CHAPTER

16

Statistics and Probability

Probability

Probability deals with how likely it is that something will happen. It is an area of mathematics with many diverse applications. Probability is used in areas ranging from weather forecasting and insurance – where it is used to calculate risk factors and premiums – to predicting the risks of new medical treatments and forecasting the effects of global warming.

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16A An introduction to probability

There are many situations in which it would be useful to be able to measure how likely (or unlikely) it is that an event will occur. We can do this in mathematics by using the idea of **probability**, which we define as a number between 0 and 1 that we assign to any **event** we are interested in. Then:

- a probability of 1 represents an event that is 'certain' or 'guaranteed to happen'
- a probability of 0 represents an event that we would describe as 'impossible' or one that 'cannot possibly occur'
- an event that has a probability $\frac{1}{2}$ is as likely to occur as not to occur
- an event that has a probability close to 0 is unlikely to occur
- an event that has a probability close to 1 is likely to occur.

In this chapter we look at methods for determining probabilities.

Sample space

A box contains 12 identical marbles numbered from 1 to 12. The box is shaken and a marble is randomly taken from it and its number noted. This is an example of doing a random **experiment**. The numbers 1, 2, …, 12 are called the **outcomes** of this experiment. The outcomes for this experiment are **equally likely**. The probability of each outcome is $\frac{1}{12}$. The complete set of possible outcomes (or sample points) for any experiment is called the **sample space** of that experiment. For example, we can write down the sample space ξ for this experiment as:

$$\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

In this chapter, all of the experiments have finite sample spaces with equally likely outcomes. For a sample space with n equally likely outcomes the probability of each outcome is $\frac{1}{n}$.

Events

An **event** is a collection of sample points. An **event** is a subset of the sample space.

Suppose that for the experiment above we are interested in getting a prime number. In this case 'the number is prime' is the event that interests us. Some of the outcomes will give rise to this event. For instance, if the outcome is 2, then the event 'the number is prime' takes place. We say that the outcome 2 is **favourable to the event** 'the number is prime'. If the outcome is 4, then the event 'the number is prime' does not occur. The outcome 4 is **not favourable to the event**.

Of the 12 possible outcomes, these are the ones that are favourable to the event 'the number is prime':

$$\{2, 3, 5, 7, 11\}$$

In many situations, 'success' means 'favourable to the event' and 'failure' means 'not favourable to the event'.

Events are named by capital letters. For instance, we talk about:

B the event 'a prime is obtained' from the experiment described above.

C the event 'obtaining an even number' when a die is tossed.

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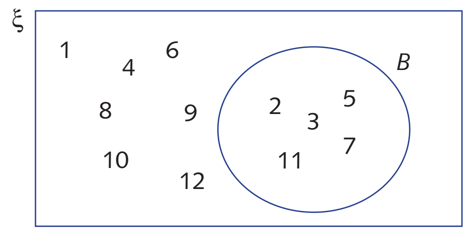
An outcome is favourable to an event if it is a member of that event. For example:

$2 \in B$ and $2 \in C$

These sample spaces and events can be illustrated with **Venn diagrams**. Venn diagrams were introduced in the context of sets in Year 7.

Here is the sample space ξ and the event B. In this context ξ is the universal set for the experiment of withdrawing a marble and observing the number on it, as described previously.



Probability of an event

The probability of the event A is written $P(A)$.

Probabilities are assigned to events in such a way that:

$0 \le P(A) \le 1$ for all events A

That is, the probability of an event is a number between 0 and 1 inclusive.

For an experiment in which all of the outcomes are equally likely:

$$\text{probability of an event} = \frac{\text{number of outcomes favourable to that event}}{\text{total number of outcomes}}$$

For the event B described on the previous page, $P(B) = \frac{5}{12}$.

In general, the probability of an event is the sum of the probabilities of the outcomes that are favourable to that event.

The total probability is 1

The sum of the probabilities of the outcomes of an experiment is 1.

For the experiment of withdrawing a marble discussed above, each outcome has probability $\frac{1}{12}$. The sum of these probabilities is 1.

The words 'random' and 'randomly'

In probability, we frequently hear these words, as in the following situation:

A classroom contains 23 students. A teacher comes into the room and chooses a student at random to answer a question about history.

What does this mean? It means that the teacher chose the student as if she knew nothing at all about the students. Another way of interpreting this is to imagine that the teacher had her eyes closed and had no idea who was in the class when she chose a student. Each student is equally likely to be chosen.

The probability of a particular student being chosen is $\frac{1}{23}$.

Example 1

A die is rolled once. Draw a Venn diagram for the experiment and circle the outcomes favourable to the event 'the number is greater than 4'. What is the probability of a number greater than 4 being obtained?

