CHAPTER

5

Number and Algebra

Linear equations and inequalities

We have seen that many real-world problems can be converted to equations. We have also seen how to solve simple equations. In this chapter, we consider a wider variety of equations and use them to solve practical problems.

Equations arise naturally when solving problems. In fact, a lot of problem-solving relies on us being able to translate a given word or real world problem into an equation, or equations, solve the equation(s) and relate the solution to the original problem. Turning a complicated problem into an equation enables us to understand and solve difficult problems. Before we reach this stage, we need to have at hand a collection of techniques for solving equations.

In earlier times, people used a number of ad hoc methods for solving equations. Only since the development of modern algebra have standard procedures and notations been introduced that enable us to solve equations quickly and efficiently.

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5A Expressions

Two examples of **linear expressions** are $2x + 3$ and $x + 7$. The expression $2x + 3$ is called **linear** because the graph of $y = 2x + 3$ is a straight line, as we saw in Year 8. We begin with some revision examples and exercises.

Example 1

Brian is 6 cm taller than Geoff. Represent this information algebraically.

Solution

Since we do not know how tall Geoff is, we call his height x cm.

Then Brian is $(x + 6)$ cm tall.

Example 2

The length of a rectangle is 3 m more than its width. The width of the rectangle is w m.

a Express the length of the rectangle in terms of w.

b Express the perimeter of the rectangle in terms of w.

Solution

a Length of rectangle $= (w + 3)$ m

b Perimeter $= w + w + (w + 3) + (w + 3)$

$= (4w + 6)$ m

Exercise 5A

Example 1

1 Fiona is 5 cm taller than Tristan. Tristan's height is h cm. What is Fiona's height?

2 Seuret weighs 2 kg less than his older sister Vivian. If Vivian weighs w kg, what is Seuret's weight?

3 When Andriana's age is doubled, the number is 3 more than Helen's age. If Andriana's age is x years, what is Helen's age?

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Example 2

4 The length of a rectangle is 5 metres more than its width.

a If w metres is the width of the rectangle, express the length of the rectangle in terms of w.

b If ℓ metres is the length of the rectangle, express the width of the rectangle in terms of ℓ.

5 In a competition, Deeksha scored 18 points more than Greta and Deirdre scored 5 points less than twice the number of points Greta scored. If a is the number of points Greta scored:

a express the number of points Deeksha scored in terms of a

b express the number of points Deirdre scored in terms of a

6 The length of a rectangular paddock is 20 m less than three times its width. If the width of the paddock is x m, express the length of the paddock in terms of x.

7 In a triathlon race, Luca ran at an average speed 5 times his average swimming speed. Also, when his average running speed was multiplied by 4, this number was 3 less than his average speed for the cycling leg.

If x km/h is Luca's average swimming speed, find expressions in terms of x for his:

a average running speed

b average cycling speed

8 Match each of the following mathematical expressions with its corresponding English expression.

a	$4 + 2x$	**i**	Six less than four times x
b	$x - 5$	**ii**	Three times one more than x
c	$2x - 4$	**iii**	Two less than one-quarter of x
d	$3(x + 1)$	**iv**	One-quarter of two less than x
e	$4x - 6$	**v**	Four less than twice x
f	$\frac{x}{4} - 2$	**vi**	Six more than half of x
g	$\frac{x - 2}{4}$	**vii**	One more than three times x
h	$x + 6$	**viii**	Five less than x
i	$3x + 1$	**ix**	Six more than x
j	$\frac{x}{2} + 6$	**x**	Four more than twice x

9 Gemma is 6 cm shorter than Gavin and 4 cm taller than Brent. If x cm represents Gemma's height, express:

a Gavin's height in terms of x

b Brent's height in terms of x

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5B Solving simple linear equations

A statement such as $x + 7 = 11$ is called an **equation** and we may **solve** the equation to find a value for x that makes the statement true. In this case, the **solution** is $x = 4$.

Reading equations

The equation $2x + 4 = 10$ can be read as 'two times a number plus 4 is equal to 10'.

The instruction 'Solve the equation $2x + 4 = 10$' can also be read as 'A number is multiplied by 2 and 4 is then added. The result is 10. Find the number.'

In this case, the number is 3. The solution is $x = 3$.

Equivalent equations

Consider these equations:

$$2x + 7 = 11 \qquad (1)$$

$$2x + 9 = 13 \qquad (2)$$

Equation (2) is obtained from equation (1) by adding 2 to each side of the equation.

So equation (1) is obtained from equation (2) by subtracting 2 from each side of the equation.

Equations (1) and (2) are said to be **equivalent equations**.

Equation (3), below, is obtained from equation (1) by subtracting 7 from each side of the equation.

$$2x = 4 \qquad (3)$$

$$x = 2 \qquad (4)$$

Equation (4) is obtained from equation (3) by dividing each side of the equation by 2. You can obtain equation (3) from equation (4) by multiplying each side of the equation by 2.

All of the above equations are **equivalent**.

Equivalent equations

- If we add the same number to, or subtract the same number from, both sides of an equation, the new equation is **equivalent** to the original equation.
- If we multiply or divide both sides of an equation by the same non-zero number, the new equation is **equivalent** to the original equation.
- Equivalent equations have exactly the same solutions.

In the following examples, equivalent equations are formed to solve the equations.

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Example 3

Solve:

a $3x + 5 = 20$ **b** $5x - 7 = 18$ **c** $3 - 2x = 15$ **d** $-3p = \frac{2}{5}$

Solution

a $3x + 5 = 20$

$3x = 15$ (Subtract 5 from both sides.)

$x = 5$ (Divide both sides by 3.)

b $5x - 7 = 18$

$5x = 25$ (Add 7 to both sides.)

$x = 5$ (Divide both sides by 5.)

c $3 - 2x = 15$

$-2x = 12$ (Subtract 3 from both sides.)

$x = -6$ (Divide both sides by -2.)

d $-3p = \frac{2}{5}$

$p = -\frac{2}{15}$ (Divide both sides by -3.)

Example 4

Solve:

a $3x + 7 = 2x + 13$ **b** $5a - 21 = 14 - 2a$

Solution

a $3x + 7 = 2x + 13$

$3x + 7 - 2x = 2x + 13 - 2x$ (Subtract $2x$ from both sides.)

$x + 7 = 13$

$x = 6$ (Subtract 7 from both sides.)

b $5a - 21 = 14 - 2a$

$7a - 21 = 14$ (Add $2a$ to both sides.)

$7a = 35$ (Add 21 to both sides.)

$a = 5$ (Divide both sides by 7.)

