CHAPTER

15

Measurement and Geometry

Areas, volumes and time

The **area** of a plane figure is a measure of the amount of the size of the interior region. Calculating areas is an important skill used by many people in their daily work. Builders and tradespeople often need to work out the areas and dimensions of the things they are building, and so do architects, designers and engineers.

In this chapter, we will revise and extend the range of figures whose areas we can calculate. This will include parallelograms, kites, trapeziums and rhombuses. Our basic approach is to dissect these figures into simpler ones whose areas we can already calculate.

When we consider solids, we use the idea of **volume** to measure the size of the interior region. We will learn how to find the volumes of a range of solids known as **prisms**, and also how to find the volume of a cylinder. Finally, we will look at how to find the surface area of a prism by looking at each of its faces.

In this chapter, we develop a number of fundamental formulas for areas and volumes. You should commit these to memory. You should also know where these formulas come from and how they work.

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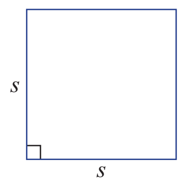
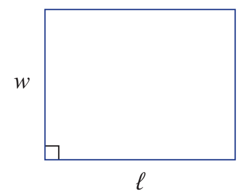
15A Review of area and length

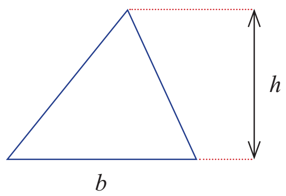
We have already met the formulas for the areas of squares, rectangles and triangles, as shown below.



Area $= s^2$



Area $= \ell w$

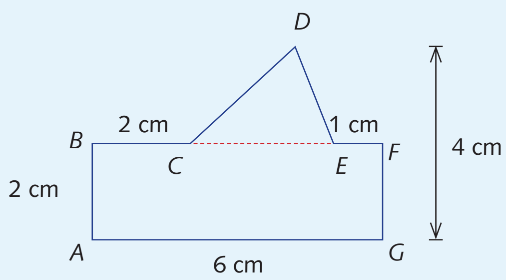


Area $= \frac{1}{2}bh$

We can use these basic shapes to find the areas of more complicated figures, just as we did in the previous chapter. If a region can be broken into non-overlapping rectangular or triangular pieces, then the area of the region is the sum of the areas of the pieces.

Example 1

The region below consists of a triangle on top of a rectangle. Find the area of the region.



Solution

Triangle CBE has base $CE = 3$ cm and height 2 cm.

$$\begin{aligned}\text{Area of triangle } CDE &= \frac{1}{2} \times 3 \times 2 \\ &= 3 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Area of rectangle } ABFG &= 6 \times 2 \\ &= 12 \text{ cm}^2\end{aligned}$$

Total area of the figure is 15 cm^2.

Sometimes a region is obtained by removing a piece (or pieces) from a larger one. In this case, we subtract the areas of the removed parts.

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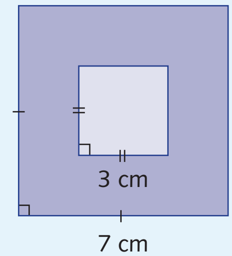
Example 2

Find the area of the figure enclosed between the two squares.



Solution

$$\begin{aligned}\text{Area} &= 7^2 - 3^2\\ &= 49 - 9\\ &= 40\ \text{cm}^2\end{aligned}$$

Area of a composite figure

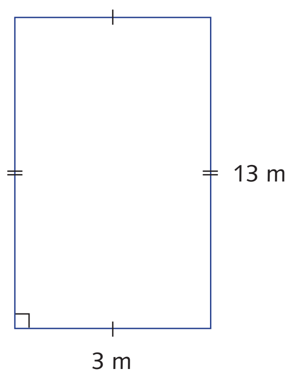
- To find the area of a composite figure, we dissect it into simpler figures and add the areas of the these pieces.
- We can also find areas by subtraction.

Exercise 15A

Example 1

1 Find the area of each figure.

a



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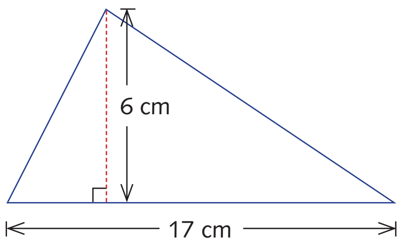

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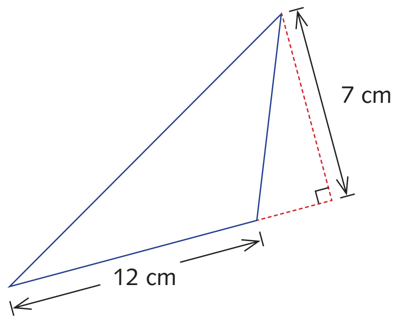

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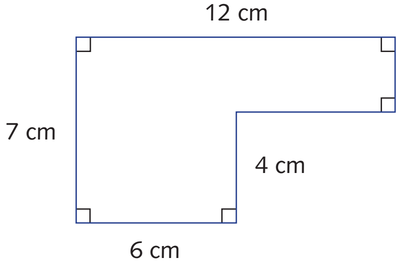

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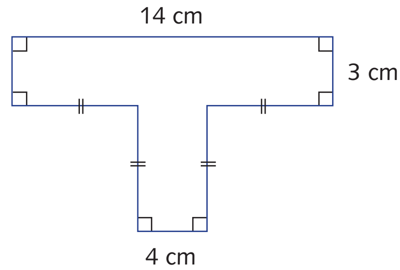
e



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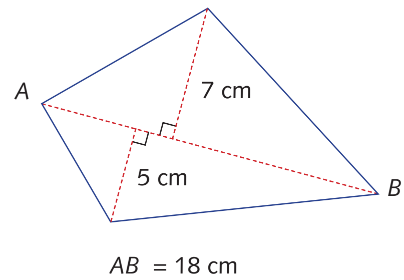
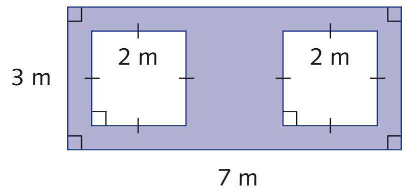


Example 2

2 Find the area of the shaded region in each figure.

a



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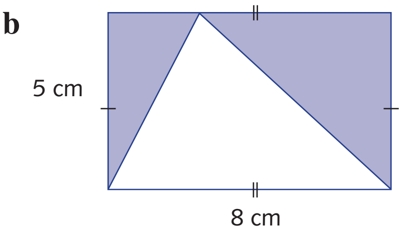

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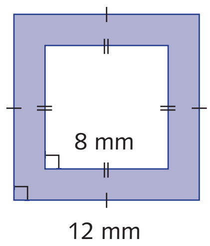

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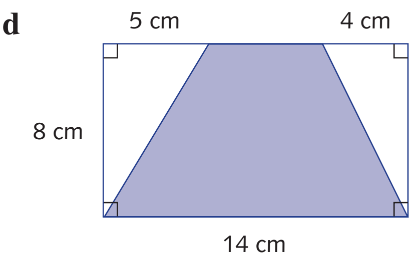
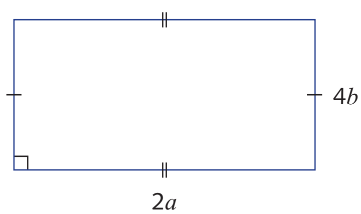


3 A rectangular orchard is 520 m long and 300 m wide. Find the area of the orchard in hectares. (Recall that 1 hectare = 10 000 m^2.)

4 The base and height of a triangle are whole numbers and its area is 12. Find all values of the base and height.

5 Write down an algebraic formula, in simplest form, for the area of each figure.

a



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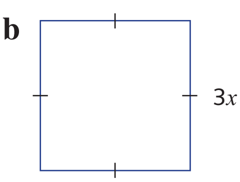

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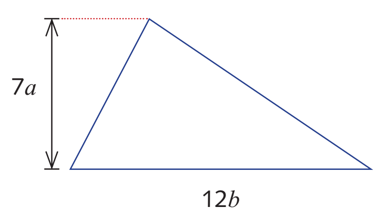

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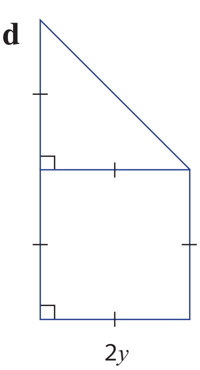

6 A room has 4 walls and 2 doorways. The walls each have dimensions 7 m by 5 m, and the doorways have dimensions $\frac{3}{4}$ m by 2 m. If the inside walls of the room must be painted, how much will this cost if the rate for painting is $5 per square metre?

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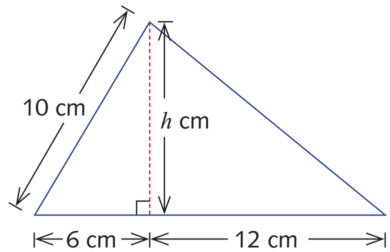
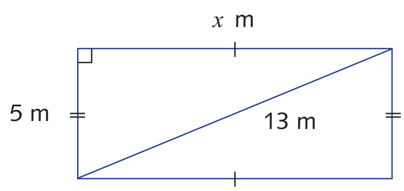
7 Use Pythagoras' theorem to find h, and hence find the area of the triangle.

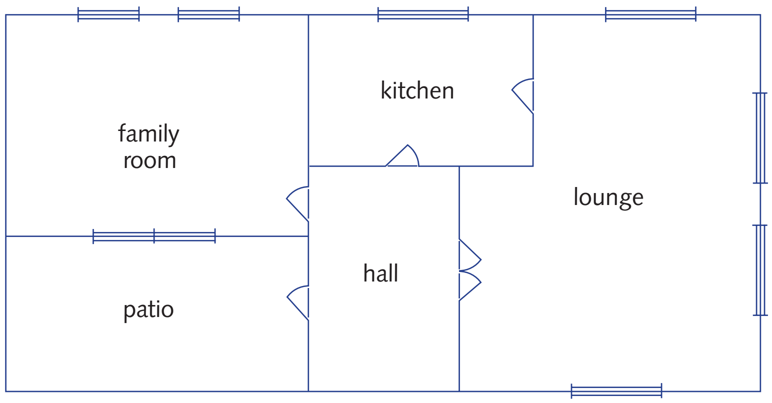


8 Find x and hence find the area of the rectangle.



9 Find the area of the floor of the room that is left uncovered if a carpet measuring 4 m by 3 m is laid in a room 5 metres square.

10 This diagram is the ground floor plan of a house drawn to a scale of 1 cm to represent 2 metres. (Take width to be left to right.)
Take measurements and answer the questions below.



a What is the length of the house?

b What is the width of the house?

c Find the length and width of the family room.

d What is the area of the kitchen?

e What is the width of the lounge at its widest point?

f What is the area of the patio?

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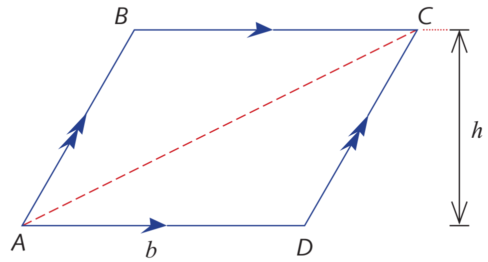
15B Areas of special quadrilaterals

Area of a parallelogram

A **parallelogram** has each pair of its opposite sides parallel, as shown below.



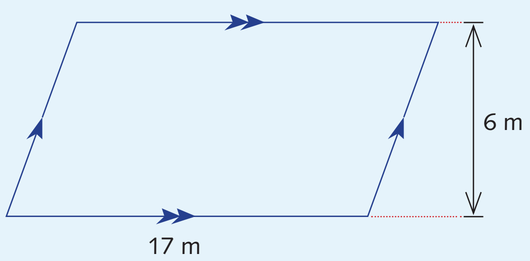
To find the area of this figure, first draw the diagonal AC to form triangles ABC and ADC.
The area of $\Delta ABC = \frac{1}{2} \times b \times h$. Similarly, the area of $\Delta ADC = \frac{1}{2} \times b \times h$.

$$\begin{aligned} \text{The area of the parallelogram } ABCD &= \text{area of } \Delta ADC + \text{area of } \Delta ABC \\ &= \frac{1}{2}bh + \frac{1}{2}bh \\ &= bh \end{aligned}$$

Area of a parallelogram = base × height = bh

Example 3

Find the area of the parallelogram shown below.



