CHAPTER

6

Number and Algebra

Formulas

The area, A square units, of a circle with radius r units is given by $A = \pi r^2$.

The volume, V cubic units, of a cylinder with radius r units and height h units is given by $V = \pi r^2 h$.

Einstein discovered the remarkable formula $E = mc^2$, where:

E = energy

m = mass

and c = the speed of light.

These are examples of formulas.

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6A Substitution into formulas

A formula relates different quantities. For instance, the formula $A = \pi r^2$ relates the radius, r units, of a circle to the area, A square units, of the circle. That is, it describes how the area of a circle depends on its radius.

Formulas with a subject

In many formulas that you see there is a single pronumeral on the left-hand side of the equal sign. This pronumeral is called the **subject** of the formula.

For instance, in the formula $E = mc^2$, E is the subject.

Substitution

If the values of all the pronumerals except the subject of a formula are known, then we can find the value of the subject by substitution.

Example 1

Find the value of the subject when the pronumerals in the formula have the values indicated.

a $F = ma$, where $a = 10$, $m = 3.5$

b $m = \dfrac{a+b}{2}$, where $a = 12$, $b = 26$

Solution

a $F = ma$
$= 10 \times 3.5$
$= 35$

b $m = \dfrac{a+b}{2}$
$= \dfrac{12+26}{2}$
$= 19$

Example 2

The formula for the circumference C of a circle of radius r is $C = 2\pi r$.

Find the value of C when $r = 20$:

a in terms of π (that is, exactly)

b correct to 2 decimal places

Solution

a $C = 2\pi r$
$= 40\pi$

b $C = 40\pi$
$= 125.663...$ (using a calculator)
≈ 125.66 (correct to 2 decimal places)

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Example 3

a The area of a triangle A cm^2 is given by $A = \frac{1}{2}bh$, where b cm is the base length and h cm is the height. Calculate the area of a triangle with base length 16 cm and height 11 cm.

b The simple interest payable when \$$P$ is invested at a rate of r% per year for t years is given by $I = \frac{Prt}{100}$. Calculate the simple interest payable when \$1000 is invested at 3.5% per year for 6 years.

Solution

a
$$A = \frac{1}{2}bh$$
$$= \frac{1}{2} \times 16 \times 11$$
$$= 88$$

The area of the triangle is 88 cm^2.

b
$$I = \frac{Prt}{100}$$
$$= \frac{1000 \times 3.5 \times 6}{100}$$
$$= 210$$

The interest payable is \$210.

Substitution into a formula

When the pronumeral whose value is to be found is not the subject of the formula, it is necessary to solve an equation to find the value of this pronumeral. This is shown in the following examples.

Example 4

For a car travelling in a straight line with initial velocity u m/s and acceleration a m/s^2, the formula for the velocity v m/s at time t seconds is $v = u + at$.

a Find u if $a = 2$, $v = 15$ and $t = 7$.

b Find a if $v = 10$, $u = 6$ and $t = 3$.

Solution

a $v = u + at$

When $a = 2$, $v = 15$ and $t = 7$.
$$15 = u + 2 \times 7$$
$$15 = u + 14$$
$$u = 1$$

The initial velocity is 1 m/s.

b $v = u + at$

When $v = 10$, $u = 6$ and $t = 3$.
$$10 = 6 + 3a$$
$$4 = 3a$$
$$a = \frac{4}{3}$$

The acceleration is $\frac{4}{3}$ m/s^2.

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Example 5

The thin lens formula states that

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v},$$

where u is the distance from the object to the lens, v is the distance of the image from the lens and f is the focal length of the lens.

a Find f if $u = 2$ and $v = 5$.

b Find u if $f = 2$ and $v = 6$.

Solution

a $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

When $u = 2$ and $v = 5$,

$$\frac{1}{f} = \frac{1}{2} + \frac{1}{5}$$

$$= \frac{7}{10}$$

Taking reciprocals of both sides of the equation gives $f = \frac{10}{7}$.

b $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

If $f = 2$ and $v = 6$,

$$\frac{1}{2} = \frac{1}{u} + \frac{1}{6}$$

$$\frac{1}{u} = \frac{1}{2} - \frac{1}{6}$$

$$= \frac{1}{3}$$

Taking reciprocals of both sides of the equation gives $u = 3$.

Example 6

The area of a circle A cm^2 is given by $A = \pi r^2$, where r cm is the radius of the circle.

