CHAPTER

4

Number and Algebra

Factorisation

The distributive law can be used to rewrite a product involving brackets as an expression without brackets. For instance, the product $3(a + 2)$ can be rewritten as $3a + 6$; this is called the expanded form of the expression, and the process is called expansion.

The process of writing an algebraic expression as a product of two or more algebraic factors is called factorisation. Factorisation is the reverse process to expansion. For example, we can write $3a + 6$ as $3(a + 2)$. This is called the factorised form.

In this chapter we will look at how to factorise expressions such as $x^2 + 7x + 12$. We recall that such an expression is called a quadratic.

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4A Factorisation using common factors

If each term in the algebraic expression to be factorised contains a **common factor**, then this common factor is a factor of the entire expression. To find the other factor, divide each term by the common factor. The common factor is placed outside brackets. For this reason the procedure is sometimes called 'taking the common factor outside the brackets'.

Example 1

Factorise $4a + 12$.

Solution

4 is a common factor of $4a$ and 12.

Thus $4a + 12 = 4(a + 3)$

Notice that the answer can be checked by expanding $4(a + 3)$.

In general, take out as many common factors as possible. The common factors may involve both numbers and pronumerals. It may be easier if you first take out the common factor of the numbers and then the common factor of the pronumerals.

Example 2

Factorise: **a** $12x^2 + 3x$ **b** $36ab - 27a$

Solution

a $12x^2 + 3x = 3(4x^2 + x)$

$= 3x(4x + 1)$

b $36ab - 27a = 9(4ab - 3a)$

$= 9a(4b - 3)$

Note: It is more efficient to take out all the common factors in one step, as shown in the following examples.

Example 3

Factorise:

a $3x + 9$ **b** $2x^2 + 4x$ **c** $-7a^2 - 49$ **d** $7a^2 + 63ab$

Solution

a $3x + 9 = 3(x + 3)$

b $2x^2 + 4x = 2x(x + 2)$

c $-7a^2 - 49 = -7 \times a^2 + (-7) \times 7$

$= -7(a^2 + 7)$

$7(-a^2 - 7)$ is also correct.

d $7a^2 + 63ab = 7a(a + 9b)$

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Example 4

Factorise:

a $5pq^2 + 10p^2q + 25p^2q^2$ **b** $16ab + 10b^2 - 2a^2b$

Solution

a $5pq^2 + 10p^2q + 25p^2q^2 = 5pq(q + 2p + 5pq)$

b $16ab + 10b^2 - 2a^2b = 2b(8a + 5b - a^2)$

Exercise 4A

1 Complete each factorisation.

a $12x = 12 \times \square$ **b** $24a = 12 \times \square$ **c** $36ab = 9a \times \square$

d $15ac = 5c \times \square$ **e** $y^2 = y \times \square$ **f** $6y^2 = 3y \times \square$

g $24a^2 = 6a \times \square$ **h** $-6b^2 = 2b \times \square$ **i** $8a^2b = 2ab \times \square$

j $4x^2y = 2x \times \square$ **k** $12m^2n = 3mn \times \square$ **l** $25a^2b^2 = 5ab \times \square$