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Exercise 5B

1 Solve these equations.

a $a + 2 = 5$ **b** $b + 7 = 19$ **c** $c - 6 = 11$

d $d - 15 = 3$ **e** $2a = 6$ **f** $3b = 9$

g $6d = 42$ **h** $-3m = 6$ **i** $-2n = 8$

j $9q = -27$ **k** $b + 7 = 29\frac{1}{2}$ **l** $3a = \frac{2}{3}$

m $x - 6 = 5\frac{1}{3}$ **n** $-4y = \frac{8}{9}$ **o** $2x = -\frac{5}{6}$

Example 3

2 Solve these equations.

a $2a + 5 = 7$ **b** $3b + 4 = 19$ **c** $3c - 1 = 20$

d $5d - 7 = 23$ **e** $4f - 3 = 13$ **f** $3g + 17 = 5$

g $5h + 21 = 11$ **h** $6a + 17 = -1$ **i** $4a + 23 = -9$

j $3a - 16 = -31$ **k** $7b - 17 = -66$ **l** $2b + 5 = 7\frac{1}{2}$

m $2x + 11 = 7\frac{1}{4}$ **n** $4m - 9 = -13\frac{4}{5}$ **o** $-2b + 4 = 9\frac{1}{4}$

3 Solve these equations.

a $2 - 3a = 8$ **b** $3 - 4b = 15$ **c** $5 - 2c = -13$

d $3 - 5d = -22$ **e** $-6 - 7e = 15$ **f** $-4 - 3f = 14$

Example 4

4 Solve these equations for x and check your answers.

a $5x + 5 = 3x + 1$ **b** $7x + 15 = 2x + 20$ **c** $9x - 7 = 7x + 3$

d $5x - 6 = x - 2$ **e** $4x + 7 = x - 2$ **f** $3x + 1 = 9 - x$

g $4x - 3 = 18 - 3x$ **h** $2x - 3 = 7 - x$ **i** $8 - 3x = 2x - 7$

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5C Equations with brackets

In this section, we look at solving equations in which brackets are involved. In previous work in this area, you have always expanded the brackets first. In some examples, we do not do this.

Example 5

Solve:

a $3(x+2)=21$ **b** $2(3-x)=12$

Solution

a $3(x+2)=21$

$x+2=7$ (Divide both sides by 3.)

$x=5$

b $2(3-x)=12$

$3-x=6$ (Divide both sides by 2.)

$-x=3$

$x=-3$

Example 6

Solve $3(x+5)=31$.

Solution

Method 1

$$\begin{aligned} 3(x+5) &= 31 \\ 3x+15 &= 31 \\ 3x &= 16 \\ x &= \frac{16}{3} \\ x &= 5\tfrac{1}{3} \end{aligned}$$

Method 2

$$\begin{aligned} 3(x+5) &= 31 \\ x+5 &= \frac{31}{3} \\ x &= \frac{16}{3} \end{aligned}$$

When brackets are involved, the equation is usually solved by expanding the brackets first. In the above example, method 1 is preferable to method 2.

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Example 7

Solve:

a $2(x + 1) + 4(x + 3) = 26$ **b** $3(a + 5) = 2(a + 6)$ **c** $3(y - 3) - 2(y - 4) = 4$

Solution

a
$$\begin{aligned} 2(x + 1) + 4(x + 3) &= 26 \\ 2x + 2 + 4x + 12 &= 26 \quad \text{(Expand the brackets.)} \\ 6x + 14 &= 26 \\ 6x &= 12 \\ x &= 2 \end{aligned}$$

b
$$\begin{aligned} 3(a + 5) &= 2(a + 6) \\ 3a + 15 &= 2a + 12 \\ a + 15 &= 12 \quad \text{(Subtract } 2a \text{ from both sides.)} \\ a &= -3 \end{aligned}$$

c
$$\begin{aligned} 3(y - 3) - 2(y - 4) &= 4 \\ 3y - 9 - 2y + 8 &= 4 \quad \text{(Expand both sets of brackets, being careful with the signs.)} \\ y - 1 &= 4 \\ y &= 5 \end{aligned}$$

Exercise 5C

Example 5

1 Solve for x.

a $2(x + 3) = 8$ **b** $3(x - 2) = 15$ **c** $4(x - 1) = 12$
d $5(x + 1) = 10$ **e** $3(5 - x) = 9$ **f** $4(8 - x) = 36$
g $-3(2x - 6) = 12$ **h** $-4(3x - 7) = -8$ **i** $-2(7x + 6) = 86$

Example 6

2 Solve for x.

a $2(x + 3) = 15$ **b** $5(x - 2) = 16$ **c** $7(3 - x) = 20$
d $4(7 - x) = 7$ **e** $-2(x - 5) = 7$ **f** $-3(2x - 1) = 2$
g $5(x - 3) = \frac{2}{3}$ **h** $2(2x - 6) = \frac{2}{6}$ **i** $-3(5x + 2) = \frac{1}{4}$

Example 7

3 Solve:

a $2(a + 1) + 4(a + 2) = 22$ **b** $4(b - 1) + 3(b + 2) = 30$
c $5(c + 2) - 2(c + 1) = 17$ **d** $4(2d + 1) - 5(d - 2) = 17$
e $2(x + 3) - 3(x - 4) = 20$ **f** $5(2y - 3) - 3(y - 5) = 21$
g $5(a + 3) = 3(2a + 1)$ **h** $5(2a - 1) = 2(3a + 2)$
i $-2(x + 4) + 3(x - 2) = 16$ **j** $-5(x + 3) - 4(x + 1) = 17$
k $\frac{1}{2}(2x + 5) + 6(x - 2) = 4\frac{1}{2}$ **l** $\frac{1}{2}(4x + 1) + 2(x - 2) = 13$

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5D Linear equations involving fractions

When there are fractions in equations, the standard procedure is to remove the fractions by multiplying both sides of the equation by an appropriate whole number. The next step is to remove the brackets.

Example 8

Solve:

a $2x + \frac{1}{2} = \frac{2}{3}$

b $3x - \frac{1}{4} = \frac{4}{5}$

Solution

a

$$2x + \frac{1}{2} = \frac{2}{3}$$

$$6\left(2x + \frac{1}{2}\right) = 6 \times \frac{2}{3}$$ (Multiply both sides by 6, the lowest common multiple of the denominators.)

$$12x + 3 = 4$$

$$12x = 1$$

$$x = \frac{1}{12}$$

b

$$3x - \frac{1}{4} = \frac{4}{5}$$

$$20\left(3x - \frac{1}{4}\right) = 20 \times \frac{4}{5}$$

$$60x - 5 = 16$$

$$60x = 21$$ (Multiply both sides by 20.)

$$x = \frac{21}{60}$$

$$= \frac{7}{20}$$

Example 9

Solve:

a $\frac{2x}{3} + \frac{1}{5} = 4$

b $10 - \frac{a+3}{4} = 6$

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Solution

a

$$\frac{2x}{3} + \frac{1}{5} = 4$$

$$15\left(\frac{2x}{3} + \frac{1}{5}\right) = 15 \times 4$$

$$15 \times \frac{2x}{3} + 15 \times \frac{1}{5} = 15 \times 4$$

$$10x + 3 = 60$$

$$10x = 57$$

$$x = \frac{57}{10} \text{ or } x = 5\tfrac{7}{10}$$

b

$$10 - \frac{a+3}{4} = 6$$

$$4 \times 10 - 4 \times \frac{a+3}{4} = 4 \times 6$$

$$40 - (a + 3) = 24$$

$$37 - a = 24$$

$$-a = -13$$

$$a = 13$$

Example 10

Solve $2.1x + 3.5 = 9.4$

Solution

$$2.1x + 3.5 = 9.4$$

$$21x + 35 = 94 \qquad \text{(Multiply both sides by 10.)}$$

$$21x = 59$$

$$x = \frac{59}{21}$$

Example 11

Solve $\frac{2x}{3} - 3 = x + \frac{3}{4}$.

