CHAPTER

18

Number and Algebra

Graphing straight lines

Graphing straight lines is contained in a topic in mathematics called **coordinate geometry**. Coordinate geometry is one of the most important and exciting ideas of mathematics. It provides a connection between algebra and geometry through graphs of lines and curves. This enables geometric problems to be solved algebraically and provides geometric insights into algebra. An immediate illustration of this will be shown in this chapter through the relation between solving linear equations and straight-line graphs.

Straight-line graphs can be used in many different practical situations. It will be seen how straight-line graphs can be used to help to understand problems involving constant rate.

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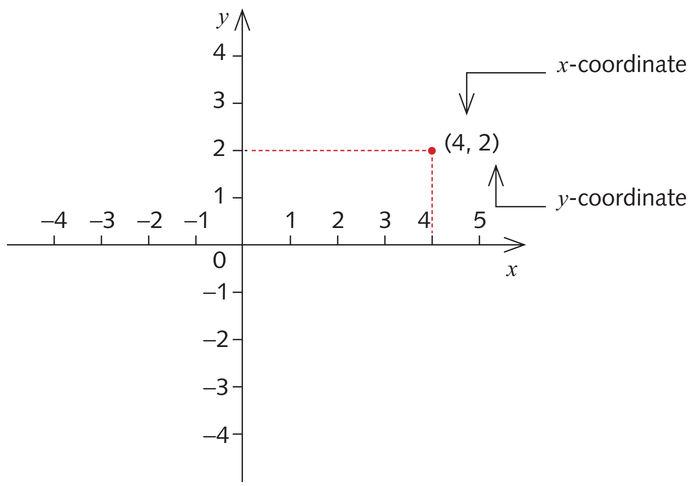
18A The Cartesian plane



The number plane was introduced in Chapter 4. This is known as the **Cartesian plane**. In the diagram above, we have labelled the horizontal axis the x-axis and the vertical axis the y-axis. With this labelling, the first coordinate of a point is the x-coordinate, and the second coordinate is the y-coordinate. For example, the point $A(4, 2)$ has x-coordinate 4 and y-coordinate 2.

Suppose that we are given a rule connecting the x and y-coordinates of a point. We can make up a table of values and plot the points on the Cartesian plane.

In the example below, what do you notice about the points? You should check, using your ruler and eye, that they lie on a line.

We can follow the same kind of procedure for any given rule – a table can be formed and the corresponding points plotted, although they do not always lie on a line.

Example 1

For each given rule, complete the table, list the coordinates of the points, and plot the points on the Cartesian plane. Draw a line through them.

a $y = -x$

x	–3	–2	–1	0	1	2	3
y							

b $y = x + 1$

x	–3	–2	–1	0	1	2	3
y							

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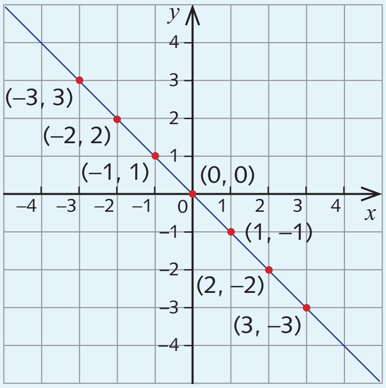
Solution

a $y = -x$

x	–3	–2	–1	0	1	2	3
y	3	2	1	0	–1	–2	–3

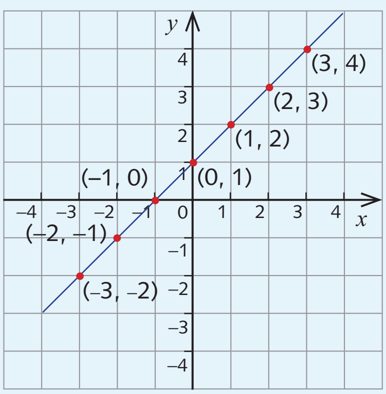
The points are $(-3,3)$, $(-2,2)$, $(-1,1)$, $(0,0)$, $(1,-1)$, $(2,-2)$ and $(3,-3)$.



b $y = x + 1$

x	–3	–2	–1	0	1	2	3
y	–2	–1	0	1	2	3	4

The points are $(-3,-2)$, $(-2,-1)$, $(-1,0)$, $(0,1)$, $(1,2)$, $(2,3)$ and $(3,4)$.



Exercise 18A

Example 1

1 For each given rule, complete the table, list the coordinates, and plot the corresponding set of points on a number plane. Draw a line through each set of points.

a $y = 3x$

x	–3	–2	–1	0	1	2	3
y							

b $y = -2x$

x	–3	–2	–1	0	1	2	3
y							

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c $y = x - 2$

x	−3	−2	−1	0	1	2	3
y							

d $y = x + 2$

x	−3	−2	−1	0	1	2	3
y							

e $y = 2x + 1$

x	−3	−2	−1	0	1	2	3
y							

f $y = 1 - x$

x	−3	−2	−1	0	1	2	3
y							

2 For each given rule, complete the table, list the coordinates, and plot the corresponding set of points on a number plane. Draw a line through each set of points.

a $y = x + \frac{1}{2}$

x	−3	−2	−1	0	1	2	3
y							

b $y = x - \frac{1}{2}$

x	−3	−2	−1	0	1	2	3
y							

c $y = 2x + \frac{1}{2}$

x	−3	−2	−1	0	1	2	3
y							

d $y = -x + \frac{1}{2}$

x	−3	−2	−1	0	1	2	3
y							

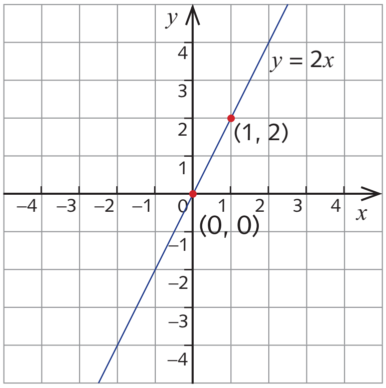
18B Drawing straight lines by plotting two points

Straight-line graphs that pass through the origin

We have seen that when we determine a table of values from a rule such as $y = 2x$ (or $y = 3x$ or $y = -x$), the points lie on a line. That is, we can draw a line through the points. The line is said to be the **graph** of $y = 2x$ (or $y = 3x$ or $y = -x$).

Since two points determine a line uniquely, we require two coordinate pairs that satisfy the equation. For $y = 2x$, the values $x = 0$ and $x = 1$ are suitable.

x	0	1
y	0	2



The coordinates $(0, 0)$ and $(1, 2)$ are plotted and the line is drawn through the points.

It is wise to use a third point as a check.
For example, $x = 2, y = 4$.