If $A = 20$, find r:

a exactly

b correct to 2 decimal places

Solution

a $A = \pi r^2$

When $A = 20$,

$$20 = \pi r^2$$

$$\frac{20}{\pi} = r^2 \quad \text{(Divide both sides of equation by } \pi\text{.)}$$

$$r = \sqrt{\frac{20}{\pi}} \quad (r \text{ is positive.})$$

b $r \approx 2.52$ (correct to 2 decimal places)

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Exercise 6A

Example 1

1 For each part, find the value of the subject when the other pronumerals have the value indicated.

a $A = \ell w$, where $\ell = 5$, $w = 8$

b $s = \dfrac{d}{t}$, where $d = 120$, $t = 6$

c $A = \dfrac{1}{2}xy$, where $x = 10$, $y = 7$

d $A = \dfrac{1}{2}(a + b)h$, where $a = 4$, $b = 6$, $h = 10$

e $t = a + (n - 1)d$, where $a = 30$, $n = 8$, $d = 4$

f $E = \dfrac{1}{2}mv^2$, where $m = 8$, $v = 4$

Example 2, 3

2 For each part, find the value of the subject when the other pronumerals have the value indicated. Calculate **a–c** correct to 3 decimal places and **d** correct to 2.

a $x = \sqrt{ab}$, where $a = 40$, $b = 50$

b $V = \pi r^2 h$, where $r = 12$, $h = 20$

c $T = 2\pi\sqrt{\dfrac{\ell}{g}}$, where $\ell = 88.2$, $g = 9.8$

d $A = P(1 + R)^n$, where $P = 10\,000$, $R = 0.065$, $n = 10$

Example 4

3 For the formula $v = u + at$, find:

a v if $u = 6$, $a = 3$ and $t = 5$

b u if $v = 40$, $a = 5$ and $t = 2$

c a if $v = 60$, $u = 0$ and $t = 5$

d t if $v = 100$, $u = 20$ and $a = 6$

4 **a** For the formula $S = 2(a - b)$, find a if $S = 60$ and $b = 10$.

b For the formula $I = \dfrac{180n - 360}{n}$, find n if $I = 120$.

c For the formula $a = \dfrac{m+n}{2}$, find m if $a = 20$ and $n = 6$.

d For the formula $A = \dfrac{PRT}{100}$, find P if $A = 1600$, $R = 4$ and $T = 10$.

e For the formula $S = 2(\ell w + \ell h + hw)$, find h if $S = 592$, $\ell = 10$ and $w = 8$.

f For the formula $s = ut + \dfrac{1}{2}at^2$, find a if $s = 1000$, $u = 20$ and $t = 5$.

g For the formula $t = a + (n - 1)d$, find n if $t = 58$, $d = 3$ and $a = 7$.

5 Given $v^2 = u^2 + 2ax$ and $v > 0$, find the value of v (correct to 1 decimal place) when:

a $u = 0$, $a = 5$ and $x = 10$

b $u = 2$, $a = 9.8$ and $x = 22$

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Example 5 **6** Given $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, find the value of f when:

a $u = 2$ and $v = 4$
b $u = 6$ and $v = 9$
c $u = 12$ and $v = 11$
d $u = 2$ and $v = 15$

7 For the formula $s = ut + \frac{1}{2}at^2$, find the value of:

a u, when $s = 10$, $t = 20$ and $a = 2$
b a, when $s = 20$, $u = 5$ and $t = 2$

Example 5 **8** Given $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, find the value of:

a u when $f = 2$ and $v = 4$
b u when $f = 3$ and $v = 4$

9 Given that $P = \frac{M+m}{M-m}$, find the value of P when:

a $M = 8$ and $m = 4$
b $M = 26$ and $m = 17$
c $M = 19.2$ and $m = 5.9$
d $M = \frac{3}{4}$ and $m = \frac{2}{5}$

Example 6 **10** The area A cm^2 of a square with side length x cm is given by $A = x^2$. If $A = 20$, find:

a the value of x
b the value of x correct to 2 decimal places.

11 For a rectangle of length ℓ cm and width w cm, the perimeter P cm is given by $P = 2(\ell + w)$.

Use this formula to calculate the length of a rectangle which has width 15 cm and perimeter 57 cm.

12 The formula for finding the number of degrees Fahrenheit (F) for a temperature given as a number of degrees Celsius (C) is $F = \frac{9}{5}C + 32$.