Solution

$$\begin{aligned} \text{Area} &= \text{base} \times \text{height} \\ &= 17 \times 6 \\ &= 102 \text{ m}^2 \end{aligned}$$

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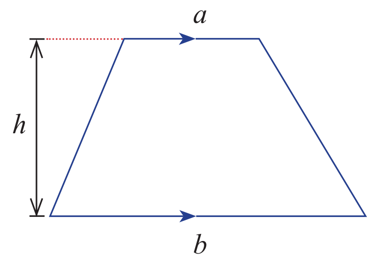
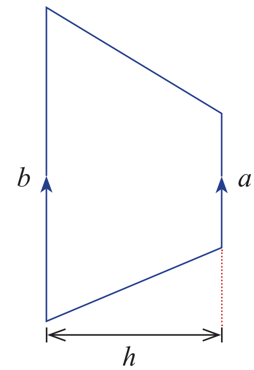
Area of a trapezium

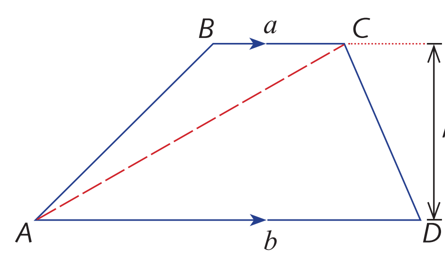
A **trapezium** is a quadrilateral with one pair of opposite sides parallel.





To find the area of a trapezium, we need to know the lengths of the two parallel sides and the perpendicular distance between these two sides, which we will call the **perpendicular height** of the trapezium.

To find the formula for the area of a trapezium first draw the diagonal AC to form triangles ABC and ADC. The height of both triangles is h.

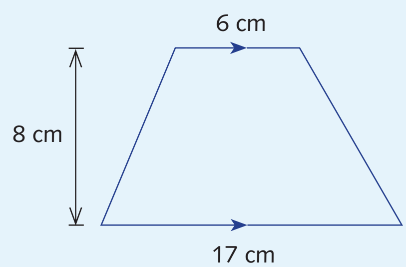


$$\begin{aligned}\text{The area of the trapezium} &= \text{area of } \Delta ABC + \text{area of } \Delta ADC\\ &= \frac{1}{2}\times a \times h + \frac{1}{2}\times b\times h\\ &= \frac{1}{2}ah + \frac{1}{2}bh\\ &= \frac{1}{2}h(a+b)\end{aligned}$$

$$\text{Area of a trapezium} = \frac{1}{2}h(a+b)$$

Example 4

Find the area of the trapezium shown opposite.



Solution

$$\begin{aligned}\text{Area} &= \frac{1}{2}h(a+b)\\ &= \frac{1}{2}\times 8\times(6+17)\\ &= 92\text{ cm}^2\end{aligned}$$

Note: Every parallelogram is a trapezium, but not every trapezium is a parallelogram.

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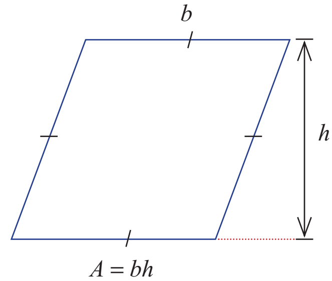
Area of a rhombus and a kite

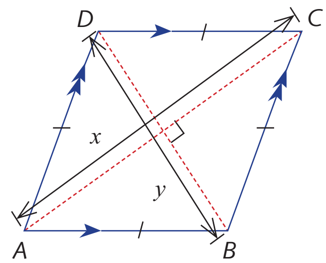
A **rhombus** is a quadrilateral with all four sides equal.



Since a rhombus is also a parallelogram, the area A is given by:

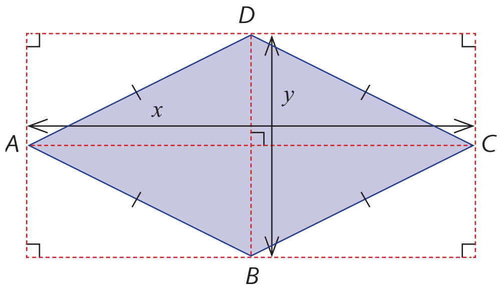
$$A = bh$$

It was shown in Chapter 13 that the diagonals of a rhombus bisect each other at right angles.



There is a formula for the area of a rhombus in terms of the product of the diagonals.

Take a rhombus and stand it on one corner. The two diagonals cut the rhombus into four right-angled triangles, which can be completed to form four rectangles inside a larger rectangle.



Since the eight triangles have the same area, the area of the rhombus is half the area of the large rectangle, which is $x \times y$. Hence if x and y are the lengths of the diagonals of a rhombus, then

$$\text{area of a rhombus} = \frac{1}{2}xy$$

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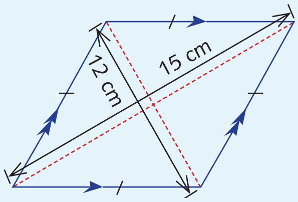
Example 5

a Find the area of the rhombus with diagonals 12 cm and 15 cm.



b A rhombus has area 144 cm^2 and one of the diagonals has length 12 cm. What is the length of the second diagonal?

Solution

a $\text{Area} = \frac{1}{2}xy$ (where x and y are the lengths of the diagonals)

$= \frac{1}{2} \times 12 \times 15$

$= 90 \text{ cm}^2$

b $144 = \frac{1}{2} \times 12 \times y$ (where y is the length of the other diagonal)

$144 = 6y$

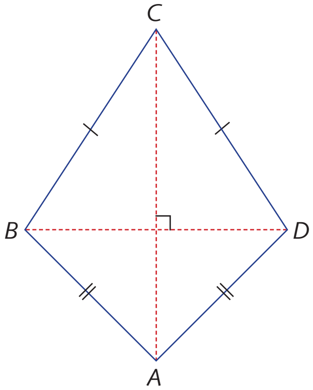
$24 = y$

The other diagonal has length 24 cm.

A **kite** is a quadrilateral that has two pairs of adjacent equal sides.

Note: A rhombus is clearly a kite but a kite is not necessarily a rhombus.

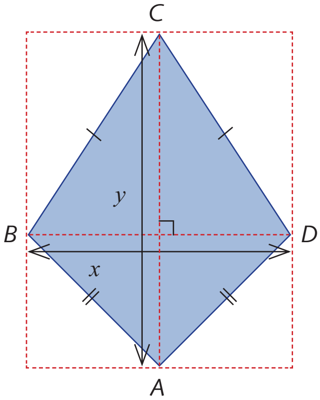
The diagonals AC and BD are perpendicular – can you show this using congruence?



As we did for the rhombus, we can complete the kite to form a rectangle whose area is twice that of the kite.

Hence for a kite with diagonals x and y

$$\text{area of a kite} = \frac{1}{2}xy$$



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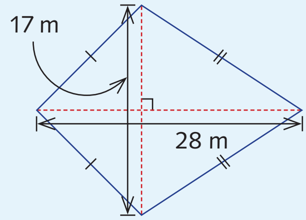
Example 6

a Find the area of a kite with diagonals 17 m and 28 m.



b A kite has area 256 cm^2 and one of its diagonals has length 8 cm. What is the length of the other diagonal?

Solution

a

$$\text{Area} = \frac{1}{2}xy$$
$$= \frac{1}{2} \times 17 \times 28$$
$$= 238 \text{ m}^2$$

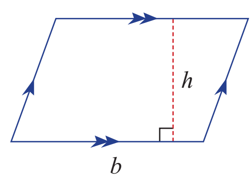
b

$$256 = \frac{1}{2} \times 8 \times y$$
$$256 = 4y$$
$$64 = y$$

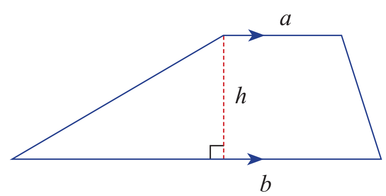
The length of the other diagonal is 64 cm.

Areas of special quadrilaterals

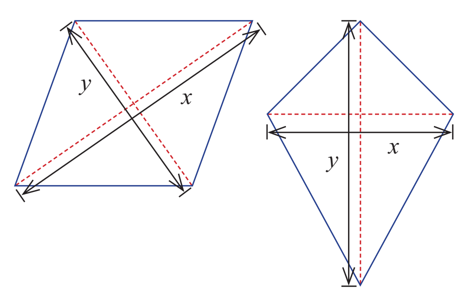
- Area of a parallelogram = bh



- Area of a trapezium = $\frac{1}{2}h(a + b)$



- Area of a rhombus or a kite = $\frac{1}{2}xy$



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Exercise 15B

1 The table below gives the bases, heights and areas of various parallelograms. Fill in the missing entries.

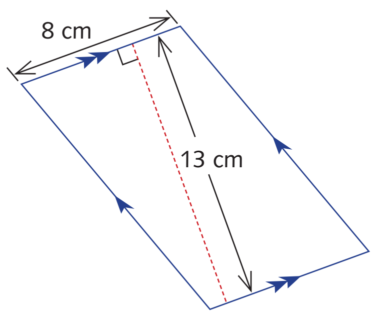
	Base	Height	Area
a	7 cm	6 cm	
b	9 cm		27 cm^2
c		17 m	85 m^2
d	12 km		144 km^2

	Base	Height	Area
e	500 km		30 000 km^2
f		50 m	6000 m^2
g	55 m		5500 m^2
h	16 cm		256 cm^2

Example 3, 4

2 Find the areas of these regions.

a



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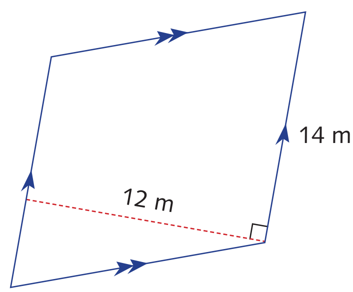

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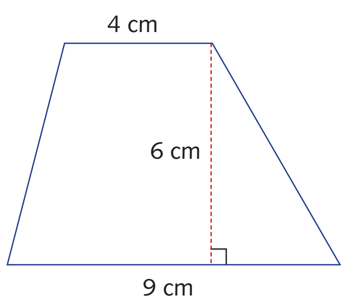

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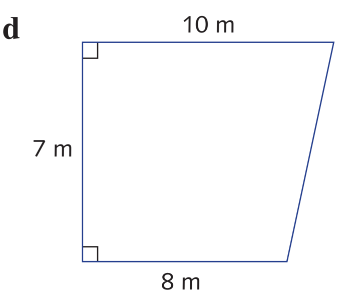

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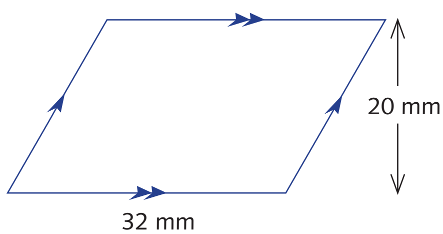

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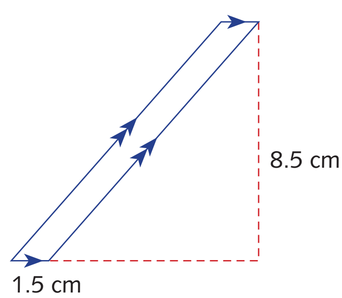

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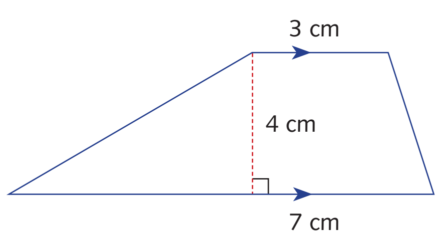

