30°

b radius 9 cm, containing an angle of 10°

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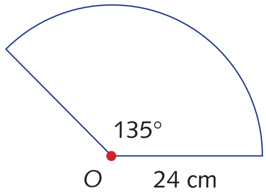
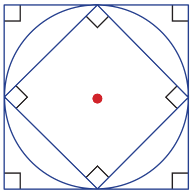
20 Find the area of the sector shown opposite. Leave your answer in terms of π.



21 A circle has radius r. Find the area of the circle and the two squares to show that $2 < \pi < 4$.

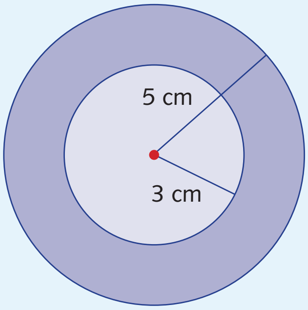


14D Areas of composite figures

We can now find the areas of more complicated figures, using the ideas of addition and subtraction of areas that we have previously used for such calculations involving rectangles and triangles.

Example 5

Find the shaded area enclosed between the circles in the diagram below. The smaller circle has radius 3 cm while the larger has radius 5 cm.



Solution

The area enclosed between the two circles is simply the area of the larger circle minus the area of the smaller one.

$$\begin{aligned}\text{Area} &= (\pi \times 5^2) - (\pi \times 3^2)\\ &= 25\pi - 9\pi\\ &= 16\pi \text{ cm}^2\end{aligned}$$

We can leave the area in terms of π, or approximate it using $\pi \approx \frac{22}{7}$, giving:

$$\begin{aligned}\text{Area} &= 16\pi\\ &\approx 16 \times \frac{22}{7}\\ &= \frac{352}{7}\\ &= 50\tfrac{2}{7} \text{ cm}^2\end{aligned}$$

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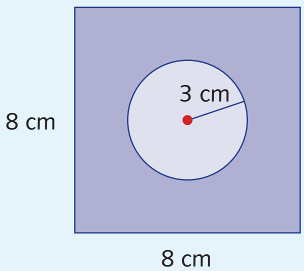
The region between two circles with a common centre is called an **annulus**, which is the Latin word for 'ring'.

Example 6

A circle of radius 3 cm is cut from a square with side length 8 cm.

Find the area of the remaining region:



a in terms of π

b using $\pi \approx \frac{22}{7}$

Solution

a Area of square $= 8 \times 8$

$= 64\text{ cm}^2$

Area of circle $= \pi \times 3^2$

$= 9\pi\text{ cm}^2$

Shaded area $= (64 - 9\pi)\text{ cm}^2$

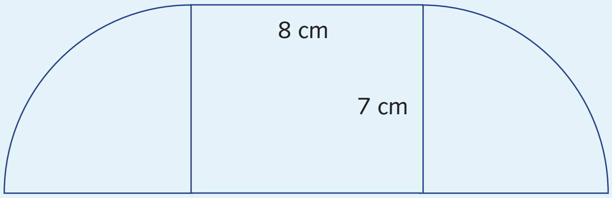
b Shaded area $\approx 64 - 9 \times \frac{22}{7}$

$= \frac{250}{7}$

$= 35\frac{5}{7}\text{ cm}^2$

Example 7

The figure below, consisting of a rectangle of length 8 cm and width 7 cm and two quarter-circles of radius 7 cm, is cut from a piece of cardboard.



a Find the area of the figure in terms of π.

b Find the approximate area of the figure, using $\pi \approx 3.14$.