Fahrenheit temperatures are still used in the USA, but in Australia we commonly use Celsius. Calculate the Fahrenheit temperatures which people in the USA would recognise for:

a the freezing point of water, 0°C
b the boiling point of water, 100°C
c a nice summer temperature of 25°C

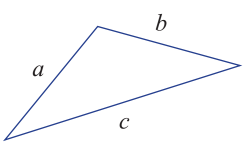
Now calculate the Celsius temperatures which people in Australia would recognise for:

d 50°F
e 104°F

13 The area A cm^2 of a triangle with side lengths a cm, b cm and c cm is given by Heron's formula

$$A^2 = s(s-a)(s-b)(s-c),$$

where $s = \frac{a+b+c}{2}$ = half the perimeter.



Find the exact areas of the triangles whose side lengths are given below.

a 6 cm, 8 cm and 10 cm
b 5 cm, 12 cm and 13 cm
c 8 cm, 10 cm and 14 cm
d 13 cm, 14 cm and 15 cm

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14 Sam throws a stone down to the ground from the top of a cliff s metres high, with an initial speed of u m/s. It accelerates at a m/s^2. The stone hits the ground with a speed of v m/s given by the formula $v^2 = u^2 + 2as$. Find the speed at which the stone hits the ground, correct to 2 decimal places, if:

a $u = 0$, $a = 9.8$ and $s = 50$ **b** $u = 5$, $a = 9.8$ and $s = 35$

15 The distance d metres Jim's car takes to stop once the brakes are applied is given by the formula $d = 0.2v + 0.005v^2$, where v km/h is the speed of the car when the brakes are applied.

Find the distance the car takes to stop if the brakes are applied when it is travelling at each of the speeds given below. Calculate your answers correct to 3 decimal places where appropriate.

a 60 km/h **b** 65 km/h **c** 70 km/h

d 80 km/h **e** 100 km/h **f** 120 km/h

6B Changing the subject of a formula

Sometimes the pronumeral whose value is to be determined is not the subject of the formula. In this situation you have a choice. You can either:

- **rearrange** the formula to make the unknown pronumeral the subject and then substitute values for the known pronumerals (method 1), or
- **substitute** the values for the known pronumerals and then solve the resulting equation for the unknown pronumeral (method 2). This was the approach taken in Section 6A.

In either case, an equation has to be solved. In method 2, the equation involves numbers. In method 1, the equation involves pronumerals.

It is preferable to use method 1 when you are asked to find several values of a pronumeral that is not the subject of the formula.

Example 7

The cost \$$C$ of hiring Scott's car is given by the formula $C = \frac{1}{4}x + 40$, where x is the number of kilometres driven. Find the number of kilometres driven by a person who is charged \$130 for hiring the car.

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Solution

Method 1

Rearrange the formula to make x the subject and then substitute the given value of C, as follows.

$$C = \frac{1}{4}x + 40$$

$$C - 40 = \frac{1}{4}x \qquad \text{(Subtract 40 from both sides of the formula.)}$$

$$x = 4C - 160 \qquad \text{(Multiply both sides of the formula by 4.)}$$

When $C = 130$, $x = 4 \times 130 - 160$
$= 360$

Thus the person drove 360 km.

Method 2

Substitute the numbers and then solve the resulting equation, giving

$$C = \frac{1}{4}x + 40$$

When $C = 130$,

$$130 = \frac{1}{4}x + 40$$

$$90 = \frac{1}{4}x \qquad \text{(Subtract 40 from both sides of the equation.)}$$

$$x = 360 \qquad \text{(Multiply both sides of the equation by 4.)}$$

Thus the person drove 360 km.

Example 8

The manager of a bed-and-breakfast guest house finds that the weekly profit $\$P$ is given by the formula

$$P = 40G - 600,$$

where G is the number of guests who stay during the week. Make G the subject of the formula and use the result to find the number of guests needed to make a profit of \$800.

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Solution

$$\begin{aligned} P &= 40G - 600 \\ P + 600 &= 40G \\ G &= \frac{P + 600}{40} \end{aligned}$$

When $P = 800$,

$$\begin{aligned} G &= \frac{800 + 600}{40} \\ &= \frac{1400}{40} = 35 \end{aligned}$$

Thirty-five guests are required to make a profit of $800.

