CHAPTER

20

Review and problem-solving

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20A Review

Chapter 10: Rates and ratios

1 Copy and complete the following equivalent ratios.

a $2:3=\square:9$ **b** $9:24=\square:8$

c $10:15=\square:4$ **d** $8:12=\square:3$

2 Express each of these ratios in simplest form.

a $5:10$ **b** $22:4$ **c** $125:750$ **d** $2:10$

e $32:72$ **f** $96:24$ **g** $225:150$ **h** $20:100$

3 In a box of toffees, the ratio of red wrappers to green wrappers is $5:6$, while the ratio of green wrappers to blue wrappers is $3:10$. Find the ratio of red wrappers to blue wrappers.

4 Divide 198 in the ratio $7:4$.

5 Divide \$1125 in the ratio $2:1$.

6 Divide \$2190 in the ratio $3:2:1$.

7 If a dozen eggs costs \$5.64, what is the cost of 30 eggs?

8 A man works for 7 hours and gets paid \$157.50.

a What is his hourly rate?

b How much does he get paid for 10 hours work?

9 A car uses 40 L of petrol to travel 320 km. How far can it travel with 50 L of petrol?

10 The cost of 1 kg of a certain type of fish is \$8.20.

a How much does 4.5 kg of this kind of fish cost?

b How many kilograms can you buy for \$49.20?

11 Express each of the following as a ratio in simplest form.

a 50 g to 200 g **b** 700 g to 1 kg

12 In a co-educational school of 1029 pupils, 504 are girls. What is the ratio of the number of girls to the number of boys?

13 The ratio of the number of birds to the number of cats owned by people in a neighbourhood is $5:3$. The number of cats is 96. How many birds are there?

14 In an apartment block, 24 apartments have two bedrooms and 32 have one bedroom. Give the ratio of the number of two-bedroom apartments to the number of one-bedroom apartments.

15 Four bricklayers can build a certain wall in 10 days. How long would it take five bricklayers to build it?

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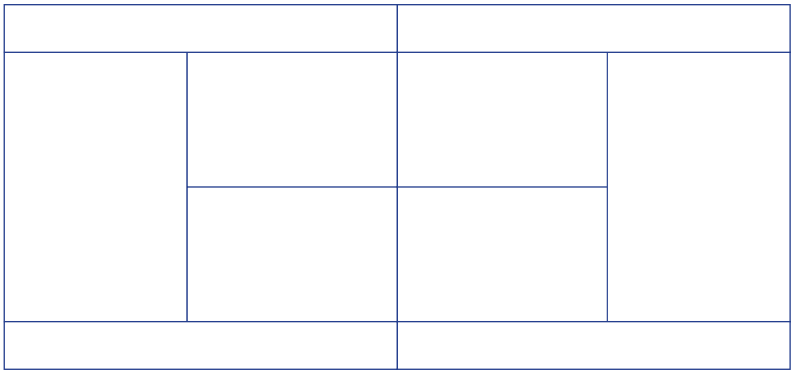
16 This plan of a tennis court is drawn to a scale of 1 cm to represent 230 cm. By taking measurements with your ruler, answer the questions below.



a What is the approximate length of the court?

b What is the approximate width of the court?

c Find the approximate distance from the base line to the service line on the same side of the net.

d What is the approximate area of the court?

e What is the approximate distance between the singles side line and doubles side line on each side of the court?

17 Convert each scale to ratios of whole numbers, reduced to lowest terms. Make sure to convert both measurements to the same unit first where necessary.

a 3 cm : 72 cm **b** 2 cm : 0.4 m **c** 5 cm : 10 mm **d** 27 cm : 3 km

18 Copy and complete each conversion of scale.

a $4:9 = 1$ cm: _____ cm **b** $4:9 = 1$ cm: _____ mm **c** $4:9 = 1$ cm: _____ m

19 A car travels at 90 km/h for 5 hours. How far does it go?

20 A train takes 5 hours to complete a journey of 400 km. What is the average speed of the train?

21 A car travels a distance of 480 km at an average speed of 60 km/h. How long did the journey take?

22 Elizabeth walks 20 km in 5 hours. How far, at the same rate, will she walk in:

a 1 hour? **b** 2 hours? **c** 3 hours?

23 An aircraft travels 6500 km in 13 hours. What is its average speed?

24 A train travels for 45 minutes at 80 km/h. How far does it go?

25 Claire travels 40 km by train in $\frac{3}{4}$ of an hour and then cycles for 10 km in 48 minutes.

a How long is she travelling in total?

b What is her average speed during the train trip?

c What is her average speed during the bike trip?

d What is her average speed over the whole trip?

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Chapter 11: Algebra – part 2