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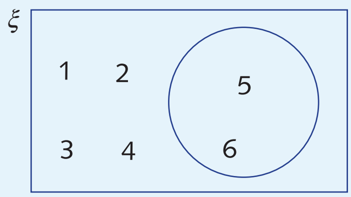
Solution

The outcomes favourable to the event are a 5 or a 6 appearing.

$$P(\text{number greater than } 4) = \frac{2}{6} = \frac{1}{3}$$



Example 2

A standard pack, or simply a pack, of playing cards consists of four suits: Hearts, Diamonds, Clubs and Spades. Each suit has 13 cards consisting of Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen and King. The pack is shuffled and a card is drawn. What is the probability of drawing:

a a King?

b a Heart?

Solution

a Let K be the event 'a King is drawn'.

$$P(K) = \frac{4}{52} = \frac{1}{13}$$

b Let H be the event 'a Heart is drawn'.

$$P(H) = \frac{13}{52} = \frac{1}{4}$$

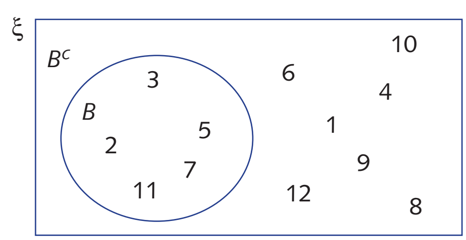
The complement of an event

Sometimes, rather than being interested in seeing if the event A happens, we are interested in the event 'not A'. For instance, we might want to know the probability that it does not rain tomorrow, or that a particular team does not win a premiership, or that the number 40 is not drawn out in a lottery draw.

The event '**not A**' consists of every possible outcome that is not in A; that is, 'not A' is everything in the sample space ξ that is outside A. The event 'not A' is called the **complement** of A and is denoted by A^c.

Recall our experiment of drawing a marble from a bag of 12 marbles, with B the event 'a prime is obtained'.

$$B^c = \{1,\ 4,\ 6,\ 8,\ 9,\ 10,\ 12\} \text{ and } P(B^c) = \frac{7}{12}$$



Notice that $P(B) = \dfrac{5}{12}$ and $P(B) + P(B^c) = 1$.

Every outcome of a sample space is contained in one of A or A^c, but not both.

Therefore $P(A) + P(A^c) = 1$ and $P(A^c) = 1 - P(A)$.

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The complement of an event

The complement of an event A is everything in the sample space ξ that does not lie in A. It is denoted by A^c and is called the event 'not A'.

$$P(A^c) = 1 - P(A)$$

Example 3

A dice is thrown and the value on the uppermost face observed. Find the probability of obtaining:

a 5 or 6

b 1, 2, 3 or 4

Solution

a Let A be the event $\{5 \text{ or } 6\}$.

$$P(A) = \frac{2}{6} = \frac{1}{3}$$

b The event $\{1, 2, 3, 4\}$ is A^c. Therefore:

$$\begin{aligned} P(A^c) &= 1 - P(A) \\ &= 1 - \frac{1}{3} \\ &= \frac{2}{3} \end{aligned}$$

When we wish to find the probability of an event, it is sometimes a smart strategy to calculate the probability of the complement, and subtract the answer from 1.

Example 4

A number is chosen from the first 100 whole numbers. That is, $\xi = \{0, 1, \ldots, 99\}$. What is the probability that the number chosen is not divisible by 7?

Solution

Let A be the event 'a number is divisible by 7'. There are 15 such numbers in the first 100 whole numbers.

$$P(A) = \frac{15}{100} = \frac{3}{20}$$

A^c is the event 'a number is not divisible by 7'.

$$P(A^c) = 1 - \frac{3}{20} = \frac{17}{20}$$

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Exercise 16A

1 One letter is chosen at random from the word SALE. What is the probability that it is A?

2 What is the probability of choosing a prime number from the numbers $5, 6, 7, 8, 9, 10$?

3 What is the probability of randomly picking the most expensive car from a range of six new and differently priced cars in a showroom?

Example 1

4 What is the probability of choosing an integer that is exactly divisible by 5 from the set $\{5, 6, 7, 8, 9, 10, 11, 12\}$?

5 In a raffle, 200 tickets are sold. If you have bought one ticket, what is the probability that you will win first prize?

6 One card is chosen at random from a pack of 52 ordinary playing cards. What is the probability that it is the Ace of Hearts?

7 A number is chosen from the first 15 positive whole numbers. What is the probability that it is exactly divisible by both 3 and 4?

Example 2

8 One card is drawn at random from a pack of playing cards. What is the probability that it is:

a an Ace? **b** a red card? **c** a Heart? **d** a picture card?

(A picture card is a King, Queen or Jack.)

9 A book of 150 pages has a picture on 20 of its pages. If one page is chosen at random, what is the probability that it has a picture on it?

10 One counter is picked at random from a bag containing 15 red counters, 5 white counters and 5 yellow counters. What is the probability that the counter removed is:

a red? **b** yellow? **c** not red?

