CHAPTER

# 16

Measurement and Geometry

# Measurement – areas, volumes and time

In this chapter we review earlier material on the areas of plane figures and the surface areas and volumes of solids. The detailed and careful development of this material was undertaken in *ICE-EM Mathematics Year 8*.

Calculating areas, volumes and surface areas is a very practical and important skill. We have mixed many of the different types of figures and solids together, so in each problem you will need to recall which formula is appropriate. We also deal with composite solids, for which more than one formula is needed.

In the final two sections of this chapter, conversion of metric units is reviewed and extended and very small and very large measurements and their units are considered.



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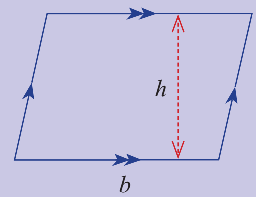
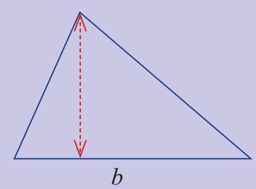
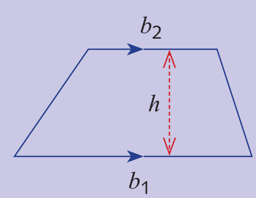
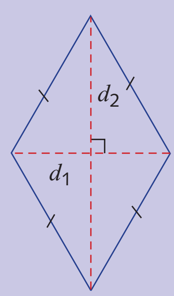
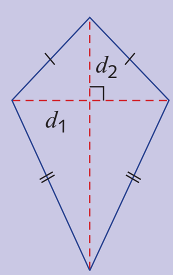
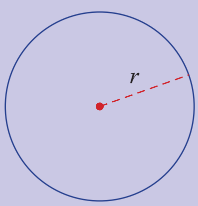
# 16A Review of area

The formulas for the areas of the standard or basic plane figures are summarised below.

| Name of figure | Diagram | Area ($A$) |
|---|---|---|
| Parallelogram |  | $A = bh$ |
| Triangle |  | $A = \frac{1}{2}bh$ |
| Trapezium |  | $A = \frac{1}{2}(b_1 + b_2)h$ |
| Rhombus |  | $A = \frac{1}{2}d_1 d_2$<br>$d_1$ and $d_2$ are the lengths of the diagonals. |
| Kite |  | $A = \frac{1}{2}d_1 d_2$<br>$d_1$ and $d_2$ are the lengths of the diagonals. |
| Circle |  | $A = \pi r^2$ |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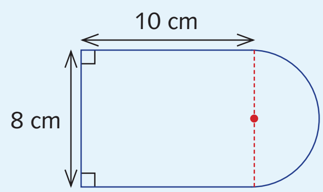
A **composite figure** involves two or more of these basic figures. Its area is usually obtained by adding or subtracting the areas of basic figures.

In all of the diagrams in this chapter, assume that all quadrilaterals are rectangles unless the context indicates otherwise.

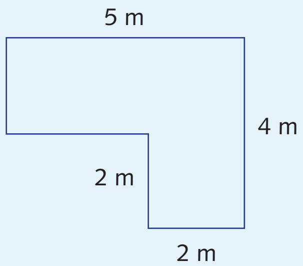
## Example 1

Calculate the area, correct to 2 decimal places, of these plane figures.

**a**




**b**




## Solution

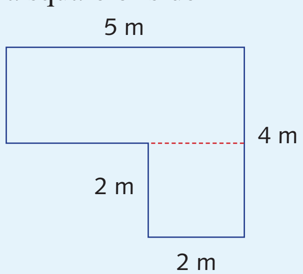
**a** The figure consists of a rectangle with dimensions 8 cm by 10 cm, and a semicircle of radius 4 cm.

So $A = 10 \times 8 + \frac{1}{2} \times \pi \times 4^2$

$= (80 + 8\pi)\ \text{cm}^2$

$\approx 105.13\ \text{cm}^2$

**b** The figure consists of a rectangle with dimensions 5 m by 2 m, and a square of side length 2 m.



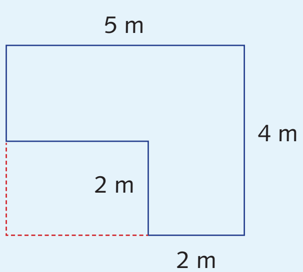

So $A = 5 \times 2 + 2 \times 2$

$= 10 + 4$

$= 14\ \text{m}^2$

**Alternative solution**

The figure can also be thought of as a rectangle with dimensions 5 m by 4 m, with a rectangle with dimensions 3 m by 2 m cut out.




So $A = 5 \times 4 - 3 \times 2$

$= 20 - 6$

$= 14\ \text{m}^2$, as before

*Note*: We read 14 $\text{m}^2$ as '14 square metres', not '14 metres squared'.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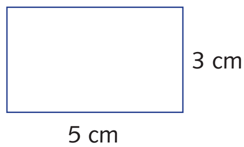
## Exercise 16A

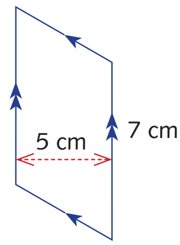
Unless otherwise specified, give the answer to each problem in exact form. If the answer involves π, also give the answers correct to 2 decimal places.

**1** Calculate the area of each figure.

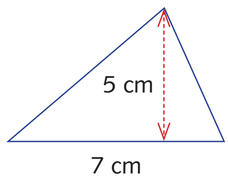
**a**



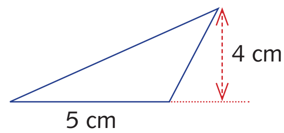

**b**



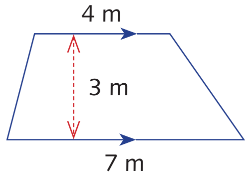

**c**



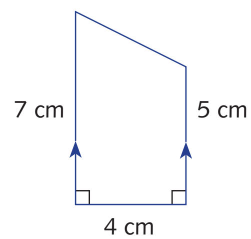

**d**



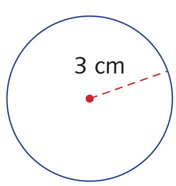

**e**



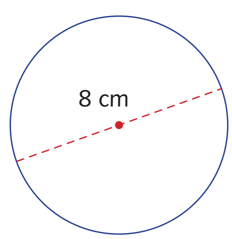

**f**



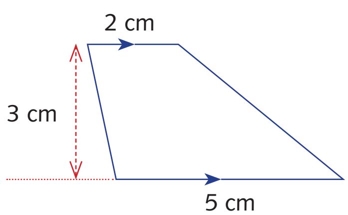

**g**



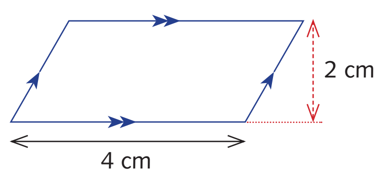

**h**




**i**

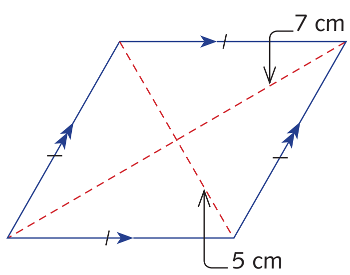



**j**

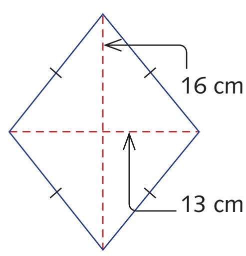



**2** Find the area of each figure. Labels indicate diagonal lengths.

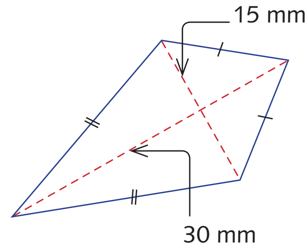
**a**




**b**




**c**




**3** Find the area of a square with diagonals of length 5 cm.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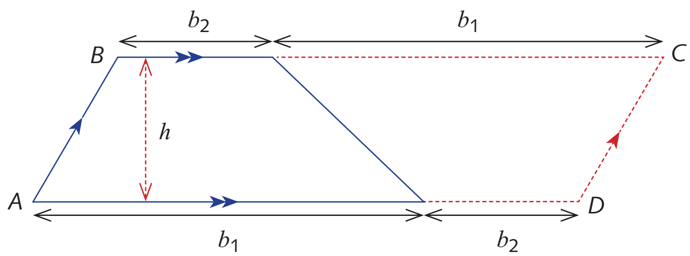
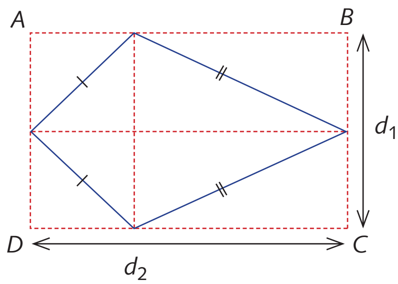
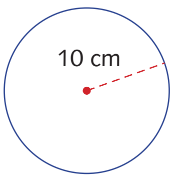
**4** Use the diagram below to derive the formula for the area of the trapezium *ABCD*.