1 Express each of these expressions as a single fraction.

a $\frac{x}{3}+\frac{x}{3}$

b $\frac{4x}{5}-\frac{2x}{5}$

c $\frac{z}{11}+\frac{6z}{11}$

d $\frac{3x}{5}+\frac{x}{2}$

e $\frac{x}{4}+\frac{6x}{7}$

f $\frac{2x}{9}-\frac{3x}{4}$

g $\frac{4x}{7}-\frac{x}{2}$

h $\frac{3x}{5}+\frac{3x}{7}$

i $\frac{2x}{5}+\frac{4x}{15}-\frac{x}{3}$

j $\frac{x}{2}-\frac{2x}{5}+\frac{x}{3}$

2 Expand the brackets and collect like terms.

a $4(x-3)+6$

b $-2(7x+3)-8x$

c $-(x-6)-5(x-2)$

d $2\left(\frac{2x}{3}+4\right)+\frac{x}{3}$

e $5\left(\frac{x}{2}+5\right)-\frac{x}{2}$

f $3\left(\frac{3x}{4}-2\right)+\frac{x}{2}$

g $3\left(\frac{2x}{5}+2\right)-\frac{3x}{7}$

h $4\left(\frac{3x}{5}+1\right)-\frac{2x}{3}$

i $6\left(\frac{4x}{7}-1\right)-\frac{3x}{11}$

3 Solve these equations for x.

a $2x+5x+1=22$

b $4(x+1)+2=15$

c $-3(2x+1)+5=17$

d $x+3=2$

e $2x+4=-2$

f $2-x=8$

g $\frac{x}{5}+4=11$

h $2x-5=-6x+7$

i $2(x-2)=11x$

j $\frac{3x}{5}+6=\frac{21}{5}$

k $2x-12=2(3+4x)$

4 Solve these equations.

a $\frac{3x}{7}+\frac{x}{7}=6$

b $\frac{4p}{9}+\frac{2p}{9}=1$

c $-\frac{x}{5}+\frac{2x}{5}=1$

d $\frac{2x}{7}+\frac{x}{5}=2$

e $\frac{m}{3}+\frac{2m}{5}=1$

f $\frac{5m}{9}-\frac{4m}{5}=1$

5 Given that $y=3x-5$, find the value of y when $x=5$.

6 **a** A number is multiplied by −5 and 3 is subtracted. The result is −4. Write an equation and find the number.

b Six is added to a number, and the result is multiplied by −3. The result is 28. Write an equation and find the number.

c Seven is subtracted from a number, and the result is multiplied by 4. The result is 15. Write an equation and find the number.

d When a number is multiplied by 5 and divided by 11, the result is 22.8. Find the number.

e If you add 13 to a number, you get the same result as when you subtract half the number from 4. What is the number?

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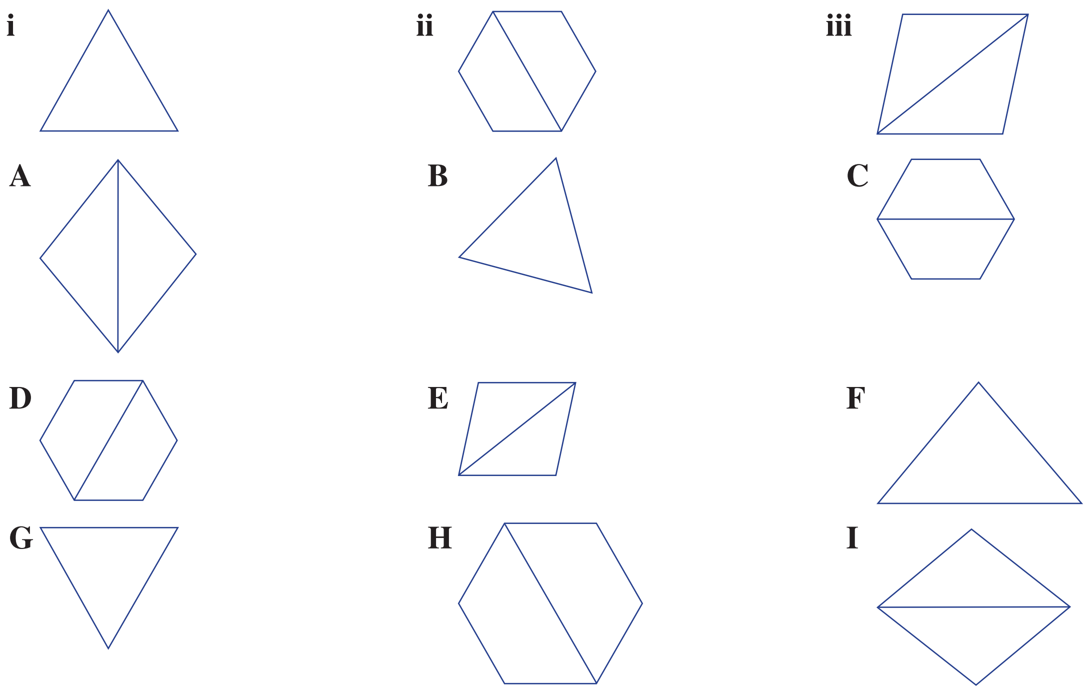
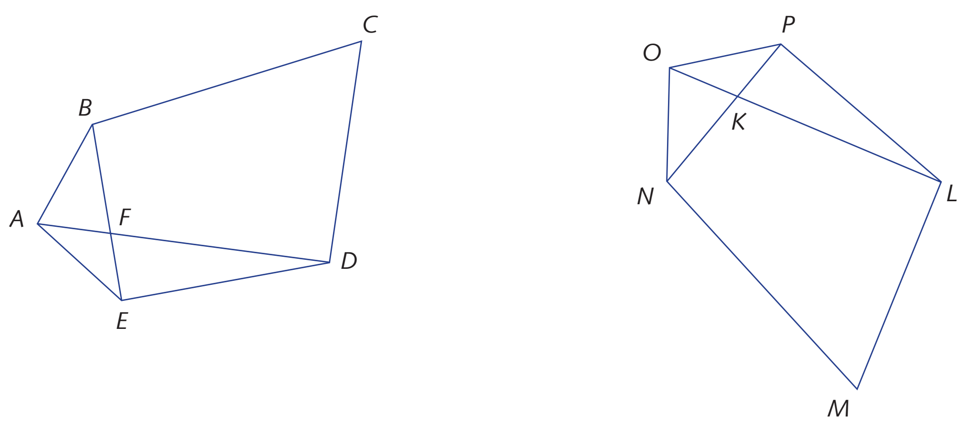
Chapter 12: Congruent triangles

1 List the figures in the collection below that are congruent to figures **i**, **ii** and **iii**.



2 Complete the pairing of matching vertices, matching sides and matching angles of these two congruent figures.



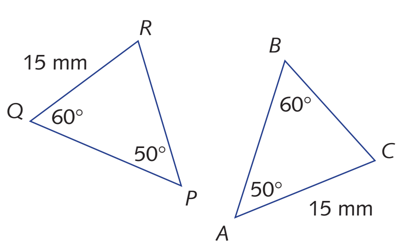
a $A \leftrightarrow$ **b** $B \leftrightarrow$ **c** $C \leftrightarrow$ **d** $D \leftrightarrow$

e $E \leftrightarrow$ **f** $F \leftrightarrow$ **g** $AD \leftrightarrow$ **h** $AB \leftrightarrow$