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Solution

a Area of rectangle $= 8 \times 7$

$= 56 \text{ cm}^2$

Area of quarter-circle $= \frac{1}{4} \times \pi \times 7^2$

$= \frac{49}{4}\pi \text{ cm}^2$

Area of figure $= 56 + 2 \times \frac{49}{4}\pi$

$= 56 + \frac{49}{2}\pi \text{ cm}^2$

b Area of figure $\approx 56 + \frac{49}{2} \times 3.14$

$= 132.93 \text{ cm}^2$

Area of an annulus

The area of an annulus is given by the formula

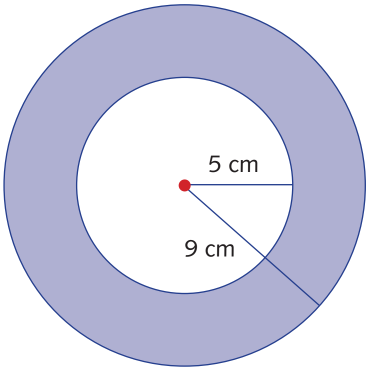
$A = \pi(R^2 - r^2)$,

where R is the radius of the outer circle, and r is the radius of the inner circle.

Exercise 14D

Example 5

1 Find the approximate value of the area of the annulus formed by a circle of radius 9 cm and one of radius 5 cm. Use $\pi \approx \frac{22}{7}$.



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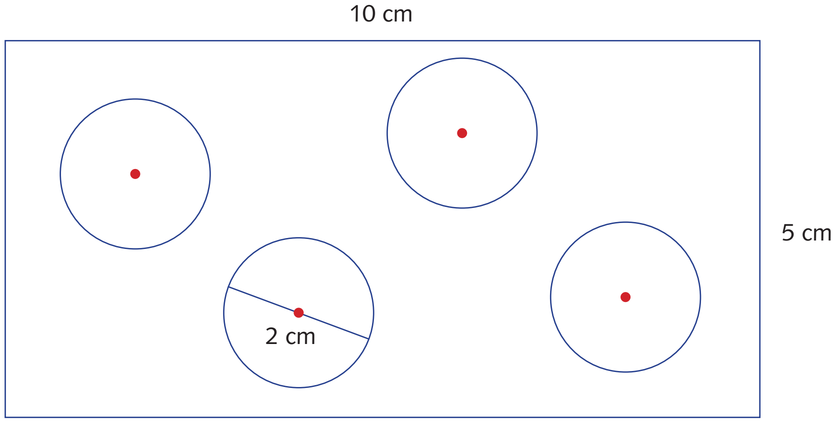
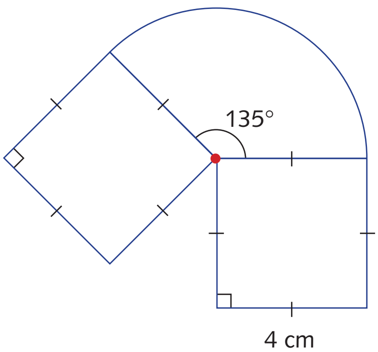
Example 6

2 Four coins, each with diameter 2 cm, are cut from a metal rectangle with side lengths that are 10 cm and 5 cm, as shown below. What is the remaining area? Give your answer in terms of π and then find an approximate value using $\pi \approx \frac{22}{7}$.



Example 7

3 The diagram to the right shows two squares and a sector. Find the area of this figure, leaving your answer in terms of π.



4 Jim's mum used a pastry cutter to cut 12 circles of dough, each of diameter 7 cm, from a rectangular slab of pastry that was 85 cm by 15 cm. What is the approximate value of the area of dough that was left? Use $\pi \approx 3.14$.

5 Which has the larger area:

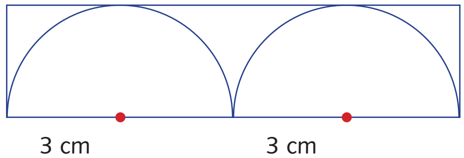
a a circle of diameter 10 cm or a square of side length 8 cm?

b a circle of radius 4 cm or a square of side length 7 cm?

c a circle of diameter 12 m or a triangle with base 13 m and height 17 m?

6 A square of side length 14 mm can just fit inside a circle of diameter 20 mm. Find the approximate value of the area of the circle that is not covered by the square. Use $\pi \approx 3.14$.

7 Two semicircles, each of radius 3 cm, are cut from a rectangle of length 12 cm, as shown below.



a What is the width of the rectangle?

b What is the area remaining? Leave your answer in terms of π.