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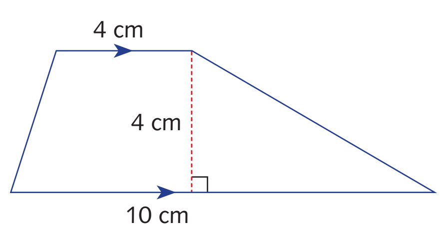

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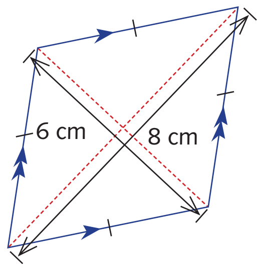
Example 5, 6

3 Find the area of these regions.

a



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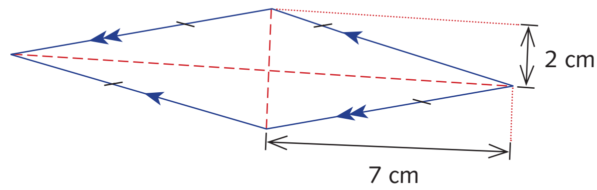

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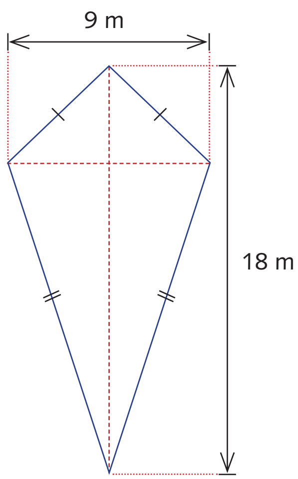

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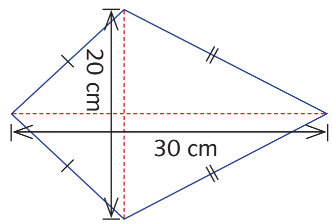
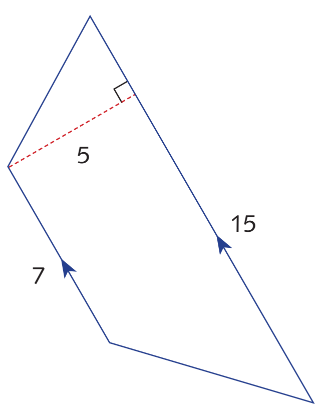
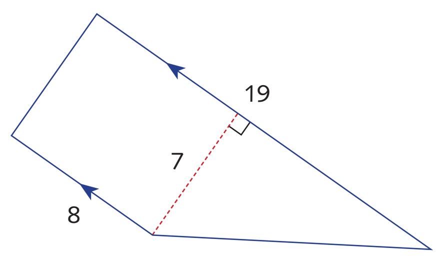


4 Find the areas of these trapeziums. All measurements are in centimetres.

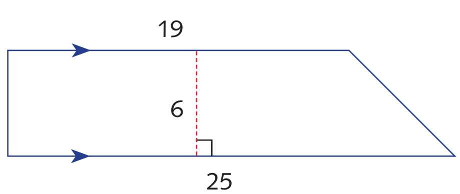
a



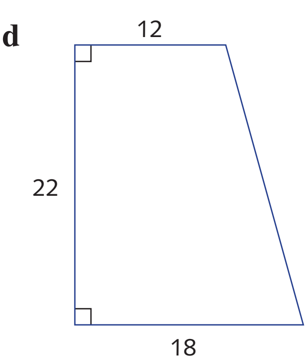
b



c



d



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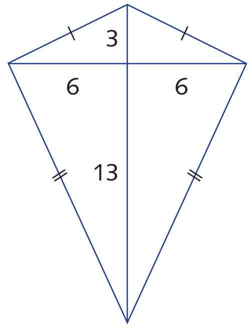
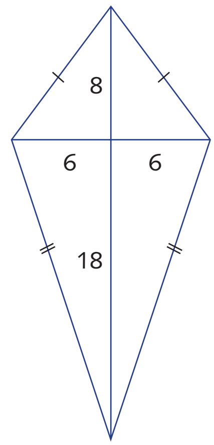
5 Find the area of each figure below.

a



b



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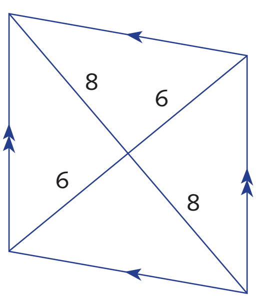

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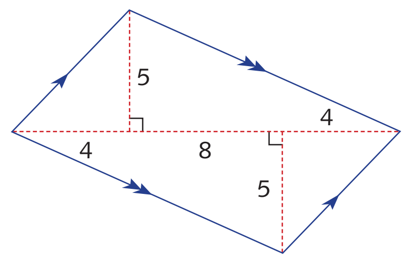
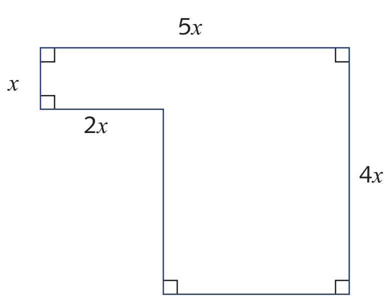
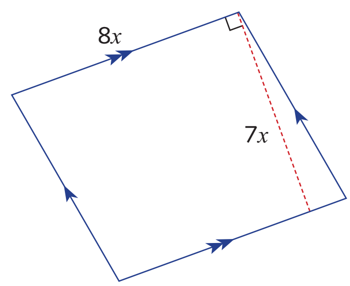


6 Find an algebraic expression, in simplest form, for the area of each figure below.

a



b



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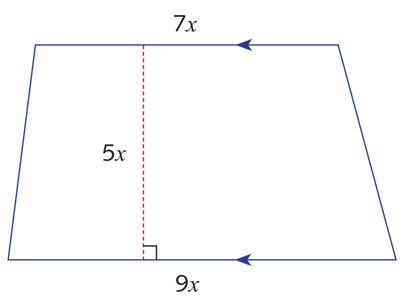

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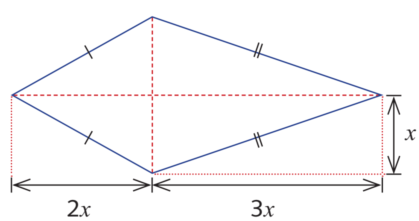
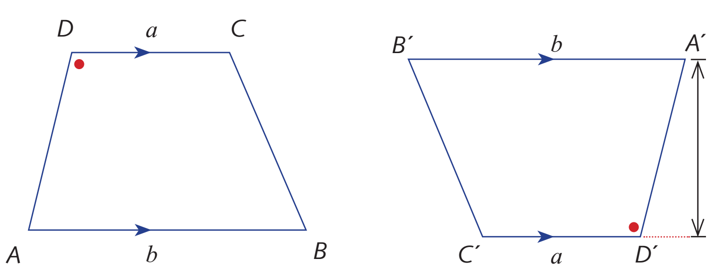


7 Prove that the area of a trapezium is $\frac{1}{2}h(a+b)$ by pasting together two identical trapeziums $ABCD$ and $A'B'C'D'$.



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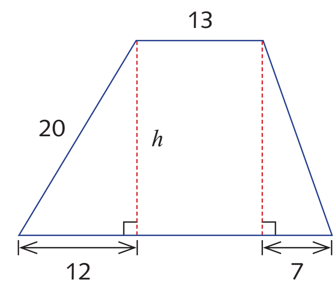
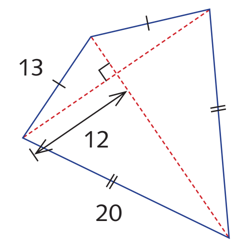
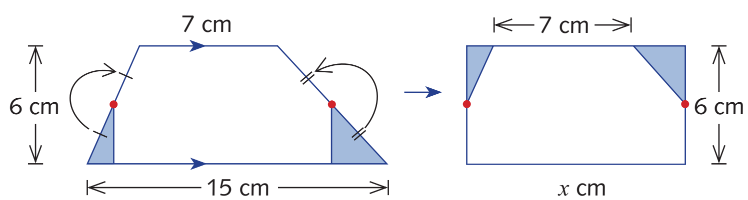
8 Use Pythagoras' theorem to find the height of this trapezium, and hence find its area.



9 Find the lengths of the diagonals of this kite, and hence find its area.



10 Here is another way of dissecting a trapezium to find its area. Take the midpoints of the two non-parallel sides and cut off the triangles, as shown. These are then moved up to form a rectangle.

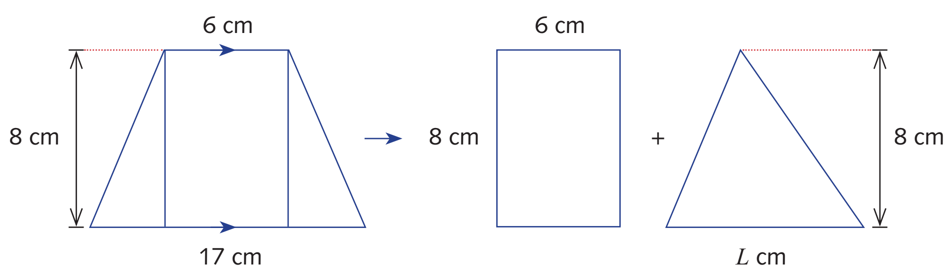


a What is the length x cm of the rectangle?

b Find the area of the trapezium, using the rectangle.

c Use the usual formula for the area of a trapezium in the first diagram to confirm your answer in part **b**.

11 Here is yet another way of dissecting a trapezium to find its area. Cut the two triangles from the end of the trapezium and rearrange them to form a rectangle and a triangle, as shown below.



a What is the value of L?

b Find the area of the trapezium.

c Can you draw a trapezium in which this method of dissection will not work?

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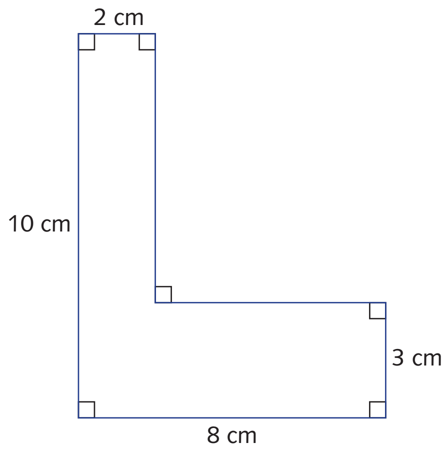
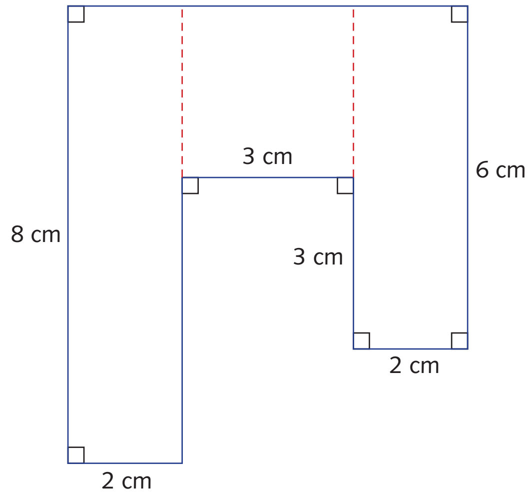
12 Find the area of the following shapes by first dividing them into rectangles.