Example 9

Given the formula $v^2 = u^2 + 2as$:

a rearrange the formula to make s the subject

b find the value of s when $u = 4, v = 10$ and $a = 2$

c find the value of s when $u = 4, v = 12$ and $a = 3$

Solution

a

$$\begin{aligned} v^2 &= u^2 + 2as \\ v^2 - u^2 &= 2as && \text{(Subtract } u^2 \text{ from both sides of the formula.)} \\ s &= \frac{v^2 - u^2}{2a} && \text{(Divide both sides of the equation by } 2a.\text{)} \end{aligned}$$

b When $u = 4, v = 10$ and $a = 2$.

$$\begin{aligned} s &= \frac{10^2 - 4^2}{2 \times 2} \\ &= \frac{100 - 16}{4} \\ &= \frac{84}{4} \\ &= 21 \end{aligned}$$

c When $u = 4, v = 12$ and $a = 3$.

$$\begin{aligned} s &= \frac{12^2 - 4^2}{2 \times 3} \\ &= \frac{144 - 16}{6} \\ &= \frac{128}{6} \\ &= 21\tfrac{1}{3} \end{aligned}$$

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Example 10

Rearrange each of these formulas to make the pronumeral in brackets the subject.

a $E = \dfrac{p^2}{2m}$ (m)

b $T = 2\pi\sqrt{\dfrac{\ell}{g}}$ (ℓ)

c $\dfrac{1}{f} = \dfrac{1}{u} + \dfrac{1}{v}$ (u)

d $P = \sqrt{h+c} - a$ (h)

Solution

a
$$E = \frac{p^2}{2m}$$
$$mE = \frac{p^2}{2}$$ (Multiply both sides of the equation by m.)
$$m = \frac{p^2}{2E}$$ (Divide both sides by E.)

b
$$T = 2\pi\sqrt{\frac{\ell}{g}}$$
$$\frac{T}{2\pi} = \sqrt{\frac{\ell}{g}}$$ (Divide both sides of the equation by 2π.)
$$\frac{\ell}{g} = \left(\frac{T}{2\pi}\right)^2$$ (Square both sides of the equation.)
$$= \frac{T^2}{4\pi^2}$$
$$\ell = \frac{T^2}{4\pi^2} \times g$$ (Multiply both sides of the equation by g.)

That is, $\ell = \dfrac{T^2 g}{4\pi^2}$

c
$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$
$$\frac{1}{f} - \frac{1}{v} = \frac{1}{u}$$ (Subtract $\dfrac{1}{v}$ from both sides.)
$$\frac{v - f}{fv} = \frac{1}{u}$$ (common denominator on LHS of equation)
$$u = \frac{fv}{v - f}$$ (Take reciprocals of both sides.)

Note: $\dfrac{1}{f} = \dfrac{1}{u} + \dfrac{1}{v}$ does not imply $f = u + v$.

d
$$P = \sqrt{h + c} - a$$
$$P + a = \sqrt{h + c}$$ (Add a to both sides of the equation.)
$$(P + a)^2 = h + c$$ (Square both sides.)
$$h = (P + a)^2 - c$$

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The previous example shows some of the techniques that can be used to rearrange a formula.

Example 11

Make q the subject of the formula $\frac{3p}{4} - \frac{5}{q} = \frac{p^2}{3q}$.

Solution

$$\frac{3p}{4} - \frac{5}{q} = \frac{p^2}{3q}$$

$$12q\left(\frac{3p}{4} - \frac{5}{q}\right) = 12q \times \frac{p^2}{3q}$$

$$3p \times 3q - 5 \times 12 = p^2 \times 4$$ (Multiply both sides by the lowest common denominator, $12q$.)

$$9pq - 60 = 4p^2$$

$$9pq = 4p^2 + 60$$

$$q = \frac{4p^2 + 60}{9p}$$

Changing the subject of a formula

When rearranging a formula, the basic strategy is to move all terms involving the new subject to one side, and all the other terms to the other side. To do this:

- fractions can be eliminated by multiplying both sides of the formula by a common denominator
- all like terms should be collected
- the same operation(s) must be performed on both sides of the formula.

Exercise 6B

Example 7

1 The profit $\$P$ made each day by a store owner who sells CDs is given by the formula $P = 5n - 150$, where n is the number of CDs sold.

a What profit is made if the store owner sells 60 CDs?

b Make n the subject of the formula.

c How many CDs were sold if the store made:

i a profit of \$275?

ii a profit of \$400?

iii a loss of \$100?

iv no profit?