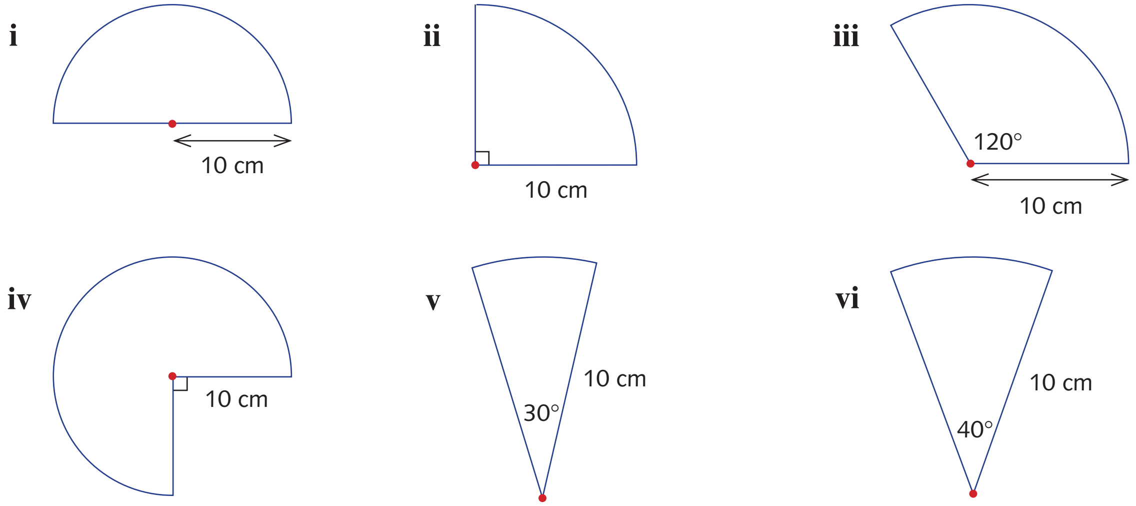

**5** Use the diagram opposite to derive the formula for the area of the kite *ABCD*.



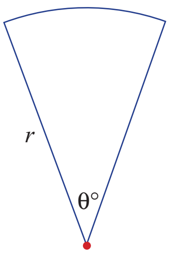

**6** **a** Find the area of the circle shown below.




**b** Find the area of each sector. (The centre of the circle is marked with a dot.)




**c** Find the area of the sector of a circle in terms of the radius $r$ and the angle $\theta$ at the centre.




**7** Calculate the radius of the circle with area:

**a** 12 $mm^2$ **b** 50 $cm^2$ **c** $9\pi$ $m^2$ **d** $25\pi$ $cm^2$

**8** Calculate the radius of a circle whose area is half that of a circle with radius 6 cm.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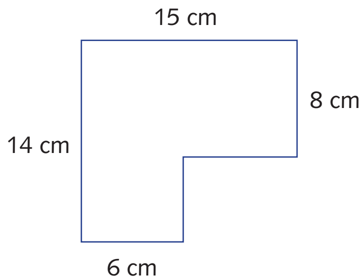
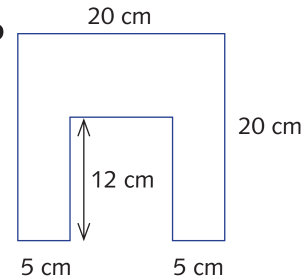
Example 1

**9** Calculate the area of each region.

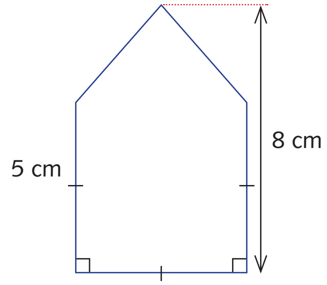
**a**



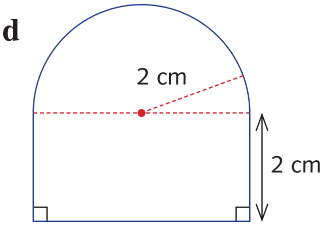

**b**



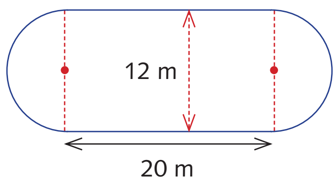

**c**



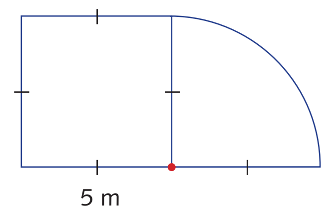

**d**



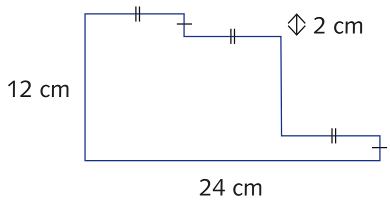

**e**



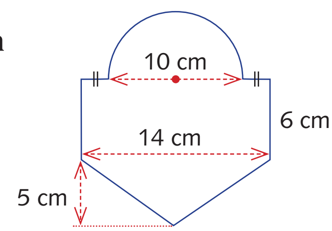

**f**



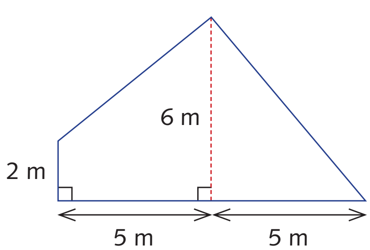

**g**



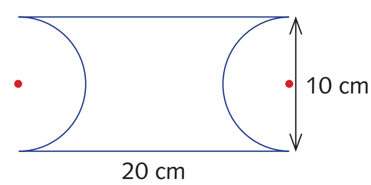

**h**



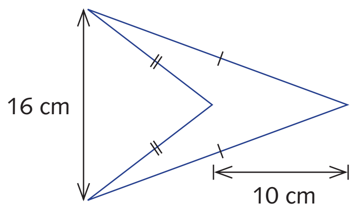

**i**



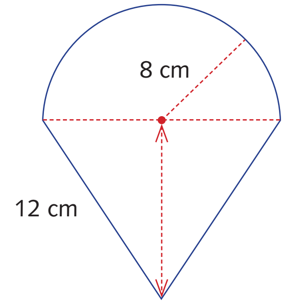

**j**



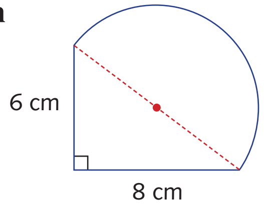

**k**



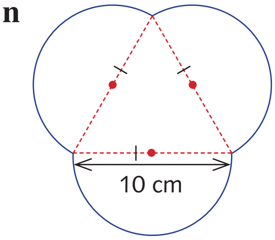

**l**




**m**




**n**



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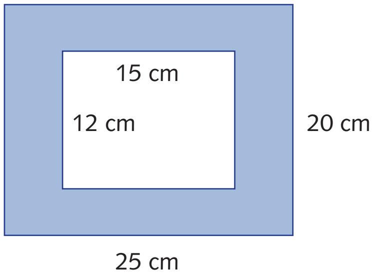
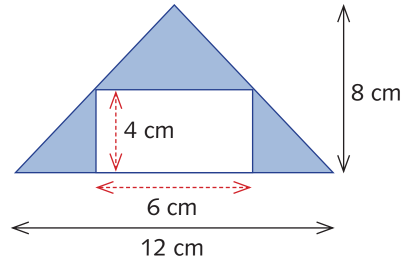
**10** Calculate the area of the shaded region in each diagram.

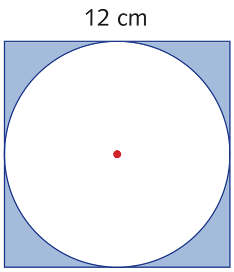
**a**



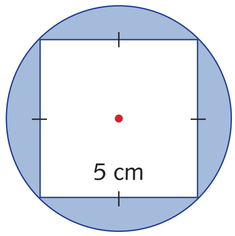

**b**



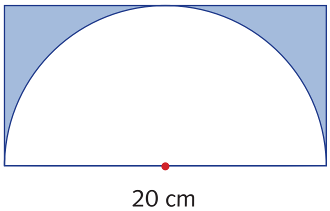

**c**



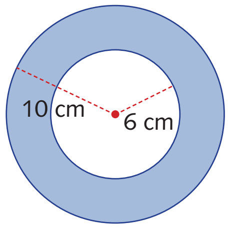

**d**



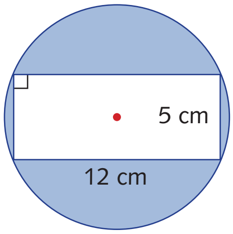

**e**



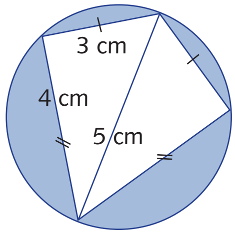

**f**



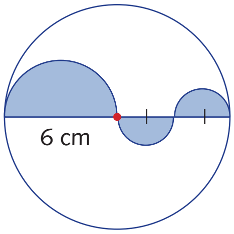

**g**



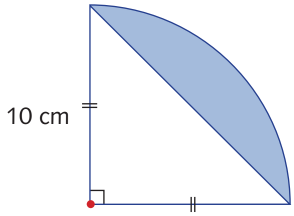

**h**




**i**



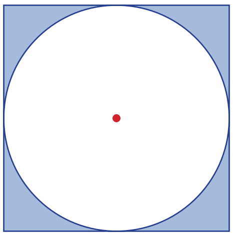

**j**




**11** In the diagram opposite:

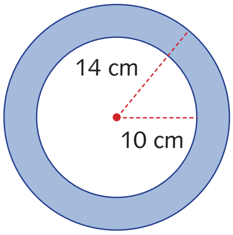
**a** Find the area of the shaded region, correct to 2 decimal places, if the square has side length 10 cm.

**b** Find the side length of the square, correct to 2 decimal places, if the area of the shaded region is 100 $cm^2$.



**12** **a** Find the area of the shaded region in the diagram opposite.

**b** A third circle is drawn with the same centre as the other two circles. Find the radius of this circle if it cuts the shaded region into halves.





