CHAPTER

20

Review and problem-solving

ICE-EM Mathematics 9 3ed ISBN 978-1-108-40432-7

20A Review

Chapter 11: Coordinate geometry

1 Find the distance between the points with the given coordinates.

a $(3,\ 2)$ and $(5,\ 10)$ **b** $(2,\ 3)$ and $(5,\ 12)$ **c** $(-1,\ 3)$ and $(0,\ 6)$

d $(3, -2)$ and $(5,\ 0)$ **e** $(-2,\ 9)$ and $(3, -1)$ **f** $(2,\ 4)$ and $(5, -2)$

2 Find the coordinates of the midpoint of the interval with the given endpoints.

a $(3,\ 2)$ and $(5,\ 10)$ **b** $(2,\ 3)$ and $(5,\ 12)$ **c** $(-1,\ 3)$ and $(0,\ 6)$

d $(3, -2)$ and $(5,\ 0)$ **e** $(-2,\ 9)$ and $(3, -1)$ **f** $(2,\ 4)$ and $(5, -2)$

3 Find the gradient of the line that passes through each pair of points.

a $(3,\ 2)$ and $(5,\ 10)$ **b** $(2,\ 3)$ and $(5,\ 12)$ **c** $(-1,\ 3)$ and $(0,\ 6)$

d $(3, -2)$ and $(5,\ 0)$ **e** $(-2,\ 9)$ and $(3, -1)$ **f** $(2,\ 4)$ and $(5, -2)$

4 A line has gradient 2 and passes through the point $(1,\ 3)$.

a Use this information to fill in the table of values below for points $(x,\ y)$ on the line.

x	0	1	2	3	4
y		3			

b Find the equation of the line.

5 A line has gradient $-\frac{1}{2}$ and passes through the point $(2,\ 6)$.

a Use this information to fill in the table of values below.

x	0	1	2	3	4
y			6		

b Find the equation of the line.

6 A line has gradient 2 and passes through the point $(-1,\ 6)$. Find the equation of the line.

7 A line passes through the points $(2,\ 4)$ and $(5,\ 6)$.

a Find the gradient of the line.

b Find the equation of the line.

8 Sketch the graph and label the x- and y-intercepts for the following lines.

a $y = 2x - 1$ **b** $y = 3x + 2$ **c** $2x - 3y = 6$ **d** $x + 4y = 5$

9 Find the gradient, x-intercept and y-intercept of the following lines.

a $y = 5x - 3$ **b** $y = 4 - x$ **c** $3x - 4y = 12$ **d** $5x + 2y = 4$

10 **a** Find the equation of the line through the point $(3,\ 2)$ parallel to the line with equation $y = 2x + 1$.

b Find the equation of the line through the point $(3,\ 2)$ perpendicular to the line with equation $y = 2x + 1$.

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Chapter 12: Probability

1 A letter is chosen at random from the word RANDOM.

a List the sample space for this experiment.

b i If B is the event 'the letter chosen is a vowel', write down the outcomes favourable to B.

ii What is the probability of B occurring?

c Find the probability that the letter chosen:

i is a consonant

ii comes after K in the alphabet

iii comes before H in the alphabet

iv has an axis of symmetry

2 The numbers 1 to 25 are written on 25 cards. If a card is selected at random, find the probability that the number on the card is:

a 14

b greater than 20

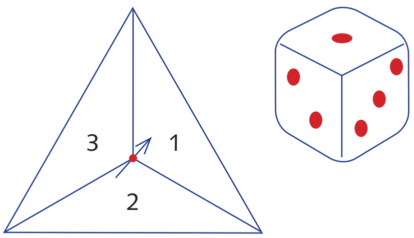
c even

d a multiple of 5

e odd and a multiple of 3

f neither even nor a multiple of 5

3 In a game, a three-sided spinner is spun and a die is rolled. The two numbers obtained are added together.



a Draw an array to show all the possible scores.