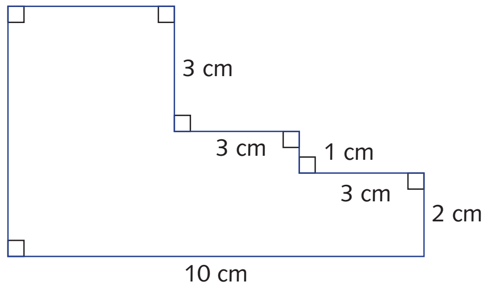
a



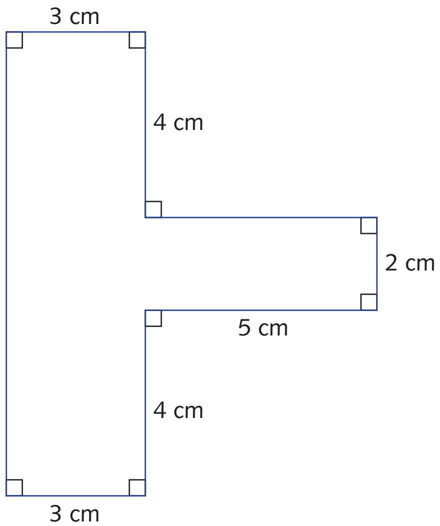
b



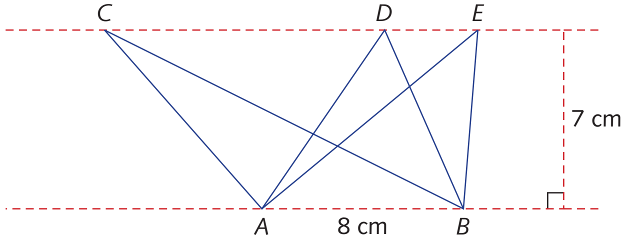
c



d



13 Write down the area of each of the triangles with base *AB*.



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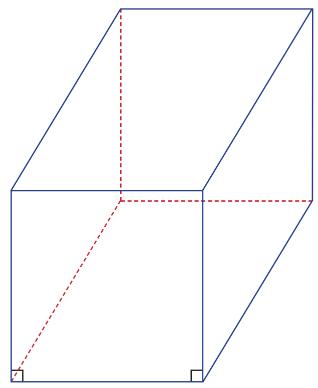
15C Volume of a rectangular prism



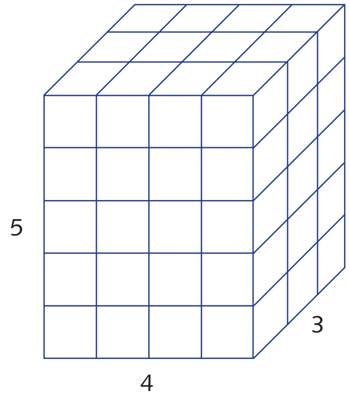
Look around the room you are in. Most rooms have a horizontal floor, and the ceiling is horizontal as well. The four walls are vertical. Here is a sketch of what the room might look like.

This is an example of a **right-rectangular prism**. Note that:

- the base is a rectangle
- if we slice it through any horizontal plane, the cross-section is a rectangle congruent to the base
- there are three right angles at each corner.

(The word 'right' is used here as a compact and convenient way of saying 'the walls are vertical'.) If the faces are all congruent then we call the solid a **cube**. An example of such a solid is a shoe box.

The **volume** of a rectangular prism is a measure of the space inside the prism.



Consider a rectangular prism of side lengths 5, 4 and 3. We call these numbers **dimensions** of the prism. (They could also be called the **length**, **width** and **height** of the prism, but since we can rotate the prism, these words are used rather loosely, so it is often better to talk about the dimensions.)

We can cut the prism up into cubes, each of side length 1, as shown. We say that the volume of each of these cubes is 1, which we read as '1 cubic unit'.

Altogether there are $5 \times 3 \times 4 = 60$ cubes, each of volume 1, so we say that the volume of the prism is 60.

In practice, the dimensions could be 3 cm, 4 cm and 5 cm. The unit cube will have volume 1 cm^3 and the box will have volume 60 cm^3.

In general, the volume of a rectangular prism is given by:

$$\begin{aligned}\text{volume of a rectangular prism} &= \text{length} \times \text{width} \times \text{height}\\ &= \ell wh\end{aligned}$$

Example 7

Find the volume of a rectangular prism with dimensions 4 cm, 16 cm and 3 cm.

Solution

$$\begin{aligned}\text{Volume} &= \ell wh\\ \text{Then } V &= 4 \times 16 \times 3\\ &= 192 \text{ cm}^3\end{aligned}$$

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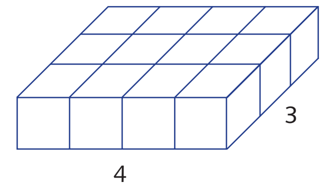
Slices and area



We can see that the rectangular prism is made up of five copies of the $4 \times 3 \times 1$ slice shown. The base area of the slice is 12 and the height is 1. We have 5 such slices in the original prism so we obtain volume $= 12 \times 5 = 60$ as before.

Thus the volume of a rectangular prism equals Ah, where A is the area of the base.

Note: The base can be any one of the three different faces of the prism.

The formulas above are still valid even when the dimensions are not whole numbers.

Right-rectangular prism

- A **right-rectangular prism** is a polyhedron in which:
  - the base is a rectangle
  - each cross-section parallel to the base is a rectangle congruent to the base
  - there are three right angles at each corner.
- Volume of a right-rectangular prism = length × width × height
  $= \ell wh$
- Volume of a right-rectangular prism = Area of base × height
  $= Ah$

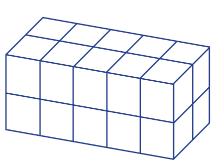
Exercise 15C

Example 7

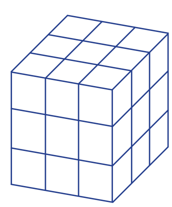
1 Find the volume of a rectangular prism whose dimensions are 12 mm, 10 mm and 7 mm.

2 If  is a 1-unit cube, find the volume of each of these figures in unit cubes.

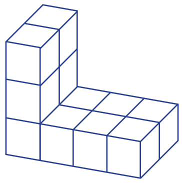
a



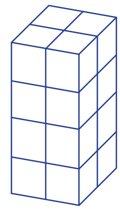
b



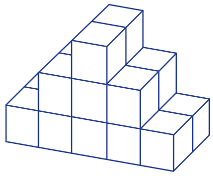
c



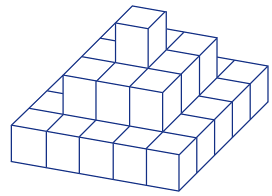
d



e



f



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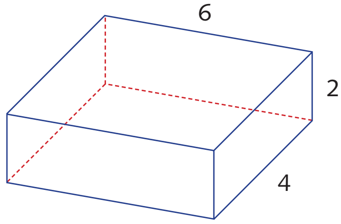
3 The table below gives the lengths, breadths, heights and volumes of various rectangular prisms. Fill in the missing entries.

	Length	Breadth	Height	Volume
a	3 cm	7 cm	6 cm	
b	5 cm	9 cm	4 cm	
c		3 m	2 m	18 m^3
d	3 m	12 m		144 m^3

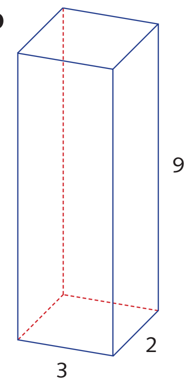
	Length	Breadth	Height	Volume
e	56 m	40 m	70 m	
f	2 mm	8 mm	16 mm	
g	32 m	4 m		1024 m^3
h	8 cm	8 cm		256 cm^3

4 Give the volume of each prism. All measurements are in cm.

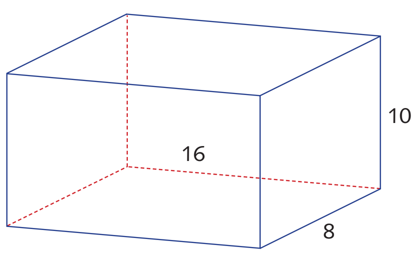
a



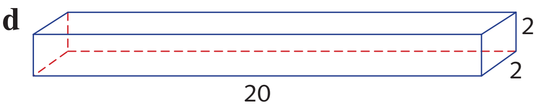
b



c



d

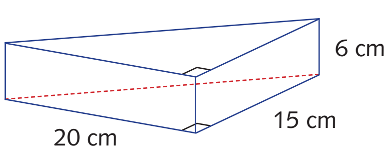


5 A swimming pool has dimensions 10 m by 12 m and is 5 m deep. What is its volume?

6 A rectangular water tank, with a base that is 1.7 m by 1.4 m, holds water to a depth of 2 m. What is the volume of the tank, in cubic metres?

7 A Rubik's cube has side length 7 cm. What is its volume?

8 A piece of cheese sold in the shops is obtained by cutting a rectangular block of cheese in two. What is the volume of the cheese?



9 A tank in the form of a rectangular prism has base dimensions 2.3 m by 1.4 m and holds water to a depth of 2 m. If the depth is increased by 3 m, what is the increase in the volume of the water?

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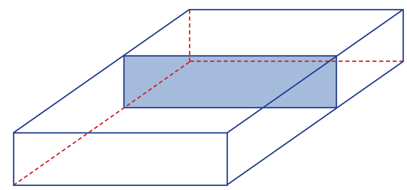
15D Volumes of other prisms

Recall that a **polyhedron** is a solid bounded by **polygons**. A **right prism** is a polyhedron that has two congruent and parallel faces called the base and top, and all its remaining faces are rectangles. A prism has **uniform cross-section**. This means that it is possible to take slices through the solid parallel to the base so that the area of each slice is always the same.

The name 'prism' comes from the Greek word *prizein*, which means 'to saw'. The idea is that if we 'saw' through a prism, the cross-sections are always the same.

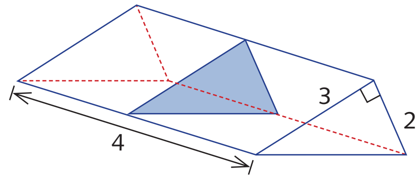
In a **rectangular prism**, the cross-section is always a rectangle.



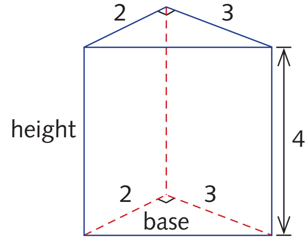
Volume of a triangular prism

In a **triangular prism**, the cross-section is always a triangle.

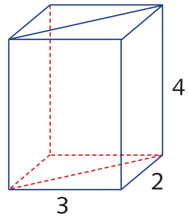
Here is a triangular prism with a cross-section that is a right-angled triangle.



We can redraw the prism as shown here.



This prism can be thought of as the rectangular prism with dimensions $2 \times 3 \times 4$ cut in half.



$$\begin{aligned}\text{Volume of triangular prism} &= \frac{1}{2} \times 2 \times 3 \times 4\\ &= 12\end{aligned}$$

In this case the base of the prism is the right-angled triangle with shorter side lengths 2 and 3.

The area of this triangle $= \frac{1}{2} \times 2 \times 3$.

The volume of the prism is given by the area of the base of the prism multiplied by the height.

$$\begin{aligned}V &= 3 \times 4\\ &= 12\end{aligned}$$

The volume of any triangular prism whose base has area A and whose height (or depth) is h is given by:

volume of a prism $= Ah$

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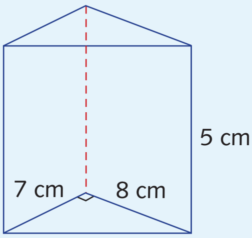
Example 8

Find the volume of the prism shown on the right.



Solution

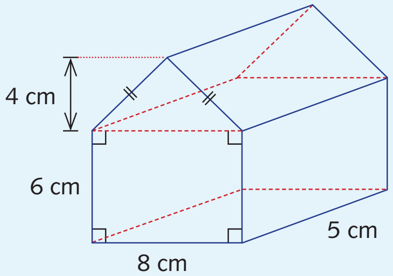
This is a triangular prism with area of base $A = \frac{1}{2} \times 8 \times 7$

$= 28 \text{ cm}^2$

$$\begin{aligned} \text{Volume} &= Ah \\ &= 28 \times 5 \\ &= 140 \text{ cm}^3 \end{aligned}$$

Example 9

Find the volume of the prism shown in the diagram.