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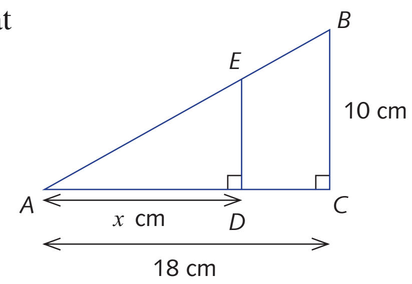
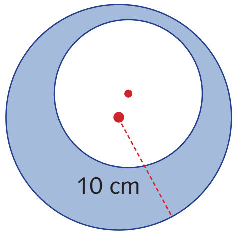
**13** In the diagram opposite, calculate the exact value of $x$ so that $DE$ divides $\Delta ABC$ into two regions of equal area.



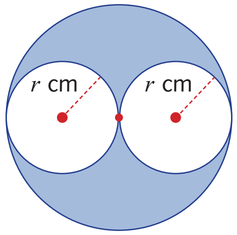

**14** In the diagram opposite, the outer circle has radius 10 cm and the area of the shaded region is equal to the area of the unshaded region. Find the radius of the small circle.




**15** In the diagram opposite:

**a** Find the area of the shaded region if $r = 6$.

**b** Find the value of $r$ if the area of the shaded region is $200\text{ cm}^2$.




# 16B Review of surface area of a prism

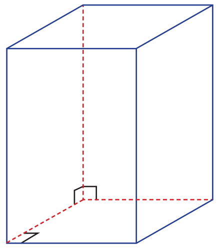
In the previous section, we reviewed the area of plane figures. Such figures are two–dimensional because they can be drawn on a flat piece of paper. The three-dimensional objects that we will now consider are called **prisms**. They have both a surface area and a volume. We begin by looking at surface areas.

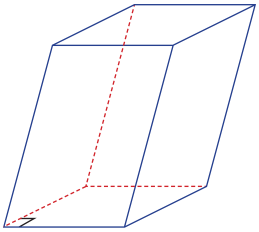
A **polyhedron** is a solid bounded by polygons.

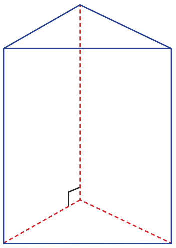
A **prism** is a polyhedron that has two congruent and parallel faces and all its remaining faces are parallelograms.

A **right prism** is a prism in which the top and bottom polygons are vertically above each other, and the vertical polygons connecting their sides are rectangles. A prism that is not a right prism is often called an **oblique prism**.

Some examples of prisms are shown below.


Right rectangular prism


Oblique rectangular prism


Right triangular prism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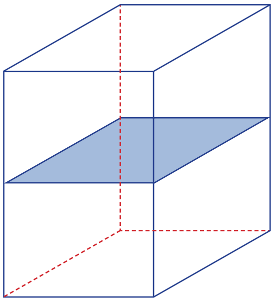
In this chapter, we will work only with right prisms. The word ‘right’ will usually be omitted.

A prism with a rectangular base is called a **rectangular prism**, while a **triangular prism** has a triangular base.

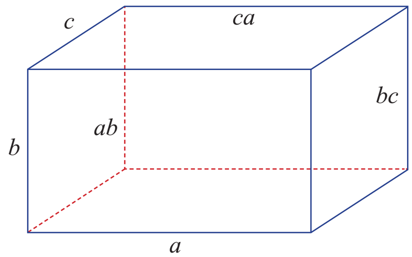


You will notice that if we slice a prism by a plane parallel to its base, the cross-section is congruent to its base and so has the same area as the base.

In this chapter, we will use the pronumeral $S$ for the surface area of a solid and $V$ for its volume.

## Surface area of a prism

The **surface area of a prism** is the sum of the areas of its faces.




A rectangular prism with dimensions $a$, $b$ and $c$ has six faces. These occur in pairs, faces with areas $ab$, $bc$ and $ca$ each occurring twice.

We can therefore write:

surface area of a rectangular prism $= S = 2(ab + bc + ca)$

We do not need to learn this formula. We can simply find the area of each face and take the sum of the areas. The same idea applies to all other types of prisms.

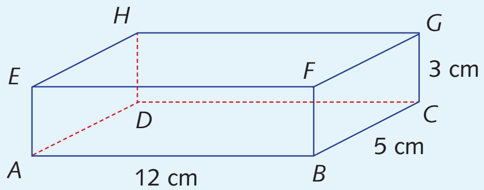
### Surface area of a prism

To find the surface area of a prism, find the area of each face and add up the areas.

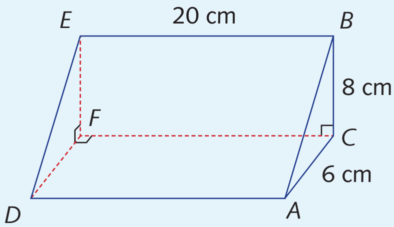
### Example 2

Find the surface area of:

**a** the rectangular prism




**b** the triangular prism




### Solution

**a** Area of the six rectangular faces $= 2(12 \times 5 + 12 \times 3 + 5 \times 3)$

$= 222 \text{ cm}^2$

(continued over page)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**b** By Pythagoras' theorem,

$$AB^2 = 6^2 + 8^2$$
$$AB = 10 \text{ cm}$$

Area of the three rectangular faces $= 8 \times 20 + 6 \times 20 + 10 \times 20$

$= 480 \text{ cm}^2$

Area of the two rectangular faces $= 2 \times \frac{1}{2} \times 6 \times 8$

$= 48 \text{ cm}^2$

Hence Surface area $= 480 + 48$

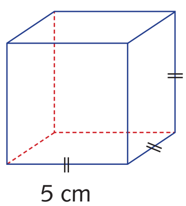
$= 528 \text{ cm}^2$

## Exercise 16B

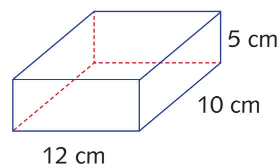
Example 2a

**1** Calculate the surface area of each prism.

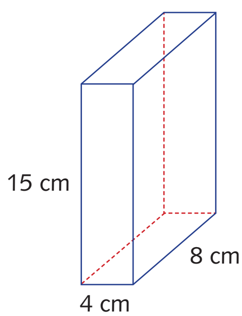
**a**



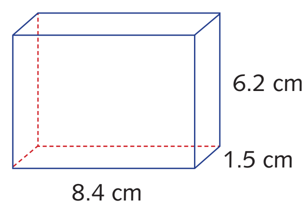

**b**



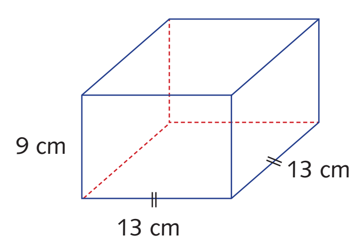

**c**




**d**




**2** An ice-cream container, open at the top, has dimensions as shown in the diagram opposite. Find the surface area of the outside of the container.

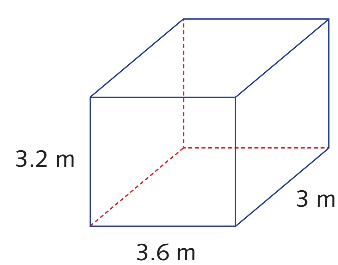



**3** A shoe box, open at the top, has length 25 cm, width 18 cm and height 9.8 cm. Find the surface area of the outside.

**4** Dianne is going to paint her room. The dimensions of the room are shown opposite. What area must Dianne paint if she paints:

**a** only the walls?

**b** the walls and the ceiling?



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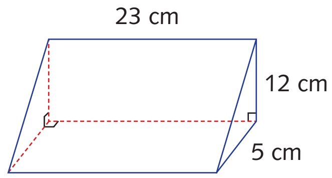
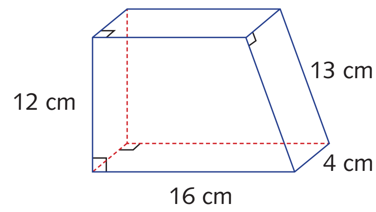
Example 2b

**5** Calculate the surface area of each solid.

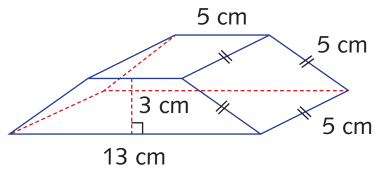
**a**



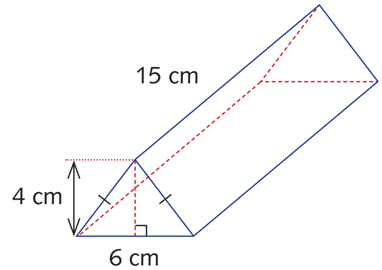

**b**



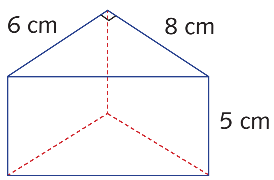

**c**



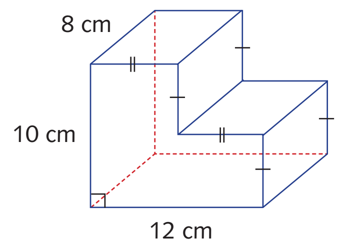

**d**



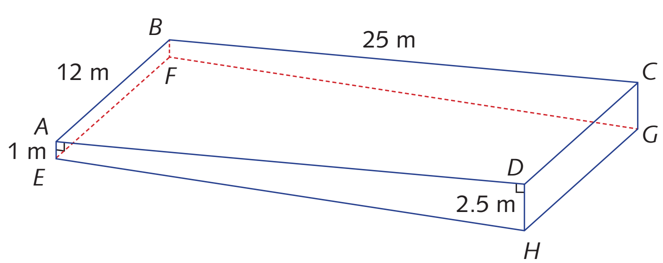

**e**




**f**




**6** A swimming pool has dimensions as shown in the diagram below.

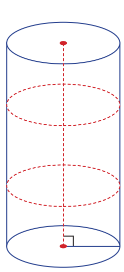



**a** Find the length *EH*, correct to the nearest millimetre.

**b** Find the approximate number of tiles, each of size 15 cm × 15 cm, required to tile the sides and bottom of the pool. (The tiler may cut the tiles and piece them together.)

