CHAPTER

# 1

Number and Algebra

# Algebra

Algebra is the language of mathematics. The development of algebra revolutionised mathematics. It is used by engineers, architects, applied scientists and economists to solve practical problems. Every time you see a formula, algebra is being used.

We are now going to review and consolidate our knowledge of algebra so that we will be able to use it in more advanced work.


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# 1A Substitution

Substitution occurs when a pronumeral is replaced by a numerical value. This allows the term or expression to be evaluated.

When evaluating an expression, it is important to remember the order in which operations are to be applied.

### Order of operations

- Evaluate expressions inside brackets first.
- In the absence of brackets, carry out operations in the following order:
  - powers
  - multiplication and division from left to right
  - addition and subtraction from left to right.

### Example 1

**a** Evaluate $2x$ when $x = 3$.

**b** Evaluate $5a + 2b$ when $a = 2$ and $b = -3$.

**c** Evaluate $2p(3p - q)$ when $p = 1$ and $q = -2$.

### Solution

**a** $2x = 2 \times 3$
$= 6$

**b** $5a + 2b = 5 \times 2 + 2 \times (-3)$
$= 10 - 6$
$= 4$

**c** $2p(3p - q) = 2 \times 1 \times [3 \times 1 - (-2)]$
$= 2 \times 5$
$= 10$

## Exercise 1A

Example 1a

**1** Evaluate $3a$ when $a = 7$.

**2** Evaluate $12x$ when $x = -5$.

Example 1b

**3** Evaluate $7m - 4n$ when:

**a** $m = 3$ and $n = 5$

**b** $m = 2$ and $n = -5$

**c** $m = -1$ and $n = 2$

**d** $m = -3$ and $n = -2$


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**4** Evaluate $a+2b-3c$ when:

**a** $a = 3, b = 5$ and $c = 2$

**b** $a = 3, b = -5$ and $c = 2$

**c** $a = -3, b = 5$ and $c = 2$

**d** $a = 3, b = 5$ and $c = -2$

**e** $a = -3, b = -5$ and $c = 2$

**f** $a = 3, b = -5$ and $c = -2$

**5** Evaluate $2x - 3y$ when:

**a** $x = \frac{1}{2}$ and $y = \frac{3}{4}$

**b** $x = -\frac{1}{3}$ and $y = \frac{1}{6}$

**c** $x = \frac{2}{5}$ and $y = -\frac{1}{4}$

**d** $x = -\frac{2}{3}$ and $y = -\frac{3}{4}$

**6** Evaluate $a+2b-3c$ when:

**a** $a = 0.3, b = 0.5$ and $c = 0.2$

**b** $a = 1.3, b = -0.5$ and $c = 1.2$

**c** $a = -2.3, b = 1.5$ and $c = 0.2$

**d** $a = 2.3, b = 2.5$ and $c = -2.3$

**7** Evaluate $p^2 - 2q$ when:

**a** $p = 3$ and $q = 2$

**b** $p = 7$ and $q = -2$

**c** $p = -7$ and $q = 2$

**d** $p = -7$ and $q = -2$

**e** $p = -\frac{1}{3}$ and $q = \frac{5}{6}$

**f** $p = -\frac{2}{5}$ and $q = \frac{1}{4}$

Example 1c

**8** Evaluate $2m(m - 3n)$ when:

**a** $m = 3$ and $n = 5$

**b** $m = 2$ and $n = -5$

**c** $m = -1$ and $n = 2$

**d** $m = -3$ and $n = -2$

**e** $m = \frac{1}{2}$ and $n = \frac{1}{3}$

**f** $m = \frac{1}{3}$ and $n = \frac{1}{2}$

**9** Evaluate $\frac{x+y}{3}$ when:

**a** $x = 6$ and $y = 5$

**b** $x = -6$ and $y = 5$

**c** $x = 6$ and $y = -5$

**d** $x = -6$ and $y = -5$

**10** Evaluate $\frac{p+2q}{3r}$ when:

**a** $p = 7, q = 4$ and $r = 2$

**b** $p = 7, q = -2$ and $r = 2$

**c** $p = -7, q = 4$ and $r = 4$

**d** $p = -7, q = -2$ and $r = 2$


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# 1B Like terms

Demitri has three pencil cases, each containing the same number, $x$, of pencils. So he has a total of $3x$ pencils. If he gets two more pencil cases with $x$ pencils in each, then he has $3x + 2x = 5x$ pencils in total.

$3x$ and $2x$ are said to be **like terms**.

If Lee has $x$ packets of chocolates each containing $y$ chocolates, then she has $x \times y = xy$ chocolates. If David has twice as many chocolates as Lee, he has $2 \times xy = 2xy$ chocolates.

Together they have $2xy + xy = 3xy$ chocolates.

$2xy$ and $xy$ are **like terms**. The pronumerals are the same and have the same indices. The **coefficients** of these terms are 2 and 1.

The terms $2x$ and $3y$ are not like terms.

Similarly, the terms $2x^2y$ and $3xy^2$ are not like terms since the indices of $x$ and $y$ differ.

### Example 2

Which of the following are pairs of like terms?

**a** $3x$ and $2x$ **b** $3m$ and $2n$ **c** $3x^2$ and $3x$

**d** $2x^2y$ and $3yx^2$ **e** $2mn$ and $3nm$ **f** $5^2y$ and $6y^2x$

### Solution

**a** $3x$ and $2x$ are like terms.

**b** $3m$ and $2n$ are not like terms. (The pronumerals are different.)

**c** $3x^2$ and $3x$ are not like terms. (The indices of $x$ are different.)

**d** $2x^2y$ and $3yx^2$ are like terms. (The multiplication order is not important.)

**e** $2mn$ and $3nm$ are like terms. (The multiplication order is not important.)

**f** $5x^2y$ and $6y^2x$ are not like terms. ($x^2y$ is not the same as $xy^2$.)

### Like terms

- **Like terms** contain the same pronumerals, with each pronumeral having the same index.
- We can add and subtract like terms.
- Unlike terms cannot be added or subtracted to form a single term.


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Consider the following examples.

### Example 3

Simplify each expression, if possible.

**a** $4a + 7a$ **b** $3x^2y + 4x^2y - 2x^2y$ **c** $5m + 6n$

**d** $9b + 2c - 3b + 6c$ **e** $3z + 5yx - z - 6xy$ **f** $6x^3 - 4x^2 + 5x^3$

### Solution

**a** $4a + 7a = 11a$

**b** $3x^2y + 4x^2y - 2x^2y = 5x^2y$

**c** $5m + 6n$. No simplification is possible because $5m$ and $6n$ are unlike terms.

**d** $9b + 2c - 3b + 6c = 9b - 3b + 2c + 6c$
$= 6b + 8c$

**e** $3z + 5yx - z - 6xy = 3z - z + 5xy - 6xy$
$= 2z - xy$

**f** $6x^3 - 4x^2 + 5x^3 = 6x^3 + 5x^3 - 4x^2$
$= 11x^3 - 4x^2$

### Example 4

Simplify each expression.

**a** $\frac{x}{2} + \frac{x}{3}$ **b** $\frac{3x}{4} - \frac{2x}{5}$

### Solution

**a** $\frac{x}{2} + \frac{x}{3} = \frac{3x}{6} + \frac{2x}{6}$
$= \frac{5x}{6}$

**b** $\frac{3x}{4} - \frac{2x}{5} = \frac{15x}{20} - \frac{8x}{20}$
$= \frac{7x}{20}$

## Exercise 1B

Example 2

**1** Which of the following are pairs of like terms?

**a** $11a$ and $4a$ **b** $6b$ and $-2b$ **c** $12m$ and $5m$

**d** $14p$ and $5p$ **e** $7p$ and $-3q$ **f** $-6a$ and $7b$

**g** $4mn$ and $7mn$ **h** $3pq$ and $-2pq$ **i** $6ab$ and $-7b$

**j** $4ab$ and $5a$ **k** $11a^2$ and $5a^2$ **l** $6x^2$ and $-7x^2$


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**m** $4b^2$ and $-3b$ **n** $2y^2$ and $2y$ **o** $-5a^2b$ and $9a^2b$