Solution

The cross-section is the front face of the prism, a triangle on a rectangle.

$$\begin{aligned} A &= \left(\frac{1}{2} \times 8 \times 4\right) + (8 \times 6) \\ &= 64 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Volume} &= Ah \\ &= 64 \times 5 \\ &= 320 \text{ cm}^3 \end{aligned}$$

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The base of the prism can be any polygon. Any polygonal prism can be be divided up into triangular prisms.

Volumes of other prisms

- A **right prism** is a polyhedron that has two congruent and parallel faces and all its remaining faces are rectangles.
- A prism has uniform cross-section.
- The volume of any prism whose base has area A and whose height is h is given by:

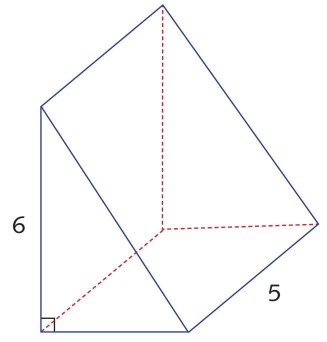
volume of a prism = Ah

Exercise 15D

Example 8, 9

1 Find the volume of each prism. All measurements are in centimetres.

a



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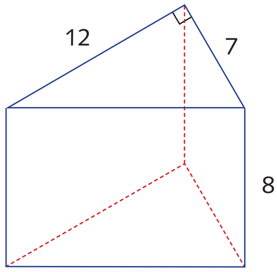

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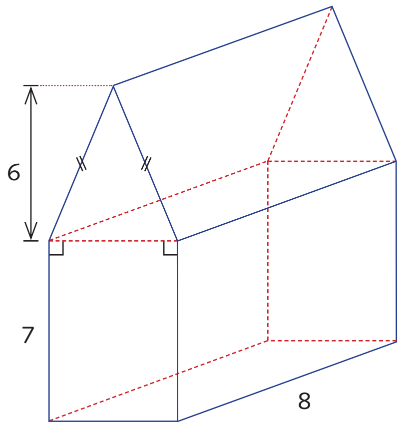

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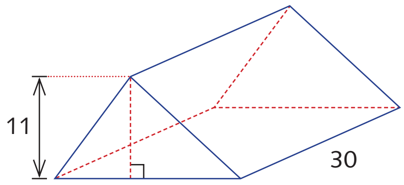
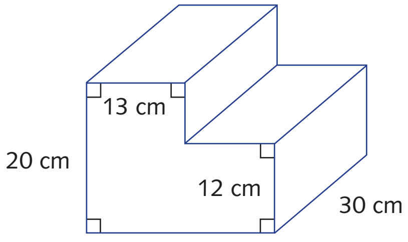


2 A small step has the shape shown opposite.

a Find the area of the front face.

b Find the volume of the step.



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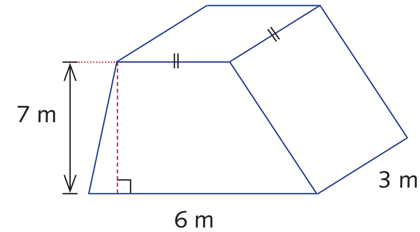
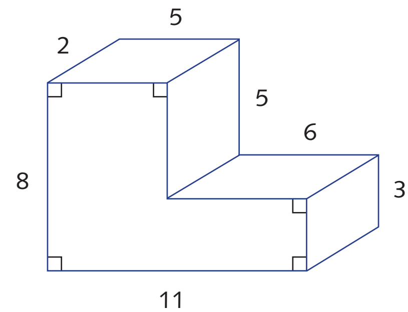
3 A large pedestal is in the shape of a prism whose front face is a trapezium.

a Find the area of the front face.

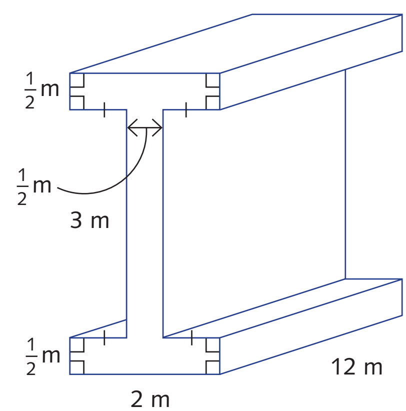
b Find the volume of the pedestal.



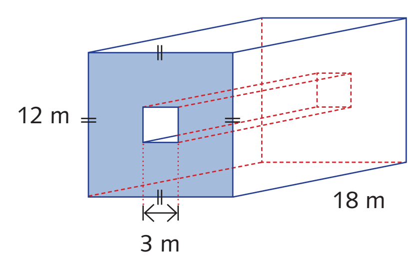
4 Find the volume of the stone block shown opposite, using the given measurements, which are in metres.



5 A steel girder has measurement in metres as shown. What is its volume?



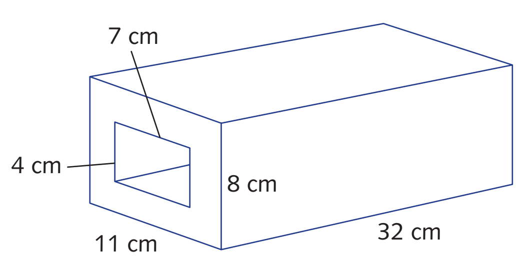
6 A 3 m by 3 m square prism is removed from a 12 m by 12 m square prism with depth 18 m. What is the volume of the remaining solid?



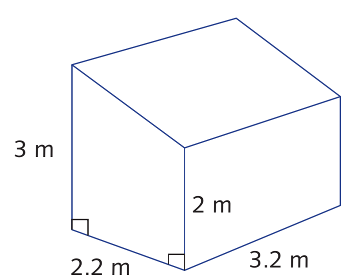
7 The front face of a prism is a parallelogram with base 8 cm and height 6 cm. If the depth is 12 m, find the volume.

8 Find the volume of each prism.

a



b



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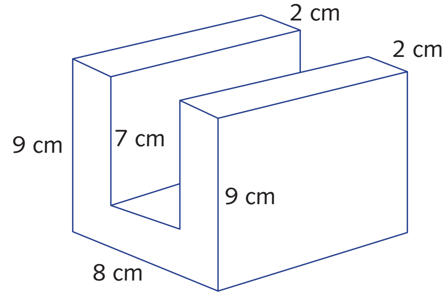
9 The volume of the prism is 792 cm^3.

Find:

a the area of the cross-section

b the length of the prism



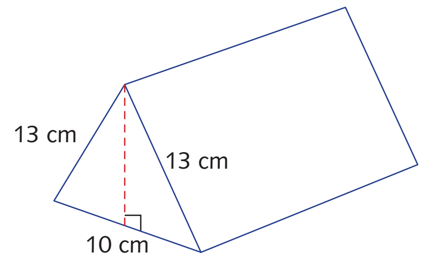
10 The volume of the prism is 4800 cm^3.

Find:

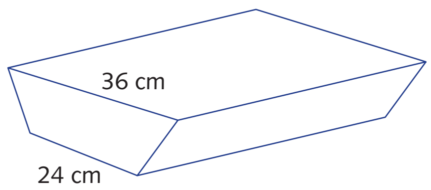
a the height of the triangle

b the area of the cross-section

c the length of the prism



11 The cross-section of the prism is a trapezium of height 5 cm. The volume of the prism is 3900 cm^3. Find the length of the prism.



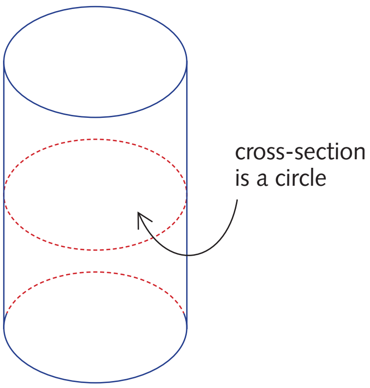
15E Volume of a cylinder

If you look in the kitchen cupboard, you will probably see a number of cans of food. In mathematics, we call these solids **cylinders**, from a Greek word meaning 'to roll'.

If we slice a cylinder parallel to its base, then each cross-section is a circle of the same size as the top and bottom.

We can use the same formula we found before to find the volume of a cylinder. Its volume is the area of the base, which is a circle, multiplied by the perpendicular height. We cannot prove this formula at this stage.

Informally, we can approximate the base by a polygon and use the formula for the area of a prism.



cylinder

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If the base circle of the cylinder has radius r, then we saw in Chapter 14 that the area of the circle is πr^2. If the height is h, then we say that the volume is:

$$\text{volume} = \pi r^2 \times h = \pi r^2 h$$

Hence:

$$\text{volume of a cylinder} = \pi r^2 h,$$

where r is the radius of the circular base and h is the perpendicular height.

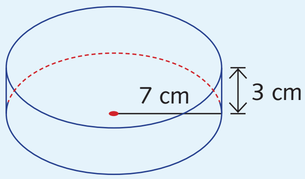
Example 10

A cylinder has base radius 7 cm and height 3 cm. Find:

a the exact volume, in terms of π

b an approximate value for the volume, using $\pi \approx \dfrac{22}{7}$

Solution



a $V = \pi r^2 h$
$= \pi \times 49 \times 3$
$= 147\pi \text{ cm}^3$

b $V = \pi r^2 h$
$\approx \dfrac{22}{7} \times 49 \times 3$
$= 462 \text{ cm}^3$

Exercise 15E

Example 10

1 For each cylinder, find:

i the exact volume, in terms of π

ii an approximate value for the volume, using $\pi \approx \dfrac{22}{7}$

a Radius 14 m, height 20 m

b Diameter 7 cm, height 16 cm

c Radius 21 mm, height 12 mm

2 A jam jar is in the shape of a cylinder. It has a base radius of $3\frac{1}{2}$ cm and a height of 15 cm. Use $\pi \approx \dfrac{22}{7}$ to find an approximate value for its volume.

3 Use $\pi \approx 3.14$ to find the approximate value of the volume of a can with base diameter 60 mm and height 20 mm.

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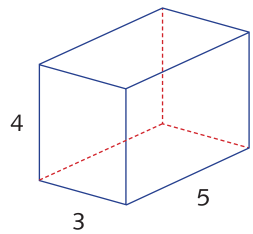
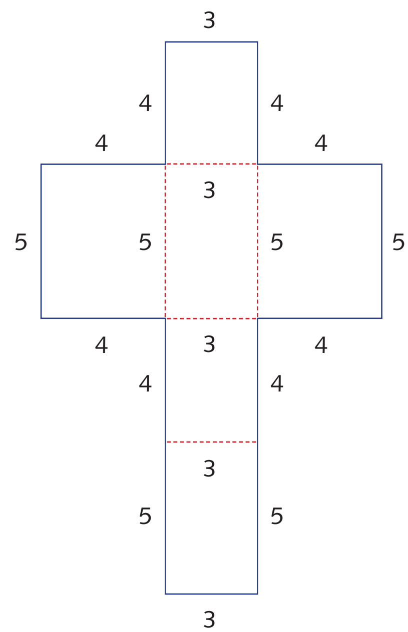
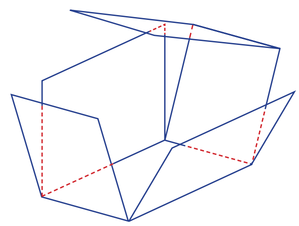
15F Surface area of a prism

Suppose we take a rectangular prism whose dimensions are 3 by 4 by 5, and open it out as shown below.







We can find the area of the flattened box by adding up the areas of the six rectangular faces. There are:

- two rectangular faces with area $3 \times 4 = 12$
- two rectangular faces with area $3 \times 5 = 15$
- two rectangular faces with area $4 \times 5 = 20$.

In total, this gives an area of $2 \times (12 + 15 + 20) = 94$.

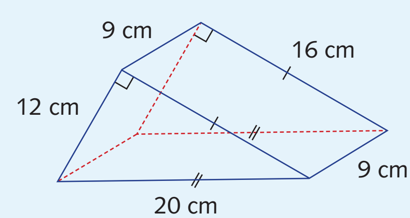
This is called the **surface area** of the box.

