Consumer arithmetic

This chapter deals with some important practical financial topics such as investing and borrowing money, income tax and GST, inflation, depreciation, profits and losses, discounts and commissions. Mention is also made of topics such as population, rainfall and the composition of materials.

When buying a car, using a credit card, or deciding how good a particular 'bargain' is, we all need to be competent in our understanding of consumer arithmetic. Otherwise, we run the serious risk of wasting our money or, even worse, being at the mercy of unscrupulous individuals. This is particularly important when we come to the 'big ticket items' such as purchasing a car or a home, or planning superannuation. It is also important when completing our tax returns.

Using a calculator and approximations

Everything in this chapter involves calculations with percentages. We are now assuming that you are using a calculator, so we have made little attempt to set questions where the numbers work out nicely. However, some of the problems can be done mentally. This chapter gives you the opportunity to learn how to efficiently use a calculator.

Nevertheless, you should always look over your work and check that the answers to your calculations are reasonable and sensible.

When the calculator displays numbers with many decimal places, you will need to round the answer in some way that is appropriate in the context of the question.

Whenever you round a number, you should be careful to use the symbol $\approx$, which means *approximately equals*, rather than the symbol $=$, which means *exactly equals*.

For example, if we are told that a grocer received \$1000 in cash payments for goods that he had sold and that he banked one-third of this, we would write:

$$\text{amount banked} = 1000 \div 3$$
$$\approx \$333.33$$

not

$$\text{amount banked} = 1000 \div 3$$
$$= \$333.3333\ldots$$

because the grocer could not have banked fractions of a cent.

3A Review of percentages

A percentage such as 35% is a rational number and can be rewritten as a decimal or as a fraction, as follows:

$$35\% = 35 \div 100 \qquad \text{or} \qquad 35\% = \frac{35}{100}$$
$$= 0.35 \qquad\qquad\qquad\qquad = \frac{7}{20}$$

In this chapter, we encourage you to write percentages as decimals rather than fractions. Decimals are more commonly used than fractions when dealing with money.

⇨ Converting a percentage to a decimal or a fraction

To convert a percentage to a decimal or a fraction, divide by 100.

For example: $42\% = 42 \div 100$
$= 0.42$

$$12\tfrac{1}{4}\% = \frac{49}{4}\%$$
$$= \frac{49}{4} \times \frac{1}{100}$$
$$= \frac{49}{400}$$

Conversely, to convert a decimal such as 1.24, or a fraction such as $\dfrac{3}{8}$, to a percentage, multiply by 100%.

$$1.24 = 1.24 \times 100\%$$
$$= 124\%$$

and

$$\frac{3}{8} = \frac{3}{8} \times \frac{100}{1}\%$$
$$= 37\tfrac{1}{2}\%$$

⇨ Converting a decimal or a fraction to a percentage

To convert a decimal or a fraction to a percentage, multiply by 100%.

Percentages of a quantity

To calculate a percentage of a quantity:

- convert the percentage to a decimal or a fraction
- multiply the quantity by the fraction or decimal.

Example 1

The Brilliant Light Bulb Company estimates that 3.5% of its light bulbs are defective. If a shop owner buys 1250 light bulbs to light the shop, how many would he expect to be defective?

Solution

Number of defective bulbs $= 1250 \times 3.5\%$
$= 1250 \times 0.035$
≈ 44 \qquad (Round 43.75 to 44.)

Calculating a percentage

We can express one quantity as a percentage of another. Remember that both quantities must be expressed in the same unit of measurement before calculating a percentage.

Example 2

A typical computer weighs about 25 kg. When it is broken down as waste, it yields about 3 g of arsenic. What percentage of the total is this?

Solution

Using grams, the computer weighs 25 000 g and the arsenic weighs 3 g.

Hence percentage of arsenic $= \dfrac{3}{25\,000} \times 100\%$

$$= \dfrac{3}{250}\%$$
$$= 0.012\%$$

Calculating a percentage

To calculate the percentage that one quantity, a, is of another quantity, b:

• first convert both quantities to the same unit of measurement
• then form the fraction $\dfrac{a}{b}$ and multiply it by 100%.

Exercise 3A

1 Express each percentage as a decimal.

   a 72%   **b** 7.6%   **c** 98%   **d** 16%

   e 8%   **f** 6.25%   **g** 175%   **h** 0.6%

   i $77\tfrac{3}{4}\%$ **j** 0.1%   **k** 142.6%   **l** $\dfrac{1}{4}\%$

2 Express each percentage as a fraction in lowest terms.

   a 35%   **b** 56%   **c** 75%   **d** $37\tfrac{1}{2}\%$

   e $33\tfrac{1}{3}\%$ **f** $16\tfrac{2}{3}\%$ **g** 7.25%   **h** 6.4%

   i 210%   **j** 125%   **k** $112\tfrac{1}{2}\%$ **l** 136%

3 Express each fraction or decimal as a percentage.

a $\dfrac{3}{5}$ **b** $\dfrac{3}{8}$ **c** $\dfrac{9}{16}$ **d** $2\tfrac{1}{4}$

e $\dfrac{7}{20}$ **f** $\dfrac{2}{3}$ **g** $\dfrac{4}{3}$ **h** $\dfrac{3}{400}$

i 0.43 **j** 0.225 **k** 0.04 **l** 0.015

m 1.2 **n** 2.03 **o** 1.175 **p** 0.0075

4 Copy and complete this table.

	Percentage	Fraction	Decimal
a	54%		
b		$\dfrac{2}{5}$	
c			0.32
d	18.5%		
e			0.06
f		$\dfrac{7}{8}$	
g			1.02
h	108.6%		
i		$1\tfrac{2}{5}$	

Example 1

5 Evaluate these amounts, correct to 2 decimal places where necessary.

a 15% of 40 **b** 57% of 1000 **c** 26% of 264

d 120% of 538 **e** 15.8% of 972 **f** 138.5% of 650

g 2.8% of 318 **h** 0.1% of 6000 **i** 150% of 846

6 Evaluate these amounts, correct to the nearest cent where necessary.

a 62% of $10 **b** 23.7% of $960 **c** 3.2% of $1500

d 110% of $1280 **e** 0.25% of $800 **f** $6\tfrac{1}{2}\%$ of $200

g $7\tfrac{3}{4}\%$ of $1000 **h** $\dfrac{1}{4}\%$ of $840 **i** 7.25% of $1600

Example 2

7 Find what percentage the first quantity is of the second quantity, correct to 1 decimal place where necessary.

a 7 km, 50 km **b** $4, $200 **c** 14 kg, 400 kg

d 70 m, 50 m **e** 15 weeks, 60 weeks **f** 60 weeks, 15 weeks

8 Find what percentage the first quantity is of the second quantity, correct to 2 decimal places where necessary. You will need to express both quantities in the same unit first.

 a 68 cents, $5.00   **b** 3.4 cm, 2 m

 c 7 g, 3 kg   **d** 8 hours, 2 weeks

 e 15 days, 3 years   **f** 250 m, 4 km

 g 4 km, 250 m   **h** 1 day, 1 year

 i 1 year, 1 day   **j** 33 weeks, 1 century

 k 56 cm, 2.4 km   **l** 5 apples, 16 dozen apples

9 Find what percentage the first quantity is of the second quantity, correct to 4 decimal places where necessary.

 a 48 mm, 1 km   **b** 1.5 hours, 3 years

 c 7.8 g, 60 kg   **d** 3.5 cents, $1400

10 There are 740 students at a primary school, 5% of whom have red hair. Calculate the number of students in the school who have red hair.

11 A sample of a certain alloy weighs 1.6 g.

 a Aluminium makes up 48% of the alloy. What is the weight of the aluminium in the sample?

 b The percentage of lead in the alloy is 0.23%. What is the weight of the lead in the sample?

12 A soccer match lasted 92 minutes (including injury time). If team A was in possession for 55% of the match, for how many minutes and seconds was team A in possession?

13 Carbon dioxide makes up 0.059% of the mass of the Earth's atmosphere. The total mass of the atmosphere is about 5 million megatonnes. What is the total mass of the carbon dioxide in the atmosphere?

14 The label on a Sunnyvale tomato paste bottle says that in every 25 g serving, there are 3.6 g of carbohydrate, 0.1 g of fat, and 105 mg of sodium.

 a Express as a percentage of the 25 g serving:

  i the mass of carbohydrate

  ii the mass of fat

  iii the mass of sodium

 b The Sunnyvale website claims that the percentage of protein is 3.2%. What mass of protein is that per 25 g serving?