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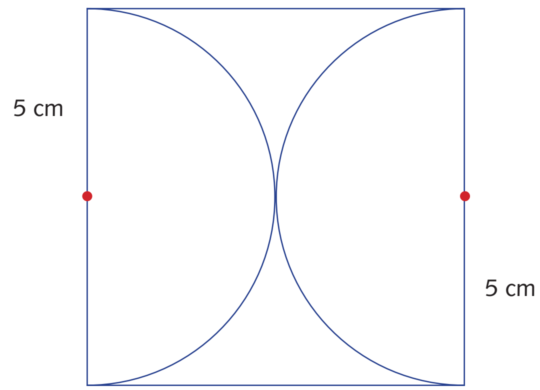
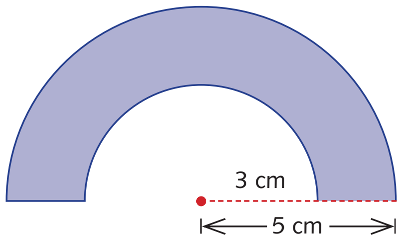
8 Two semicircles, each of radius 5 cm, are cut from a square, as shown below.

a What is the side length of the square?

b What is the approximate value of the remaining area? Use $\pi \approx 3.14$.



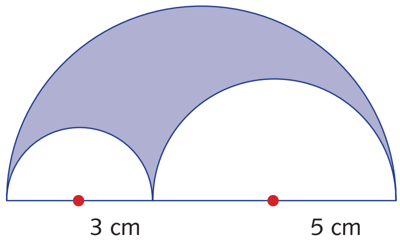
9 Find the approximate value of the shaded area shown opposite. Use $\pi \approx \frac{22}{7}$.



10 Two semicircles, of radii 3 cm and 5 cm, are cut from a larger semicircle, as shown opposite.

a What is the radius of the largest semicircle?

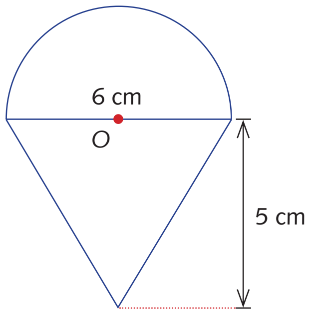
b What is the approximate value of the remaining area? Use $\pi \approx 3.14$.



11 An athletics track is made up of two parallel straight sections that are 102 metres long and 62 metres apart, with two semicircular ends. Calculate the approximate distance travelled by an athlete who runs one lap of this track. Use $\pi \approx \frac{22}{7}$.

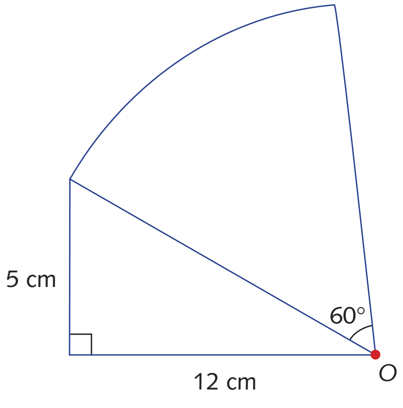
12 Find the area of the following figures. Give your answers in terms of π.

a



A semicircle on a triangle.

b



A sector of a circle, centre O, on a right-angled triangle.