# 16C Surface area of a cylinder

A **cylinder** is a solid that has parallel circular discs of equal radius at the top and the bottom. Each cross-section parallel to the top is a circle with the same radius, and the centres of these circular cross-sections lie on a straight line, called the **axis of the cylinder**.



We will use a dot (•) to indicate the centre of the circular base or top.

As we did with prisms, we find the surface area of a cylinder by adding up the area of the curved section of the cylinder, and the area of the two circles.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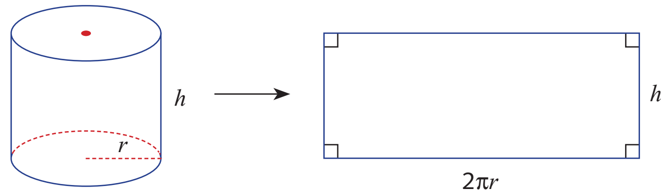
## Area of the curved surface

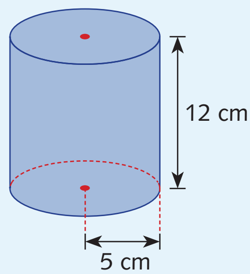
Suppose that we have a cylinder with base radius $r$ and height $h$. If we roll it along a flat surface through one revolution, as shown in the diagram, the curved surface traces out a rectangle. The height of the rectangle is the height of the cylinder, while the length of the rectangle is the circumference of the circle, which is $2\pi r$, so the area of the curved part is $2\pi rh$.

Area of curved surface of cylinder $= 2\pi rh$




### Example 3

Calculate the total surface area, correct to 2 decimal places, of a cylinder with base radius 5 cm and height 12 cm.




### Solution

$$\begin{aligned}
\text{Area of curved surface} &= 2\pi rh \\
&= 2\pi \times 5 \times 12 \\
&= 120\pi \text{ cm}^2 \\
\text{Area of each circular end} &= \pi \times 5^2 \\
&= 25\pi \text{ cm}^2 \\
\text{Surface area of the cylinder} &= 120\pi + 25\pi + 25\pi \\
&= 170\pi \text{ cm}^2 \\
&\approx 534.07 \text{ cm}^2
\end{aligned}$$

You should generally work in terms of $\pi$ and only approximate in the last step.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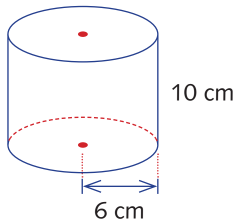
# Exercise 16C

Unless otherwise specified, give the answer to each problem in exact form and also correct to 2 decimal places.

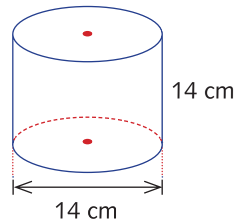
Example 3

**1** Calculate the surface area of each cylinder.

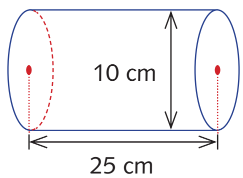
**a**



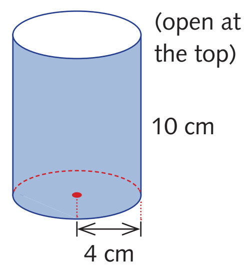

**b**



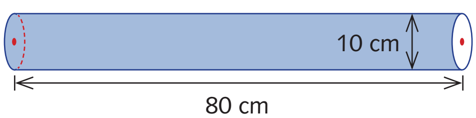

**c**




**d**




**2** The outside surface of a piece of open pipe with diameter 10 cm and length 80 cm is to be painted. Calculate the area to be painted.



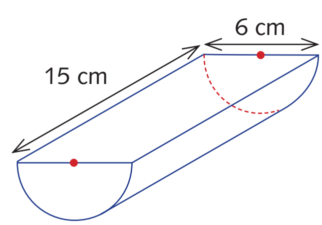

**3** For the solid half-cylinder opposite, find:

**a** the area of the top

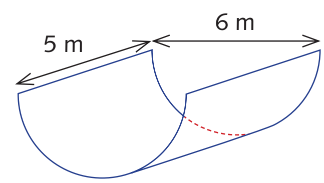
**b** the area of the two ends

**c** the curved surface area

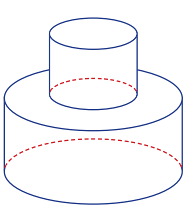
**d** the total surface area




**4** A skating ramp is in the shape of a half-cylinder. If the two sides are 6 m apart and the length of the ramp is 5 m, find the area of the ramp.




**5** A cylinder of radius 3 cm and height 4 cm is mounted on a cylinder of radius 6 cm and height 5 cm. Find the area of the visible surfaces, if the larger cylinder is sitting on a table.

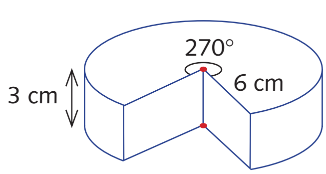


**6** The surface area of a cylinder is 25.13 $m^2$ and the radius of the base is 1 m. Find the height of the cylinder, correct to 2 decimal places.

**7** **a** The radius of a cylinder is equal to its height, and it has a total surface area of 500 $cm^2$. Find its height.

**b** The height of a cylinder is equal to its diameter, and it has a total surface area of 500 $cm^2$. Find the radius.

**8** A sector of cheese is cut from a cylindrical block of cheese. Find the surface area of the remaining piece, correct to 2 decimal places.





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# 16D Volumes

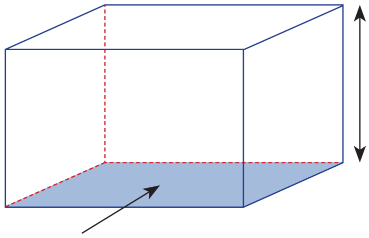
## Volumes of right prisms

Recall that a right prism is a polyhedron that has two congruent and parallel faces, and all its remaining faces are rectangles. A prism has a uniform cross-section.

## Volume of a rectangular prism

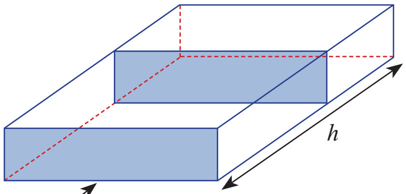
We have seen earlier that the volume of a right rectangular prism is given by:

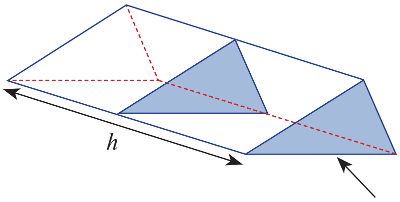
$$\begin{aligned}\text{volume of a right rectangular prism} &= \text{area of base} \times \text{height}\\ &= A \times h\\ V &= Ah\end{aligned}$$




The same formula holds for any prism. If $A$ is the area of the cross-section and $h$ is the height, then:

$$\text{Volume} = Ah$$

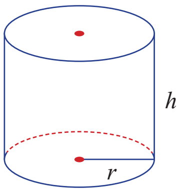


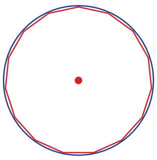



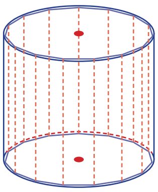

*Note*: In the last two diagrams, the prisms are not resting on their bases. Regardless of the orientation, 'height' ($h$) always represents the distance between the congruent faces of the right prism aligning with the uniform cross-section. When dealing with triangular prisms, don't confuse the '$h$' in the formula for the area of a triangle with the 'height' in the formula, $V = Ah$.

## Volume of a cylinder

A cylinder has a circular base. Suppose that we now draw a polygon with a large number of sides inside the base circle, with the vertices on the circle. The volume of the corresponding prism will be the area of the base multiplied by the height, so it seems reasonable that the volume of the cylinder should also be the area of its base multiplied by the height.







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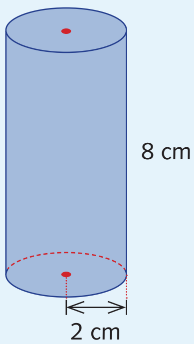
Thus the volume of a cylinder with radius $r$ and height $h$ is equal to the area of the base, $\pi r^2$, multiplied by the height $h$:

$$\text{volume of a cylinder} = \pi r^2 h$$

**Example 4**

Calculate the volume of a cylinder with radius 2 cm and height 8 cm, correct to 2 decimal places.




**Solution**

$$\begin{aligned} V &= \text{area of base} \times \text{height} \\ &= \pi \times 2^2 \times 8 \\ &= 32\pi \text{ cm}^3 \\ &\approx 100.53 \text{ cm}^3 \end{aligned}$$

*Note*: This is read as '100.53 cubic centimetres', not '100.53 centimetres cubed'.