15 Mt Kosciusko has a height of 2228 m, while the height of Mt Everest is 8848 m. Calculate your answers to this question correct to 3 decimal places.

    a What percentage is the height of Mt Everest of the height of Mt Kosciusko?

    b The Earth's radius is about 6400 km. What percentage of the radius of the Earth is the height of Mt Everest?

16 In the Federal Parliament, there are 150 members in the House of Representatives, of whom 37 are from Victoria.

    a Correct to 1 decimal place, what percentage of members are from Victoria?

    b The population of Australia is about 22.6 million. What percentage of Australians are members of the House of Representatives?

17 The distance by air from Melbourne to Darwin is 3346 km, and from Melbourne to Singapore it is 6021 km. What percentage, correct to the nearest percent, is:

    a the Melbourne–Darwin distance of the Melbourne–Singapore distance?

    b the Melbourne–Singapore distance of the Melbourne–Darwin distance?

3B Using percentages

Percentages are used extensively in finance, and are common in many other practical situations. The rest of this chapter will give examples of a few well-known situations, concentrating on financial applications. If you read a newspaper for a few days, you will find a great variety of further interesting uses of percentages.

First, however, we will introduce another important method that will be used with percentages throughout this chapter.

Reversing the process to find the original amount

Suppose that 15% of the total mass of a chicken roll is actually chicken. What mass of chicken rolls can be made with 600 g of chicken?

    Mass of chicken = mass of rolls × 15%

Reversing this:

    mass of rolls = mass of chicken ÷ 15%

                = 600 ÷ 0.15

                = 4000 g

Hence 4 kg of chicken rolls can be made with 600 g of chicken.

➡️ **Finding the original amount**

- To find, for example, 15% of a given amount, multiply by 15%.
- Conversely, to find the original amount given 15% of it, divide by 15%.

Example 3

Joshua saves 12% of his after-tax salary every week. If he saves $90 a week, what is his after-tax salary?

Solution

Savings = after-tax salary × 12%

Reversing this:

$$\text{after-tax salary} = \text{savings} \div 12\%$$
$$= 90 \div 0.12$$
$$= \$750$$

Example 4

Sterling silver is an alloy that is made up of 92.5% by mass silver and 7.5% copper.

a How much sterling silver can be made with 5 kg of silver and unlimited supplies of copper?

b How much sterling silver can be made with 5 kg of copper and unlimited supplies of silver?

Solution

a Mass of silver = mass of sterling silver × 92.5%

Reversing this:

$$\text{mass of sterling silver} = \text{mass of silver} \div 92.5\%$$
$$= 5 \div 0.925$$
$$\approx 5.405 \text{ kg}$$

b Mass of copper = mass of sterling silver × 7.5%

Reversing this:

$$\text{mass of sterling silver} = \text{mass of copper} \div 7.5\%$$
$$= 5 \div 0.075$$
$$\approx 66.667 \text{ kg}$$

Note: One reason for the choice of rounding to 3 decimal places is that there are 1000 g in a kilogram. That is, we are calculating to the nearest gram.

Commission

If a person takes a painting to a gallery to be sold, the gallery will usually charge the vendor or seller a percentage of the selling price as the fee for exhibiting, advertising and selling the painting. Such a fee is called a **commission**, and it applies whenever an agent sells goods or services such as a house or a car on behalf of someone else.

Example 5

The Eureka Gallery charges a commission of 9.2%.

a The Australian painting *Showing the Flag at Bakery Hill* was sold recently for $180 000. How much did the Gallery receive, and how much was left for the seller?

b The Gallery received a commission of $7912 for selling the painting *Ned at the Glen*. What was the selling price of the painting, and what did the seller actually receive?

Solution

a Commission = 180 000 × 9.2%
$$= 180\,000 \times 0.092$$
$$= \$16\,560$$

Amount received by seller = 180 000 − 16 560
$$= \$163\,440$$

b Commission = selling price × 9.2%

Reversing this:
selling price = commission ÷ 9.2%
$$= 7912 \div 0.092$$
$$= \$86\,000$$

Amount received by seller = 86 000 − 7912
$$= \$78\,088$$

Profit

Businesses aim to make a profit on their investments. A profit equation can be formulated as:

profit = total revenue (sales) − total costs

Profit and loss as percentages

Is an annual profit of $20 000 a great performance or a poor performance? For a business with annual turnover of $100 000, such a profit would be considered very large. For a business with annual turnover of $100 000 000, however, it would be considered a very poor performance.

For this reason, it is often relevant to express profit or loss as a percentage of the total costs or the annual turnover.

Example 6

The Budget Shoe Shop spent $6\,600\,000$ last year buying shoes and paying salaries and other expenses. They made a 2% profit on these costs.

a What was their profit last year?

b What was the total of their sales?

c In the previous year, their costs were $5\,225\,000$ and their sales were only $5\,145\,000$. What percentage loss did they make on their costs?

Solution

a Profit $= 6\,600\,000 \times 2\%$

$\qquad = 6\,600\,000 \times 0.02$

$\qquad = \$132\,000$

b Total sales $=$ total costs $+$ profit

$\qquad = 6\,600\,000 + 132\,000$

$\qquad = \$6\,732\,000$

c Last year, loss $=$ total costs $-$ total sales

$\qquad = 5\,225\,000 - 5\,145\,000$

$\qquad = \$80\,000$

Percentage loss $= \left(\dfrac{80\,000}{5\,225\,000} \times \dfrac{100}{1} \right)\%$

$\qquad \approx 1.53\%$

Example 7

Joe's tile shop made a profit of 5.8% on total costs last year. If the actual profit was $83\,000$, what were the total costs, and what were the total sales?

Solution

$\qquad$ Profit $=$ costs $\times 5.8\%$

Reversing this, costs $=$ profit $\div 5.8\%$

$\qquad = 83\,000 \div 0.058$

$\qquad \approx \$1\,431\,034$, correct to the nearest dollar.

Hence, total sales $=$ profit $+$ costs

$\qquad \approx 83\,000 + 1\,431\,034$

$\qquad = \$1\,514\,034$

Income tax

Income tax rates are often **progressive**. This means that the more you earn, the higher the rate of tax on each extra dollar you earn.

Example 8

Income tax in the nation of Immutatia is calculated as follows.

- There is no tax on the first $12 000 that a person earns in any one year.
- From $12 001 to $30 000, the tax rate is 15c for each dollar over $12 000.
- From $30 001 to $75 000, the tax rate is 25c for each dollar over $30 000.
- Over $75 000, the tax rate is 35c for each dollar over $75 000.

Find the income tax payable by a person whose taxable income for the year is:

a $10 600   **b** $25 572   **c** $62 300   **d** $455 000

Solution

a There is no tax on an income of $10 600.

b Tax on first $12 000 = $0

Tax on remaining $13 572 = $13 572 \times 0.15$
$$= \$2035.80$$

This is the total tax payable.

c Tax on first $12 000 = $0

Tax on next $18 000 = $18 000 \times 0.15$
$$= \$2700$$

Tax on remaining $32 300 = $32 300 \times 0.25$
$$= \$8075$$

Total tax $= 2700 + 8075$
$$= \$10\,775$$

d Tax on first $12 000 = $0

Tax on next $18 000 = $2700   (see part **c**)

Tax on next $45 000 = $45 000 \times 0.25$
$$= \$11\,250$$

Tax on remaining $380 000 = $380 000 \times 0.35$
$$= \$133\,000$$

Total tax $= 2700 + 11\,250 + 133\,000$
$$= \$146\,950$$

Exercise 3B

Note that some of the questions can be done using the unitary method. For example, Question **5** is one such question.

Example 3, 4

1 a Five per cent of a particular amount is $12. Find the amount by dividing $12 by 5% = 0.05.

   b Check your answer to part **a** by taking 5% of it.

2 a Twenty-two per cent of a sand pile is 284 kg. Find the mass of the sand pile.

   b Check your answer to part **a** by taking 22% of it.

3 Find the quantity, given that:

   a 2% of it is $12

   b 6% of it is 750 g

   c 30% of it is 36 minutes

   d 90% of it is 54 cm

4 In each part, find the price if:

   a a deposit of $360 is 30% of the price

   b a deposit of $168 is 15% of the price

5 Find the original quantity, correct to a suitable number of decimal places, if:

   a 23% of it is 100 kg

   b 0.2% of it is 4 mm

   c 0.92% of it is 1.86 hectares

   d 97% of it is $700

6 Cameron and Wendy together earn $1156 per week after tax. Of this, they pay $460 off their mortgage, $185 for groceries, and $260 for their car and transport, and they save $124.

   a Express each amount as a percentage of their weekly income, correct to the nearest 1%.

   b Find how much is unaccounted for in the list above, and what percentage it is of their weekly income.