b If the spinner is spun and the dice is rolled, find the probability that the score obtained is:

i 7

ii greater than 5

iii odd

iv a multiple of 3

4 Two students, Annie and Bianca, each choose a number between 1 and 5 inclusive. By listing all the possible outcomes, find the probability that:

a they both choose an even number

b Bianca chooses a larger number than Annie

c the product of their numbers is greater than 10

d the sum of their numbers is less than 6

5 Two fair dice are rolled and the uppermost numbers are noted.

a Represent the sample space by drawing an array and listing all the possible outcomes.

b Find the probability of obtaining:

i a double 6

ii at least one 6

iii a double of any number

iv a total of 10 or more

6 A fair coin is tossed four times.

a Draw a tree diagram to represent the outcomes of this experiment and list the sample space.

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b Find the probability of getting:

i four tails **ii** three tails and one head **iii** fewer than three tails

7 In a group of 200 students, 60 study geography, 80 study economics and 70 study neither.

a Represent this information in a Venn diagram.

b If a student is selected at random from the group, what is the probability that the student studies:

i geography? **ii** geography and economics?

iii at least one of these subjects?

8 A girl performs the following experiment. She draws a marble from a bag containing several different coloured marbles and notes its colour. She then replaces the marble. She repeats the experiment 50 times and keeps tally of the number of marbles of each colour. Her results are given below.

Colour	Red	Blue	Yellow	Green
Number drawn	6	17	19	8

a Using the table, estimate the probability that the next marble drawn is:

i red **ii** blue **iii** not green **iv** yellow or blue

b Which of the following statements do you think are true? Justify your answer.

i The bag contains 50 marbles.

ii The bag contains 4 marbles.

iii The bag contains twice as many blue marbles as green marbles.

iv The bag contains equal numbers of blue and yellow marbles.

v The bag contains only red, blue, yellow and green marbles.

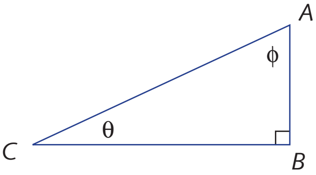
Chapter 13: Trigonometry

1 In the triangle opposite, name:

a the hypotenuse **b** the side opposite θ

c the side opposite ϕ **d** the side adjacent to θ

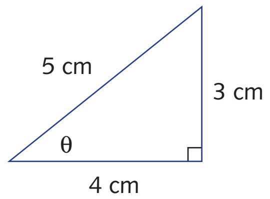
e the side adjacent to ϕ



2 In each of the following triangles, write down the value of:

i $\sin\theta$ **ii** $\cos\theta$ **iii** $\tan\theta$

a



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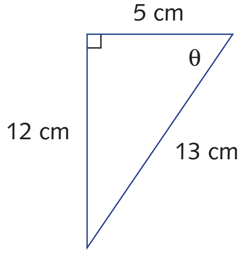

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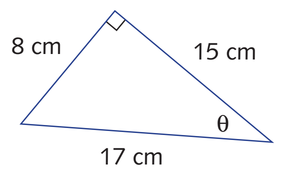

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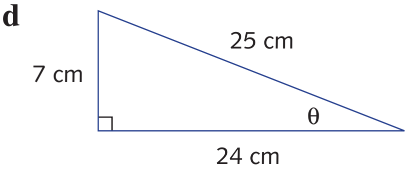


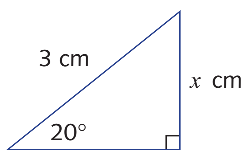
3 Find, correct to 4 decimal places:

a cos 15° **b** sin 86° **c** tan 64°

4 Find θ, to the nearest degree:

a $\tan \theta = 2$ **b** $\sin \theta = 0.2$ **c** $\cos \theta = 0.34$

5 Find the values of the pronumerals, correct to 2 decimal places.

a


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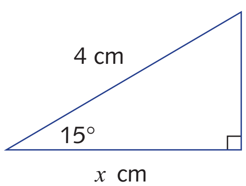