**p** $6mn^2$ and $11mn^2$ **q** $6\ell^2s$ and $17\ell s^2$ **r** $12d^2e$ and $14de^2$

**s** $-4x^2y^2$ and $12x^2y^2$ **t** $-5a^2bc^2$ and $8a^2bc^2$ **u** $6a^2bc$ and $6a^2bc^2$

Example 3a, b

**2** Simplify each expression by collecting like terms.

**a** $7a + 2a$ **b** $8b + 3b$ **c** $11c - 6c$

**d** $15d - 8d$ **e** $6x^2 + 4x^2$ **f** $9a^2 - 6a^2$

**g** $4a + 5a - 6a$ **h** $7f - 3f + 9f$ **i** $6m - 4m - 9$

**j** $8p + 10p - 17p$ **k** $9a^2 + 5a^2 - 12a^2$ **l** $17m^2 - 14m^2 + 8m^2$

**m** $6ab + 9ab - 12ab$ **n** $14a^2d - 10a^2d - 6a^2d$

**3** Copy and complete:

**a** $2a + ... = 7a$ **b** $5b - ... = 2b$ **c** $8mn + ... = 12mn$

**d** $11pq - ... = 6pq$ **e** $4x^2 + ... = 7x^2$ **f** $6m^2 - ... = m^2$

**g** $8ab - ... = -2ab$ **h** $5m^2n - ... = -6m^2n$ **i** $-7a^2b + ... = a^2b$

**j** $-6\ell m + ... = \ell m$

Example 3d, e

**4** Simplify each expression by collecting like terms.

**a** $5c - 2d + 8c + 6d$ **b** $7m + 2n + 5m - n$ **c** $8p + 6 + 3p - 2$

**d** $9 - 2m + 5 + 7m$ **e** $8m - 6n - 2n - 7m$ **f** $10ab + 11b - 12b + 3ab$

**g** $6a^2 - 2 + 7ab - 9b$ **h** $-3x^2 + 5x - 2x^2 + 7x$ **i** $4p^2 - 3p - 8p - 3p^2$

Example 3f

**5** Simplify each expression by collecting like terms.

**a** $19xy + 6yx - 4xy$ **b** $8xy^2 + 9xy^2 - y^2x$ **c** $-4x^2 + 3x^2 - 3y - 7y$

**d** $6v^2z - 11z + 7v^2z - 14z$ **e** $6yz - 11x + 10zy + 15x$ **f** $7x^3 + 6x^2 - 4y^3 - x^2$

**g** $8x^2 - 12x^2 + y^2 + 12y^2$ **h** $2x^2 + x^2 - 5xy + 7yx$ **i** $-3ab^2 + 4a^2b - 5ab^2 + a^2b$

Example 4a

**6** Simplify each expression.

**a** $\frac{x}{4} + \frac{x}{3}$ **b** $\frac{a}{2} + \frac{a}{5}$ **c** $\frac{c}{6} + \frac{c}{7}$

**d** $\frac{z}{3} + z$ **e** $\frac{c}{5} + \frac{c}{10}$ **f** $\frac{x}{4} + x$

**7** Simplify each expression.

**a** $\frac{z}{2} - \frac{z}{3}$ **b** $\frac{z}{3} - \frac{z}{5}$ **c** $\frac{x}{7} - \frac{x}{8}$

**d** $x - \frac{x}{3}$ **e** $\frac{x}{4} - \frac{x}{8}$ **f** $c - \frac{c}{7}$


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Example 4b

**8** Simplify each expression.

**a** $\frac{2x}{3} + \frac{x}{4}$ **b** $\frac{5x}{7} + \frac{x}{3}$ **c** $\frac{3x}{4} + \frac{x}{2}$

**d** $\frac{5x}{3} + \frac{x}{2}$ **e** $\frac{7x}{11} - \frac{x}{2}$ **f** $\frac{2x}{3} - \frac{x}{2}$

**g** $\frac{5x}{11} - \frac{2x}{3}$ **h** $\frac{7x}{3} + \frac{5x}{4}$ **i** $2x - \frac{7x}{4}$

**9** In each part, the expression in each box is obtained by adding the expressions in the two boxes directly below it. Fill in the contents of each box. (The first one has been done for you.)

**a**

| | | |
|---|---|---|
| | $9a + 14b$ | |
| $5a + 5b$ | | $4a + 9b$ |
| $2a + b$ | $3a + 4b$ | $a + 5b$ |

**b**

| | | |
|---|---|---|
| | | |
| | | |
| $m + 3n$ | $2m + n$ | $4m + 3n$ |

**c**

| | | |
|---|---|---|
| | | |
| | | |
| $2p - 3q$ | $4p + q$ | $5p - 2q$ |

**d**

| | | |
|---|---|---|
| | | |
| | | $6a + 2b$ |
| $3a + 2b$ | | $4a + b$ |

**e**

| | | |
|---|---|---|
| | $11x + 2y$ | |
| | | $6x - 3y$ |
| $4x - y$ | | |

**f**

| | | |
|---|---|---|
| | $7d + e$ | |
| | | $4d - 5e$ |
| | $5d + 3e$ | |

# 1C Multiplication and division

Any two terms, like or unlike, can be multiplied together to produce a single term. This is different from addition and subtraction, where only like terms can be combined into a single term.

For a single term, it is conventional to use alphabetical order when you simplify your answer.

For example, write $3abc$ instead of $3bca$, and write $4x^2yz$ instead of $4yx^2z$.

### Example 5

Simplify each expression.

**a** $4 \times 3a$ **b** $2d \times 5e$ **c** $4m \times 5m$

**d** $3p \times 2pq$ **e** $3x \times (-6)$ **f** $-5ab \times (-3bc)$


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**Solution**

**a** $4 \times 3a = 12a$ **b** $2d \times 5e = 10de$ **c** $4m \times 5m = 20m^2$

**d** $3p \times 2pq = 6p^2q$ **e** $3x \times (-6) = -18x$ **f** $-5ab \times (-3bc) = 15ab^2c$

**Example 6**

Simplify each expression.

**a** $24x \div 6$ **b** $3 \times 12a \div 4$ **c** $-18x^2 \div (-3)$

**Solution**

**a** $24x \div 6 = 4x$

**b** $3 \times 12a \div 4 = 36a \div 4$
$= 9a$

**c** $-18x^2 \div (-3) = 6x^2$

**Example 7**

Simplify each expression.

**a** $\dfrac{15a}{3}$ **b** $\dfrac{12x}{21}$ **c** $\dfrac{-24xy}{6y}$

**Solution**

**a** $\dfrac{15a}{3} = 5a$ **b** $\dfrac{12x}{21} = \dfrac{4x}{7}$ **c** $-\dfrac{24xy}{6y} = -4x$

**Multiplication and division**

- To multiply fractions, multiply the numerators and multiply the denominators.
- When the numerator and denominator have a common factor, the factor should be cancelled; that is, the numerator and the denominator should be divided by the common factor.

**Example 8**

Rewrite each expression as a single fraction.

**a** $\dfrac{2a}{5} \times \dfrac{a}{4}$ **b** $\dfrac{3x}{7} \times \dfrac{5y}{12}$ **c** $\dfrac{4p}{9} \times \dfrac{3}{2p}$ **d** $\dfrac{15}{x} \times \dfrac{2}{3x}$


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**Solution**

**a** $\frac{2a}{5} \times \frac{a}{4} = \frac{\cancel{2} \times a \times a}{5 \times \cancel{4}_{2}}$ (Multiply the numerators and the denominators.)

(Divide 2 into the numerator and the denominator.)

$= \frac{a \times a}{5 \times 2}$

$= \frac{a^2}{10}$

**b** $\frac{3x}{7} \times \frac{5y}{12} = \frac{\cancel{3} \times x \times 5 \times y}{7 \times \cancel{12}_{4}}$ (Multiply the numerators and the denominators.)

$= \frac{5 \times x \times y}{7 \times 4}$

$= \frac{5xy}{28}$

**c** $\frac{4p}{9} \times \frac{3}{2p} = \frac{{}^{2}\cancel{4} \times \cancel{p} \times \cancel{3}}{{}_{3}\cancel{9} \times \cancel{2} \times \cancel{p}}$

$= \frac{2}{3}$

**d** $\frac{15}{x} \times \frac{2}{3x} = \frac{{}^{5}\cancel{15} \times 2}{x \times \cancel{3} \times x}$

$= \frac{5 \times 2}{x \times x}$

$= \frac{10}{x^2}$

Recall that to divide by a fraction, we multiply by its reciprocal.

**Example 9**

Rewrite each expression as a single fraction.

**a** $\frac{2x}{3} \div \frac{3x}{5}$

**b** $\frac{6a}{7b} \div \frac{2ab}{3}$

**Solution**

**a** $\frac{2x}{3} \div \frac{3x}{5} = \frac{2\cancel{x}}{3} \times \frac{5}{3\cancel{x}}$

$= \frac{2 \times 5}{3 \times 3}$

$= \frac{10}{9}$

**b** $\frac{6a}{7b} \div \frac{2ab}{3} = \frac{{}^{3}\cancel{6}\,\cancel{a}}{7b} \times \frac{3}{\cancel{2}\,\cancel{a}b}$

$= \frac{3 \times 3}{7 \times b \times b}$

$= \frac{9}{7b^2}$

Your answer should always be expressed in simplest form – that is, the highest common factor of the numerator and denominator is one.