7 What percentage of the total cost is a deposit of:

   a $33 on a television valued at $550?

   b $124.10 on a stove valued at $1460?

Example 5a

8 A book dealer sells rare books and charges a commission of 8% on the selling price. Find, correct to the nearest cent, the commission charged on a book that sells for these prices, and the amount that the seller eventually receives.

   a $400   **b** $1300   **c** $575   **d** $142.50

Example 5b

9 Shara the stockbroker charges 0.15% commission on all shares that she sells for clients. In each case, find the price at which a parcel of shares was sold if her commission was:

   a $30.00   **b** $67.35   **c** $384.75   **d** $36.51

10 Jeff works as a salesman selling second-hand tractors. He is paid a salary of $35 000 a year, together with a 6% commission on all the sales he makes. Find his total annual income if his sales for the year were:

   a $20 000   **b** $1 000 000   **c** $126 000   **d** $3 458 000

11 A salesperson is paid a commission on her monthly sales. What is the percentage commission if she receives a payment of:

    a $168 on sales of $1200   **b** $540 on sales of $6000

    c $1530 on sales of $18 000   **d** $1596 on sales of $42 000

12 Find the selling price if the commission and the commission rate are as given.

    a Commission $35, rate 7%   **b** Commission $646, rate 3.4%

    c Commission $16 586.96, rate 5.2%   **d** Commission $3518.61, rate 11.4%

13 Find, to 1 decimal place, the percentage profit or loss on costs in these situations.

    a Costs $16 000 and sales $18 000

    b Costs $162 000 and sales $150 000

    c Costs $2 800 000 and sales $3 090 000

    d Costs $289 000 000 and sales $268 000 000

Example 6 **14** The Secure Locksmith Company had sales last year of $568 000 and costs of $521 000.

    a What was their profit?

    b What was the profit as a percentage of the cost price? (Calculate the percentage correct to 2 decimal places.)

    c In the previous year, they made a loss of 4% of their costs of $250 000. Find their loss and their sales.

Example 7 **15** **a** A company made a profit of $18 000, which was a 2.4% profit on its costs. Find the costs and the total sales.

    b A company made a loss of $657 000, which was a 4.5% loss on its costs. Find the costs and the total sales.

    c A company made a loss of $250 800, which was a 3.8% loss on its costs. Find the costs and the total sales.

Example 8 **16** This question uses the income tax rates in the nation of Immutatia, which are as follows.

    • There is no tax on the first $12 000 that a person earns in any one year.

    • From $12 001 to $30 000, the tax rate is 15c for each dollar over $12 000.

    • From $30 001 to $75 000, the tax rate is 25c for each dollar over $30 000.

    • Over $75 000, the tax rate is 35c for each dollar over $75 000.

    a Find the income tax payable on:

        i $8000   **ii** $14 000   **iii** $36 000   **iv** $200 000

    b What percentage, to 2 decimal places, of each person's income was paid in income tax in parts **i**–**iv** of part **a**?

    c Find the income if the income tax on it was:

        i $1260   **ii** $3420   **iii** $13 950   **iv** $14 650

3C Simple interest

When money is lent by a bank, whoever borrows the money normally makes a payment, called **interest**, for the use of the money.

The amount of interest paid depends on:

- the **principal**, which is the amount of money borrowed
- the **rate** at which interest is charged
- the **time** for which the money is borrowed.

Conversely, if a person invests money in a bank or elsewhere, the bank pays the person interest because the bank uses the money to finance its own investments.

This section will deal only with **simple** interest. In simple interest transactions, interest is paid on the original amount borrowed.

Formula for simple interest

How much simple interest will I pay altogether if I borrow $4000 for 10 years at an interest rate of 7% per annum? (The phrase *per annum* is Latin for 'for each year'; it is often abbreviated to p.a.)

Last year you probably learned to set out the working for simple interest in two successive steps, something like this:

$$\text{Interest paid at the end of each year} = 4000 \times 7\%$$
$$= 4000 \times 0.07$$
$$= \$280$$

$$\text{Total interest paid over 10 years} = 280 \times 10$$
$$= \$2800$$

This working can all be done in one step if we can develop a suitable formula.

Suppose I borrow $P for T years at an interest rate R. Using the same two-step approach as before:

$$\text{Interest paid at the end of each year} = P \times R$$

$$\text{Total interest paid over T years} = P \times R \times T$$
$$= PRT$$

This gives us the well-known **simple interest formula**:

$$I = PRT \qquad (\text{Interest} = \text{principal} \times \text{rate} \times \text{time})$$

Using this formula, the calculation can now be set out in one step:

$$I = PRT$$
$$= 4000 \times 7\% \times 10 \qquad (\textit{Note}: \text{The interest rate } R \text{ is } 7\%, \text{ not } 7.)$$
$$= 4000 \times 0.07 \times 10$$
$$= \$2800$$

Note: The interest rate is given per year, so the time must also be written in years. In some books R is written as $r\%$.

Example 9

Find the simple interest on $8000 for eight years at 9.5% p.a.

Solution

$I = PRT$
$= 8000 \times 9.5\% \times 8$
$= 8000 \times 0.095 \times 8$
$= \$6080$

Reverse use of the simple interest formula

There are four pronumerals in the formula $I = PRT$. If the values of any three are known, then substituting into the simple interest formula allows the fourth value to be found.

Example 10

Jessie borrows $3000 from her parents to help buy a car. They agree that she should only pay simple interest. Five years later she pays them back $3600, which includes simple interest on the loan. What was the interest rate?

Solution

The total interest paid was $600, the principal was $3000 and the time was 5 years.

$I = PRT$
$600 = 3000 \times R \times 5$
$600 = 15\,000 \times R$
$R = \dfrac{600}{15\,000} \times 100\%$ (Interest rates are normally written as percentages.)
$= 4\%$

Simple interest formula

- Suppose that a principal P is invested for T years at an interest rate R p.a. Then the total interest I is given by:

    $I = PRT$

    Remember that R is a percentage. If the interest rate is 5%, then $R = 0.05$.
- If the interest rate R is given per year, the time T must be given in years.
- The formula has four pronumerals. If any three are known, the fourth can be found by substitution.

Exercise 3C

Example 9

1 $12 000 is invested at 7% p.a. simple interest for five years.

a How much interest will be earned each year?

b Find how much interest will be earned over the five-year period.

2 $2000 is invested at 6.75% p.a. simple interest for three years.

a How much interest will be earned each year?

b Find how much interest will be earned over the three-year period.

3 Find the total simple interest earned in each of these investments.

a $400 for three years at 6% p.a.

b $850 for six years at 4.5% p.a.

c $15 000 for 12 years at 8.4% p.a.

4 Find the time T for $2000 of simple interest on a principal of $8000 at a rate of 5% p.a.

Example 10

5 Find the rate R p.a. for $7200 of simple interest on a principal of $8000 over 12 years.

6 Find the principal P for $3500 of simple interest at a rate of 7% p.a. over 10 years.

7 Calculate the missing entries for these simple interest investments.

	Principal	Rate p.a.	Time in years	Total interest
a	$10 000	8%		$3200
b	$4 400 000	$7\frac{1}{2}$%		$3 960 000
c	$5000		6	$1350
d	$260 000		8	$83 200
e		6%	5	$900
f		3.6%	4	$115.20

8 Sartoro invested $80 000 in a building society that pays 6.5% p.a. simple interest. Over the years, the investment has paid him $57 200 in interest. How many years has he had the investment?

9 Madeline has received $168 000 in total simple interest payments on an investment of $400 000 that she made six years ago. What rate of interest has the bank been paying?

10 An investor wishes to earn $240 000 interest over a five-year period from an account that earns 12.5% p.a. simple interest. How much does the investor have to deposit into the account?

11 Regan has arranged to borrow $10 000 at 9.5% p.a. for four years. She will pay simple interest to the bank every year for the loan, with the principal remaining unchanged. How much interest will Regan pay over the four years of the loan?

12 Tyler intends to live on the interest on an investment with the bank at 8.6% p.a. simple interest. She will receive $68 000 simple interest every year from the investment. How much money has she invested?

3D Percentage increase and decrease

When a quantity is increased or decreased, the change is often expressed as a percentage of the original amount.

This section introduces a concise method of solving problems about percentage increase and decrease. This method will be applied in various ways throughout the remaining sections of the chapter.

Percentage increase

This evening's news reported that shares in the Consolidated Nail Factory Pty Ltd were selling at $12.00 yesterday, but rose 14.5% today.