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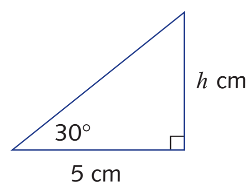

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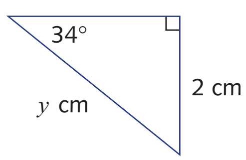

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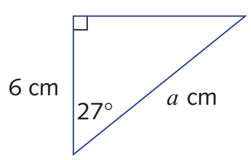

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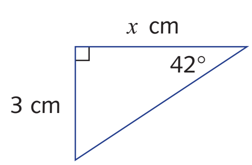
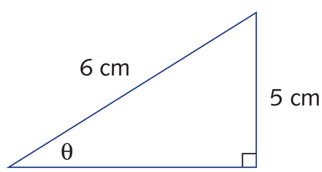


6 Find θ, correct to the nearest degree.

a


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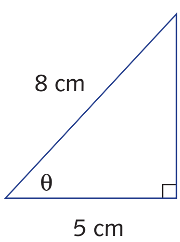

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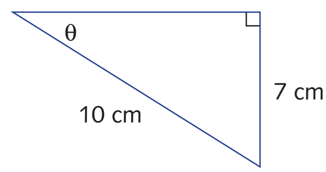

7 A road makes an angle of 6° with the horizontal. How much does it rise over a distance of 800 m along the road? Give your answer to the nearest metre.

8 A straight slide is 3.5 m long and has a vertical ladder 2.1 m high. Find the angle the slide makes with the ground, giving your answer to the nearest degree.

9 A ladder of length 4.5 m is leaning against a vertical wall. If the top of the ladder is 3 m above the ground, find:

a the angle the ladder makes with the wall, giving your answer to the nearest degree

b the distance the foot of the ladder is from the wall, giving your answer to the nearest centimetre.

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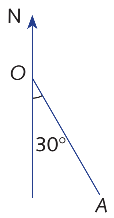
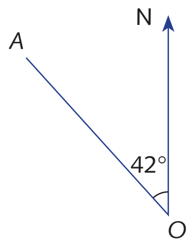
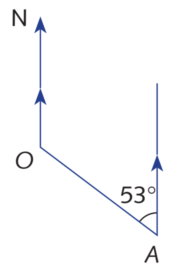
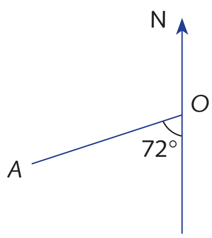
10 Give the true bearing of A from O.

a 

b 

c 

d 

11 A plane flies on a bearing of 142°T for 400 km. How far south of its starting point is the plane now? (Give your answer in kilometres, correct to 1 decimal place.)

12 A ship travels on a bearing of 320°T until it is 50 km west of its starting point.

a How far did it travel? Give your answer correct to 1 decimal place.

b How far north of its starting point is it? Give your answer correct to 1 decimal place.

13 A man is watching a ship from the top of a scenic lookout. His eye level is 30 m above sea level. If the angle of depression from the man to the ship is 8°, how far from the foot of the lookout is the ship? Give your answer to the nearest metre.

14 A bird-watcher is lying on the ground 15 m from a vertical tree that contains a bird's nest. If the nest is 8 m up the tree, what is the angle of elevation from the watcher to the nest? Give your answer to the nearest degree.

15 The angle of depression of a yacht from the top of the cliff is 44°. If the yacht sails 200 m further away from the cliff, the angle of depression is now 38°. How high is the cliff? Give your answer correct to the nearest metre.

Chapter 14: Simultaneous linear equations

1 Find the coordinates of the point of intersection of the lines $y = 2x - 4$ and $x + y = 7$. Sketch the graphs, labelling the point of intersection and the x- and y-intercepts of each line.