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## Exercise 1C

Example 5

**1** Simplify:

**a** $2 \times 3a$ **b** $4b \times 5$ **c** $4a \times 3b$ **d** $5c \times 2d$

**e** $4f \times (-5g)$ **f** $-3m \times 4n$ **g** $-2p \times (-3q)$ **h** $-6\ell \times (-5n)$

**i** $a \times a$ **j** $m \times m$ **k** $2a \times 4a$ **l** $-2m \times (-4m)$

**m** $7a \times 8ab$ **n** $2p \times 3pq$ **o** $-2mn \times 5n$ **p** $-6cd \times (-2de)$

**2** Copy and complete:

**a** $8a \times \ldots = 16a$ **b** $9b \times \ldots = 18b$ **c** $8a \times \ldots = 16ab$

**d** $5m \times \ldots = 15mn$ **e** $3a \times \ldots = 12a^2$ **f** $6p \times \ldots = 30p^2$

**g** $-5b \times \ldots = 10b^2$ **h** $-3\ell \times \ldots = 24\ell^2$ **i** $4m \times \ldots = 12m^2n$

**j** $4d \times \ldots = 28d^2e$ **k** $-3ab \times \ldots = 15ab^2c$ **l** $-2de \times \ldots = 10d^2ef$

Example 6

**3** Simplify:

**a** $15x \div 5$ **b** $27y \div 3$ **c** $24a^2 \div 8$

**d** $32m \div 16$ **e** $3 \times 12t \div 9$ **f** $7 \times 15p \div 21$

**g** $21t \div 12 \times 4$ **h** $24x \div 8 \times 3$ **i** $18y \div 6 \times 2$

**j** $-18x^2 \div 9$ **k** $-16a^2 \div (-4)$ **l** $-24x^2 \div (-8)$

Example 7

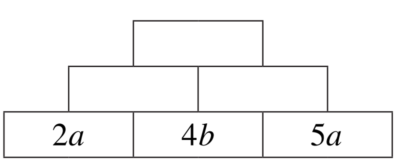
**4** Simplify each expression by cancelling common factors.

**a** $\frac{4x}{6}$ **b** $\frac{3a}{9}$ **c** $-\frac{12m}{18}$ **d** $\frac{14p}{21}$ **e** $\frac{22x^2}{33}$

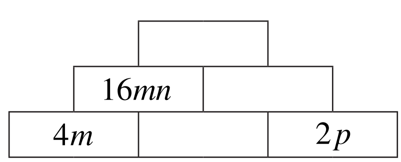
**f** $\frac{15xy}{20}$ **g** $-\frac{3ab}{b}$ **h** $\frac{12ab}{4a}$ **i** $\frac{2xy}{6xy}$ **j** $-\frac{4xy}{8x}$

**5** In each part, the expression in each box is obtained by multiplying together the terms in the two boxes directly below it. Fill in the empty boxes.

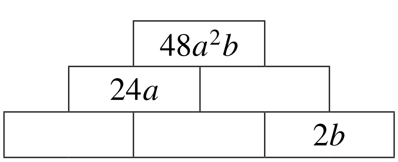
**a**



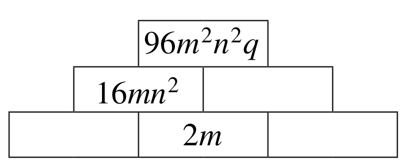

**b**




**c**




**d**





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Example 8

**6** Rewrite each expression as a single fraction. Cancel common factors first.

**a** $\frac{2b}{3} \times \frac{9}{4}$ **b** $\frac{3x}{5} \times \frac{2}{3}$ **c** $\frac{2y}{5} \times \frac{y}{4}$

**d** $\frac{3b}{4a} \times \frac{2ab}{9}$ **e** $\frac{2}{5a} \times \frac{1}{4a}$ **f** $\frac{p}{6q} \times \frac{9p}{4q}$

**g** $\frac{20x}{3y} \times \frac{6}{5xy}$ **h** $\frac{14mp}{9p} \times \frac{3np}{7p}$ **i** $\frac{2yz}{5xz} \times \frac{3xy}{4yz}$

Example 9

**7** Rewrite each expression as a single fraction. Simplify your answer by cancelling common factors.

**a** $\frac{2b}{3} \div \frac{4}{9}$ **b** $\frac{3x}{5} \div \frac{3}{4}$ **c** $\frac{2y}{5} \div \frac{y}{4}$

**d** $\frac{p}{6} \div \frac{9p}{4}$ **e** $\frac{5}{6a} \div \frac{1}{4a}$ **f** $\frac{p}{6q} \div \frac{4p}{9q}$

**g** $\frac{8x}{5} \div 4$ **h** $\frac{9y}{2} \div 18$ **i** $\frac{3a}{4} \div \frac{5a}{2}$

**j** $\frac{4m}{5n} \div \frac{12mn}{7}$ **k** $\frac{5p}{6} \div \left(-\frac{10p}{3}\right)$ **l** $-\frac{9y}{4xy} \div \frac{3x}{16y}$

# 1D Simple expansion of brackets

This section looks at how algebraic products that contain brackets can be rewritten in different forms. Removing brackets is called **expanding**.

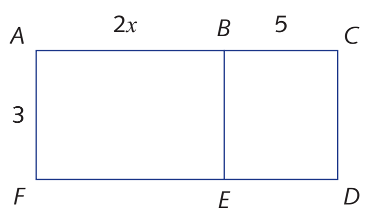
We are familiar with the distributive law for numbers from Year 7 and Year 8.

The distributive law can be illustrated using a diagram, as shown below.

Area of rectangle $ACDF$ = area of rectangle $ABEF$ + area of rectangle $BCDE$

That is, $3(2x + 5) = 3 \times 2x + 3 \times 5$

$= 6x + 15$



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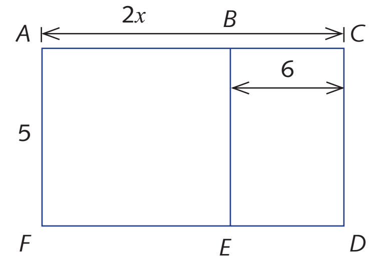
Area of rectangle $ABEF$ = area of rectangle $ACDF$ − area of rectangle $BCDE$

That is, $5(2x - 6) = 5 \times 2x - 5 \times 6$

$= 10x - 30$




## Simple expansion of brackets

- In general: $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$
- Each term in the brackets is multiplied by the term outside the brackets.

### Example 10

Expand the brackets.

**a** $2(a + 3)$ **b** $3(x - 2)$ **c** $4(2m - 7)$

**d** $p(p + n)$ **e** $-2(x - 4)$ **f** $-5(3x + y)$

#### Solution

**a** $2(a + 3) = 2a + 6$ **b** $3(x - 2) = 3x - 6$ **c** $4(2m - 7) = 8m - 28$

**d** $p(p + n) = p^2 + np$ **e** $-2(x - 4) = -2x + 8$ **f** $-5(3x + y) = -15x - 5y$

### Example 11

Expand and simplify each expression.

**a** $5(a + 1) + 6$ **b** $4(2b - 1) + 7$ **c** $6(d + 5) - 3d$

#### Solution

**a** $5(a + 1) + 6 = 5a + 5 + 6$
$= 5a + 11$

**b** $4(2b - 1) + 7 = 8b - 4 + 7$
$= 8b + 3$

**c** $6(d + 5) - 3d = 6d + 30 - 3d$
$= 3d + 30$

The distributive law can be used to simplify expressions that initially appear quite complicated.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## Example 12

Expand and collect like terms for each expression.

**a** $2(b + 5) + 3(b + 2)$

**b** $3(x - 2) - 2(x + 1)$

**c** $5(a + 1) - 2(a - 4)$

**d** $3a(2a + 3b) + 2a(3a - 2b)$

### Solution

**a** $2(b + 5) + 3(b + 2) = 2b + 10 + 3b + 6$
$= 5b + 16$

**b** $3(x - 2) - 2(x + 1) = 3x - 6 - 2x - 2$
$= x - 8$

**c** $5(a + 1) - 2(a - 4) = 5a + 5 - 2a + 8$
$= 3a + 13$

*Note*: When expanding the second bracket, be **very careful**. The $-2$ is multiplied by each term in the second bracket, and $-2 \times (-4) = +8$.

**d** $3a(2a + 3b) + 2a(3a - 2b) = 6a^2 + 9ab + 6a^2 - 4ab$
$= 12a^2 + 5ab$

## Example 13

Expand and simplify each expression.

**a** $\frac{3}{5}\left(6x + \frac{7}{3}\right)$

**b** $\frac{4}{3}(6x + 11) + \frac{2}{3}$

### Solution

**a** $\frac{3}{5}\left(6x + \frac{7}{3}\right) = \frac{3}{5} \times 6x + \frac{\cancel{3}^{1}}{5} \times \frac{7}{\cancel{3}_{1}}$
$= \frac{18x}{5} + \frac{7}{5}$

**b** $\frac{4}{3}(6x + 11) + \frac{2}{3} = \frac{4}{3} \times 6x + \frac{4}{3} \times 11 + \frac{2}{3}$
$= 8x + \frac{44}{3} + \frac{2}{3}$
$= 8x + \frac{46}{3}$

*Note*: It is neither necessary nor desirable to write $\frac{46}{3}$ as $15\frac{1}{3}$ in an algebraic expression.


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## Exercise 1D

Example 10a, b, c

**1** Expand each expression using the distributive law. For parts **a**, **e**, and **i** check your answers by substituting values for the pronumeral.

**a** $5(x + 3)$ **b** $2(b + 7)$ **c** $3(2a + 1)$

**d** $6(2d + 5)$ **e** $4(a - 7)$ **f** $3(b - 5)$

**g** $6(d - 2)$ **h** $8(f - 4)$ **i** $2(4f - 5)$

**j** $3(2g - 6)$ **k** $5(3p - 2)$ **l** $6(5q - 1)$

Example 10e, f

**2** Expand each expression using the distributive law. For parts **a**, **e**, **i** and **m** check your answers by substituting values for the pronumeral.