The point $(2, 4)$ lies on the line.

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Example 2

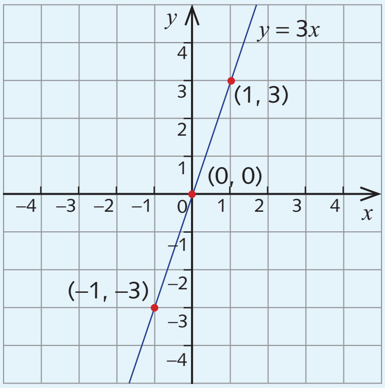
Draw the graph of $y = 3x$.

Solution

First make a table of values. Only two points are required but a third point is calculated as a check.

x	–1	0	1
y	–3	0	3

Now draw the line through the points.



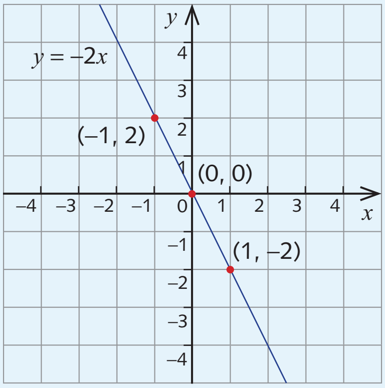
Example 3

Draw the graph of $y = -2x$.

Solution

First make a table of values. Only two points are required but a third point is calculated as a check.

x	–1	0	1
y	2	0	–2



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Straight-line graphs that do not pass through the origin

Not all straight-line graphs pass through the origin. In Section 18A, you plotted points for graphs such as $y = x + 1$ and $y = x + \frac{1}{2}$. Look back at those diagrams and check that the lines through the plotted points do not pass through the origin.

Scales

In the previous examples, we have chosen to use 0.5 cm to represent 1 unit. (We have done this to save space, but you may find it more convenient to use 1 cm to represent 1 unit.) We say that we have used the **scale** '0.5 cm represents 1 unit'. Up until now we have used the same scale on both the x- and y-axes, but sometimes it is helpful to use different scales on the two axes. Here is an example to show this.

Example 4

Draw the graph of $y = 5x + 10$.

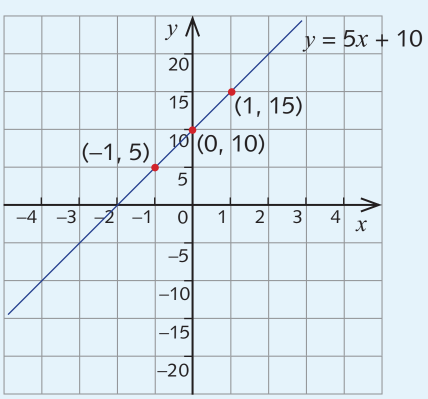
Solution

First make a table of values.

x	–1	0	1
y	5	10	15

We choose the scale '0.5 cm represents 1 unit' on the x-axis, and the scale '0.5 cm represents 5 units' on the y-axis. (*Reason*: The y values change by 5 when the x values change by 1.)

Here is the straight-line graph.



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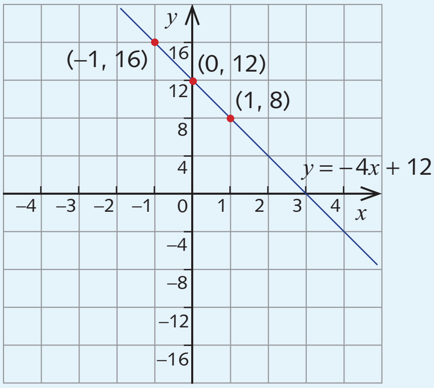
Example 5

Draw the graph of $y = -4x + 12$. Choose suitable scales for the two axes.

Solution

x	-1	0	1
y	16	12	8

We choose the scale '0.5 cm represents 1 unit' on the x-axis, and the scale '0.5 cm represents 4 units' on the y-axis.

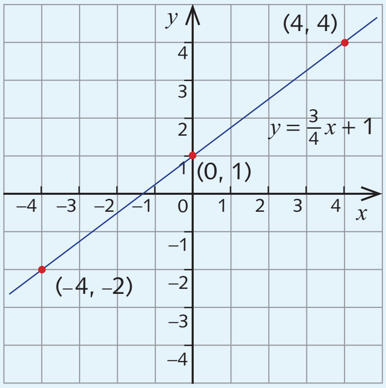


Example 6

Draw the graph of $y = \frac{3}{4}x + 1$. Use the standard scale of '0.5 cm represents 1 unit' on both the x- and y-axes.

Solution

x	-4	0	4
y	-2	1	4



Exercise 18B

Example 2, 3

1 On a single set of axes, draw the graphs of:

a $y = x$ **b** $y = 2x$ **c** $y = 4x$ **d** $y = -3x$ **e** $y = -4x$

2 On a single set of axes, draw the graphs of:

a $y = -x$ **b** $y = 3x$ **c** $y = 5x$ **d** $y = \frac{1}{2}x$ **e** $y = -\frac{1}{2}x$

(*Hint*: For parts **d** and **e**, use the x-values -2, 0 and 2.)

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Example 4

3 **a** Use the same set of axes with the same scales to draw the graphs of:

i $y = 2x$ **ii** $y = 2x - 1$ **iii** $y = 2x + 1$

iv $y = 2x + 4$ **v** $y = 2x + 3$

b What is the relationship between these straight-line graphs?

Example 5, 6

4 **a** Use the same set of axes to draw the graphs of:

i $y = 2x + 2$ **ii** $y = -x + 2$ **iii** $y = \frac{1}{2}x + 2$

iv $y = -\frac{1}{4}x + 2$ **v** $y = 3x + 2$ **vi** $y = 2 - 3x$

vii $y = 2 - \frac{1}{3}x$

b What is the relationship between these straight-line graphs?

18C Points and lines

We will now use the ideas developed in the algebra chapters of this book to answer some questions about straight-line graphs. We will use both substitution and solving equations to discover some further facts about straight-line graphs.

Finding the y-coordinate given the x-coordinate

We can find the y-coordinate of any point on the graph, corresponding to a given x value. To do this, we substitute the x-coordinate into the equation of the line, as shown in the following example. This will give us the coordinates of a point on the line.

Example 7

Find the coordinates of the points on the graph for the given x value for the straight-line graph of $y = 4x - 5$.

a $x = -1$ **b** $x = 0$ **c** $x = 20$

Solution

a $y = 4 \times (-1) - 5$
$= -4 - 5$
$= -9$

The y-coordinate is -9.

