CHAPTER

10

Review and problem-solving

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10A Review

Chapter 1: Algebra

1 Evaluate $3x + 2y^2$ when:

a $x = 2$ and $y = 3$ **b** $x = 5$ and $y = 2$

c $x = -23$ and $y = -3$ **d** $x = \frac{1}{2}$ and $y = \frac{-3}{5}$

2 Simply each of these expressions by collecting like terms.

a $3a + 2b\text{-}a + 4b$ **b** $5x^2y - 3xy + 7xy - x^2y$

c $7m + 12n^2 + 2n^2 - 9m$ **d** $p^2 - 6p - p + 15$

3 Simplify:

a $7ab \times 2a$ **b** $-3x \times -2y$ **c** $\frac{20xy}{5x}$ **d** $25a \div 5 \times 3$

4 Write each expression as a single fraction.

a $\frac{a}{5} - \frac{2a}{3}$ **b** $\frac{3x}{8} - \frac{2x}{5}$ **c** $\frac{a}{5} \times \frac{2a}{3}$

d $\frac{a}{2b} \times \frac{2ab}{7}$ **e** $\frac{3x}{4} \div \frac{6x}{7}$ **f** $\frac{ab}{3} \div \frac{6b}{b}$

5 Expand:

a $3(a + 4)$ **b** $6(x - 1)$ **c** $2(3b + 2)$ **d** $5(4d - 1)$

e $-3(3d - 2)$ **f** $-2(5\ell - 4)$ **g** $-2x(3x + 1)$ **h** $4x(2x + 3)$

6 Expand and collect like terms for each of these expressions.

a $3(a + 2) + 4(a + 5)$ **b** $4(2x - 1) + 3(3x + 2)$

c $5(3d - 2) + 4(2d - 7)$ **d** $8(4e + 3) - 5(e - 1)$

e $6(f - 2) - 3(2f - 5)$ **f** $2x(x + 4) + 3(x - 2)$

g $x(3x + 2) - 4x(2x - 3)$ **h** $2x(5x + 4) - 6x(3x - 7)$

7 Simplify:

a $\frac{x + 1}{4} + \frac{x + 3}{3}$ **b** $\frac{x - 2}{2} + \frac{x - 1}{3}$ **c** $\frac{2x + 1}{3} - \frac{x + 1}{4}$ **d** $\frac{3x - 1}{4} - \frac{2x - 1}{6}$

8 Expand and simplify:

a $(x + 3)(x + 5)$ **b** $(x + 7)(x - 3)$ **c** $(x - 3)(x + 8)$

d $(2x + 1)(3x - 2)$ **e** $(4x + 3)(3x + 5)$ **f** $(5x - 2)(2x + 3)$

g $(x + 5)(x - 5)$ **h** $(2x + 3)(2x - 3)$ **i** $(3x - 5)(3x + 5)$

j $(x + 7)^2$ **k** $(2x - 5)^2$ **l** $(3x - 4)^2$

m $(x + 2)^2 - (x - 4)^2$ **n** $(2x + 3)^2 - (2x - 3)^2$

o $(x + 1)(2x + 3) + (2x - 1)(3x + 2)$ **p** $(x + 2)(2x - 5) - (3x + 1)(2x - 4)$

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9 Copy and complete:

a $(x + 3)(x + \ldots) = x^2 + 10x + \ldots$

b $(x + 2)(x + \ldots) = x^2 - x - \ldots$

c $(x + 6)(\ldots + \ldots) = x^2 + 11x + 30$

d $(x + 4)(\ldots + \ldots) = x^2 + 10x + 24$

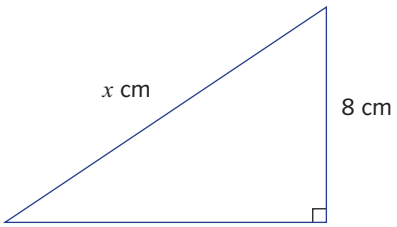
e $(2x - 1)(x + \ldots) = 2x^2 + \ldots - 6$

f $(3x + 2)(\ldots + \ldots) = 6x^2 + \ldots + 14$

Chapter 2: Pythagoras' theorem and surds

1 For each of these right-angled triangles, find the value of the pronumeral, correct to 1 decimal place.

a



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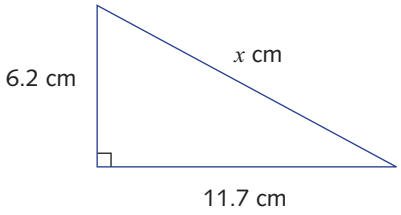

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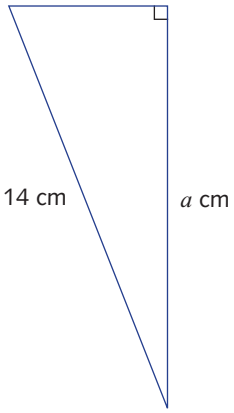

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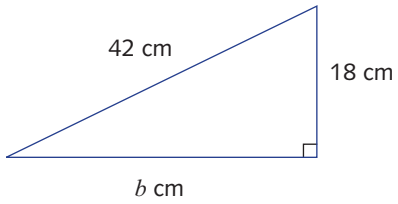

2 The lengths of the sides of a triangle are 8.2 cm, 11.6 cm and 14.3 cm. Is the triangle right-angled?

3 In each part below, the two shorter side lengths of a right-angled triangle are given. State the length of the hypotenuse.

a 3 cm, 4 cm

b 5 cm, 12 cm

c 4 cm, 7.5 cm

d 0.3 cm, 0.4 cm

e 1 cm, 2.4 cm

f 12 cm, 22.5 cm

4 A gardener is designing a rectangular lawn $ABCD$. If $AB = 4.2$ m and $BC = 3.15$ m, how far apart should A and C be to ensure $\angle ABC = 90°$?

5 A plane takes off and after climbing on a straight line path for a distance of 1 km, it has flown a horizontal distance of 900 m. What is the plane's altitude, correct to the nearest metre?

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6 Simplify each of these surds.

a $\sqrt{20}$ **b** $\sqrt{75}$ **c** $2\sqrt{18}$ **d** $4\sqrt{50}$ **e** $5\sqrt{108}$ **f** $9\sqrt{27}$

7 Write each number as the square root of a whole number.

a $2\sqrt{3}$ **b** $3\sqrt{2}$ **c** $10\sqrt{5}$ **d** $4\sqrt{7}$

8 Simplify:

a $4\sqrt{2} + 7\sqrt{2}$ **b** $8\sqrt{3} - 5\sqrt{3}$ **c** $4\sqrt{2} \times 5\sqrt{3}$ **d** $3\sqrt{5} \times 4\sqrt{7}$

e $\sqrt{18} + \sqrt{32}$ **f** $\sqrt{27} - \sqrt{12}$ **g** $4\sqrt{12} + 3\sqrt{75}$ **h** $8\sqrt{50} - 2\sqrt{98}$

9 Expand and simplify:

a $\sqrt{2}(\sqrt{3} + \sqrt{10})$ **b** $\sqrt{3}(4\sqrt{3} - 5)$ **c** $3\sqrt{5}(2\sqrt{2} - 4\sqrt{5})$

d $2\sqrt{2}(3\sqrt{3} + 4\sqrt{2})$ **e** $(2\sqrt{3} + 1)(3\sqrt{3} - 2)$ **f** $(4\sqrt{2} + 3)(5\sqrt{2} - 7)$

g $(3\sqrt{2} - 1)^2$ **h** $(\sqrt{5} + 1)^2$ **i** $(2\sqrt{5} + 7\sqrt{2})(2\sqrt{5} - 7\sqrt{2})$

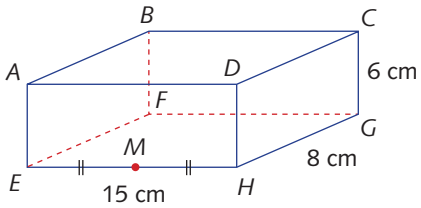
10 Express each number with a rational denominator.

a $\dfrac{3}{\sqrt{3}}$ **b** $\dfrac{2\sqrt{5}}{\sqrt{5}}$ **c** $\dfrac{2}{4\sqrt{3}}$ **d** $\dfrac{5\sqrt{3}}{3\sqrt{2}}$

11 Express each number with a rational denominator.

a $\dfrac{3\sqrt{2}}{\sqrt{5} + 2}$ **b** $\dfrac{\sqrt{3}}{2\sqrt{3} - 1}$ **c** $\dfrac{3\sqrt{2} + 1}{\sqrt{5} + 2}$ **d** $\dfrac{\sqrt{2} + 1}{\sqrt{3} + \sqrt{2}}$

12 For the rectangular prism to the right, calculate the length of each of these intervals. Give your answers as surds in simplest form.



a *EG* **b** *EC* **c** *HC*

d *GM* **e** *CM* **f** *AM*

13 A vase is in the shape of a cylinder with base radius 4 cm and height 10 cm. What is the length, correct to 1 decimal place, of the longest flower stem that can just fit in the vase?

Chapter 3: Consumer arithmetic

1 Express each percentage as a fraction in its simplest form.

a 18% **b** 64% **c** 2.6% **d** 8.5% **e** $37\frac{1}{2}\%$ **f** $6\frac{2}{3}\%$

2 Express each percentage as a decimal.

a 8% **b** 27% **c** 9.6% **d** 45.8% **e** $12\frac{1}{4}\%$ **f** $38\frac{1}{2}\%$

3 Express each rational number as a percentage.

a $\dfrac{2}{5}$ **b** $\dfrac{5}{8}$ **c** 0.61 **d** 0.02 **e** $\dfrac{4}{7}$ **f** $\dfrac{5}{9}$

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4 Copy and complete the following table.