**a** $-2(a + 4)$ **b** $-3(b + 6)$ **c** $-6(2a + 7)$

**d** $-3(4b + 9)$ **e** $-2(3a - 1)$ **f** $-6(4b - 7)$

**g** $-5(2b - 7)$ **h** $-7(3b - 2)$ **i** $-4(3b - 5)$

**j** $-5(4b - 7)$ **k** $-9(3x + 2)$ **l** $-12(4y - 5)$

**3** Expand each expression using the distributive law.

**a** $\frac{1}{2}(2x + 6)$ **b** $\frac{1}{3}(9x + 18)$ **c** $\frac{2}{3}(12p + 6)$

**d** $\frac{3}{4}(20q + 40)$ **e** $\frac{1}{2}(6x - 24)$ **f** $-\frac{1}{2}(10\ell - 6)$

**g** $-\frac{1}{4}(16p - 20)$ **h** $-\frac{2}{3}(9m - 15)$ **i** $-\frac{4}{5}(25n - 100)$

**4** Expand:

**a** $\frac{2}{3}\left(6x + \frac{3}{4}\right)$ **b** $\frac{1}{2}\left(5y + \frac{2}{5}\right)$ **c** $\frac{3}{5}\left(\frac{x}{6} + \frac{1}{3}\right)$

**d** $\frac{2}{7}\left(\frac{3x}{4} + \frac{5}{6}\right)$ **e** $\frac{1}{3}\left(\frac{y}{2} - \frac{3}{4}\right)$ **f** $-\frac{3}{5}\left(\frac{a}{3} - \frac{2}{3}\right)$

Example 10d

**5** Expand:

**a** $a(a + 4)$ **b** $b(b + 7)$ **c** $c(c - 5)$

**d** $2g(3g - 5)$ **e** $4h(5h - 7)$ **f** $2i(5i + 7)$

**g** $3j(4j + 7)$ **h** $-k(5k - 4)$ **i** $-\ell(3\ell - 1)$

**j** $-2m(5m - 4)$ **k** $-3n(5n + 7)$ **l** $-4x(3x - 5)$

**6** Expand:

**a** $3a(2a + b)$ **b** $4c(2c - d)$ **c** $5d(2d - 4e)$

**d** $2p(3q - 5r)$ **e** $-3x(2x + 5y)$ **f** $-2z(3z - 4y)$


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**g** $2a(3 + 4ab)$ **h** $5m(2m - 4n)$ **i** $5x(2xy + 3)$

**j** $3p(2 - 5pq)$ **k** $-6y(2x - 3y$ **l** $-10b(3a - 7b)$

**7** A student expanded brackets and obtained the following answers. In each part, identify and correct the student's incorrect answer and write the correct expansion.

**a** $4(a + 6) = 4a + 6$ **b** $5(a + 1) = 5a + 6$ **c** $-3(p - 5) = -3p - 15$

**d** $a(a + b) = 2a + ab$ **e** $2m(3m + 5) = 6m + 10m$ **f** $4a(3a + 5) = 7a^2 + 20a$

**g** $3a(4a - 7) = 12a^2 - 7$ **h** $-6(x - 5) = 6x + 30$ **i** $3x(2x - 7y) = 6x^2 - 21y$

Example 11

**8** Expand and collect like terms for each expression.

**a** $8(a + 2) + 7$ **b** $5(b + 3) + 10$ **c** $2(c + 7) - 9$

**d** $5(g + 2) + 8g$ **e** $4(h + 1) + 3h$ **f** $6(i - 5) - 3i$

**g** $2(j - 3) - j$ **h** $2a(4a + 3) + 7a$ **i** $5b(2b - 3) + 6b$

**j** $2a(4a + 3) + 7a^2$ **k** $3b(3b - 5) - 7b^2$ **l** $5(2 - 4p) + 20p$

**m** $4(1 - 3q) + 15q$ **n** $2a(3a + 2b) - 6a^2$ **o** $7m(4m - 3n) + 30mn$

**9** Expand and collect like terms for each expression. Express the answer as a single fraction.

**a** $\frac{2}{3}(x + 3) + \frac{x}{6}$ **b** $\frac{1}{4}(x + 2) + \frac{x}{3}$ **c** $\frac{3}{5}(x - 1) + \frac{2x}{3}$

**d** $\frac{5}{6}(x - 4) + \frac{3x}{4}$ **e** $\frac{3}{7}(3x + 5) + \frac{x}{3}$ **f** $\frac{1}{2}(4x - 1) + \frac{2x}{5}$

**g** $\frac{3}{4}(2x + 1) - \frac{x}{3}$ **h** $-\frac{1}{2}(3x + 2) - \frac{2x}{5}$ **i** $-\frac{2}{3}(4x - 3) - \frac{x}{4}$

Example 12

**10** Expand and collect like terms for each expression.

**a** $2(y + 1) + 3(y + 4)$ **b** $3(x - 1) + 4(x + 3)$ **c** $2(3b - 2) + 5(2b - 1)$

**d** $3(a + 5) - 2(a + 7)$ **e** $5(b - 2) - 4(b + 3)$ **f** $2(3y - 4) - 3(2y - 1)$

**g** $x(x - 2) + 3(x - 2)$ **h** $2p(p + 1) - 5(p + 1)$ **i** $4y(3y - 5) - 3(3y - 5)$

**j** $2x(2x + 1) - 7(x + 1)$ **k** $2p(3p + 1) - 4(2p + 1)$ **l** $3z(z + 4) - z(3z + 2)$

**m** $3y(y - 4) + y(y - 4)$ **n** $4z(4z - 2) - z(z + 2)$

**11** **a** Copy and complete:

**i** $12 \times 99 = 12 \times (100 - 1) = 12 \times \ldots - 12 \times \ldots = \ldots - \ldots = \ldots$

**ii** $14 \times 53 = 14 \times (50 + \ldots) = 14 \times \ldots + 14 \times \ldots = \ldots + \ldots = \ldots$

**b** Use a similar technique to the one used in part **a** to calculate the value of each product.

**i** $14 \times 21$ **ii** $17 \times 101$ **iii** $70 \times 29$

**iv** $8 \times 121$ **v** $13 \times 72$ **vi** $17 \times 201$



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12 Expand:

**a** $x(x^2 + 3)$ **b** $x(x^2 + 2x + 1)$ **c** $2x(x^2 - 3x)$

**d** $2x^2(3 - x)$ **e** $5a(3a + 1)$ **f** $6a^2(1 + 2a - a^2)$

# 1E Binomial products

In the previous section, we learnt how to expand a product containing one pair of brackets, such as $3(2x - 4)$. Now we are going to learn how to expand a product with two pairs of brackets, such as $(x + 2)(x + 5)$. Such expressions are called **binomial products**.

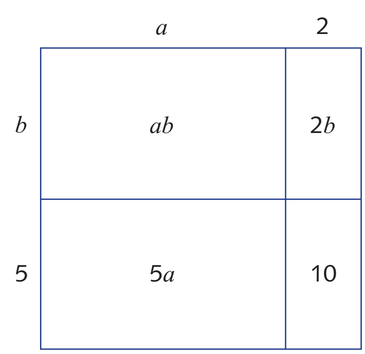
We first look at expanding the expression $(a + 2)(b + 5)$ using the distributive law.

The expression in the second pair of brackets, $(b + 5)$, is multiplied by both $a$ and $2$, giving:

$$\begin{aligned}(a + 2)(b + 5) &= a(b + 5) + 2(b + 5)\\ &= ab + 5a + 2b + 10\end{aligned}$$

Each term in the second pair of brackets is multiplied by each term in the first, and the sum of these is taken.

The procedure can be illustrated with the following diagram.




Area of the rectangle = sum of the areas of the smaller rectangles.

That is, $(a + 2)(b + 5) = ab + 5a + 2b + 10$.

## Binomial products

- In general:

$$\begin{aligned}(a + b)(c + d) &= a(c + d) + b(c + d)\\ &= ac + ad + bc + bd\end{aligned}$$

- Each term in the second pair of brackets is multiplied by each term in the first, making a total of four applications of multiplication.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## Example 14

Expand $(x + 4)(x + 5)$.

### Solution

The expansion can be completed using a table.

| | $x$ | $4$ |
|---|---|---|
| $x$ | $x^2$ | $4x$ |
| $5$ | $5x$ | $20$ |

$$\begin{aligned}(x + 4)(x + 5) &= x(x + 5) + 4(x + 5)\\ &= x^2 + 5x + 4x + 20\\ &= x^2 + 9x + 20\end{aligned}$$

The $x^2$ and $9x$ are unlike terms and cannot be added.

To perform a quick check, substitute $x = 1$:

$$\begin{aligned}(x + 4)(x + 5) &= (1 + 4)(1 + 5)\\ &= 5 \times 6\\ &= 30\end{aligned}$$

and $x^2 + 9x + 20 = 1^2 + 9 \times 1 + 20a$

$$\begin{aligned}&= 1 + 9 + 20\\ &= 30\end{aligned}$$

Remember that this is not a proof that the expansion is correct. It means that it is probably correct.

With practice you should be able to complete the expansion in one step.

## Example 15

Expand and collect like terms for each expression.