Rather than calculating the price increase and adding it on, the calculation can be done in one step by using the fact that the new price is $100\% + 14.5\% = 114.5\%$ of the old price.

$$\begin{aligned}
\text{New price} &= \text{old price} \times 114.5\% \\
&= 12 \times 114.5\% \\
&= 12 \times 1.145 \\
&= \$13.74
\end{aligned}$$

Example 11

The number of patients admitted to St Spyridon's Hospital this year suffering from pneumonia is 56% greater than the number admitted for this condition last year.
If 245 pneumonia patients were admitted last year, how many were admitted this year?

Solution

This year's total is $100\% + 56\% = 156\%$ of last year's total.

$$\begin{aligned}
\text{This year's total} &= 245 \times 156\% \\
&= 245 \times 1.56 \\
&\approx 382 \ (\text{correct to the nearest whole number})
\end{aligned}$$

Inflation

The prices of goods and services in Australia usually increase by a small amount every year.

This gradual rise in prices is called **inflation**, and is measured by taking the average percentage increase in the prices of a large range of goods and services.

Other things such as salaries and pensions are often adjusted automatically every year to take account of inflation.

Example 12

The war-ravaged nation of Zerbai is experiencing inflation of 35% p.a. as a result of overspending on its navy and air force. Inflation of 35% means that, on average, prices are increasing by 35% every year.

a If the price of water is adjusted in line with inflation, what will an annual bill of $600 become in the next year?

b What should an annual salary of $169 000 in one year increase to in the following year if it is adjusted to keep pace with inflation?

Solution

Next year's prices are 100% + 35% = 135% of last year's prices.

a Next year's bill = 600×1.35

= \$810

b Next year's salary = $169\,000 \times 1.35$

= \$228 150

Percentage decrease

The same method can be used to calculate percentage decreases. For example, suppose that 35% of a farmer's sheep station, which has an area of 7500 hectares, went under water during the recent floods.

We can calculate how much land remained above the water for his stock to graze:

100% − 35% = 65% of his land remained above water.

Area remaining above water = $7500 \times 65\%$

$= 7500 \times 0.65$

$= 4875$ hectares

Example 13

The company that Yuri Ivanov works for is going through hard times and has decreased all its salaries by 12%. Yuri is attempting to cut every one of his expenses by the same percentage.

a His family's weekly grocery bill averages 450 roubles. What should he try to reduce the weekly price of his groceries to?

b His monthly rental is 18 000 roubles. If he moves apartments, what monthly rental should he try to find?

Solution

Yuri's new salary is 100% − 12% = 88% of his original salary.

a New weekly grocery bill = 450×0.88

= 396 roubles

b New monthly rental = $18\,000 \times 0.88$

= 15 840 roubles

> **→ Percentage increase and decrease**
>
> - To increase an amount by, for example, 15%, multiply by $1 + 0.15 = 1.15$.
> - To decrease an amount by, for example, 15%, multiply by $1 - 0.15 = 0.85$.

Finding the rate of increase or decrease

Example 14

Suppose that the rainfall has increased from 480 mm p.a. to 690 mm p.a. What rate of increase is this?

Solution

Method 1

Find the actual increase by subtraction, and then express the increase as a percentage of the original rainfall.

Increase $= 210$ mm

$$\text{Percentage increase} = \frac{\text{increase}}{\text{original rainfall}} \times 100\%$$
$$= \frac{210}{480} \times 100\%$$
$$= 43.75\%$$

Method 2

Express the new value as a percentage of the original value, and then subtract 100%.

$$\frac{\text{new rainfall}}{\text{old rainfall}} = \frac{690}{480} \times 100\%$$
$$= 143.75\%$$

So the rainfall has increased by 43.75%.

Note: Percentage decrease is sometimes represented as a negative percentage increase or, in other contexts, as a negative percentage change. This understanding is consistent with the formula, amount of change = new amount – old amount, where the new amount is less than the old amount in situations of decrease.

Reversing the process to find the original amount

Example 15

The Wind Energy Company recently announced that this year's profit of $1 400 000 constituted a 35% increase on last year's profit. What was last year's profit?

Solution

This year's profit is 100% + 35% = 135% of last year's profit.

Hence this year's profit = last year's profit × 1.35

Reversing this, last year's profit = this year's profit ÷ 1.35

$$= 1\,400\,000 \div 1.35$$

$$\approx \$1\,037\,037, \text{ correct to the nearest dollar}$$

Thus, to find the original amount, we divide by 1.35, because dividing by 1.35 is the reverse of multiplying by 1.35.

Exactly the same principle applies when an amount has been decreased by a percentage.

Example 16

The price of shares in the Fountain Water Company has decreased by 15% over the last month to $52.70. What was the price a month ago?

Solution

The new share price is 100% − 15% = 85% of the old share price.

Hence   new price = old price × 0.85

Reversing this,   old price = new price ÷ 0.85

$$= 52.70 \div 0.85$$

$$= \$62$$

Finding the original amount

• To find the original amount after an increase of, for example, 15%, divide by 1.15.
• To find the original amount after a decrease of, for example, 15%, divide by 0.85.

Discounts

It is very common for a shop to **discount** the price of an item. This is done to sell stock of a slow-moving item more quickly, or simply to attract customers into the shop.

Discounts are normally expressed as a percentage of the original price.

Example 17

The Elegant Shirt Shop is closing down and has discounted all its prices by 35%.

a What is the discounted price of a shirt whose original price is:

   i $120?   **ii** $75?

b What was the original price of a shirt whose discounted price is $92.30?

Solution

a The discounted price of each item is $100\% - 35\% = 65\%$ of the old price.

  i   Hence discounted price $=$ old price $\times 0.65$

$$= 120 \times 0.65$$
$$= \$78$$

  ii   discounted price $= 75 \times 0.65$

$$= \$48.75$$

b From part **a**, discounted price $=$ old price $\times 0.65$

Reversing this,   old price $=$ discounted price $\div 0.65$

$$= 92.30 \div 0.65$$
$$= \$142$$

The GST

In 1999, the Australian Government introduced a Goods and Services Tax, or GST for short. This tax applies to nearly all goods and services in Australia.

The current rate is 10% on the pre-tax price of the goods or service.

- When GST applies, it is added to the pre-tax price. This is done by multiplying the pre-tax price by 1.10.

- Conversely, if a quoted price already includes GST, the pre-tax price is obtained by dividing the quoted price by 1.10.

Example 18

The current GST rate is 10% of the pre-tax price.

  a If a domestic plumbing job costs \$630 before GST, how much will it cost after adding GST, and how much tax is paid to the Government?

  b I paid \$70 for petrol recently. What was the price before adding GST, and what tax was paid to the Government?

Solution

The after-tax price is 110% of the pre-tax price.

a   After-tax price $= 630 \times 1.10$

$$= \$693$$

Tax $= 693 - 630$

$$= \$63$$

Note: 10% of \$630 is \$63.

b Pre-tax price $= 70 \div 1.10$ (Divide by 1.10 to reverse the process.)

$$\approx \$63.64$$

$$\text{Tax} \approx 70 - 63.64$$

$$= \$6.36$$

Exercise 3D

Example 11

1 Traffic on all roads has increased by an average of 8% during the past 12 months. By multiplying by $108\% = 1.08$, estimate the number of vehicles now on a road given the number of vehicles the road carried a year ago was:

    a 10000 per day   **b** 80000 per day   **c** 148000 per day

Example 12

2 Prices have increased with inflation by an average of 3.8% since the same time last year. Find today's price for an item that one year ago cost:

    a $200   **b** $1.68   **c** $345000   **d** $9430

Example 13

3 Rainfall across one state has decreased over the last five years by about 24%. By multiplying by $76\% = 0.76$, estimate, correct to the nearest 10 mm, the annual rainfall this year at a place where the rainfall five years ago was:

    a 1000 mm   **b** 250 mm   **c** 680 mm   **d** 146 mm

Example 14

4 Admissions to different wards of St Luke's Hospital mostly rose from 2006 to 2007, but by quite different percentage amounts. Find the percentage increase or decrease in wards where the numbers during 2006 and 2007, respectively, were:

    a 50 and 68   **b** 120 and 171   **c** 92 and 77   **d** 24 and 39

5 In another state, the percentage decrease in rainfall over the last five years has been quite variable, and in some cases, rainfall has actually increased. Find the rate of decrease or increase if the annual rainfall five years ago and now are, respectively:

    a 500 mm and 410 mm   **b** 920 mm and 960 mm

    c 140 mm and 155 mm   **d** 420 mm and 530 mm

Example 15

6 Radix Holdings Pty Ltd recently issued bonus shares to its shareholders. Each shareholder received an extra 12% of the number of shares currently held. Find the original holding of a shareholder who now holds:

    a 672 shares   **b** 4816 shares   **c** 1000 shares   **d** 40200 shares

Example 17

7 A clothing store is offering a 15% discount on all its summer stock. What is the discounted price of an item with original price:

    a $80?   **b** $48?   **c** $680?   **d** $1.60?