2 Fill in the blanks by finding the missing factors.

a $12a + 18 = 6 \times \square$ **b** $15p - 10 = 5 \times \square$

c $20mn - 15n = 5 \times \square$ **d** $20mn - 15n = 5n \times \square$

e $a^2 + 4a = (a + 4) \times \square$ **f** $b^2 - 10b = b \times \square$

g $6yz^2 - 18yz = 3z \times \square$ **h** $6yz^2 - 18yz = yz \times \square$

i $6yz^2 - 18yz = -3yz \times \square$ **j** $6yz^2 - 18yz = 6yz \times \square$

Example 1

3 Factorise:

a $6x + 24$ **b** $5a + 15$ **c** $ac + 5c$ **d** $a^2 + a$

e $y^2 + xy$ **f** $4x + 24$ **g** $7a - 63$ **h** $9a + 36$

i $y^2 - 3y$ **j** $-14a - 21$ **k** $-6y - 9$ **l** $-4 - 12b$

Example 2

4 Factorise:

a $4ab + 16a$ **b** $12a^2 + 8a$ **c** $18m^2n + 9mn^2$

d $15a^2b^2 + 10ab^2$ **e** $4a^2 + 6a$ **f** $8a^2 + 12ab$

5 Factorise:

a $3b - 6b^2$ **b** $4x^2 - 6xy$ **c** $9mn - 12m^2n$

d $18y - 9y^2$ **e** $4a - 6ab^2$ **f** $6xy - 4x^2$

g $14mn^2 - 21m^2n$ **h** $6pq^2 - 21qp^2$ **i** $10ab^2 - 25a^2b$

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Example 3

6 Factorise:

a $-10b^2 + 5b$ **b** $-16a^2b - 8ab$ **c** $-x^2y - 3xy$

d $-4pq + 16p^2$ **e** $-5x^2y + 30x$ **f** $18p^2 - 4pq$

g $-8a^2b^2 - 2ab$ **h** $12xy^2 - 3x^2y$ **i** $-25m^2n^2 - 10mn^2$

7 In each part, an expression for the area of the rectangle has been given. Find an expression for the missing side length.

a $2a + 3$

Area $= 8a + 12$

b $3a$

Area $= 9a + 6ab$

c 5

Area $= 10b + 15$

d a

Area $= 2a + a^2$

e $6a$

Area $= 12a^2 + 6ab$

f $3q^2 - 2$

Area $= 24p^2q^2 - 16p^2$

Example 4

8 Factorise:

a $4a^2b - 2ab + 8ab^2$ **b** $4m^2n - 4mn + 16n^2$

c $7ab + 14a^2 + 21b$ **d** $2m^2 + 4mn + 6n$

e $5a^2b + 3ab + 4ab^2$ **f** $6a + 8ab + 10ab^2$

g $5p^2q^2 + 10pq^2 + 15p^2q$ **h** $5\ell^2 - 15\ell m - 20m^2$

4B Factorisation using the difference of two squares

You will recall from Section 1G the important identity

$$(a + b)(a - b) = a^2 - b^2,$$

which is called the difference of two squares. We can now use this result the other way around to factorise an expression that is the difference of two squares.

That is:

$$a^2 - b^2 = (a + b)\,(a - b)$$

That is, the factors of $a^2 - b^2$ are $a + b$ and $a - b$.

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Example 5

Factorise:

a $x^2 - 9$ **b** $25 - y^2$ **c** $4x^2 - 9$

Solution

a $x^2 - 9 = x^2 - 3^2$

$= (x + 3)(x - 3)$

Check the answer by expanding $(x + 3)(x - 3)$ to see that $x^2 - 9$ is obtained.

b $25 - y^2 = 5^2 - y^2$

$= (5 + y)(5 - y)$

c $4x^2 - 9 = (2x)^2 - 3^2$

$= (2x + 3)(2x - 3)$

Example 6

Factorise:

a $3a^2 - 27$ **b** $-16 + 9x^2$

Solution

a $3a^2 - 27 = 3(a^2 - 9)$

$= 3(a^2 - 3^2)$

$= 3(a + 3)(a - 3)$

b $-16 + 9x^2 = 9x^2 - 16$

$= (3x)^2 - 4^2$

$= (3x - 4)(3x + 4)$

Factorisation using the difference of two squares identity

$a^2 - b^2 = (a + b)(a - b)$

Exercise 4B

Example 5

1 Factorise:

a $x^2 - 16$ **b** $x^2 - 49$ **c** $a^2 - 121$ **d** $d^2 - 400$

e $(2x)^2 - 25$ **f** $(3x)^2 - 16$ **g** $(4x)^2 - 1$ **h** $(5m)^2 - 9$

i $9x^2 - 4$ **j** $16y^2 - 49$ **k** $100a^2 - 49b^2$ **l** $64m^2 - 81p^2$

m $1 - 4a^2$ **n** $9 - 16y^2$ **o** $25a^2 - 100b^2$ **p** $-9 + x^2$

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Example 6a

2 Factorise:

a $3x^2 - 48$ **b** $4x^2 - 100$ **c** $5x^2 - 45$ **d** $6x^2 - 24$

e $10x^2 - 1000$ **f** $7x^2 - 63$ **g** $8x^2 - 50$ **h** $12m^2 - 75$

i $3 - 12b^2$ **j** $20 - 5y^2$ **k** $27a^2 - 12b^2$ **l** $16x^2 - 100y^2$

m $45m^2 - 125n^2$ **n** $27a^2 - 192l^2$ **o** $-8x^2 + 32y^2$ **p** $-200p^2 + 32q^2$