**a** $(x + 3)(x - 2)$

**b** $(x - 2)(x + 5)$

**c** $(x - 4)(x - 3)$

**d** $(2y + 1)(3y - 4)$

### Solution

**a** $$\begin{aligned}(x + 3)(x - 2) &= x^2 - 2x + 3x - 6\\ &= x^2 + x - 6\end{aligned}$$

**b** $$\begin{aligned}(x - 2)(x + 5) &= x^2 + 5x - 2x - 10\\ &= x^2 + 3x - 10\end{aligned}$$

**c** $$\begin{aligned}(x - 4)(x - 3) &= x^2 - 3x - 4x + 12\\ &= x^2 - 7x + 12\end{aligned}$$

**d** $$\begin{aligned}(2y + 1)(3y - 4) &= 6y^2 - 8y + 3y - 4\\ &= 6y^2 - 5y - 4\end{aligned}$$


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## Exercise 1E

Example 14

**1** Expand and collect like terms. For parts **a** and **d**, substitute a value for the pronumeral as a check.

**a** $(x + 3)(x + 4)$ **b** $(a + 5)(a + 8)$ **c** $(a + 3)(a + 9)$

**d** $(3 + x)(5 + x)$ **e** $(2 + x)(x + 1)$ **f** $(a + 8)(9 + a)$

Example 15a, b, c

**2** Expand and collect like terms. For parts **a** and **d**, substitute a value for the pronumeral as a check.

**a** $(x - 5)(x - 1)$ **b** $(x - 3)(x - 2)$ **c** $(p - 6)(p + 4)$

**d** $(x - 7)(x + 6)$ **e** $(x + 11)(x - 3)$ **f** $(x + 3)(x - 8)$

**g** $(x + 6)(x - 9)$ **h** $(x - 11)(x - 7)$ **i** $(x + 7)(x - 4)$

Example 15d

**3** Expand and collect like terms.

**a** $(2x + 3)(3x + 4)$ **b** $(3x + 2)(5x + 4)$ **c** $(5x + 1)(x + 2)$

**d** $(2a - 5)(a - 3)$ **e** $(3a - 1)(2b + 5)$ **f** $(4m + 3)(2m - 1)$

**g** $(2p + 5)(3p - 2)$ **h** $(5x - 2)(3x - 8)$ **i** $(2x - 7)(3x - 1)$

**j** $(6x - 5)(x + 2)$ **k** $(7x + 9)(x - 5)$ **l** $(2b + 3)(4b - 2)$

**4** Expand and collect like terms.

**a** $(2a + b)(a + 3b)$ **b** $(m + 3n)(2m + n)$ **c** $(4c + d)(2c - 3d)$

**d** $(4x + 5y)(2x - y)$ **e** $(3x - 2a)(x + 5a)$ **f** $(3x - y)(2x + 5y)$

**g** $(2a + b)(3b - a)$ **h** $(2p + 5q)(3q - 2p)$ **i** $(2p - 5q)(3q - 2p)$

**5** Write down each expansion, if possible, in one step.

**a** $(x + 2)(x + 5)$ **b** $(x + 3)(x - 7)$ **c** $(x - 6)(x - 4)$

**d** $(2x + 1)(x + 3)$ **e** $(3x + 2)(x + 5)$ **f** $(4x + 1)(3x - 1)$

**g** $(2x - 3)(3x - 5)$ **h** $(5x - 2)(3x + 7)$ **i** $(4x + 3)(2x - 1)$

**j** $(4x - 7)(3x + 5)$ **k** $(2x - 1)(2x + 1)$ **l** $(x - 4)(2x + 5)$

**6** Zak expanded a product using the distributive law but, unfortunately, he erased the terms in the second pair of brackets in each case. Fill in the missing terms in the second pair of brackets.

**a** $(x + 6)(\ldots\ldots) = x^2 + 7x + 6$ **b** $(x + 5)(\ldots\ldots) = x^2 + 8 + 15$

**c** $(x + 7)(\ldots\ldots) = x^2 + 5x - 14$ **d** $(x + 3)(\ldots\ldots) = x^2 - 2x - 15$

**e** $(2x + 1)(\ldots\ldots) = 6x^2 - x - 2$ **f** $(3x + 4)(\ldots\ldots) = 3x^2 + x - 4$

**g** $(4x + 3)(\ldots\ldots) = 8x^2 - 22x - 21$ **h** $(3x + 1)(\ldots\ldots) = 15x^2 - 13x - 6$

**7** Fill in the blanks.

**a** $(x + 5)(x + 7) = x^2 + \ldots x + \ldots$ **b** $(x + \ldots)(x + 6) = x^2 + 9x + \ldots$

**c** $(x + 4)(x - \ldots) = x^2 - 2x - \ldots$ **d** $(2x + 3)(x + \ldots) = 2x^2 + 7x + \ldots$


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e $(4x - 1)(x - \ldots) = 4x^2 - \ldots x + 3$

f $(\ldots x + 1)(3x - 5) = 6x^2 - \ldots x - \ldots$

g $(\ldots x + \ldots)(2x + 5) = 4x^2 + 12x + \ldots$

h $(\ldots x - 3)(\ldots x + \ldots) = 12x^2 - x - 6$

**8** Expand and collect like terms.

a $(x + 3)(x - 3)$ b $(x + 3)(x + 3)$ c $(x - 5)(x - 5)$

d $(7x - 1)(7x + 1)$ e $(2x - 5)(x + 3)$ f $(7x + 1)(7x + 1)$

g $(2 - x)(x + 2)$ h $(2x + 3)(2x + 3)$ i $(5a - 1)(5a + 1)$

**9** Expand and collect like terms.

a $\left(\frac{a}{2} + 2\right)\left(\frac{a}{3} + 1\right)$ b $\left(\frac{2b}{3} + 2\right)\left(\frac{b}{5} - 2\right)$ c $\left(\frac{2x}{5} + \frac{1}{2}\right)\left(\frac{x}{5} - 2\right)$

d $\left(\frac{y}{4} + 3\right)\left(\frac{y}{3} - \frac{3}{4}\right)$ e $\left(\frac{3m}{4} + 1\right)\left(\frac{2m}{3} - 3\right)$ f $\left(\frac{5b}{4} + \frac{1}{5}\right)\left(\frac{b}{5} - \frac{1}{2}\right)$

# 1F Perfect squares

A **perfect square** is an expression such as $(x + 3)^2$, $(x - 5)^2$ or $(2x + 7)^2$. The expansions of these have a special form.

$$\begin{aligned}(x + 3)^2 &= (x + 3)(x + 3)\\ &= x(x + 3) + 3(x + 3)\\ &= x^2 + 3x + 3x + 9\\ &= x^2 + 6x + 9\end{aligned}$$

Note that the form of the answer is $x^2$ + twice $(3 \times x) + 3^2$.

$$\begin{aligned}(x - 5)^2 &= (x - 5)(x - 5)\\ &= x^2 - 5x - 5x + 25\\ &= x^2 - 10x + 25\\ &= x^2 + \text{twice}(-5 \times x) + (-5)^2\end{aligned}$$

Another example:

$$\begin{aligned}(3x + 7)^2 &= 3x(3x + 7) + 7(3x + 7)\\ &= 9x^2 + 21x + 21x + 7^2\\ &= 9x^2 + 42x + 49\\ &= (3x)^2 + \text{twice}(7 \times 3x) + 7^2\end{aligned}$$


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## Perfect squares

- In general:

  $(a + b)^2 = a^2 + 2ab + b^2$

- Similarly:

  $(a - b)^2 = a^2 - 2ab + b^2$

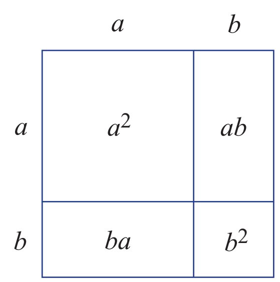
- To expand $(a + b)^2$, take the sum of the squares of the terms and add twice the product of the terms.

The result can be illustrated geometrically.

Area of square $= (a + b)^2$

This is the same as the sum of the areas of rectangles $= a^2 + ab + ba + b^2$

$= a^2 + 2ab + b^2$




### Example 16

Expand:

**a** $(x + 3)^2$ **b** $(x - 6)^2$

#### Solution

**a** $(x + 3)^2 = x^2 + 2 \times 3x + 9$

$= x^2 + 6x + 9$

**b** $(x - 6)^2 = x^2 - 2 \times 6x + 36$

$= x^2 - 12x + 36$

### Example 17

Expand each expression.

