CHAPTER

17

Number and Algebra

Quadratic equations

We have been solving linear equations for some time. We will now learn how to apply factorisation to solve quadratic equations.

Quadratic equations turn up routinely in mathematics. Being able to solve them is a fundamental skill. For example, finding the distance a rocket or cricket ball will travel involves quadratic equations.

The ancient Babylonians were solving quadratic equations more than 5000 years ago!

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17A Solution of simple quadratic equations

In Chapter 15 we factorised quadratic expressions. For example:

$x^2 - 2x = x(x - 2)$ $\quad$ $x^2 - 16 = (x - 4)(x + 4)$

$x^2 - x - 12 = (x - 4)(x + 3)$ $\quad$ $x^2 - 4x + 4 = (x - 2)^2$

$2x^2 + x - 6 = (2x - 3)(x + 2)$

The following are examples of **quadratic equations**.

$x(x - 2) = 0 \quad x^2 - x - 12 = 0 \quad x^2 + 6x = 8$

Solving a quadratic equation means finding the values of x that satisfy the equation. One important method for solving a quadratic equation is factorising and then using the following simple idea:

If the product of two numbers is zero, at least one of the numbers is zero.

In symbols:

If $ab = 0$, then $a = 0$ or $b = 0$. (Both a and b may equal zero.)

To solve a quadratic equation:

- Move all the terms to the left-hand side of the equation, leaving zero on the right-hand side.
- Factorise the expression as a product of two factors.
- Equate each factor to zero and solve each linear equation.

The solutions to these linear equations are the solutions to the quadratic equation.

In Example 1 the quadratic equations are given in factorised form.

Example 1

Solve each equation.

a $(x - 3)(x + 4) = 0$ $\quad$ **b** $x(x + 5) = 0$

c $(3x + 4)(x + 7) = 0$ $\quad$ **d** $(3 - 2x)(4x + 5) = 0$

Solution

a
$$(x - 3)(x + 4) = 0$$
$$x - 3 = 0 \text{ or } x + 4 = 0$$
$$x = 3 \text{ or } x = -4$$

b
$$x(x + 5) = 0$$
$$x = 0 \text{ or } x + 5 = 0$$
$$x = 0 \text{ or } x = -5$$

c
$$(3x + 4)(x + 7) = 0$$
$$3x + 4 = 0 \text{ or } x + 7 = 0$$
$$x = -\frac{4}{3} \text{ or } x = -7$$

d
$$(3 - 2x)(4x + 5) = 0$$
$$3 - 2x = 0 \text{ or } 4x + 5 = 0$$
$$x = \frac{3}{2} \text{ or } x = -\frac{5}{4}$$

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Example 2

Solve each equation.

a $x^2 - 2x = 0$

b $x^2 - 2x = 48$

c $x^2 = 3x + 10$

d $x^2 - 7x = 18$

Solution

In each case we move all terms to the left-hand side, and factorise the quadratic expression.

a
$$\begin{aligned} x^2 - 2x &= 0 \\ x(x-2) &= 0 \\ x = 0 \text{ or } x - 2 &= 0 \\ x = 0 \text{ or } x &= 2 \end{aligned}$$

b
$$\begin{aligned} x^2 - 2x &= 48 \\ x^2 - 2x - 48 &= 0 \\ (x-8)(x+6) &= 0 \\ x = 8 \text{ or } x &= -6 \end{aligned}$$

c
$$\begin{aligned} x^2 &= 3x + 10 \\ x^2 - 3x - 10 &= 0 \\ (x-5)(x+2) &= 0 \\ x = 5 \text{ or } x &= -2 \end{aligned}$$

d
$$\begin{aligned} x^2 - 7x &= 18 \\ x^2 - 7x - 18 &= 0 \\ (x-9)(x+2) &= 0 \\ x = 9 \text{ or } x &= -2 \end{aligned}$$

Notice that in these examples we obtain two solutions to the original equation. We can check they are solutions by substitution. For example, for part **c** above:

If $x = 5$:
$$\begin{aligned} \text{LHS} &= 5^2 = 25 \\ \text{RHS} &= 3 \times 5 + 10 \\ &= 25 \end{aligned}$$
Therefore LHS = RHS

If $x = -2$:
$$\begin{aligned} \text{LHS} &= (-2)^2 = 4 \\ \text{RHS} &= 3 \times -2 + 10 \\ &= 4 \end{aligned}$$
Therefore LHS = RHS

We have checked that $x = 5$ and $x = -2$ are solutions to the quadratic equation $x^2 = 3x + 10$.

Numerical common factor

If there is a **numerical factor** common to all of the coefficients in the equation, we can divide both sides of the equation by that common factor.

Example 3

Solve each equation.

a $6x^2 = 18x$

b $2x^2 - 10x + 12 = 0$

c $-x^2 + 9x - 18 = 0$

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Solution

a

$$6x^2 = 18x$$

$$x^2 = 3x \quad \text{(Divide both sides of the equation by 6.)}$$

$$x^2 - 3x = 0$$

$$x(x - 3) = 0$$

$$x = 0 \text{ or } x - 3 = 0$$

$$x = 0 \text{ or } x = 3$$

Note: In part **a** we cannot divide both sides of the equation by x since we would lose the solution $x = 0$.

b

$$2x^2 - 10x + 12 = 0 \quad \text{(Divide both sides of the equation by 2.)}$$

$$2(x^2 - 5x + 6) = 0$$

$$x^2 - 5x + 6 = 0$$

$$(x - 3)(x - 2) = 0$$

$$x - 3 = 0 \text{ or } x - 2 = 0$$

$$x = 3 \text{ or } x = 2$$

c

$$-x^2 + 9x - 18 = 0$$

$$-1(x^2 - 9x + 18) = 0$$

$$x^2 - 9x + 18 = 0 \quad \text{(Divide both sides of the equation by } -1.\text{)}$$

$$(x - 3)(x - 6) = 0$$

$$x - 3 = 0 \text{ or } x - 6 = 0$$

$$x = 3 \text{ or } x = 6$$

Equations involving a difference of squares

Example 4

Solve $18 - 2y^2 = 0$.

Solution

$$18 - 2y^2 = 0$$

$$9 - y^2 = 0 \quad \text{(Divide both sides of the equation by } -2.\text{)}$$

$$(y - 3)(y + 3) = 0$$

$$y = 3 \text{ or } y = -3$$

This problem could also be solved as follows:

$$18 - 2y^2 = 0$$

$$18 = 2y^2$$

$$y^2 = 9$$

$$y = 3 \text{ or } y = -3$$

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Solution of simple quadratic equations

- To solve a quadratic equation:
  - If there is a numerical common factor, divide both sides of the equation by the factor.
  - Move all terms to the left-hand side of the equation, leaving zero on the right-hand side.
  - If possible, factorise the resulting expression as a product of two factors.
  - Equate each factor to zero and solve each equation.
- Solutions to the equations of the form $x^2 = a$, $a \geq 0$ are $x = \sqrt{a}$ or $x = -\sqrt{a}$.
- Not all quadratic equations have solutions. For example, $x^2 + 5 = 0$ has no solutions since $x^2 + 5 \geq 5$ for all values of x. Hence $x^2 + 5$ will never equal zero.