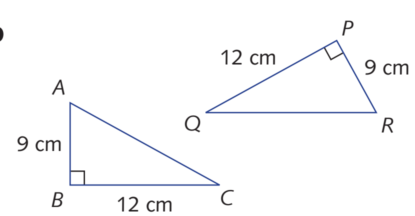
i $CD \leftrightarrow$ **j** $\angle ABC \leftrightarrow$ **k** $\angle BED \leftrightarrow$ **l** $\angle AFB \leftrightarrow$

3 In each part below, say whether the two triangles are congruent. If they are, write a congruence statement, including the appropriate congruence test.

a



b



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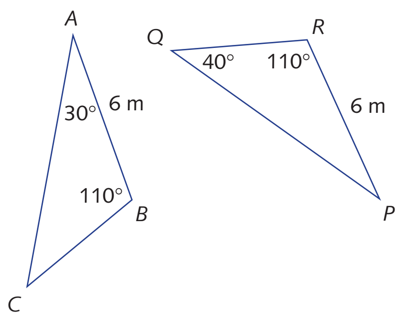
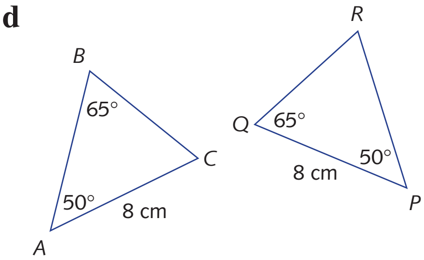
c

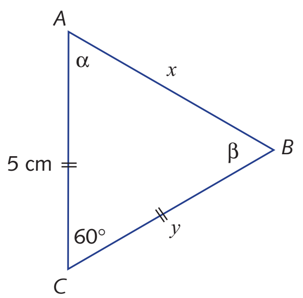


d

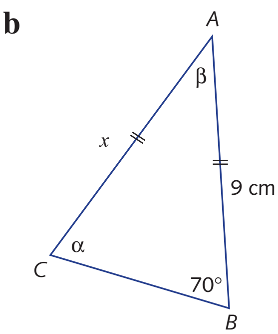


4 Find the values of x, y, α, β, and γ, giving reasons.

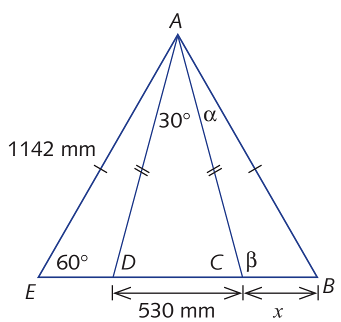
a



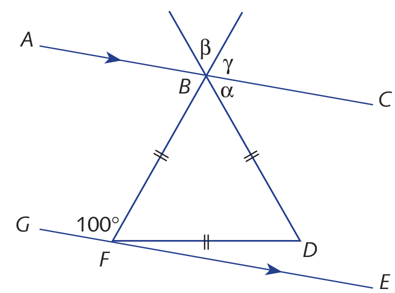
b



c



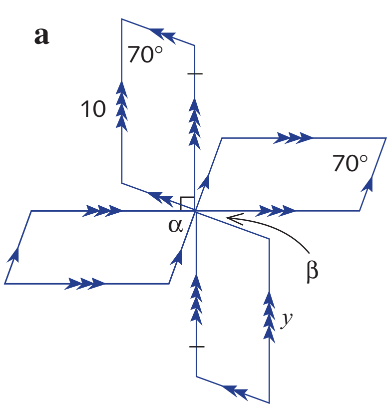
d



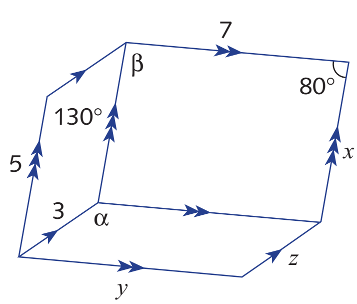
Chapter 13: Congruence and special quadrilaterals

1 Use the properties of a parallelogram to find the values of x, y, z, α, and β in the diagrams below.

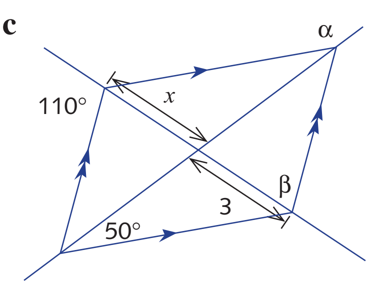
a



b



c



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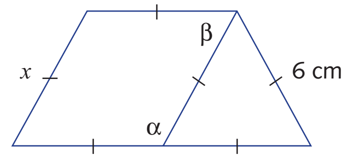
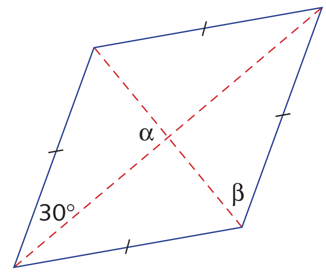
2 Use the properties of a rhombus to find the values of x, y, z, α, and β in the diagrams below.

