CHAPTER

17

Number and Algebra

Formulas and factorisation

Formulas are widely used in many fields of work and study, including science, medicine, finance and engineering. We also commonly use formulas when we are working with spreadsheets on a computer.

In the course of this chapter, topics in algebra that have been studied earlier will be revised. Substitution into formulas often results in an equation that needs to be solved.

We introduce another important technique, factorisation, which is the opposite of expanding of brackets.

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17A Formulas

Formulas have been used in earlier chapters. For example:

- the area A of a rectangle is given by the formula $A = \ell w$, where ℓ and w are the length and width of the rectangle
- the volume V of a cylinder is given by the formula $V = \pi r^2 h$, where r is the radius of the cylinder and h the height of the cylinder.

The subject of a formula

In the first formula above, A is the **subject** of the formula; in the second formula, V is the subject. It is called the subject as it is expressed in terms of the other pronumerals.

In Example 1, the value of the subject of the formula is to be found, given the values of the other pronumerals in the formula.

Example 1

Given the formula $P = 2\ell + 2w$, find the value of P if $\ell = 10$ and $w = 5$.

Solution

$$\begin{aligned} P &= 2\ell + 2w \\ &= 2 \times 10 + 2 \times 5 \\ &= 30 \end{aligned}$$

If we are given a set of values that includes the value of the subject of the formula, we will need to solve an equation to determine the unknown value.

Example 2

Given the formula $y = mx + c$, find:

a the value of x if $y = 3$, $m = 2$ and $c = -4$

b the value of m if $y = 13$, $x = 2$ and $c = 3$

Solution

a $y = mx + c$

$3 = 2x - 4$ (Substitute $y = 3$, $m = 2$ and $c = -4$.)

Solve the equation for x.

$2x = 7$ (Add 4 to both sides of the equation.)

$x = 3\frac{1}{2}$ (Divide both sides of the equation by 2.)

(*continued over page*)

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b $y = mx + c$

$13 = 2m + 3$ (Substitute the values.)

Solve the equation for m.

$2m = 10$ (Subtract 3 from both sides of the equation.)

$m = 5$ (Divide both sides of the equation by 2.)

Example 3

Given the formula $P = 2(\ell + w)$, find the value of w if $P = 21$ and $\ell = 5$.

Solution

$P = 2(\ell + w)$

$21 = 2(5 + w)$ (Substitute the values.)

$21 = 10 + 2w$ (Expand the brackets.)

$2w + 10 = 21$

$2w = 11$ (Subtract 10 from both sides of the equation.)

$w = \frac{11}{2}$ (Divide both sides of the equation by 2.)

$= 5\frac{1}{2}$

Example 4

The formula for the surface area S cm^2 of a rectangular prism is $S = 2(\ell w + \ell h + hw)$, where ℓ, w and h are the length, width and height of the prism, in cm. If $S = 60$, $\ell = 5$ and $w = 4$, find h, the height of the prism.

Solution

$60 = 2(5 \times 4 + 5 \times h + 4 \times h)$ (Substitute in values.)

$60 = 40 + 10h + 8h$

$20 = 10h + 8h$ (Subtract 40 from both sides of the equation.)

$20 = 18h$

$\frac{10}{9} = h$ (Divide both sides of the equation by 18.)

$h = 1\frac{1}{9}$

The height of the rectangular prism is $1\frac{1}{9}$ cm.

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Exercise 17A

Example 1

1 The area of a triangle, A cm^2, is given by the formula $A = \frac{bh}{2}$, where b cm is the length of the base and h cm is the height of the triangle.

a Find the area when $b = 6$ and $h = 7$.

b Find the length of the base if the area is 72 cm^2 and the height is 8 cm.

Example 2

2 Given the formula $y = mx + c$:

a find y if $m = 6$, $x = 3$ and $c = -10$

b find y if $m = -2$, $x = 3$ and $c = 7$

c find x if $m = 8$, $y = 20$ and $c = 5$

d find x if $m = -4$, $y = 10$ and $c = 2$

3 Temperature is measured using two different scales. The formula for changing the temperature in degrees Fahrenheit (°F) to degrees Celsius (°C) is $C = \frac{5}{9}(F - 32)$, where C is the temperature in °C and F is the temperature in °F.