11 If you bought 10 raffle tickets and a total of 400 were sold, what would be the probability that you won first prize?

Example 3

12 The numbers on a roulette wheel go from 0 to 36. The wheel is spun. What is the probability that when it stops it will be pointing to:

a an even number? **b** an odd number?

c a number less than 10, excluding zero?

Example 4

13 A number is chosen at random from the first 20 positive whole numbers. What is the probability that it is not a prime number?

14 A card is drawn at random from a pack of cards pack of 52 playing cards. What is the probability that it is not a 2?

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15 One letter is chosen from the letters of the alphabet. What is the probability that it is not a vowel?

16 A box of 60 coloured crayons contains a mixture of colours. Only 10 of the crayons are red. If one crayon is chosen at random, what is the probability that it is not red?

17 A number is chosen at random from the first 10 positive whole numbers. What is the probability that it is not exactly divisible by 3?

18 One letter is chosen at random from the word ALPHABET. What is the probability that it is not a vowel?

19 In a raffle, 500 tickets are sold. If you bought 20 tickets, what is the probability that you will not win first prize?

20 If you roll an ordinary six-sided die, what is the probability that you will not get a score of 5 or more?

21 There are 200 packets hidden in a lucky dip. Five packets contain $1 and the rest contain 5 cents. What is the probability that you will not draw out a packet containing $1?

22 When a pack of playing cards is cut, what is the probability that the card showing is not a picture card (picture cards being Jacks, Queens and Kings)?

23 A letter is chosen at random from the letters of the word SUCCESSION. What is the probability that the letter is:

a N? **b** S? **c** a vowel? **d** not S?

24 A card is drawn at random from a pack of playing cards. What is the probability that it is:

a an Ace? **b** a Spade? **c** not a Club? **d** not a 7 or an 8?

25 A bag contains a set of snooker balls (15 red balls and one each of white, yellow, green, brown, blue, pink and black). What is the probability that one ball selected at random is:

a red? **b** not red? **c** black? **d** neither red nor white?

26 There are 60 cars in a station car park. Of the cars, 22 are Australian-made, 28 are Japanese-made and the rest are European. What is the probability that the first car to leave is:

a Japanese? **b** not Australian? **c** European? **d** American?

27 A whole number is chosen from the first 30 positive whole numbers. What is the probability that:

a it is divisible by 5? **b** it is divisible by 5 and 3?

c it is divisible by 5 but not by 3? **d** it is even and divisible by 5?

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16B 'Or' and 'and'

Sometimes, rather than just considering a single event, we are interested in two or more events happening. Suppose a number is chosen at random from the first 20 positive whole numbers.

Let A be the event 'the number is even'.

Let B be the event 'the number is divisible by 5'.

In this experiment the sample space is:

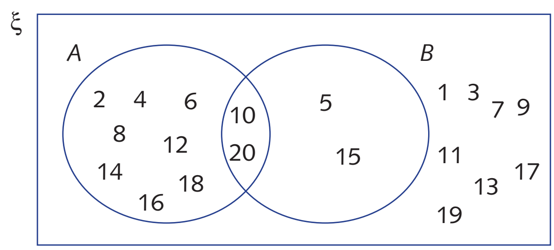
$$\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$$

and the events we are looking at are:

$$A = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$$

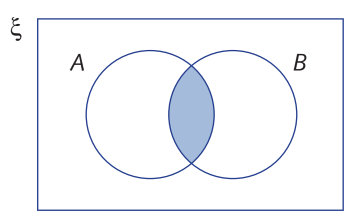
$$B = \{5, 10, 15, 20\}$$

The Venn diagram illustrating these events is as shown. Venn diagrams were introduced in Year 7.

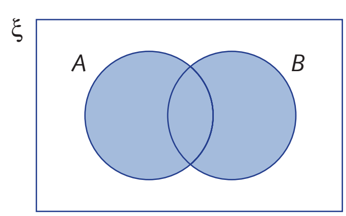


The event 'a number that is even **and** divisible by 5 is chosen' is the **intersection** of the sets A and B and is written as $A \cap B$. The event $A \cap B$ is often called 'A and B'.

The event 'an even number **or** a number divisible by 5 is chosen' is the **union** of the sets A and B and is written as $A \cup B$. The event $A \cup B$ is often called 'A or B'.



$A \cap B$ is shaded



$A \cup B$ is shaded

For an outcome to be in the event $A \cup B$, it must be in **either** the set of outcomes for A or the set of outcomes for B. Of course, it can be in both sets.

For an outcome to be in the event $A \cap B$, it must be in **both** the set of outcomes for A and the set of outcomes for B.

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Example 5

Consider the experiment 'rolling a 10-sided die numbered 1, 2,…, 10'. Let A be the event 'the number obtained is greater than 6' and let B be the event 'the number obtained is even'. Show the events on a Venn diagram, and find their intersection.