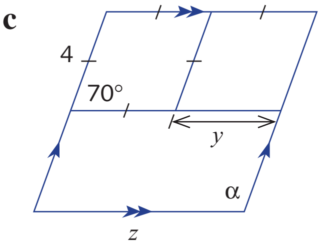
a



b

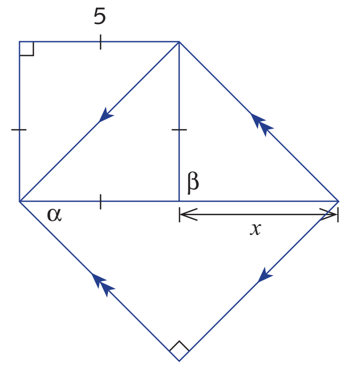


c

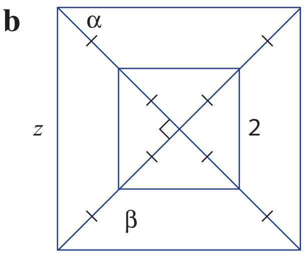


3 Find the values of x, z, α, and β in the diagrams below.

a



b



4 Each of the groups in the column on the left matches with at least one of the descriptions on the right. Can you match each group on the left with a description so that each description is used only once?

	Quadrilateral		Description
A	Parallelogram	I	Opposite sides are parallel and all four angles are right angles
B	Rhombus	II	Opposite sides are parallel
C	Rectangle	III	Opposite sides are parallel and all four angles are right angles and all four sides are equal
D	Square	IV	Opposite sides are parallel and all four sides are equal

5 **a** Complete this sentence by inserting the word ‘square’ or ‘rhombus’.

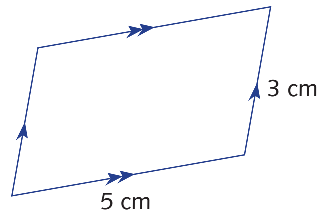
A ‘________’ is always a ‘________’ but a ‘________’ is not necessarily a ‘________’.

b Complete this sentence by inserting the word ‘square’ or ‘rectangle’.

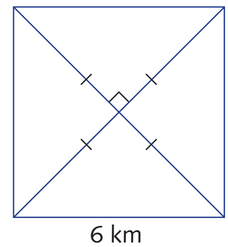
A ‘________’ is always a ‘________’ but a ‘________’ is not necessarily a ‘________’.

6 Find the perimeter of each parallelogram.

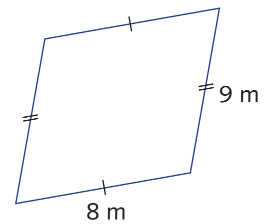
a



b



c



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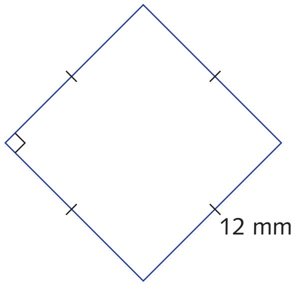
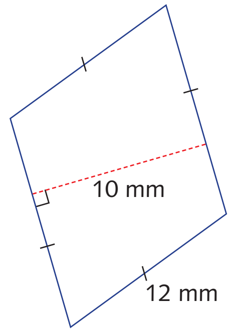
7 Find the area of each parallelogram.

a



b



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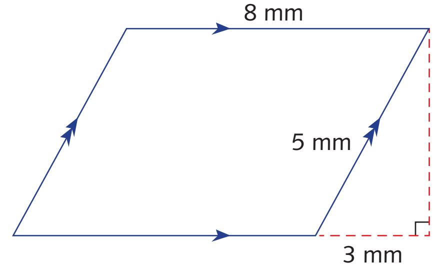


Chapter 14: Circles

1 Use your compasses, ruler and protractor to draw:

a a sector of a circle with diameter 10 cm and containing an angle of 130°

b a sector of a circle with radius 4 cm and containing an angle of 60°

c a semicircle with diameter 7 cm

d a quadrant with radius 3 cm

2 Find the circumferences of the two circles specified below. In each case, give the answer:

i in terms of π

ii as an approximate value, using $\pi \approx \frac{22}{7}$

iii as an approximate value, using $\pi \approx 3.14$

a Diameter 8 mm **b** Radius 3 m

3 Find the approximate value of the area of each of the circles in Question **2**, using $\pi \approx \frac{22}{7}$.

4 Find, for a sector with radius 12 cm and containing an angle of 72°:

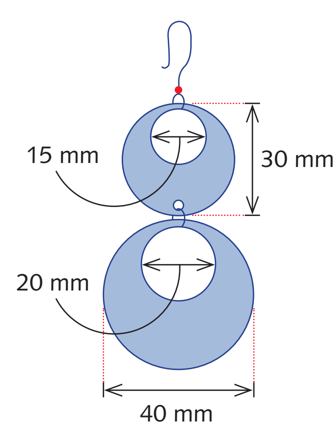
a the perimeter of the sector

b the area of the sector

Give each answer in terms of π.

5 Find the area of the annulus formed by two concentric circles, one of diameter 9 m and the other of diameter 5 m. Leave your answer in terms of π.

6 Find the approximate value of the area of plastic, represented by the shaded area in the figure opposite, needed to make the piece of jewellery shown. Use $\pi \approx 3.14$.



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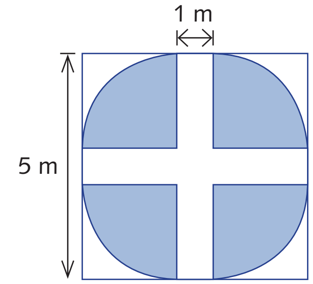
7 Clive has a square area of garden with side length 5 m. He is planning to design a vegetable garden with two straight paths, each of width 1 m, across the garden. The paths are to be at right angles to each other, as shown. The diagram is symmetric. He also plans to make four vegetable garden beds in the shape of quadrants, as shown, and to put pavers in the rest of the garden.

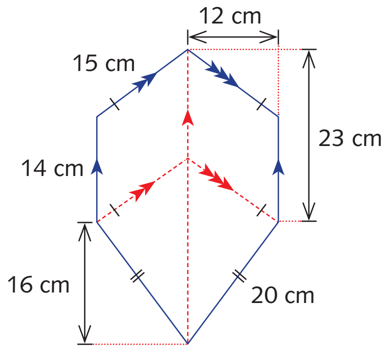
What will be the value of the area of the vegetable garden beds once the paths and pavers have been laid? Give your answer in square metres, in terms of π.