The coordinates are $(-1, -9)$.

b $y = 4 \times 0 - 5$
$= -5$

The y-coordinate is -5.

The coordinates are $(0, -5)$.

c $y = 4 \times 20 - 5$
$= 75$

The y-coordinate is 75.

The coordinates are $(20, 75)$.

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Finding the x-coordinate given the y-coordinate

This involves solving an equation, as shown in the following example.

Example 8

For the straight-line graph with equation $y = 4x - 5$, find the coordinates of the points on the line with y-coordinate:

a $y = 11$ **b** $y = 0$

Solution

a For $y = 11$, $4x - 5 = 11$

$\boxed{+5}$ $4x = 16$ (Add 5 to both sides of the equation.)

$\boxed{\div 4}$ $x = 4$ (Divide both sides of the equation by 4.)

The x-coordinate is 4.

The coordinates are $(4, 11)$.

b For $y = 0$, $4x - 5 = 0$

$\boxed{+5}$ $4x = 5$ (Add 5 to both sides of the equation.)

$\boxed{\div 4}$ $x = \frac{5}{4}$ (Divide both sides of the equation by 4.)

The x-coordinate is $\frac{5}{4}$.

The coordinates are $\left(\frac{5}{4}, 0\right)$.

Checking that a point lies on the graph

We check that a point lies on a line by seeing if the coordinates satisfy the equation.

Example 9

Check whether or not each of the following points lie on the line with equation $y = 2x + 3$.

a $(3, 9)$ **b** $(-2, 7)$ **c** $\left(-\frac{2}{3}, 0\right)$

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Solution

a When $x = 3,\ y = 2 \times 3 + 3$

$= 9$

So the point (3,9) lies on the line.

b When $x = -2,\ y = 2 \times -2 + 3$

$= -1$

This does not agree with the given y value, so the point (–2, 7) does not lie on the line $y = 2x + 3$.

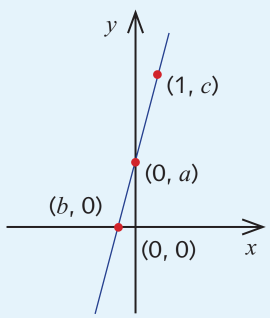
c For $x = -\frac{2}{3},\ y = 2 \times \frac{-2}{3} + 3$

$= 1\frac{2}{3}$

This does not agree with the given y value, so the point $\left(-\frac{2}{3}, 0\right)$ does not lie on the line $y = 2x + 3$.

Example 10

The graph of $y = 4x + 3$ is shown opposite. Find the values of a, b and c.



Solution

When the x-coordinate is 1,

$y = 4 \times 1 + 3$

$= 7$

so $c = 7$

When the x-coordinate is 0,

$y = 4 \times 0 + 3$

$= 3$

so $a = 3$

When the y-coordinate is 0,

$4x + 3 = 0$

$4x = -3$ (Subtract 3 from both sides.)

$x = -\frac{3}{4}$ (Divide both sides of the equation by 4.)

so $b = -\frac{3}{4}$

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Exercise 18C

Example 7

1 For the straight-line graph of $y = 2x - 3$, find the coordinates of the point on the line with x-coordinate:

a $x = -1$ **b** $x = 0$ **c** $x = 20$

2 For the straight-line graph of $y = -4x + 5$, find the coordinates of the point on the line with x-coordinate:

a $x = 1$ **b** $x = 0$ **c** $x = 2$

3 For the straight-line graph of $y = x - 5$, find the coordinates of the point on the line with x-coordinate:

a $x = -1$ **b** $x = 0$ **c** $x = 20$

4 For the straight-line graph of $y = -2x - 3$, find the coordinates of the point on the line with x-coordinate:

a $x = -1$ **b** $x = 0$ **c** $x = 15$

Example 8

5 For the straight-line graph with equation $y = 3x - 2$, find the x-coordinate corresponding to each of these y-coordinates.

a $y = 4$ **b** $y = 0$ **c** $y = 19$ **d** $y = -34$ **e** $y = -50$

6 For the straight-line graph with equation $y = -3x + 2$, find the x-coordinate corresponding to each of these y-coordinates.

a $y = 6$ **b** $y = 0$ **c** $y = 26$ **d** $y = -34$ **e** $y = -50$

Example 9

7 Check whether or not each point lies on the line with equation $y = 2x - 1$.

a $(3, 9)$ **b** $(-2, -5)$ **c** $(-1, -3)$ **d** $(4, 10)$

8 Check whether or not each point lies on the line with equation $y = -2x + 3$.

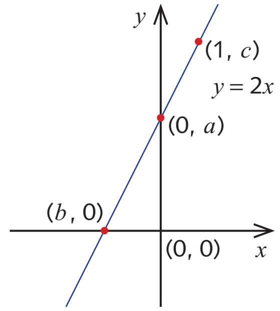
a $(3, 9)$ **b** $(-2, 7)$ **c** $(-1, 5)$ **d** $(4, -5)$

9 Check whether or not each point lies on the line with equation $y = -6x$.

a $(0, 0)$ **b** $(1, 6)$ **c** $(-1, 6)$ **d** $(4, 11)$

Example 10

10 The graph of $y = 2x + 3$ is shown below. Find the values of a, b and c.



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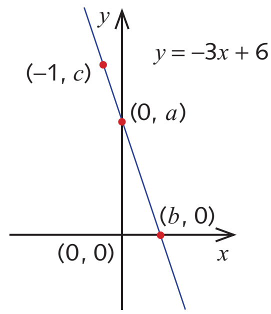
11 The graph of $y = -3x + 6$ is shown below. Find the values of a, b and c.



12 The x-coordinate of a particular point on the line $y = 5 - 3x$ is 10. Write down the y-coordinate.

13 The y-coordinate of a particular point on the line $y = 10 + 2x$ is 72. Write down the x-coordinate.

14 If the points $(-1, a)$, $(b, 15)$, and $(c, -20)$ lie on the line with equation $y = -5x$, find the values of a, b and c.

15 If the points $(3, a)$, $(-12, b)$ and $(c, -12)$ lie on the line with equation $y = -\frac{2}{3}x$, find the values of a, b and c.

18D The y-intercept and the gradient of a line

The y-intercept

The **y-intercept** of a line is the y-value of the point where the line cuts the y-axis. If a point is on the y-axis, then its x-coordinate is 0. This means we can find the y-intercept by substituting $x = 0$ into the equation.

Example 11

Find the y-intercept of the line with equation $y = 2x + 1$.

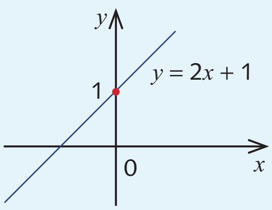
Solution

To find the y-intercept, we put $x = 0$.