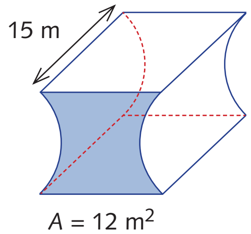
## Some related solids

Prisms and cylinders are solids with constant cross-sectional areas. The volume, $V$, of any solid with a constant cross-sectional area is given by exactly the same formula

$$V = \text{area of base} \times \text{height}$$

For example, for the solid to the right:

$$\begin{aligned} V &= \text{area of base} \times \text{height} \\ &= 12 \times 15 \\ &= 180 \text{ m}^3 \end{aligned}$$




**Volumes**

- Volume of a solid with constant cross-sectional area $A$ and height $h$ is $Ah$.
- Volume of a cylinder = area of base × height:

$$V = \pi r^2 h$$

A composite solid involves two or more of the basic solids. Its volume is usually obtained by adding or subtracting the volumes of basic solids.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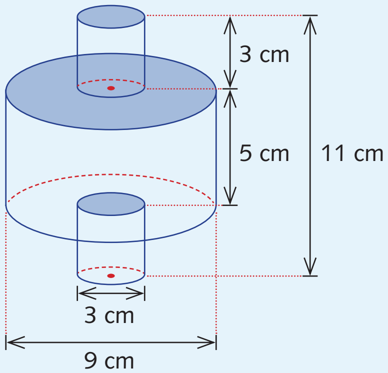
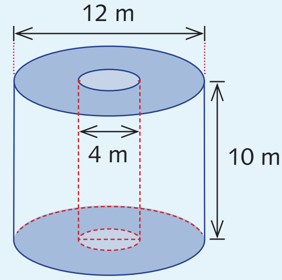
## Example 5

Find the volume of each solid in terms of $\pi$ and then correct to 2 decimal places.

**a**




**b**




### Solution

**a** Volume of large cylinder $= \pi \times 4.5^2 \times 5$

$= 101.25\pi \text{ cm}^3$

Volume of each small cylinder $= \pi \times 1.5^2 \times 3$

$= 6.75\pi \text{ cm}^3$

Hence volume of solid $= 101.25\pi + 6.75\pi + 6.75\pi$

$= 114.75\pi \text{ cm}^3$

$\approx 360.50 \text{ cm}^3$

**b** Volume = volume of large cylinder − volume of small cylinder

$= \pi \times 6^2 \times 10 - \pi \times 2^2 \times 10$

$= 360\pi - 40\pi$

$= 320\pi \text{ m}^3$

$\approx 1005.31 \text{ m}^3$

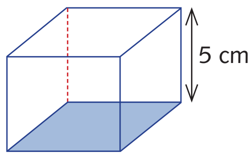
## Exercise 16D

Unless otherwise specified, give the answer to each problem in exact form and also correct to 2 decimal places.

Example 4

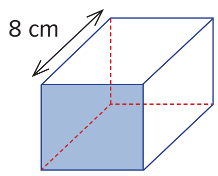
**1** Calculate the volume of each solid. The area, $A$, of the shaded face is given.

**a**



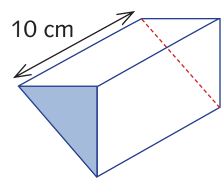

$A = 24 \text{ cm}^2$

**b**




$A = 30 \text{ cm}^2$

**c**




$A = 18 \text{ cm}^2$



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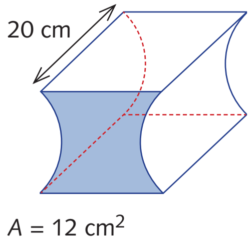
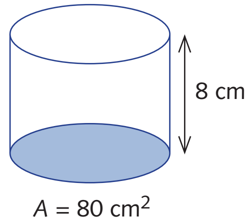
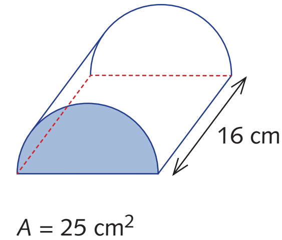
**d** 


**e** 


**f** 


**2** Use the result

volume = area of base × height

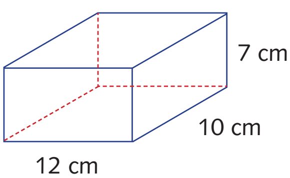
to express the volume of each solid, in terms of the given pronumerals.

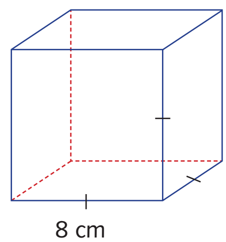
**a** The volume of a cube of side length $a$

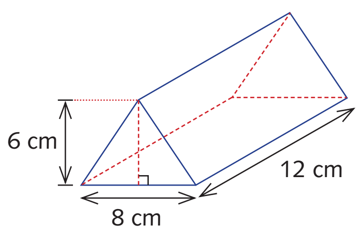
**b** The volume of a rectangular prism with length $a$, width $b$ and height $c$

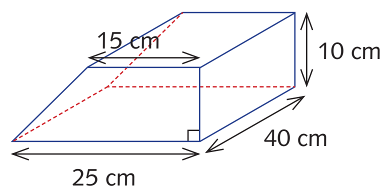
**c** The volume of a cylinder with radius $a$ and height $b$

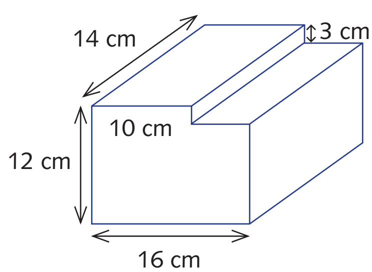
**3** Calculate the volume of each prism.

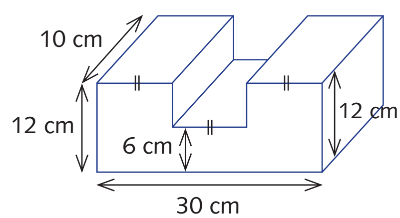
**a** 

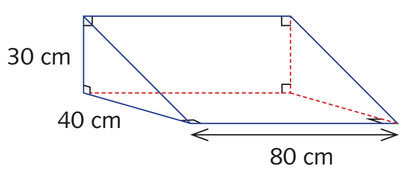

**b** 

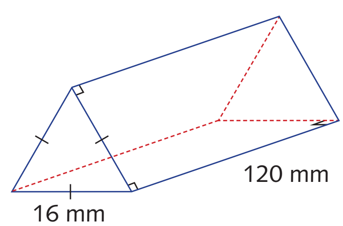

**c** 

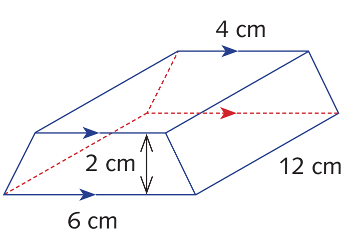

**d** 

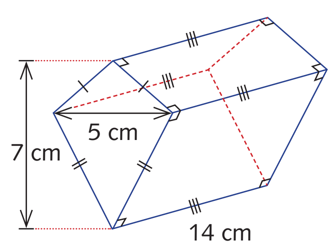

**e** 


**f** 


**g** 


**h** 


**i** 


**j** 



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**4** **a** Find the area of the base of a prism whose volume is 240 cm$^3$, given that its height is 8 cm.

**b** Find the height of a rectangular prism if its square base has sides of length 8.5 cm and its volume is 255 cm$^3$.

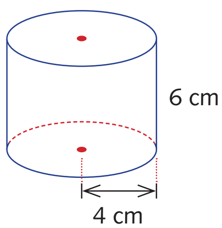
**c** Find the length of the side of a cube whose volume is 15.625 m$^3$.

**d** Find the dimensions of a square-based prism if its height is twice the side length of the square and the volume is 1228.25 mm$^3$.

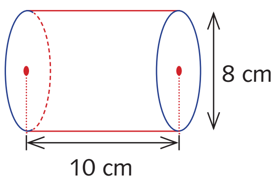
Example 4, 5

**5** Find the volume of each solid.

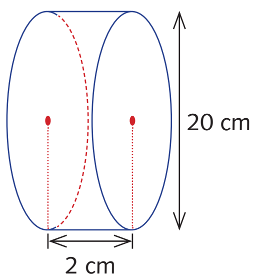
**a**



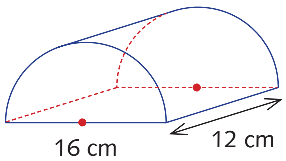

**b**



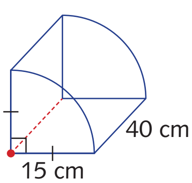

**c**



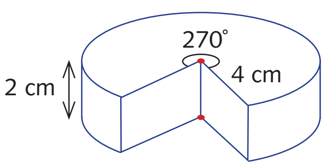

**d**



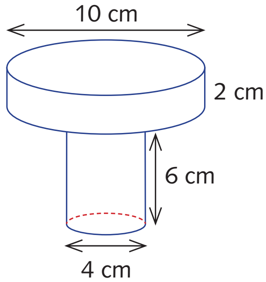

**e**



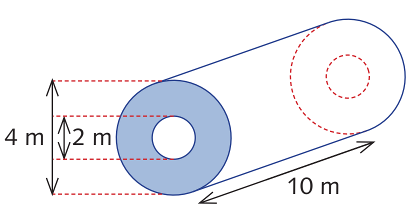

**f**




**g**




**h**




**6** **a** A cylinder with base radius 4 cm has a volume of 200 cm$^3$. Find its height.

**b** A cylinder of height 8 cm has a volume of 375 cm$^3$. Find its base radius.

**c** A cylinder of volume 1000 cm$^3$ is such that its base radius is equal to its height. Find its height.

**d** A cylinder of volume 500 cm$^3$ is such that its diameter is equal to its height. Find its height.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# 16E Conversion of units

We often need to convert from one unit into another. We recall:

1 cm = 10 mm    1 m = 100 cm    1 km = 1000 m

Just as lengths can be converted from one unit into another, so can areas and volumes. We can use the basic length conversions above to obtain area and volume conversions. The underpinning reasoning comes from the fact that area is measured in squares and volume in cubes. A detailed discussion of this can be found in *ICE-EM Mathematics Year 8*.