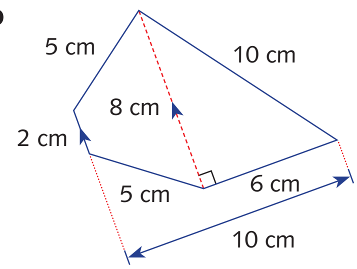
Chapter 15: Areas, volumes and time

1 Find the perimeter and area of each of these figures.
Give answers to **d**, **e** and **f** in terms of π.

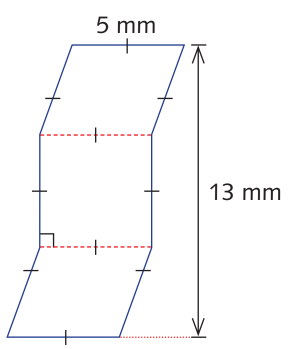
a



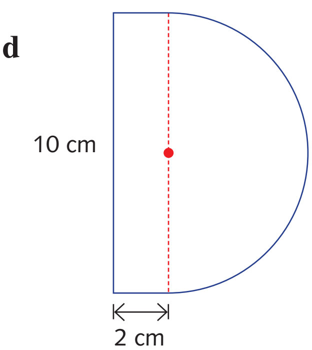
b



c

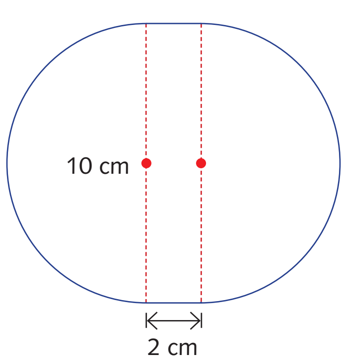


d



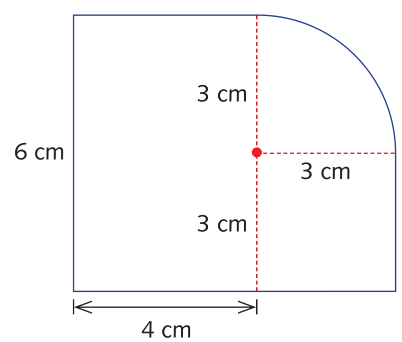
A semicircle and rectangle

e



Two semicircles and rectangle

f



Quadrant of circle and rectangles

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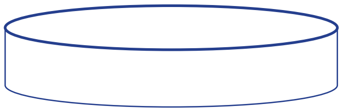
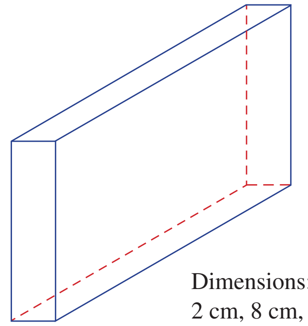
2 Find the volumes of these solids. Each solid is a prism or a cylinder.
Give your answer for **a** and **d** in terms of π.

a



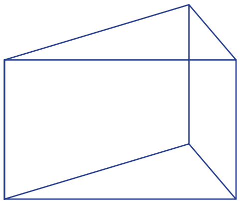
Radius 6 cm
Height 2.1 cm

b



Dimensions:
2 cm, 8 cm, 12 cm

c

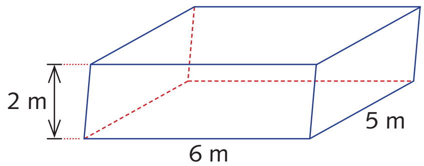


Area of cross-section 5 cm^2
Height 6.2 cm

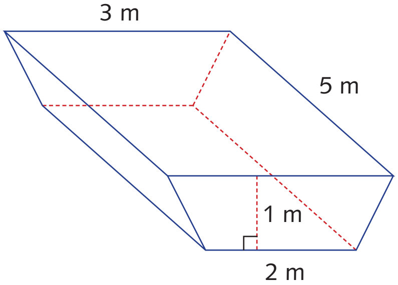
d

Radius 2 cm
Length 5 cm

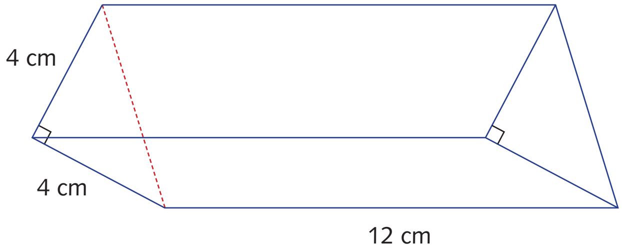
e



f



3 Find the surface area and volume of the prism shown below. The cross-section is a right-angled isosceles triangle.



Chapter 16: Probability

1 A bag contains three red beads, two blue beads and four yellow beads. If one bead is drawn at random from the bag, find the probability that the bead is:

a red

b red or blue

c red or yellow or blue

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2 A two-digit number – that is, a number from 10 to 99 – is chosen at random. Find the probability that the number:

a is greater than 72

b contains the digit 6 at least once (that is, a number such as 64, or 16 or 66)

3 A letter is chosen at random from the word RANDOM.

a List the sample space for this experiment.

b i If E is the event ‘the letter chosen is a vowel’, write down the outcomes that satisfy E.

ii What is the probability of E occurring?

4 The numbers 1 to 25 are written on 25 cards. If a card is selected at random, find the probability that the number on the card is:

a 14

b greater than 20

c even

d a multiple of 5

e odd and a multiple of 3

f neither even nor a multiple of 5

5 In a group of 200 students, 60 study geography, 80 study economics and 70 study neither.

a Represent this information on a Venn diagram.

b If a student is selected at random from the group, what is the probability that the student studies:

i geography?

ii geography and economics?

iii at least one of these subjects?