Then $y = 2 \times 0 + 1$

$= 1$

Hence the y-intercept is 1.



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Example 12

Find the y-intercept of the line with equation $y = 3x - 5$.

Solution

Substitute $x = 0$ into the equation.

Then $y = 3 \times 0 - 5$

$= -5$

The y-intercept is -5.

Gradient

Here are two pictures of a car going up a hill.

We can see that the first hill is steeper than the second one. How can we measure the steepness of a hill?

There is a simple way to do this. We measure how far up in metres the car moves in the vertical direction (rise) for each 1 m that it moves in the horizontal direction (run).

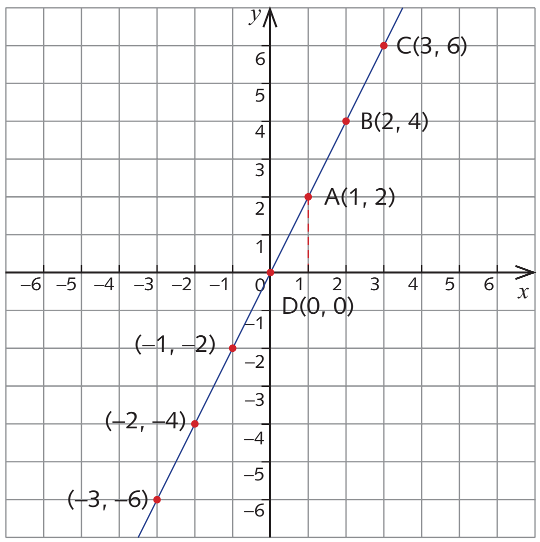
We then say the gradient of the hill $= \frac{\text{rise}}{\text{run}}$.

Using this idea we can easily see that the gradient of the left-hand hill is greater than the right-hand hill.

We use a similar idea to measure the slope of a straight-line graph.

Lines through the origin with positive gradient

The graph shown is $y = 2x$.



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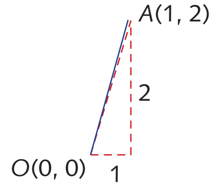
Look at the interval joining $O(0,0)$ to $A(1,2)$:



Using the same language as we did for the hill:

run = 1 and rise = 2

The gradient of the interval is $\frac{2}{1} = 2$.

Now look at the interval joining $A(1,2)$ to $B(2,4)$:

run = 1 and rise = 2

So the gradient of the interval is 2.

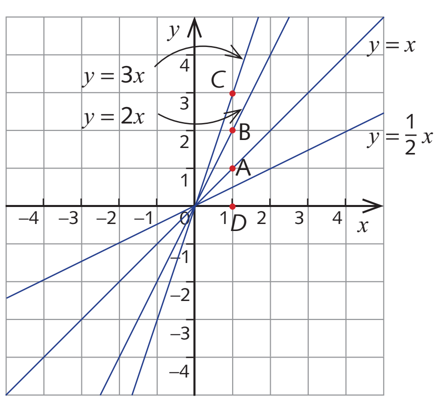
Finally, for the interval joining $A(2,4)$ to $C(3,6)$:

run = 1 and rise = 2

So the gradient of the interval is 2.

In each case the gradient is the same. We say that the **gradient of the line** is the change in y as we move 1 unit to the right. It doesn't matter which point we start from.

The diagram below shows the lines $y = x$, $y = 2x$, $y = 3x$ and $y = \frac{1}{2}x$. The steepest of these lines is $y = 3x$, while the least steep is $y = \frac{1}{2}x$.



What makes one line steeper than another? Suppose we go from the origin to the point A on the line $y = x$. To reach A from the origin, we need to move 1 unit to the right in the horizontal direction, and then 1 unit upwards in the vertical direction. So gradient = 1.

To reach B from the origin, we need to move 1 unit to the right in the horizontal direction, and then 2 units upwards in the vertical direction. So gradient = 2.

To reach C from the origin, we need to move 1 unit to the right in the horizontal direction, and then 3 units upwards in the vertical direction. So gradient = 3.

From this we can see:

The gradient of $y = \frac{1}{2}x$ is $\frac{1}{2}$.

The gradient of $y = x$ is 1.

The gradient of $y = 2x$ is 2.

The gradient of $y = 3x$ is 3.

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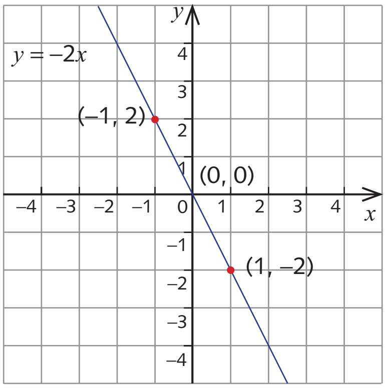
Positive gradient

Lines sloping up from left to right have positive gradient.

Lines through the origin with negative gradient

The graph of $y = -2x$ is shown.



Start at the point $(-1, 2)$. We move 1 unit to the right and 2 units down to arrive at the origin, so:

$$\text{rise} = -2$$

The run is the change in x-coordinate as we move from left to right on the graph, so:

$$\text{run} = 1$$

Hence, the gradient $= \frac{-2}{1} = -2$

The gradient of $y = -\frac{1}{2}x$ is $-\frac{1}{2}$.

Similarly:

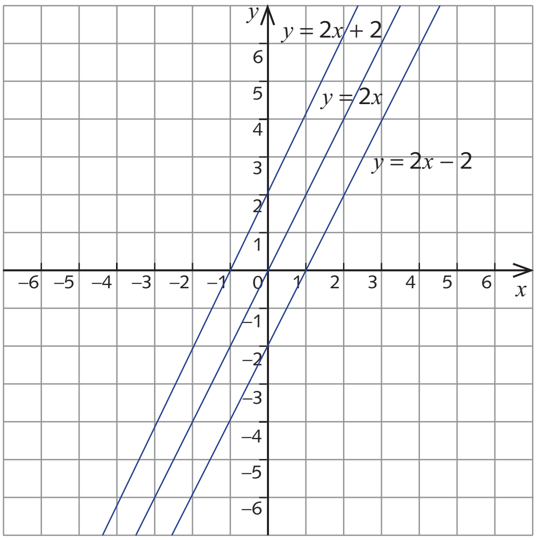
- The gradient of $y = -x$ is -1.
- The gradient of $y = -2x$ is -2.
- The gradient of $y = -3x$ is -3.

Negative gradient

Lines sloping down from left to right have negative gradient.

Straight lines not passing through the origin

The graphs of $y = 2x$, $y = 2x - 2$ and $y = 2x + 2$ are shown in the graph to the right.