Exercise 17A

Example 1

1 Solve the equations.

a $x(x - 2) = 0$ **b** $(x + 1)(x - 1) = 0$ **c** $2x(x + 1) = 0$

d $(3x - 1)(2x + 5) = 0$ **e** $(1 - x)(3 - 2x) = 0$ **f** $(x + 1)^2 = 0$

g $x(x - 3) = 0$ **h** $(x - 3)(x - 4) = 0$ **i** $x(x + 5) = 0$

j $(3x - 2)(x + 2) = 0$ **k** $(2 + x)(3 - x) = 0$ **l** $(x - 3)^2 = 0$

Example 2

2 Solve each quadratic equation. Check your solutions in parts **c**, **e**, **f** and **l**.

a $x^2 + 3x + 2 = 0$ **b** $x^2 + 7x = -12$ **c** $y^2 - 7y + 10 = 0$

d $x^2 + 13x - 30 = 0$ **e** $b^2 - 13b - 14 = 0$ **f** $z^2 - 27z - 90 = 0$

g $x^2 = -5x - 4$ **h** $x^2 = 19x - 18$ **i** $x^2 + x - 12 = 0$

j $x^2 - 16x + 28 = 0$ **k** $x^2 + 3x - 10 = 0$ **l** $x^2 + x = 90$

3 Solve each quadratic equation.

a $x^2 - 5x = 0$ **b** $x^2 + 7x = 0$ **c** $3x^2 - 6x = 0$

d $5x^2 - x = 0$ **e** $4x^2 = -10x$ **f** $x^2 + 4x = 0$

g $x^2 = 6x$ **h** $4x^2 + 12x = 0$ **i** $3x - 2x^2 = 0$

j $5x - 15x^2 = 0$ **k** $9x^2 - 18x = 0$ **l** $49x^2 = 7x$

Example 3

4 Solve the quadratic equations.

a $5x^2 - 10x = 0$ **b** $2x^2 + 5x = 0$ **c** $4x^2 = -12x$

d $3x = 2x^2$ **e** $5x = 15x^2$ **f** $3a^2 + 15a + 18 = 0$

g $4r^2 - 4r - 24 = 0$ **h** $5c^2 + 40c + 60 = 0$ **i** $3a^2 - 48a + 192 = 0$

j $2n^2 - 22n + 60 = 0$ **k** $4s^2 = 36s + 280$ **l** $-x^2 + 24x + 25 = 0$

m $-x + 42 = x^2$ **n** $-2x^2 + 10x + 48 = 0$ **o** $3x^2 - 48x = -84$

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Example 4

5 Solve each quadratic equation.

a $x^2 - 1 = 0$ **b** $x^2 - 25 = 0$ **c** $4x^2 - 1 = 0$

d $2x^2 - 50 = 0$ **e** $x^2 + 6x + 9 = 0$ **f** $x^2 + 10x + 25 = 0$

g $x^2 - 12x + 36 = 0$ **h** $x^2 - 16 = 0$ **i** $16x^2 - 25 = 0$

j $9 - x^2 = 0$ **k** $72 - 2y^2 = 0$ **l** $x^2 - 4x + 4 = 0$

m $x^2 - 2x + 1 = 0$ **n** $x^2 = 6x + 27$ **o** $x^2 = 8x + 33$

17B Solution of quadratic equations when the coefficient of x^2 is not 1

In Chapter 15 we factorised quadratics for which the coefficient of x^2 is not 1. We will use this technique of factorisation in the following example.

Example 5

Solve each quadratic equation.

a $6x^2 + 7x + 2 = 0$ **b** $12x^2 = 23x - 5$

Solution

a $6x^2 + 7x + 2 = 0$

Find two numbers whose product is $6 \times 2 = 12$ and whose sum is 7. They are 4 and 3.

$(6x^2 + 4x) + (3x + 2) = 0$ (Split the middle term.)

$2x(3x + 2) + 1(3x + 2) = 0$ (Use grouping.)

$(3x + 2)(2x + 1) = 0$

$3x + 2 = 0$ or $2x + 1 = 0$

$x = -\frac{2}{3}$ or $x = -\frac{1}{2}$

Note: It does not matter in which order we split the middle term. In the above example, we could write $4x + 3x$ or $3x + 4x$, and factorise in pairs. Try it for yourself.

b $12x^2 = 23x - 5$

$12x^2 - 23x + 5 = 0$

Find two numbers whose product is $12 \times 5 = 60$ and whose sum is –23. The numbers are –20 and –3.

$12x^2 - 20x - 3x + 5 = 0$

$4x(3x - 5) - 1(3x - 5) = 0$

$(3x - 5)(4x - 1) = 0$

$3x - 5 = 0$ or $4x - 1 = 0$

$x = \frac{5}{3}$ or $x = \frac{1}{4}$

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Quadratic equations of the form $ax^2 + bx + c = 0$

- If the coefficients have a numerical common factor, divide both sides of the equation by that factor.
- To factorise a quadratic expression such as $ax^2 + bx + c$, find two numbers α and β whose product is ac and whose sum is b. Write the middle term as $\alpha x + \beta x$ and factorise by grouping.

Exercise 17B

1 Solve the quadratic equations.

a $4x(6x + 12) + 8(6x + 12) = 0$ **b** $3x(2x - 4) + 6(2x - 4) = 0$

c $5x(3x - 6) + 10(3x - 6) = 0$ **d** $2x(x - 5) + 12(x - 5) = 0$

Example 5a

2 Solve the quadratic equations.

a $2x^2 - 5x - 3 = 0$ **b** $6x^2 + x - 2 = 0$ **c** $8x^2 - 14x + 3 = 0$

d $3x^2 + 5x - 2 = 0$ **e** $6x^2 - 11x - 10 = 0$ **f** $10x^2 + 11x - 6 = 0$

g $6x^2 + 5x + 1 = 0$ **h** $6x^2 - 7x - 10 = 0$ **i** $12x^2 + 5x - 2 = 0$

j $10x^2 + 31x + 15 = 0$ **k** $12x^2 - 8x - 15 = 0$ **l** $15x^2 + 13x + 2 = 0$

Example 5b

3 Solve the quadratic equations.

a $7x^2 - 16x = 15$ **b** $2x^2 + 3x = 2$ **c** $6x^2 = 7x + 3$

d $6x^2 + 5x = 6$ **e** $7x^2 = 78x - 11$ **f** $4x^2 = 3x + 1$

g $3x^2 = 8x + 3$ **h** $5x^2 - 23x = 84$ **i** $4x^2 - 15 = 4x$

j $12x^2 = 4x + 5$ **k** $3x^2 - 4 = 4x$ **l** $6x = 2x^2 - 8$

4 Solve the quadratic equations.

a $9x^2 + 4x - 5 = 0$ **b** $5x^2 + 7x - 12 = 0$

c $10x^2 - 22x + 4 = 0$ **d** $-8a^2 - 24a + 14 = 0$

e $-4a^2 + 8a + 21 = 0$ **f** $6a^2 - 56a + 50 = 0$

g $5x^2 + 26x + 24 = 0$ **h** $6x^2 + 11x - 10 = 0$

i $3x^2 - 41x - 60 = 0$ **j** $-8x^2 + 4x + 12 = 0$

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17C Quadratics in disguise

In many mathematical problems and applications, equations arise that do not initially appear to be quadratic equations. We often need to rearrange an equation to put it into the standard form $ax^2 + bx + c = 0$.

We start with two simple examples.