	Percentage	Fraction	Decimal
a	25%		
b		$\frac{3}{10}$	
c			0.26
d		$\frac{2}{3}$	
e	8%		
f			0.075

5 Calculate:

a 8% of 120 **b** 16% of 54 **c** 85% of \$400 **d** $9\frac{1}{2}$% of \$6000

6 There are 650 students at a high school, 54% of whom are boys. How many boys are at the school?

7 Netball is played by 6% of Australians. If the population of Australia is 22 500 000, how many Australians play netball?

8 In a class of 25 students, 8 travel to school by train. What percentage of the class travel to school by train?

9 In a survey of 1200 adults, it was discovered that 114 of them were unemployed. What percentage of the adults surveyed were unemployed?

10 Find the new value if:

a 80 is increased by 40% **b** 150 is increased by 6%

c 240 is decreased by 12% **d** 160 is decreased by 4%

11 During a sale, the price of a sofa bed is reduced by 20%. If the original price of the bed was \$650, what is its sale price?

12 A salesperson is given a salary increase of 4%. If her existing weekly salary is \$640, what will her new weekly salary be?

13 Joe's Electrical Store is having an 8% discount sale. The sale price of some items is given below. Calculate the price of the items before they were reduced.

a Heater \$276 **b** Vacuum cleaner \$138

c Dishwasher \$690 **d** Microwave \$132.80

14 The enrolment of a school increased from 680 to 740. Calculate the percentage increase, correct to 2 decimal places.

15 During a sale the price of a suit is reduced from \$420 to \$370. Calculate the percentage discount, correct to 1 decimal place.

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16 What single percentage change, correct to 2 decimal places, is equivalent to each of these multiple changes?

a A 6% increase followed by a 12% increase

b A 10% increase followed by a 10% decrease

c A 16% decrease followed by a 8% decrease

d A 12% decrease followed by a 14% increase

17 Over the course of a year an employee is given successive salary increases of 4%, 6% and 5%.

a If the employee's original monthly salary was \$2600, what is the employee's salary after the three increases?

b What single percentage change is equivalent to the three successive salary increases?

18 To obtain a bonus, a salesperson's sales must increase by 20% in a two-month period. If the salesperson's sales increase by 8% in the first month, by what percentage must they increase in the second month to ensure the bonus is obtained?

19 Mia invests \$6000 in the bank. How much will she have in her account after three years if the bank pays:

a 8% simple interest p.a. **b** 4% compound interest p.a.

20 The value of a new car depreciates at a compound rate of 6% each year. If the car has an initial value of \$19 960, calculate its value after:

a one year **b** five years **c** 10 years

Chapter 4: Factorisation

1 Factorise:

a $5a + 10$ **b** $6c - 8$ **c** $9d - 24$ **d** $3e^2 + 9e$

e $6f^2 + 10f$ **f** $-3h^2 - 15h$ **g** $4a^2b + 6ab^2$ **h** $9mn^2 + 12mn$

2 Factorise:

a $x^2 + 7x + 12$ **b** $x^2 - 9x + 18$ **c** $x^2 - 5x - 6$ **d** $x^2 + 3x - 28$

e $x^2 - 11x + 30$ **f** $x^2 - 14x + 24$ **g** $x^2 - 6x - 55$ **h** $3x^2 + 6x + 9$

i $4x^2 - 8x + 12$ **j** $x^2 - 100$ **k** $9x^2 - 16y^2$ **l** $1 - 16a^2$

3 Write each expression as a simplified single fraction.

a $\dfrac{1}{(x-1)^2} \div \dfrac{1}{x^2-1}$ **b** $\dfrac{x-4}{x^2+2x+1} \times \dfrac{x+1}{x^2-16}$

c $\dfrac{m-2}{4m} \times \dfrac{m}{m-2}$ **d** $\dfrac{p+1}{8(p-1)} \times \dfrac{4(p-1)}{(p+1)(p+2)}$

e $\dfrac{4}{a} \div \dfrac{2}{a^2}$ **f** $\dfrac{5a-7}{2a+4} \times \dfrac{12}{10a-14}$

g $\dfrac{x^2+3x-4}{2x-2} \times \dfrac{6x-12}{x-1}$ **h** $\dfrac{x^3}{y^2} \div \dfrac{x}{2y^3}$

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Chapter 5: Linear equations and inequalities

1 Write each of the following statements using symbols.

a two more than x

b four less than b

c half of c

d three times d

e one more than twice e

f two less than one-third of f

g twice three more than s

h half of one less than h

2 Solve these equations.

a $4a = 36$

b $\frac{b}{4} = 2$

c $5x - 3 = 42$

d $3x + 19 = 46$

e $7 - 2y = -15$

f $8 - 3m = 2$

g $4(2x - 3) = 84$

h $7(1 - x) = 42$

i $5x - \frac{1}{2} = 17$

j $\frac{2x}{3} + \frac{1}{4} = 1$

k $3 - \frac{x}{5} = 4$

l $2 - \frac{3x}{4} = 5$

3 Solve the following equations.

a $2x + 7 = x + 9$

b $3x + 11 = 4x + 27$

c $2x - 9 = 3x + 15$

d $4x - 5 = 8x - 13$

e $5(x + 1) = 2(3x + 1)$

f $4(2x - 3) = 3(x + 4)$

g $2(x + 1) + 3(x + 2) = 17$

h $4(x - 3) - 3(2x + 1) = 18$

i $\frac{2a + 1}{3} = \frac{a - 1}{4}$

j $\frac{a - 3}{2} + 1 = \frac{a - 1}{4}$

k $\frac{3a}{2} + 1 = \frac{a + 1}{3}$

l $\frac{a - 1}{5} = \frac{2a + 3}{4}$

4 When $1\frac{1}{2}$ is added to twice a certain number, the result is $4\frac{3}{4}$. What is the number?

5 Gina is 4 years older than her sister. When their ages are added together the result is 22. How old is Gina?

6 Frankie buys four pencils and two rulers at the newsagent at a total cost of \$5.62. If each ruler costs 35 cents more than each pencil, what is the cost of a pencil?

7 A woman is three times as old as her daughter. In four years' time she will be only two and a half times as old as her daughter. What is the woman's present age?

8 In an 80 km biathlon, a competitor completes the bicycle leg in 2 hours and the running leg in 40 minutes. If the competitor cycles 20 km/h faster than he runs, at what speed does he cycle?

9 Solve each equation for x.

a $7x + a = 3$

b $ax - b = c$

c $\frac{x}{a} + b = c$

d $a - x = b$

e $a - \frac{x}{b} = c$

f $x(a + b) = c$

g $a(x + b) = cx$

h $\frac{x}{a} + \frac{b}{c} = 0$

i $\frac{x}{a} - \frac{b}{c} = d$

j $\frac{x}{a} + b = \frac{cx}{d} + e$

10 If $ax + b = c$:

a solve the equation for x

b use the formula you found in part **a** to solve for x:

i $2x + 1 = 5$ **ii** $3x - 5 = 7$ **iii** $-3x - 1 = 5$

iv $\frac{x}{2} + 1 = 3$ **v** $\frac{x}{3} - 2 = -3$ **vi** $\frac{x}{5} - \frac{1}{2} = 2$

11 If $\frac{x}{a} + b = \frac{c}{d}$:

a solve the equation for x

b use the formula you found in part **a** to solve for x:

i $\frac{x}{2} + 3 = \frac{3}{7}$ **ii** $\frac{x}{2} - \frac{1}{2} = \frac{1}{2}$ **iii** $3x - \frac{1}{3} = 3$

12 Solve each equation for x.

a $(x + a)(x - a) = (x - b)^2$ **b** $\frac{bx}{1 + bx} + \frac{ax}{a + ax} = 2$

13 Solve these inequalities.

a $2x + 9 < 23$ **b** $1 - 5x \geq 21$ **c** $3d + 5 \leq 5d + 7$

d $2(3q + 1) \leq 18$ **e** $\frac{\ell + 2}{3} \leq \frac{\ell - 5}{7}$ **f** $\frac{2(m + 1)}{3} + 1 \leq \frac{m - 4}{2}$

Chapter 6: Formulas

1 Find the value of the subject of each of the following formulas when the pronumerals have the values indicated.

a $V = \ell bh$, when $\ell = 20$, $b = 4.6$ and $h = 2.8$

b $s = ut + \frac{1}{2}at^2$, when $u = 4.6$, $a = 9.8$ and $t = 4$

c $E = \frac{p^2}{2m}$, when $p = 14.6$ and $m = 8$

d $A = P\left(1 + \frac{r}{100}\right)^n$, when $p = 1000$, $r = 8$ and $n = 4$

2 Rewrite each formula so that the pronumeral shown in the box is the subject.

a $A = \pi ab$ $\boxed{a}$ **b** $A = \pi r(r + \ell)$ $\boxed{\ell}$

c $S = \frac{n(a - 1)}{2}$ $\boxed{a}$ **d** $\ell = a + (n - 1)d$ $\boxed{n}$

e $v = at + bt$ $\boxed{t}$ **f** $ap = b^2 - bp$ $\boxed{p}$

g $\frac{1}{a} = \frac{1}{b} + \frac{1}{c}$ $\boxed{b}$ **h** $a = \frac{h}{R - r}$ $\boxed{r}$

i $T = 2\pi\sqrt{\frac{\ell}{g}}$ $\boxed{\ell}$ **j** $m = \frac{3x}{may^2}$ $\boxed{y}$

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3 **a** Given that $v = u + at$, find the value of a when $v = 26$, $u = 4$ and $t = 11$.

b Given that $F = \dfrac{GMm}{V^2}$, find the value of G when $F = 200$, $M = 10$, $m = 4$ and $V = 2$.

c Given that $P = \dfrac{1}{r} + \dfrac{1}{q}$, find the value of r when $P = 8.6$ and $q = 0.4$.