Notice that they are parallel and therefore have the same gradient.

You can check this by choosing intervals on the lines $y = 2x - 2$ and $y = 2x + 2$ and calculating the gradient.

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In summary, to find the gradient of a line we follow these steps.

- Find a point on the line.
- Move along the line by moving 1 unit to the right in the horizontal direction.
- Find the change in the y-coordinate that has occurred. This gives the gradient of the line.

Example 13

Find the gradient of the line $y = 3x - 1$.

Solution

Substitute an x value to find one point on the line.

If we put $x = 1$, this gives $y = -2$. So $(1, 2)$ is a point on the line.

Substitute another x value to find a second point on the line.

If we put $x = 2$, this gives $y = 5$. So $(2, 5)$ is another point on the line.

The change in the y-coordinates is $5 - 2 = 3$ when we move 1 unit in the positive x-direction. Hence, the gradient of the line is 3.

Example 14

Find the gradient of the line with equation $y = -3x + 5$.

Solution

First, put $x = 0$. This gives $y = 5$, so $(0, 5)$ is a point on the line.

Now move 1 unit to the right in the horizontal direction.

We get $x = 1$ and $y = -3 \times 1 + 5 = 2$. So $(1, 2)$ is another point on the line.

The change in y is $2 - 5 = -3$, so the gradient is -3.

Exercise 18D

Example 11, 12

1 Find the y-intercept of each line by substituting $x = 0$.

a $y = 2x + 3$ **b** $y = 4x - 7$ **c** $y = -2x + 5$

d $y = -4x - 8$ **e** $y = 5x$

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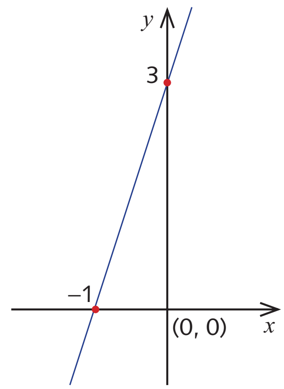
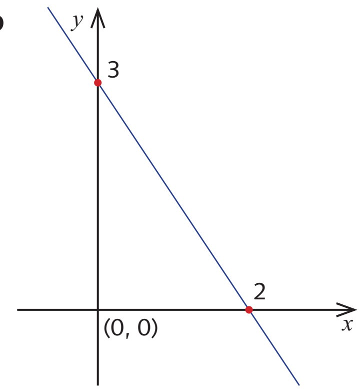
2 Read off the y-intercept from each of these graphs.

a



b



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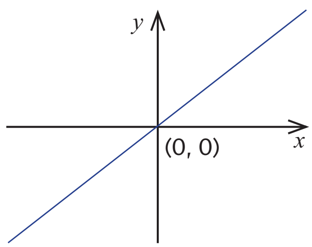

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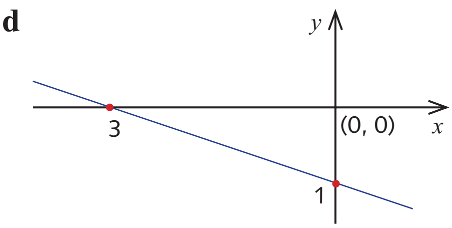

Example 13

3 For each of the four equations shown below:

i make up a table of values for $x = -1, 0, 1$

ii draw the graph of the line

iii find the gradient of the line

a $y = 2x$ **b** $y = 2x + 1$ **c** $y = 3x - 2$ **d** $y = x + 3$

4 For each of the four equations shown below:

i find the points on the line that have x-values of 1 and 2

ii hence find the gradient of the line

a $y = 3x$ **b** $y = 4x - 1$ **c** $y = 2x - 5$ **d** $y = x + 2$

5 **a** Find the gradient of the line $y = 3x - 1$ and the gradient of the line $y = 2x + 4$.

b Which of the lines in part **a** is steeper?

Example 14

6 For each of the four equations shown below:

i make up a table of values for $x = 0, 1, 2$

ii draw the graph of the line

iii find the gradient of the line

a $y = -2x$ **b** $y = -x + 1$ **c** $y = -3x + 5$ **d** $y = -2x + 3$

7 For each line given below, find the points on the line that have x-values of 1 and 2, and hence find the gradient of the line.

a $y = -3x$ **b** $y = -4x + 2$ **c** $y = -2x + 5$ **d** $y = -x + 2$

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18E More on gradients

In Examples 13 and 14 in the previous section, we saw that the gradient of the line $y = 3x - 1$ is 3 and the gradient of the line $y = -3x + 5$ is -3.

From these and other examples we have done, you may have guessed that the gradient of the line written in the form $y = mx + c$ is the number m in front of the x, which is called the **coefficient** of x. If so, you are correct! Well done.

Equations of the form $y = mx + c$

- The graph of each equation of the form $y = mx + c$ is a straight line.
- The number m gives the **gradient** of the line and c gives the **y-intercept**.

Example 15

Find the gradient and y-intercept of the lines:

a $y = 7x - 4$

b $y = -x + 3$

Solution

a The gradient of the line $y = 7x - 4$ is 7. The y-intercept is -4.

b The gradient of the line $y = -x + 3$ is -1. The y-intercept is 3.

We can now quickly compare the steepness of any two lines that have positive gradients.

Example 16

Which of the lines $y = 4x - 6$ and $y = 6x - 4$ is steeper?

Solution

The gradient of the line $y = 4x - 6$ is 4.

The gradient of the line $y = 6x - 4$ is 6.

Hence, the line $y = 6x - 4$ is steeper.

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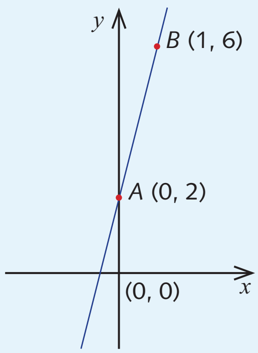
Example 17

The line shown opposite has equation $y = mx + 2$.
Find the value of m.



Solution

The points $A(0,2)$ and $B(1,6)$ lie on the line. The change in the y-coordinate, as x changes from 0 to 1, is 4, so the gradient of the line is 4. Hence, $m = 4$.