Solution

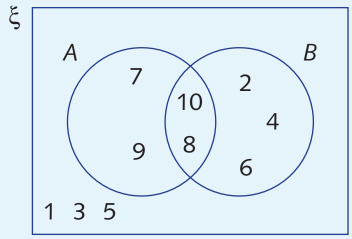
A consists of the outcomes 7, 8, 9, 10.

That is, $A = \{7, 8, 9, 10\}$

B consists of the outcomes 2, 4, 6, 8, 10.

That is, $B = \{2, 4, 6, 8, 10\}$

So, $A \cap B = \{8, 10\}$



Example 6

A person is asked to think of a number between 1 and 10 (inclusive). What is the probability that the number chosen is:

a even?

b greater than 7?

c less than 5?

d greater than 7 or less than 5?

e even or greater than 7?

f even and greater than 7?

g even and less than 5?

h greater than 7 and less than 5?

Solution

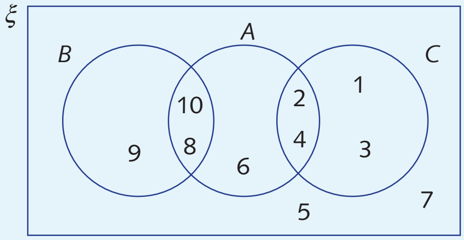
Let A be the event 'number chosen is even'

B be the event 'number chosen is greater than 7'

C be the event 'number chosen is less than 5'.

Then the outcomes that are favourable to these events are:

$A = \{2, 4, 6, 8, 10\}$ $B = \{8, 9, 10\}$ $C = \{1, 2, 3, 4\}$



a $P(A) = \frac{5}{10} = \frac{1}{2}$ **b** $P(B) = \frac{3}{10}$ **c** $P(C) = \frac{4}{10} = \frac{2}{5}$

(*continued over page*)

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d $B \cup C = \{1, 2, 3, 4, 8, 9, 10\}$

Thus $P(B \cup C) = \frac{7}{10}$ (There is no overlap of B and C.)

e $A \cup B = \{2, 4, 6, 8, 9, 10\}$

Hence, $P(A \cup B) = \frac{6}{10} = \frac{3}{5}$

f $A \cap B = \{10, 8\}$

Hence, $P(A \cap B) = \frac{2}{10} = \frac{1}{5}$

g $A \cap C = \{2, 4\}$

Hence, $P(A \cap C) = \frac{2}{10} = \frac{1}{5}$

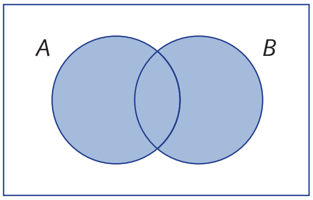
h There are no numbers common to both B and C.

Hence, $P(B \cap C) = 0$

That is, it is impossible to find a number that is both greater than 7 and less than 5 at the same time.

Note: The previous example shows that $P(A \cup B)$ may not be the same as $P(A) + P(B)$. This is because A and B may overlap.

The problem can be done without a Venn diagram, but a Venn diagram makes it easier to list the set.



We recall that for a finite set S, the symbol $|S|$ stands for the number of elements in S. The number of elements in $A \cup B$ is:

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$|A \cap B|$ is subtracted as the sum $|A| + |B|$ includes the elements in $A \cap B$ twice.

Divide both sides of this equation by $|\xi|$, the number of elements in the sample space.

$$\frac{|A \cup B|}{|\xi|} = \frac{|A|}{|\xi|} = \frac{|B|}{|\xi|} - \frac{|A \cap B|}{|\xi|}$$

For a sample space with equally likely outcomes this means:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

This result is true for all sample spaces.

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'Or' and 'and'

- For an outcome to be in the event $A \cup B$, it must be in either the set of outcomes for A or the set of outcomes for B, and of course it can be in both sets. $A \cup B$ is called the union of A and B.
- For an outcome to be in the event $A \cap B$, it must be in both the set of outcomes for A and the set of outcomes for B. $A \cap B$ is called the intersection of A and B.
- For any two events, A and B:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Exercise 16B

Example 5

1 Consider the random experiment of rolling a fair six-sided die. Let A be the event 'an even number is rolled'. Let B be the event 'a number less than 3 is rolled'. Illustrate with a Venn diagram.

2 A number is chosen at random from the numbers 1, 2, 3, …, 20. Let A be the event 'the number chosen is a multiple of 3' and B be the event 'the number chosen is greater than 12'. Illustrate with a Venn diagram.

3 A fair six-sided die is thrown and the uppermost number noted. Let A be the event 'the number is odd' and B be the event 'the number is greater than 3'. Illustrate with a Venn diagram.