## Area conversions

To obtain the conversion factor for areas, we square the corresponding conversion factor for lengths.

### Example 6

Convert each measurement into square centimetres.

**a** $2.5\ \text{m}^2$    **b** $3600\ \text{mm}^2$

#### Solution

**a**

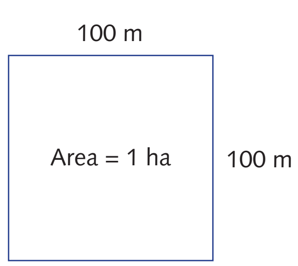
$$\begin{aligned} 1\,\text{m}^2 &= 100^2\ \text{cm}^2 \\ &= 10\,000\ \text{cm}^2 \\ \text{so } 2.5\,\text{m}^2 &= 2.5 \times 10\,000\ \text{cm}^2 \\ &= 25\,000\ \text{cm}^2 \end{aligned}$$

**b**

$$\begin{aligned} 1\ \text{cm}^2 &= 10^2\ \text{mm}^2 \\ &= 100\ \text{mm}^2 \\ \text{so } 3600\ \text{mm}^2 &= \frac{3600}{100}\ \text{cm}^2 \\ &= 36\ \text{cm}^2 \end{aligned}$$

## Hectares

A **hectare** (ha) is the area enclosed by a square with a side length of 100 m.
That is,




$$\begin{aligned} 1\ \text{ha} &= 100 \times 100\ \text{m}^2 \\ &= 10\,000\ \text{m}^2 \\ &= 10^4\ \text{m}^2 \\ 100\ \text{ha} &= 100 \times 10^4\ \text{m}^2 \\ &= 10^6\ \text{m}^2 \\ &= 1\ \text{km}^2 \end{aligned}$$

The metric system was introduced into Europe by Napoleon in the early nineteenth century, and was adopted by Australia in the 1960s. Until that time the Imperial system was used and land was measured in acres. In the metric system the hectare is the unit used for land measurement.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**Example 7**

The area of a cattle station in outback Australia is 200 000 ha. Calculate:

**a** the area in square metres

**b** the area in square kilometres

**c** the dimensions of the station, if it is a square, correct to 1 decimal place

**d** the dimensions of the station, if it is a rectangle and one side length is 50 km

**Solution**

**a**

$$\begin{aligned} 1 \text{ ha} &= 100 \times 100 \text{ m}^2 \\ &= 10\,000 \text{ m}^2 \\ \text{so} \quad 200\,000 \text{ ha} &= 200\,000 \times 10\,000 \text{ m}^2 \\ &= 2\,000\,000\,000 \text{ m}^2 \\ &= 2 \times 10^9 \text{ m}^2 \end{aligned}$$

**b**

$$\begin{aligned} 1 \text{ km}^2 &= 100 \text{ ha} \\ \text{so} \quad 200\,000 \text{ ha} &= 2000 \text{ km}^2 \end{aligned}$$

**c** Let $x$ km be the side length of the square.

$$\begin{aligned} \text{so} \quad x^2 &= 2000 \\ \text{hence} \quad x &\approx 44.7 \end{aligned}$$

Thus the station is approximately 44.7 km by 44.7 km.

**d** Area = length × width

2000 = 50 × width

Width = 40 km

Thus the station is 40 km by 50 km.

## Volume conversions

To obtain a conversion factor for volumes, we cube the corresponding conversion factor for lengths.

**Example 8**

Convert each measurement into the units indicated in brackets.

**a** $2760 \text{ mm}^3 \left(\text{cm}^3\right)$ **b** $0.27 \text{ m}^3 \left(\text{cm}^3\right)$ **c** $256\,000 \text{ cm}^3 \left(\text{m}^3\right)$ **d** $0.59 \text{ cm}^3 \left(\text{mm}^3\right)$


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**Solution**

**a**

$$\begin{aligned} 10\text{ mm} &= 1\text{ cm} \\ \text{so} \quad 10^3\text{ mm}^3 &= 1\text{ cm}^3 \\ \text{hence} \quad 2700\text{ mm}^3 &= \frac{2760}{1000}\text{ cm}^3 \\ &= 2.76\text{ cm}^3 \end{aligned}$$

**b**

$$\begin{aligned} 1\text{ m} &= 100\text{ cm} \\ \text{so} \quad 1\text{ m}^3 &= 100^3\text{ cm}^3 = 10^6\text{ cm}^3 \\ \text{hence} \quad 0.27\text{ m}^3 &= 0.27 \times 10^6\text{ cm}^3 \\ &= 270\,000\text{ cm}^3 \end{aligned}$$

**c** From part **b**: $10^6\text{ cm}^3 = 1\text{ m}^3$

$$\begin{aligned} \text{hence} \quad 256\,000\text{ cm}^3 &= \frac{256\,000}{1\,000\,000}\text{ m}^3 \\ &= 0.256\text{ m}^3 \end{aligned}$$

**d** From part **a**: $1\text{ cm}^3 = 1000\text{ mm}^3$

$$\begin{aligned} \text{hence} \quad 0.59\text{ cm}^3 &= 0.59 \times 1000\text{ mm}^3 \\ &= 590\text{ mm}^3 \end{aligned}$$

## Liquids

In many situations, particularly when liquids are involved, litres are used as the unit of volume.

A **litre** (1 L) is equal to $1000\text{ cm}^3$.

Thus, a litre is the volume of a cube of side length 10 cm. Also, a cubic metre is 1000 L.

$$\begin{aligned} 1\text{ L} &= 1000\text{ cm}^3 \\ 1\text{ m}^3 &= 10^6\text{ cm}^3 \\ &= 1000\text{ L} \end{aligned}$$

A **millilitre** (1 mL) is equal to $\frac{1}{1000}$ of a litre and is thus equal to $1\text{ cm}^3$.

$$1\text{ mL} = \frac{1}{1000}\text{ L} = 1\text{ cm}^3$$

**Example 9**

A large water trough in the shape of a rectangular prism has internal dimensions of 3 m by 0.6 m by 0.5 m. How many litres of water does the trough hold when full?


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**Solution**

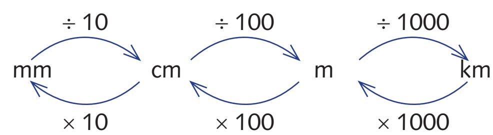
Since $1000\ \text{cm}^3 = 1\ \text{L}$, the volume is best calculated in cubic centimetres.

$$\begin{aligned} V &= 300\ \text{cm} \times 60\ \text{cm} \times 50\ \text{cm} \\ &= 900\ 000\ \text{cm}^3 \\ &= 900\ \text{L} \end{aligned}$$

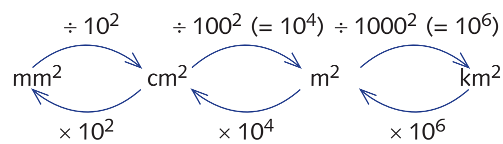
Hence the water trough holds 900 L.

## Conversion of units

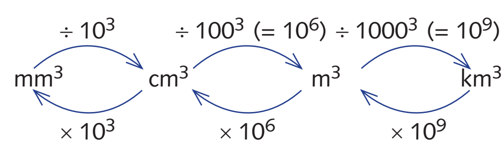
**Length**




**Area**




**Volume**




**Area of land**

$1\ \text{ha} = 10^4\ \text{m}^2$ $\quad$ $1\ \text{km}^2 = 100\ \text{ha}$

**Litres**

$1\ \text{L} = 1000\ \text{mL}$ $\quad$ $1\ \text{mL} = 1\ \text{cm}^3$ $\quad$ $1\ \text{m}^3 = 1000\ \text{L}$

# Exercise 16E

Example 6

**1** Convert each measurement into the unit indicated in brackets.

**a** $300\ \text{mm}^2\ (\text{cm}^2)$ **b** $3.1\ \text{m}^2\ (\text{cm}^2)$ **c** $0.5\ \text{m}^2\ (\text{mm}^2)$

**d** $0.6\ \text{cm}^2\ (\text{mm}^2)$ **e** $0.36\ \text{km}^2\ (\text{m}^2)$ **f** $2800\ \text{cm}^2\ (\text{m}^2)$

Example 7

**2** A rectangular piece of land measures 260 m by 430 m. Calculate the area of the land in:

**a** square metres **b** hectares **c** square kilometres



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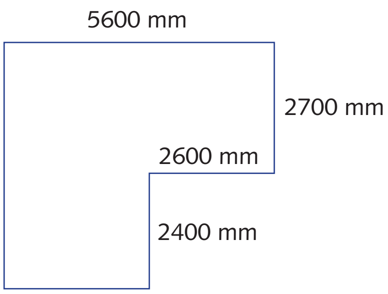
**3** A rectangular piece of land has an area of 2.7 ha. If the block of land is 135 m wide, how long is the block of land?