**a** $(2x + 3)^2$ **b** $(ax - b)^2$

#### Solution

**a** $(2x + 3)^2 = (2x)^2 + 2 \times 2x \times 3 + 3^2$

$= 4x^2 + 12x + 9$

**b** $(ax - b)^2 = (ax)^2 - 2 \times ax \times b + b^2$

$= a^2x^2 - 2abx + b^2$

The expansion of a perfect square can be performed mentally. With practice you will be able to write down the answer without a middle step.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## Exercise 1F

Example 16a

**1** Expand each expression. Check your answers to parts **a**, **c** and **e** by substitution.

**a** $(x+1)^2$ **b** $(x+5)^2$ **c** $(x+6)^2$ **d** $(x+20)^2$

**e** $(a+8)^2$ **f** $(2+x)^2$ **g** $(9+x)^2$ **h** $(x+a)^2$

Example 16b

**2** Expand each expression. Check your answers to parts **a**, **c** and **e** by substitution.

**a** $(x-4)^2$ **b** $(x-7)^2$ **c** $(x-6)^2$ **d** $(x-1)^2$

**e** $(x-5)^2$ **f** $(x-20)^2$ **g** $(x-11)^2$ **h** $(x-a)^2$

Example 17a

**3** Expand each expression.

**a** $(3x+2)^2$ **b** $(2a+b)^2$ **c** $(2a+3b)^2$ **d** $(3a+4b)^2$

**e** $(2x+3y)^2$ **f** $(2a+3)^2$ **g** $(3x+a)^2$ **h** $(5x+4y)^2$

Example 17b

**4** Expand each expression.

**a** $(3x-2)^2$ **b** $(4x-3)^2$ **c** $(2a-b)^2$ **d** $(2a-3b)^2$

**e** $(3a-4b)^2$ **f** $(2x-3y)^2$ **g** $(3c-b)^2$ **h** $(4x-5)^2$

**5** Expand:

**a** $\left(\frac{x}{2}+3\right)^2$ **b** $\left(\frac{x}{3}-2\right)^2$ **c** $\left(\frac{2x}{5}-1\right)^2$ **d** $\left(\frac{3x}{4}+\frac{2}{3}\right)^2$

**6** Maria has two pieces of paper. One is 10 cm × 10 cm and the other is 6 cm × 6 cm.

**a** Is it possible for these two square pieces of paper to cover a square of size 16 cm × 16 cm completely?

**b** How does the answer to part **a** show that $(10+6)^2 \neq 10^2 + 6^2$?

**c** How much more paper is needed to cover the 16 cm × 16 cm square?

**d** How does your answer to part **c** reinforce the result that

$$(10+6)^2 = 10^2 + 2 \times 10 \times 6 + 6^2\,?$$

**7** Tom sees the following written on the board in a mathematics classroom:

‘Now, $6 + 4 = 10$

Squaring both sides gives

$$(6+4)^2 = 10^2$$

so, $6^2 + 4^2 = 10^2$

That is, $36 + 16 = 100$’

Tom, realising that $36 + 16 \neq 100$, tells the teacher there is a mistake. Can you explain what mistake had been made?



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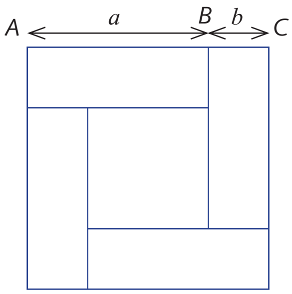
**8** The figure opposite is made from rectangles and a square, with $AB = a$ and $BC = b$.




**a** Explain how the diagram geometrically proves that $(a - b)^2 + 4ab = (a + b)^2$.

**b** By expanding, show algebraically that $(a - b)^2 + 4ab = (a + b)^2$.

**9** Evaluate the following squares, using the method shown in the example below. (You may be able to do these mentally.)

$$\begin{aligned} 21^2 &= (20 + 1)^2 \\ &= 20^2 + 2 \times 20 \times 1 + 1^2 \\ &= 400 + 40 + 1 \\ &= 441 \end{aligned}$$

**a** $31^2$ **b** $19^2$ **c** $42^2$ **d** $18^2$ **e** $51^2$

**f** $101^2$ **g** $99^2$ **h** $201^2$ **i** $301^2$ **j** $199^2$

**10** Evaluate the following using $(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$.

**a** $(1.01)^2$ **b** $(0.99)^2$ **c** $(4.01)^2$ **d** $(4.02)^2$

**e** $(0.999)^2$ **f** $(1.02)^2$ **g** $(3.01)^2$ **h** $(0.98)^2$

**11** Expand and collect like terms.

**a** $(x - 2)^2 + (x - 4)^2$ **b** $(x - 2)^2 + (x + 2)^2$ **c** $(2x - 3)^2 + (x - 1)^2$

**d** $(2x + 5)^2 + (2x - 5)^2$ **e** $(x - 4)^2 + (x + 5)^2$ **f** $(2x - 4)^2 + (x - 3)^2$

**12** Expand and collect like terms.

**a** $x^2 + (x + 1)^2 + (x + 2)^2 + (x + 3)^2$

**b** $x^2 + (x - 1)^2 + (x - 2)^2 + (x - 3)^2$

**c** $x^2 - (x - 1)^2 + (x - 2)^2 - (x - 3)^2$

**13** Expand:

**a** $\left(\frac{x}{2} + 1\right)^2 + \left(\frac{x}{2} - 1\right)^2$ **b** $\left(\frac{x}{3} + 3\right)^2 + \left(\frac{2x}{3} + 1\right)^2$

**c** $\left(\frac{3x}{4} + 2\right)^2 + \left(\frac{x}{2} + 3\right)^2$ **d** $\left(\frac{2x}{5} - \frac{1}{4}\right)^2 + \left(\frac{x}{5} + \frac{1}{2}\right)^2$

**14** Identify which of the following is not a perfect square expansion.

**a** $16x^2 - 8x + 1$ **b** $\frac{9x^2}{4} + 15x + 25$ **c** $4x^2 - 3x + \frac{9}{4}$ **d** $\frac{x^2}{4} + \frac{x}{3} - \frac{1}{9}$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# 1G Difference of two squares

In this section, we look at a special type of expansion: one that produces the **difference of two squares**.

As an example, consider $(x + 2)(x - 2)$.

$$\begin{aligned}(x + 2)(x - 2) &= x^2 - 2x + 2x - 2^2\\ &= x^2 - 2^2\end{aligned}$$

Similarly,

$$\begin{aligned}(2x + 3)(2x - 3) &= 4x^2 - 6x + 6x - 3^2\\ &= (2x)^2 - 3^2\end{aligned}$$

Note that:

- the product $(x + 2)(x - 2)$ is of the form:

  (sum of two terms) × (difference of those terms)
- the answer is of the form:

  (first term) squared – (second term) squared

In general:

$$\begin{aligned}(a + b)(a - b) &= a^2 - ab + ab - b^2\\ &= a^2 - b^2\end{aligned}$$

**Example 18**

Expand:

**a** $(x - 5)(x + 5)$

**b** $(3x - 4)(3x + 4)$

**Solution**

**a** Using the result $(a + b)(a - b) = a^2 - b^2$, we get

$(x - 5)(x + 5) = x^2 - 25$

**b** $$\begin{aligned}(3x - 4)(3x + 4) &= (3x)^2 - 4^2\\ &= 9x^2 - 16\end{aligned}$$

**Difference of two squares**

In general:

$(a + b)(a - b) = a^2 - b^2$


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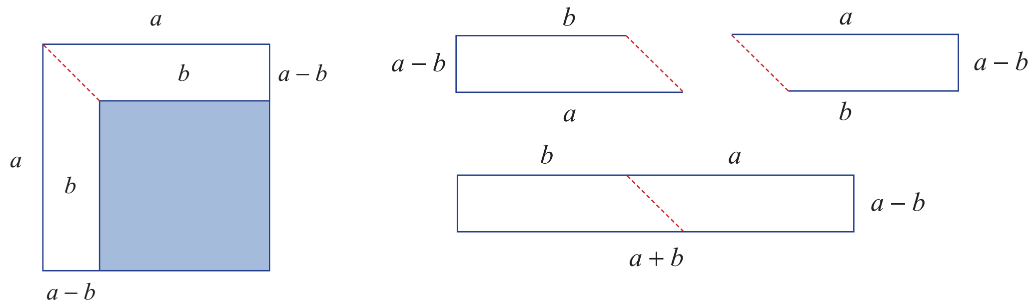
The result is illustrated by the following diagrams.




The diagram on the left shows that the unshaded region has area $a^2 - b^2$.

The diagram on the right, above, shows that the unshaded region has area $(a + b)(a - b)$.

## Exercise 1G

Example 18a

**1** Expand by using the difference of two squares.

**a** $(x - 4)(x + 4)$ **b** $(x - 7)(x + 7)$ **c** $(a - 1)(a + 1)$

**d** $(a - 9)(a + 9)$ **e** $(c - 3)(c + 3)$ **f** $(d - 2)(d + 2)$

**g** $(z - 7)(7 + z)$ **h** $(10 + x)(10 - x)$ **i** $(x - 5)(x + 5)$

Example 18b

**2** Expand:

**a** $(2x - 1)(2x + 1)$ **b** $(3x - 2)(3x + 2)$ **c** $(4a - 5)(4a + 5)$

**d** $(3x - 5)(3x + 5)$ **e** $(2x + 7)(2x - 7)$ **f** $(5a + 2)(5a - 2)$

**g** $(2r - 3s)(2r + 3s)$ **h** $(2x + 3y)(2x - 3y)$ **i** $(5a + 2b)(5a - 2b)$

**3** Expand by using the difference of two squares.

**a** $\left(\frac{2x}{3} + 1\right)\left(\frac{2x}{3} - 1\right)$ **b** $\left(\frac{x}{2} + 3\right)\left(\frac{x}{2} - 3\right)$

**c** $\left(\frac{x}{3} + \frac{1}{2}\right)\left(\frac{x}{3} - \frac{1}{2}\right)$ **d** $\left(\frac{2x}{5} + \frac{3}{4}\right)\left(\frac{2x}{5} - \frac{3}{4}\right)$

**4** The following are either difference of squares expansions or perfect square expansions. Which are which?

**a** $a^2 - 1$ **b** $a^2 - 2a + 1$ **c** $x^2 - 9$

**d** $4x^2 - 25$ **e** $4a^2 + 12a + 9$ **f** $9a^2 - 6a + 1$

**5** Evaluate the following products, using the method shown in the example below. (You may be able to do these mentally.)

$$\begin{aligned} 51 \times 49 &= (50 + 1)(50 - 1) \\ &= 50^2 - 1^2 \\ &= 2499 \end{aligned}$$

**a** $21 \times 19$ **b** $31 \times 29$ **c** $18 \times 22$ **d** $32 \times 28$

**e** $17 \times 23$ **f** $59 \times 61$ **g** $101 \times 99$ **h** $102 \times 98$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**6** Evaluate using the difference of two squares identity.