Find the temperature in °C corresponding to a temperature of:

a 100°F

b 212°F

c 0°F

d –40°F

Example 3

4 Given the formula $T = a + (n - 1)d$:

a find T if $a = 6$, $n = 10$ and $d = 2$

b find T if $a = 6$, $n = 10$ and $d = -2$

c find a if $T = 300$, $n = 5$ and $d = 2$

d find n if $T = 49$, $d = 3$ and $a = 10$

e find d if $T = 35$, $n = 6$ and $a = 5$

f find d if $T = -35$, $n = 6$ and $a = -5$

5 The average a of two numbers m and n is given by the formula $a = \frac{m + n}{2}$.

a Find a if $m = 2$ and $n = 8$.

b Find a if $m = 6$ and $n = -10$.

c Find m if $a = 6$ and $n = 4$.

d Find a if $m = -2$ and $n = -6$.

e Find m if $a = 0$ and $n = -9$.

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6 The formula for the interior angle I of a regular polygon with n sides is $I = \frac{180n - 360}{n}$, where I is measured in degrees.

a Find the size of each interior angle of:

i a pentagon (5 sides) **ii** a hexagon (6 sides) **iii** a dodecagon (12 sides)

b Find the number of sides of a polygon for which the size of each interior angle is:

i 135° **ii** 144°

7 If $s = 2(a - b)$, find:

a s when $a = 5$ and $b = -2$

b a when $s = 10$ and $b = 6$

c b when $a = -2$ and $s = 6$

8 Given that $A = \frac{PRT}{100}$, find:

a A when $P = 200$, $R = 4$ and $T = 6$

b P when $A = 1200$, $R = 3$ and $T = 10$

c T when $A = 14\,000$, $P = 12\,000$ and $R = 5$

9 The area of a rectangle is x cm^2. The length of the rectangle is 4 cm.

a Write an expression in terms of x for the width of the rectangle.

b Write a formula for the perimeter P cm of the rectangle in terms of x.

c Find P if:

i $x = 10$ **ii** $x = 24$

d Find x if:

i $P = 20$ **ii** $P = 40$

Example 4

10 The formula for the surface area S cm^2 of a rectangular prism is $S = 2(\ell w + \ell h + hw)$, where ℓ, w and h are the length, width and height of the prism, measured in cm. If $S = 80$, $\ell = 5$ and $w = 2$, find h, the height of the prism.

11 If $c = a^2 + b^2$, find:

a the value of c when $a = 2$ and $b = 1$

b the value of c when $a = 3$ and $b = 4$

c the value of c when $a = 5$ and $b = 12$

12 If $c = \sqrt{a^2 + b^2}$, find the value of c when:

a $a = 5$ and $b = 12$

b $a = 3$ and $b = 4$

c $a = 24$ and $b = 7$

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17B Expansion and factorisation

We have learned how to expand brackets. For example:

$2(x+3)=2x+6$ and $3(x-4)=3x-12$

Example 5

Expand the brackets.

a $3(3x+5)$ **b** $-4(7-2x)$ **c** $3x(x+2)$

d $ab(2a+b)$ **e** $-3x^2(x+3)$

Solution

a $3(3x+5)=9x+15$

b $-4(7-2x)=-28+8x$

c $3x(x+2)=3x^2+6x$

d $ab(2a+b)=2a^2b+ab^2$

e $-3x^2(x+3)=-3x^3-9x^2$

There are many situations where one needs to reverse this process.

For example:

$2x+6=2(x+3)$ and $3x-12=3(x-4)$

The expression $2x+6$ has two **terms**, $2x$ and 6. The common factor of these terms is 2.

We write:

$$\begin{aligned}2x+6&=2\times x+2\times 3\\&=2(x+3)\end{aligned}$$

- To go from the left-hand side of this equation to the right-hand side, you **factorise** $2x+6$.
- To go from the right-hand side of this equation to the left-hand side, you **expand the brackets**.

The new procedure is called **factorisation** and it is intrinsically harder than expanding brackets.

Highest common factor

The expression $12x^2+9x$ has two terms, $12x^2$ and $9x$.

- They have a common factor 3.
- They have a common factor x.
- They have a common factor $3x$.
- They have no other common factors.

In this case, $3x$ is the highest common factor (HCF), and we write $12x^2+9x=3x(4x+3)$.