Example 6

4 A person is asked to think of a number between 1 and 10 (inclusive). Find the probability that the number is:

a odd
b greater than 6
c less than 4
d greater than 6 or less than 4
e odd or greater than 6
f odd and greater than 6
g odd and less than 4
h greater than 6 and less than 4

5 A fair six-sided die is thrown and the uppermost number is noted. Find the probability that the number is:

a even
b a 6
c less than or equal to 3
d even and a 6
e even or a 6
f less than or equal to 3 and a 6
g less than or equal to 3 or a 6
h even and less than or equal to 3
i even or less than or equal to 3

6 In a certain experiment, two events A and B satisfy $P(A) = 0.6$, $P(B) = 0.3$ and $P(A \cap B) = 0.2$. Find $P(A \cup B)$.

7 For two events C and D, $P(C) = 0.7$, $P(C \cap D) = 0.3$ and $P(C \cup D) = 0.9$. Find $P(D)$.

8 For two events E and F, $P(E) = 0.85$, $P(F) = 0.72$ and $P(E \cup F) = 0.95$. Find $P(E \cap F)$.

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16C Two-way tables

Students in a city school carried out a survey of the students in their school, investigating whether or not they ate breakfast. The survey presented the question 'Do you eat breakfast on a regular basis?' and the students recorded the results along with gender of each respondent.

The results were organised in a table.

	Male	Female	Total
Eat breakfast regularly	320	410	730
Do not eat breakfast regularly	300	200	500
Total	620	610	1230

This table is called a **two-way table**. It is a very good format for answering questions that involve 'and' and 'or'.

For example, for a student selected at random from the school:

$$P(\text{male } \textit{and} \text{ 'eats breakfast regularly'}) = \frac{320}{1230} = \frac{32}{123}$$

For a two-by-two table, the two-way table takes the following form in general.

	A	A^c	Total
B	$\lvert A \cap B \rvert$	$\lvert A^c \cap B \rvert$	$\lvert B \rvert$
B^c	$\lvert B^c \cap A \rvert$	$\lvert B^c \cap A \rvert$	$\lvert B^c \rvert$
Total	$\lvert A \rvert$	$\lvert A^c \rvert$	

Read this table comparing it with the table on eating breakfast.

It is worth constructing the associated probability table for this.

	A	A^c	Total
B	$P(A \cap B)$	$P(A^c \cap B)$	$P(B)$
B^c	$P(B^c \cap A)$	$P(B^c \cap A)$	$P(B^c)$
Total	$P(A)$	$P(A^c)$	1

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Example 7

A school has 1200 students. The table below gives information about whether or not each student plays a musical instrument.

	Boys	Girls
Plays a musical instrument	325	450
Does not play an instrument	225	200

Each student has a school number between 1 and 1200.

Each student's number is written on a card and the 1200 cards are placed in a hat. One card is randomly pulled out of the hat. What is the probability that the number pulled out belongs to:

a a boy?

a a girl?

c a student who plays a musical instrument?

d a boy who plays a musical instrument?

Solution

a Number of students = 1200
Number of boys = 325 + 225
= 550

$$P(\text{boy's number}) = \frac{550}{1200} = \frac{11}{24}$$

b Number of students = 1200
Number of girls = 450 + 200
= 650

$$P(\text{girl's number}) = \frac{650}{1200} = \frac{13}{24}$$

c Number of students who play a musical instrument = 325 + 450

$$P(\text{musician's number}) = \frac{775}{1200} = \frac{31}{48}$$

d $$P(\text{boy musician's number}) = \frac{325}{1200} = \frac{13}{48}$$

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Example 8

A survey of 200 people was carried out to determine hair and eye colour. The results are shown in the table below.

	Fair	Brown	Red	Black
Blue	25	9	6	18
Brown	16	16	18	22
Green	15	17	22	16

What is the probability that a person chosen at random from this group has:

a fair *or* brown hair?

b blue *or* brown eyes?

c red hair *and* green eyes?

Solution

a $P(\text{fair } or \text{ brown hair}) = \dfrac{25+16+15+9+16+17}{200} = \dfrac{98}{200} = \dfrac{49}{100}$

b $P(\text{blue } or \text{ brown eyes}) = \dfrac{25+9+6+18+16+16+18+22}{200} = \dfrac{65}{100} = \dfrac{13}{20}$

c $P(\text{red hair } and \text{ green eyes}) = \dfrac{22}{200} = \dfrac{11}{100}$

Exercise 16C

Example 7

1 The patrons in a cinema each receive a numbered ticket when entering. The following two-way table describes the audience in terms of the categories Child–Adult and Male–Female.

	Male	Female	Total
Child	15	23	38
Adult	24	30	54
Total	39	53	92

A ticket is drawn at random from a hat. The winner will receive free popcorn and soft drink. What is the probability that the number drawn is held by:

a a male?

b a female?

c a child?

d a male child?

e a female child?

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2 Five thousand drivers were questioned and classified according to age and number of accidents in the last year. The results are in the table below.