(As a check, if $y = 6$ and $x = 1$, then $6 = m + 2$)

Exercise 18E

Example 15

1 Write down the gradient of each line.

a $y = 7x$

b $y = 9x + 2$

c $y = -12x + 3$

d $y = -x + 8$

2 Write down the gradient of each line.

a $y = 5 - 7x$

b $y = -3x + 12$

c $y = 6 - 11x$

d $y = 9 - x$

Example 16

3 For each pair of lines, state which is steeper.

a $y = 5x$, $y = 4x - 2$

b $y = 7x + 2$, $y = 9x - 5$

c $y = 3 + 2x$, $y = 3x + 2$

4 **a** For each line given below, state whether it slopes upwards or downwards.

i $y = -7x$

ii $y = 4x - 9$

iii $y = -9x + 8$

iv $y = -x + 12$

b For each equation in part **a**, find the value of m.

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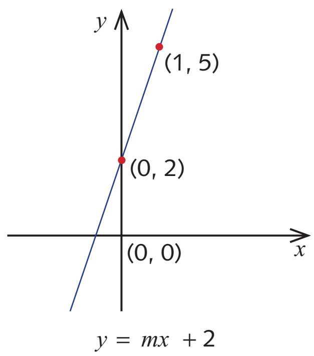
Example 17

5 In each problem below, find the value of m.

a



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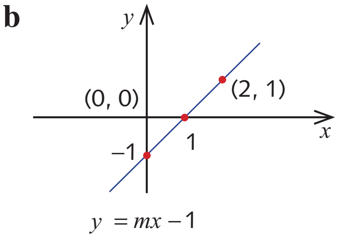
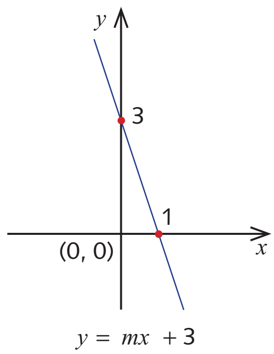
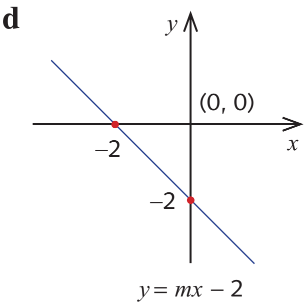


c



d



6 Find the value of m for the line that connects the points with coordinates $(3,-2)$ and $(0,7)$.

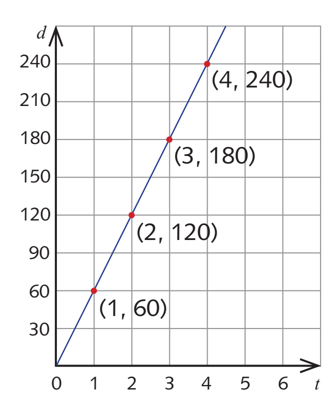
7 Find the value of m for the line that connects the points with coordinates $(-2,0)$ and $(5,14)$.

18F Applications to constant rate problems

The idea of a rate was introduced in Section 10B. For example, 60 km/h and 50 litres/minute are examples of constant rates. We can use straight-line graphs to help us solve problems involving constant rates. In Section 10C, we looked at the familiar example of an object moving at constant speed.

For example, suppose that a car is travelling at 60 km/h. We can plot a table of values for the distance d (in km) travelled by the car after t hours.

t (hours)	0	1	2	3	4
d (kilometres)	0	60	120	180	240



We can see from the table that the equation relating d and t is $d = 60t$.

We take the horizontal axis to be the t-axis and the vertical axis to be the d-axis. The graph opposite is drawn for t-values from 0 to 4 and d takes values from 0 to 240.

The graph has a gradient of 60 and passes through the origin.
The gradient gives the speed of the car, in km/h.

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Example 18

A car is travelling at a constant speed of 100 km/h.

a What is the formula for the distance d (in km) travelled by the car in t hours?

b What is the gradient of the straight-line graph of d against t?

Solution

a In 1 hour, the car travels 100 km.

In 2 hours, the car travels 200 km.

The formula is $d = 100t$.

b The gradient of the straight line graph is 100.

Constant rate

Questions involving a constant rate give rise to straight-line graphs. The gradient of the line is the constant rate.

Example 19

A cylindrical tank can hold a maximum of 40 litres of water. It has 10 litres of water in it to start with. Water is flowing slowly in at a rate of 5 litres per minute.

a Prepare a table of values showing how much water is in the container at 1-minute intervals from 0 up to 6 minutes.

b Plot the graph of the volume V (in litres) of water in the tank against time t (in minutes) since the start.

c Give the formula for V in terms of t.

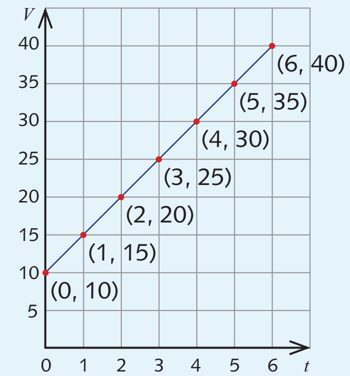
Solution

a

t (minutes)	0	1	2	3	4	5	6
V (litres)	10	15	20	25	30	35	40

b (The scale on the t-axis is '0.5 cm represents 1 minute' and the scale on the V-axis is '0.5 cm represents 5 litres'.)

c From the graph, the gradient is 5 and the V-axis intercept is 10 litres. The formula is $V = 5t + 10$, where t takes values from 0 to 6 minutes inclusive.



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Exercise 18F

Example 18

1 Maria decides to ride her bicycle to the next town, which is 60 km away. She rides at 15 km/h.

a Let t denote the number of hours that have elapsed since she set out. Prepare a table of values showing how far (d km) she is from her starting point at 1-hour intervals up to $t = 4$ hours.

b Plot the points (t, d) from your table of values and draw the graph.

Note: For travel graphs it is usual to plot distance travelled against time.

c Give the formula for d in terms of t.

d Use the formula to find how far Maria has ridden after $3\frac{1}{4}$ hours.

Example 19

2 A tank can hold a maximum of 40 litres of water. Initially it has 15 litres of water in it. Water is flowing slowly in at a rate of 5 litres per minute. Let V be the volume (in litres) of water in the tank t minutes from the start.

a Prepare a table showing the values of V at 1-minute intervals up to $t = 5$.

b Plot the graph of V against t from your table of values.

c Give the formula for V in terms of t.

d Use the formula to find the volume of water in the tank after $2\frac{1}{2}$ minutes.

3 We have a pile of matchsticks. We form the letter L with squares made up of matchsticks, as shown in the diagrams below.