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Example 8

2 The cost $\$C$ of hiring a reception room for a function is given by the formula $C = 12n + 250$, where n is the number of people attending the function.

a Rearrange the formula to make n the subject.

b How many people attended the function if the cost of hiring the reception room was:

i \$730? **ii** \$1090? **iii** \$1210? **iv** \$1690?

3 Given the formula $v = u + at$:

a rearrange the formula to make u the subject

b find the value of u when:

i $v = 20$, $a = 2$ and $t = 5$ **ii** $v = 40$, $a = -6$ and $t = 4$

c rearrange the formula to make a the subject

d find the value of a when:

i $v = 20$, $u = 15$ and $t = 2$ **ii** $v = -26.8$, $u = -14.4$ and $t = 2$

iii $v = \frac{1}{2}$, $u = \frac{2}{3}$ and $t = \frac{5}{6}$

e rearrange the formula to make t the subject and find t when $v = 6$, $u = 7$ and $a = -3$.

4 Given the formula $t = a + (n - 1)d$:

a rearrange the formula to make a the subject

b find the value of a when:

i $t = 11$, $n = 4$ and $d = 3$ **ii** $t = 8$, $n = 5$ and $d = -3$

c rearrange the formula to make d the subject

d find the value of d when:

i $t = 48$, $a = 3$ and $n = 16$ **ii** $t = 120$, $a = -30$ and $n = 101$

e rearrange the formula to make n the subject and find the value of n when $t = 150$, $a = 5$ and $d = 5$

Example 10

5 Rearrange each of these formulas to make the pronumeral in brackets the subject.

a $y = mx + c$ (c) **b** $y = mx + c$ (x)

c $A = \frac{1}{2}bh$ (x) **d** $C = 2\pi r$ (r)

e $P = A + 2\ell h$ (l) **f** $s = ut + \frac{1}{2}at^2$ (a)

g $A = 2\pi r^2 + 2\pi rh$ (h) **h** $V = \frac{1}{3}\pi r^2 h$ (h)

i $s = \frac{n}{2}(a + \ell)$ (a) **j** $S = \frac{n}{2}(a + \ell)$ (n)

k $V = \pi r^2 + \pi rs$ (s) **l** $E = mgh + \frac{1}{2}mv^2$ (h)

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6 The formula for the sum S of the interior angles in a convex n-sided polygon is $S = 180(n - 2)$. Rearrange the formula to make n the subject and use this to find the number of sides in the polygon if the sum of the interior angles is:

a 1080° **b** 1800° **c** 3240°

Example 9

7 The kinetic energy E joules of a moving object is given by $E = \frac{1}{2}mv^2$, where m kg is the mass of the object and v m/s is its speed.

Rearrange the formula to make m the subject and use this to find the mass of the object when its energy and speed are, respectively:

a 400 joules, 10 m/s **b** 28 joules, 4 m/s **c** 57.6 joules, 2.4 m/s

8 When an object is shot up into the air with a speed of u metres per second, its height above the ground h metres and time of flight t seconds are related (ignoring air resistance) by $h = ut - 4.9t^2$.

Find the speed at which an object was fired if it reached a height of 27.5 metres after 5 seconds.

Example 11

9 Rearrange each of these formulas to make the pronumeral in brackets the subject. (All pronumerals represent positive numbers.)

a $c = a^2 + b^2$ (a) **b** $K = 3\ell^2 + 4m$ (ℓ)

c $x = \sqrt{ab}$ (b) **d** $d = 8\sqrt{\frac{h}{5}}$ (h)

e $T = \frac{2\pi}{n}$ (n) **f** $D = \frac{m}{v}$ (v)

g $E = \frac{m}{2r^2}$ (r) **h** $T = \frac{a}{4r^2}$ (r)

10 Temperatures can be measured in either degrees Fahrenheit or degrees Celsius. To convert from one scale to the other, the following formula is used: $F = \frac{9}{5}C + 32$.

a Rearrange the formula to make C the subject.

b On a particular day in Melbourne, the temperature was 28°C. What is this temperature measured in Fahrenheit?

c In Boston, USA, the minimum overnight temperature was 4°F. What is this temperature measured in Celsius?

d What number represents the same temperature in °C and °F?

e An approximate conversion formula, used frequently when converting oven temperatures, is $F = 2C + 30$. Use this to convert these temperatures:

i an oven temperature of 180°C **ii** an oven temperature of 530°F

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6C Constructing formulas

In this section we shall learn how to create formulas from given information.