2 Use the substitution method to solve the following pairs of simultaneous equations.

a $y = 3x - 5$
$x + y = 3$

b $x - 2y = 7$
$x = 3y + 1$

c $y = 3 - 5x$
$y = 4x - 3$

3 Use the elimination method to solve the following pairs of simultaneous equations.

a $2x + y = 4$
$x - y = 5$

b $5x - 2y = 3$
$x - y = 2$

c $2x + 3y = 8$
$3x - 4y = 10$

d $5x - 4y = 13$
$2x - 7y = 6$

4 The equations of two lines are $y = mx + 7$ and $2x - 5y = 10$.

a For what value of m do the two lines not intersect?

b For what values of m do the two lines intersect?

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5 The lines with equations $y = 3x - k$ and $y = mx - 2$ intersect at $(2, 3)$. Find m and k.

6 Find the area of the triangle formed by the lines $x + 3y = 6$, $y = 5 - x$ and:

a the x-axis

b the y-axis

7 In a farmyard containing pigs and chickens, there are 40 heads and 144 legs. How many pigs are in the yard?

8 Tibor is currently 7 times his son's age. In 8 years' time, he will be 3 times his son's age. What is Tibor's current age?

9 The lines with equations $y = x + 2$, $y = 4 - x$ and $y = 2x + c$ are concurrent. Find the point of concurrency and hence find c.

10 Solve the following simultaneous equations.

a $ax + by = c$ $\quad (a^2 + b^2 \neq 0)$
$bx - ay = d$

b $2ax + y = c$
$by + x = d$ $\quad \left(ab \neq \frac{1}{2}\right)$

Chapter 15: Further factorisation

1 Factorise:

a $4x - 100$ b $7x^2 - 49$ c $2y - 16y^2$ d $10a - 16a^2$

2 Factorise:

a $x^2 - 100$ b $x^2 - 49$ c $9x^2 - 16y^2$ d $1 - 16a^2$

3 Factorise:

a $3x^2 - 27$ b $2x^2 - 8$ c $18x^2 - 32y^2$ d $2 - 18a^2$

4 Factorise:

a $x^2y^2 - 4$ b $x^2 - 64y^2$ c $8y^2 - 32$ d $2a^2 - 32$

5 Factorise:

a $(x + 2)^2 - 4$ b $(y + 2)^2 - 9$ c $3(x + 2)^2 - 27$ d $(a - 1)^2 - 1$

6 Factorise:

a $x^2 + 7x + 12$ b $x^2 + 9x + 18$ c $x^2 - 5x - 6$

d $x^2 + 3x - 28$ e $x^2 - 11x + 30$ f $x^2 - 14x + 24$

g $x^2 + 3x - 70$ h $x^2 - 6x - 55$ i $3x^2 + 6x + 9$

j $4x^2 - 8x + 12$ k $-x^2 - x + 6$ l $-2x^2 - 3x + 44$

7 Factorise by grouping:

a $8xy + 2y + 12x + 3$

b $6ax + 15x - 4a - 10$

c $6ac + 2bd - 3bc - 4ad$

d $2m^2 - mp + 6mn - 3np$

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8 Factorise:

a $2x^2 + 9x + 10$ **b** $6x^2 + 19x + 10$ **c** $6x^2 + 13x - 5$

d $8x^2 - 10x - 3$ **e** $21x^2 - 53x - 8$ **f** $6x^2 - 7x - 20$

g $4x^2 - 12x + 9$ **h** $9x^2 + 24x + 16$ **i** $4x^2 - 8x - 12$

9 Simplify:

a $\frac{1}{(x-1)^2} \div \frac{1}{x^2-1}$ **b** $\frac{x-4}{x^2+2x+1} \times \frac{x+1}{x^2-16}$

c $\frac{m-2}{4m} \times \frac{m}{m-2}$ **d** $\frac{p+1}{8(p-1)} \times \frac{4(p-1)}{(p+1)(p+2)}$