Example 17

8 A shoe store is offering a 35% discount at its end-of-year sale. Find the original price of an item whose discounted price is:

a $1820 **b** $279.50 **c** $1.56 **d** $20.80

9 A research institute is trying to find out how much water Lake Grendel had 1000 years ago. The lake now contains 24 000 megalitres, but there are various conflicting theories about the percentage change over the last 1000 years. Find how much water the lake had 1000 years ago, correct to the nearest 10 megalitres, if in the last 1000 years the volume has:

a risen by 80% **b** fallen by 28% **c** risen by 140% **d** fallen by 4%

Example 15, 16

10 Mr Brown has a spreadsheet showing the value at which he bought his various parcels of shares, the value at 31 December last year, and the percentage increase or decrease in their value. (Decreases are shown with a negative sign.) Unfortunately, a virus has corrupted one entry in each row of his spreadsheet. Help him by calculating the missing values, correct to the nearest cent, and the missing percentages, correct to 2 decimal places.

	Value at purchase	Value at 31 December	Percentage increase
a	$12 000		30%
b	$28 679.26		−62%
c	$5267.70		289.14%
d		$72 000	20%
e		$26 000	−22%
f		$112 000	346.5%
g		$15 934	−91.38%
h	$60 000	$81 000	
i	$98 356.68	$14 321.57	
j	$14 294.12	$2314.65	

Example 18

11 The GST is a tax on most goods and services at the rate of 10% of the pre-tax price.

a Find the after-tax price on goods or services whose pre-tax price is:

i $170 **ii** $4624 **iii** $68 920 **iv** $6.80

b Find the pre-tax price on goods or services whose after-tax price is:

i $550 **ii** $7821 **iii** $192 819 **iv** $5.28

c Find the after-tax price on goods or services on which the GST is:

i $60 **ii** $678.20 **iii** $54 000 **iv** $0.93

12 a A shirt originally priced at $45 was increased in price by 100%. What percentage discount will restore it to its original price?

b The daily passenger total of the Route 58 bus was 460, and in one year, it increased by 24%. What percentage decrease next year would restore it to its original passenger total?

c Shafqat had savings of $6000, but he spent 35% of this last year. By what percentage of the new amount must he increase his savings to restore them to their original value?

d The profit of the Arborville Gelatine Factory was $86400, but it then decreased by 42%. By what percentage must the profit increase to restore it to its original value?

13 a Find, correct to 2 decimal places, the percentage decrease necessary to restore a quantity to its original value if it has been increased by:

   i 10%   **ii** 22%   **iii** 240%   **iv** 2.3%

  b Find, correct to 2 decimal places, the percentage increase necessary to restore a quantity to its original value if it has been decreased by:

   i 10%   **ii** 22%   **iii** 75%   **iv** 2.3%

3E Repeated increase and decrease

The method introduced in the last section becomes very useful when two or more successive increases or decreases are applied, because the original amount can simply be multiplied successively by two or more factors. Here is a typical example.

Repeated increase

Example 19

The population of Abelsburg in the census three years ago was 46430. In the three years after the census, however, its population has risen by 6.2%, 8.5% and 13.1%, respectively.

a What was its population one year after the census?
b What was its population two years after the census?
c What is its population now, three years after the census?
d What was the percentage increase in population over the three years, correct to the nearest 0.1%?

Solution

a One year after the census, the population was 106.2% of its original value.
Hence population after one year $= 46430 \times 1.062$
$$\approx 49309, \text{ correct to the nearest whole number}$$

b Two years afterwards, the population was 108.5% of its value one year afterwards.
Hence population after two years $= (46430 \times 1.062) \times 1.085$
$$\approx 53500, \text{ correct to the nearest whole number}$$

Note: Do not use the approximation from part **a** to calculate part **b**. Either start the calculation again, or use the unrounded value from part **a**.

(continued over page)

c Three years afterwards, the population was 113.1% of its value two years afterwards.

Hence population after three years $= (46\,430 \times 1.062 \times 1.085) \times 1.131$

$\approx 60\,508$, correct to the nearest whole number

d Population three years afterwards

$=$ original population $\times 1.062 \times 1.085 \times 1.131$

$\approx$ original population $\times 1.303$

$\approx$ original population $\times 130.3\%$

Hence the population has increased over the three years by about 30.3%.

Note: The percentage increase of 30.3% is significantly larger than the sum of the three percentage increases,

$6.2\% + 8.5\% + 13.1\% = 27.8\%$.

Note: The answer to part **d** does not depend on what the original population was.

Repeated decrease

The same method can be applied just as easily to percentage decreases, as demonstrated in the next example.

Example 20

Teresa invested $75\,000 from her inheritance in a mining company that has not been very successful. In the first year, she lost 55% of the money, and in the second year, she lost 72% of what remained.

a How much does she have left after one year?

b How much does she have left after two years?

c What percentage of the original inheritance has she lost over the two years?

Solution

a One year later, the percentage remaining was $100\% - 55\% = 45\%$.

Hence amount left after one year $= 75\,000 \times 0.45$

$= \$33\,750$

b Two years later, she had $100\% - 72\% = 28\%$ of what she had after one year.

Hence amount left after two years $= 75\,000 \times 0.45 \times 0.28$

$= \$9450$

c Amount left after two years $=$ original investment $\times 0.45 \times 0.28$

$=$ original investment $\times 0.126$

$=$ original investment $\times 12.6\%$

So she has lost $100\% - 12.6\% = 87.4\%$ of her investment over the two years.

Combinations of increases and decreases

Some problems involve both increases and decreases. They can be solved in exactly the same way.

Example 21

The volume of water in the Welcome Dam has varied considerably over the last three years. During the first year the volume rose by 27%, then it fell 43% during the second year, and it rose 16% in the third year.

a What is the percentage increase or decrease over the three years, correct to the nearest 1%?

b If there were 366 500 megalitres of water in the dam three years ago, how much water is in the dam now, correct to the nearest 500 megalitres?

Solution

a Final volume = original volume × 1.27 × 0.57 × 1.16

$\qquad\qquad\quad$ ≈ original volume × 0.84

Since 0.84 < 1, the volume has decreased. The percentage decrease is about 100% − 84% = 16% over the three years.

b Final volume = original volume × 1.27 × 0.57 × 1.16

$\qquad\qquad\quad$ = 366 500 × 1.27 × 0.57 × 1.16

$\qquad\qquad\quad$ ≈ 308 000 megalitres, correct to the nearest 500 megalitres.

This time the sum of the percentages is 27% − 43% + 16% = 0%, but the volume of water has changed.

Repeated increases and decreases

- To apply successive increases of, for example, 15%, 24% and 38% to a quantity, multiply the quantity by 1.15 × 1.24 × 1.38.

- To apply successive decreases of, for example, 15%, 24% and 38% to a quantity, multiply the quantity by 0.85 × 0.76 × 0.62.

Reversing the process to find the original amount

As shown before, division reverses the process and allows us to find the original amount, as in the following example.

Example 22

A clothing shop discounted a shirt by 45% a month ago, and has now discounted the reduced price by 20%.

a What was the total discount on the shirt?

b If the shirt is now selling for $61.60, what was the original price of the shirt?

Solution

a After the first discount, the price was 100% – 45% = 55% of the original price.
After the second discount, the price was 100% – 20% = 80% of the reduced price.
Thus final price = original price × 0.55 × 0.80
= original price × 0.44

So the total discount is 100% – 44% = 56%.

b Reversing this, original price = final price ÷ 0.44
= 61.60 ÷ 0.44
= $140

➡ Reversing repeated increases and decreases

- To find the original quantity after successive increases of, for example, 15%, 24% and 38%, divide the final quantity by (1.15 × 1.24 × 1.38).
- To find the original quantity after successive decreases of, for example, 15%, 24% and 38%, divide the final quantity by (0.85 × 0.76 × 0.62).

Successive divisions

Calculations involving brackets can be tricky to handle when using the calculator.

For example, working with the figures in the summary box above, suppose that a population has increased by 15%, 24% and 38% in three successive years and is now 50 000. Then:

original population = 50 000 ÷ (1.15 × 1.24 × 1.38)
≈ 25 408

We suggest that it is easier to avoid brackets and divide 50 000 successively by 1.15, 1.24 and 1.38. In effect, the working then goes like this:

original population = 50 000 ÷ (1.15 × 1.24 × 1.38)
= 50 000 ÷ 1.15 ÷ 1.24 ÷ 1.38
≈ 25 408

Try the calculation both ways and see which you find more natural.