Similarly we say that $3ab^2$ is the highest common factor of $21a^2b^2$ and $15ab^2$ since 3 is the HCF of 21 and 15, a is the highest power of a which divides both expressions and b^2 is the highest power of b which divides both expressions.

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Example 6

Find the highest common factor of each pair of terms.

a $3x, 5x$ **b** $2x, 8x$ **c** $8x, 12$ **d** $15x, 20y$

e $2x^2, 6x$ **f** $12ab^2, 6b$ **g** $3x^3, 4x^2$

Solution

a The highest common factor of $3x$ and $5x$ is x.

b $8x = 4 \times 2x$

The highest common factor of $2x$ and $8x$ is $2x$.

c $8x = 4 \times 2x$ and $12 = 4 \times 3$

The highest common factor of $8x$ and 12 is 4.

d $15x = 5 \times 3x$ and $20y = 5 \times 4y$

The highest common factor of $15x$ and $20y$ is 5.

e $2x^2 = 2x \times x$ and $6x = 2x \times 3$

The highest common factor of $2x^2$ and $6x$ is $2x$.

f $12ab^2 = 6b \times 2ab$

The highest common factor of $12ab^2$ and $6b$ is $6b$.

g The highest common factor of $3x^3$ and $4x^2$ is x^2.

Example 7

Factorise:

a $3x + 12$ **b** $4z^2 + 3z$ **c** $-4x + 8$

Solution

a $3x + 12 = 3 \times x + 3 \times 4$
$= 3(x + 4)$

b $4z^2 + 3z = z \times 4z + z \times 3$
$= z(4z + 3)$

c $-4x + 8 = 4(-x + 2)$ *or* $-4x + 8 = -4(x - 2)$

(Either of these answers is acceptable.)

Example 8

Factorise:

a $2x + 4y$ **b** $2xy - 4y$ **c** $6ab^2 - ab$

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Solution

a $2x + 4y = 2 \times x + 2 \times 2y$
$= 2(x + 2y)$

b $2xy - 4y = 2y \times x - 2y \times 2$
$= 2y(x - 2)$

c $6ab^2 - ab = ab \times 6b - ab \times 1$
$= ab(6b - 1)$

Example 9

Factorise:

a $abc + 4a^2bc$

b $2x^3y - 6y + 14xy^2$

Solution

a $abc + 4a^2bc = abc \times 1 + abc \times 4a$
$= abc(1 + 4a)$

b $2x^3y - 6y + 14xy^2 = 2y \times x^3 - 2y \times 3 + 2y \times 7xy$
$= 2y(x^3 - 3 + 7xy)$