	Younger than 28	28 or older	Total
No accidents	650	850	1500
One or more accidents	2500	1000	3500
Total	3150	1850	5000

A driver from the group of 5000 is chosen at random. What is the probability that the driver:

a is younger than 28?

b had no accidents and is younger than 28?

c is 28 years or older and has had one or more accidents?

3 The eye colour and gender of 300 people were recorded. The results are shown in the table below.

Eye colour / Gender	Blue	Brown	Green	Grey
Male	40	50	10	20
Female	80	70	10	20

What is the probability that a person chosen at random from the sample:

a has blue eyes?

b is male?

c is male and has green eyes?

d is female and does not have blue eyes?

e has blue eyes or is female?

f is male or does not have green eyes?

4 A bowl contains green and red normal jelly beans and green and red double-flavoured jelly beans. The number of each type is given in the following table.

	Green	Red
Normal jelly bean	39	54
Double-flavoured jelly bean	27	24

A jelly bean is randomly taken out of the bowl. Find the probability that:

a it is a double-flavoured jelly bean

b it is a green jelly bean

c it is a green normal jelly bean

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Example 8

5 A survey of 400 people was carried out to determine hair and eye colour. The results are shown in the table below.

Hair colour / Eye colour	Fair	Brown	Red	Black
Blue	50	18	12	36
Brown	32	32	36	44
Green	30	34	44	32

What is the probability that a person chosen at random from this group has:

a blue eyes?

b red hair?

c fair *or* brown hair?

d blue *or* brown eyes?

e red hair *and* green eyes?

f eyes that are not green?

g hair that is not red?

h fair hair *and* blue eyes?

i eyes that are not blue *or* hair that is not fair?

6 The 330 subjects volunteering for a medical study are classified by gender and blood pressure (high, normal and low). The results are shown in the table below.

	H	N	L
M	176	44	20
F	22	44	24

If a subject is selected at random, find:

a $P(\mathrm{N})$

b $P(\mathrm{F} \cap \mathrm{H})$

c $P(\mathrm{F} \cup \mathrm{H})$

7 A survey of 3000 people was carried out to determine which type of breakfast they had. The results are shown in the table below.

Breakfast	Qld	NSW	Vic
Only toast	500	300	120
Only cereal	320	320	360
Cooked breakfast	300	340	440

What is the probability that a person chosen at random from this group:

a eats only toast for breakfast *and* comes from Victoria?

b eats only toast *or* only cereal?

c eats a cooked breakfast *and* comes from Queensland?

d eats only toast?

e comes from NSW?

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16D Further uses of Venn diagrams

Sometimes, instead of listing the outcomes of the experiment in the appropriate events on a Venn diagram, the number of outcomes in each event is written on the diagram. Consider the following examples.

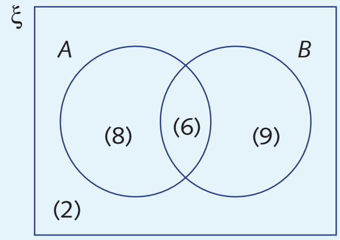
Example 9

In a group of 25 families, 14 own a Playstation, 15 own an Xbox, and 6 own both a Playstation and an Xbox. Represent this information on a Venn diagram.

Solution

Let A be the event 'family owns a Playstation' and B be the event 'family owns an Xbox'. There are:

- 6 elements in $A \cap B$
- $14 - 6 = 8$ elements in A but not B
- $15 - 6 = 9$ elements in A^c but not B
- 2 elements not in A or B since $6 + 8 + 9 + 2 = 25$



Example 10

In a class of 28 students, 20 study French and 15 study chemistry. Each student in the class studies either French or chemistry.

a Represent this information on a Venn diagram.

b One student is selected at random from the group. What is the probability that the student studies:

i both French and chemistry? **ii** chemistry but not French?

Solution

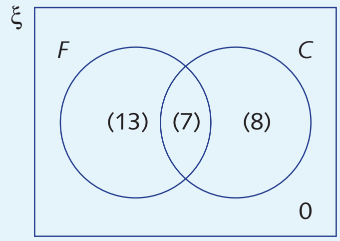
a Let F be the event 'student studies French' and C be the event 'student studies chemistry'. Twenty students study French and 15 study chemistry in a class of 28.

$$|F \cup C| = |F| + |C| - |F \cap C|$$

$$28 = 15 + 20 - |F \cap C|$$

Therefore, $|F \cap C| = 7$

Then $20 - 7 = 13$ students study French but not chemistry, and $15 - 7 = 8$ students study chemistry but not French.



b **i** Using the Venn diagram, $P(F \cap C) = \frac{7}{28} = \frac{1}{4}$

ii Using the Venn diagram, $P(F^c \cap C) = \frac{8}{28} = \frac{2}{7}$

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Exercise 16D

Example 9

1 In a group of 40 children, 12 like both heavy metal and hip-hop; 8 children like heavy metal but not hip-hop; 14 children like hip-hop but not heavy metal; and the rest like neither. Represent this information on a Venn diagram.