Example 6b

3 Factorise:

a $-25 + x^2$ **b** $-4 + 9x^2$ **c** $-9x^2 + 4$

d $-81x^2 + 16$ **e** $-100x^2 + 9$ **f** $-18 + 50x^2$

g $-12x^2 + 27$ **h** $-36x^2 + 400$ **i** $-175 + 28x^2$

4 Use the factorisation of the difference of two squares to evaluate the following. One has been done for you.

$$17^2 - 3^2 = (17 + 3)(17 - 3)$$
$$= 20 \times 14$$
$$= 280$$

a $23^2 - 7^2$ **b** $23^2 - 3^2$ **c** $36^2 - 6^2$

d $94^2 - 6^2$ **e** $1.8^2 - 0.2^2$ **f** $28^2 - 2.2^2$

g $11.3^2 - 8.7^2$ **h** $92.6^2 - 7.4^2$ **i** $3.214^2 - 2.214^2$

5 **a** Evaluate:

i $4^2 - 3^2$ **ii** $5^2 - 4^2$ **iii** $6^2 - 5^2$ **iv** $7^2 - 6^2$

v $8^2 - 7^2$ **vi** $9^2 - 8^2$ **vii** $10^2 - 9^2$ **viii** $101^2 - 100^2$

b What do you notice?

c Use the factorisation of $(n + 1)^2 - n^2$ to prove the result of part **b**.

6 The following leads you through the geometrical proof that $a^2 - b^2 = (a - b)(a + b)$.

In the diagram opposite, square $ECFG$, of side length b, is cut out of square $ABCD$, which has side length a.

a What is the area of hexagon $ABEGFD$?

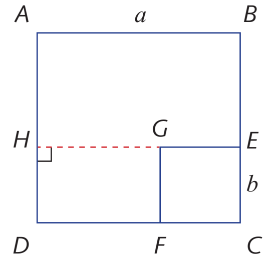
b What is the length of BE?

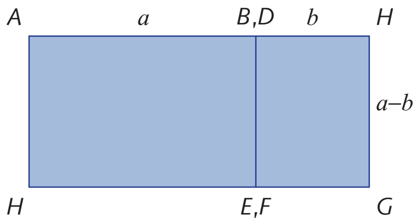
c What is the length of DF?

d The rectangle $HGFD$ is moved so that DF is placed on top of BE (see diagram).

i What is the area of the large shaded rectangle?

ii What have we proved?





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4C Factorisation of simple quadratics

A **simple quadratic** expression is an expression of the form $x^2 + bx + c$, where b and c are given numbers. When we expand $(x + 3)(x + 4)$, we obtain a simple quadratic.

$$\begin{aligned} x(x+4) + 3(x+4) &= x^2 + 4x + 3x + 12 \\ &= x^2 + 7x + 12 \end{aligned}$$

We want to develop a method for reversing this process.

In the expansion $(x+3)(x+4) = x^2 + 7x + 12$, notice that the coefficient of x is $3 + 4 = 7$. The term that is independent of x, the constant term, is $3 \times 4 = 12$. This suggests a method of factorising.

In general, when we expand $(x + p)(x + q)$, we obtain

$$x^2 + px + qx + pq = x^2 + (p+q)x + pq$$

The coefficient of x is the sum of p and q, and the constant term is the product of p and q.

Factorisation of simple quadratics

To factorise a simple quadratic, look for two numbers that add to give the coefficient of x, and that multiply together to give the constant term.

For example, to factorise $x^2 + 8x + 15$, we look for two numbers that multiply to give 15 and add to give 8. Of the pairs that multiply to give 15 $(15 \times 1,\ 5 \times 3, -5 \times (-3)$ and $-15 \times (-1))$, only 5 and 3 add to give 8.

Therefore, $x^2 + 8x + 15 = (x + 3)(x + 5)$

The result can be checked by expanding $(x + 3)(x + 5)$.

Example 7

Factorise $x^2 - 3x - 18$.

Solution

We are looking for two numbers that multiply to give -18 and add to give -3. The numbers -6 and 3 satisfy both conditions.

$x^2 - 3x - 18 = (x - 6)(x + 3)$

Consider factorising $x^2 - 36$.

Since the constant term is -36 and the coefficient of x is 0, we are looking for two numbers that multiply to give -36 and add to give 0. The numbers -6 and 6 satisfy both conditions.

Thus $x^2 - 36 = (x - 6)(x + 6)$.