**4** A cattle station has an area of 260 $km^2$. What is its area in hectares?

**5** One acre is approximately 0.4 ha. A rectangular block of land measures $50 \text{ m} \times 150 \text{ m}$. Calculate the area of the block of land in:

**a** hectares  **b** acres (approximately)

**6** Robyn intends to paint the ceiling of her living room. A plan of the room is drawn with measurements. One litre of paint will cover 12 $m^2$. Calculate:




**a** the area of the ceiling in square metres

**b** the amount of paint needed to put one coat of paint on the ceiling

Example 8

**7** Convert each measurement into the units indicated in brackets.

**a** $5760 \text{ mm}^3 (\text{cm}^3)$  **b** $0.56 \text{ m}^3 (\text{cm}^3)$  **c** $0.62 \text{ m}^3 (\text{L})$

**d** $2600 \text{ cm}^3 (\text{L})$  **e** $52\,000 \text{ mm}^3 (\text{mL})$  **f** $960 \text{ L} (\text{m}^3)$

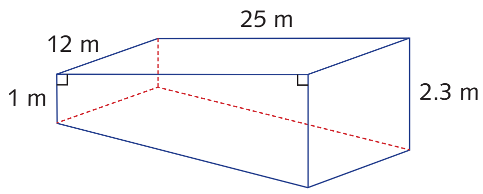
Example 9

**8** A large tank in the shape of a rectangular prism has internal dimensions $3 \text{ m} \times 2 \text{ m} \times 1.5 \text{ m}$. How many litres does the tank hold when full?

**9** A cylindrical water tank has a diameter of 3 m and a height of 2 m. Calculate, to the nearest 100 mL, the volume of the tank in:

**a** cubic metres  **b** litres

**10** A school swimming pool has dimensions as shown in the diagram opposite. How long would it take to fill this pool if the pump can deliver 1500 L of water a minute?




**11** A medicine bottle is in the shape of a cylinder with internal base diameter 50 mm and internal height 80 mm. If the normal dosage of medicine is 15 mL, how many dosages can be obtained from the bottle? (Assume that initially the bottle is completely full.)

**12** A swimming pool is in the shape of a rectangular prism. The pool is 12 m long, 4 m wide and 1.5 m deep. The pool is to be lined with tiles.

**a** How many tiles are needed to line the pool if each tile is $200 \text{ mm} \times 200 \text{ mm}$.

**b** A path 1 m wide is to be laid around the pool. How many pavers are needed to make the path if the size of each of the pavers is $500 \text{ mm} \times 500 \text{ mm}$?

**c** When the pool is completed, it is filled at 400 L/h. How long will it take to fill the pool to 10 cm below the top?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# 16F Small and large units of measurement

In Chapter 8 we introduced scientific notation as a useful way of expressing very small or very large numbers. Our metric system has prefixes that allow us to describe small or large amounts succinctly. You have already seen how the metric system uses powers of 10.

In the metric system:

- the prefix 'kilo' corresponds to the power $10^3$
- the prefix 'milli' corresponds the power $10^{-3}$

Thus the units of length, mass and volume:

- 1 kilogram = $10^3$ grams, 1 kilometer = $10^3$ metres and 1 kilolitre = $10^3$ litres
- 1 milligram = $10^{-3}$ grams, 1 millimetre = $10^{-3}$ metres and 1 millilitre = $10^{-3}$ litres

The prefixes and the associated powers of 10 in the table below are used with our metric system to help to express the size of very small or very large quantities.

| Prefix | Power | Symbol | Prefix | Power | Symbol |
|---|---|---|---|---|---|
| kilo | $10^3$ | K | milli | $10^{-3}$ | m |
| mega | $10^6$ | M | micro | $10^{-6}$ | μ |
| giga | $10^9$ | G | nano | $10^{-9}$ | n |
| tera | $10^{12}$ | T | pico | $10^{-12}$ | p |
| peta | $10^{15}$ | P | femto | $10^{-15}$ | f |

The following facts illustrate the use of some of these prefixes:

- A large dam has capacity 392 813 megalitres. This is written as 392 813 ML.
- Human hair ranges from 18 micrometres (18 μm) to 180 micrometres (180 μm) in width.
- An average piece of paper is 90 micrometres (90 μm) thick.
- Atoms are between 62 (62 pm) and 520 picometres (520 pm) in diameter.
- The subatomic particle, the proton, is estimated to have a radius of 0.8768 femtometres (0.8768 fm).
- The minimum distance between the Earth and Jupiter is 591 gigametres (591 Gm).
- The average distance between Saturn and the Sun is 1.079 terametres (1.079 Tm).
- The angstrom (Å) is often used in the natural sciences to express the sizes of atoms, the lengths of chemical bonds and the wavelengths of electromagnetic radiation:

$$1 \text{ angstrom} = 10^{-1} \text{ nanometres} = 10^{-10} \text{ metres}$$

## Computer memory

Prefixes are used to describe digital memory storage capacity. The **byte** is the fundamental unit used to measure storage capacity.

In computer science, storage capacity occurs in powers of 2, but there has been some attempt to make this binary conversion conform to the usual use of the prefixes.


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| Storage capacity | Closest power of 10 bytes | Actual number of bytes |
|---|---|---|
| kilobyte (kB) | $10^3$ | $2^{10} = 1024 = 1.024 \times 10^3$ |
| megabyte (MB) | $10^6$ | $2^{20} = 1\,048\,576 = 1.048\,576 \times 10^6$ |
| gigabyte (GB) | $10^9$ | $2^{30} = 1\,073\,741\,824 = 1.073\,741\,824 \times 10^9$ |
| terabyte (TB) | $10^{12}$ | $2^{40} = 1\,099\,511\,627\,776 = 1.099\,511\,627\,776 \times 10^{12}$ |

## Time

Seconds are part of the metric system and can be combined with the prefixes given above:

$$1 \text{ microsecond} = 10^{-6} \text{ seconds}$$
$$1 \text{ nanosecond} = 10^{-9} \text{ seconds}$$
$$1 \text{ picosecond} = 10^{-12} \text{ seconds}$$

Some examples include:

- A camera flash illuminates for approximately 1000 microseconds.
- 2.68 microseconds were subtracted from the Earth day after the 2004 Indian Ocean earthquake.
- The time for fusion reaction in a hydrogen bomb is 20 to 24 nanoseconds.
- The time for light to travel 1 millimetre in a vacuum is approximately 3.3365 picoseconds.

## Light years

- A light year is the distance that light travels in one year ($365\frac{1}{4}$ days). It is most commonly used in astronomy for measuring distances to stars and other galaxies.
- A light year is about $9\,460\,000\,000\,000\,000$ metres $= 9.46 \times 10^{15}$ metres $=$ 9.46 petametres
- The nearest star to our solar system is Proxima Centuri, which is about 4.2 light years away. The centre of our galaxy is about 30 000 light years away.

## Exercise 16F

**1** Convert each measurement into metres, expressing your answer in scientific form.

**a** 23 millimetres **b** 67 picometres **c** 456 picometres
**d** 25 micrometres **e** 93 nanometres **f** 651 nanometres

**2** Convert each measurement into microseconds.

**a** One day **b** One week **c** 14 hours 25 seconds **d** 0.65 seconds

**3** Convert each measurement into nanoseconds.

**a** One day **b** One week **c** 14 hours 35 seconds **d** 672 microseconds


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4 A cube holds 1 kilolitre of water. What is the length of a side of the cube? (Remember: $1 \text{ litre} = 1000 \text{ cm}^3$)

5 A reservoir holds 765 910 megalitres of water. How many litres is this? Express your answer in scientific form.

6 The half-life of a copernicum-277 atom is 240 microseconds. Express this in seconds using scientific notation.

7 The following was stated in a newspaper: ‘One million megalitres of pollution spewed onto the Great Barrier Reef.’

**a** How many litres is this? **b** How many gigalitres is this?

8 The volume of a dam on a farm is estimated using the following formula.

$$\text{Volume } (\text{m}^3) = 0.4 \times \text{surface area} \times \text{depth}$$

0.4 is a conversion factor that takes into account the slope of the sides of dams.

If the shape of the dam surface is a circle of diameter 40 m and the depth is 4 m, calculate the volume of water held in the dam in megalitres.

9 Light travels 1 millimetre in 3.3 picoseconds.

**a** How long does it take for light to travel 1 metre?

**b** How long does it take for light to travel 1 kilometre?

**c** How far does light travel in 1 nanosecond?

# Review exercise

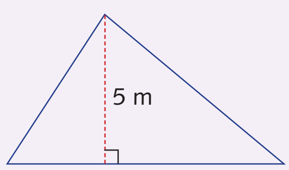
1 Convert each measurement to the given units.

**a** 1.2 cm into m **b** $0.23 \text{ m}^2$ into $\text{cm}^2$ **c** $0.55 \text{ km}^2$ into ha

**d** $0.35 \text{ cm}^3$ to $\text{mm}^3$ **e** $840 \text{ cm}^3$ to L

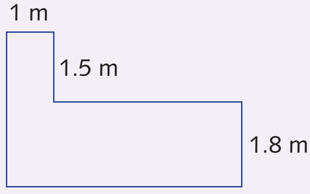
2 Find the area of each figure.

**a**

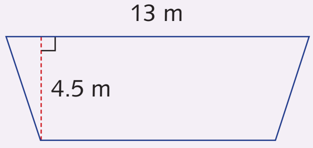



**b**


**c**




**d**





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**3** Convert each measurement to the units shown in brackets.