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Review exercise

1 Use your compasses, ruler and protractor to draw:

a a circle with radius 3 cm **b** a circle with diameter 8 cm

c a sector with radius 5 cm and an angle of 110° at the centre

d a semicircle with radius 4 cm **e** a quadrant with radius 5 cm

2 Find the circumference of each of these circles. Leave your answers in terms of π.

a Radius 2 m **b** Diameter 86 mm

c Radius 12 mm **d** Radius 187 cm

3 Use $\pi \approx 3.14$ to find the approximate value of the circumference of each circle in Question **2**.

4 Use $\pi \approx \frac{22}{7}$ to find the approximate value of the circumference of each of these circles.

a Radius 14 mm **b** Diameter 35 cm **c** Radius 42 m

5 Find the approximate value of the perimeter of each figure, using $\pi \approx 3.14$.

a A semicircle with diameter 9 cm

b A quadrant with radius 2 cm

c A quadrant with radius 5 cm

d A semicircle with radius 8 mm

e A sector with radius 14 cm containing an angle of 120°

6 Find the area of each of the following circles, giving your answers in terms of π.

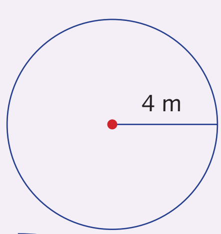
a Radius 21 mm **b** Diameter 14 m

c Radius 63 cm **d** Radius 35 m

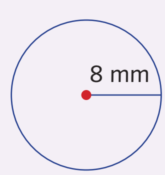
7 Use $\pi \approx \frac{22}{7}$ to find the approximate value of the area of each of the circles in Question **6**.

8 Find the approximate value of the area of each figure shown below. Use $\pi \approx 3.14$.

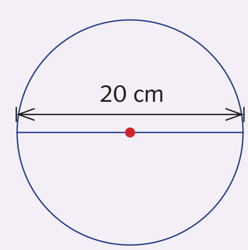
a



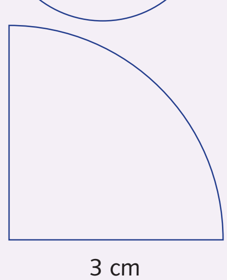
b



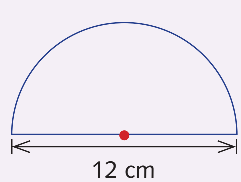
c



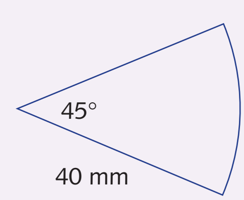
d



e



f



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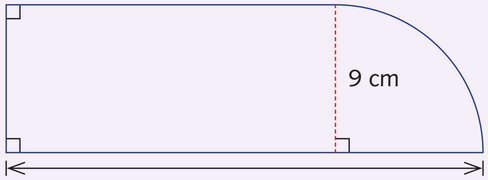
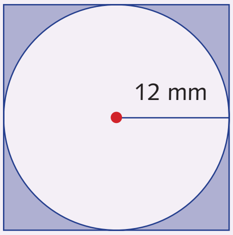
9 Find the approximate value of the area of the annulus formed by a circle of radius 10 cm and a circle radius 4 cm. Use $\pi \approx \frac{22}{7}$.

10 Find the area of the figure shown below, leaving your answer in terms of π.

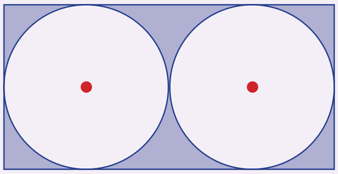


11 Find the approximate value of the shaded area for each of these figures. Use $\pi \approx 3.14$.

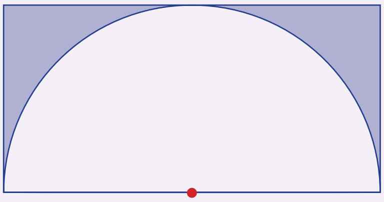
a



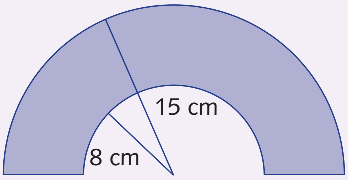
b



c



d

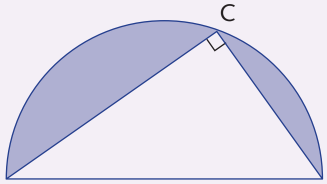


12 A tyre has a diameter of 52 cm. Calculate the circumference of the tyre. Give your answer in terms of π.

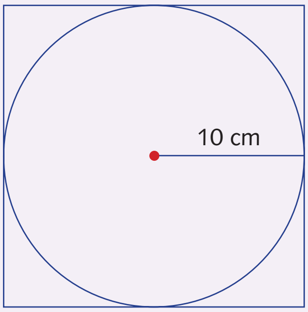
13 Meredith has a circular table with diameter 1.2 m. What is the approximate area of a circular tablecloth that would cover the table and have 5 cm hanging over the edge? Use $\pi \approx 3.14$.