Example 6

Solve each equation.

a $x(x + 5) = 6$

b $\frac{x(x-3)}{2} + x = 1$

Solution

a
$$\begin{aligned} x(x+5) &= 6 \\ x^2 + 5x - 6 &= 0 \\ (x+6)(x-1) &= 0 \\ x + 6 = 0 \text{ or } x - 1 &= 0 \\ x = -6 \text{ or } x &= 1 \end{aligned}$$

We can check that these are the correct solutions by substitution.

If $x = -6$, LHS $= -6(-6 + 5) = 6 =$ RHS

If $x = 1$, LHS $= 1(1 + 5) = 6 =$ RHS

b
$$\begin{aligned} \frac{x(x-3)}{2} + x &= 1 \\ x(x-3) + 2x &= 2 \quad \text{(Multiply both sides by 2.)} \\ x^2 - 3x + 2x &= 2 \\ x^2 - x - 2 &= 0 \\ (x+1)(x-2) &= 0 \\ x + 1 = 0 \text{ or } x - 2 &= 0 \\ x = -1 \text{ or } x &= 2 \end{aligned}$$

Some equations involve fractions in which the pronumeral appears in the denominator. We always assume that the pronumeral cannot take a value that makes the denominator equal to zero. It is important to check that your answers are in fact correct solutions to the given equation.

Example 7

Solve:

a $x = \frac{3x - 2}{x}$

b $\frac{x-2}{3} = \frac{5}{x}$

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Solution

a

$$x = \frac{3x-2}{x}$$

$$x^2 = 3x - 2 \quad \text{(Multiply both sides by } x\text{.)}$$

$$x^2 - 3x + 2 = 0$$

$$(x-1)(x-2) = 0$$

$$x - 1 = 0 \text{ or } x - 2 = 0$$

$$x = 1 \text{ or } x = 2$$

b Method 1

$$\frac{x-2}{3} = \frac{5}{x}$$

$$3x \times \frac{x-2}{3} = 3x \times \frac{5}{x}$$

(Multiply both sides of the equation by $3x$.)

$$x(x-2) = 15$$

$$x^2 - 2x = 15$$

$$x^2 - 2x - 15 = 0$$

$$(x+3)(x-5) = 0$$

$$x + 3 = 0 \text{ or } x - 5 = 0$$

$$x = -3 \quad \text{or} \quad x = 5$$

Method 2

$$\frac{x-2}{3} \times \frac{5}{x}$$

$$x(x-2) = 3 \times 5$$

(This is called cross-multiplication.)

We must check mentally that these solutions satisfy the original equation.

Note: **Cross-multiplication** is an effective method in the solution of such equations. It is justified by Method 1 in the above example.

Example 8

Solve $(x-2)(2x+5) = 2x+5$.

Solution

Method 1

$$(x-2)(2x+5) = 2x+5$$

$$2x^2 - 4x + 5x - 10 = 2x + 5$$

$$2x^2 + x - 10 = 2x + 5$$

$$2x^2 - x - 15 = 0$$

$$(2x+5)(x-3) = 0$$

$$2x + 5 = 0 \text{ or } x - 3 = 0$$

$$x = -\frac{5}{2} \text{ or } x = 3$$

Method 2

$$(x-2)(2x+5) = 2x+5$$

$$(x-2)(2x+5) - (2x+5) = 0$$

$$(2x+5)(x-2-1) = 0$$

$$(2x+5)(x-3) = 0$$

$$2x + 5 = 0 \text{ or } x - 3 = 0$$

$$x = -\frac{5}{2} \text{ or } x = 3$$

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Example 9

Solve the equation $\frac{x+4}{2} - \frac{3}{x-3} = 1$.

Solution

$$\frac{x+4}{2} - \frac{3}{x-3} = 1$$

$$(x+4)(x-3) - 6 = 2x - 6 \qquad \text{(Multiply both sides by } 2(x-3).)$$

$$x^2 + x - 12 - 6 = 2x - 6$$

$$x^2 - x - 12 = 0$$

$$(x-4)(x+3) = 0$$

$$x = 4 \text{ or } x = -3$$

Exercise 17C

Example 6a

1 Solve the equations.

a $x(x-6) = -8$ **b** $x(x-7) = -10$ **c** $x(x-5) = -4$

d $x(x-8) = -15$ **e** $x(x-3) = 4$ **f** $x(x-6) = 27$

g $(x-3)(x-4) = 12$ **h** $(x+7)(x-2) + 14 = 0$

i $(x+2)(x-6) - 65 = 0$ **j** $(x-1)(x-2) = 20$

Example 6b, 7

2 Solve the quadratic equations.

a $x + 5 = \frac{14}{x}$ **b** $\frac{15}{x} = x - 2$ **c** $\frac{6}{x} - x = 1$

d $x + \frac{6}{x} = 7$ **e** $x + \frac{32}{x} = 18$ **f** $\frac{x+1}{3} = \frac{10}{x}$

g $\frac{x+1}{4} = \frac{5}{x}$ **h** $x - \frac{14}{x} = 5$ **i** $\frac{x^2 - 4x}{12} = \frac{x-4}{2}$

j $\frac{1}{x+1} = \frac{5x-6}{6x}$ **k** $\frac{2x+1}{3} = \frac{-(51+32x)}{3(2x-3)}$ **l** $x + \frac{1}{x} = \frac{13}{6}$

m $\frac{3}{x-1} = \frac{3x-2}{x(x-2)}$ **n** $\frac{35-4x}{5} = \frac{36-5x}{5x}$

Example 8

3 Solve the quadratic equations.

a $x(x+3) = x(2x-4)$ **b** $(x+3)(2x-1) = (2x-1)(3x-4)$

c $3x(x+3) = 2x(2x+5)$ **d** $4(x+3)(2x-3) = 2(x+3)(5x+8)$

e $(5x+2)(x-7) = (x-7)(2x+1)$

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4 Solve the quadratic equations.

a $x + 4 = \frac{12}{x}$

b $x - 3 = \frac{10}{x}$

c $x - \frac{35}{x} = 2$

d $x = \frac{18}{x} - 7$

e $(x + 2)(x + 4) = 3$

f $(x - 2)(x + 5) = 8$

g $(2x + 1)(x + 5) = 18$

h $(2x - 3)(x + 2) = 4$

i $x = \frac{5}{x} - 4$

j $x = \frac{2}{x - 1}$

k $x = \frac{7}{2x + 3} + 1$

l $x = \frac{13}{3x - 1} - 3$

Example 9

5 Solve the quadratic equations.

a $\frac{2x}{3} + \frac{x - 1}{15} = \frac{2}{3} + \frac{22}{x}$

b $\frac{x}{2} + \frac{x - 4}{x - 4} = \frac{x}{3}$

c $\frac{x + 1}{7} + \frac{8}{x - 2} = 3$

d $\frac{x - 1}{x + 2} - \frac{x - 3}{x - 4} = -\frac{2}{3}$

e $x(x + 9) + 3 = x(2 - 3x)$

f $3x(x - 2) = x(x + 1)$

17D Applications of quadratic equations

When we apply mathematics to practical problems, we often arrive at equations that we have to solve. In many cases these equations are quadratic equations.

Some of the solutions to the quadratics may not be relevant to the problem. For example, a quadratic equation may yield negative or fractional solutions, which may not make sense as solutions to the original problem. This point is illustrated in the next two examples.

Example 10

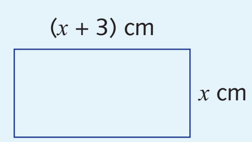
A rectangle has one side 3 cm longer than the other. The rectangle has area 28 cm^2.
How long is the shorter side?