Example 12

Find a formula for n, the number of cents in x dollars and y cents.

Solution

In $\$x$ there are $100x$ cents.

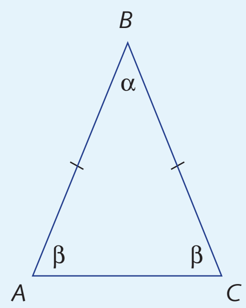
In $\$x$ and y cents there are $(100x + y)$ cents.

The formula is $n = 100x + y$.

Example 13

Here is an isosceles triangle with equal base angles marked.

Find a formula for β in terms of α.



Solution

$$\alpha + 2\beta = 180 \qquad \text{(angle sum of triangle)}$$
$$2\beta = 180 - \alpha$$
$$\beta = \frac{180 - \alpha}{2}$$

Exercise 6C

Example 12

1 Construct a formula for:

a D in terms of n, where D is the number of degrees in n right angles

b c in terms of D, where c is the number of cents in $\$D$

c m in terms of h, where m is the number of minutes in h hours

d d in terms of m, where d is the number of days in m weeks

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2 Construct a formula for:

a the number of centimetres n in p metres

b the number of millilitres s in t litres

c the number of centimetres q in $5p$ metres

d the number of grams x in $\frac{y}{2}$ kilograms

3 Find a formula for:

a the number of cents z in x dollars and y cents

b the number of minutes x in y minutes and z seconds

c the number of hours x in y minutes and z seconds

d the cost $\$m$ of 1 book if 20 books cost $\$c$

e the cost $\$n$ of 1 suit if 5 suits cost $\$m$

f the cost $\$m$ of 1 tyre if x tyres cost $\$y$

g the cost $\$p$ of n suits if 4 suits cost $\$k$

h the cost $\$q$ of x cars if 8 cars cost $\$b$

Example 13

4 Find a formula relating x and y for each of these statements, making y the subject.

a y is three less than x.

b y is four more than the square of x.

c y is eight times the square root of one-fifth of x.

d x and y are supplementary angles.

e A car travelled 80 km in x hours at an average speed of y km/h.

f A car used x litres of petrol on a trip of 80 km and the fuel consumption was y litres/100 km.

5 Find a formula relating the given pronumerals for each of these statements.

a The number of square cm x in y square metres

b The selling price $\$S$ of an article with an original price of $\$m$ when a discount of 20% is given

c The length c cm of the hypotenuse and the lengths a cm and b cm of the other two sides in a right-angled triangle

d The area A cm^2 of a sector of a circle with a radius of length r cm and angle θ at the centre of the circle

e The distance d km travelled by a car in t hours at an average speed of 75 km/h

f The number of hectares h in a rectangular paddock of length 400 m and width w m

6 In each part, find a formula from the information given.

a A hire car firm charges \$20 per day plus 40 cents per km. What is the total cost $\$C$ for a day in which x km was travelled?

b If there are 50 litres of petrol in the tank of a car and petrol is used at the rate of 4 litres per day, what is the number of litres y that remains after x days?

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c Cooking instructions for a forequarter of lamb are as follows: preheat oven to 220°C and cook for 45 min per kg plus an additional 20 min. What is the formula relating the cooking time T minutes and weight w kg?

d In a sequence of numbers the first number is 2, the second number is 4, the third is 8, the fourth is 16, etc. Assuming the doubling pattern continues, what is the formula you would use to calculate t, the nth number?

e A piece of wire of length x cm is bent into a circle of area A cm^2. What is the formula relating A and x?

7 Gareth the gardener has a large rectangular vegetable patch and he wishes to put in a path around it using concrete pavers that measure 50 cm × 50 cm. The path is to be 1 paver wide. Let n be the number of pavers required. If the vegetable patch measures x metres by y metres, find a formula for n in terms of x and y.

Review exercise

1 If $s = \frac{n}{2}(2a + (n-1)d)$:

a find the value of s when $n = 10$, $a = 6$ and $d = 3$

b find the value of a when $s = 350$, $n = 20$ and $d = 4$

c find the value of d when $s = 460$, $n = 10$ and $a = 10$

2 The formula for the geometric mean m of two positive numbers a and b is $m = \sqrt{ab}$.

a Find m if $a = 16$ and $b = 25$.

b Find a if $m = 7$ and $b = 16$.