Solution

$$\frac{2x}{3} - 3 = x + \frac{3}{4}$$

$$8x - 36 = 12x + 9 \qquad \text{(Multiply both sides by 12.)}$$

$$-4x = 45$$

$$x = -\frac{45}{4}$$

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Example 12

Solve $\frac{a+5}{4} = \frac{a+3}{3}$.

Solution

$$\frac{a+5}{4} = \frac{a+3}{3}$$

$3(a + 5) = 4(a + 3)$ (Multiply both sides by 12.)

$3a + 15 = 4a + 12$

$15 = a + 12$

$3 = a$

$a = 3$

Solving linear equations

To solve linear equations:

- Remove all fractions by multiplying both sides of the equation by the lowest common multiple of the denominators.
- Remove all brackets.
- Collect like terms and solve the equation.

Exercise 5D

Example 8

1 Solve:

a $2x - \frac{1}{2} = \frac{1}{4}$ **b** $3x + \frac{3}{2} = \frac{5}{3}$ **c** $\frac{4}{3} - 2y = \frac{1}{2}$

d $\frac{7}{2} - 3y = \frac{2}{3}$ **e** $\frac{2}{3} + 4x = \frac{2}{3}$ **f** $-\frac{3}{4} - 5y = \frac{2}{5}$

g $-2x + \frac{1}{3} = \frac{1}{5}$ **h** $\frac{2}{3} + 3x = \frac{1}{5}$ **i** $-3y - \frac{2}{5} = \frac{1}{4}$

Example 9a

2 Solve:

a $\frac{a}{3} + 5 = 3$ **b** $\frac{3a}{4} - \frac{4}{5} = \frac{2}{3}$ **c** $\frac{b}{3} - 5 = 3$

d $\frac{3b}{7} + 6 = 2$ **e** $2 - \frac{x}{3} = 6$ **f** $3 - \frac{3x}{4} = 6$

g $\frac{2}{3}(m - 3) = 1$ **h** $3\left(\frac{m}{5} + 2\right) = 2$ **i** $-3\left(\frac{x}{6} + 1\right) = 4$

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Example 9b

3 Solve:

a $\frac{2y+1}{3}+4=7$ **b** $\frac{2x+5}{3}=9$ **c** $\frac{5p-2}{4}-1=6$

d $\frac{3y-1}{2}+2=9$ **e** $\frac{2x-3}{5}=3$ **f** $\frac{4a+3}{5}-2=1$

g $2+\frac{2x-1}{5}=5$ **h** $18-\frac{7x+2}{3}=8$ **i** $1-\frac{5y-3}{4}=-2$

Example 10

4 Solve:

a $e+1.8=2.9$ **b** $f+3.6=7.5$ **c** $g-2.8=3.8$

d $1.2u=15.6$ **e** $3.6r=9$ **f** $-4.7x=49.35$

g $1.2x+4=10$ **h** $3.8x-7=8.2$ **i** $4.6x-2.6=25$

Example 11

5 Solve these equations for x and check your answers.

a $\frac{3x}{4}-1=\frac{x}{2}+3$ **b** $\frac{x}{3}+2=\frac{4x}{3}+3$ **c** $\frac{5x}{6}-3=7-\frac{x}{3}$

d $\frac{x}{3}-4=6-\frac{2x}{5}$ **e** $\frac{5}{3}-\frac{x}{2}=\frac{3x}{4}+\frac{7}{6}$ **f** $\frac{11}{12}-\frac{5x}{6}=\frac{3x}{4}-\frac{2}{3}$

6 Solve for x.

a $1.6x+10=0.9x+12$ **b** $5.9x-7=2.4x+35$ **c** $8.3x+12=36-1.7x$

d $4.8-1.3x=23+1.3x$ **e** $1.5x+3.9=6.7-0.5x$ **f** $11.8x+7.6=61.6-1.7x$

Example 12

7 Solve for a.

a $\frac{a+3}{2}=\frac{a+1}{5}$ **b** $\frac{a+1}{3}=\frac{2a-1}{7}$

c $\frac{2a+1}{3}=\frac{3a+1}{4}$ **d** $\frac{a+1}{2}+1=\frac{a-1}{5}$

e $\frac{3a+2}{4}+2=a$ **f** $\frac{2a+1}{2}+\frac{a}{3}=4$

g $\frac{3a-2}{4}=\frac{a-5}{2}$ **h** $\frac{2a-1}{3}-2=\frac{a+3}{4}$

i $\frac{4a-1}{3}+a=2$ **j** $\frac{a}{2}+\frac{a-1}{3}=\frac{a+1}{4}$

8 Solve:

a $5a+9=24$ **b** $3b-7=32$

c $\frac{c}{3}-2=4$ **d** $\frac{d}{2}+6=3$

e $\frac{4e}{3}+\frac{1}{2}=2$ **f** $\frac{2f}{3}-1=\frac{3}{7}$

g $2g+5=7g-6$ **h** $4h-2=5-3h$

i $2(i-1)=5(i+6)$ **j** $3(j+2)=2(2j-1)$

k $2(k+1)-3(k-2)=7$ **l** $4(\ell-1)+3(\ell+2)=8$

m $\frac{m+1}{3}=\frac{m-2}{5}$ **n** $\frac{2n-1}{3}=\frac{4n+1}{5}$

o $\frac{2q-2}{5}+1=4q$ **p** $\frac{2r+1}{3}+2=\frac{3r-1}{4}$

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5E Using linear equations to solve problems

Problem-solving often involves introducing algebra, translating the problem into an equation and then solving the equation. An important first step is to introduce an appropriate pronumeral for one of the unknown quantities.

Example 13

Three children earn weekly pocket money. Andrew earns \$2 more than Gina, and Katya earns twice the amount Gina earns. The total of the weekly pocket money is \$22.

a How much money does Gina earn?

b How much money do Andrew and Katya earn?

Solution

a Let $\$m$ be the amount of pocket money Gina earns in a week.

Then Andrew earns $\$(m + 2)$ and Katya earns $\$(2m)$,

so $m + (m + 2) + 2m = 22$ (The total weekly pocket money is \$22.)

$$4m + 2 = 22$$

$$4m = 20$$

$$m = 5$$

So Gina earns \$5 per week.

b Andrew earns \$7 and Katya earns \$10 per week.

Example 14

Ali and Jasmine each have a number of swap cards. Jasmine has 25 more cards than Ali, and in total the two children have 149 cards.

a How many cards does Ali have?

b How many cards does Jasmine have?

Solution

a Let x be the number of cards Ali has.

Jasmine has $(x + 25)$ cards.

Total number of cards is 149,

so $x + (x + 25) = 149$

$$2x + 25 = 149$$

$$2x = 124$$

$$x = 62$$

So Ali has 62 cards.

b Jasmine has $62 + 25 = 87$ cards.

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Harder examples involving rates

Speed is one of the most familiar rates. In problems involving speed, we use the relationship:

$$\textit{average speed} = \frac{\textit{distance travelled}}{\textit{time taken}}$$

or
$$\textit{time taken} = \frac{\textit{distance travelled}}{\textit{average speed}}$$

or $\textit{distance travelled} = \textit{average speed} \times \textit{time taken}$

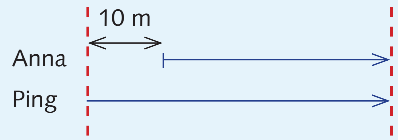
Example 15

Ping and Anna compete in a handicap sprint race. Anna starts the race 10 m ahead of Ping. Ping runs at an average speed that is 20% faster than Anna's average speed. The two sprinters will be level in the race after 9 seconds. Find the average speed of:

a Anna **b** Ping

Solution

a The diagram below represents the situation.