Solution

Let x cm be the length of the shorter side. The other side has length $(x + 3)$ cm.

Area of rectangle is:

$$\begin{aligned} x(x + 3) &= 28 \\ x^2 + 3x - 28 &= 0 \\ (x - 4)(x + 7) &= 0 \\ x - 4 = 0 \text{ or } x + 7 &= 0 \\ x = 4 \text{ or } x &= -7 \end{aligned}$$



Since length is positive, $x = -7$ makes no sense. Hence the shorter side has length 4 cm and the longer side is 7 cm.

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Example 11

Each term in the sequence 5, 9, 13, 17, … is obtained by adding 4 to the previous number. The sum, S, of the first n terms in this sequence is given by $S = 2n^2 + 3n$. How many terms must we add to make a sum of 90?

Solution

$S = 90$, so $\quad 2n^2 + 3n = 90$

$2n^2 + 3n - 90 = 0$ (Find two numbers with product $2 \times (-90) = -180$ and sum 3. The numbers are 15 and -12.)

$$2n^2 + 15n - 12n - 90 = 0$$
$$n(2n + 15) - 6(2n + 15) = 0$$
$$(2n + 15)(n - 6) = 0$$
$$2n + 15 = 0 \text{ or } n - 6 = 0$$
$$n = -\frac{15}{2} \text{ or } n = 6$$

Since n is a positive whole number, the solution $n = -\frac{15}{2}$ does not make sense here.
So $n = 6$.

Check: $5 + 9 + 13 + 17 + 21 + 25 = 90$

Exercise 17D

Although some of these problems can be solved by inspection, in each case, introduce a pronumeral, write down a quadratic equation and solve it.

Example 10

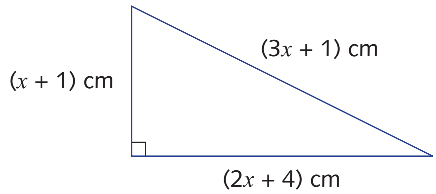
1 The product of a certain whole number and four more than that number is 140. Find the number.

2 The product of a certain whole number and three less than that number is 108. Find the number.

3 The product of a certain integer and nine more than that integer is 10. What can the integer be?

4 The product of a certain integer and twice that integer is 32. What can the integer be?

5 Find the value of x in the diagram opposite.



Example 11

6 The sum of n terms in the sequence 23, 19, 15, 11, … is given by the rule $S = 25n - 2n^2$. How many terms must we add to make a sum of 72?

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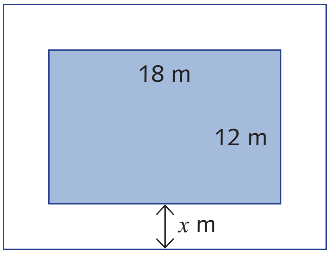
7 A metal sheet is 50 cm wide and 60 cm long. It has squares cut out of the corners so that it can be folded to form a box with a base area of 1200 cm^2. Find the length of the side of the squares.

8 A rectangular lawn is 18 m long and 12 m broad. The lawn is surrounded by a path of width x m. The area of the path is equal to the area of the lawn. Find x.



9 A man travelled from town A to town B at a speed of V km/h. The distance from A to B is 120 km. The man returned from B to A at a speed of $(V - 2)$ km/h. He took two hours longer on the return journey. Find V.

10 Sally rides to school each morning. She discovers that if she were to ride the 10 km to school at a speed 10 km/h faster than she normally rides, she would get to school 10 minutes earlier. Find Sally's normal riding speed. (Let V km/h be Sally's normal speed.)

11 Giorgia travelled 108 km and found that she could have made the journey in four and a half hours less time had she travelled at a speed 2 km/h faster. What was Giorgia's speed?

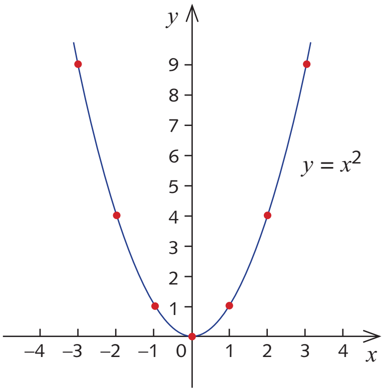
12 A, B and C do a piece of work together. A could have done it alone in six hours longer, B in 15 hours longer and C in twice the time. How long did it take all three to do the work together?

17E Graphs of quadratics

In this section we plot graphs of quadratics. First, we complete a table of values for $y = x^2$ for x between –3 and 3.

x	–3	–2	–1	0	1	2	3
$y = x^2$	9	4	1	0	1	4	9

If we plot the corresponding points and join them with a smooth curve, we obtain the graph to the right.



This graph is called a **parabola**. The point $(0, 0)$ is called the **vertex** or **turning point** of the parabola. The vertex corresponds to the minimum value of y, since x^2 is positive except when $x = 0$.

The graph is symmetrical about the y-axis (the line $x = 0$). This line is called the **axis of symmetry** of the parabola.

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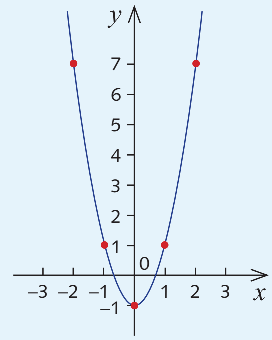
Example 12

Plot the graph of $y = 2x^2 - 1$ for $-2 \le x \le 2$. Find the axis of symmetry and the coordinates of the vertex.

Solution

A suitable table of values is:

x	–2	–1	0	1	2
$y = 2x^2 - 1$	7	1	–1	1	7



Notice that the y values are the same for $x = -1$ and $x = 1$.

The equation of the axis of symmetry is $x = 0$. When $x = 0$, $y = -1$, so the vertex has coordinates $(0, -1)$.

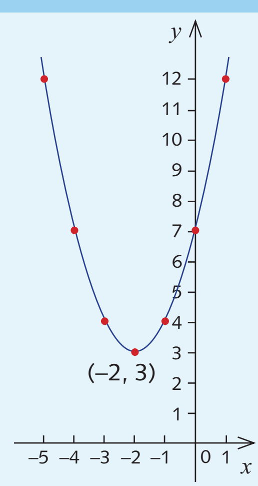
Example 13

Plot the graph of $y = x^2 + 4x + 7$ for $-5 \le x \le 1$.

Solution

A suitable table of values is:

x	–5	–4	–3	–2	–1	0	1
$y = x^2 + 4x = 7$	12	7	4	3	4	7	12



Plot the points and join them with a smooth curve. The graph shown opposite is obtained.

This parabola is just like the parabola $y = x^2$, except that it has been translated.

The equation of the axes of symmetry is $x = -2$.

The vertex has coordinates $(-2, 3)$.

Graphs of quadratics

- The graph of $y = x^2$ is called a parabola.
- The point $(0, 0)$ is called the vertex.
- The graph is symmetrical about the y-axis.