3 If $x = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$:

a find x if $b = 4$, $a = 1$ and $c = -24$

b find c if $a = 1$, $x = 6$ and $b = 2$

4 A pillar is in the shape of a cylinder with a hemispherical top. If r metres is the radius of the base and h metres is the total height, the volume V cubic metres is given by the formula $V = \frac{1}{3}\pi r^2(3h - r)$.

a Rearrange the formula to make h the subject.

b Find the height of the pillar, correct to the nearest centimetre, if the radius of the pillar is 0.5 m and the volume is 10 m^3.

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5 Rearrange each of these formulas to make the pronumeral in brackets the subject. (All of the pronumerals represent positive numbers.)

a $A = \ell \times w$ (ℓ) **b** $C = 2\pi r$ (r)

c $V = \pi r^2 h$ (h) **d** $A = 2h(\ell + b)$ (b)

e $A = 4\pi r^2$ (r) **f** $w = 10\sqrt{\frac{x}{a}}$ (x)

g $w = \sqrt{\frac{3V}{\pi h}}$ (V) **h** $\frac{1}{x} + \frac{1}{y} = \frac{2}{z}$ (y)

6 If a stone is dropped off a cliff, the number of metres it has fallen after a certain number of seconds is found by multiplying the square of the number of seconds by 4.9.

a Find the formula for the distance d metres fallen by the stone in t seconds.

b Find the distance fallen in 1.5 seconds.

7 If $t = \sqrt{\frac{M}{M - m}}$:

a express the formula with m as the subject

b express the formula with M as the subject

c find the value of M if $m = 3$ and $t = \sqrt{2}$.

8 The total surface area $S\,\text{cm}^2$ of a cylinder is given in terms of its radius r cm and height h cm by the formula $S = 2\pi r(r + h)$.

a Express this formula with h as the subject.

b What is the height of such a cylinder if the radius is 7 cm and the total surface area is 500 cm^2? Calculate your answer in centimetres, correct to 2 decimal places.

9 The sum S of the squares of the first n whole numbers is given by the formula $S = \frac{n(n+1)(2n+1)}{6}$. Find the sum of the squares of:

a the first 20 whole numbers

b all the numbers from 5 to 21 inclusive

10 **a** For the formula $T = 2\pi\sqrt{\frac{W}{gF}}$, make F the subject.

b For the formula $P = \frac{\pi r x}{180} + 2r$, make x the subject.

c For the formula $D = \sqrt{\frac{f+x}{f-x}}$, make x the subject.

11 Cans in a supermarket are displayed in a triangular stack with one can at the top, two cans in the second row from the top, three cans in the third row from the top, and so on. What is the number of cans in the display if the number of rows is:

a 4? **b** 5? **c** n? **d** 35?

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12 Rearrange each of these formulas to make the pronumeral in brackets the subject. Assume all pronumerals take non-negative values.

a $M = \sqrt{\frac{ab}{t} + a^2}$ (t)

b $V = \frac{1}{3}a^2\sqrt{\ell^2 - \frac{a^2}{2}}$ (ℓ)

c $E = \frac{w^2 a}{(w^2 + m)b^3}$ (a)

d $T = 2\pi\sqrt{\frac{\ell + r}{g}}$ (r)

e $x = a\sqrt{\frac{k}{ph + y}}$ (h)

f $v^2 = u^2 + 2as$ (u)

13 The volume V cm^3 of metal in a tube is given by the formula $V = \pi\ell(r^2 - (r - t)^2)$ where ℓ cm is the length, r cm is the radius of the outside surface and t cm the thickness of the material.

a Find V if $\ell = 50$, $r = 5$ and $t = 0.25$.

b Make r the subject of the formula.

14 For the formula $I = \frac{180n - 360}{n}$:

a find I when $n = 6$

b make n the subject of the formula and find n when $I = 108$

15 Find the formula connecting x and y for each of these statements making y the subject.

a y is four more than twice the square of x.

b x and y are complementary angles.

c A car travelled x km in y hours at a speed of 100 km/h.

d A car travelled 100 km in y hours at a speed of x km/h.