Using the power button on the calculator

When a quantity is repeatedly increased or decreased by the same percentage, the power button makes calculations quicker. Make sure that you can use it correctly by experimenting with simple calculations like $3^4 = 81$ and $2^5 = 32$.

Example 23

The drought in Paradise Valley has been getting worse for years. Each year for the last five years, the rainfall has been 8% less than the previous year's rainfall.

a What is the percentage decrease in rainfall over the five years?

b If the rainfall this year was 458 mm, what was the rainfall five years ago?

Solution

Each year the rainfall is 92% of the previous year's rainfall.

a Final rainfall = original rainfall × 0.92 × 0.92 × 0.92 × 0.92 × 0.92

$$= \text{original rainfall} \times (0.92)^5$$
$$\approx \text{original rainfall} \times 0.659$$

So rainfall has decreased by about $100\% - 65.9\% = 34.1\%$ over the five years.

b From part **a**, final rainfall = original rainfall × $(0.92)^5$

Reversing this, original rainfall = final rainfall ÷ $(0.92)^5$

$$= 458 \div (0.92)^5$$
$$\approx 695 \text{ mm}$$

Exercise 3E

Example 20

1 Oranges used to cost $2.80 per kg, but the price has increased by 5%, 10% and 12% in three successive years. Multiply by $1.05 \times 1.1 \times 1.12$ to find their price now.

2 The dividend per share in the Electron Computer Software Company has risen over the last four years by 10%, 15%, 5% and 12%, respectively. Find the latest dividend received by a shareholder whose dividend four years ago was:

a $1000   **b** $1678   **c** $28.46   **d** $512.21

3 Rates in Bullimbamba Shire have risen 7% every year for the last seven years. Find the rates now payable by a landowner whose rates seven years ago were:

a $1000   **b** $346   **c** $2566.86   **d** $788.27

Example 20

4 A tree, whose original foliage was estimated to have a mass of 1500 kg, lost 20% of its foliage in a storm, then lost 15% of what was left in a storm the next day, then lost 40% of what was left in a third storm. Estimate the remaining mass of foliage.

5 Shares in the Metropolitan Brickworks have been falling by 23% per year for the last five years. Find the present worth of a parcel of shares whose original worth five years ago was:

a $1000   **b** $120 000   **c** $25 660   **d** $3 860 000

Example 21

6 a A shirt is discounted by 50% and the resulting price is then increased by 50%. By what percentage is the price increased or decreased from its original value?

   b The price of a shirt is increased by 50% and the resulting price is then decreased by 50%. By what percentage is the price increased or decreased from its original value?

   c Can you explain the relationship between your answers to parts **a** and **b**?

Example 22

7 A book shop has a 50% sale on all stock, and has a container of books with the sale price reduced by a further factor of 16%.

   a What was the total discount on each book in the container?

   b If a book in the container is now selling for $10.50, what was its original price?

8 Calculate the total increase or decrease in a quantity when:

   a it is increased by 20% and then decreased by 20%

   b it is increased by 80% and then decreased by 80%

   c it is increased by 10% and then decreased by 10%

   d it is increased by 30% and then decreased by 30%

Example 23

9 The price of gemfish has been rising. The price has risen by 10%, 15% and 35% in three successive years, and they now cost $24 per kg. Find:

   a the price one year ago   **b** the price two years ago

   c the original price three years ago

10 The crime rate in Gotham City has been rising each decade. In the last four decades the number of robberies has risen by 64%, 223%, 75% and 12%. If there are now 958 robberies per year, find how many robberies per year there were:

   a one decade ago   **b** two decades ago

   c three decades ago   **d** four decades ago

11 A particular strain of bacteria increases its population on a certain prepared Petri dish by 34% every hour. Calculate the size of the original population four hours ago if there are now 56 000 bacteria.

12 Flash Jim is desperate to attract customers to his used car yard. He has cut prices recently by 5%, then by 10%, then by 24%. Find, correct to the nearest $100, the original price of a used car now priced at:

   a $10 000   **b** $35 000   **c** $4600   **d** $76 800

13 The radioactivity of any sample of the element strontium-90 decreases by 90.75% every century. Find the percentage reduction in radioactivity over each of the periods given below. (Calculate percentages correct to 3 decimal places.)

   a Two centuries   **b** Three centuries   **c** Five centuries

14 Here is a table of the annual inflation rate in Australia in the years ending 30 June 2001 to 30 June 2006 (from the Reserve Bank of Australia website).

Year	2001	2002	2003	2004	2005	2006
Inflation rate	6.0%	2.8%	2.7%	2.5%	2.5%	4.0%

Suppose the salary for certain jobs at Company X rises on the 1 July every year, in line with Australia's inflation rate for the financial year just past (ending 30 June).

a A junior secretary earned $40 000 from 1 July 2000 to 30 June 2001. Determine how much someone in that position would earn from:

   i   1 July 2001 to 30 June 2002

   ii 1 July 2006 to 30 June 2007

b A team manager was on an annual salary of $100 000 from 1 July 2006 to 30 June 2007. Determine how much someone in that position would earn:

   i   in the previous financial year

   ii from 1 July 2003 to 30 June 2004

15 At the start of the trading day, shares of a particular stock decrease in value by 20%. However, by the end of the day the shares 'recover' and record a 15% increase from its lowest value. Determine the percentage decrease in the value of the shares over the course of the day.

3F Compound interest

In all the examples in this section, the interest is compounded annually. This means that at the end of each year, the interest earned is added to the principal invested or borrowed. That increased amount then becomes the amount on which interest is earned in the following year. This is called **compound interest**.

Example 24

Gail has invested $100 000 for six years with the Mountain Bank. The bank pays her interest at the rate of 7.5% p.a., compounded annually.

a How much will the investment be worth at the end of one year?

b How much will the investment be worth at the end of two years?

c How much will the investment be worth at the end of six years?

d What is the percentage increase on her original investment at the end of six years?

e What is the total interest earned over the six years?

f What would the simple interest on the investment have been, assuming the same interest rate of 7.5% p.a.?

Solution

Each year the investment is worth 107.5% of its value the previous year.

a   Amount at the end of one year $= 100\,000 \times 1.075$

$$= \$107\,500$$

b   Amount at the end of two years $= 100\,000 \times 1.075 \times 1.075$

$$= 100\,000 \times (1.075)^2$$

$$= \$115\,562.50$$

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c Amount at the end of six years $= 100\,000 \times (1.075)^6$

$$\approx \$154\,330.15$$

d Final amount $=$ original amount $\times (1.075)^6$

$$\approx \text{original amount} \times 1.5433$$

So the total increase over six years is about 54.33%.

e Total interest $\approx 154\,330.15 - 100\,000$

$$= \$54\,330.15$$

f Simple interest $= PRT$

$$= 100\,000 \times 0.075 \times 6$$

$$= \$45\,000$$

Note: Compound interest for two or more years is always greater than simple interest for two or more years.

Compound interest on a loan

Exactly the same principles apply when someone borrows money from a bank and the bank charges compound interest on the loan. If no repayments are made, the amount owing compounds in the same way, and can grow quite rapidly.

This is shown in the following example.

Example 25

Hussain is setting up a plumbing business and needs to borrow $150\,000 from a bank to buy a truck and other equipment. The bank will charge him interest of 11% p.a., compounded annually. Hussain will pay the whole loan off all at once four years later.

a How much will Hussain owe the bank at the end of four years?

b What is the percentage increase in the money owed at the end of four years?

c What is the total interest that Hussain will pay on the loan?

d What would the simple interest on the loan have been, assuming the same interest rate of 11% p.a.?

Solution

Each year Hussain owes 111% of what he owed the previous year.

a Amount at the end of four years $= 150\,000 \times (1.11)^4$

$$\approx \$227\,710.56$$

b Final amount $=$ original amount $\times (1.11)^4$

$$\approx \text{original amount} \times 1.5181$$

So the total increase over four years is about 51.81%.

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c Total interest $\approx 227\,710.56 - 150\,000$

$\qquad\qquad\qquad = \$77\,710.56$

d Simple interest $= PRT$

$\qquad\qquad\qquad\quad = 150\,000 \times 0.11 \times 4$

$\qquad\qquad\qquad\quad = 66\,000$

Note: Making no repayments on a loan that is accruing compound interest is a risky business practice because, as this example makes clear, the amount owing grows with increasing rapidity as time goes on. This is particularly relevant to credit card debt.