4 **a** Conversion of a given temperature from the Fahrenheit scale to the Centigrade scale is done by means of the formula

$$C = \frac{5(F - 32)}{9}$$

i The melting point of gold is 2280°F. Convert this into Celsius (correct to the nearest degree).

ii A healthy human being's temperature is approximately 36.9°C. What is the equivalent on the Fahrenheit scale, correct to 1 decimal place?

b A formula used in life insurance is $Q = \dfrac{2m}{2 + m}$.

i Calculate Q if $m = -0.7$ (correct to 4 significant figures).

ii Calculate m if $Q = 3$.

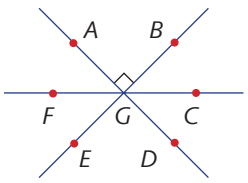
5 For $x = -\dfrac{a}{2} - \dfrac{b}{c}$, find the exact value of x if:

a $a = 2$, $b = -2$, $c = -1$ **b** $a = \sqrt{2}$, $b = \sqrt{2}$, $c = 4$

c $a = 0$, $b = 0$, $c = 5$ **d** $a = \pi$, $b = -\pi$, $c = 7$

Chapter 7: Congruence and special quadrilaterals

1 In the diagram opposite, state whether the following angles are acute, obtuse, right or straight.



a $\angle BGC$ **b** $\angle AGE$ **c** $\angle FGD$

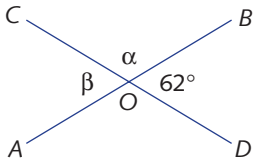
d $\angle EGB$ **e** $\angle CGD$ **f** $\angle EGD$

2 Given that $ABC = 46°$, $\angle PQR = 134°$, $\angle LMN = 43°$, $\angle DEF = 44°$ and $\angle XYZ = 43°$, which two angles are:

a complementary? **b** supplementary?

3 Calculate the values of the pronumerals.

a



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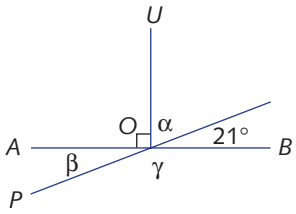

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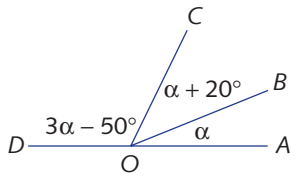

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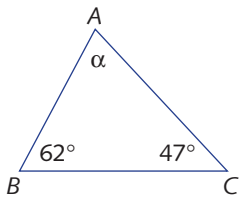
4 Calculate the values of the pronumerals.

a



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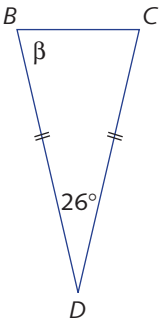

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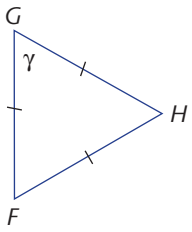

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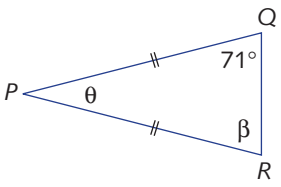

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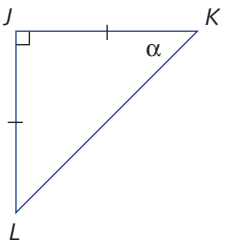

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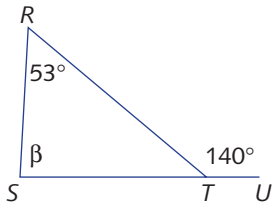
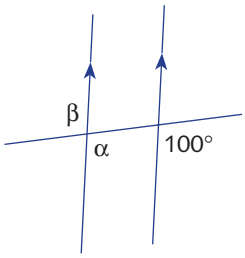
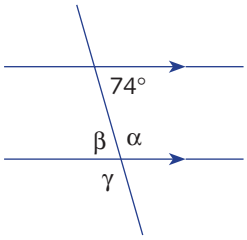


5 Find the values of the pronumerals.

a



b



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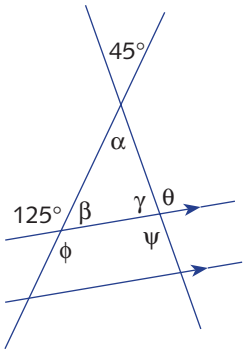

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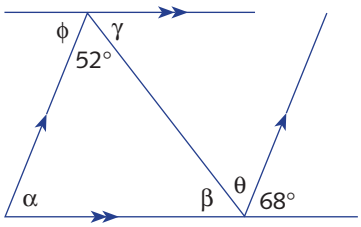
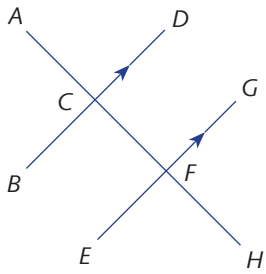


6 In the diagram opposite, which angle is:



a corresponding to $\angle ACB$?

b vertically opposite $\angle BCF$?

c co-interior to $\angle CFE$?

d corresponding to $\angle CFG$?

e alternate to $\angle CFE$?

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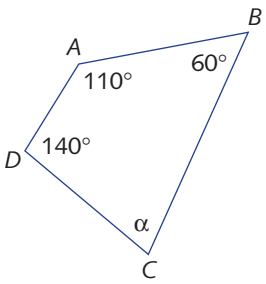
7 Find the values of the pronumerals.

a



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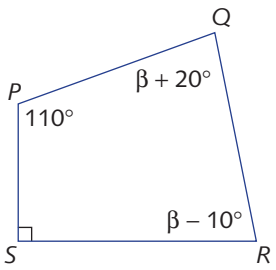

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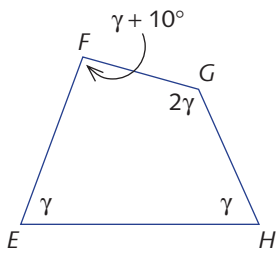
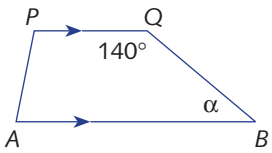


8 Find the values of the pronumerals.

a



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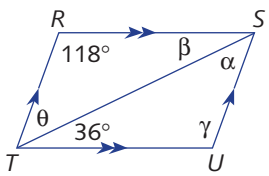

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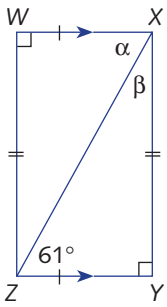

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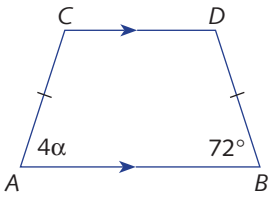
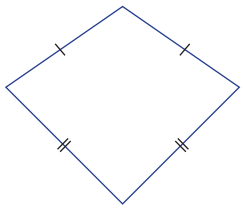


9 State the type of quadrilateral for each of the following.

a



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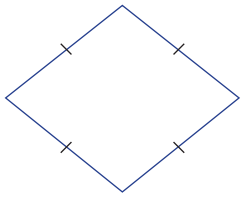

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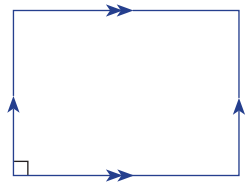

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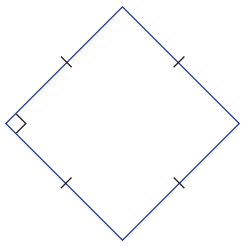
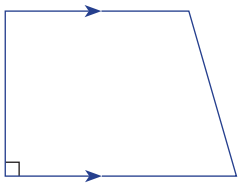
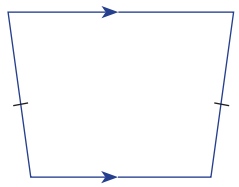


e

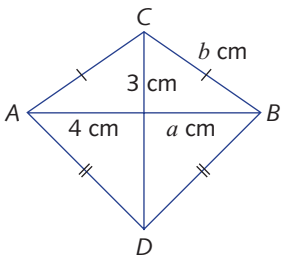


f



10 Find the values of the pronumerals.

a



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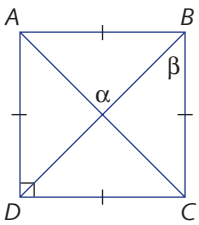

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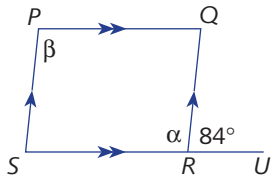

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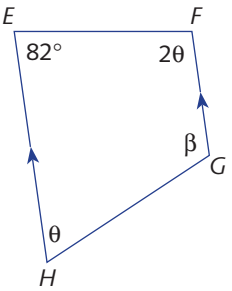
d



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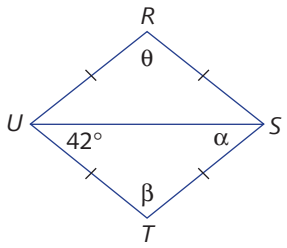

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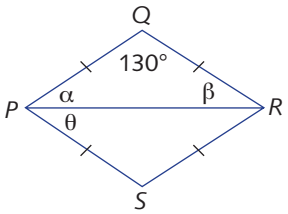


11 From the list – trapezium, isosceles trapezium, parallelogram, rhombus, rectangle, square, kite – name the quadrilaterals that have:

a opposite angles equal

b diagonals of equal length

c diagonals that intersect at 90°

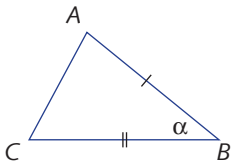
d both diagonals as angle bisectors

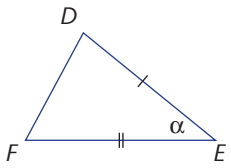
e opposite sides parallel

f adjacent angles supplementary

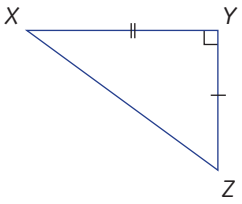
12 Each pair of triangles is congruent. State the congruence test and name the two triangles with the vertices in correct order.