In practice, the dimensions will be measured in centimetres and the surface area in cm^2. The surface area of a prism is the sum of the areas of its faces. A rectangular prism with dimensions a, b and c has surface area:

$$\text{surface area of a rectangular prism} = 2(ab + ac + bc)$$

Example 11

Find the surface area of the triangular prism shown opposite.



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Solution

$$\text{Area of front} = \frac{1}{2} \times 12 \times 16 = 96\ \text{cm}^2$$

$$\text{Area of back} = 96\ \text{cm}^2$$

$$\text{Area of the three rectangular faces} = (9 \times 20) + (9 \times 12) + (9 \times 16) = 432\ \text{cm}^2$$

$$\text{Total surface area} = 96 + 96 + 432 = 624\ \text{cm}^2$$

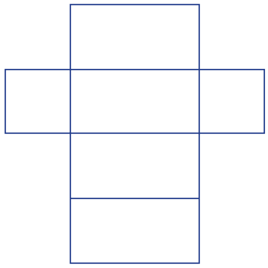
Surface area of a prism

The surface area of a prism is the sum of the areas of its faces.

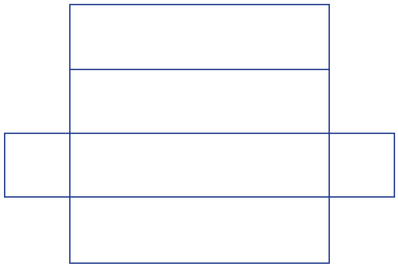
Exercise 15F

1 In each case below, the surface of a solid has been cut open and opened up as shown. Draw the original solid.

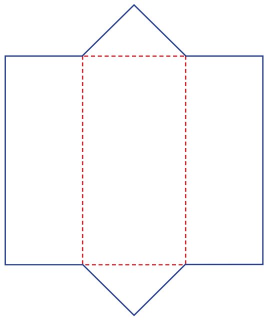
a



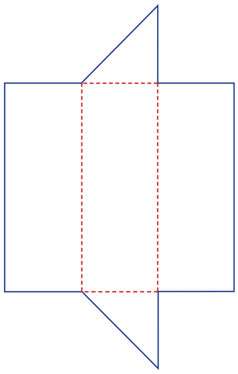
b



c



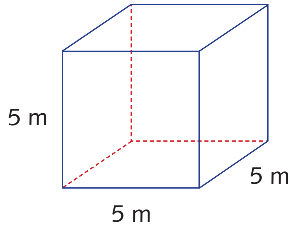
d



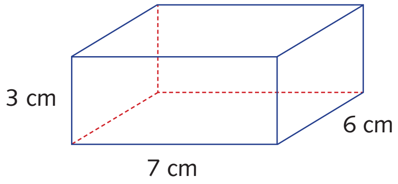
Example 11

2 Find the surface area of each rectangular prism.

a



b



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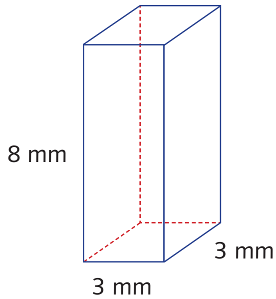
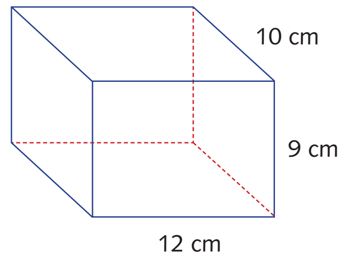
c



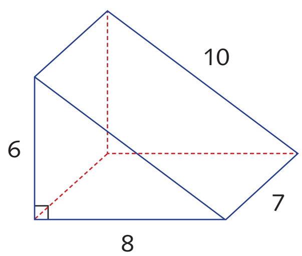
d



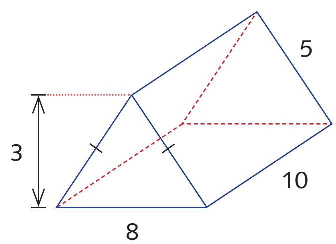
Example 11

3 Find the surface area of each triangular prism. All measurements are in centimetres.

a

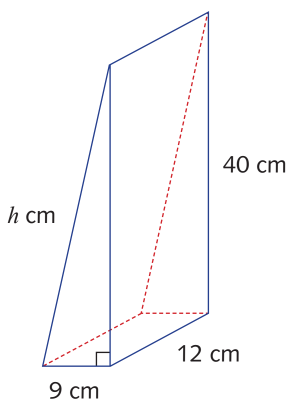


b

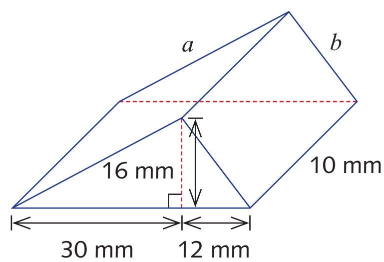


4 Use Pythagoras' theorem to find the unknown lengths in the figures below and then calculate their surface areas.

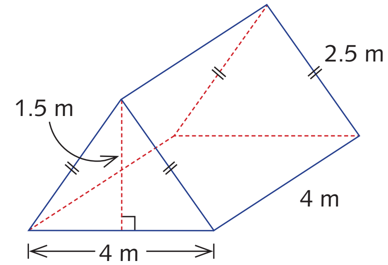
a



b

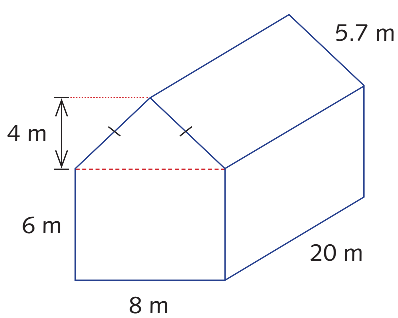


5 A tent made from calico, including the ground sheet, is in the shape of a triangular prism, with dimensions as shown. How much calico is needed to make the tent?

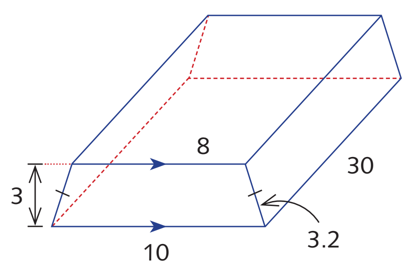


6 Find the surface area of each prism.

a



b



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15G Conversion of units

We often need to convert from one standard unit to another.

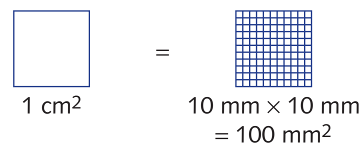
We recall:

$1\text{ cm} = 10\text{ mm} \qquad 1\text{ m} = 100\text{ cm} \qquad 1\text{ km} = 1000\text{ m}$

Just as lengths can be converted from one unit to another, so can areas and volumes. We can use the basic length conversions above to obtain area and volume conversions.

Area conversions

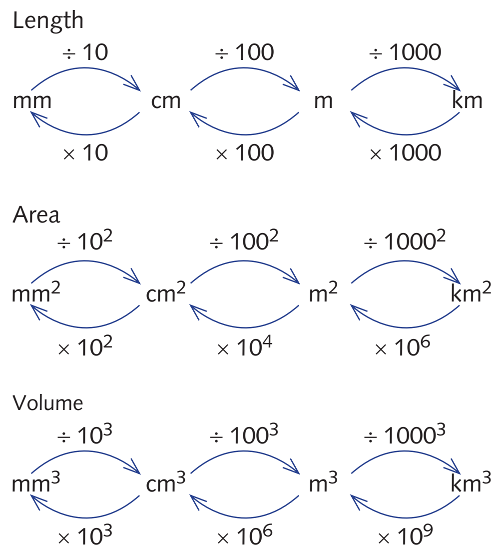
A square of side length 1 cm has area 1 cm^2.
Since 1 cm = 10 mm, the area of this square is also $10 \times 10 = 100\text{ mm}^2$.
Hence $1\text{ cm}^2 = 100\text{ mm}^2$



Similarly, we have:

$$1\text{ m}^2 = 100^2\text{ cm}^2 = 10\,000\text{ cm}^2 = 1\,000\,000\text{ mm}^2$$
$$1\text{ km}^2 = 1000^2\text{ m}^2 = 1\,000\,000\text{ m}^2$$

Hence, to obtain the conversion factor for areas, we square the corresponding conversion factor for lengths.



Example 12

Convert each of the following measurements to cm^2.

a 2.5 m^2

b 3600 m^2

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Solution

a

$$\begin{aligned} 1\text{ m}^2 &= 100^2\text{ cm}^2 \\ &= 10\,000\text{ cm}^2 \\ \text{so } 2.5\text{ m}^2 &= 2.5 \times 10\,000\text{ cm}^2 \\ &= 25\,000\text{ cm}^2 \end{aligned}$$

b

$$\begin{aligned} 1\text{ cm}^2 &= 10^2\text{ mm}^2 \\ &= 100\text{ mm}^2 \\ \text{so } 3600\text{ mm}^2 &= \frac{3600}{100}\text{ cm}^2 \\ &= 36\text{ cm}^2 \end{aligned}$$

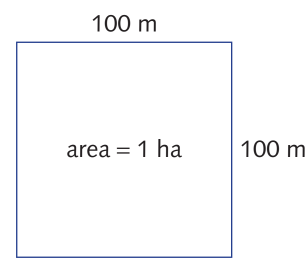
Hectares

The metric system was introduced in Europe by Napoleon in the early nineteenth century, and was adopted by Australia in 1966. Up until that time the Imperial system was used and land was measured in acres. To include a similar sized unit of area in the metric system, the following unit of area is defined.

A **hectare** (ha) is the area enclosed by a square with side length 100 m.

That is,

$$\begin{aligned} 1\text{ ha} &= (100 \times 100)\text{ m}^2 \\ &= 10\,000\text{ m}^2 \\ &= 10^4\text{ m}^2 \\ 100\text{ ha} &= 100 \times 10^4\text{ m}^2 \\ &= 10^6\text{ m}^2 \\ &= 1\text{ km}^2 \end{aligned}$$



Example 13

The area of a cattle station in outback Australia is 200 000 ha.

Calculate:

a the area in m^2

b the area in km^2

c the dimensions of the station, if it is a square, to one decimal place

d the dimensions of the station, if it is a rectangle and one side length is 50 km

Solution

a

$$\begin{aligned} 1\text{ ha} &= (100 \times 100)\text{ m}^2 \\ &= 10\,000\text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{so } 200\,000\text{ ha} &= (200\,000 \times 10\,000)\text{ m}^2 \\ &= 2\,000\,000\,000\text{ m}^2 \\ &= (2 \times 10^9)\text{ m}^2 \end{aligned}$$

b Since $1\text{ km}^2 = 100$ ha

$200\,000\text{ ha} = 2000\text{ km}^2$

c Let x km be the side length of the square,

so $x^2 = 2000$

Hence, $x \approx 44.7$ (to one decimal place)

So the station is approximately 44.7 km by 44.7 km.

d $\dfrac{2000}{50} = 40$

The station is 40 km by 50 km.

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Volume conversions

A cube of side length 1 cm has volume 1 cm^3 (this is read as, 1 cubic centimetre). Since 1 cm = 10 mm, the volume of this cube is also $10 \times 10 \times 10 = 1000\ mm^3$.

Hence $1\ cm^3 = 1000\ mm^3$.

Similarly, we have :

$$1\ m^3 = 100^3\ cm^3 = 1\,000\,000\ cm^3 = 10^6\ cm^3$$
$$1\ km^3 = 1000^3\ m^3 = 1\,000\,000\,000\ m^3 = 10^9\ m^3$$

Hence, to obtain the conversion factor for volumes, we cube the corresponding conversion factor for lengths.