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Note: It is not really sensible to do the example in this way – it is best to recognise $x^2 - 36$ as a difference of squares. However, it does show that the method of factorisation in this section is consistent with the earlier technique.

Example 8

Factorise $x^2 + 8x + 16$.

Solution

We are looking for two numbers that multiply to give 16 and add to give 8. The numbers 4 and 4 satisfy both conditions.

Thus $x^2 + 8x + 16 = (x + 4)(x + 4)$

$= (x + 4)^2$

Exercise 4C

Example 7

1 Factorise these quadratic expressions.

a $x^2 + 5x + 6$

b $x^2 + 11x + 18$

c $x^2 + 7x + 10$

d $x^2 + 11x + 30$

e $x^2 + 9x + 14$

f $x^2 + 19x + 90$

g $x^2 + 9x + 20$

h $x^2 + 7x + 12$

i $x^2 + 12x + 32$

j $x^2 + 13x + 40$

k $x^2 + 20x + 75$

l $x^2 + 28x + 27$

m $x^2 + 15x + 56$

n $x^2 + 18x + 56$

2 Factorise these quadratic expressions.

a $x^2 - 5x + 6$

b $x^2 - 14x + 33$

c $x^2 - 17x + 30$

d $x^2 - 13x + 42$

e $x^2 - 9x + 14$

f $x^2 - 47x + 90$

g $x^2 - 15x + 44$

h $x^2 - 25x + 100$

i $x^2 - 18x + 80$

j $x^2 - 21x + 80$

k $x^2 - 14x + 40$

l $x^2 - 11x + 24$

m $x^2 - 30x + 56$

n $x^2 - 14x + 24$

3 Factorise these quadratic expressions.

a $x^2 + x - 6$

b $x^2 - 8x - 33$

c $x^2 + x - 30$

d $x^2 - 19x - 42$

e $x^2 - 5x - 14$

f $x^2 - 9x - 90$

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g $x^2 - 7x - 44$ **h** $x^2 + 15x - 100$

i $x^2 + 2x - 80$ **j** $x^2 - 7x - 60$

k $x^2 + 3x - 40$ **l** $x^2 - 10x - 24$

m $x^2 + 4x - 21$ **n** $x^2 + 2x - 15$

o $x^2 - x + 56$ **p** $x^2 + 5x - 24$

4 Factorise these quadratic expressions.

a $x^2 - 3x + 2$ **b** $x^2 + 8x + 12$

c $x^2 - 3x - 10$ **d** $x^2 + 11x + 30$

e $x^2 - 5x - 14$ **f** $x^2 - 9x - 90$

g $x^2 - 5x + 4$ **h** $x^2 - 7x - 18$

i $x^2 - x - 12$ **j** $x^2 - 11x + 28$

k $x^2 + 3x - 10$ **l** $x^2 + x - 90$

Example 8

5 Factorise these quadratic expressions.

a $x^2 + 6x + 9$ **b** $x^2 + 14x + 49$

c $x^2 - 10x + 25$ **d** $x^2 - 18x + 81$

e $x^2 + 10x + 25$ **f** $x^2 + 12x + 36$

g $x^2 + 30x + 225$ **h** $x^2 - 16x + 64$

i $x^2 - 20x + 100$ **j** $x^2 - 8x + 16$

4D Factorisation using perfect squares

In Chapter 4, Example 8, we used the methods discussed previously to show that $x^2 + 8x + 16 = (x + 4)^2$. This form of quadratic factorisation is called a perfect square.

We recall from Chapter 1 that a perfect square is an expression such as $(x + 3)^2$, $(x - 5)^2$ or $(2x + 7)^2$.

The expansion of a perfect square has a special form. For example:

$$\begin{aligned}(x + 3)^2 &= (x + 3)(x + 3)\\ &= x^2 + 6x + 9\\ &= x^2 + 2 \times (3x) + 3^2\end{aligned}$$

Note that the constant term 9 is the square of half the coefficient of x.

The quadratic $x^2 + 10x + 25$ is a perfect square since the ‘constant term is equal to the square of half of the coefficient of x’. So:

$$x^2 + 10x + 25 = (x + 5)^2$$

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We recognise a perfect square such as this in the following way: the constant term is the square of half of the coefficient of x. For example:

$$x^2 + 12x + 36 = (x + 6)^2 \qquad x^2 + 7x + \frac{49}{4} = \left(x + \frac{7}{2}\right)^2$$

$$x^2 - 14x + 49 = (x - 7)^2 \qquad x^2 - 9x + \frac{81}{4} = \left(x - \frac{9}{2}\right)^2$$

Factorising using the perfect square identities

- In general, $a^2 + 2ab + b^2 = (a + b)^2$
- Similarly, $a^2 - 2ab + b^2 = (a - b)^2$

When a quadratic expression has the form of a perfect square, factorisation can occur immediately by application of the relevant identity.