14 A CD has diameter 11.8 cm, and the hole in the centre has diameter 1.4 cm. Find the approximate area of the CD, using $\pi \approx 3.14$.

15 AB is the diameter of the semicircle shown opposite. If $AC = 4$ cm, $CB = 3$ cm and $\angle ACB = 90°$, find the area of the shaded region. Give your answer in terms of π.



16 In the diagram opposite, the circle with radius 10 cm touches the insides of the square. Find the ratio of the area of the square to the area of the circle. Give your answer in terms of π.



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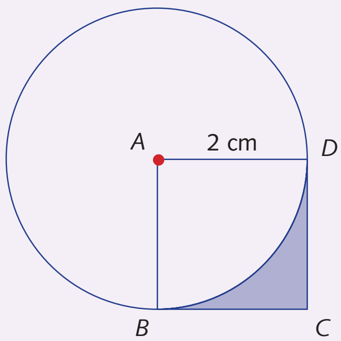
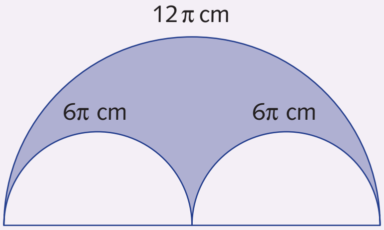
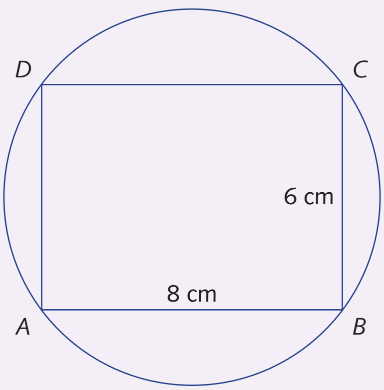
17 The circle in the diagram has centre A and radius 2 cm. If $ABCD$ is a square, find the area of the shaded region. Give your answer in terms of π.



18 The arc lengths of three semicircles are given. Find the area of the shaded region.

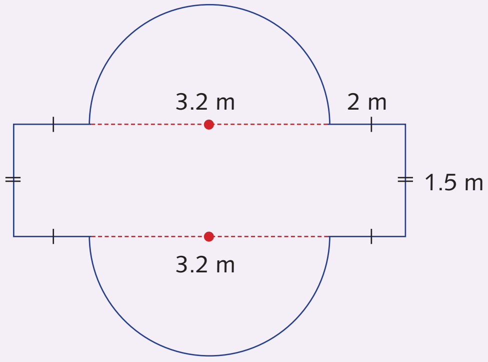


19 In the diagram to the right, $ABCD$ is a rectangle in which $AB = 8$ cm and $BC = 6$ cm. Find the area of the circle in cm^2.

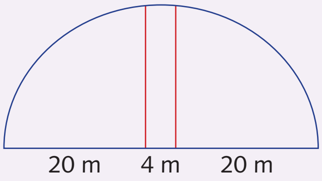


20 A Milo tin has a diameter of 10 cm. How far will it move if it is rolled through six revolutions?

21 Find the approximate value of the perimeter of the figure to the right, using $\pi \approx 3.14$.



22 A hockey goal 'circle' is made up of a rectangle and two quadrants, as shown below. Find its approximate area, using $\pi \approx 3.14$.