Let x be Anna's average speed, measured in m/s.

$$\text{Ping's average speed is } 120\% \text{ of } x = \frac{120x}{100}$$
$$= \frac{6x}{5} \text{ m/s}$$

We use distance = average speed × time taken

After 9 seconds, Anna has run a distance of $9x$ m and Ping has run a distance of $\frac{6x}{5} \times 9 = \frac{54x}{5}$ m.

Anna started 10 m in front of Ping. So when they are level,

$$9x + 10 = \frac{54x}{5}$$
$$45x + 50 = 54x$$
$$50 = 9x$$
$$x = \frac{50}{9}$$
$$x = 5\tfrac{5}{9}$$

So Anna runs at an average speed of $5\frac{5}{9}$ m/s.

b Ping runs at $\frac{6x}{5} = \frac{6}{5} \times \frac{50}{9} = 6\frac{2}{3}$ m/s.

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Example 16

For a training run, a triathlete covers 50 km in $4\frac{1}{4}$ hours. She runs part of the way at a speed of 10 km/h, cycles part of the way at a speed of 40 km/h and swims the remaining distance at a speed of $2\frac{1}{2}$ km/h. The athlete runs for twice the time it takes to complete the cycle leg. How long did she take to complete the cycle leg?

Solution

Let t hours be the time for the cycle leg.

Then $2t$ hours is the time for the running leg and $(4\frac{1}{4} - t - 2t)$ hours is the time for the swim leg.

Now, distance of run + distance of cycle + distance of swim = total distance,

$$\begin{aligned} \text{so} \quad 10 \times 2t + 40 \times t + 2\tfrac{1}{2} \times \left(4\tfrac{1}{4} - t - 2t\right) &= 50 \\ 20t + 40t + \frac{5}{2}\left(\frac{17}{4} - 3t\right) &= 50 \\ 60t + \frac{85}{8} - \frac{15t}{2} &= 50 \\ 480t + 85 - 60t &= 400 \\ 420t &= 315 \\ t &= \frac{315}{420} \\ t &= \frac{3}{4} \end{aligned}$$

The athlete takes $\frac{3}{4}$ hour, or 45 minutes, to complete the cycle leg.

Exercise 5E

In each of the following, form an equation and solve.

1 Jacques thinks of a number x. When he adds 17 to his number, the result is 32. What is the value of x?

2 When 16 is added to twice Simone's age, the answer is 44. How old is Simone?

3 When 14 is added to half of Suzette's weight in kilograms, the result is 42. How much does Suzette weigh?

4 Yolan buys 8 pens and receives 80 cents change from \$20.00. How much does a pen cost, assuming each pen costs the same amount?

5 If the sum of $2p$ and 19 is the same as the sum of $4p$ and 11, find the value of p.

6 If the sum of half of q and 6 is equal to the sum of one-third of q and 2, find the value of q.

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7 Derek is presently 20 years older than his daughter, Alana.

a If x represents Alana's present age, express each of the following in terms of x.

i Derek's present age

ii Alana's age in 12 years' time

iii Derek's age in 12 years' time

b If Derek's age 12 years from now is twice Alana's age 12 years from now, find their present ages.

8 Alan, Brendan and Calum each have a number of plastic toys from a fast food store. Brendan has 5 more toys than Alan, and Calum has twice as many toys as Alan.

a If x represents the number of toys Alan has, express each of the following in terms of x:

i the number of toys Brendan has

ii the number of toys Calum has

iii the total number of toys the three boys have

b If the boys have 37 toys in total, determine how many toys each boy has.

9 The length of a swimming pool is 2 m more than four times its width.

a If x metres represents the width of the pool, express the length of the pool in terms of x.

b If the perimeter of the pool is 124 m find the length and width of the pool.

10 Ms Minas earns \$3600 more than Mr Brown, and Ms Lee earns \$2000 less than Mr Brown.

a If \$$x$ represents Mr Brown's salary, express the salary of:

i Ms Minas in terms of x

ii Ms Lee in terms of x

b If the total of the three incomes is \$151 600, find the income of each person.

11 Dudley has a number of 10-cent and 20-cent coins. If their total value is \$6, how many are there of each coin if there are:

a equal numbers of each coin?

b twice as many 10-cent coins as there are 20-cent coins?

c twice as many 20-cent coins as there are 10-cent coins?

12 The distance between two towns, A and B, is 350 km. Find x if:

a the trip takes x hours at an average speed of 80 km/h

b the trip takes $3\frac{1}{2}$ hours at an average speed of x km/h

c the trip takes a total of 3 hours, travelling x hours at 120 km/h and the remaining time at 60 km/h

13 A classroom has a length 6 m shorter than its width and a perimeter of 30 m. Find the length and width of the classroom.

14 Of the total amount of money in his wallet, a man gave one-third to his daughter and one-quarter to his son. After this he only had \$250 left. How much was originally in his wallet?

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15 A student has an average mark of 68 from 10 tests. What mark must be gained in the next test to raise his average to 70?

16 Luisa travels 25 km in $3\frac{3}{4}$ hours. She walks part of the way at 4 km/h and cycles the rest at 12 km/h. How far did she walk?

17 A man is four times as old as his son. In five years he will be only three times as old as his son. What is the man's present age?

18 A bottle of cordial contains 5 litres, of which 10% is pure fruit juice. How many litres of water must be added to dilute it to 4% fruit juice?

19 A coffee blender mixes two types of coffee. Type *A* costs \$13 per kg and type *B* costs \$18 per kg. How many grams of each type of coffee should be blended so that 1 kg costs \$15?

20 A collection of coins consisting of 10-cent, 20-cent and 50-cent pieces has a value of \$4.50. The number of 20-cent pieces is twice the number of 10-cent pieces and the number of 50-cent pieces is 3 less than twice the number of 10-cent pieces. How many 10-cent, 20-cent and 50-cent pieces are there in the collection?

21 If 4 litres of laboratory alcohol is 80% pure (80% alcohol, 20% water), how many litres of water must be added so that the resulting mixture will contain 30% alcohol?

22 A rectangular garden pond has a length 60 cm more than its width. Let the width of the pond be w cm.

a Find the length of the pond in terms of w.

b If the length of the pond is 150 cm, what are the dimensions of the pond?

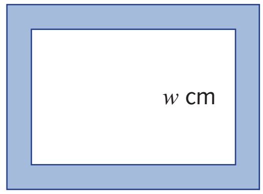
The pond has a w wide path around its perimeter.

c Express, in terms of w, the:

i length

ii width

of the outer rectangle formed by the path.



d Express, in terms of w, the area of the path.

e If the width of the pond is 120 cm, find the area of the path.

f If the area of the path is 5.6 m^2, find the width of the pond.

23 A father is concerned about his daughter's progress in mathematics. In order to encourage her, he agrees to give her 10 cents for every problem she solves correctly and to penalise her 15 cents for every problem she gets wrong. The girl completed 22 problems for homework. Let the number of questions she got right be x.

a How much money does the father have to give his daughter for getting x questions correct?

b How much does the daughter have to pay in penalties for the incorrect questions?

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c Using the fact that the girl made a profit of 20 cents, write down an equation involving x.

d How many questions did the girl get correct?