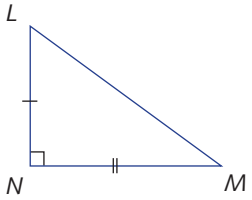
a





b





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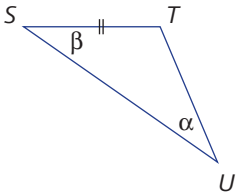
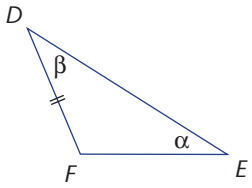
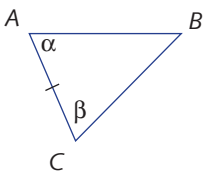
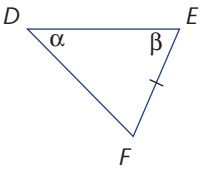


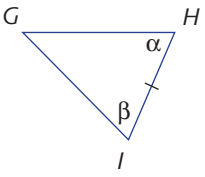


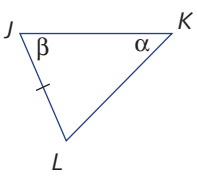
13 In each part, name the triangle congruent to ΔABC. State the congruence test used.

a









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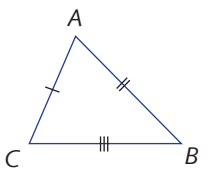

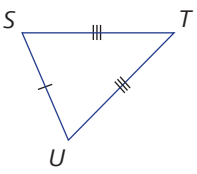

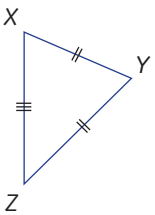

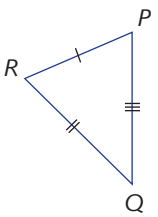

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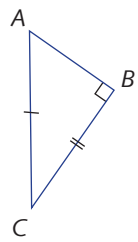
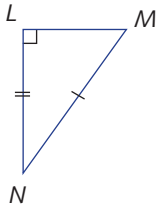
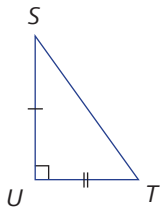
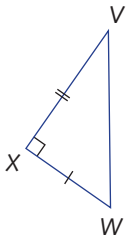
c









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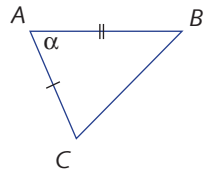
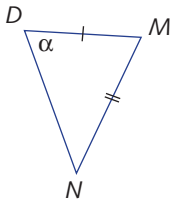
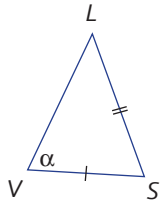
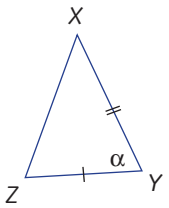


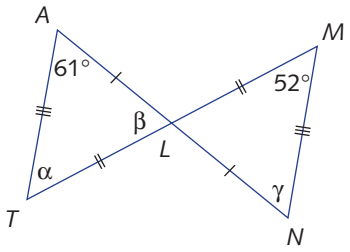




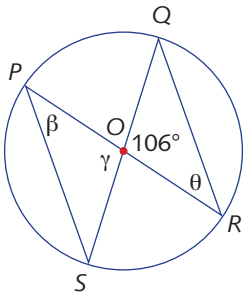


14 Each pair of triangles is congruent. Name the two triangles with the vertices in the correct order and then find the value of each pronumeral.

a

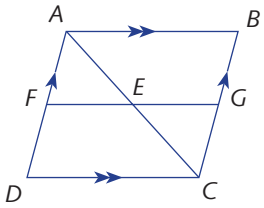


b

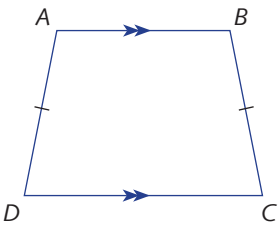


(*O* is the centre of the circle.)

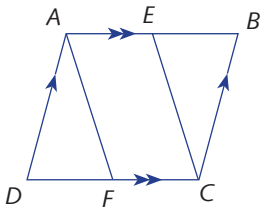
15 In the parallelogram *ABCD*, *F* is the midpoint of *AD* and *G* is the midpoint of *BC*. If *AC* and *FG* intersect at *E*, prove that *E* is the midpoint of *AC*.



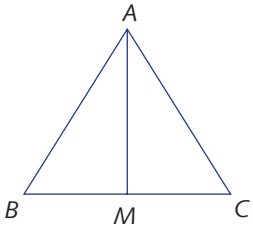
16 In the isosceles trapezium *ABCD*, *E* is the midpoint of *AB* and *F* is the midpoint of *CD*. Prove that $EF \perp DC$.
(*Hint*: Join *AF* and *BF*.)



17 In the parallelogram *ABCD*, *E* is the midpoint of *AB* and *F* is the midpoint of *CD*. Prove that *AECF* is a parallelogram.



18 In the diagram to the right, *ABC* is isosceles with $AB = AC$ and *M* is the midpoint of the interval *BC*. Prove that $\angle ABC = \angle ACB$.
(Only congruence is to be used.)



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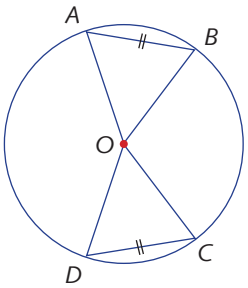
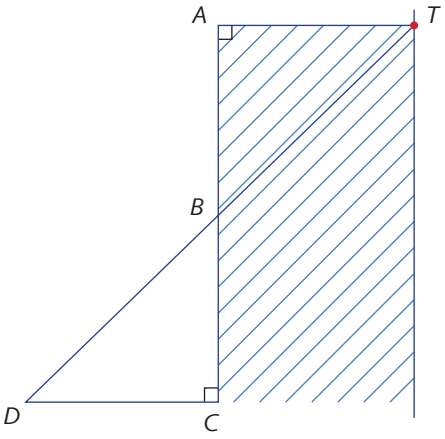
19 AB and CD are chords of a circle with centre O. If $AB = CD$, prove that $\angle AOB = \angle COD$.



20 A scout wants to estimate the width of a river. On the opposite river bank there is a tree T. The scout marks a point A on his side of the river directly opposite T. He then goes 6 paces along the river bank perpendicular to AT and marks a point B, then continues a further 6 paces to mark a point C. The scout then walks away from the river, at a right angle to BC, to a point D where B and T are in a line. From C to D is 7 paces.

a Approximately how many paces wide is the river?

b If the scout's pace is 0.8 m long, approximately how wide is the river in metres?



Chapter 8: Index laws

1 Express in index form.

a $3 \times 3 \times 3 \times 3$ **b** $a \times a \times a \times a \times a \times a$

2 Evaluate:

a 6^3 **b** 5^4 **c** 2^7 **d** 4^5

3 Evaluate:

a 2.6^3 **b** 1.7^2 **c** 8.9^4 **d** 0.047^2

4 Simplify, writing each answer in index form.

a $5^6 \times 5^7$ **b** $3^5 \times 3^7$ **c** $2x^4 \times 5x^2$ **d** $3m^4 \times 5m^6$

e $\dfrac{5^7}{5^2}$ **f** $\dfrac{24a^6}{16a^2}$ **g** $\dfrac{15n^{10}}{10n^7}$ **h** $(5^3)^2$

i $(2n^6)^3$ **j** $2a^0$ **k** $(3ab^2)^3$ **l** $(5x^2y)^0$

5 Simplify:

a $\dfrac{m^5n^4}{m^2n} \times \dfrac{m^6n^7}{m^4n^8}$ **b** $\dfrac{p^4q^5}{pq^2}$ **c** $(3m^2n)^3 \times 2m^4n$

d $\dfrac{(2m^4n)^3}{4m^5n}$ **e** $\dfrac{16q^4r^2}{3q^2} \div \dfrac{8qr^3}{pq^5r^4}$ **f** $\dfrac{7x^2y^4}{2xy} \div \dfrac{21xy^5}{4x^3y^6}$

6 Express with a positive index and evaluate as a fraction.

a 4^{-2} **b** 10^{-4} **c** 8^{-1} **d** $\left(\dfrac{2}{3}\right)^{-2}$ **e** $\left(\dfrac{3}{4}\right)^{-3}$

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7 Express with positive indices.