Example 9

Factorise:

a $x^2 + 8x + 16$ **b** $x^2 - 10x + 25$ **c** $x^2 + 11x + \frac{121}{4}$

Solution

a $x^2 + 8x + 16 = x^2 + 2 \times 4x + 4^2$
$= (x + 4)^2$

b $x^2 - 10x + 25 = x^2 - 2 \times 5x + 5^2$
$= (x - 5)^2$

c $x^2 + 11x + \frac{121}{4} = x^2 + 2 \times \frac{11}{2}x + \left(\frac{11}{2}\right)^2$
$= \left(x + \frac{11}{2}\right)^2$

Exercise 4D

Example 9

1 Factorise using the appropriate perfect square identity.

a $x^2 + 12x + 36$ **b** $x^2 - 8x + 16$ **c** $x^2 + 10x + 25$

d $a^2 - 4a + 4$ **e** $m^2 - 26m + 169$ **f** $a^2 + 28a + 196$

g $x^2 - 9x + \frac{81}{4}$ **h** $x^2 + 13x + \frac{169}{4}$ **i** $x^2 - 11x + \frac{121}{4}$

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2 Copy and complete:

a $x^2 + 8x + 16 = (x + \_\_\_\_\_)$

b $x^2 - 10x + \_\_\_\_\_\_ = (x - 5)^2$

c $x^2 - \_\_\_\_\_ + \_\_\_\_\_\_ = (x - 9)^2$

d $x^2 + \_\_\_\_\_ + \_\_\_\_\_\_ = (x + \frac{7}{2})^2$

e $x^2 - 9x + \_\_\_\_\_\_ = (x - \_\_\_\_\_)^2$

f $x^2 - \frac{5x}{2} + \_\_\_\_\_\_ = (x - \_\_\_\_\_\_)^2$

3 Identify the simple quadratic expression that cannot be factorised as a perfect square.

a **i** $x^2 + 4x + 4$ **ii** $x^2 - 6x + 12$ **iii** $x^2 - 12x + 36$ **iv** $x^2 - 10x + 25$

b **i** $x^2 + 6x + 9$ **ii** $x^2 + 5x + \frac{25}{4}$ **iii** $x^2 - 8x - 16$ **iv** $x^2 - 14x + 49$

c **i** $x^2 + \frac{2x}{3} + \frac{1}{9}$ **ii** $x^2 - 3x + \frac{9}{4}$ **iii** $x^2 - \frac{5x}{3} + \frac{25}{36}$ **iv** $x^2 + \frac{7x}{4} + \frac{49}{16}$

d **i** $x^2 - \frac{11x}{2} + \frac{121}{16}$ **ii** $x^2 - \frac{4x}{5} + \frac{4}{25}$ **iii** $x^2 - \frac{3x}{2} - \frac{9}{16}$ **iv** $x^2 - \frac{9x}{2} + \frac{81}{16}$

4 A brick company provides rectangular and square pavers.

a Draw a diagram to show how two different square pavers of side lengths a and b respectively, and two identical rectangular pavers with dimensions $a \times b$, can be arranged into a square.

b How many of each type of paver enables you to pave a square area of side length $a + 3b$? Draw a diagram to illustrate how this can be done.

4E Quadratics with common factors

Sometimes a common factor can be taken out of a quadratic expression so that the expression inside the brackets becomes a simple quadratic that can be factorised.