6 The table shows the type of accommodation and car ownership status of 150 university students.

Accommodation	**Car**	**No car**
Residential college	10	50
Apartment off campus	30	40
Living at home	15	5
Total	**55**	**95**

For a randomly selected student from this university, what is the probability that the student:

a lives in a residential college and has a car?

b has a car?

c lives at home and has a car?

d has no car?

e lives in an apartment off campus and does not have a car?

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Chapter 17: Formulas and factorisation

1 Find the value of each of these expressions for $a = -4$.

a $10 + 2a$ **b** $5 - 2a$ **c** $94 + a^2$

2 Given that $y = 3x - 5$, find the value of y when $x = 5$.

3 Given that $v = u + at$, find the value of:

a v when $u = 4$, $t = 12$ and $a = 6$

b a when $v = 24$, $u = 4$ and $t = 10$

c t when $v = 5$, $u = 12$ and $a = 14$

4 A rectangle is 4 m longer than it is wide. Let the width be w metres.

a Write an expression in terms of w for the length, ℓ, of the rectangle.

b Write an expression in terms of w for the perimeter, P, of the rectangle. Find P and ℓ when $w = 5$.

c Write an expression for the area, A, of the rectangle.

d Find A when $w = 3$.

e Find w when $P = 32$.

f Find w when $A = 45$.

5 Find the highest common factor of each pair of terms.

a $8, 4$ **b** $5x, 5y$ **c** $14x, 63x$ **d** $17y,\ 51y^2$

e $18ab^2, 9ab$ **f** $15x^2y, 25yx^2$ **g** $27a^2b^2, 9a^2b$ **h** $8a^3, 9b^2$

6 Factorise by taking the highest common factor.

a $3y + 6$ **b** $8m - 56$ **c** $-2 + 44p$

d $80y + 5$ **e** $-12z + 36$ **f** $5x^2 - x$

g $12y^2 + 4$ **h** $-42x + 36x^2$ **i** $12ab^2 + 2ab$

j $8a + 4ab$ **k** $10xy + 5xy^2$ **l** $25mnp - 5m^2p$

7 Expand:

a $2(m + 3)$ **b** $-2(3x - 4)$ **c** $-4(3x - 2)$

d $(3x + 2)(x - 5)$ **e** $a(a^2 + 3)$ **f** $(2 - x)(2 + x)$

g $(2z + 1)(2z + 4)$ **h** $(2z - 3)(2z + 3)$ **i** $(x + 2)^2$

j $(2b + 3)^2$ **k** $(2s - 6)(2s + 3)$ **l** $(2a + b)(2a - 3b)$

8 Factorise:

a $x^2 + 6x + 9$ **b** $a^2 + 4a + 4$ **c** $x^2 - 7x + 12$

d $x^2 + 4x + 3$ **e** $a^2 - 4a + 4$ **f** $x^2 + 9x + 14$

g $x^2 - 5x - 14$ **h** $a^2 - 2a - 24$ **i** $a^2 - 5a + \frac{25}{4}$

j $x^2 - 7x + 6$ **k** $x^2 - x - 30$ **l** $x^2 - 4x - 21$

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Chapter 18: Graphing straight lines

1 Complete each table of values for the given formula. List the coordinates of the points you get from the table and plot them on a number plane, drawing a line through the points.

a $y = -3x$

x	–3	–2	–1	0	1	2	3
y							

b $y = 3x - \frac{1}{2}$

x	–3	–2	–1	0	1	2	3
y							

2 **a** On a single set of axes, draw the graphs of:

i $y = 3x$ **ii** $y = 3x + 2$ **iii** $y = 3x - 1$

b State the gradient of each of the graphs in part **a**.

3 **a** On a single set of axes, draw the graphs of:

i $y = x$ **ii** $y = 4x$ **iii** $y = -2x$

iv $y = \frac{1}{2}x$ **v** $y = -\frac{3}{4}x$

b For each line in part **a**, find the points on the line with x values of 1 and 2, and hence find the gradient of the line.

c For each line in part **a**, state whether the line slopes upwards or downwards.

d Which of the lines in part **a** has:

i the steepest upwards slope? **ii** the steepest downwards slope?

4 **a** On a single set of axes, draw the graphs of:

i $y = 4x + 1$ **ii** $y = -2x + 1$ **iii** $y = \frac{2}{3}x + 1$

b State the value of the slope m for each of the lines in part **a**.

5 Calculate the slope of the line that passes through the points shown in the table below.

x	–3	–2	–1	0	1	2	3
y	6	3	0	–3	–6	–9	–12

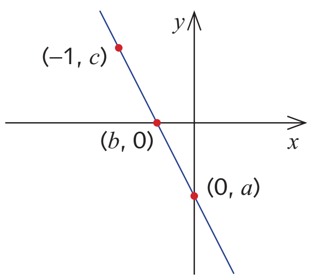
6 For the straight-line graph of $y = 3x - 2$, find the y-coordinate of the point on the line with x-coordinate:

a $x = -2$ **b** $x = 0$ **c** $x = 12$

7 Check whether or not each of these points lies on the line with equation $y = -5x + 2$.

a $(2, 8)$ **b** $(-2, 8)$ **c** $(2, -8)$ **d** $(-2, -8)$

8 The graph of $y = -4x - 2$ is shown opposite. Find the values of a, b and c.



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9 **a** The x -coordinate of a particular point on the line $y = 7 - 2x$ is 8. Write down the y-coordinate.

b The y-coordinate of a particular point on the line $y = 5 + 3x$ is 17. Write down the x-coordinate.

10 If the points $(2, a)$, $(-1, b)$ and $(c, -6)$ lie on the line with equation $y = -\frac{3}{4}x$, find the values of a, b and c.