Challenge exercise

1 Rearrange each of these formulas to make the pronumeral in brackets the subject. (All of the pronumerals represent positive numbers.)

a $P = \frac{M + m}{M - m}$ (M)

b $F = \frac{MP}{M + \ell}$ (M)

c $\frac{1}{S} = \frac{1}{R} + \frac{1}{T}$ (R)

d $E = \frac{R}{i} + P$ (i)

e $T = t - \frac{h}{\ell}$ (ℓ)

f $I = \frac{M}{4}\left(R^2 + \frac{L^2}{3}\right)$ (L)

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g $L = \ell\sqrt{1 + \dfrac{v^2}{c^2}}$ (v)

h $T = 2\pi\left(\dfrac{1+e}{1-e}\right)$ (e)

i $v = c\left(\dfrac{1}{r} - \dfrac{1}{s}\right)$ (r)

j $P = 2\pi\sqrt{\dfrac{h+k}{g}}$ (k)

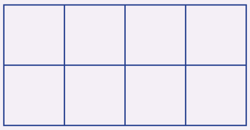
k $P = p\sqrt{1 + \dfrac{1}{\ell}}$ (ℓ)

l $A = \pi(R^2 - r^2)$ (R)

m $(a+b)^2 + c^2 = (a-d)^2$ (a)

n $(M-m)^2 + p^2 = M^2$ (M)

2 Sophie is playing with a box of connecting rods which can be joined together to produce a rectangular grid. The grid shown opposite is of size 4×2 (that is, length $= 4$, width $= 2$) and uses 22 rods.



a How many rods are needed to make these grids?

i 1×2 **ii** 3×2 **iii** 10×2 **iv** $n \times 2$

b How many rods are needed to make these grids?

i 1×3 **ii** 2×3 **iii** 10×3 **iv** $n \times 3$

c How many rods are needed to make a grid of size $n \times m$?

d In the 4×2 grid shown above, how many rods have been placed:

i vertically? **ii** horizontally?

e In an $n \times m$ grid, how many rods have been placed:

i vertically? **ii** horizontally? **iii** in total?

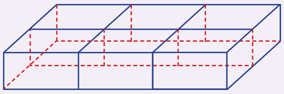
f Does your answer to part **e iii** agree with your answer to part **c**?

g Sophie makes a 4×7 grid. How many rods did she use?

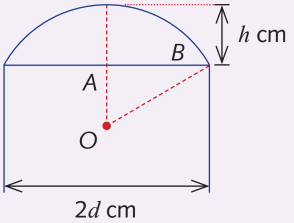
h If she used 97 rods to make a grid of width 6, what was the length of the grid?

i If she has 100 rods, can she make a grid that uses all 100 rods? If so, what size grid can she make?

j If the rods can be joined to make a three-dimensional grid, find the formula for the number of rods required to make a grid as shown of size $m \times n \times p$ (that is, length $= m$, width $= n$, height $= p$). The diagram shows a rectangular prism measuring 3 rods by 2 rods by 1 rod.



3 A builder wishes to place a circular cap of a given height above an existing window. To do this he needs to know the location of the centre of the circle (the cap is not necessarily a semicircle) and the radius of the circle. O is the centre of the required circle, the radius of the required circle is r cm, the width of the window is $2d$ cm and the height of the circular cap is h cm.



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a Express each of these in terms of r, d and h.

i AB **ii** OA

b Show that $r = \dfrac{h^2 + d^2}{2h}$.

c If the window is 120 cm wide and the cap is 40 cm high, find:

i the radius of the circle

ii how far below the top of the window the centre of the circle must be placed

d If the builder used a circle of radius 50 cm and this produced a cap of height 20 cm, what was the width of the window?

4 A group of n people attend a club meeting. Before the meeting begins, they all shake hands with each other. Write a formula to find H, the number of handshakes exchanged.

5 A cyclic quadrilateral has all its vertices on a circle. Its area A is given by Brahmagupta's formula

$$A^2 = (s - a)(s - b)(s - c)(s - d)$$

where a, b, c and d are the side lengths of the quadrilateral and

$s = \dfrac{a + b + c + d}{2}$ is the 'semi-perimeter'. Find the exact area of a cyclic quadrilateral with side lengths:

a 4, 5, 6, 7 **b** 7, 4, 4, 3 **c** 8, 9, 10, 13

d 39, 52, 25, 60 **e** 51, 40, 68, 75

6 **a** The population of a town decreases by 2% each year. The population was initially P, and is Q after n years. What is the formula relating Q, P and n?

b The population of a town decreases by 5% each year. The percentage decrease over a period of n years is a%. What is the formula relating a and n?

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