> ### Compound interest
>
> Suppose that an amount P is invested at an interest rate, say 7%, compounded annually. The interest is calculated each year on the new balance. The new balance is obtained by adding on 7% of the balance of the previous year. Thus:
>
> $\qquad$ amount after four years $= P \times 1.07 \times 1.07 \times 1.07 \times 1.07$
>
> $\qquad\qquad\qquad\qquad\qquad\qquad\quad = P \times (1.07)^4$

Reversing the process to find the original amount

As always, division reverses the process to find the original amount, as in the following example.

Example 26

Eleni wants to borrow money for three years to start a business, and then pay all the money back, with interest, at the end of that time. The bank will not allow her final debt, including interest, to exceed $300\,000$. Interest is 9% p.a., compounded annually. What is the maximum amount that Eleni can borrow?

Solution

Each year Eleni will owe 109% of what she owed the previous year.

Hence $\qquad\qquad$ final debt $=$ original debt $\times 1.09 \times 1.09 \times 1.09$

$\qquad\qquad\qquad$ final debt $=$ original debt $\times (1.09)^3$

Reversing this, original debt $=$ final debt $\div (1.09)^3$

$\qquad\qquad\qquad\qquad\qquad = 300\,000 \div (1.09)^3$

$\qquad\qquad\qquad\qquad\qquad \approx \$231\,655$

Exercise 3F

Example 24

1 a Christine invested $100 000 for six years at 5% p.a. interest, compounded annually.

    i By multiplying by 1.05, find the value of the investment after one year.

    ii By multiplying by $(1.05)^2$, find the value of the investment after two years.

    iii By multiplying by $(1.05)^6$, find the value of the investment after six years.

    iv Find the percentage increase in the investment over the six years.

    v Find the total interest earned over the six years.

  b Find the simple interest on the principal of $100 000 over the six years at the same rate of 5% p.a.

Example 25

2 a Gary has borrowed $200 000 for six years at 8% p.a. interest, compounded annually, in order to start his indoor decorating business. He intends to pay the whole amount back, plus interest, at the end of the six years.

    i Find the amount owing after one year.

    ii Find the amount owing after two years.

    iii Find the amount owing after six years.

    iv Find the percentage increase in the loan over the six years.

    v Find the total interest charged over the six years.

  b Find the simple interest on the principal of $200 000 over the six years at the same rate of 8% p.a.

3 A couple take out a housing loan of $320 000 over a period of 20 years. They make no repayments over the 20-year period of the loan. Compound interest is payable at $6\frac{1}{2}$% p.a., compounded annually. How much would they owe at the end of the 20-year period, and what is the total percentage increase?

4 The population of a city increases annually at a compound rate of 3.2% for five years. If the population is 21 000 initially, what is it at the end of the five-year period, and what is the total percentage increase?

5 a Find the compound interest on $1000 at 5% p.a., compounded annually for 200 years.

  b Find the simple interest on $1000 at 5% p.a. for 200 years.

6 $10 000 is borrowed for five years and compound interest at 10% p.a. is charged by the lender.

  a How much money is owed to the lender after the five-year period?

  b How much of this amount is interest?

Example 26

7 Money borrowed at 8% p.a. interest, compounded annually, grew to $100 000 in four years.

  a Find the total percentage increase.

  b Hence find the original amount invested.

8 Suzette wants to invest a sum of money now so that it will grow to $180 000 in 10 years' time. How much should she invest now, given that the interest rate is 6% compounded annually?

9 A bank offers 8% p.a. compound interest. How much needs to be invested if the investment is to be worth $100 000 in:

a 10 years?   **b** 20 years?   **c** 25 years?   **d** 100 years?

10 A man now owes the bank $56 000, after having taken out a loan five years ago. Find the original amount that he borrowed if the rate of interest per annum, compounded annually, has been:

a 3%   **b** 5.6%   **c** 9.25%   **d** 15%

11 Mr Brown has had further difficulties with the virus that attacks his spreadsheet entries. Here is the remains of a spreadsheet that he prepared in answer to questions from business friends. The spreadsheet calculated interest compounded annually, on various amounts, at various interest rates, for various periods of time. Help him reconstruct the missing entries.

	Principal	Rate p.a.	Time in years	Final amount	Total interest
a	$4000	6%	20		
b	$10 000	8.2%	15		
c	$2 000 000	4.8%	10		
d		6%	20	$4000	
e		8.2%	15	$10 000	
f		4.8%	10	$2 000 000	

12 Ms Smith invested $50 000 at 6% p.a. interest, compounded annually, for four years. The tax department wants to know exactly how much interest she earned each year. Calculate these figures for Ms Smith.

13 Mrs Robinson has taken out a loan of $300 000 at 8% p.a. interest, compounded annually, for four years. She wants to know exactly how much interest she will be charged each year so that she can include it as a tax deduction in her income tax return. Calculate these figures for Mrs Robinson.

14 **a** Find the total percentage growth in a compound interest investment:

  i   at 15% for two years   **ii** at 10% for three years

  iii at 6% for five years   **iv** at 5% for six years

  v   at 3% for 10 years   **vi** at 2% for 15 years

  b What do you observe about these results?

15 A couple took out a six-year loan to start a business. For the first three years, compound interest of 8% p.a. was charged. For the second three years, compound interest of 12% p.a. was charged. Find the total percentage increase in the amount owing.

16 One six-year loan attracts compound interest calculated at 2%, 4%, 6%, 8%, 10% and 12% in successive years. Another six-year loan attracts compound interest calculated at 12%, 10%, 8%, 6%, 4% and 2% in successive years. Find the total percentage increase in money owing in both cases, compare the two results, and explain what has happened.

17 An investment at an interest rate of 10% p.a., compounded annually, returned interest of $40 000 after five years. Calculate the original amount invested.

3G Depreciation

Depreciation occurs when the value of an asset reduces as time passes. For example, a company may buy a car for $40 000, but after four years the car will be worth a lot less, because the motor will be worn, the car will be out of date, the body and interior may have a few scratches, and so forth.

Accountants usually make the assumption that an asset such as a car depreciates at the same rate every year. This rate is called the **depreciation rate**. It works like compound interest. The depreciation is applied each year to the current value. In the following example, the depreciation rate is taken to be 20%.

Example 27

The Medicine Home Delivery Company bought a car four years ago for $40 000, and assumed that the value of the car would depreciate at 20% p.a.

a What value did the car have at the end of two years?
b What value does the car have now, after four years?
c What is the percentage decrease in value over the four years?
d What is the average depreciation on the car over the four years? (Express your answer in dollars per year)

Solution

The value each year is taken to be $100\% - 20\% = 80\%$ of the value in the previous year.

a Value at the end of two years $= 40\,000 \times 0.80 \times 0.80$
$$= 40\,000 \times (0.80)^2$$
$$= \$25\,600$$

b Value at the end of four years $= 40\,000 \times 0.80 \times 0.80 \times 0.80 \times 0.80$
$$= 40\,000 \times (0.80)^4$$
$$= \$16\,384$$

(continued over page)

c Final value = original value $\times (0.80)^4$

= original value $\times 0.4096$

Hence the percentage decrease over four years is $100\% - 40.96\% = 59.04\%$

d Depreciation over four years = $40\,000 - 16\,384$

= $\$23\,616$

Average depreciation per year = $23\,616 \div 4$

= $\$5904$ per year

Depreciation

Suppose that an asset with original value P depreciates at, for example, 7% every year. To find the depreciated value, decrease the current value by 7% each year. Thus:

value after four years = $P \times 0.93 \times 0.93 \times 0.93 \times 0.93$

= $P \times (0.93)^4$

Reversing the process to find the original amount

If we are given a depreciated value and the rate of depreciation, we can find the original value by division.

Example 28

A school buys new computers every four years. At the end of the four years, it offers them for sale to the students on the assumption that they have depreciated at 35% p.a. (per annum). The school is presently advertising some computers at $400 each.

a What did each computer cost the school originally?

b What is the average depreciation on each computer, in dollars per year?

Solution

a Each year a computer is worth $100\% - 35\% = 65\%$ of its value the previous year.
Hence final value = original value $\times 0.65 \times 0.65 \times 0.65 \times 0.65$

final value = original value $\times (0.65)^4$

Reversing this, original value = final value $\div (0.65)^4$

= $400 \div (0.65)^4$

$\approx \$2241$

b Depreciation over four years $\approx 2241 - 400$

= $\$1841$

Average depreciation per year $\approx 1841 \div 4$

$\approx \$460$

Exercise 3G

Note: The depreciation rates in this exercise are taken from the Australian Taxation Office's Schedule of Depreciation. These are intended for income tax purposes. A company may have reasons to use different rates.