Exercise 17B

Example 5

1 Expand the brackets.

a $9(2x + 5)$ **b** $2(5 + 3x)$ **c** $7(2x - 11)$ **d** $5(2x + 10)$

e $-4(11 + 2x)$ **f** $-6(3x - 5)$ **g** $-5(x - 1)$ **h** $7(-3x - 4)$

i $5x(x + 2)$ **j** $3x(x - 6)$ **k** $-7x(2x - 11)$ **l** $5x(10 - x)$

m $ab(2a + b)$ **n** $3x^2(x + 3)$ **o** $-4x^2(3 - 2x)$ **p** $-5x(2x + 3)$

q $ab^2(a + 3)$ **r** $pq(p + q)$ **s** $-xy(3x + 2y)$ **t** $xy^2(2x - 2y)$

Example 6

2 Find the highest common factor of each pair of terms.

a $4, 12$ **b** $18x, 27x$ **c** $12x^2, 6x$ **d** $24ab^2, 3b$

e $15x, 18$ **f** $14x^2, 70$ **g** $30x, 10y$ **h** $56a, 84$

i $15x, 4y$ **j** $16x^3, 5x^2$ **k** $20xy, y^2$

3 Factorise:

a $2a + 4$ **b** $-2a + 4$ **c** $-2x - 4$ **d** $20a + 30$

e $-56x + 96$ **f** $5 - 15x$ **g** $20 - 25x$ **h** $100 - 25x$

i $-56x - 80$ **j** $18 - 90z$

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Example 7

4 Factorise:

a $5x+60$ **b** $6m^2+3m$ **c** $-3x+6$ **d** $6x^2+3x$

e $8-4x$ **f** $8x-16x^2$ **g** $-5x+10x^2$ **h** $70+10a$

i $3x+30$ **j** $5m^2+15m$ **k** $-3x^2+6x$ **l** $4x^2+12x$

m $25-5x^2$ **n** $4x-16x^2$ **o** $-m+10m^2$ **p** $-70n+10n^2$

q $4x^3-60x$ **r** $18m^3+3m^2$ **s** $-3x+6x^2$ **t** $-6x^2-3x$

u $8x^2-4x$ **v** $3x-15x^2$ **w** $-15x+10x^2$ **x** $70a^2+10a$

Example 8

5 Factorise:

a $3x+9y$ **b** $xy+4x$ **c** $2x-8y$ **d** $6x+8xy$

e $3a-12ab$ **f** $10m+8mn$ **g** $12xy+10x^2y$ **h** $pq+qr$

Example 9

6 Factorise:

a $3xyz-12xy^2z$ **b** $16a^2bc+4ab^3c$

c $8m^2np+20m^2n^3p^4$ **d** $9p^2q-3pq+12pq^2$

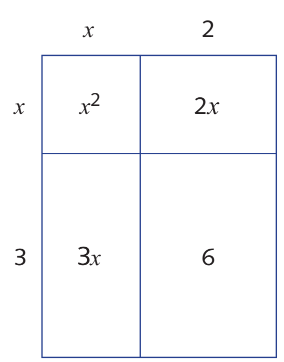
e $5a^2b+20abc+15ab^2c$

17C Binomial products

How do we expand an expression of the form $(x+2)(x+3)$?

An expression such as $(x+2)(x+3)$ is called a **binomial product**, as each factor of the product has two terms in it. To expand a binomial expression, we multiply each term in the second bracket by each term in the first, and then collect like terms.

$$\begin{aligned}(x+2)(x+3) &= x(x+3)+2(x+3)\\ &= x^2+3x+2x+6\\ &= x^2+5x+6\end{aligned}$$



The expansion can be illustrated using an area diagram, as shown above.

Example 10

Expand and simplify:

a $x(x+4)$ **b** $(x+3)(x+4)$ **c** $(2x+5)(x+4)$

d $(x-5)(x+6)$ **e** $(x-3)^2$ **f** $(x-4)(x+4)$

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Solution

a $x(x+4) = x^2 + 4x$

b $(x+3)(x+4) = x(x+4) + 3(x+4)$
$= x^2 + 4x + 3x + 12$
$= x^2 + 7x + 12$

c $(2x+5)(x+4) = 2x(x+4) + 5(x+4)$
$= 2x^2 + 8x + 5x + 20$
$= 2x^2 + 13x + 20$

d $(x-5)(x+6) = x(x+6) - 5(x+6)$
$= x^2 + 6x - 5x - 30$
$= x^2 + x - 30$

e $(x-3)^2 = (x-3)(x-3)$
$= x(x-3) - 3(x-3)$
$= x^2 - 3x - 3x + 9$
$= x^2 - 6x + 9$

f $(x-4)(x+4) = x(x+4) - 4(x+4)$
$= x^2 + 4x - 4x - 16$
$= x^2 - 16$

Exercise 17C

Example 10

1 Expand and simplify:

a $x(x-6)$ **b** $x(2x+1)$ **c** $2x(x-4)$
d $(x+2)(x-3)$ **e** $(x+8)(x-7)$ **f** $(x+10)(x-5)$
g $(m+2)(m-3)$ **h** $(z+6)(z-6)$ **i** $(a+10)(a+2)$
j $(n-8)(n+5)$ **k** $(z+6)(z+2)$ **l** $(2c+3)(5c+6)$
m $(2a-5)(a+10)$ **n** $(2y-7)(y+10)$ **o** $(2s-6)(s+8)$
p $(y-9)^2$ **q** $(x+5)^2$ **r** $(2x-3)^2$

2 Expand and simplify:

a $(x+2)^2$ **b** $(x-4)^2$ **c** $(2x-7)^2$
d $(4a-2)^2$ **e** $(3m-5)^2$ **f** $(4x-6)^2$
g $(3m-6)^2$ **h** $(4-h)^2$ **i** $(3m-4)^2$