11 Find the y-intercept of each of these lines by substituting $x = 0$.

a $y = 8x + 1$ **b** $y = -4x - 3$ **c** $y = -\frac{1}{2}x + 5$

12 Complete the table of values and plot the graph for each of the given formulas, drawing a smooth curve through the points.

a $y = 4 - x^2$

x	–3	–2	–1	0	1	2	3
y							

b $y = (x + 1)^2$

x	–3	–2	–1	0	1	2	3
y							

c $y - x^3 - 1$

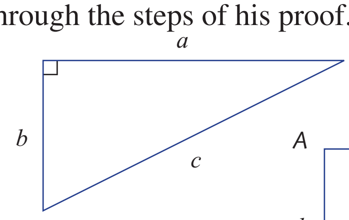
x	–2	–1	0	1	2
y					

20B Problem-solving

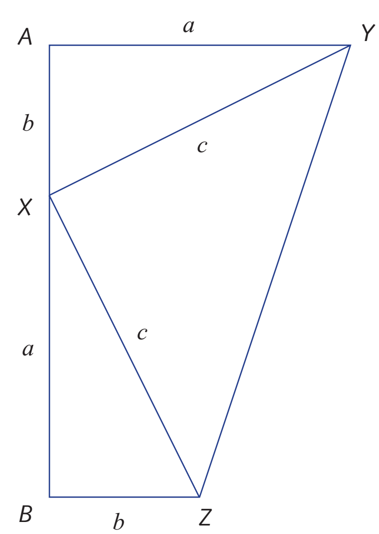
Pythagoras' theorem

The American president James Garfield (1831–1881) gave a simple proof of Pythagoras' theorem. In this problem, you will be working through the steps of his proof.

Begin by drawing a right-angled triangle with side lengths a, b and c, as shown opposite.



Now draw the triangle again to form a trapezium, as shown opposite.



1 Explain why this shape is a trapezium.

2 Explain why $\angle YXZ$ is a right angle.

3 Write down the area of the trapezium, in terms of a and b, using the formula for the area of a trapezium.

4 Find the area of the trapezium, in terms of a, b and c, by adding up the areas of the three triangles.

5 Explain why $\frac{1}{2}(a + b)(a + b) = \frac{1}{2}(2ab + c^2)$.

6 Use the result in step **5** to prove Pythagoras' theorem.

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Scones

A chef has a large ball of dough. Its volume is 5000π cm^3. The chef intends to roll the dough out into a circular slab of thickness 2 cm, and then cut it into as many scones as she can, using a round cutter of diameter 5 cm.

1 Find the diameter of the rolled-out slab of dough.

2 Find how many scones the chef will be able to make using the dough from the dough-ball, assuming that when she finishes cutting out the first lot of scones, she collects the scraps of dough, rolls them out again and keeps cutting, and that she continues this process until there is either no dough left or not enough to make another scone. (The thickness is always 2 cm.)

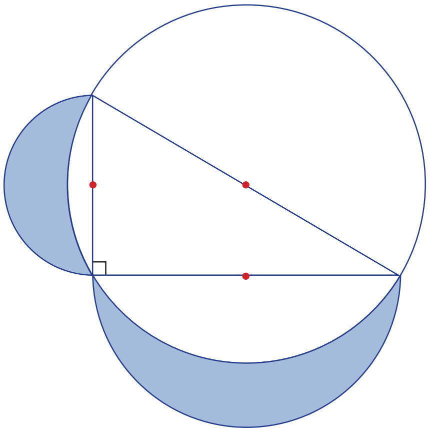
A litre

We wish to build a rectangular prism of height 10 cm and volume 1000 cm^3 (= 1 L). All dimensions are to be integer number of centimetres. One such container is a cube with edges 10 cm long. Find all the others.

Circles

Start with a right-angled triangle. Construct three circles, using the three sides as diameters so that each circle has the midpoint of a side as its centre, as shown opposite. Assume that the area of the triangle is 36 cm^2. Find the combined area of the shaded regions of the two smaller circles that lie outside the largest circle.

What is surprising about the result?



A quadrilateral

We are given a quadrilateral and are told that three of its sides are of equal length and the fourth side is the same length as each of the diagonals. Find the interior angles of the quadrilateral.

Unusual numbers

1 What number gives the same answer when it is subtracted from 10 as it does when it is multiplied by 10?

2 What number gives the same answer when it is divided by 10 as it does when it is added to 10?

3 What numbers give the same result when they are divided into 10 as they do when they are multiplied by 10? (There are two such numbers.)

4 What numbers give the same result when they are divided into 10 as they do when they are subtracted from 10? (Again, two numbers are possible. Give your answers correct to three decimal places. Some trial and error may be needed here.)

Which fits better?

Which fits better – a square peg in a round hole or a round peg in a square hole? Explain your assumptions and answer mathematically.

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Racing and chasing

A runaway dog, called Fido, is running at 3 m/s and is being chased by his owner, who is riding her bicycle at 5 m/s. How long will it be before she catches up with the dog?

The answer to this question is: 'It all depends on how far she is behind Fido to start with'. Obviously, if the distance between them is 100 m she will take a lot longer to catch up than if she starts only 20 m behind. Five times as long, in fact.

This question is one involving the concept of **closing speed**. By finding the difference between the front-runner's speed and the chaser's speed, the question can be solved very easily. It makes use of the fact that, since speed = distance ÷ time, then it follows that time = distance ÷ speed.

The owner is cycling at (5 – 3) m/s = 2 m/s faster than the dog. This is the speed at which she is closing the distance between herself and Fido. You can imagine that the distance between them is being 'eaten up' at the rate of 2 m/s. If you do this, it is obvious that, if Fido is 100 metres ahead when the chase begins, he will be 98 m ahead after 1 second, 96 m ahead after 2 seconds, and after (100 m) ÷ (2 m/s) = 50 s, he will be 0 m ahead. With a 20-m head start, the chase will only last 10 seconds.