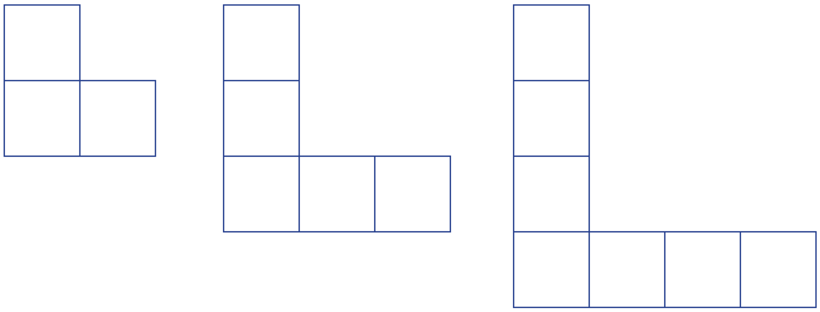

Diagram 1 Diagram 2 Diagram 3 (and so on)

a Copy and complete the table below, where n is the number of the diagram.

Diagram number (n)	Number of matches (M)
1	10
2	
3	
4	
5	
6	

b Find a formula for M in terms of n.

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c Plot the points (n, M) for values of n from 1 to 10, using your table of values. (Draw a line through the points you plot, although in this question doing so does not make *practical* sense – why not?)

d Find the value of M for $n = 12$.

e What is the value of n for the diagram that uses 124 matches?

18G Not all graphs are straight lines

Up to this point, we have considered only straight-line graphs. However, there are many kinds of equations that do not produce straight lines when plotted, as the following example shows. You will often encounter these kinds of equations as you progress in your study of mathematics.

Example 20

For the rule $y = x^2$:

a calculate the table of values for the x values $-3, -2, -1, 0, 1, 2, 3$

b plot the points

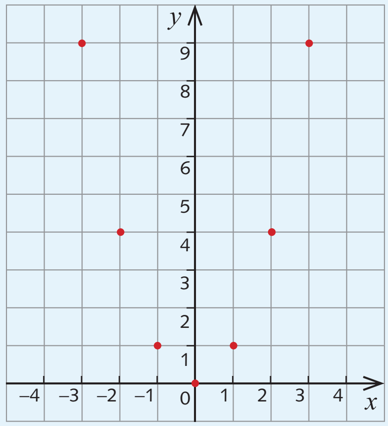
c join the points with a smooth curve

Solution

a

x	–3	–2	–1	0	1	2	3
y	9	4	1	0	1	4	9

b



You can see immediately that the points do not lie on a straight line.

c Draw a smooth freehand curve through the points. Compare your curve with your neighbour's.

The curve, $y = x^2$, is called a **parabola**.

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Exercise 18G

Example 20

1 For each given rule, complete the table, decide on the set of values for the y-axis, and plot the points. Draw a smooth freehand curve through the points.

a $y = x^2 - 4$

x	−3	−2	−1	0	1	2	3
y							

b $y = x^3$

x	−3	−2	−1	0	1	2	3
y							

c $y = x^3 + 1$

x	−3	−2	−1	0	1	2	3
y							

d $y = (x - 3)^2$

x	0	1	2	3	4	5	6
y							

e $y = (x + 3)^2$

x	−6	−5	−4	−3	−2	−1	0
y							

f $y = x^3 - 4x$

x	−3	−2	−1	0	1	2	3
y							

g $y = x^3 - 9x$

x	−4	−3	−2	−1	0	1	2	3	4
y									

h $y = x^2 - 2x$

x	−2	−1	0	1	2	3	4
y							

i $y = 9 - x^2$

x	−6	−5	−4	−3	−2	−1	0	1	2
y									

Review exercise

1 On a single set of axes, draw the graphs of:

a $y = x$ **b** $y = -5x$ **c** $y = -\frac{1}{4}x$

d $y = \frac{2}{3}x$ **e** $y = -\frac{5}{4}x$

2 On a single set of axes, draw the graphs of:

a $y = 3x - 6$ **b** $y = -x + 4$ **c** $y = \frac{1}{2}x + 4$

d $y = -\frac{1}{4}x - 2$ **e** $y = -3x + 2$

3 Find the y-intercept of each of these lines by substituting $x = 0$ into the equation.

a $y = x - 4$ **b** $y = 3x + 10$

c $y = 4 - 3x$ **d** $y = 5x - 10$

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4 For each line given below, find the points on the line with x-values of 1 and 2, and hence calculate the gradient of the line.

a $y = 3x - 2$ **b** $y = 2x - 8$ **c** $y = -5x + 6$

5 **a** For each line given below, state whether it slopes upwards or downwards.

i $y = 2x$ **ii** $y = -3x$ **iii** $y = 4x - 6$ **iv** $y = 2 - x$

b Each equation above has the form $y = mx + b$, where m and b are numbers. For each equation, find the value of b.

6 Check whether or not each of these points lies on the line with equation $y = 2x + 3$.

a $(3, 9)$ **b** $(-2, -1)$ **c** $(-1, -5)$ **d** $(-4, -5)$

7 The x-coordinate of a point on the line $y = 10 - 3x$ is 3. Write down the y-coordinate.

8 The y-coordinate of a point on the line $y = 8 + 2x$ is 72. Write down the x-coordinate.

9 If the points $(-1, a)$, $(b, 5)$ and $(c, -20)$ lie on the line with equation $y = 5x + 5$, find the values of a, b and c.

10 Write down the gradients and y-axis intercepts for the lines with the equations given below.

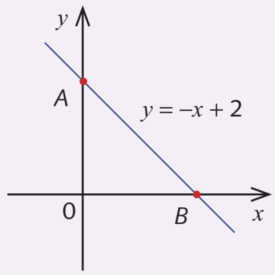
a $y = 5x - 8$

b $y = 3x + 7$

c $y = -3x + 11$

11 If the points $(-2, a)$, $(b, 16)$ and $(c, 28)$ lie on the line with equation $y = 3x + 7$, find the values of a, b and c.

12 The equation of the line passing through points A and B in the diagram below is $y = -x + 2$.



a Find the coordinates of points A and B.

b Find the length of the line interval AB.

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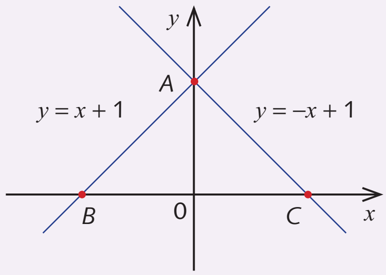
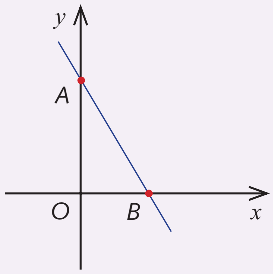
13 The graphs of $y = x + 1$ and $y = -x + 1$ are shown below.



a Find the coordinates of points A, B and C.

b Find the area of triangle ABC.

14 The line through points A and B in the diagram below has equation $y = -5x + c$, where c is a positive number. The area of triangle OAB is 10 cm^2.



a Find the value of c.

b Find the coordinates of A and B.