3 Expand and simplify:

a $(a-6)(a+6)$ **b** $(x-5)(x+5)$ **c** $(2x-4)(2x+4)$
d $(5s-3)(5s+3)$ **e** $(6m-2)(6m+2)$ **f** $(3n-7)(3n+7)$
g $(6c-7)(6c+7)$ **h** $(7v+3)(7v-3)$ **i** $(5-m)(5+m)$

4 Expand and simplify:

a $(m+7)(m-7)$ **b** $(7b+6)(4b+3)$ **c** $(a+2b)(a-2b)$
d $(m-n)(m-2n)$ **e** $(m-n)(m+n)$ **f** $(a+b)^2$
g $(2a-b)(a+3b)$ **h** $(g-3h)(g+3h)$ **i** $(a-b)^2$

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17D Factorisation of simple quadratics

A **monic quadratic** expression is an expression of the form $x^2 + bx + c$, where b and c are given numbers. When we expand $(x+3)(x+4)$, we obtain a monic quadratic:

$$\begin{aligned} x(x+4)+3(x+4) &= x^2+4x+3x+12 \\ &= x^2+7x+12 \end{aligned}$$

We want to develop a method for reversing this process.

In the expansion $(x+3)(x+4) = x^2 + 7x + 12$, notice that the coefficient of x is $3+4=7$. The term that is independent of x, the constant term, is $3 \times 4 = 12$. This suggests a method of factorising.

In general, when we expand $(x+p)(x+q)$, we obtain

$$x^2 + px + qx + pq = x^2 + (p+q)x + pq$$

The coefficient of x is the sum of p and q, and the constant term is the product of p and q.

Factorisation of simple quadratics

To factorise a simple quadratic, look for two numbers that add to give the coefficient of x, and that multiply together to give the constant term.

You can check your answer by expansion.

To factorise $x^2 + 8x + 15$, we look for two numbers that multiply to give 15 and add to give 8. Of the pairs that multiply to give $15 (15 \times 1, 5 \times 3, -5 \times (-3)$ and $-15 \times (-1))$, only 5 and 3 add to give 8.

Therefore, $x^2 + 8x + 15 = (x+3)(x+5)$.

The result can be checked by expanding $(x+3)(x+5) = x(x+5) + 3(x+5)$.

Example 11

Factorise:

a $x^2 + 6x + 8$ **b** $x^2 - 6x + 8$ **c** $x^2 - 3x - 18$

Solution

a We are looking for two numbers with product 8 and sum 6. The numbers are 2 and 4.
$x^2 + 6x + 8 = (x+2)(x+4)$

b We are looking for two numbers with product 8 and sum −6. The numbers are −2 and −4.
$x^2 - 6x + 8 = (x-2)(x-4)$

c We are looking for two numbers with product −18 and sum −3. The numbers are −6 and 3.
$x^2 - 3x - 18 = (x-6)(x+3)$

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Example 12

Factorise $x^2 - 36$.

Solution

We are looking for two numbers with product –36 and sum 0. The numbers are –6 and 6.

Thus $x^2 - 36 = (x - 6)(x + 6)$

Example 13

Factorise $x^2 + 8x + 16$.

Solution

We are looking for two numbers with product 16 and sum 8. The numbers are 4 and 4.