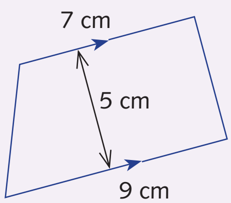
**a** 20 cm (mm) **b** 3200 mm (m) **c** 5 ha $(\text{m}^2)$

**d** 2000 $\text{cm}^2$ $(\text{m}^2)$ **e** 3 $\text{cm}^2$ $(\text{mm}^2)$ **f** 3.2 L $(\text{cm}^3)$

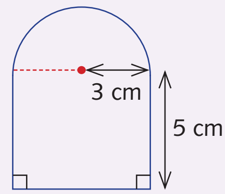
**g** 0.5 $\text{m}^3$ $(\text{cm}^3)$ **h** 2 $\text{m}^3$ (L) **i** 25 000 $\text{m}^2$ (ha)

**4** Calculate the area of each figure.

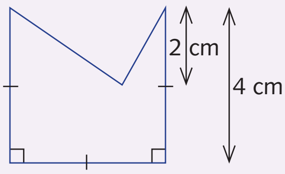
**a**



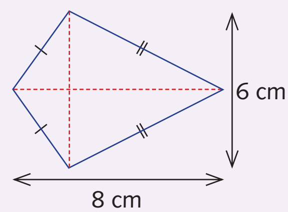

**b**




**c**




**d**

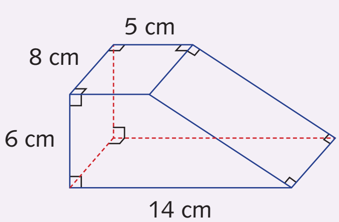



**5** For each of the following solids, calculate:

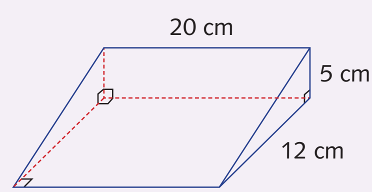
**i** the surface area

**ii** the volume

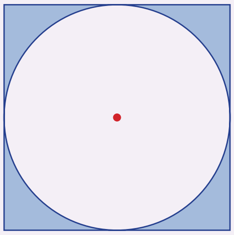
**a**




**b**




**6** If the square in the shape opposite has an area of 64 $\text{cm}^2$, find the area of the shaded region.



**7** A rectangular garden plot 8 m by 3 m is surrounded by a concrete path 1.2 m wide and laid to a depth of 8 m.

**a** Find the area of the path.

**b** Find the volume of concrete required.

**c** Find the cost of the path if the concrete costs $75 per cubic metre.

**8** Find the total surface area of a solid cylinder of radius 5 cm and height 10 cm.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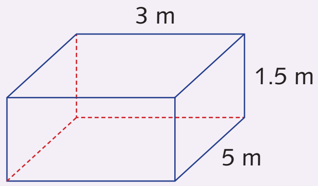
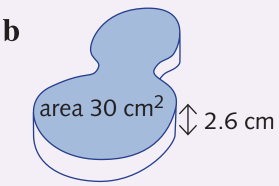
**9** Find the volume of each solid.

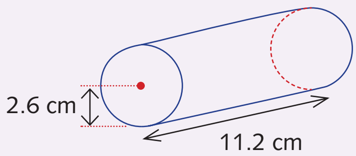
**a**



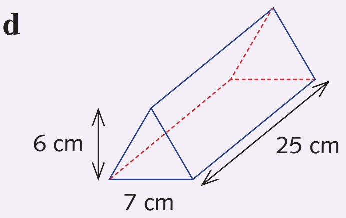

**b**




**c**

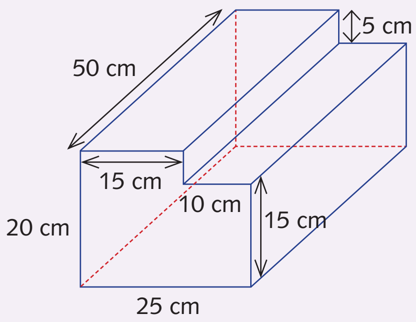



**d**

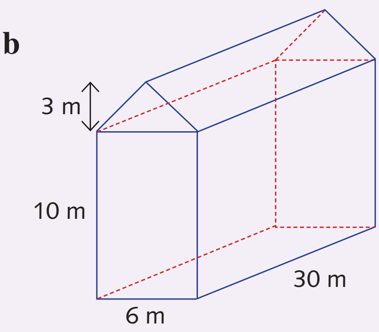



**10** Find the volume of each solid.

**a**



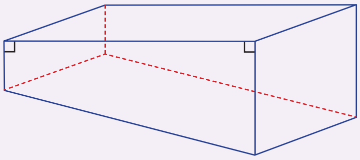

**b**




# Challenge exercise

**1** A rectangular house that measures 25 m by 15 m has a water tap at one corner of the house. A hose 20 m long is connected to the tap. What is the largest area of lawn over which the hose can reach?

**2** A swimming pool is in the shape of a trapezoidal prism, as shown. The pool is 10 m long, 5 m wide, 1 m deep at one end and 2 m deep at the other end. The pool is filled with water until it is 10 cm from the top.



**a** Calculate, to the nearest litre, the volume of water in the pool.

**b** After a group of children played in the pool, the level of the water is 0.15 m below the top. How much water, correct to the nearest litre, splashed out of the pool?



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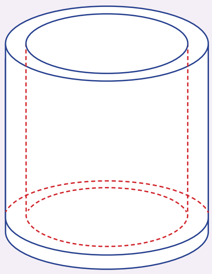
**3** A foam drink-can holder is in the shape of a cylinder. The inside radius is $5.5$ cm and the inside height is $11$ cm. The foam is $1$ cm thick and forms the curved surface of the cylinder. The base is cut out of foam $1$ cm thick.

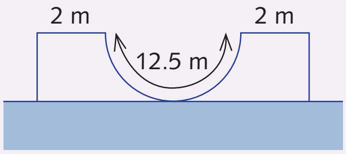
**a** Calculate the volume inside the cylinder in cubic centimetres.

**b** Calculate the volume of foam in the drink-can holder in cubic centimetres.

**c** If a cylindrical drink can perfectly fills the holder, find the total surface area of the exposed foam in square centimetres.



**4** A skateboard ramp, a cross-section of which is shown opposite, consists of a half-cylinder with platforms of width 2 m. The curved part of the ramp has length 12.5 cm.




**a** Find the radius of the cylinder, correct to 2 decimal places.

**b** Find, correct to 2 decimal places, the total width of the ramp, including the platform.

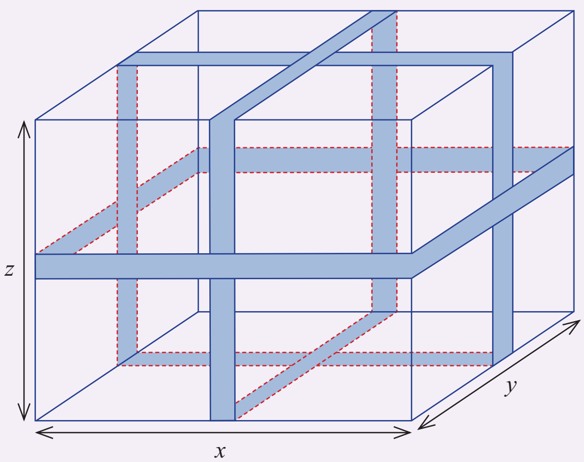
**c** If the ramp is 8 m long, find the surface area of the curved part of the ramp.

**5** **a** A rectangular prism has faces of area $66$, $48$ and $88$. Find the volume of the prism.

**b** A box in the shape of a rectangular prism is strapped as shown. The length of the straps are $100$, $60$ and $80$.

**i** Find the dimensions of the box.

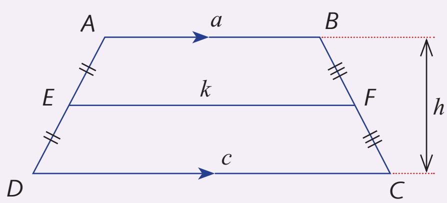
**ii** Find the volume of the box.




**6** Let $ABCD$ be a trapezium with parallel sides $AB$ and $CD$. Let $E$ and $F$ be the midpoints of $AD$ and $BC$ respectively. Let $k$ be the length of $EF$. Let $AB = a$ and $CD = c$.

**a** Prove that $k = \dfrac{a+c}{2}$.

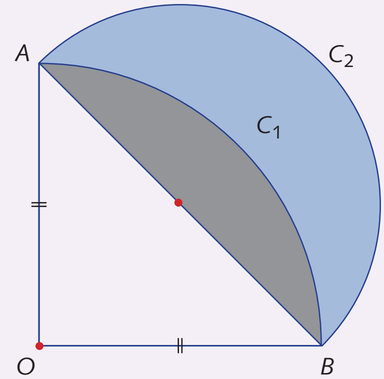
**b** Deduce that the area of the trapezium is $kh$, where $h$ is the distance between the parallel sides.




**7** **a** Let $ABCD$ be a convex quadrilateral. A line through the vertex $A$, parallel to the diagonal $BD$, meets $CD$ (extended) at $E$. Show that the area of triangle $BCE$ equals the area of $ABCD$.

**b** Let $ABCDE$ be a convex pentagon. Construct a triangle with the same area as this pentagon.

**8** $OA$ and $OB$ are radii of a circle $C_1$. A circle $C_2$ is drawn with diameter $AB$ as shown. Find the ratio of the area (blue) to the area (grey).





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