Example 10

Factorise:

a $3x^2 + 9x + 6$ **b** $6x^2 - 54$ **c** $-x^2 - x + 2$

Solution

a $3x^2 + 9x + 6 = 3(x^2 + 3x + 2)$ (Take out the common factor.)
$= 3(3x + 2)(3x + 1)$

b $6x^2 - 54 = 6(x^2 - 9)$
$= 6(x + 3)(x - 3)$

c $-x^2 - x + 2 = -(x^2 + x - 2)$ (Factor -1 from each term.)
$= -(x + 2)(x - 1)$

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Exercise 4E

Example 10a

1 Factorise:

a $2x^2 + 14x + 24$ **b** $3x^2 + 24x + 36$ **c** $3x^2 - 27x + 24$

d $4x^2 - 24x + 36$ **e** $7x^2 + 14x + 7$ **f** $5x^2 - 5x - 30$

g $4x^2 - 4x + 48$ **h** $2x^2 - 18x + 36$ **i** $5x^2 + 40x + 35$

j $3x^2 + 9x - 120$ **k** $3x^2 - 3x - 90$ **l** $5x^2 + 60x + 180$

m $2x^2 - 4x - 96$ **n** $5x^2 + 65x + 180$ **o** $3x^2 + 30x - 72$

p $3x^2 - 18x + 27$ **q** $5x^2 - 20x + 20$ **r** $3x^2 - 24x + 36$

Example 10b

2 Factorise:

a $4x^2 - 16$ **b** $2x^2 - 18$ **c** $3x^2 - 48$

d $3a^2 - 27$ **e** $6x^2 - 600$ **f** $3a^2 - 27b^2$

g $27x^2 - 3y^2$ **h** $45 - 5b^2$ **i** $12 - 3m^2$

j $128 - 2x^2$ **k** $\frac{1}{2}a^2 - 2b^2$ **l** $27x^2 - \frac{1}{3}y^2$

m $\frac{1}{4}a^2 - 9$ **n** $\frac{1}{5}x^2 - 20$ **o** $\frac{1}{4}x^2 - y^2$

Example 10c

3 Factorise:

a $-x^2 - 8x - 12$ **b** $12 - 11x - x^2$ **c** $7 - 6x - x^2$

d $9 + 8x - x^2$ **e** $-x^2 - 4x - 4$ **f** $-x^2 - 14x - 45$

g $-x^2 + 3x + 40$ **h** $42 + x - x^2$ **i** $22x - x^2 - 40$

j $11x - x^2 - 24$ **k** $-3x^2 - 30x + 72$ **l** $-56 - x^2 - 15x$

m $-16x - 63 - x^2$ **n** $-x^2 - 35 + 12x$ **o** $7x + 18 - x^2$

Review exercise

1 Factorise:

a $4x + 16$ **b** $7x - 21$ **c** $6a - 9$

d $4ab + 7a$ **e** $6pq - 11p$ **f** $5mn - 10n$

g $4uv - 8v$ **h** $a^2 + 9a$ **i** $4m^2n - 12mn$

j $a^2b - 4ab^2$ **k** $3pq - 6p^2$ **l** $6p^3q - 18pq$

2 Factorise:

a $x^2 - 9$ **b** $x^2 - 16$ **c** $9a^2 - 25$

d $16m^2 - 1$ **e** $9 - 4b^2$ **f** $100 - 81b^2$

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g $16x^2 - y^2$ **h** $2m^2 - 50$ **i** $3a^2 - 27$
j $1 - 36b^2$ **k** $4y^2 - \frac{1}{4}$ **l** $p^2q^2 - 1$

3 Factorise:

a $x^2 + 8x + 12$ **b** $x^2 + 9x + 18$ **c** $x^2 + 11x + 30$
d $x^2 - 11x + 24$ **e** $x^2 - 10x + 24$ **f** $x^2 - 14x + 24$
g $x^2 - 25x + 24$ **h** $x^2 + x - 20$ **i** $x^2 - 2x - 48$
j $x^2 - 4x - 12$ **k** $x^2 + 3x - 40$ **l** $x^2 - 7x - 8$
m $x^2 - x - 132$ **n** $a^2 + 19a + 60$ **o** $x^2 - 50x + 96$

4 Factorise:

a $a^2 - 22a + 121$ **b** $m^2 - 14m + 49$ **c** $s^2 + 8s + 16$
d $a^2 + 24a + 144$ **e** $a^2 - 12a + 36$ **f** $z^2 - 40z + 400$
g $x^2 + 5x + \frac{25}{4}$ **h** $y^2 - \frac{2y}{3} + \frac{1}{9}$ **i** $a^2 + \frac{3a}{2} + \frac{9}{16}$

5 Factorise:

a $2x^2 + 18x + 40$ **b** $3x^2 - 30x + 63$ **c** $5x^2 - 50x + 120$
d $2x^2 + 8x - 90$ **e** $3x^2 - 6x - 105$ **f** $2x^2 - 6x - 260$