Thus $x^2 + 8x + 16 = (x + 4)(x + 4)$

$= (x + 4)^2$

Exercise 17D

Example 11a

1 Factorise these quadratic expressions.

a $x^2 + 4x + 3$ **b** $x^2 + 5x + 4$ **c** $x^2 + 7x + 6$

d $x^2 + 5x + 6$ **e** $x^2 + 11x + 24$ **f** $x^2 + 13x + 36$

g $x^2 + 10x + 16$ **h** $x^2 + 11x + 30$ **i** $x^2 + 12x + 20$

j $x^2 + 14x + 48$ **k** $x^2 + 16x + 63$ **l** $x^2 + 18x + 80$

m $x^2 + 11x + 28$ **n** $x^2 + 16x + 55$ **o** $x^2 + 12x + 32$

Example 11b

2 Factorise these quadratic expressions.

a $x^2 - 6x + 5$ **b** $x^2 - 7x + 12$ **c** $x^2 - 8x + 15$

d $x^2 - 9x + 14$ **e** $x^2 - 10x + 24$ **f** $x^2 - 12x + 35$

g $x^2 - 12x + 32$ **h** $x^2 - 9x + 18$ **i** $x^2 - 13x + 12$

j $x^2 - 15x + 14$ **k** $x^2 - 13x + 42$ **l** $x^2 - 17x + 72$

m $x^2 - 15x + 56$ **n** $x^2 - 19x + 90$ **o** $x^2 - 17x + 52$

Example 11c

3 Factorise these quadratic expressions.

a $x^2 - x - 6$ **b** $x^2 + 2x - 15$ **c** $x^2 + 3x - 10$

d $x^2 - 6x - 16$ **e** $x^2 - 4x - 12$ **f** $x^2 + 2x - 24$

g $x^2 + x - 30$ **h** $x^2 - 3x - 18$ **i** $x^2 - 7x - 18$

j $x^2 + x - 20$ **k** $x^2 + 3x - 40$ **l** $x^2 - 4x - 5$

m $x^2 - 4x - 45$ **n** $x^2 + x - 42$ **o** $x^2 - x - 20$

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Example 12

4 Factorise these quadratic expressions.

a $x^2 - 4$
b $y^2 - 16$
c $x^2 - 1$
d $m^2 - 25$
e $p^2 - 9$
f $x^2 - 144$

Example 13

5 Factorise these quadratic expressions.

a $x^2 + 14x + 49$
b $a^2 + 10a + 25$
c $x^2 - 6x + 9$
d $x^2 - 12x + 36$
e $a^2 - 10a + 25$
f $x^2 + 6x + 9$

6 Factorise these quadratic expressions.

a $x^2 + 13x + 36$
b $x^2 + 15x + 36$
c $x^2 + 20x + 36$
d $x^2 + 12x + 36$
e $x^2 - 15x + 36$
f $x^2 - 13x + 36$
g $x^2 - 2x - 24$
h $x^2 + 5x - 24$
i $x^2 + 10x - 24$
j $x^2 - 23x - 24$
k $x^2 - x - 42$
l $x^2 - x - 30$
m $x^2 + x - 132$
n $x^2 + x - 30$
o $x^2 - 2x - 48$
p $x^2 + 8x - 48$
q $x^2 + 22x - 48$
r $x^2 - 47x - 48$
s $x^2 - 14x + 48$
t $x^2 + 16x + 48$
u $x^2 + 14x + 40$
v $x^2 - 22x + 40$
w $x^2 - 3x - 40$
x $x^2 + 18x - 40$
y $x^2 - 3x - 28$
z $x^2 + 12x - 28$

Review exercise

1 a Given the formula $C = \frac{5}{9}(F - 32)$:

i find C if $F = 27$
ii find F if $C = 40$

b Given the formula $A = \frac{1}{2}(a + b)h$:

i find A if $a = 2$, $b = 5$ and $h = 3$
ii find h if $A = 20$, $a = 3$ and $b = 7$

2 Find the highest common factor of each pair of terms.

a $10, 5$
b $7x, 7y$
c $12x, 28x$
d $21y, 42y^2$

3 Factorise:

a $2x + 6$
b $7x - 49$
c $-3 + 42x$
d $100x + 5$
e $-14x + 28$
f $8x^2 - x$
g $30x^2 + 12$
h $-20y + 25y^2$

4 Factorise:

a $4x + 12y$
b $xy + 6x$
c $15x - 10y$
d $7x + 14xy$
e $12a - 4ab$
f $6m + 9mn$
g $14xy + 4x^2y$
h $10pq + 8qr$

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5 Factorise:

a $4xyz - 6xy^2z$ **b** $25a^2bc + 10ab^3c$ **c** $9m^2np + 27m^2n^3p^4$

d $22p^2q - 2pq + 4pq^2$ **e** $6a^2b + 8abc + 4ab^2c$ **f** $4x^2yz - 8xy^2z - 12xyz^2$