Example 14

Convert each of the following measurements to the units indicated in the brackets.

a $2760\ mm^3$ (cm^3)

b $0.27\ m^3$ (cm^3)

c $256\,000\ cm^3$ (m^3)

d $0.59\ cm^3$ (mm^3)

Solution

a
$$\begin{aligned} 10\ mm &= 1\ cm \\ \text{so } 10^3\ mm^3 &= 1\ cm^3 \\ 1000\ mm^3 &= 1\ cm^3 \end{aligned}$$
$$\begin{aligned} \text{Hence, } 2760\ mm^3 &= \frac{2760}{1000}\ cm^3 \\ &= 2.76\ cm^3 \end{aligned}$$

b
$$\begin{aligned} 1\ m &= 100\ cm \\ \text{so } 1\ m^3 &= 100^3\ cm^3 \\ 1\ m^3 &= 1\,000\,000\ cm^3 \\ &= 10^6\ cm^3 \end{aligned}$$
$$\begin{aligned} \text{Hence, } 0.27\ m^3 &= (0.27 \times 10^6)\ cm^3 \\ &= 2.7 \times 10^5\ cm^3 \\ &= 270\,000\ cm^3 \end{aligned}$$

c
$$\text{From } \mathbf{b},\ 10^6\ cm^3 = 1\ m^3$$
$$\begin{aligned} \text{Hence, } 256\,000\ cm^3 &= \frac{256\,000}{1\,000\,000}\ cm^3 \\ &= 0.256\ m^3 \end{aligned}$$

d
$$\text{From } \mathbf{a},\ 1\ cm^3 = 1000\ mm^3$$
$$\begin{aligned} \text{Hence, } 0.59\ cm^3 &= 0.59 \times 1000\ mm^3 \\ &= 590\ mm^3 \end{aligned}$$

Litres

The following units of volume are used when measuring liquids.

A **litre** (1 L) is equal to 1000 cm^3.

$$\begin{aligned} \text{That is, } 1\ L &= 1000\ cm^3 \\ 1\ m^3 &= 10^6\ cm^3 \\ &= 1000\ L \end{aligned}$$

A **millilitre** (1 mL) is equal to $\frac{1}{1000}$ of a litre and is thus equal to 1 cm^3.

That is, $1\ mL = \frac{1}{1000} L = 1\ cm^3$.

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Example 15

A large water trough in the shape of a rectangular prism has internal dimensions of 3 m by 0.6 m by 0.5 m. How many litres of water does the trough hold when full?

Solution

Since $1000\ \text{cm}^3 = 1$ litre, the volume is best calculated in cubic centimetres.

Now $100\ \text{cm} = 1\ \text{m}$

$$\begin{aligned}\text{Volume} &= 300\ \text{cm} \times 60\ \text{cm} \times 50\ \text{cm}\\ &= 900\,000\ \text{cm}^3\\ &= 900\ \text{litres}\end{aligned}$$

Hence, the water trough holds 900 litres.

Conversion of units

Length

$1\ \text{cm} = 10\ \text{mm}$	$1\ \text{m} = 100\ \text{cm}$	$1\ \text{km} = 1000\ \text{m}$

Area

$1\ \text{cm}^2 = 100\ \text{mm}^2$	$1\ \text{m}^2 = 10^4\ \text{cm}^2$	$1\ \text{km}^2 = 10^6\ \text{m}^2$

Volume

$1\ \text{cm}^3 = 1000\ \text{mm}^3$	$1\ \text{m}^3 = 10^6\ \text{cm}^3$	$1\ \text{km}^3 = 10^9\ \text{m}^3$

Area of land

$1\ \text{ha} = 10^4\ \text{m}^2$	$1\ \text{km}^2 = 100\ \text{ha}$

Litres (measurement of liquids)

1 litre = 1000 millilitres	$1\ \text{mL} = 1\ \text{cm}^3$	$1\ \text{m}^3 = 1000\ \text{L}$

Exercise 15G

Example 12

1 Convert each measurement to cm^2.

a $300\ \text{mm}^2$ **b** $0.7\ \text{m}^2$ **c** $90\ \text{mm}^2$ **d** $3.1\ \text{m}^2$

2 Convert each measurement to mm^2.

a $2\ \text{cm}^2$ **b** $0.5\ \text{m}^2$ **c** $0.6\ \text{cm}^2$ **d** $2.3\ \text{m}^2$

3 Convert each measurement to m^2.

a $2.4\ \text{km}^2$ **b** $36\,000\ \text{cm}^2$ **c** $0.36\ \text{km}^2$ **d** $2800\ \text{cm}^2$

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4 A table top measures 900 mm × 1150 mm.

a Calculate the area of the table top in square millimetres.

b Express your answer to part **a** in square centimetres.

c Give the dimensions of the table top in centimetres.

d Using the dimensions found in **c**, calculate the area of the table top in square centimetres.

e Do your answers to part **b** and part **d** agree?

f Express the area of the table top in square metres.

g Give the dimensions of the table top in metres.

h Using the dimensions found in part **g**, calculate the area of the table top in square metres.

i Check that your answers to part **f** and part **h** agree.

Example 13

5 A rectangular piece of land measures 260 m by 430 m. Calculate the area of the land in:

a m^2

b hectares

6 A rectangular piece of land has an area of 2.7 ha. If the block of land is 135 metres wide, how long is the block of land?

7 A cattle station has an area of 260 km^2. What is this area in hectares?

8 A farm has an area of 480 ha. Calculate the area in:

a m^2

b km^2

9 One acre is approximately 0.4 hectares. A rectangular block of land measures 50 m × 150 m. Calculate the area of the block of land in:

a hectares

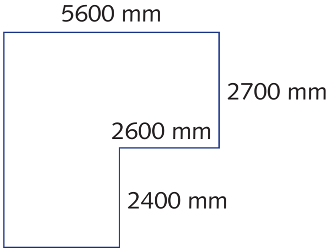
b acres (approximately)

10 A classroom measures 6400 mm × 7800 mm. Carpet is to be laid on the floor at a cost of $45 per square metre. Calculate:

a the floor area in square metres

b the cost of the carpet for the classroom

11 Robyn intends to paint the ceiling of her living room. A plan of the room is drawn with measurements. One litre of paint will cover 12 m^2. Calculate:



a the area of the ceiling in square metres

b the amount of paint needed to put one coat of paint on the ceiling

12 Convert 1 m^2 to:

a cubic centimetres

b litres

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Example 14 **13** Convert each of the following measurements to the units indicated in the brackets.

a 5760 mm^3 (cm^3) **b** 0.56 m^3 (cm^3)

c $756\,000 \text{ cm}^3$ (m^3) **d** 0.59 cm^3 (mm^3)

14 Convert each measurement to the units in the brackets.

a 0.62 m^3 (L) **b** 2600 cm^3 (L) **c** $52\,000 \text{ mm}^3$ (mL)

d 2.7 L (cm^3) **e** 960 L (m^3) **f** 26 mL (mm^3)

Example 15 **15** A large tank in the shape of a rectangular prism has internal dimensions $3 \text{ m} \times 2 \text{ m} \times 1.5 \text{ m}$. How many litres does the tank hold when full?

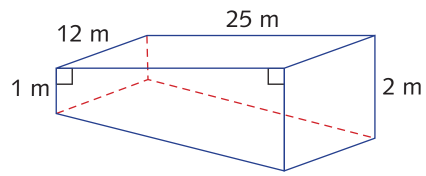
16 A cylindrical water tank has a diameter of 3 m and a height of 2 m. Calculate, to the nearest 100 mL, the volume of the tank in:

a m^3

b litres

17 A cylindrical water tank has diameter 4 m and a height of 2.5 m. If the tank is initially full, for how many days can a family use this water tank if their daily consumption of water is 600 litres? (Assume no water enters the tank during the time period.)

18 A school swimming pool has dimensions as shown. How long would it take to fill this pool if the pump can deliver 1500 litres of water a minute?



19 A medicine bottle is in the shape of a cylinder with base diameter 50 mm and height 80 mm. If the normal dosage of medicine is 15 mL, how many dosages can be obtained from the bottle? (Assume that initially the bottle is completely full.)

20 A swimming pool is in the shape of a rectangular prism. The pool is 12 metres long, 4 metres wide and 1.5 metres deep. The pool is to be lined with tiles.

a How many tiles are needed to line the pool if each tile is:

i $100 \text{ mm} \times 100 \text{ mm}$?

ii $200 \text{ mm} \times 200 \text{ mm}$?

iii $200 \text{ mm} \times 100 \text{ mm}$?

b A path 1 metre wide is to be put around the pool. How many pavers are needed to make the path if each of the pavers is of size:

i $1 \text{ m} \times 1 \text{ m}$?

ii $500 \text{ mm} \times 500 \text{ mm}$?

iii $250 \text{ mm} \times 250 \text{ mm}$?

c When the pool is completed, it is filled at 400 litres/hour. How long will it take to fill the pool to 10 cm below the top?

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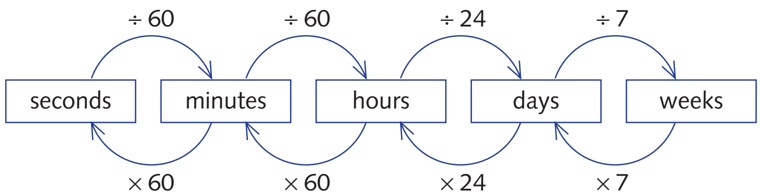
15H Time

Time is different from other measurements because it is not based on powers of 10.

- There are 60 seconds in 1 minute.
- There are 60 minutes in 1 hour.
- There are 24 hours in 1 day.
- There are 7 days in a week.



There are two ways of recording the time of day:

- using **a.m.** and **p.m**. For example, 5:30 p.m. means 5 hours 30 minutes after midday, and 12:30 a.m. means 30 minutes after midnight. Remember that a.m. stands for *ante meridiem*, which is Latin for ‘being before noon’, and p.m. stands for *post meridiem*, which is Latin for ‘being after noon’.
- using the **24-hour system**. For example, 2235 means 22 hours and 35 minutes after midnight. This is the same as 10:35 p.m. 0530 means 5 hours 30 minutes after midnight or 5:30 a.m. 1730 is the same as 5:30 p.m. Note that there is no need to indicate morning by using a.m., or afternoon by using p.m., when using 24-hour time.

Example 16

A truck driver drove for 5 hours 45 minutes on Monday, 4 hours 50 minutes on Tuesday and 6 hours 30 minutes on Wednesday. What was the total time she spent driving?

Solution

5 h	45 m	(Add the minutes, add the hours.)
+ 4 h	50 m	
+ 6 h	+ 30 m	
15 h	125 m	

$$\begin{aligned}\text{Total time spent driving} &= \text{15 hours + 2 hours + 5 minutes}\\ &\ \text{(Convert minutes total to hours and minutes.)}\\ &= \text{17 hours 5 minutes}\end{aligned}$$

Duration or elapsed time

Calculating elapsed time is a skill that is made interesting by the fact that time is based on the numbers 24 and 60.

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Example 17

a Jane left home at 3:54 p.m. to travel to her aunt's house. She arrived at 5:40 p.m. How long did her journey take?

b Jane's father drove from their home to her aunt's house and it took him 48 minutes. How much longer did Jane take to arrive?

Solution

a There are 6 minutes to 4 p.m., and 1 hour and 40 minutes to 5:40 p.m.

So Jane's total travel time = 6 minutes + 1 hour 40 minutes
= 1 hour 46 minutes

b We need to work out the difference between Jane's travel time of 1 hour 46 minutes and her father's time of 48 minutes.

12 minutes are needed to build up from 48 minutes to 1 hour, and then there are 46 minutes after that.

Total = 12 + 46
= 58 minutes

Jane took 58 minutes longer to arrive than her father.

Example 18

Marathon runners take around 3 hours to run 42 km. If a runner started at 11:23:00 a.m. (23 minutes past 11, no seconds) and finished at 2:40:49 p.m., what was the time taken to complete the marathon?