**a** $1.01 \times 0.99$ **b** $5.01 \times 4.99$ **c** $8.02 \times 7.98$

**d** $1.05 \times 0.95$ **e** $10.01 \times 9.99$ **f** $20.1 \times 19.9$

**7** Look at the figure opposite.

**a** What is the length of $AB$?

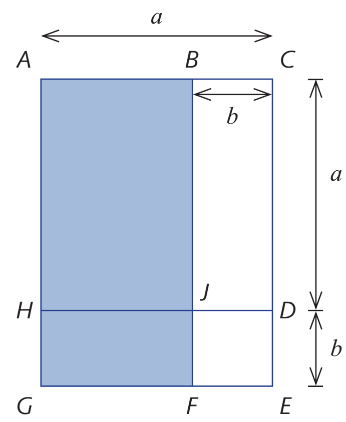
**b** What is the length of $AG$?

**c** What is the area of the shaded region?

**d** Explain why:

area $ABFG$ = area $ACDH$ + area $HDEG$ − area $BCDJ$ − area $JDEF$

**e** What has this got to do with the difference of squares expansions?




# 1H Miscellaneous questions

In this section we present some harder practice questions.

### Example 19

Expand and collect like terms.

**a** $(x + 2)^2 - (x + 1)^2$ **b** $(x + 6)^2 - (x - 6)^2$

### Solution

**a** $(x + 2)^2 - (x - 1)^2 = x^2 + 4x + 4 - (x^2 + 2x + 1)$
$= 2x + 3$

**b** $(x + 6)^2 - (x - 6)^2 = x^2 + 12x + 36 - (x^2 - 12x + 36)$
$= 24x$

## Exercise 1H

Example 19

**1** Expand and collect like terms in each case.

**a** $(x + 2)^2 - (x + 4)^2$ **b** $(x + 6)^2 - (x + 5)^2$

**c** $(a - 4)^2 - (a - 4)(a + 4)$ **d** $(a + 6)(a - 5) - (a + 3)(a - 2)$

**e** $(3x + 2)(x + 1) - (x + 2)(x + 3)$ **f** $(2b - 5)(3b + 2) + (b + 5)(b + 6)$

**g** $(a + 2b)(2a - b) - (a + b)^2$ **h** $(2x + y)(y - 2x) + (2x - y)^2$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**2** Evaluate $3m^3 - 2n$ when:

**a** $m = 2$ and $n = 3$

**b** $m = -4$ and $n = -5$

**c** $m = -\frac{1}{2}$ and $n = \frac{3}{4}$

**d** $m = \frac{1}{3}$ and $n = \sqrt{2}$

**3** Evaluate $\frac{2x^2 + y}{y^2}$ when:

**a** $x = 3$ and $y = 4$

**b** $x = -\frac{1}{2}$ and $y = \frac{3}{2}$

**c** $x = \sqrt{3}$ and $y = -2$

**d** $x = \sqrt{6}$ and $y = -4$

**4** Evaluate $a + 2b - c$ when:

**a** $a = \frac{1}{4}$, $b = \frac{1}{2}$ and $c = \frac{3}{8}$

**b** $a = \frac{b}{3}$, $b = -\frac{1}{4}$ and $c = \frac{5b}{6}$

**c** $a = \frac{1}{c}$, $b = \frac{3c}{2}$ and $c = \frac{3}{8}$

**d** $a = c^2$, $b = \frac{1}{c^2}$ and $c = \frac{2}{3}$

**5** Evaluate $(x^2 - 2)^2$ when:

**a** $x = 2$ **b** $x = -2$ **c** $x = \frac{2}{3}$ **d** $x = -\sqrt{2}$ **e** $x = \sqrt{3}$

**6** Simplify each expression by collecting like terms.

**a** $\frac{3}{7}a^2 + \frac{2}{3}a^2$ **b** $\frac{3a}{b} + \frac{5a}{2b}$ **c** $\frac{2x}{y^3} + \frac{7x}{3y^3}$ **d** $\frac{2a^2b}{5} - \frac{2a^2b}{5} + \frac{6m}{n^2}$

**7** Copy and complete:

**a** $\frac{2a^2b}{5} - \ldots = \frac{3a^2b}{10}$

**b** $\frac{2x^2}{y} - \ldots = \frac{5x^2}{3y}$

**c** $\frac{3m}{2n^2} + \ldots = \frac{7m}{2n^2}$

**d** $\frac{p^3}{2n^2} - \ldots = -\frac{p^3}{n^2}$

**8** For each value of $x$, find $x^2$ and simplify.

**a** $x = \sqrt{5}$ **b** $x = 3a$ **c** $x = -2b$ **d** $x = a + b$ **e** $x = 3m - 2n$

**9** For each value of $x$, find $x^2 + 3x$ and simplify.

**a** $x = 3a$ **b** $x = -2b$ **c** $x = a - b$ **d** $x = 2m + n$

**10** For each value of $x$, find $(x - 1)^2$ and simplify.

**a** $x = 3a$ **b** $x = -2b$ **c** $x = 2a + 1$ **d** $x = -4b + 2$

**11** Expand and collect like terms in each case.

**a** $x(2x^2 + 3x + 4)$

**b** $3a(a^2 - 4a + 1)$

**c** $(m + 2)(m^2 + 3m + 2)$

**d** $(p - 3)(2p^2 + 5p + 3)$

**e** $(x - 1)(x^2 + x + 1)$

**f** $(x + 1)(x^2 - x + 1)$


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**12** If $x = 1, y = 3, z = 5$ and $w = 0$, find the exact value of $\sqrt{3xy} + \sqrt{3xz} + \sqrt{3wy}$.

**13** Expand and collect like terms.

**a** $(x + y - z)(x - y + z)$ **b** $(a - b + c)(a + b - c)$ **c** $(2x - y + z)(2x + y - z)$

**d** $(x + z + 1)^2$ **e** $(x + y + z)^2$ **f** $(x + y - z)^2$

**14** Find $\dfrac{2ab}{a + b}$ if:

**a** $a = 2$ and $b = 3$ **b** $a = 10$ and $b = 50$ **c** $a = \dfrac{1}{2}$ and $b = \dfrac{1}{3}$

**d** $a = \dfrac{1}{3}$ and $b = \dfrac{1}{4}$ **e** $a = \dfrac{2}{3}$ and $b = \dfrac{3}{4}$ **f** $a = \dfrac{1}{x}$ and $b = \dfrac{1}{y}$