6 Expand the brackets and simplify in each case.

a $x(x+15)$ **b** $x(6x+7)$ **c** $5x(2x-1)$

d $(x+6)(x+4)$ **e** $(x+4)(x-2)$ **f** $(2x-1)(x+5)$

g $(x-4)(4x+1)$ **h** $(3x-2)(5x+1)$ **i** $(x-6)(2x-3)$

7 Expand the brackets and simplify in each case.

a $(x+1)^2$ **b** $(x+2)^2$ **c** $(x+9)^2$

d $(x-1)^2$ **e** $(x-5)^2$ **f** $(x-4)^2$

g $(2x+1)^2$ **h** $(3x-2)^2$ **i** $(9-x)^2$

8 The formula used in an experiment is $E = \frac{w}{w+x}$.
Find the value of E when $w = 30$ and $x = 18$.

9 For the formula $s = ut + \frac{1}{2}at^2$, find:

a the value of s, when $u = 4$, $t = 3$ and $a = 6$

b the value of u, when $s = 100$, $t = 5$ and $a = 4$

c the value of a, when $s = 200$, $t = 10$ and $u = 6$

10 Factorise:

a $x^2 + 8x + 12$ **b** $x^2 + 9x + 18$ **c** $x^2 + 11x + 30$

d $x^2 + 11x + 28$ **e** $x^2 - 11x + 24$ **f** $x^2 - 10x + 24$

g $x^2 - 14x + 24$ **h** $x^2 - 25x + 24$ **i** $x^2 + x - 20$

j $x^2 - 2x - 48$ **k** $x^2 - 4x - 12$ **l** $x^2 + 3x - 40$

m $x^2 - 7x - 8$ **n** $x^2 - x - 132$ **o** $x^2 + 15x - 100$

11 Factorise these quadratic expressions.

a $x^2 - 64$ **b** $y^2 - 81$ **c** $x^2 - 121$

d $m^2 - 169$ **e** $p^2 - 225$ **f** $x^2 - 196$

12 Factorise these quadratic expressions.

a $x^2 + 10x + 25$ **b** $a^2 + 40a + 400$ **c** $x^2 - 16x + 64$

d $x^2 - 2x + 1$ **e** $a^2 - 40a + 400$ **f** $x^2 + 16x + 64$

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Challenge exercise

1 a Expand $(e+4)^2$.

b Use this result to expand:

i $(e+4)^3$

ii $(e+4)^4$

2 Expand:

a $(a+b+1)(a+b-1)$

b $(x+a-3)(x-a-3)$

c $(x^2+x+1)(x^2-x+1)$

d $(x-1)(x-1)(x^2+1)$

e $(x-1)(x^2+x+1)$

f $(x+1)(x^2-x+1)$

3 Draw diagrams (involving rectangles) to show that:

a $(x+4)^2 = x^2+8x+16$

b $(a+b)^2 = a^2+2ab+b^2$

4 **Heron's formula** for the area A of a triangle with side lengths a, b and c is given by

$A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \dfrac{a+b+c}{2}$

a Find A for $a = 3$, $b = 4$ and $c = 5$.

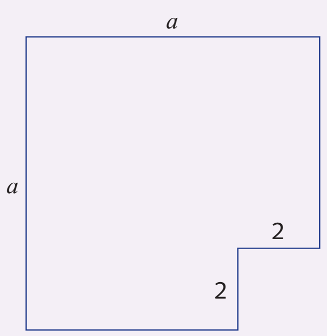
b Find A for $a = 13$, $b = 14$ and $c = 15$.

c Find A for $a = 193$, $b = 194$, $c = 195$.

Hint: Factorise A^2 into its prime factors.

5 a The figure opposite represents a square piece of paper with a corner cut out. Make a single straight cut to reshape the paper to form a rectangle with length $a+2$ and width $a-2$. This illustrates $a^2-4 = (a+2)(a-2)$.

b Similarly, illustrate the identity $a^2-b^2 = (a-b)(a+b)$.



6 In Question **5b** we saw that $(a-b)(a+b) = a^2-b^2$. We can use this identity to perform arithmetic short cuts. For example:

$$\begin{aligned} 49 \times 51 &= (50-1)(50+1) \\ &= 50^2 - 1^2 \\ &= 2500 - 1 \\ &= 2499 \end{aligned}$$

Use this trick to find:

a 19×21

b 29×31

c 48×52

d 67×73

e 99×101

f 202×198

7 a Expand and simplify $(n+1)^2 - n^2$.

b Using your answer, write the number 51 as the difference of two square numbers.

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