24 During a power failure, Sonia lights two identical candles of length 10 cm at 7 p.m. One candle burns out by 11 p.m. and the other candle burns out at 12 midnight. Assume that the length of each candle goes down by a constant rate.

a How high is each candle at 8 p.m.?

b How high is each candle at 9 p.m.?

c How high is each candle t hours after 7 p.m.?

d At what time is one candle twice as high as the other candle?

5F Literal equations

In this chapter, we have discussed methods for solving linear equations.

Sometimes equations arise in which some of the coefficients are pronumerals. These are called **literal equations**. We need to be told which pronumeral we are solving for.

Example 17

Solve:

a $ax + b = c$ for x

b $a(x + b) = c$ for x

Solution

a
$$\begin{aligned} ax + b &= c \\ ax &= c - b \quad \text{(Subtract } b \text{ from both sides.)} \\ x &= \frac{c - b}{a} \quad \text{(Divide both sides by } a\text{.)} \end{aligned}$$

b
$$\begin{aligned} a(x + b) &= c \\ ax + ab &= c \\ ax &= c - ab \\ x &= \frac{c - ab}{a} \end{aligned}$$

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Example 18

Solve $mx - n = nx + m$ for x.

Solution

$$mx - n = nx + m$$

$$mx - nx - n = m \quad \text{(Subtract } nx \text{ from both sides.)}$$

$$mx - nx = m + n \quad \text{(Add } n \text{ to both sides.)}$$

$$x(m - n) = m + n \quad \text{(Factorise the left-hand side.)}$$

$$x = \frac{m + n}{m - n} \quad \text{(Divide both sides by } m - n\text{.)}$$

Exercise 5F

Example 17

1 Solve each of these equations for x.

a $x + b = c$

b $x - d = e$

c $p - x = q$

d $-x + m = n$

e $cx = b$

f $c - bx = e$

g $a(x + b) = c$

h $m(nx + p) = n$

i $\frac{x}{a} = b$

j $\frac{x + a}{b} = c$

k $\frac{x}{a} + b = c$

l $\frac{mx}{n} = p$

m $\frac{ax}{b} + c = d$

n $\frac{ax + b}{c} = d$

o $\frac{mx - n}{n} = m$

p $\frac{x}{f} + \frac{g}{h} = k$

q $\frac{x}{a} - \frac{a}{b} = b$

r $\frac{x}{b} - b = \frac{a}{b}$

Example 18

2 Solve each of these equations for x.

a $ax + b = cx + d$

b $mx + n = nx - m$

c $a(x + b) = cx + d$

d $a(x - b) = c(x - d)$

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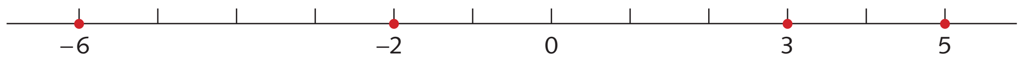
5G Inequalities

The numbers -6, -2, 3 and 5 are shown on a number line.



Using the 'greater than' sign, we write $5 > 3$ because 5 is to the right of 3. Similarly, $3 > -2$ and $-2 > -6$.

We can also use the 'less than' sign and write $3 < 5, -2 < 3$ and $-6 < -2$. Notice that every negative number is less than 0. For example, $-6 < 0$.

The symbols ≤ and ≥

The symbol ≤ means 'is less than or equal to'.

The symbol ≥ means 'is greater than or equal to'.

The statement '$4 \le 6$' means that either '$4 < 6$' or '$4 = 6$' is true. It is correct because the statement '$4 < 6$' is true. It does not matter that the second statement, '$4 = 6$', is false.

It is also correct to say that $3 \ge 3$, because the statement '$3 = 3$' is true. Once again, it does not matter that the statement '$3 > 3$' is false.

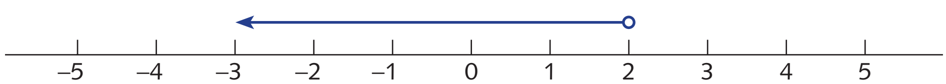
Statements such as '$-6 < 2$', '$5 > 3$', '$4 \le 6$' and '$3 \ge 3$' are called **inequalities**.

Sets and intervals

We will need ways of describing and graphing various collections of numbers.

An example is the collection of all numbers that are less than 2.

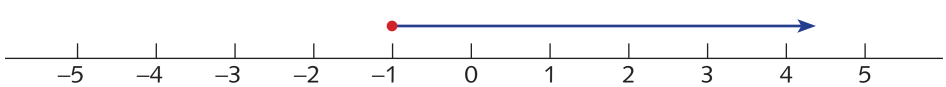
On the number line, we graph this in the following way.



The open dot, is used to indicate that 2 is not in the collection. The collection we are interested in is the interval that begins at 2 (but does not include 2) and extends indefinitely to the left, as indicated. The arrow head means 'goes on indefinitely'.

Another example is the collection of all numbers that are greater than or equal to -1.

On the number line, we graph this in the following way.



Here, the filled dot indicates that -1 is included in the interval. The interval begins at and includes -1, and extends indefinitely to the right.

The word 'set' means the same as 'collection'. Sets are indicated by curly brackets, {,}. For example, 'the set of numbers x that are less than 2' can be written as $\{x: x < 2\}$. When we write $\{x: x \ge 1\}$, it represents 'the set of all numbers x that are greater than or equal to 1'.

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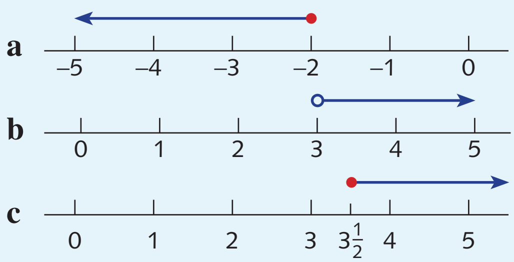
Example 19

Graph each set on the number line.

a $\{x: x \le -2\}$ **b** $\{x: x > 3\}$ **c** $\left\{x: x \ge 3\frac{1}{2}\right\}$