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Exercise 17E

Example 12

1 Plot the graph of each of the following for $-3 \le x \le 3$. In each case, give the coordinates of the vertex.

a $y = x^2 + 1$ b $y = x^2 - 1$ c $y = x^2 - 4$

d $y = x^2 + 2$ e $y = x^2 - 3$ f $y = x^2 + 3$

Example 13

2 Plot each of the following graphs for the given x values.

a $y = x^2 - 4x + 3,\ 0 \le x \le 4$ b $y = x^2 + 2x - 3, -4 \le x \le 2$

c $y = 2x^2 - 2, -2 \le x \le 2$ d $y = x^2 + x - 2, -3 \le x \le 2$

e $y = x^2 - 2x - 8, -3 \le x \le 5$ f $y = x^2 - 2x, -2 \le x \le 4$

g $y = x^2 + 3x, -4 \le x \le 1$ h $y = 2x^2 - 2x, -1 \le x \le 2$

3 Plot each of the following graphs for the given x values.

a $y = -x^2, -3 \le x \le 3$ b $y = 2 - x^2, -3 \le x \le 3$

c $y = x - x^2, -2 \le x \le 3$ d $y = 2x - x^2, -1 \le x \le 3$

4 a Plot the graph of $y = x^2 - x - 2$ for $-2 \le x \le 3$.

b Solve the equation $x^2 - x - 2 = 0$.

c How would you read the solution to the equation $x^2 - x - 2 = 0$ from the graph?

5 a Plot the graph of $y = x^2 - x - 6$ for $-3 \le x \le 4$.

b Solve the equation $x^2 - x - 6 = 0$.

c How would you read the solution to the equation $x^2 - x - 6 = 0$ from the graph?

6 a Plot the graph of $y = x^2 + 2x - 3$ for $-4 \le x \le 2$.

b Solve the equation $x^2 + 2x - 3 = 0$.

c How would you read the solution to the equation $x^2 + 2x - 3 = 0$ from the graph?

17F Solving quadratic equations by completing the square

In all the examples so far, the quadratic expressions factorised nicely and gave us integer or rational solutions. This is not always the case. For example, $x^2 - 7 = 0$ has solutions $x = \sqrt{7}$ and $x = -\sqrt{7}$.

Here are the steps for solving the quadratic $x^2 + 4x - 9 = 0$ using the method of completing the square.

We first complete the square on the left-hand side.

$$x^2 + 4x + 4 - 4 - 9 = 0 \quad \text{(Add and subtract the square of half the coefficient of } x.)$$
$$(x + 2)^2 - 13 = 0$$
$$(x + 2)^2 = 13$$
$$x + 2 = \sqrt{13} \text{ or } x + 2 = -\sqrt{13}$$
$$x = -2 + \sqrt{13} \text{ or } x = -2 - \sqrt{13}$$

These two numbers are the solutions to the original equation as each step is reversible. Checking by substitution is hard. It is more efficient to check your calculation.

- Quadratic equations with integer coefficients can have:
  - integer or rational solutions; for example, $4x^2 - 1 = 0$ has solutions $\frac{1}{2}$ and $-\frac{1}{2}$.
  - solutions involving square roots; for example, $x^2 - 7 = 0$ has solutions $-\sqrt{7}$ and $\sqrt{7}$.
  - no solution; for example, $x^2 + 1 = 0$ has no solutions.
- The method of completing the square enables us to solve equations whose solutions involve square roots.

Example 14

Solve:

a $x^2 - 6x - 1 = 0$

b $x^2 + 8x + 2 = 0$

Solution

a
$$x^2 - 6x - 1 = 0$$
$$x^2 - 6x + 9 - 9 - 1 = 0$$
$$(x - 3)^2 - 10 = 0$$
$$(x - 3)^2 = 10$$
$$x - 3 = \sqrt{10} \text{ or } x - 3 = -\sqrt{10}$$
$$x = 3 + \sqrt{10} \text{ or } x = 3 - \sqrt{10}$$

b
$$x^2 + 8x + 2 = 0$$
$$x^2 + 8x + 16 - 16 + 2 = 0$$
$$(x + 4)^2 - 14 = 0$$
$$(x + 4)^2 = 14$$
$$x + 4 = \sqrt{14} \text{ or } x + 4 = -\sqrt{14}$$
$$x = -4 + \sqrt{14} \text{ or } x = 4 - \sqrt{14}$$

It is not always the case that the coefficient of x is an even number. If the coefficient of x is odd, fractions will arise, but the method is the same.

Example 15

Solve $x^2 - 5x - 3 = 0$.

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Solution

$$x^2 - 5x - 3 = 0$$

$$x^2 - 5x + \frac{25}{4} - \frac{25}{4} - 3 = 0 \quad \text{(Add and subtract the square of half the coefficient of } x.)$$

$$\left(x^2 - 5x + \frac{25}{4}\right) - \frac{37}{4} = 0$$

$$x^2 - 5x + \frac{25}{4} = \frac{37}{4}$$

$$\left(x - \frac{5}{2}\right)^2 = \frac{37}{4}$$

$$x - \frac{5}{2} = \frac{\sqrt{37}}{2} \text{ or } x - \frac{5}{2} = -\frac{\sqrt{37}}{2}$$

$$x = \frac{5 + \sqrt{37}}{2} \text{ or } x = \frac{5 - \sqrt{37}}{2}$$

The method of completing the square works just as well when the roots are rational.

Example 16

Solve $x^2 - 5x + 6 = 0$.

Solution

$$x^2 - 5x + 6 = 0$$

$$\left(x^2 - 5x + \frac{25}{4}\right) - \frac{25}{4} + 6 = 0$$

$$\left(x - \frac{5}{2}\right)^2 - \frac{1}{4} = 0$$

$$x - \frac{5}{2} = \sqrt{\frac{1}{4}} \text{ or } x - \frac{5}{2} = -\sqrt{\frac{1}{4}}$$

$$x = \frac{5}{2} + \frac{1}{2} \text{ or } x = \frac{5}{2} - \frac{1}{2}$$

$$x = 3 \text{ or } x = 2$$

Alternatively

$$x^2 - 5x + 6 = 0$$

$$(x - 3)(x - 2) = 0$$

$$x - 3 = 0 \text{ or } x - 2 = 0$$

$$x = 3 \text{ or } x = 2$$

There are quadratic equations that have no solutions. Consider, for example, $x^2 - 6x + 12 = 0$:

$$x^2 - 6x + 12 = 0$$

$$(x^2 - 6x + 9) - 9 + 12 = 0$$

$$(x - 3)^2 + 3 = 0$$

$$(x - 3)^2 = -3$$

Since $(x - 3)^2 \geq 0$ for all values of x, there is no solution to the equation $(x - 3)^2 = -3$.

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Solution of quadratic equations by completing the square

- To solve a quadratic equation with the coefficient of x^2 equal to 1 by completing the square, we:
  - add and subtract the square of half the coefficient of x
  - complete the square
  - solve for x.
- The solutions of a quadratic equation often involve surds.