a a^{-2} **b** b^{-6} **c** $a^{-2}b^{-4}$ **d** $m^{-1}n^{-3}$ **e** $\dfrac{1}{x^{-4}}$

f $\dfrac{2}{y^{-3}}$ **g** $3m^{-2}$ **h** $5m^{-2}$ **i** $\dfrac{m^{-4}}{n^{-6}}$ **j** $\dfrac{3p^{-1}}{q^{-2}}$

8 Simplify, expressing your answer with positive indices.

a $x^{-4}y^{-2} \times x^2y^{-6}$ **b** $p^{-5}q^{-3} \times pq^{-2}$ **c** $4n^{-2}b \times 3a^4b^{-6}$

d $2\ell^2m^{-1} \times 5\ell^{-4}m^{-2}$ **e** $\dfrac{56t^{-4}}{8t^2}$ **f** $\dfrac{25\ell^{-2}}{15\ell^{-6}}$

g $\dfrac{28x^6y^2}{21x^{10}y}$ **h** $\dfrac{16u^{-2}v}{12u^{-6}v^{-7}}$ **i** $(2mn^{-1})^{-2}$

j $(3m^{-1}n^2)^{-4}$ **k** $(2a^{-4}b^{-1})^{-3} \times 3a^6b^{-2}$ **l** $(a^{-1}b^2)^{-4} \times (a^2b^{-3})^3$

m $\dfrac{(3a^{-2}b^{-1})^{-3}}{a^2b^{-4}} \times \dfrac{a^{-3}b^4}{12a^6b^{-2}}$ **n** $\dfrac{(a^{-1}b^4)^{-3}}{(a^2b^{-1})^4} \times \dfrac{a^{-2}b}{(ab^{-1})^3}$ **o** $\dfrac{(a^2)^4}{b^4} \div \dfrac{ab^{-2}}{(b^{-1})^3}$

p $\dfrac{ab^{-1}}{4a^2} \div \dfrac{(2a^{-3}b)^{-2}}{12a^{-1}b^3}$ **q** $\dfrac{(4xy)^0}{(4xy)^{-1}} \times \dfrac{3x^2y^{-6}}{(x^{-2}y^3)^{-4}}$ **r** $\dfrac{2m^{-3}n^{-2}}{(2mn^4)^{-2}} \times \dfrac{3m^{-3}}{(mn^2)^0}$

9 Evaluate without using a calculator.

a $9^{\frac{1}{2}}$ **b** $8^{\frac{1}{3}}$ **c** $16^{\frac{1}{4}}$ **d** $243^{\frac{1}{5}}$

e $4^{\frac{3}{2}}$ **f** $27^{\frac{2}{3}}$ **g** $49^{\frac{3}{2}}$ **h** $8^{\frac{4}{3}}$

10 Simplify, expressing the answer with positive indices.

a $\sqrt[3]{8a^2}$ **b** $\sqrt[5]{32a^5b}$ **c** $\sqrt[4]{16a^2}$ **d** $\sqrt[3]{5x^5} \times \sqrt[3]{25x^3}$

e $3x^{\frac{3}{2}} \times 2x^{\frac{5}{2}}$ **f** $\left(32a^{10}b^5\right)^{\frac{2}{5}}$ **g** $\dfrac{\left(12x^3y^5\right)^{\frac{2}{3}}}{(4y)^{\frac{1}{3}}}$ **h** $6a^{\frac{3}{2}}b^{\frac{1}{3}} \times 2a^{\frac{1}{2}}b^{\frac{5}{3}}$

11 Write in scientific notation (standard form).

a 21 000 **b** 410 **c** 61 000 000 000

d 2400 **e** 0.0062 **f** 0.0471

g 0.000 0007 **h** 0.000 38 **i** 46 **j** 2.9

12 Write as a decimal.

a 7.2×10^4 **b** 3.8×10^2 **c** 9.7×10^{-1} **d** 2.06×10^{-2}

e 1.52×10^2 **f** 4.07×10^3 **g** 1.6×10^{-4} **h** 8.7×10^{-2}

13 Simplify, giving your answer in standard form.

a $\left(5 \times 10^2\right) \times \left(4 \times 10^6\right)$ **b** $\left(6 \times 10^{-4}\right) \div \left(2 \times 10^{-3}\right)$

c $\left(4.2 \times 10^3\right)^2$ **d** $\dfrac{(3 \times 10^{-2})^2}{1.8 \times 10^{-4}}$

14 Write in standard form, correct to the number of significant figures indicated in the brackets.

a 241 000 (2) **b** 627 (1) **c** 0.000 649 26 (3)

d 0.008 716 (2) **e** 27 654 321 (4) **f** 0.267 83 (3)

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15 Give a 1 significant figure estimate for:

a 210000×0.00613 **b** $470000000 \div 216000$

c $\dfrac{0.000473}{0.00991}$ **d** $(0.00314)^2$

16 Evaluate, giving your answer to the number of significant figures indicated in the brackets.

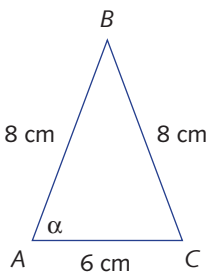
a 4.671×1.304 (2) **b** $\dfrac{18.6 \times 1.75}{0.26}$ (1) **c** $\dfrac{87.01 \times 0.003}{2.765}$ (2)

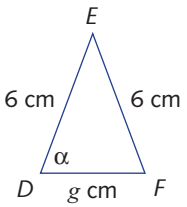
d $\dfrac{\sqrt{47.8}}{2.73}$ (4) **e** $\dfrac{18.7^2 + 9.21^2}{18.7^2 - 9.21^2}$ (4) **f** $\dfrac{0.03^2}{\sqrt{0.03}}$ (2)

Chapter 9: Enlargements and similarity

1 For each part, write a similarity statement with the vertices in the correct order. Then find the value of each unknown side length.

a





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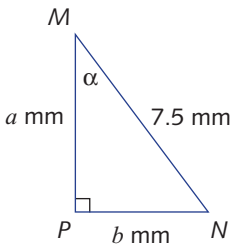
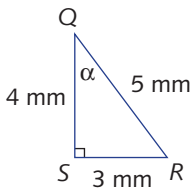
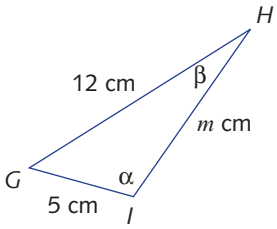
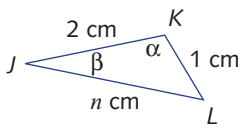


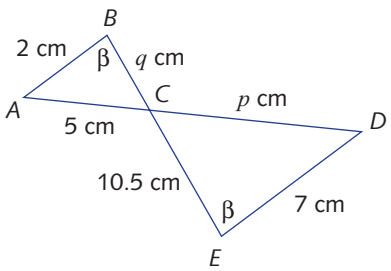


c



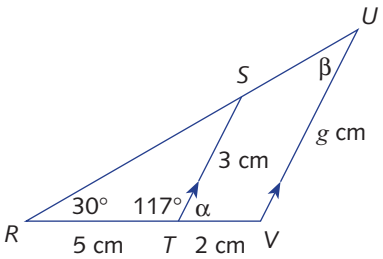


d



2 Name each pair of similar triangles, giving the similarity test used, and then find the value of each pronumeral.

a



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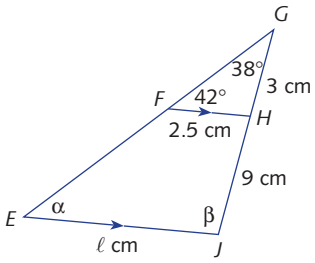

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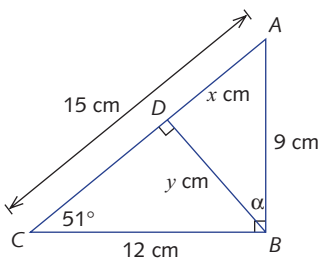
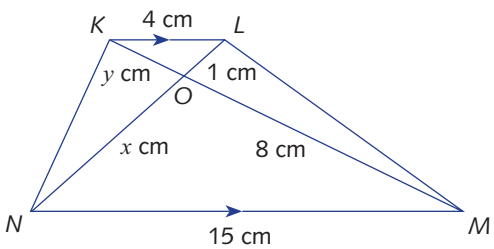


d



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3 At a particular time of the day, a tree casts a shadow of length 5.2 m and a 30 cm ruler casts a shadow of length 16 cm. Find the height of the tree.

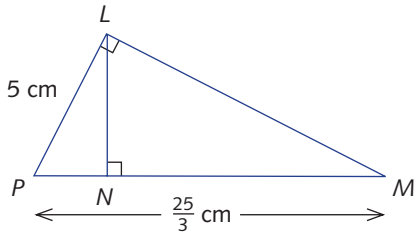
4 A girl is walking along a footpath and notices that the end of her shadow coincides with the end of the shadow of a lamp post. If the girl is 1.4 m tall and she is standing 5 metres from the lamp post, find the height of the lamp post if the length of her shadow is 80 cm.