2 A survey of 50 families showed that 28 owned a cat, 30 owned a dog and 15 owned both a dog and a cat.

a Represent this information on a Venn diagram.

b One family is selected at random from the group. What is the probability that the family owns:

i a cat but not a dog? **ii** a dog but not a cat?

iii neither a dog nor a cat?

Example 10

3 In a class of 26 students, 18 study chemistry and 12 study economics. Each student studies either chemistry or economics.

a Represent this information on a Venn diagram.

b One student is selected at random. What is the probability that the student studies:

i chemistry and economics? **ii** only economics?

iii only chemistry?

4 In a youth club of 50 people, 18 play basketball, 28 play soccer and 10 play neither sport.

a Represent this information on a Venn diagram.

b One person is selected at random. What is the probability that the person chosen plays:

i basketball and soccer? **ii** soccer but not basketball?

iii only basketball?

5 In a group of 50 students, 30 study mathematics, 25 study physics and 20 study both.

a Represent this information on a Venn diagram.

b One student is selected at random from the group. What is the probability that the student studies:

i mathematics but not physics? **ii** physics but not mathematics?

iii neither physics nor mathematics?

6 In a group of 100 students, 50 study history, 30 study English literature and 20 study both.

a Represent this information on a Venn diagram.

b If a student is selected at random from the group, what is the probability that the student studies:

i at least one of these subjects?

ii history but not English literature?

iii history, given that the student also studies English literature?

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Review exercise

1 Alex has seven different caps in the bottom of his wardrobe. Four of them are black, two of them are blue and one is red. If he pulls out a cap at random, what is the probability it is blue?

2 A nurseryman has azaleas at a special low price because they have lost their labels and they are not yet in flower. From a trolley that has 15 pink azaleas and eight red azaleas, what is the probability of choosing:

a a pink azalea?

b a red azalea?

3 Chelsea has 12 ribbons in her top drawer. Four of them are pink, three are blue, three are green and two are white. If she chooses one ribbon at random, what is the probability that it is green?

4 A debating team consists of four boys and eight girls. If one of the team is chosen at random to be the leader, what is the probability that the leader is a girl?

5 A basketball team consists of five players: Adams, Brown, Cattogio, O'Leary and Nguyen. If a player is chosen at random, what is the probability that his name starts with a vowel?

6 Slips of paper numbered 1, 2, 3, ..., 10 are placed in a hat and one is drawn at random. What is the probability that the number on the slip of paper is not a multiple of three?

7 Heidi chooses, at random, one shape from the following set: equilateral triangle, square, parallelogram, rectangle, circle. What is the probability that the chosen shape has exactly four axes of symmetry?

8 A basket contains the following balls: two AFL footballs, three soccer balls, a basketball, a rugby ball and two tennis balls. If Liam chooses a ball to play with, at random, what is the probability that he chooses a round ball?

9 A jar contains 27 balls. Twenty of the balls have a star on them and 10 of the balls have an elephant printed on them. Every ball has at least one of these symbols on it.

a Draw a Venn diagram to illustrate this.

b If one ball is withdrawn at random, what is the probability of choosing:

i a ball with an elephant printed on it?

ii a ball with a star and an elephant printed on it?

iii a ball with an elephant printed once but without a star?

10 In a group of 200 students, 100 study geography, 60 study mathematics and 40 study both.

a Represent this information on a Venn diagram.

b If a student is selected at random from the group, what is the probability that the student studies:

i at least one of these subjects?

ii geography but not mathematics?

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11 A card is drawn at random from a well-shuffled pack of playing cards. Find the probability that the card chosen:

a is a Heart

b is a court card (that is, a King, Queen or Jack)

c has a face value between 2 and 8 inclusive

d is a Heart or a court card

e is a Heart and a court card

f has a face value between 2 and 8 inclusive and is a court card

g has a face value between 2 and 8 inclusive or is a court card

12 A survey of 150 people was carried out to determine eye colour and gender. The results are shown in the table below.

Eye colour	Male	Female
Blue	20	40
Brown	25	35
Green	15	15

What is the probability of a person chosen at random:

a having blue eyes?

b being male?

c being male *and* not having blue eyes?

d being female *and* not having blue eyes?

e having blue eyes *or* being female?

f being male *or* not having green eyes?

13 A survey of 100 people was carried out to determine which hand they preferred to use and their gender. The results are shown in the table below.

Preferred hand	Male	Female
Left	15	22
Right	33	30

What is the probability of person chosen at random being:

a left-handed?

b female?

c male *and* right-handed?

d male *or* right-handed?

e left-handed *and* female?

f left-handed *or* female?