Solution

The time taken to complete the marathon is calculated by building up to the next whole minute or hour.

	Time	Hour	Minutes	Seconds
Start time	11:23:00 a.m.			
Build up to	12:00:00 p.m.		37 minutes	
Build up to	2:00:00 p.m.	2 hours		
Build up to	2:40:49 p.m.		40 minutes	49 seconds
Total		2 hours	77 minutes	49 seconds
Total time taken to complete the marathon (convert minutes to hours and minutes).		3 hours	17 minutes	49 seconds

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Time

- Measurement of time is based on the numbers 7, 24 and 60.
- Building up to whole minutes, hours or days is helpful when calculating time differences and when adding times.

Exercise 15H

1 Convert each of these times to 24-hour time.

a 4:30 p.m.
b 11 a.m.
c 6:49 p.m.
d Six thirty-five in the evening
e Twenty-five to five in the morning
f Five past four in the afternoon
g Three minutes to midnight

2 Write each of these 24-hour times in 12-hour time using a.m. and p.m.

a 1400
b 0800
c 0835
d 2230

Example 16

3 Phil worked for 25 days 18 hours in January, 23 days 22 hours in February and 24 days 11 hours in March. What was the total time he worked?

Example 17

4 David travelled for 5 days 23 hours by boat, then 16 hours by plane and finally 3 days 22 hours by train to be home for his mother's birthday. How long did his journey take?

5 What is the elapsed time, in days, hours and minutes, between:

a 12 noon and 4:45 p.m.?
b 6 p.m. and 11:30 p.m.?
c 1100 and 1830?
d ten past seven in the morning and four-thirty in the afternoon of the same day?
e 3:35 a.m. on Tuesday and 6:20 a.m. on the next Wednesday?
f 1:32 a.m. on Friday and 2:50 p.m. on the following Sunday?
g 2 p.m. on Thursday and 7:25 a.m. on the following Tuesday?

6 If I started a train trip at 1530 and finished it at 0725 the next day, how long did the journey take?

Example 18

7 Three people ran the New York marathon; their times were 2:45:42 (2 hours 45 minutes 42 seconds), 2:43:57 and 2:42:15. What were the time differences between the first and second competitors, and between the second and third?

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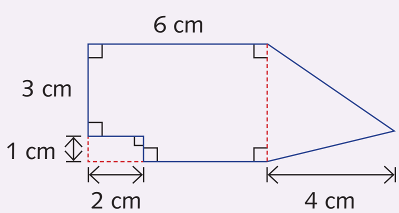
8 Suppose a child is born at 2249 on one day and another child is born at 0318 the next day. How much older is the first child?

9 At the end of 2010, the men's world record time for the 100 m sprint was 9:58 seconds, while the women's record was 10:49 seconds. How much faster was the men's record?

10 Sarah takes 4 hours and 55 minutes to complete a 200-page novel, while Derek takes 5 hours and 12 minutes. Reading at the same speeds, how much faster is Sarah than Derek, in seconds, if they both read a 300-page novel?

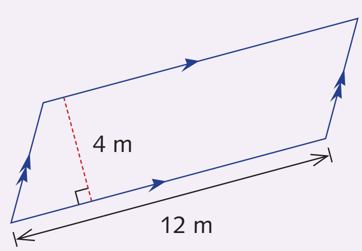
Review exercise

1 Find the area of the region shown below.

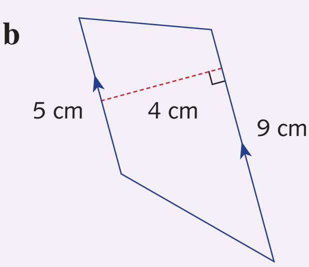


2 Find the area of each region.

a



b



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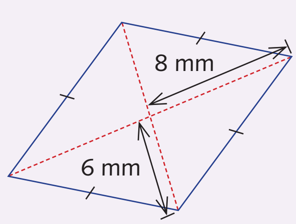
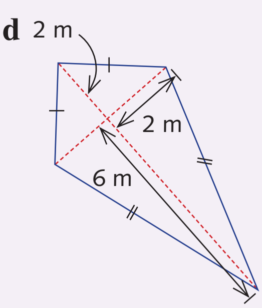


d



3 What is the volume of a rectangular fish tank with dimensions 90 cm, 45 cm and 60 cm?

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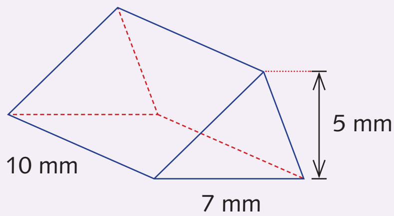
4 Find the volume of each prism.

a



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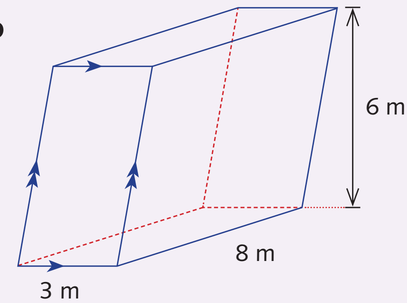

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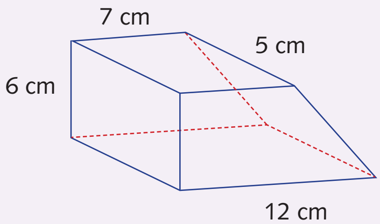

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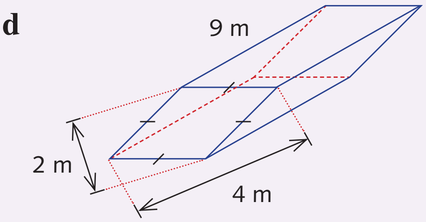

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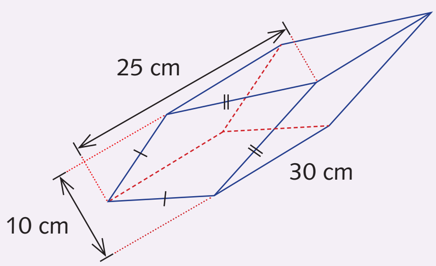


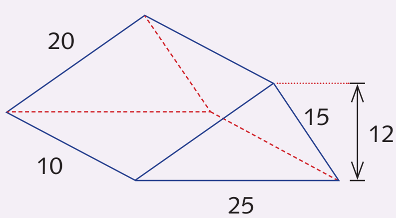
5 What are the exact volumes of the cylinders with the following measurements?

a Radius 5 m, height 4 m

b Diameter 6 cm, height 7 cm

6 Find the surface area of each prism.

a



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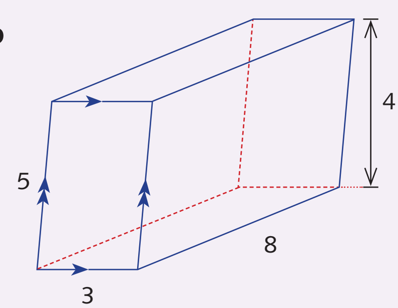

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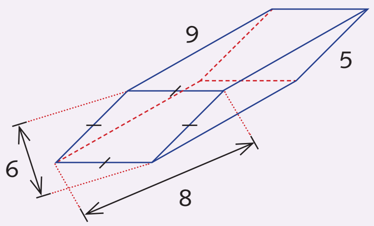

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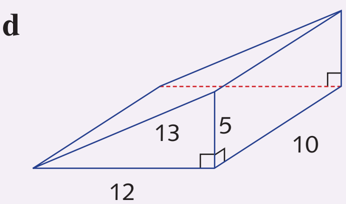

7 Convert each measurement to cm^2.

a $3.5\ m^2$ **b** $7200\ mm^2$ **c** $4\ m^2$ **d** $67\,000\ mm^2$

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8 Convert 450 000 hectares to:

a m^2 b km^2

9 How many litres does a cylinder of radius 20 cm and height 1 m hold?

10 How many litres does a rectangular prism with dimensions 20 cm × 80 cm × 50 cm hold?

11 What is the elapsed time, in days, hours and minutes, between:

a 12 noon and 7:45 p.m.?

b 1 p.m. and 11:30 p.m.?

c 1115 and 1830?

d twenty past seven in the morning and five-thirty in the afternoon of the same day?

e 2:35 a.m. on Tuesday and 7:35 a.m. on the next Wednesday?

f 2:36 a.m. on Friday and 2:51 p.m. on the following Sunday?

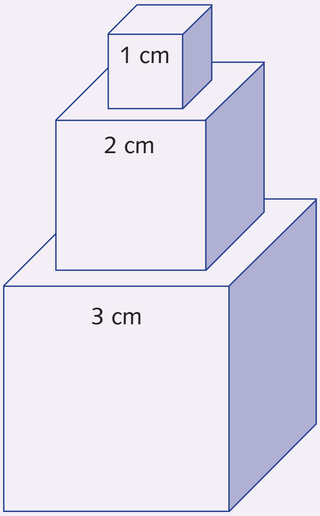
g 1 p.m. on Thursday and 7:36 a.m. on the following Tuesday?

Challenge exercise

1 The areas of three faces of a rectangular prism are 10 cm^2, 14 cm^2 and 35 cm^2. Find the volume of the rectangular prism.

2 A cylindrical container of diameter 80 cm and height h cm is filled with water. How many cylinders of diameter 20 cm and height h cm would you need to hold the same amount of water?

3 Three cubes, with side lengths of 1 cm, 2 cm and 3 cm, are glued together as shown opposite. Find the surface area of the object.



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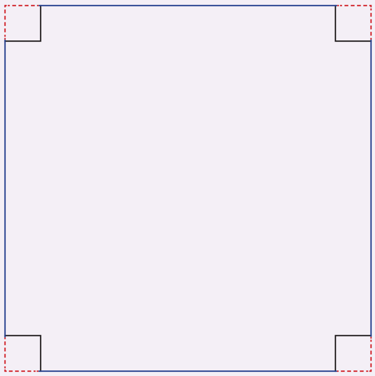
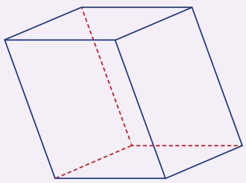
4 A square piece of card has area 49 cm^2. Squares of side length 1 cm are cut from each corner of the card, as shown opposite. Find the volume of the open box formed by turning up the flaps and gluing.



5 In the diagram below, points X, Y and Z are the midpoints of the edges they lie on. A triangular prism is formed by cutting through the points X, Y and Z. Find the volume of the remaining solid.

6 A cube with edge length 6 cm is made from cubes with edge length 1 cm. The 1-cm cubes making up the larger cube are then rearranged to form two rectangular prisms. The base of one rectangular prism is $8 \text{ cm} \times 5 \text{ cm}$ and the base of the other is $6 \text{ cm} \times 4 \text{ cm}$. What is the height of each rectangular prism?

7 Prisms whose bases are horizontal but whose sides are not vertical are called **oblique** prisms. An oblique rectangular prism is an example of a **parallelepiped**.

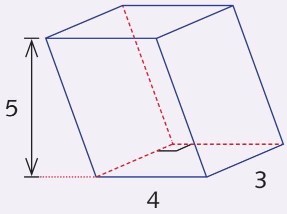


The formula we found earlier:

$$\text{volume} = \text{area of base} \times (\text{perpendicular}) \text{ height}$$

works for oblique prisms as well.

Find the volume of the parallelepiped shown opposite.



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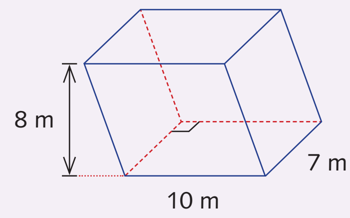
8 Find the volume of the parallelepiped shown below.



9 A cube of side length 3 cm is made from 27 cubes with sides of length 1 cm. Three cubes are removed so that there is a hole from one face to the opposite face. What is the surface area of the resulting object?

10 How many solid rectangular prisms of dimensions 1 cm $\times$ 2.5 cm $\times$ 1.6 cm can be cast from 1 m^3 of steel?

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