5 In the diagram to the right:

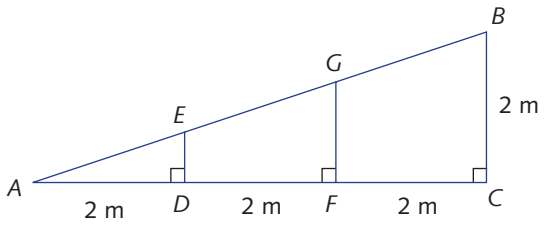
a find LM

b find LN

c find MN



6 A builder is constructing a ramp AB, as drawn in the diagram, with $BC = 2$ metres and $AC = 6$ metres. Two vertical supports ED and GF need to be placed at 2-metre intervals.



a Find:

i DE **ii** FG

b Find AB.

c The builder needs to construct two timber frames (as drawn in the diagram) to support the ramp.

i What length of timber, to the nearest metre, will he need?

ii Timber costs \$4.60 per metre and is only sold in lengths which are a multiple of one metre. How much will the timber cost the builder?

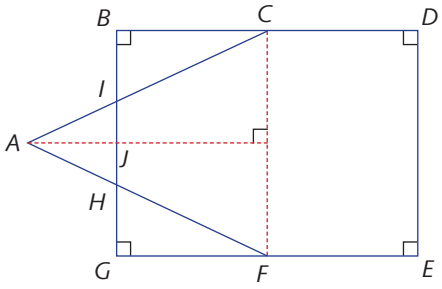
7 A ladder leans against a vertical wall. The base of the ladder is 2.4 metres from the wall and the top of the ladder is 3.2 metres above the ground. A man of height 1.7 metres climbs 1 metre up the ladder and stands in a vertical position.

a How long is the ladder?

b How far off the ground is the top of the man's head?

c How far from the wall is the man?

8 An engineer is building a trailer. The tray $BDEG$ of the trailer is 2 metres long and 1.5 metres wide – that is, $BD = 2$ m and $DE = 1.5$ m. The intervals AC and AF represent the draw-bar of the trailer, where $AC = AF$, C is the midpoint of BD and F is the midpoint of GE.



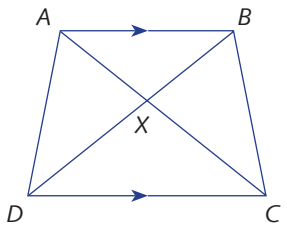
a As part of the design, $AJ = 1$ metre. Find, correct to the nearest millimetre:

i AC **ii** BI

b The engineer changes the distance of A from BG so that $BI : IJ = 7 : 8$. Find, correct to the nearest millimetre:

i BI **ii** AJ **iii** AC

9 $ABCD$ is an isosceles trapezium with $AB \parallel CD$ and $AD = BC$. The diagonals of $ABCD$ intersect at X.

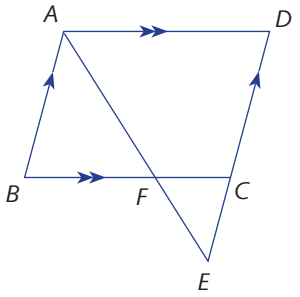


a Prove that ΔABX is similar to ΔCDX.

b Given that $AB = 4$ cm, $BX = 2.5$ cm and $DC = 8$ cm, calculate:

i AX **ii** DX

10 **a** Prove that ΔAFB is similar to ΔEFC.



b Given that $AD = 10$ cm, $CD = 4$ cm and $CE = 1$ cm, calculate FC.

c Name two other pairs of similar triangles.

11 AB and DC are the parallel sides of a trapezium and are of such length that when diagonal DB is constructed, $\angle DAB = \angle DBC$. Prove that $AB \times BC = DB \times AD$.

12 $ABCD$ is a parallelogram with E on BC such that $BE = 2EC$. The line AE and the line DC intersect at F.

a Prove that $AB = 2CF$. **b** Prove that $AF = 3EF$.

10B Problem-solving

1 Mahmoud wants to buy a new car, which is valued at $45 000. He could buy the car through an agreement with the car dealer or by borrowing the money from a bank.

a At the bank, Mahmoud asks for a loan of $45 000. The bank offers a loan at 7% p.a. simple interest to be repaid in equal monthly instalments for 5 years. Calculate:

i the amount of interest Mahmoud would pay

ii the total amount to be paid by Mahmoud

iii the amount to be paid each month

b At 'BJ's Car Sales', Mahmoud is offered the following deal: pay $5000 in cash now and then 60 monthly instalments of $934, starting in a month's time.

Calculate:

i the total amount it would cost Mahmoud to buy the car this way

ii the rate of interest charged per annum, at simple interest

c Who offers the better deal, the car dealer or the bank?

2 The workers at a production factory are negotiating a pay rise with the management. The union wants a pay increase of 2% for the first year followed by a further pay increase of 4% in the second year, with no further increase for two more years. The management offers

a 5% pay increase for the first year with no further increase for three more years. Riley is currently paid $30 000 a year.

a Under the union's claim:

i what would Riley's salary be in the first year?

ii what would Riley's salary be in the second year?

iii how much would Riley earn during the four years?

b Under the management's offer:

i what would Riley's salary be in the first year?

ii how much would Riley earn during the four years?

c Which package would be better for Riley?

3 A container holds 1.25 litres of a fruit juice drink. This drink is 25% fruit juice and the remainder is water.

a If 0.5 litres of the drink is poured out and replaced by 50% fruit juice drink, find the percentage of the contents of the container that is now fruit juice.

b If x litres of the drink is poured out and replaced by 50% fruit juice drink, find the percentage of the drink in the container that is now fruit juice.

c How many litres of the fruit juice drink has to be poured out and replaced by the 50% fruit juice drink for the percentage to increase to 41%?

4 Two cars leave Roma on the Warrego Highway travelling in the same direction. One car leaves the town 30 minutes before the other. The first car averages 80 km/h while the second car averages 96 km/h.

a How far apart are the cars when the first car is:

i 50 km from Roma?

ii x km from Roma, assuming that the first car is still in front of the second car, and that the second car has left Roma?

b How far from Roma does the second car catch up to the first?

5 The pilot of an ultra-light aircraft has determined that the plane averages 75 km/h in still air.

a If she flies out for 20 km and then returns by the same route, how long will she take?

b If she flies out for 1 hr 42 min and back for 1 hr 42 min, how far does she travel in total?

c If there is a 40 km/h headwind as the pilot flies out and a 40 km/h tailwind on return, what is the longest time the pilot can fly out to be able to return in 4 hours? (Answer in hours and minutes.)

6 The cash price of a car is advertised as $32 000. Alternatively you can buy the car with a deposit of 40% and repayments of $377.60 per month for 5 years to a finance company.

a What is the price paid for the car under the alternative scheme?

b What is the amount of interest paid to the finance company under the alternative scheme?

c What is the rate of simple interest p.a. that will apply in the alternative scheme?

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d The car depreciates in value by 12% per year.

i Find the value of the car at the end of the 5 years.

ii Calculate your average total cost per month over the 5-year period if you use the Finance Company option. Include interest and depreciation, and assume that it costs \$1000 per year for comprehensive insurance and registration, \$200 per month for petrol and \$800 per year for servicing.

7 **a** To fill my tank with petrol costs \$55.00 at \$1.10 per litre. What is the price in dollars per litre if, after a price rise, it costs \$62.50 to fill the tank with the same amount of petrol?

b A person purchases 50 litres at \$1.12 per litre. Some time later the price has increased to \$1.40 per litre. How many litres can now be purchased for the same amount of money?

c Let \$$C$ be the cost of buying V litres of petrol at R dollars per litre. Find the formula relating C, V and R.

8 George runs twice as fast as he walks. When going to school one day, he walks for twice the time that he runs and takes a total of 20 minutes to get to school.

a If x km/h is George's walking speed, find the distance that he travels to school in terms of x.

b The next day George runs for twice the time that he walks.

i If t hours is the time it takes George to get to school, find the distance to school in terms of x and t.

ii Hence find how many minutes it takes him to get to school on the second day.

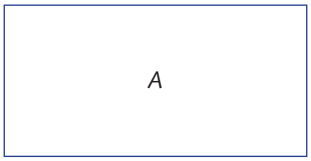
9 A car can travel 100 km on 11 litres of petrol when its speed is 50 km/h, but when its speed is 90 km/h it consumes 15 litres per 100 km.

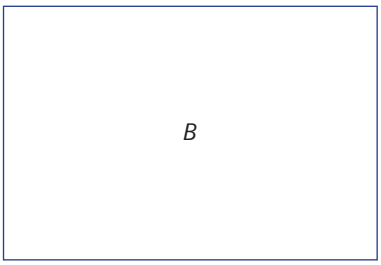
a If the car travels 28 km at 50 km/h and an additional 130 km at 90 km/h, how many litres of petrol will it consume?

b The car travels x km at 50 km/h and $5x$ km at 90 km/h. Find a formula for y, the number of litres of petrol it consumes on the journey.

10 In rectangle A, the length is 5 m more than the width.

Rectangle B is obtained from rectangle A by increasing the width of A by 50% and increasing the length of A by 20%.





a If the width of A is x m, find, in terms of x:

i the length of A

ii the length and width of B

iii the perimeter of A, expressed in simplest form

iv the perimeter of B, expressed in simplest form

v the difference between the perimeters of A and B, expressed in simplest form

b If the difference between the perimeters of A and B is 23 m, what is the value of x?

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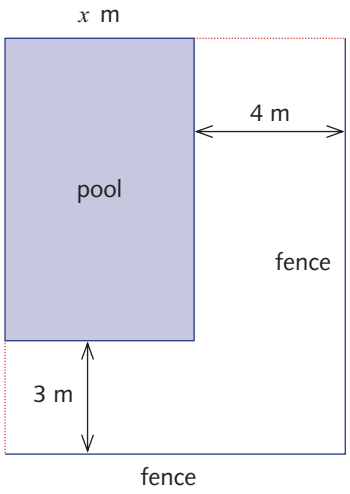
11 Andrew has been contracted by a family to build a fence around two sides of their swimming pool. The swimming pool is in the shape of a rectangle and it is twice as long as it is wide. Council regulations require the fence to be a certain distance from the pool, as shown in the diagram.



a If the width of the pool is x m, express each of the following in terms of x.