Example 27

1 The landlord of a large block of home units purchased washing machines for its units four years ago for $400 000, and is assuming a depreciation rate of 30%.

  a By multiplying by 0.70, find the value after one year.

  b By multiplying by $(0.70)^2$, find the value after two years.

  c By multiplying by $(0.70)^3$, find the value after three years.

  d By multiplying by $(0.70)^4$, find the value after four years.

  e What is the percentage decrease in value over the four years?

  f What is the average depreciation on the washing machines, in dollars p.a. over the four years?

2 The Hungry Hour Cafe purchased an air-conditioning system six years ago for $250 000, and is assuming a depreciation rate of 20%.

  a Find the value after one year.

  b Find the value after two years.

  c Find the value after six years.

  d What is the percentage decrease in value over the six years?

  e What is the average depreciation, in dollars p.a., on the air-conditioning system over the six years?

3 A business spent $560 000 installing alarms at its premises and then depreciated them at 20% p.a. Find the value after five years, and the percentage depreciation of their value.

4 The population of a sea lion colony decreases at a compound rate of 2% p.a. for 10 years. If the population is 8000 initially, what is it at the end of the 10-year period?

5 The Northern Start Bus Company bought a bus for $480 000, depreciated it at 30% p.a., and sold it again seven years later for $60 000. Was the price that they obtained better or worse than the depreciated value, and by how much?

6 The Backyard Rubbish Experts bought a fleet of small trucks for $1 340 000 and depreciated them at 22.5% p.a. Five years later they sold them for $310 000. Was the price that they obtained better or worse than the depreciated value, and by how much?

7 A landlord spent $3400 on vacuum cleaners for his block of home units and depreciated them for taxation purposes at 25% p.a. Find their value at the end of each of the first three years, and the amount of the depreciation that the landlord could claim against his taxable income for each of those three years.

8 Lara and Kate each received $100 000 from their parents. Lara invested the money at 6.2% p.a. compounded annually, whereas Kate bought a luxury car that depreciated at a rate of 20% p.a. What were the values of their investments at the end of five years?

9 Taxis depreciate at 50% p.a., and other cars depreciate at 22.5% p.a.

   a What is the total percentage depreciation on each type of vehicle after seven years?

   b What is the difference in value, to the nearest dollar, after seven years of a fleet of taxis and a fleet of other cars, if both fleets cost $1 000 000?

Example 28

10 Mr Wong's 10-year-old used car is worth $4000, and has been depreciating at 22.5% p.a. (Calculate amounts of money in whole dollars.)

   a Use division by 0.775 to find how much it was worth a year ago.

   b Find how much it was worth two years ago.

   c Find how much it was worth 10 years ago.

   d What is the total percentage depreciation on the car over the 10-year period?

   e What was the average depreciation in dollars per year over the 10-year period?

11 St Scholasticus Grammar School bought photocopying machines six years ago, which it then depreciated at 25% p.a. They are now worth $72 000.

   a How much were they worth one year ago?

   b How much were they worth two years ago?

   c How much were they worth six years ago?

   d What is the total percentage depreciation on them over the six-year period?

   e What was the average depreciation in dollars per year over the six-year period?

12 Ms Wu's seven-year-old car is worth $5600, and has been depreciating at 22.5% p.a. Calculate your answers to the nearest dollar.

   a How much was it worth four years ago?

   b How much was it worth seven years ago?

   c Ms Wu, however, only bought the car four years ago, at its depreciated value at that time. What has been Ms Wu's average depreciation in dollars over the four years she has owned the car?

   d What was the average depreciation in dollars over the first three years of the car's life?

13 I take 900 mL of a liquid and dilute it with 100 mL of water. Then I take 900 mL of the mixture and again dilute it with 100 mL of water. I repeat this process 20 times.

   a What proportion of the original liquid remains in the mixture at the end?

   b How much mixture should I take if I want it to contain 20 mL of the original liquid?

14 I take a sealed glass container and remove 80% of the air. Then I remove 80% of the remaining air. I do this process six times altogether. What percentage of the original air is left in the container?

Review exercise

1 Sarah decides to spend 40% of her weekly earnings on social activities, give 15% to her mother to repay a loan, and save the rest. She earns $84 a week.

   a How much does Sarah spend each week on social activities?

   b What percentage of her weekly earnings does she save?

2 Nick's share portfolio consists of shares, with value $10 000, in the banking industry, $3000 in mining shares and $15 000 in the gold market.

   a What percentage of his share portfolio is made up of shares in the mining sector?

   b What percentage of Nick's share portfolio are not banking shares?

3 A real estate agent charges a commission of 8.9% on every property sale.

   a If a house sells for $540 000, how much commission will the real estate agent receive, and how much is left for the seller?

   b If the real estate agent receives a commission of $8455 for selling a house, what was the selling price of the house, and what did the seller actually receive?

4 It cost the owners of the Corner Newsagency $3 500 000 to run their business last year. They recorded a profit of 4.5%.

   a What was their profit last year?

   b What was the total of their sales?

   c In the previous year, their costs were $2 750 000 and their sales were only $2 635 000. What percentage loss did they make on their costs?

5 Grant earned $1260 interest on money he had invested four years ago at a simple interest rate of 4.5% p.a. How much did Grant originally invest?

6 A country is experiencing inflation of 12% p.a.

   a If the price of bread is adjusted in line with inflation, what will an annual bread bill of $2500 become in the next year?

   b If Janienne earns $72 000 in one year and $78 000 the next, is her salary increase keeping pace with inflation?

7 A regional country medical centre has lost the services of one of its 16 doctors due to retirement, and has been unable to replace her.

   a What percentage loss is represented by the retiring doctor?

   b If the medical centre treated 400 patients per day last year and need to reduce this number by the percentage found in part **a**, how many patients per day will they be able to treat in the coming year?

8 A shop made a profit of 6.2% on total costs last year. If the actual profit was $156 000, what were the total costs and what were the total sales?

9 In the January sales, the Best Dress shop has discounted all its prices by 18%.

    a What is the discounted price of a dress with a marked price of $240?

    b What was the original price of a dress with a discounted price of $49.20?

10 The number of books in a local library varies from year to year. Three years ago, the number fell by 25%, then it rose 41% the following year, and finally rose 8% last year.

    a What is the percentage increase or decrease over the three years, correct to the nearest 1%?

    b If there were 429 000 books in the library three years ago, approximately how many books are in the library now?

11 The original asking price for a farm dropped by 30% a year ago, but did not attract a buyer. The price has now been further reduced by 15%.

    a By what percentage has the original asking price been reduced?

    b If the farm is now for sale at $2 677 500, what was the original asking price of the farm?

12 The height of a mature tree is measured on the same day each year. Each year for the last six years, the growth has been 9% less than the previous year's growth.

    a What is the percentage decrease in growth over the six years, correct to the nearest percent?

    b If the growth this year was 320 mm, what was the growth six years ago, correct to the nearest millimetre?

13 Sam's investment of $50 000 for five years earns her interest at the rate of 6.3% p.a., compounded annually.

    a How much will the investment be worth at the end of six years?

    b What is the percentage increase of her original investment at the end of six years?

    c What is the total interest earned over the six years?

    d What would the simple interest on the investment have been, assuming the same interest rate of 6.3% p.a.?

14 A company buys new company cars every three years. At the end of the three years, it offers them for sale to the employees on the assumption that they have depreciated at 30% p.a. The company is presently advertising some cars at $30 000 each.

    a What did each car cost the company originally, correct to the nearest thousand dollars?

    b What is the average depreciation in dollars p.a., correct to the nearest hundred dollars, on each car over the three-year period?

Challenge exercise

1 The population of a town decreases by 15% during 2010. What percentage increase, correct to 2 decimal places, is necessary during 2011 for the population to be restored to the level it was at immediately before the decrease in 2010?

2 The length of a rectangle is increased by 15% and the width is decreased by 11%. What is the exact percentage change in the area?

3 The radius of a circular pool of water increases by 8%. What is the exact percentage change in the area of the pool of water?

4 The area of a circular pool of oil is increased by 8%. What is the percentage increase of the radius?

5 A man earns a salary of $1440 for a 44-hour week. His weekly salary is increased by 10% and his hours are reduced by 10%. Find his new hourly salary.

6 In a particular country in 2010, 15% of the population is unemployed and 85% is employed. In 2011, 10% of the people unemployed became employed and 10% of those employed became unemployed. What percentage of the population is employed now?

7 The number of trees on Green Plateau fell by 5% every year for 10 years. Then the numbers rose by 5% every year for 20 years. What was the total percentage gain or loss of trees over the 30-year period?

8 Particular shares were released in the stock market and lost, per day, an average of 2.23% of their original value over the first four days. Over the first day, the shares increased in value by 15%, and over the second day, a further 10% increase was recorded. However, a 20% decrease in the share value occurred on the third day. What percentage decrease was recorded over the fourth day?