Exercise 17F

1 Solve the equations.

a $x^2 - 5 = 0$ **b** $x^2 - 11 = 0$ **c** $x^2 - 12 = 0$

d $2x^2 - 6 = 0$ **e** $50 - 5x^2 = 0$ **f** $40 - 8x^2 = 0$

g $(x - 2)^2 = 5$ **h** $(x + 3)^2 = 6$ **i** $(x + 2)^2 = 8$

j $\left(x + \frac{7}{2}\right)^2 = 9$ **k** $(x - 2)^2 = 16$ **l** $(x + 4)^2 - 7 = 0$

m $\left(x - \frac{11}{2}\right)^2 - \frac{7}{4} = 0$ **n** $\left(x - \frac{1}{2}\right)^2 = \frac{11}{4}$ **o** $\left(x + \frac{15}{4}\right)^2 - \frac{7}{4} = 0$

Example 14

2 Solve the equations by completing the square.

a $x^2 + 2x - 1 = 0$ **b** $x^2 + 4x + 1 = 0$ **c** $x^2 - 12x + 23 = 0$

d $x^2 + 6x + 7 = 0$ **e** $x^2 - 8x - 1 = 0$ **f** $x^2 + 10x + 12 = 0$

g $x^2 + 8x + 6 = 0$ **h** $x^2 + 6x - 1 = 0$ **i** $x^2 + 6x + 1 = 0$

j $x^2 + 8x - 2 = 0$ **k** $x^2 + 8x + 2 = 0$ **l** $x^2 + 20x - 20 = 0$

Example 15

3 Solve the equations by completing the square.

a $x^2 + x - 1 = 0$ **b** $x^2 - 3x + 1 = 0$ **c** $x^2 - 5x - 1 = 0$

d $x^2 + 3x - 2 = 0$ **e** $x^2 + 5x + 1 = 0$ **f** $x^2 - 6x + 2 = 0$

g $x^2 + 10x + 23 = 0$ **h** $x^2 + 9x + 4 = 0$ **i** $x^2 + 3x - 11 = 0$

j $x^2 + 11x + 10 = 0$ **k** $x^2 + 7x + 10 = 0$ **l** $x^2 + 7x - 5 = 0$

4 For each equation, complete the square and show that the equation has no solution.

a $x^2 + 8x + 18 = 0$ **b** $x^2 - 4x + 6 = 0$ **c** $x^2 + 2x + 4 = 0$

d $x^2 + x + 2 = 0$ **e** $x^2 + 2x + 2 = 0$ **f** $x^2 + 4x + 5 = 0$

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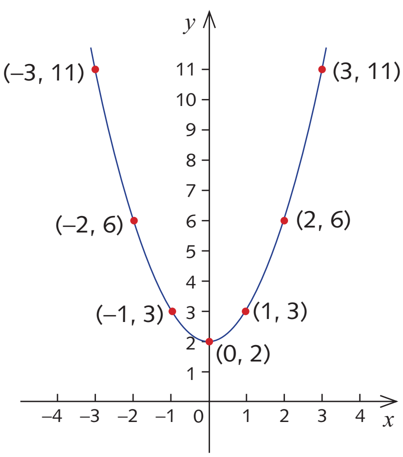
17G Sketching graphs of the form $y = x^2 + bx + c$

In Section 17E we plotted parabolas. Important features of a parabola include the vertex or turning point and the axis of symmetry. The graphs were plotted using a set of suggested x-values. In all cases the vertex occurred midway between the greatest x-value and the least x-value. What can we do when these hints are not provided?

Consider the following graphs. A connection exists between the equation and the location of the vertex of its corresponding graph.

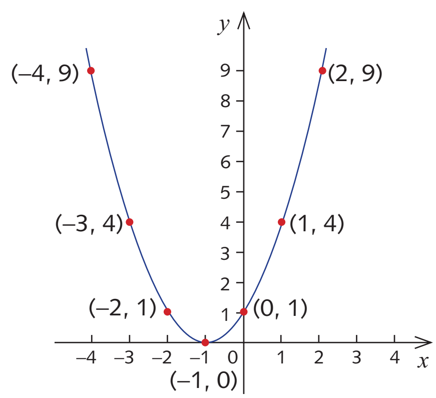
$y = x^2 + 2$

x	–3	–2	–1	0	1	2	3
y	11	6	3	2	3	6	11



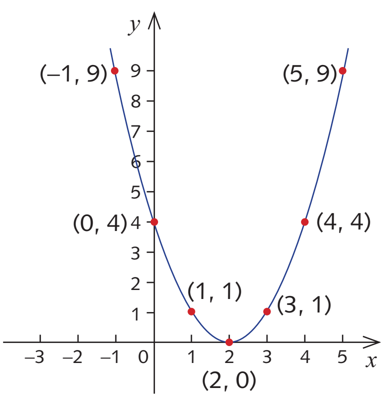
$y = (x + 1)^2$

x	–4	–3	–2	–1	0	1	2
y	9	4	1	0	1	4	9



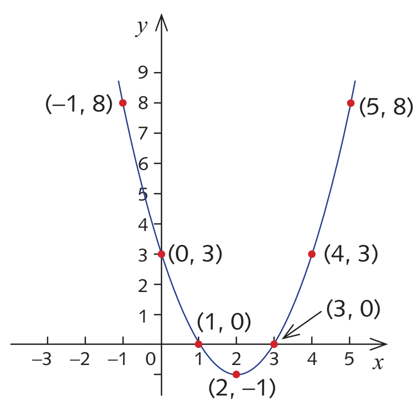
$y = (x - 2)^2$

x	–1	0	1	2	3	4	5
y	9	4	1	0	1	4	9



$y = (x - 2)^2 - 1$

x	–1	0	1	2	3	4	5
y	8	3	0	–1	0	3	8



The connection between rule and graph, evident above, is summarised below.

Graphs of the form $y = (x - h)^2 + k$ are parabolas with a vertex, or turning point, at (h, k) and axis of symmetry, $x = h$.

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These graphs are congruent to the shape of the basic parabola $y = x^2$, but have been subjected to a horizontal shift of h units and vertical shift of k units. These types of graph transformations are called 'translations'. A formal treatment of translations, and other graph transformations, is given in *ICE-EM Mathematics Year 10*.

Example 17

Find the turning point and axis of symmetry for the following quadratic graphs.

a $y = (x + 3)^2$

b $y = x^2 - 1$

c $y = (x - 1)^2 - 2$

d $y = (x + 1)^2 + 3$

Solution

a $y = (x + 3)^2$

$= (x - (-3))^2 + 0$

Turning point is $(-3,\ 0)$.
Axis of symmetry is $x = -3$.

b $y = x^2 - 1$

$= (x - 0)^2 - 1$

Turning point is $(0,\ -1)$.
Axis of symmetry is $x = 0$ (the y-axis).

c $y = (x - 1)^2 - 2$

Turning point is $(1, -2)$.
Axis of symmetry is $x = 1$.

d $y = (x + 1)^2 + 3$

$= (x - (-1))^2 + 3$

Turning point is $(-1,\ 3)$.
Axis of symmetry is $x = -1$.

By completing the square, any quadratic equation of the form $y = x^2 + bx + c$ can be converted to the form $y = (x - h)^2 + k$.

We begin the process by adding then subtracting $\left(\frac{b}{2}\right)^2$.

Example 18

Convert the following to the form $y = (x - h)^2 + k$.

a $y = x^2 + 4x + 1$

b $y = x^2 - 3x + 4$

Solution

a $y = x^2 + 4x + 1$

$= \left(x^2 + 4x + \left(\frac{4}{2}\right)^2\right) - \left(\frac{4}{2}\right)^2 + 1$

$= (x^2 + 4x + 4) - 4 + 1$

$= (x + 2)^2 - 3$

b $y = x^2 - 3x + 4$

$= \left(x^2 - 3x + \left(\frac{-3}{2}\right)^2\right) - \left(\frac{-3}{2}\right)^2 + 4$

$= \left(x^2 - 3x + \frac{9}{4}\right) - \frac{9}{4} + 4$

$= \left(x - \frac{3}{2}\right)^2 + \frac{7}{4}$

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Parabola sketching

Any quadratic graph should be sketched with the following features labelled:

1 the vertex (or the turning point)
2 the y-intercept
3 the x-intercept(s), where they exist.