- **i** The length of the pool
- **ii** The length of the fence that is parallel to the width of the pool
- **iii** The length of the fence that is parallel to the length of the pool
- **iv** The total length of the fence

Andrew gives the family a quote for the job. He says that it will cost \$6.00 per metre to build the section of the fence that is parallel to the pool's width, and the section parallel to the pool's length will cost \$8.00 per metre.

b Express the total cost \$$C$ of the fence in terms of x.

The family intend to pave the area between the pool and the fence. They need to calculate the area between the pool edge and the fence to determine the cost of the paving.

c **i** Express the area covered by the pool and the fence enclosure in terms of x.

- **ii** Show that the area to be paved is equal to $(11x + 12)$ m^2.
- **iii** If the width of the pool is 4 m, find the area to be paved.
- **iv** If the area to be paved is 61.5 m^2, calculate the width and length of the pool.

12 Two trains travel between towns A and B. They leave at the same time with one train travelling from A to B and the other from B to A. They arrive at their destinations one hour and four hours respectively, after passing one another at point P. The slower train travels at 35 km/h.

a How far does the slower train travel after they pass?

b If the faster train travels at x km/h, how far, in terms of x, does the faster train travel after they pass?

c Hence find the number of hours, in terms of x, that the faster train takes to reach point P. Repeat for the slower train to find a different expression, in terms of x, for the same time.

d Hence find the speed of the faster train.

13 Cam purchased a number of identical items, each costing \$$x$. The total cost was \$40.

a How many items were purchased?

b How many items could be purchased for \$40 if the price of each individual item increased by 40 cents? (Calculate your answer in terms of x.)

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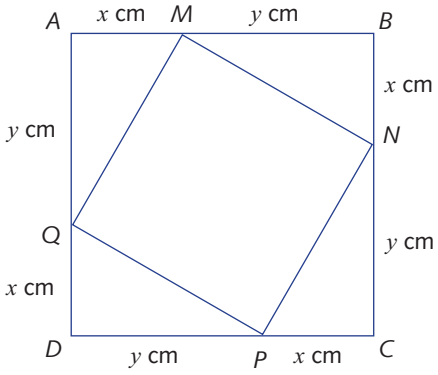
c If the number of items obtained for \$40 at the cheaper price was five more than the number obtained at the dearer price, write down an equation involving x.

d Hence find x and state the original cost of each item.

14 $ABCD$ is a square of side length 21 cm. $AM = x$ cm and $MB = y$ cm.

a Find y in terms of x.

Triangles AMQ, BNM, CPN and DQP are congruent, right-angled triangles.



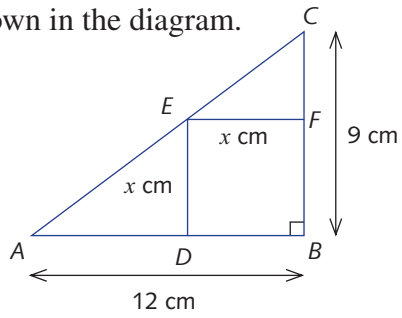
b Find the area of $MNPQ$ in terms of x.

c i If the area of $MNPQ$ is 225 cm^2, find x.

ii In this case, find the dimensions of the triangles.

d If the area of the square $MNPQ$ is half the area of the square $ABCD$, find the value of x.

15 **a** In the right-angled triangle ABC is a square $BDEF$ as shown in the diagram.

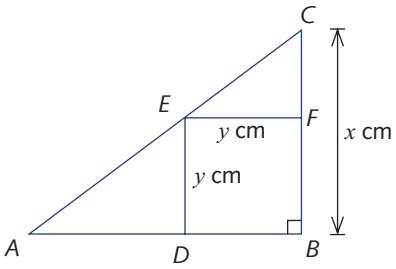


i Why is ΔEFC similar to ΔABC?

ii Hence, find x.

iii Hence, find the area of the square $BDEF$ as a percentage of the area of ΔABC, correct to 1 decimal place.

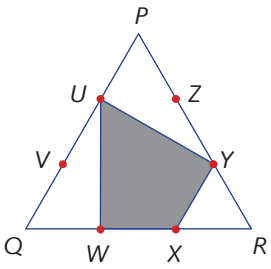
b In this diagram, the square $BDEF$ is inside ΔABC as shown. If $BC = x$ cm, $EF = y$ cm and $AB = 2BC$:



i find the relationship between x and y

ii hence find the area of the square $BDEF$ as a percentage of the area of ΔABC, correct to 1 decimal place

16 Let PQR be an equilateral triangle with sides of length 3 units. U, V, W, X, Y and Z divide the sides into unit lengths.



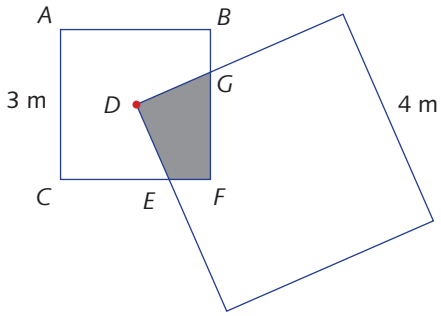
a Prove that ΔQUW is congruent to ΔPYU.

b Show area $\Delta UQW = \frac{2}{9} \times$ area ΔPQR.

c Show area $\Delta XYR = \frac{1}{9} \times$ area ΔPQR.

d Find the area of the shaded region in terms of the area of ΔPQR.

17 A 3-metre square and a 4-metre square overlap as shown in the diagram. D is the centre of the 3-metre square.



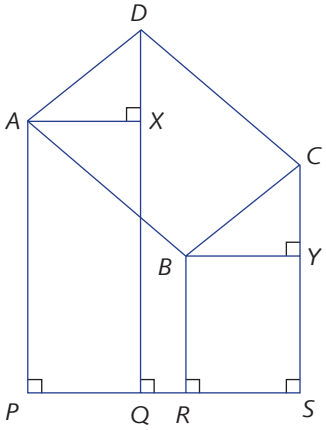
a Draw the interval DY where Y is on GF and $DY \perp GF$. Draw the interval DX where X is on CF and $DX \perp CF$.

b Show that $\Delta DXE \equiv \Delta DYG$.

c Explain why area $DEFG$ = area $DXFY$.

d Find the area of the shaded region.

18 $ABCD$ is a parallelogram where $AP = 12$, $DQ = 16, CS = 10, PQ = 5$ and $QR = 2$.



a Prove that $\Delta ADX \equiv \Delta BCY$.

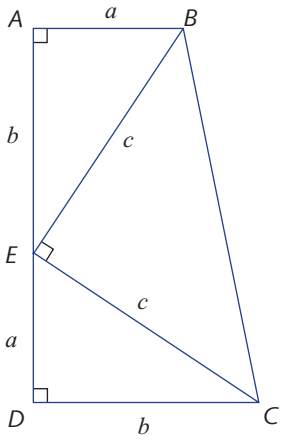
b Explain why $BY = 5$.

c Find the length of the interval BR.

d Find the area of $BRSC$.

e Find the area of the parallelogram $ABCD$.

19 Triangles ABE and DEC are congruent, right-angled triangles with side lengths as shown.



a i Find the area of triangle ABE.

ii Find the area of triangle BEC.

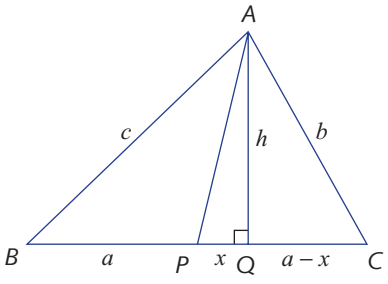
iii Use the results of **i** and **ii** to find the area of the trapezium $ABCD$.

b Use the formula to find the area of trapezium $ABCD$.

c Use your results from parts **a** and **b** to show that $a^2 + b^2 = c^2$, hence proving Pythagoras' theorem.

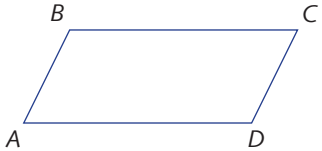
20 a ABC is a triangle and AQ is an altitude where Q lies between B and C. P is the midpoint of BC.

i Use Pythagoras' theorem in triangles ABQ and ACQ to find relationships for c^2 and b^2 in terms of a, x and h.



ii Prove that $AB^2 + AC^2 = 2BP^2 + 2AP^2$

b Use part **a** to prove that for any parallelogram $ABCD$ $2AB^2 + 2BC^2 = AC^2 + BD^2$



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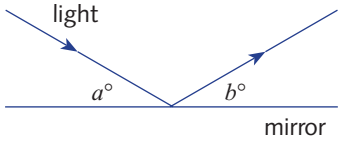
How does a sextant work?

A sextant is a navigational device used by sailors to determine the angle between the horizon and a reference point, usually a star. Knowing this angle allows the sailor to determine the latitude of the vessel's position.

A sextant uses the idea of double reflection to measure angles. The concept was first conceived by Sir Isaac Newton in 1699, but the first sextants were not commercially produced for about another 20 years.