**15** Find $\dfrac{a}{c}$ and $\dfrac{a - b}{b - c}$ if:

**a** **i** $a = 4, b = 2$ and $c = \dfrac{4}{3}$ **ii** $a = \dfrac{3}{4}, b = \dfrac{3}{5}$ and $c = \dfrac{1}{2}$

**iii** $a = 3$, $b = \dfrac{3}{2}$ and $c = 1$

**b** In each of the above, show that $\dfrac{1}{c} - \dfrac{1}{b} = \dfrac{1}{b} - \dfrac{1}{a}$.

**16** Simplify by first removing brackets.

**a** $a - (b - c) + a + (b - c) + b - (c + a)$

**b** $3a - 2b - c - (4a - 3b + 5c)$

**c** $a - (b + a - (b + a))$

**d** $-(-(a - b - c))$

**e** $-(-2x - (3y - (2x - 3y) + (3x - 2y)))$

**17** Expand and collect like terms.

**a** $2[(a + b)(a - b) + (a + c)(a - c) + a(b + c)] + (a - b)^2 + (a - c)^2 + (b - c)^2$

**b** $(a - b)(a^2 + a + b) + (a + b)(a^2 - a + b)$

**c** $\left(\dfrac{1}{a} - \dfrac{1}{b}\right)^2 \left(\dfrac{1}{a} + \dfrac{1}{b}\right)^2$

**d** $\left(a + \dfrac{1}{a}\right)^2 \left(a - \dfrac{1}{a}\right)^2$

**e** $\left(2a + \dfrac{1}{a}\right)^2 - \left(a - \dfrac{1}{a}\right)^2$

**f** $\left(a + \dfrac{1}{b}\right)\left(b - \dfrac{1}{a}\right)$

**g** $(a + b)^2 - [2b(a + b) - (a + b)(b - a)]$


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# Review exercise

**1** Evaluate $8m - 3n$ when:

**a** $m = 3$ and $n = 5$ **b** $m = 2$ and $n = -5$

**c** $m = -1$ and $n = 2$ **d** $m = -3$ and $n = -2$

**2** Evaluate $2a + 3b - 3c$ when:

**a** $a = 3, b = 5$ and $c = 2$ **b** $a = 3, b = -5$ and $c = 2$

**c** $a = -3, b = 5$ and $c = 2$ **d** $a = 3, b = 5$ and $c = -2$

**e** $a = -3, b = -5$ and $c = 2$ **f** $a = 3, b = -5$ and $c = -2$

**3** Evaluate $x^2 - 2x$ when:

**a** $x = 3$ **b** $x = -2$ **c** $x = -7$

**d** $x = \frac{2}{3}$ **e** $x = -\frac{2}{3}$ **f** $x = -1.1$

**4** Simplify:

**a** $7a + 2a$ **b** $8b + 3b$ **c** $9a - 6a$

**d** $7a + 5a - 6a$ **e** $7x - 3x + 9x$ **f** $7a^2 + 3a^2 - 12a^2$

**5** Copy and complete:

**a** $2a + \ldots = 11a$ **b** $5b - \ldots = 3b$ **c** $3mn + \ldots = 16mn$

**d** $11pq - \ldots = 5pq$ **e** $8ab - \ldots = -5ab$ **f** $3m^2n - \ldots = -8m^2n$

**g** $-11a^2b + \ldots = 2a^2b$ **h** $-7\ell m + \ldots = 4\ell m$ **i** $14xy^2 - \ldots = -7xy^2$

**6** Expand each expression and simplify if possible.

**a** $3(x + 2)$ **b** $4(b + 6)$ **c** $5(3b - 2)$

**d** $-6(2d + 5)$ **e** $2(4x - 15)$ **f** $3(4g - 6)$

**g** $-3(2p + 2)$ **h** $-5(3b - 5)$ **i** $-5(4b - 7)$

**j** $15(g + 2) - 8g$ **k** $5(h + 1) + 6h$ **l** $16(x - 1) - 13x$

**m** $2(y - 1) - y$ **n** $5a(2a + 3) - 7a$ **o** $-5b(2b - 3) + 16b$

**p** $2(3z - 1) - 4(2y - 1)$ **q** $z(z - 1) + 3(z - 2)$ **r** $6y(2y - 5) - 5(2y - 5)$

**7** Expand each expression and collect like terms.

**a** $(x + 2)(x + 4)$ **b** $(a - 11)(a - 4)$ **c** $(a - 5)(a - 5)$

**d** $(p - 7)(p + 4)$ **e** $(5x + 2)(2x + 2)$ **f** $(2x - 1)(4x - 5)$

**g** $(4x - 6)(11x + 1)$ **h** $(4x + 3)(2x - 1)$ **i** $(7a - 1)(7a + 3)$


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**8** Expand:

**a** $(x + 11)^2$ **b** $(x + 6)^2$ **c** $(x - 15)^2$ **d** $(x - 10)^2$

**e** $(x - 2y)^2$ **f** $(2a + 5b)^2$ **g** $(5x + 2)^2$ **h** $(5x - 6)^2$

**9** Expand:

**a** $(x - 6)(x + 6)$ **b** $(z - 7)(z + 7)$ **c** $(p - 1)(p + 1)$

**d** $(5x - 1)(5x + 1)$ **e** $(7x - 5)(7x + 5)$ **f** $(10 - 3a)(10 + 3a)$

**g** $(5a + 2b)(5a - 2b)$ **h** $(12x + y)(12x - y)$ **i** $(8x + 3a)(8x - 3a)$

**10** Expand and collect like terms.

**a** $(3x + y)(2x - y)$ **b** $(3x - 4a)(3x + 2a)$ **c** $(3c + 4b)(5c + 4b)$

**d** $(3x + 5y)^2$ **e** $(a - 2b)^2$ **f** $(5\ell + 2m)^2$

**g** $(3x - y)(3x + y)$ **h** $(5m + 2n)(5\mathrm{m} - 2\mathrm{n})$ **i** $(3x + 5a)^2$

**11** Expand and collect like terms.

**a** $\left(\frac{a}{2} + 1\right)\left(\frac{a}{3} - 2\right)$ **b** $\left(\frac{2x}{3} + 6\right)(x - 4)$ **c** $\left(\frac{a}{2} - 1\right)\left(\frac{a}{2} + 1\right)$

**d** $\left(\frac{3x}{5} - 1\right)\left(\frac{2x}{5} + 3\right)$ **e** $\left(\frac{5b}{6} - 3\right)(b + 2)$ **f** $(a - 6)\left(\frac{a}{3} + 4\right)$


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# Challenge exercise

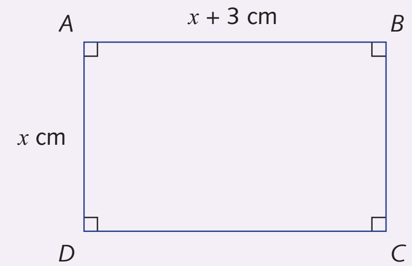
**1** **a** Show that the perimeter of the rectangle opposite is $(4x + 6)$ cm.

**b** Find the perimeter of the rectangle if $AD = 2$ cm.

**c** Find the value of $x$ if the perimeter is 36 cm.

**d** Find the area of the rectangle $ABCD$ in terms of $x$.

**e** Find the area of the rectangle if $AB = 6$ cm.

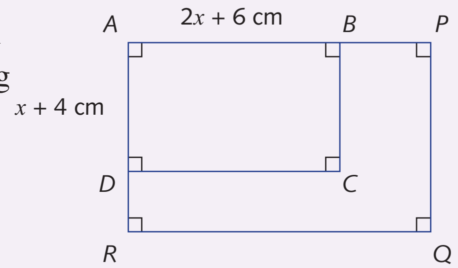



**2** Rectangle $ABCD$ is $(x + 4)$ cm wide and $(2x + 6)$ cm long. A new rectangle, $APQR$, is formed by increasing the length of each side of $ABCD$ by 50%.

**a** Find $AP$ in terms of $x$.

**b** Find $AR$ in terms of $x$.

**c** Find the area of $APQR$ in terms of $x$.




**d** Show that the difference between the areas of the two rectangles is $\frac{5}{2}x^2 + \frac{35}{2}x + 30$.

**e** What fraction of the area $ABCD$ is the answer in part **d**?

**3** We know that $(a + b)(c + d) = ac + ad + bc + bd$.

Expand $(a + b + c)(d + e + f)$. Draw a diagram to illustrate your answer.

**4** 8, 9, 10 and 11 are four consecutive whole numbers.

**a** **i** What is the product of the outer pair $(8 \times 11)$?

**ii** What is the product of the inner pair $(9 \times 10)$?

**b** Find the difference between the answers to parts **i** and **ii**.

**c** If the smallest of the four consecutive whole numbers is denoted by $n$, express each of the other numbers in terms of $n$.

**d** Express the following in terms of $n$:

**i** the product of the outer pair

**ii** the product of the inner pair

**iii** the difference between the answers to part **ii** and part **i**.

**5** 6, 8, 10 and 12 are four consecutive even whole numbers.

**a** **i** What is the product of the outer pair $(6 \times 12)$?

**ii** What is the product of the inner pair $(8 \times 10)$?

**b** If the smallest of the four consecutive even numbers is denoted by $n$, express each of the other numbers in terms of $n$.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**c** Express the following in terms of $n$:

**i** the product of the outer pair

**ii** the product of the inner pair

**iii** the difference between the answers to part **ii** and part **i**.

**d** What happens if you take four consecutive odd whole numbers and complete parts **a, b** and **c** with these numbers?

**6** **a** 7, 8 and 9 are consecutive whole numbers. $8^2 - 7 \times 9 = 1$.
Prove that this result holds for any three consecutive numbers $n - 1$, $n$ and $n + 1$.
That is, 'If the product of the outer numbers is subtracted from the square of the middle number, the result is one'.

**b** $a - d$, $a$ and $a + d$ are three numbers. Show that if the product of the outer pair is subtracted from the square of the middle term, the result is $d^2$.

**7** Expand and collect like terms.

**a** $(x^2 + x + 1)(x^2 - x + 1)$

**b** $(x + 1)(x^2 - x + 1)$

**c** $(x^5 - 1)(x^5 + 1)$

**8** Expand and collect like terms.

**a** $(x - 1)(x^2 + x + 1)$

**b** $(x - 1)(x^4 + x^3 + x^2 + x + 1)$

**c** What do you expect the result of expanding $(x - 1)(x^9 + x^8 + \ldots + 1)$ will be?

**9** **a** Show that $(a^2 + b^2)(c^2 + d^2) = (ac - bd)^2 + (ad + bc)^2$.

**b** Hence write $(3^2 + 1^2)(5^2 + 2^2)$ as the sum of two squares.

**10** **a** Evaluate each of the following.

**i** $1 \times 2 \times 3 \times 4 + 1$

**ii** $2 \times 3 \times 4 \times 5 + 1$

**iii** $3 \times 4 \times 5 \times 6 + 1$

**iv** $4 \times 5 \times 6 \times 7 + 1$

**b** Expand $(n^2 + n - 1)^2$.

**c** Show that $(n - 1)n(n + 1)(n + 2) + 1 = (n^2 + n - 1)^2$.

(Note that we have proved that the product of four consecutive integers plus one is a perfect square.)


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