In Chapter 11, we found the axis intercepts of linear graphs. The y-intercept was found by substituting $x = 0$ into the rule and solving for y. The x-intercept was found by substituting $y = 0$ into the rule and solving for x. This is no different for quadratic rules, or indeed the equation for any other type of graph.

Example 19

Sketch the following quadratic graphs.

a $y = (x + 2)^2 + 3$ **b** $y = (x - 1)^2 - 3$ **c** $x^2 + 4x - 5$

Solution

a $y = (x + 2)^2 + 3$

The turning point is $(-2, 3)$.

When $x = 0$,

$y = (0 + 2)^2 + 3$
$= 4 + 3$
$= 7$

$(0, 7)$ is the y-intercept.

There are no x-intercepts.

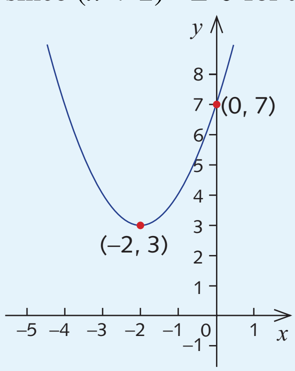
This can be confirmed algebraically.

When $y = 0$

$0 = (x + 2)^2 + 3$

$(x + 2)^2 = -3$

But this equation has no solutions since $(x + 2)^2 \geq 0$ for all x.



b $y = (x - 1)^2 - 3$

The turning point is $(1, -3)$.

When $x = 0$,

$y = (0 - 1)^2 - 3$
$= 1 - 3$
$= -2$

$(0, -2)$ is the y-intercept.

When $y = 0$,

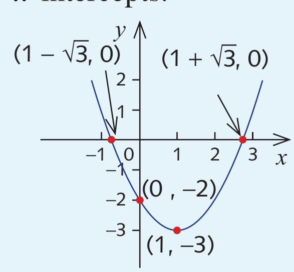
$0 = (x - 1)^2 - 3$

$(x - 1)^2 = 3$

$\therefore x - 1 = \sqrt{3}$ or $x - 1 = -\sqrt{3}$

$\therefore x = 1 + \sqrt{3}$ or $x = 1 - \sqrt{3}$

$(1 + \sqrt{3}, 0)$ and $(1 - \sqrt{3}, 0)$ are x-intercepts.



Note: $1 - \sqrt{3} \approx -0.7$. Use your calculator to find the approximate values for any x-intercepts involving surds.

(continued over page)

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c $y = x^2 + 4x - 5$ (Convert to the form $y = (x - h)^2 + k$.)

$$y = x^2 + 4x + \left(\frac{4}{2}\right)^2 - \left(\frac{4}{2}\right)^2 - 5$$

$$= (x^2 + 4x + 4) - 4 - 5$$

$$= (x + 2)^2 - 9$$

The turning point is $(-2, -9)$.

When $x = 0$, $y = -5$.

The y-intercept is $(0, -5)$.

When $y = 0$,

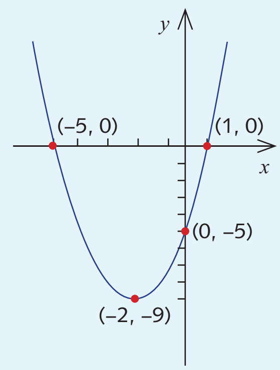
$0 = (x + 2)^2 - 9$

$\therefore (x + 2)^2 = 9$

$\therefore x + 2 = 3$ or $x + 2 = -3$

$\therefore x = 1$ or $x = -5$

The x-intercepts are $(1, 0)$ and $(-5, 0)$.



Exercise 17G

Example 17

1 Find the turning point and axis of symmetry for the following quadratic graphs.

a $y = x^2 - 1$ **b** $y = (x - 1)^2$ **c** $y = (x + 2)^2 - 1$

d $y = x^2 + 4$ **e** $y = (x + 3)^2$ **f** $y = (x + 1)^2 + 2$

g $y = (x - 2)^2 - 2$ **h** $y = 3 + (x - 1)^2$ **i** $y = -2 + (x - 3)^2$

Example 18

2 Convert the following to the form $y = (x - h)^2 + k$.

a $y = x^2 - 4x + 2$ **b** $y = x^2 + 8x + 21$ **c** $y = x^2 - 6x - 15$

d $y = x^2 + 5x - 1$ **e** $y = x^2 - 3x + 5$ **f** $y = x^2 - 7x - 10$

3 Solve the following equations, where solution(s) exist.

a $x^2 - 4 = 0$ **b** $x^2 - 11 = 0$ **c** $(x + 1)^2 - 4 = 0$

d $(x + 3)^2 + 5 = 0$ **e** $(x + 2)^2 - 9 = 0$ **f** $(x - 3)^2 - 5 = 0$

g $(x + 1)^2 - 12 = 0$ **h** $(x - 7)^2 + 1 = 0$ **i** $(x + 4)^2 - 18 = 0$

j $x^2 + 3 = 0$ **k** $25 - (x - 3)^2 = 0$ **l** $\left(x - \frac{7}{2}\right)^2 - \frac{25}{4} = 0$

m $\left(x + \frac{3}{2}\right)^2 - \frac{7}{4} = 0$ **n** $\left(x - \frac{5}{3}\right)^2 - \frac{40}{9} = 0$ **o** $\left(x + \frac{3}{4}\right)^2 + \frac{9}{4} = 0$

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Example 19a, b

4 Sketch the following parabolas, labelling the turning point and any intercepts.

a $y = (x + 1)^2 + 4$ b $y = (x - 3)^2$ c $y = x^2 - 5$

d $y = x^2 + 1$ e $y = (x - 3)^2 + 1$ f $y = (x - 2)^2 - 1$

g $y = (x + 2)^2 - 4$ h $y = (x + 1)^2 - 9$ i $y = (x + 1)^2 - 3$

j $y = (x + 4)^2$ k $y = \left(x - \frac{3}{2}\right)^2 - \frac{25}{4}$ l $y = \left(x + \frac{7}{3}\right)^2 - \frac{16}{9}$

Example 19c

5 Sketch the following parabolas, labelling the turning points and any intercepts.

a $y = x^2 + 2x + 6$ b $y = x^2 - 6x + 10$ c $y = x^2 - 4x + 7$

d $y = x^2 + 2x - 3$ e $y = x^2 + 4x + 4$ f $y = x^2 - 8x - 20$

g $y = x^2 + 2x - 1$ h $y = x^2 - 6x - 3$ i $y = x^2 + 10x + 20$

j $y = x^2 + 5x - 6$ k $y = x^2 - 3x - 10$ l $y = x^2 - x + 1$

m $y = x^2 + 3x + 4$ n $y = x^2 - 5x + 3$ o $y = x^2 + 7x + 2$

Review exercise

1 Solve the equations.

a $(x - 5)(x + 3) = 0$ b $x(x - 5) = 0$ c $x(x + 7) = 0$

d $(2x - 8)(x - 8) = 0$ e $(3x + 8)(x + 13) = 0$ f $(7 - 2x)(11x - 7) = 0$

2 Solve the equations.

a $x^2 - 3x + 2 = 0$ b $x^2 - 7x + 12 = 0$ c $x^2 - 11x + 10 = 0$

d $x^2 + 29x - 30 = 0$ e $x^2 - 5x - 14 = 0$ f $x^2 - x - 90 = 0$

g $x^2 - 5x - 24 = 0$ h $x^2 - 11x + 18 = 0$ i $x^2 - x - 12 = 0$

j $x^2 - 11x + 28 = 0$ k $x^2 + 9x - 10 = 0$ l $x^2 + x - 110 = 0$

3 Solve the equations.

a $x^2 + 18x + 81 = 0$ b $9x^2 - 16 = 0$ c $4x - x^2 = 0$

d $3x^2 - 12x - 36 = 0$ e $x^2 - 8x + 16 = 0$ f $9f^2 - 36f + 11 = 0$

g $12y^2 + 21 = -32y$ h $7x^2 = 28$ i $x^2 - 64 = 0$

4 Solve the equations.

a $5d^2 - 10 = 0$ b $\frac{2y^2}{3} - 5 = 0$ c $\frac{3(x - 10)^2}{5} - 12 = 0$

d $y^2 - 8y + 3 = 0$ e $1 = m^2 - m$ f $3n + 3 = n^2$

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5 Plot each graph for the x-values specified.