6 Factorise:

a $25a^2 - 16b^2$ **b** $a^2 + 14a + 49$ **c** $a^2 - a - 20$
d $1 - 36m^2$ **e** $4 - 9x^2y^2$ **f** $\frac{1}{9} - \frac{a^2}{25}$
g $m^2 - \frac{1}{4}$ **h** $x^3 - 49xy^2$ **i** $3a^2 - 75$
j $b^2 - 20b + 96$ **k** $n^2 - 31n + 150$ **l** $m^2 + 20m + 91$
m $a^2 - 7ab - 98b^2$ **n** $m^2 - 4m - 165$ **o** $x^2 + 3xy - 4y^2$
p $20m^2n - 5n^3$ **q** $a^2 - 2a - 63$ **r** $5q^2 - 5pq - 30p^2$
s $x^2 - 3x - 130$ **t** $42 - x - x^2$ **u** $x^2 + 7x - 18$

Challenge exercise

1 Factorise:

a $x^4 - 1$ **b** $x^4 - 16$ **c** $x^2 - 3$
d $x^2 - 5$ **e** $x^2 - 2\sqrt{2}x + 2$ **f** $x^2 + 2\sqrt{2}x + 2$
g $x^2 + x + \frac{1}{4}$ **h** $x^2 + 3x + \frac{9}{4}$ **i** $x^2 - x + \frac{1}{4}$
j $x^2 - 3x + \frac{9}{4}$ **k** $x^2 - xy - 2y^2$ **l** $x^2 + xy - 2y^2$

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2 Simplify:

a $\dfrac{x^2+x-2}{x^2-x-20} \times \dfrac{x^2+5x+4}{x^2-x} \div \left(\dfrac{x^2+3x+2}{x^2-2x-15} \times \dfrac{x+3}{x^2}\right)$

b $\dfrac{x^2-64}{x^2+24x+128} \times \dfrac{x^2+12x-64}{x^2-16} \div \dfrac{x^2-16x+64}{x^2-10x+16}$

c $\dfrac{x^2-18x+80}{x^2-5x-50} \times \dfrac{x^2-6x-7}{x^2-15x+56} \div \dfrac{x-1}{x+5}$

3 Factorise:

a $(x+2)^2-8x$

b $(x-3)^2+12x$

c $(x+a)^2-4ax$

d $(x-a)^2+4ax$

4 Simplify $\left(\dfrac{a}{b}+\dfrac{c}{d}\right) \div \left(\dfrac{a}{b}-\dfrac{c}{d}\right)$.

5 Factorise:

a $(x^2+1)^2-4$

b $(x^2-2)^2-4$

c $x^2+4x+4-y^2$

d $x^2+8x+16-a^2$

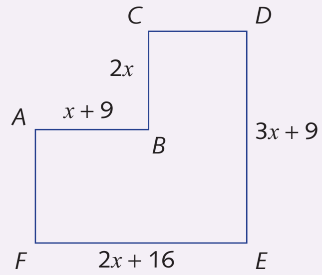
e $m^2-2m+1-n^2$

f $p^2-5p+\dfrac{25}{4}-q^2$

6 **a** By adding and subtracting $4x^2$, factorise x^4+4.

b Factorise x^4+4a^4.

7 A swimming pool is designed in an L-shape with dimensions in metres as shown.



The pool is enlarged or reduced depending on the value of x.

a Find, in terms of x, the length of:

i AF

ii CD

b Show that the perimeter is equal to $(10x+50)$ m.

c What is the perimeter if $x = 3$?

d Find the area of the swimming pool in terms of x. Expand and simplify your answer.

e It is decided that a square swimming pool would be a better use of space.

i By factorising your answer to part **d**, find the dimensions, in terms of x, of a square swimming pool with the same area as the L-shaped swimming pool.

ii What is the perimeter, in terms of x, of this square swimming pool?

8 For positive whole numbers a and b, prove that:

a if $\dfrac{a}{b} < 1$ then $\dfrac{a+1}{b+1} > \dfrac{a}{b}$

b if $\dfrac{a}{b} > 1$ then $\dfrac{a+1}{b+1} < \dfrac{a}{b}$

9 A right-angled triangle has a hypotenuse of length b cm and one other side of length a cm. If $b-a=1$, find the length of the third side in terms of a and b.

10 Factorise $\left(1+\dfrac{y^2+z^2-x^2}{2yz}\right) \div \left(1-\dfrac{x^2+y^2-z^2}{2xy}\right)$.

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