15 **a** The line with equation $y = mx + 3$ is parallel to the line with equation $y = -5x$. State the value of m.

b Find the y-coordinate of the point on the line with equation $y = -5x + 4$ for which the x-coordinate is 7.

c Find the x-coordinate of the point on the line with equation $y = -5x + 10$ for which the y-coordinate is 0.

d Find the x-coordinate of the point on the line with equation $y = -5x - 2$ for which the y-coordinate is 13.

Challenge exercise

1 A man is walking home at 6 km/h. He starts at a point 18 km from his home. Draw a graph representing his trip home. State the gradient and vertical axis intercept, and give a formula that describes the trip.

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2 A tank can hold up to 50 litres of water. It is full to start with. Water is flowing slowly out at a rate of 5 litres per minute.

a Prepare a table of values showing how much water is in the tank at 1-minute intervals.

b Plot the graph of the volume V (in litres) of water in the tank against time t (in minutes) since the start.

c Give the formula for V in terms of t.

3 The towns Cunadilla and Frevnelle are 110 km apart. David lives in Cunadilla, and Lilly lives in Frevnelle. They decide to meet on the road between the two towns. Lilly cycles at 15 km/h and David cycles at 10 km/h. They both leave their towns at 11 a.m.

Let D and L be the respective distances of David and Lilly from Cunadilla at time t hours after 11 a.m.

a Write down a formula for D in terms of t.

b Write down a formula for L in terms of t.

c Draw up a table of values for each formula. For David, consider values of t between 0 and 11. For Lilly, consider values of t between 0 and 7.

d On the same set of axes, draw the graphs of D against t, and L against t.

e At what time do the two friends meet?

f How far from Cunadilla do they meet?

4 In a 100-m handicap race, Priscilla is given a 5-m start over Wendy. That is, when the starter's gun is fired, Priscilla starts 5 m ahead of the starting line for the 100 m. Wendy starts from the starting line. Priscilla runs at 7.5 m/s and Wendy runs at 8 m/s.

a Write down a formula for P, the distance in metres of Priscilla from the starting line t seconds after the starter's gun is fired.

b Write down a formula for W, the distance in metres of Wendy from the starting line t seconds after the starter's gun is fired.

c Draw up a table of values for each formula.

d On the same set of axes, draw the graphs of P against t, and W against t.

e How long does it take for Wendy to overtake Priscilla?

f How far has each of the girls run when Priscilla is overtaken?

g What start should Priscilla be given if we want Wendy to catch up to her at the 100-m mark?

5 **a** Prepare a table of values for the graph of $y = 3x$.

b Plot the points from your table of values.

c Draw the graph of $y = 3x$.

d On the same set of axes, draw the graph of $y = 3x + 3$.

e **i** Describe a translation that moves the graph of $y = 3x$ to the graph of $y = 3x + 3$.

ii Find a second translation that does this.

iii How many such translations are there?

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f What is the equation of the graph obtained by reflecting the line $y = 3x$ in:

i the x-axis? **ii** the y-axis?

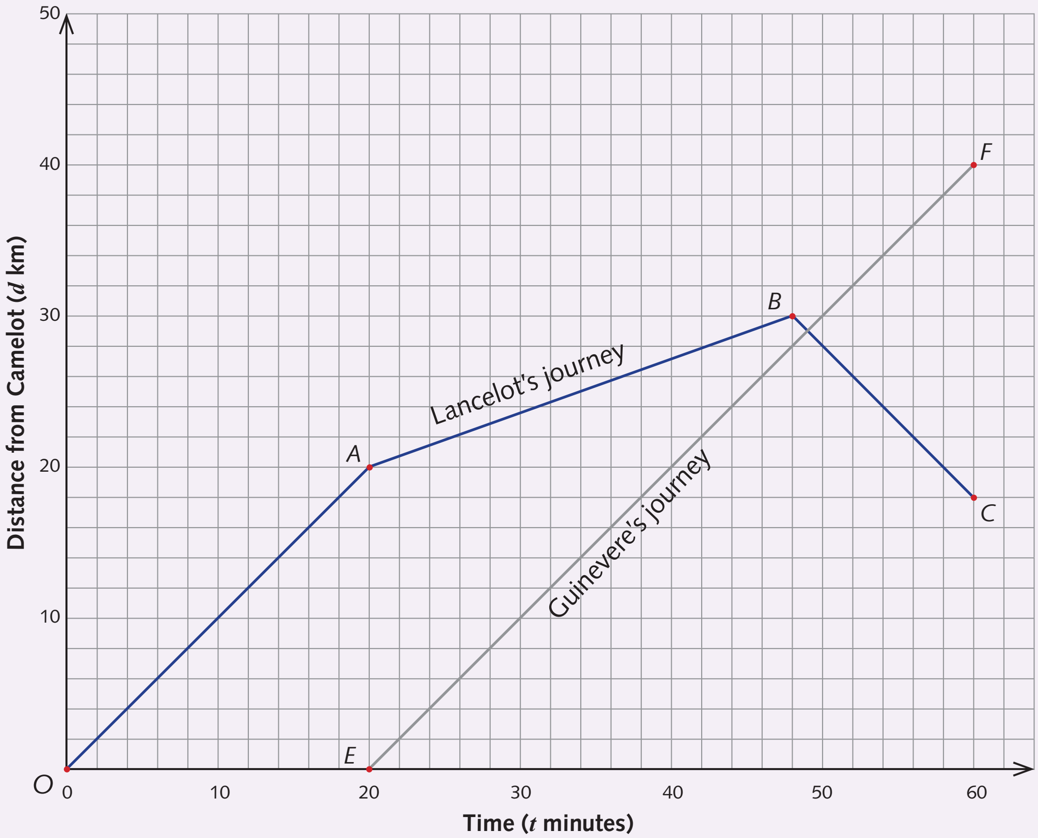
g Draw the graphs of the equations in part **f**.

h What is the equation of the graph obtained by rotating the line $y = 3x$:

i 180° about the origin in an anticlockwise direction?

ii 90° about the origin in an anticlockwise direction?

6 The line graphs below show the journeys of Lancelot and Guinevere along a road leading out of Camelot. The vertical axis shows the distance from Camelot and the horizontal axis the time taken after Lancelot leaves Camelot.



a State the gradient of each line interval.

i OA **ii** AB **iii** BC **iv** EF

b Give the speed of Lancelot for the different stages of his journey.

c Find the equation of each stage of Lancelot's journey.

d Find the equation of Guinevere's journey.

e If Lancelot continues back to Camelot at the same speed, when will he be back in Camelot?

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