We can also use a similar idea to find out where the chase ends. Since the chase lasts for 50 seconds, the owner will travel (50 s) × (5 m/s) = 250 m until she catches the dog. (Fido will have run (50 s) × (3 m/s) = 150 m, which, when added to his 100 m start, will also be 250 m from where his owner started the chase.)

Try these questions, using similar ideas.

1 In a cross-country race, Manus sees a friend 70 m ahead of him. He knows that his friend runs at a steady speed of 2.8 m/s in races of this length. Manus wishes to catch up with him, so he speeds up to 3.2 m/s.

a How long will it take Manus to catch up with his friend?

b If Manus is a kilometre from the finish when he sets out after his friend, how far from the finish line will he overtake him?

2 The leading driver in an Indy car race is having engine trouble. He has had to cut his speed to 140 km/h but is 500 m ahead of the car in second place. The driver in second place maintains a speed of 150 km/h. How long does it take for the second driver to catch up with the leader?

3 At 7:30 a.m. a moving van (van 1) leaves Melbourne and heads for Mildura, 550 km away. At 10 a.m., another van (van 2) leaves Bendigo, 150 km from Melbourne on the Melbourne–Bendigo–Mildura road. It averages 90 km/h. Van 1 travels at an average speed of 80 km/h, but stops for lunch for an hour at Sea Lake, 360 km from Melbourne. Van 2 fills up at Ouyen, 100 km before Mildura. This takes 40 min.

Draw up a time/place table showing how far from Melbourne each van is at the times when they start, stop and pass each other. Then answer these questions.

a How far ahead is van 1 when van 2 leaves Bendigo?

b When and where does van 2 pass van 1?

c When does van 2 leave Ouyen, and when it does, where is van 1?

d At what time does each van reach Mildura, to the nearest minute?

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Closing the gap

When objects are travelling towards each other, the distance between them gets 'eaten up' quickly. If a car is travelling at 100 km/h towards a truck that is travelling at 60 km/h, the car and truck have a closing speed of 160 km/h. In other words, the vehicles will be 160 km closer to each other after 1 hour, provided they are at least 160 km apart in the first place. When two objects are travelling towards each other, their closing speed is found by adding their individual speeds.

1 A car and a truck are approaching each other on a straight road. They are 20 km apart and the truck is travelling at 60 km/h. Find how long it takes for the car to pass the truck if the car maintains a steady speed of:

a 90 km/h

b 100 km/h

c 80 km/h.

2 Jill and Sophie both leave home at 8:25 a.m. and walk to school. Jill walks at a speed of 75 m/min and Sophie takes her time and walks at 60 m/min. They meet at school at 8:40 a.m.

a How far does each girl live from the school?

b Suppose Jill and Sophie start at two points on the same road and walk towards each other at the above speed and from the same times. How far apart are the points?

3 A meteor is heading straight for Earth at a speed of 90 000 km/h. A rocket is launched towards the meteor at a speed through space of 20 000 km/h. If it takes the rocket two days to collide with and destroy the meteor, how far from Earth is it when the rocket is launched, and how far from Earth does the collision take place?

20C Fibonacci sequences

A Fibonacci sequence is a sequence $F_1, F_2, F_3, \ldots$ of numbers in which each term from the third one onwards is the sum of the two terms that immediately precede it. You have to have two numbers to start with, F_1 and F_2. These are called the **seeds**. Then:

$$F_3 = F_2 + F_1,$$

$$F_4 = F_3 + F_2,$$

and so on. The classic Fibonacci sequence has 1 and 1 as its seeds. Its first 10 terms are:

$$1, 1, 2, 3, 5, 8, 13, 21, 34 \text{ and } 55$$

Use a calculator where appropriate in the following.

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Activity 1

Write out the classic Fibonacci sequence as far as its 25th term, F_{25}. Before you calculate F_{11}, make a rough guess of what the value of F_{25} will be. See how good your guess turns out to be.

Activity 2

Pick any two numbers as seeds and work out the first 20 terms for that Fibonacci sequence. Pick entirely different seed numbers from the person beside you, and keep your list at least reasonably neat, as we will be coming back to it in a little while.

Activity 3

Swap your two seed numbers from Activity 2 around and figure out the first 20 terms in the new Fibonacci sequence. (If, for example, your sequence in Activity 2 started 6, 11, 17, 28, 45, …, your new sequence will start 11, 6, 17, 23, 40, …) Yes, you do get quite different numbers from the ones in Activity 2.

Activity 4

It is now time to make a few observations about your Fibonacci sequences.

- The classic sequence (in Activity 1) has two odd seeds. This gives a certain pattern of odd and even terms throughout the sequence. What happens if you start with two even seeds or an odd and an even seed? Explain.
- Compare the 10th terms in each of the sequences you generated in Activities 2 and 3. Which one is larger? Compare the 20th terms as well. Can you explain what is happening?
- Use a calculator to divide the term F_{10} in the first sequence by the term F_9 immediately before it. Write your answer down. Then do the same with the second sequence. Now repeat the calculations but with F_{20} and F_{19} for both sequences. Do you notice anything interesting? Did any of the other students who are doing this activity get the same number? They should all have found the same answer, although there may be very small differences in the sixth decimal places.
- For the classic Fibonacci sequence, the first two terms larger than 1000000 are $F_{32} = 1\,346\,269$ and $F_{33} = 2\,178\,309$. Use these two values to see if what you noticed in the previous ratio calculations also holds for high-order terms in the classic Fibonacci sequence.

The number you obtained (to a good approximation) in the ratio calculations is famous and interesting enough to deserve its own Greek letter. It is called Φ (*phi*, pronounced to rhyme with 'spy') and is known as the **golden mean** or **golden ratio**. It is a very interesting number with a long history.

Search Google and you will discover some amazing facts about Φ. It appears in many different ways in geometry and architecture.

Now try calculating these values and see what you notice about them.

a Φ^2 **b** $\frac{1}{\Phi}$ **c** $(2\Phi - 1)^2$

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