a $y = x^2 - 2x - 3, \ -2 \le x \le 4$ **b** $y = x^2 + 2x - 3, \ -4 \le x \le 2$

c $y = x^2 - 3, \ -2 \le x \le 2$ **d** $y = x^2 + 2x - 8, \ -5 \le x \le 3$

6 Solve the equations.

a $x^2 - \frac{8x}{3} = 1$ **b** $x + \frac{8x}{3} - \frac{13}{6} = 0$ **c** $\frac{7}{3x - 4} - \frac{2}{x + 2} = 1$

7 Solve the equations by completing the square.

a $x^2 + 6x - 2 = 0$ **b** $x^2 - 4x - 4 = 0$ **c** $x^2 - 10x + 20 = 0$

d $x^2 + 5x + 3 = 0$ **e** $x^2 - 7x + 5 = 0$ **f** $x^2 + 3x - 7 = 0$

8 Solve the equations.

a $x^2 + 3x - 40 = 0$ **b** $x^2 - 3x - 40 = 0$ **c** $x^2 + 4x - 12 = 0$

d $x^2 + 4x + 3 = 0$ **e** $x^2 - 12x + 32 = 0$ **f** $x^2 - 14x + 32 = 0$

g $x^2 + 8x + 15 = 0$ **h** $x^2 + 8x + 14 = 0$ **i** $x^2 - 12x - 27 = 0$

j $x^2 - 12x - 20 = 0$ **k** $x^2 + 7x + 5 = 0$ **l** $x^2 + 5x - 10 = 0$

9 Solve the equations.

a $\frac{3x + 1}{4x + 7} = 1 - \frac{6}{x + 7}$ **b** $\frac{4}{x - 1} - \frac{5}{x + 2} = \frac{3}{x}$

c $\frac{x + 4}{x - 4} + \frac{x - 3}{x + 3} = \frac{16}{3}$ **d** $\frac{x + 3}{2x - 7} = \frac{2x - 1}{x - 3}$

10 Sketch the graphs, labelling all key features.

a $y = x^2 - 3$ **b** $y = (x + 1)^2$ **c** $y = (x - 1)^2 - 9$

d $y = x^2 - 2x + 4$ **e** $y = x^2 - 4x + 3$ **f** $y = x^2 + 6x + 4$

11 Find two numbers, the sum of whose squares is 74 and whose sum is 12.

12 The perimeter of a rectangular field is 500 m and its area is 14 400 m^2. Find the lengths of the sides.

13 The base and height of a triangle are $x + 3$ and $2x - 5$. If the area of the triangle is 20, find x.

14 Two positive numbers differ by 7 and the sum of their squares is 169. Find the numbers.

15 A rectangular field, 70 m long and 50 m wide, has a path of uniform width around it. If the area of the path is 1024 m^2, find the width of the path.

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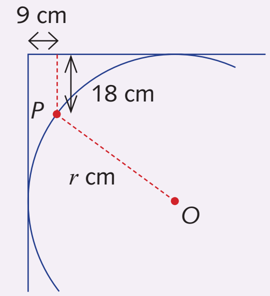
Challenge exercise

1 Solve the following:

a $\dfrac{x-\sqrt{x+1}}{x+\sqrt{x+1}}=\dfrac{11}{5}$ **b** $\dfrac{1}{2x-1}=\dfrac{2x+1}{2x^2+5x-3}$

2 A table with a circular top is placed in the corner of a rectangular room so that it touches the two walls. A point, P, on the edge of the table, as shown in the opposite diagram, is 18 cm from one wall and 9 cm from the other wall.



a If the radius of the table top is r cm, show that r satisfies the equation $r^2-54r+405=0$.

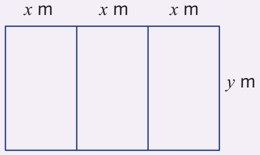
b Find the radius of the table.

c How far is the centre of the table from the corner of the room?

3 While wrapping his son's Christmas present, which is in the shape of a rectangular box, a parent notices that if the length of the box was increased by 2 cm, the width increased by 3 cm and the height increased by 4 cm, the box would become a cube and its volume would be increased by 1207 cm^3.

Find the length of the edge of the resulting cube and the dimensions of the box.

4 A gardener decides to subdivide a rectangular garden bed of area 30 m^2 into three equal sections.

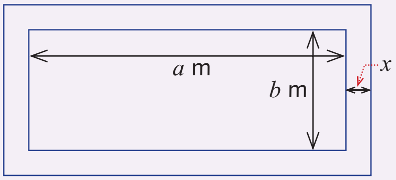


He places edging along the outside of the garden bed and as dividers between each section. It takes 32 m of edging. Each section of garden has length y metres (which is also the length of each divider) and width x metres.

a Express the area of the entire garden bed in terms of x.

b Find the dimensions of the garden bed.

5 A rectangular lawn a metres long and b metres wide has a path of uniform width x metres around it.



a Find the area of the path in terms of a, b and x.

b Let $a=28$ and $b=50$.

i Find the area of the path in terms of x.

ii If the area of the path is 160 m^2, find the value of x.

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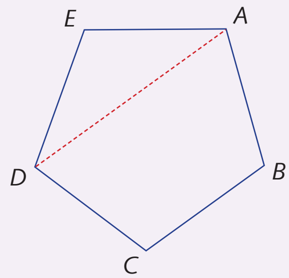
6 A diagonal of a polygon is a line segment joining a vertex to a non-adjacent vertex. In the diagram below, AD is a diagonal of the polygon $ABCDE$.



a How many diagonals has:

i a quadrilateral? **ii** a pentagon? **iii** a hexagon?

iv a decagon? **v** a 20-sided polygon?

b Explain why the number of diagonals in an n-sided polygon is $\frac{n(n-3)}{2}$.

c A particular polygon has 405 diagonals. How many sides does it have?

d What polygon has between 1900 and 2000 diagonals?

7 At a party, each guest shakes hands with each other guest.

a How many hand shakes are there if there are:

i 2 guests? **ii** 3 guests? **iii** 4 guests? **iv** 10 guests?

b At a party with n guests, explain why there are $\frac{n(n-1)}{2}$ hand shakes.

c At Bill's party there were 190 hand shakes. How many people were at the party?

8 A right-angled triangle has a hypotenuse of length 41 cm and an area of 180 cm^2. What are the lengths of the other two sides?

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