Solution



Exercise 5G

1 Copy and insert > or < to make each statement true.

a 7 … 2 **b** 3 … –4 **c** –4 … –2

d –54 … –500 **e** –6 … 0 **f** –13 … –45

g 21 … 40 **h** –2 … 5 **i** 99 … –100

2 Copy and insert ≤ or ≥ to make each statement true.

a –7 … –2 **b** 5 ... –7 **c** –5 … –5

d –10 … –50 **e** 0 … 0 **f** –23 … –45

g 12 … 26 **h** 9 … 9 **i** 98 … 89

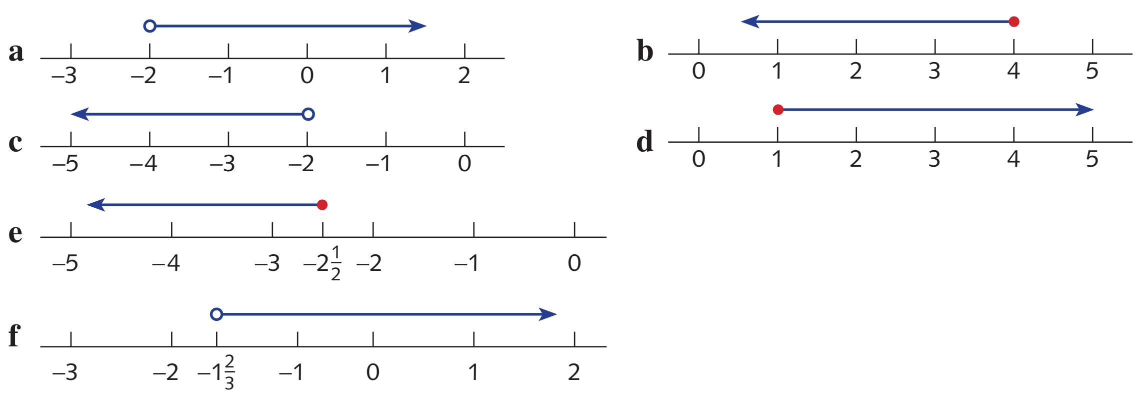
Example 19

3 Graph each set on the number line.

a $\{x: x > 10\}$ **b** $\{x: x \le 0\}$ **c** $\{x: x < -4\}$

d $\{x: x \ge -2\}$ **e** $\left\{x: x < 2\frac{1}{2}\right\}$ **f** $\left\{x: x \le -1\frac{1}{2}\right\}$

4 Use set notation to describe each interval.



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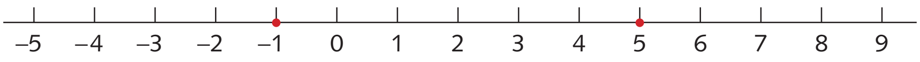
5H Solving linear inequalities

We first look at the effects of addition, subtraction, multiplication and division on inequalities.

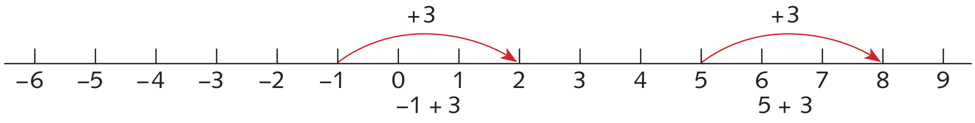
Addition and subtraction

We know that $-1 < 5$.

On the number line:

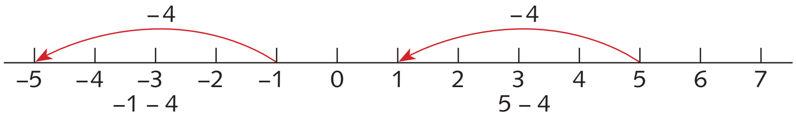


If we add 3 to each number, each moves 3 to the right.



$$-1 + 3 < 5 + 3$$

If we subtract 4 from each number, each moves 4 to the left.



$$-1 - 4 < 5 - 4$$

So we see that in each case the inequality still holds.

Addition and subtraction

- If we add the same number to both sides of an inequality, then the resulting inequality is true.
- If we subtract the same number from both sides of an inequality, then the resulting inequality is true.

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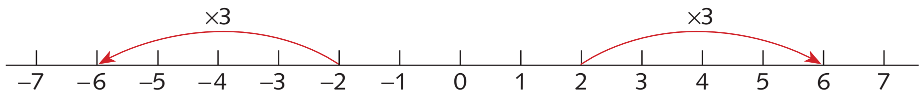
Multiplication and division

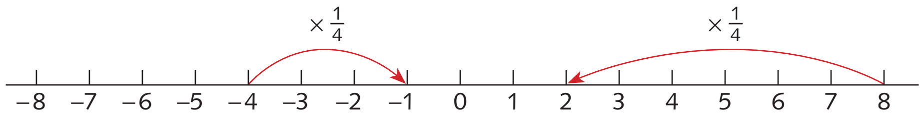
Multiplication by a positive number

If we multiply any number by 3, it moves to 3 times the distance it was **from 0** and stays on the same side of 0. The effect of multiplying both sides of an inequality by 3 is shown in a diagram.



$$-2 < 2 \text{ so } 3 \times -2 < 3 \times 2$$

If we multiply by $\frac{1}{4}$, then the new number is $\frac{1}{4}$ of the distance **from 0**, but still on the same side of 0. The effect on an inequality can be seen in a diagram.

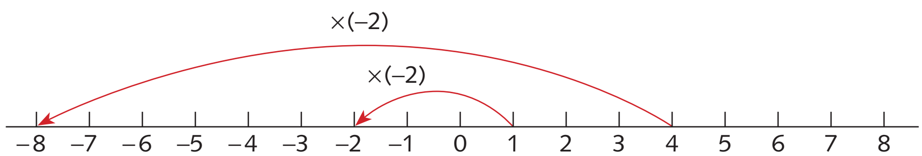


$$-4 < 8 \text{ so } \frac{1}{4} \times -4 < \frac{1}{4} \times 8$$

So the inequality remains true if we multiply both sides by a positive number.

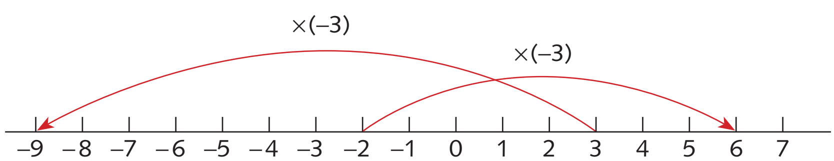
Multiplication by a negative number

We know that $1 < 4$. If we multiply both numbers by -2, they move to the opposite side of 0, and are twice as far from 0 as they were before.



$$1 < 4 \text{ but } -2 \times 1 > -2 \times 4$$

Similarly,

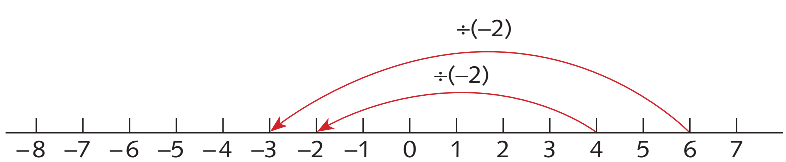


$$3 > -2 \text{ but } -3 \times 3 < -3(-2)$$

That is, in both cases the inequality sign is *reversed*.

Division by a negative number

These diagrams explain what happens to an inequality when we divide both sides by a negative number.



$$4 < 6 \text{ but } 4 \div (-2) > 6 \div (-2)$$

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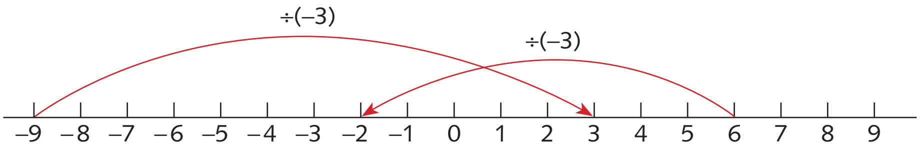

$6 > -9$ but $6 \div (-3) < -9 \div (-3)$

Multiplication and division

- If we multiply or divide both sides of an inequality by a positive number, then the resulting inequality is true.
- If we multiply or divide both sides of an inequality by a negative number, then we must reverse the inequality sign to make the resulting inequality true.

Linear inequalities

Solving the linear inequality $4x - 5 < 3$ means finding all of the values for x that satisfy that inequality.

It is easy to find some numbers that satisfy the inequality. For example, if $x = -2$,

$$4x - 5 = -13 < 3$$

Similarly, if $x = 0$,

$$4x - 5 = -5 < 3$$

However, $x = 4$ does not satisfy the inequality, because when $x = 4$,

$$4x - 5 = 11 > 3$$

So $x = -2$ and $x = 0$ satisfy the inequality, but $x = 4$ does not.