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Challenge exercise

1 Seventy-five students all own a dog, a cat or a bird. They were asked which pets they own. The replies gave the following information:
37 own a bird, 33 own a cat, 40 own a dog, 16 own both a cat and a bird, 11 own both a dog and a cat, 12 own both a bird and a dog.

a Draw a Venn diagram representing this information.

b How many students own a bird, a dog and a cat?

c A student is chosen at random and asked which pets they own. What is the probability that they own:

i a cat and a dog only?

ii a bird and a dog only?

iii a bird, a dog and a cat?

iv a bird but not a cat?

v a dog but not a bird?

2 A bag contains 1000 balls, some of which are red, some blue and the rest yellow.
The probability of drawing a red ball is $\frac{1}{25}$ and the probability of drawing a yellow ball is $\frac{7}{20}$. Find:

a the number of yellow balls

b the number of blue balls

c the number of yellow balls that need to be removed to make the probability of drawing a red ball $\frac{1}{20}$

d the number of yellow balls (from the original 1000) that need to be removed so that the probability of drawing a yellow ball is $\frac{1}{6}$

3 A box contains 3 black and 1 yellow ball. A second box contains 2 black and 2 yellow balls. A ball is taken from the first box and put in the second. A ball is then withdrawn from the second box. Find the probability that the ball taken from the second box is:

a black

b yellow

4 A three-digit number is chosen at random.

a Describe the sample space and draw a Venn diagram.

Find the probability that the number is divisible by:

b 5

c 7

d 3

e 5 and 7

f 5 or 7

g 3 and 5 and 7

h 3 and 5 but not 7

i 5 and 7 but not 3

j 3 or 5 or 7

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5 A bag contains 120 balls, some of which are red and the rest black. The probability of a ball, drawn at random, being red is $\frac{2}{15}$. Find:

a the number of red balls in the bag

b the number of black balls in the bag

c the number of red balls that should be added to the bag to change the probability of obtaining a red ball to $\frac{1}{2}$

6 Find the probability that a three-digit number chosen at random is divisible by:

a 5 **b** 3 **c** 15 **d** 3 or 5

7 A bag contains 120 balls, some of which are red, some blue and the rest yellow. The probability of drawing a blue ball is $\frac{1}{12}$ and the probablity of drawing a red ball is $\frac{2}{5}$. Find:

a the number of yellow balls in the bag

b the number of yellow balls that should be removed from the bag so that the probability of drawing a yellow ball is $\frac{1}{3}$

8 A game is played in which a counter is placed on the square C.

A	B	C	D	E

A coin is tossed. If it comes down 'head', the counter is moved one square to the right. If the coin comes down 'tail', the counter is moved one square to the left. Find the probability that, after two tosses of the coin, the counter will be on square:

a A **b** C **c** E

9 Jim has \$512 and plays a game in which he has an equal chance of winning or losing. He wins four times and loses four times in random order. He bets half the money he has left at each stage. A win gives him twice what he bets. How much does Jim have at the end?

10 In a school of 650 everybody studies French (F), German (G) or Indonesian (I). All but 41 study French, 12 study French and Indonesian but not German. Thirteen study Indonesian and German but not French and the same number study German only. Twice as many study French and German but not Indonesian as study all three. The number studying Indonesian only is the same as the total studying both French and German. What is the probability that a student chosen at random studies French only?

11 A teacher presents three problems to a class. Sixty per cent of them solve problem 1. Forty per cent solve problem 2 and 55% solve problem 3. Also,

- 15% solve problems 1 and 2 only
- 35% solve problems 1 and 3 only
- 15% solve problems 2 and 3 only
- 5% solve all three problems.

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A student is chosen at random. What is the probability that they solve:

a exactly two of the problems?

b none of the problems?

12 The 28 students in a class all buy music CDs. The categories they choose are rock (R), folk (F) and classical (C). One student buy classical CDs only. Nine students buy folk CDs only and five students but rock CDs only. Also,

$|R \cap C| = 12$, $|C \cap F \cap R| = |F^c|$ and $|F \cap R| > |F \cap C|$

A student is selected at random. Find the probability that the student studies both classical and folk.

13 Weather records for July 2010 in Sydney showed that the month had no hot, calm, dry days. Of the 31 days, 7 were wet and cold but not windy, 4 were wet and windy but not cold, 8 were cold and windy but dry. 16 days were windy, 22 were cold and 2 were wet but not cold or windy. If a day is chosen at random. What is the probability that the day was:

a cold, wet and windy?

b cold but calm and dry?

14 Use Venn diagrams to simplify each of the following events or express in a different form.

a $(A \cap B^c) \cup (A \cap B)$

b $(A \cup B)^c$

c $(A \cap B)^c$

d $(A^c \cap B) \cap (A \cap B^c)$

e $((A^c \cap B^c) \cup (A \cap B^c))^c$

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