e $\frac{4}{a} \div \frac{2}{a^2}$ **f** $\frac{5a-7}{2a+4} \times \frac{12}{10a-14}$

g $\frac{2x^2+3x-2}{(2x-1)^2}$ **h** $\frac{(x+3)(x-2)}{x+6} \div \frac{x+3}{x+6}$

i $\frac{x^2+3x-4}{2x-4} \times \frac{6x-12}{x-1}$ **j** $\frac{2x^2-6x}{x^2-1} \div \frac{x^2-x-6}{x^2-x-2}$

10 Express each of the following as a single fraction.

a $\frac{x+1}{4} + \frac{x+3}{3}$ **b** $\frac{x-2}{2} + \frac{x-1}{3}$ **c** $\frac{2x+1}{3} - \frac{x+1}{4}$ **d** $\frac{3x-1}{4} - \frac{2x-1}{6}$

11 Express the following as a single fraction.

a $\frac{2}{x} + \frac{1}{x+1}$ **b** $\frac{4}{x-1} + \frac{2}{x+2}$ **c** $\frac{x}{x-1} + \frac{2x}{x+3}$

d $\frac{x+2}{x+3} - \frac{x-1}{x-2}$ **e** $\frac{2}{2x+1} - \frac{x-4}{x-3}$ **f** $\frac{x-1}{3x+2} - \frac{x-2}{2x-1}$

12 Express the following as a single fraction.

a $\frac{4}{x} + \frac{3}{x^2}$ **b** $\frac{5}{x+1} - \frac{2}{(x+1)(x+2)}$

c $\frac{2}{x+3} + \frac{x+2}{x^2-9}$ **d** $\frac{3}{x(x-1)} + \frac{2}{x(x+1)}$

e $\frac{4}{x^2+4x+4} - \frac{2}{x^2+x-2}$ **f** $\frac{3}{x^2-x-6} + \frac{4}{x^2-4x+3}$

g $\frac{x}{2-x} + \frac{2}{x^2-4}$ **h** $\frac{3}{x^2-1} - \frac{2}{1-x}$

13 Complete the square, expressing each of the following in the form $(x+h)^2 + k$.

a $x^2 + 4x + 2$ **b** $x^2 + 6x + 6$ **c** $x^2 - 2x - 6$ **d** $x^2 + 3x - 8$

e $x^2 - x - 1$ **f** $x^2 - 10x + 20$ **g** $x^2 + 3x - 7$ **h** $x^2 - 6x - 5$

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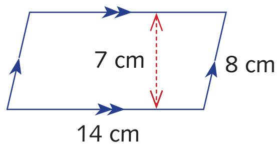
14 Factorise by first competing the square (surds could be involved in the final expression).

a $x^2 + 4x - 1$ **b** $x^2 + 2x - 2$ **c** $x^2 - 6x + 1$ **d** $x^2 + 5x + 2$

Chapter 16: Measurement – areas, volumes and time

1 Calculate the area of the following shapes.

a



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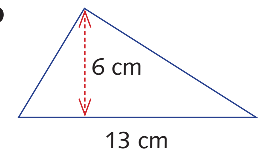
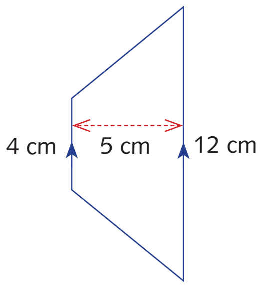


c



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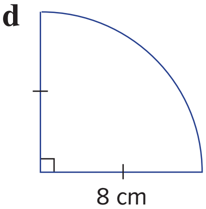
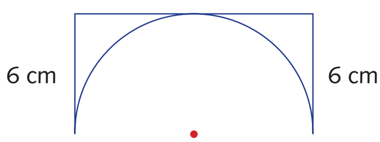