There is a systematic way of solving linear inequalities, as shown in the following examples. We often graph the solution set on the number line.

The method for solving linear inequalities is the same as that for solving linear equations, except for multiplying and dividing both sides by a negative number.

Example 20

a Solve the inequality $4x - 5 < 3$.

b Graph the solution set on the number line.

Solution

a If x satisfies the inequality

$$4x - 5 < 3$$

then $4x < 8$ (Add 5 to both sides.)

so $x < 2$ (Divide both sides by 4.)

(continued over page)

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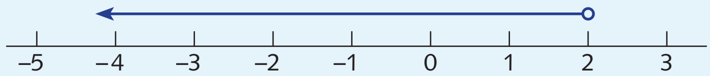
Conversely, if $x < 2$, then $4x < 8$ and $4x - 5 < 8 - 5 = 3$.

The solution is $\{x: x < 2\}$.

b The solution set is indicated on the number line.



Example 21

Solve each of the following inequalities.

a $-2x \leq 6$

b $-\frac{x}{3} > 4$

Solution

a $-2x \leq 6$

$x \geq -3$ (Divide both sides by –2 and ***reverse*** the inequality sign.)

b $-\frac{x}{3} > 4$

$x < -12$ (Multiply both sides by –3 and ***reverse*** the inequality sign.)

In Example 20, '$x < 2$', '$4x < 8$' and '$4x - 5 < 3$' are examples of **equivalent** inequalities. Equivalent inequalities have exactly the same solutions.

Two standard methods of setting out equivalent inequalities are shown in the next example.

Example 22

a Solve the inequality $2x + 3 < 3x - 4$.

b Graph the solution set on the number line.

Solution

a Method 1

$2x + 3 < 3x - 4$

$3 < x - 4$ (Subtract $2x$ from both sides.)

$7 < x$ (Add 4 to both sides.)

so $x > 7$

(continued over page)

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Method 2

$2x + 3 < 3x - 4$

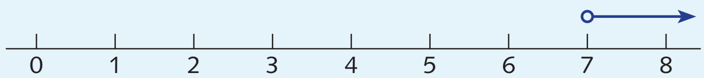
$2x < 3x - 7$ (Subtract 3 from both sides.)

$-x < -7$ (Subtract $3x$ from both sides.)

$x > 7$ (Multiply by -1 and reverse the inequality sign.)

The solution set is $\{x: x > 7\}$.

b The solution set is as shown:



Solving linear inequalities

- Multiply out brackets and multiply through by the lowest common denominator of fractions.
- Move all of the terms involving x to one side and all of the constant terms to the other side of the inequality sign.
- Simplify.
- Divide by the coefficient of x, remembering to reverse the inequality sign if the coefficient is negative.

Exercise 5H

1 **a** If $x < 6$, how many different values can x take?

b Give three values of x that satisfy $x > -1$.

c Give three non-whole numbers that satisfy $x > 2$.

Example 20

2 Solve each of these inequalities. Graph each solution set to parts **a–f** on a number line.

a $x + 3 \geq 7$ **b** $x - 2 < 3$ **c** $x + 4 \geq -8$

d $x - 10 > -6$ **e** $x - 5 > -12$ **f** $2x \geq 6$

g $3x > -15$ **h** $\frac{x}{5} \geq 4$ **i** $\frac{x}{2} \leq -3$

3 Solve:

a $2x + 1 \geq 5$ **b** $4x - 6 \leq -2$ **c** $\frac{2x}{3} > 5$

d $\frac{4x}{7} \leq -2$ **e** $3(x + 5) \geq 9$ **f** $2(5x - 2) > 5$

g $\frac{x}{3} - \frac{1}{2} \geq 1$ **h** $\frac{2x}{5} + \frac{1}{4} > 4$ **i** $2(x - 3) \leq 5$

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Example 21

4 Solve:

a $-4x \le 20$

b $-10x \ge 130$

c $-12x > -42$

d $-\frac{x}{2} \le 5$

e $-\frac{x}{7} \ge 4$

f $-\frac{x}{5} > 4$

g $-\frac{x}{12} \ge -8$

h $-\frac{x}{2} \ge -8$

i $2 - \frac{x}{9} > 5$

5 Solve:

a $3 - 2x > 5$

b $2 - 5x \le -8$

c $4(7 - x) < 5$

d $6 - \frac{2x}{3} < 4$

e $4 - \frac{2x}{5} \ge 6$

f $8 - \frac{3x}{7} \ge 2$

Example 22

6 Solve:

a $\frac{x+3}{2} \le \frac{3-x}{2}$

b $\frac{2x-1}{3} - \frac{3x+2}{4} > 3$

7 Solve:

a $1.2x + 6.8 \le 15.2$

b $2.4 - 0.7x \le 12.9$

c $1.6(x + 7) \le 1.5(x - 3)$

d $2.8(x - 4) > 1.3(x + 3.5)$

8 Solve:

a $2x - 14 \le 8$

b $-5x + 3 \ge 78$

c $\frac{2x+1}{6} > -3$

d $-\frac{x+2}{3} \le 7$

e $\frac{x-8}{2} - \frac{2x}{3} \ge 3$

f $\frac{x}{4} > -\frac{x+12}{5}$

9 When 5 is added to twice p, the result is greater than 17. What values can p take?

10 When 16 is subtracted from half of q, the result is less than 18. What values can q take?

11 When $2p$ is subtracted from 10, the result is greater than or equal to 4. What values can p take?

12 The sum of $4d$ and 6 is greater than the sum of $2d$ and 18. What values can d take?

13 A number a is increased by 3 and this amount is then doubled. If the result of this is greater than a, what values can a take?

14 Two car hire firms offer the following deals.

Movit: $25 plus 6 cents per km

Rentlow: $20 plus 8 cents per km

a If a client is to drive x km, express the cost of hiring a car from:

i Movit, in terms of x

ii Rentlow, in terms of x

b For what distances does Movit offer the better deal?

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Review exercise

1 David weighs 5 kg less than Christos. If Christos weighs w kg, express David's weight in terms of w.

2 When John's age is doubled, the number is 5 more than Kayla's age. If John is x years old, what is Kayla's age in terms of x?

3 The length of a rectangle is 10 m greater than the width of the rectangle.

a If w m is the width of the rectangle, express the length of the rectangle in terms of w.

b If the length of the rectangle is ℓ m, express the width of the rectangle in terms of ℓ.