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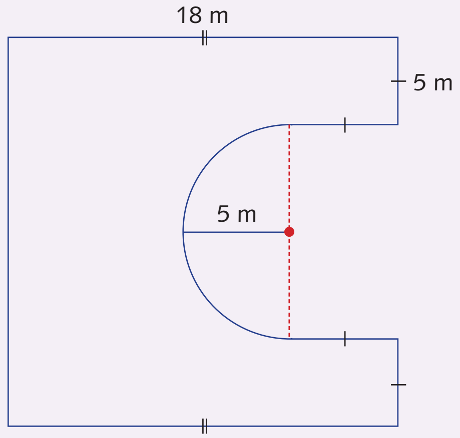
Challenge exercise

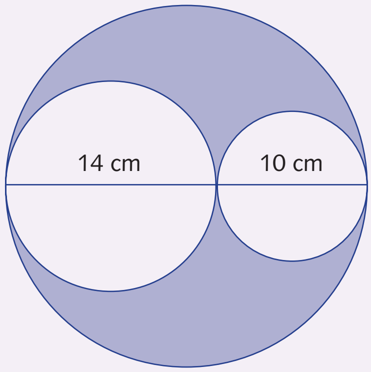
1 For the figure shown below, use $\pi \approx 3.14$ to find the approximate value of:

a the perimeter

b the area



2 Find the shaded area of the figure below, leaving your answer in terms of π.



3 A circular swimming pool has a diameter of 10 m and is 1.3 m deep. The interior of the pool was covered with square tiles of side length 2 cm. Approximately how many tiles were required?

4 Suppose we roll a circle of radius 1 cm around a larger circle of radius 4 cm without slipping.

a How many times has the small circle rotated in completing exactly one revolution of the larger circle?

b What happens when the smaller circle rolls on the inside of the larger circle?

5 The hypotenuse of a right-angled triangle is 15 cm. A circle of radius 2 cm is drawn inside the triangle so that it touches each of the three sides. What is the perimeter of the triangle?

6 By how much does the circumference of a circle increase if we increase its diameter by π units?

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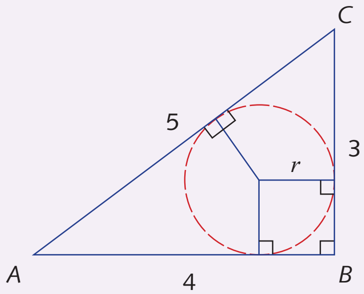
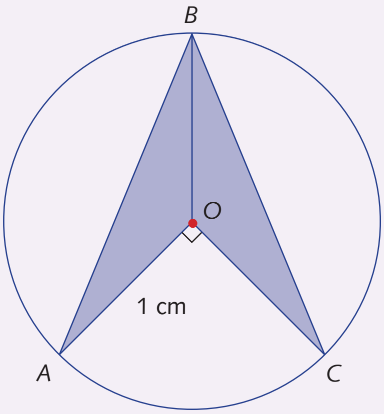
7 An arc of a circle C_1 subtends an angle of 60° at the centre, and has the same length as an arc of a circle C_2 subtending an angle of 45° at the the centre. Find the ratio of the area of C_1 to C_2.

8 **a** *ABC* is a right-angled triangle with a right angle at *B*. The circle shown is the 'incircle' of the triangle. Find the radius, *r*, of the incircle.

b Find the radius of the incircle of a right-angled triangle with sides 5, 12 and 13.



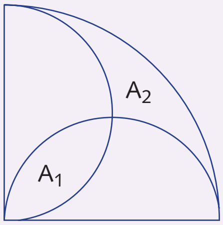
9 Triangles *BOC* and *BOA* in the diagram on the right are congruent and $\angle AOC = 90°$. The radius of the circle centre *O* is 1 cm. Find the area of the shaded region.



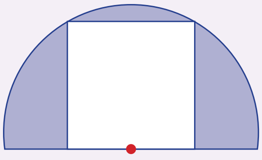
10 The vertices of a square lie on a circle of radius 4 cm. Find the ratio of the area of the circle to the area of the square in terms of π.

11 Two semicircles are drawn in a quadrant of radius 2 cm.

Find the ratio of the area (A_1) : area (A_2).



12 The vertices of a square lie on the boundary of a semicircle of radius 10 cm. Find the shaded area shown in terms of π.



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