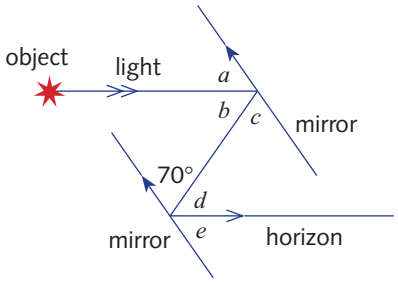
In this investigation, you will discover how a sextant works and how it can be used to measure the angle between the horizon and an object in the sky. To do this, the following important result about reflection is needed.

When light is reflected by a mirror, the angle of the incoming ray of light is equal to the outgoing angle of the ray of light. In the diagram opposite, $a = b$.



A sextant uses double reflection to measure angles.

1 In the diagram opposite, the two mirrors are parallel.



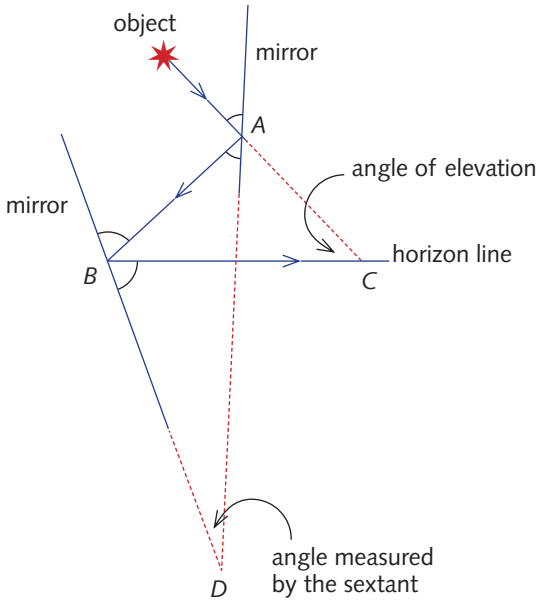
a Find the value of each pronumeral.

b Does $b = d$?

c By changing the angle of size $70°$ to other angles and finding b and d in each case, is it always true that $b = d$?

d What does the fact that $b = d$ tell you about the line from the object and the horizon?

2 Suppose that the two mirrors are no longer parallel. The angle measured by the sextant is the angle between the two mirrors ($\angle ADB$ in this diagram).



a Find the angle of elevation, given that $\angle CBD = 70°$ and $\angle ADB = 25°$.

b Find the angle of elevation, given that $\angle CBD = 64°$ and $\angle ADB = 39°$.

c By investigating other values of $\angle CBD$ and $\angle ADB$, can you guess a connection between the angle measured on the sextant and the angle of elevation?

d Find the angle of elevation if $\angle CBD = y°$ and $\angle ADB = x°$.

e Does this general result agree with your conjecture of part **iv**?

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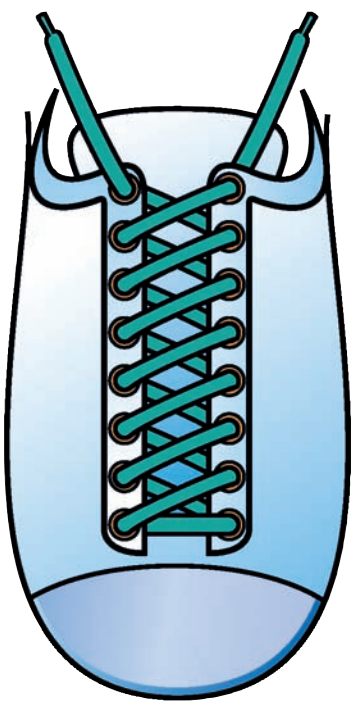
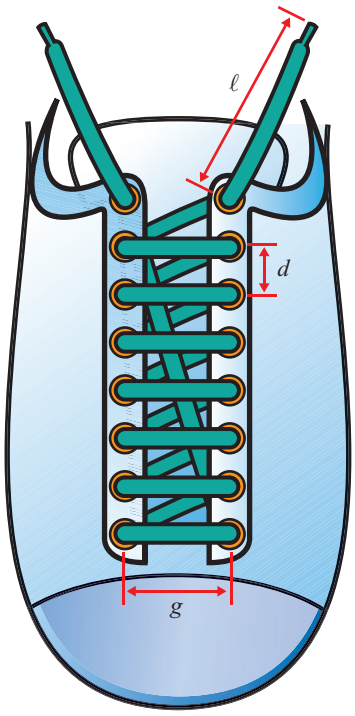
How long is a piece of string?

A shoe manufacturer is trying to calculate how long a shoe lace needs to be for a pair of shoes. There are at least two ways to lace shoes (as shown below).



standard

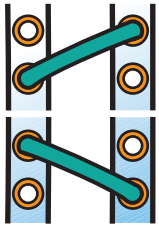


shoe store

The length of shoe lace required will depend on:

- the number n of pairs of eyelets
- the distance d between successive eyelets
- the length g of the gap between corresponding left and right eyelets
- the length ℓ of the end of each shoe lace.

1 Consider the standard lacing pattern.



a How many diagonal segments (as shown in these diagrams) are there if n is equal to:

i 3? **ii** 4? **iii** 5?

b How is the number of diagonal segments related to n?

c How long is each diagonal segment? Calculate your answer in terms of d and g.

d What is the total length of lacing required for the standard lacing pattern? Calculate your answer in terms of n, d, ℓ and g.

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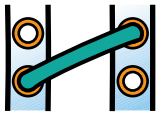
2 Consider the shoe store lacing pattern.

a How many horizontal segments (as shown here) are there if n is equal to:

i 3? **ii** 4? **iii** 5?



b How is the number of horizontal segments related to n?

c How many diagonal segments like the one shown in this diagram are there, if n is equal to:

i 3? **ii** 4? **iii** 5?

d How is the number of diagonal segments related to n?

e What is the length of each horizontal segment?

f What is the length of each diagonal segment? Give your answer in terms of d and g.

g How far is it from the bottom pair of eyelets to the top pair of eyelets if n equals:

i 3? **ii** 4? **iii** 5?

h What is the distance from the bottom pair of eyelets to the top pair in terms of d and n?

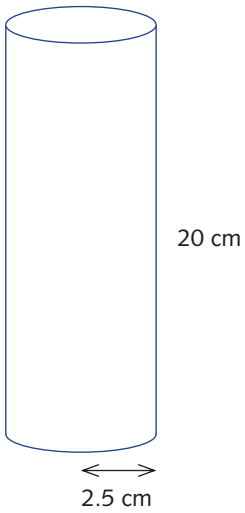
i What is the length of the long diagonal segment? Give your answer in terms of n, d, ℓ and g.

j What is the total length of lacing required for the shoe store lacing pattern? Give your answer in terms of n, d, ℓ and g.

3 By investigating the total length of lace required for each lacing pattern for different values of n, g, ℓ and d, determine which pattern requires less shoe lace.

Packaging

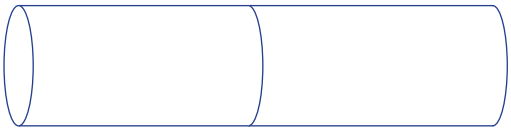
In a factory, balls are packed in canisters, which are then wrapped in bundles of six with shrink-wrap film. To make the bundle as secure as possible, the *total volume* of the pack, including any air space, needs to be as small as possible. If the volume was not as small as possible, the shrink-wrap film could be pulled tighter, thus decreasing the volume and making the package more secure.



In this project, you will investigate how cylindrical canisters can be packed so as to minimise the volume of the pack.

You will need the following information. The canisters are cylindrical, with a base radius of 2.5 cm and a height of 20 cm.

The final package can be no longer than 20 cm – that is, the cans cannot be stacked end-to-end like this.



In the following, we consider cross-sections of the bundles of canisters. When canisters are bundled, two types of region are important. These are shown below by the shading. The radius of each circle is 2.5 cm.

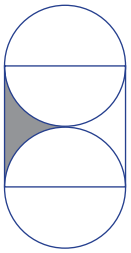
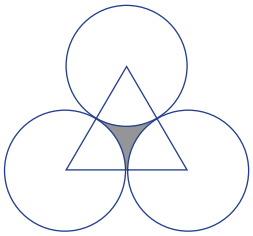
Diagram 1 

Diagram 2 

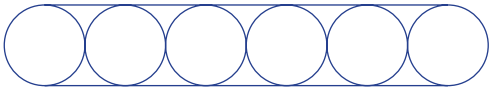
1 **a** Calculate the exact area of the shaded region in diagram 1.

b **i** What is the side length of the triangle shown in diagram 2?

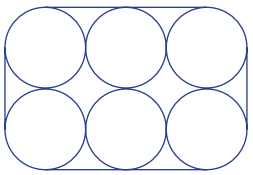
ii What is the height of the triangle? Calculate an exact answer.

iii What is the exact area of the shaded region in diagram 2?

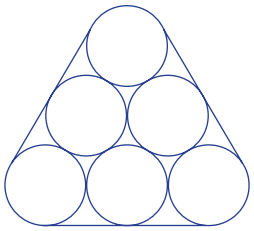
2 **a** Suppose that the six canisters are packed in a row. What is the total volume of the package? Calculate your answer to 3 decimal places.



b Suppose that the six canisters are packed in two rows of three. What is the total volume of the package? Calculate your answer to 3 decimal places.



c Suppose that the six canisters are packed as shown in the diagram opposite. What is the total volume of the package? Calculate your answer to 3 decimal places.



d Which is the best packaging: **a**, **b** or **c**?

3 Instead of being packaged in bundles of six, the company now decides to package the canisters in bundles of seven. How should the canisters be packed so as to minimise the volume?

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