4 Solve these equations.

a $a + 7 = 5$ **b** $-3m = 18$ **c** $4q = -27$

d $6a + 21 = -1$ **e** $7a - 26 = -35$ **f** $7b - 27 = -16$

g $-6 - 8e = 24$ **h** $-5 - 7f = 24$ **i** $5x - 4 = 11 - x$

j $18 - 7x = 3x - 15$ **k** $3 - 2x = 7 - 4x$ **l** $-5p + 8 = 6 - 7p$

5 Solve these equations.

a $5(x + 3) = 18$ **b** $5(x - 2) = 15$

c $12(x - 1) = 96$ **d** $7(x + 1) = 10$

e $7(3 - x) = 20$ **f** $6(11 - x) = 17$

g $6(a + 1) - 4(a + 2) = 24$ **h** $5(b - 1) - 8(b - 2) = 30$

i $5(a + 3) = 7(a + 11)$ **j** $15(3a - 1) = 2(6a + 7)$

k $-3(x - 2) = 2(3x - 1)$ **l** $-4(5m + 6) = -3(4 - 5m)$

6 Solve:

a $11 - 3.6c = 3.8$ **b** $12.6 - 4.5\ell = -5.4$ **c** $1.6(x + 7) = 17.6$

d $2.8(x - 4) = 16.8$ **e** $4.3(x + 11) = 53.75$ **f** $3.5(8 - x) = 29.5$

7 Solve these equations.

a $\frac{x}{3} - 2 = -7$ **b** $\frac{x}{2} + 6 = -3$

c $\frac{5p - 2}{4} + \frac{1}{2} = -2$ **d** $\frac{5p}{4} - 1 = \frac{3}{7}$

e $3x + 11 = 7x - 23$ **f** $4x - 2 = 15 - 3x$

g $-2(z - 1) = 5(z + 6)$ **h** $5(y + 2) = 2(2y - 1)$

i $12(k + 1) - 3(k - 2) = -7$ **j** $4(\ell - 1) - 3(\ell + 2) = 18$

k $\frac{m + 1}{7} = \frac{2m - 2}{7}$ **l** $\frac{2n - 1}{3} = \frac{5n + 1}{7}$

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m $\dfrac{2y+1}{3} + 7 = 11$

n $\dfrac{7-4x}{4} - 1 = 6$

o $\dfrac{3s-2}{5} + 1 = 6s$

p $\dfrac{2x+1}{3} + 7 = \dfrac{3x-1}{4}$

q $\dfrac{2a+5}{6} + a = 4$

r $\dfrac{x+2}{5} + \dfrac{x-1}{2} = \dfrac{x+1}{3}$

8 The length of a rectangular lawn is 15 m more than four times its width.

a If x m is the width of the lawn, express the length of the lawn in terms of x.

b If the perimeter of the lawn is 265 m, find the length and width of the lawn.

9 Mr Guernsey earns \$14 800 more than Mr Jersey, and Mrs Mann earns \$22 000 less than Mr Jersey.

a If \$$x$ represents Mr Jersey's salary, express the salary of:

i Mr Guernsey in terms of x

ii Mrs Mann in terms of x

b If the total of the three incomes is \$351 600, find the income of each person.

10 A person has a number of 10-cent and 20-cent coins. If their total value is \$18, how many are there of each coin if there are:

a equal numbers of each coin?

b twice as many 10-cent coins as there are 20-cent coins?

c twice as many 20-cent coins as there are 10-cent coins?

11 The distance between two towns A and B is 450 km. Find x if:

a the trip takes x hours at an average speed of 90 km/h

b the trip takes $5\frac{1}{4}$ hours at an average speed of x km/h

c the trip takes 5 hours, travelling x hours at 110 km/h and the remaining time at 60 km/h

12 A room has a length 8 m shorter than its width and a perimeter of 80 m. Find the length and width of the room.

13 Solve these equations for x.

a $\dfrac{mx+n}{a} = b$

b $m(x-n) = p$

c $a(bx-c) = d$

d $mx - n = px + q$

e $\dfrac{x}{a} + \dfrac{b}{c} = d$

f $\dfrac{m(x+n)}{a} = b$

14 Copy each statement and insert the correct symbol, ≥ or ≤.

a 11 ... 5

b −8 … −12

c −14 … −16

d −8 … −7

e −11 … 5

f −10 … −100

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15 Graph each set on the number line.

a $\{x: x > 2\}$

b $\{x: x < 3\}$

c $\{x: x < -1\frac{1}{2}\}$

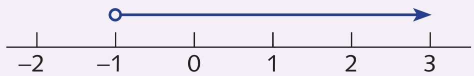
d $\{x: x \geq 4\}$

e $\{x: x \geq \frac{1}{3}\}$

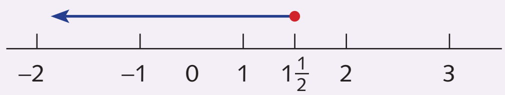
f $\{x: x \leq 2\frac{1}{4}\}$

16 Use set notation to describe each interval.

a



b



17 Solve these inequalities.

a $x + 7 \geq 17$

b $x - 7 \leq -4$

c $-2x \geq 16$

d $\frac{x}{3} < -5$

e $5x + 3 \geq 13$

f $\frac{x}{5} - \frac{1}{2} \geq 2$

g $\frac{2x}{11} + \frac{3}{5} > 4$

h $6(x - 3) \leq 15$

i $4(7 - x) < 36$

j $7 - 4x \leq -12$

k $11 - 3x > 18$

l $15 - \frac{x}{18} < 14$

m $10(x + 7) \leq 15(x - 3)$

n $2(x - 4) > 8(x + 3.5)$

o $3(5 - x) > 8(x - 3.2)$

p $6(1 - x) \leq 2.4(15 - 12x)$

18 Solve these inequalities.

a $\frac{x-3}{3} \leq \frac{3+x}{2}$

b $\frac{2x+3}{2} - \frac{x-4}{3} > 2$

c $\frac{4-x}{2} + \frac{3-x}{4} < 1$

d $\frac{3-2x}{2} \geq \frac{7-10x}{5}$

e $\frac{5-2x}{3} - \frac{5+2x}{4} \geq -1$

f $\frac{2x-1}{7} \geq \frac{2x+3}{4}$

g $\frac{6x+1}{3} > \frac{3x-1}{2} + 3$

h $\frac{6-3x}{2} \geq \frac{3x-6}{5}$

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Challenge exercise

1 At 2 p.m., two aeroplanes leave airports 2880 km apart and fly towards one another. The average speed of one plane is twice that of the other. If they pass each other at 5 p.m., what is the average speed of each plane?

2 When a mathematics teacher was asked her age she replied, ‘One-fifth of my age three years ago, when added to half my age last year, gives my age 11 years ago.’ How old is she?

3 \$420 is divided between A, B and C. B receives \$20 less than A, and C receives half as much as A and B together. How much does each receive?

4 In a printing works, 75 000 leaflets are run off by two printing presses in $18\frac{3}{4}$ hours. One press delivers 200 more leaflets per hour than the other. Find the number of leaflets produced per hour by each of the machines.

5 Ten cm^3 of silver and 5 cm^3 of copper weigh 150 g. One cubic centimetre of silver weighs $1\frac{1}{6}$ times as much as 1 cm^3 of copper. Find the weight of 1 cm^3 of each of the metals.

6 A river flows at 5 km/h. A boat goes upstream half as fast as downstream. What is the speed of the boat in still water?

7 Twenty litres of milk with unknown butter-fat content from vat A is mixed with 10 litres of milk with 3% butter-fat from vat B to produce milk with $3\frac{3}{4}\%$ butter-fat content. Find the percentage of butter-fat in vat A.

8 The ends of a bar have to be machined down until the bar is 3 m long. After 5% of the length had been removed, it was found that 0.5% of the new length still had to come off. What length was removed at each machining and what was the original length of the bar?

9 A salesman makes a trip to visit a client. The traffic he encounters keeps his average speed to 40 km/h. On the return trip, he takes a route 6 km longer, but he averages 50 km/h. If he takes the same time each way, how long was the total journey?

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