2 Calculate the area of the following shapes.

a



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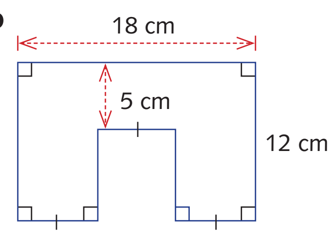

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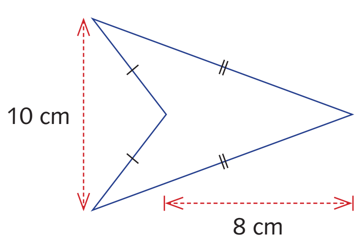

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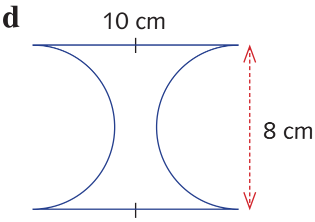
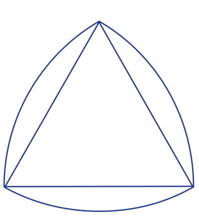


3 A company logo consists of an equilateral triangle with an arc of a circle drawn on each side using the opposite vertex as centre. If the length of each side of the equilateral triangle is 6 cm, find:



a the height of the triangle (giving an exact answer)

b the area of the logo, correct to 2 decimal places

4 Calculate the surface area of:

a a cube with side length 1.2 m

b a rectangular prism with side lengths 5 cm, 4 cm and 2 cm

c a cylinder with radius 4 cm and height 5 cm, correct to 1 decimal place

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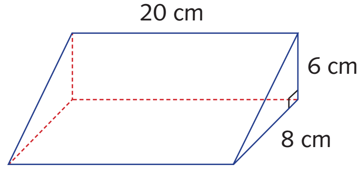
5 Calculate the surface area of the following shapes.

a



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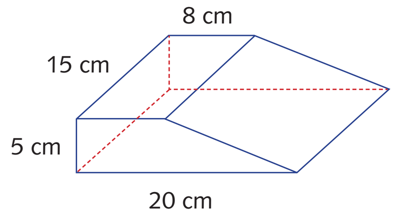
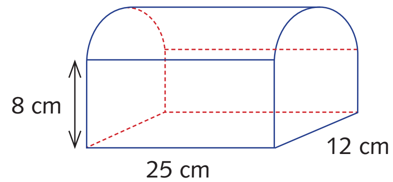
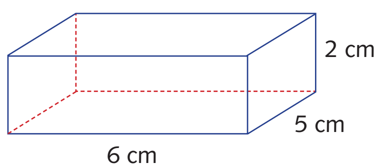


6 A tool box has the shape shown below. Calculate its surface area, correct to 1 decimal place.

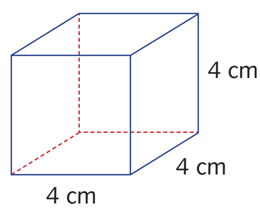


7 Calculate the volume of each of the following solids.

a



b



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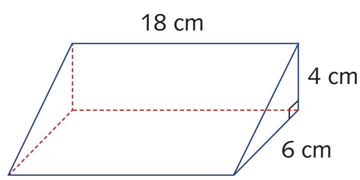

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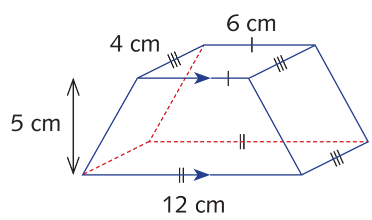

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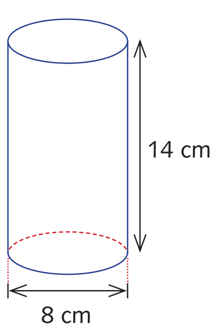

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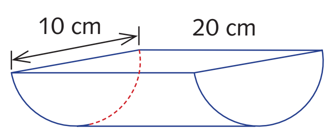

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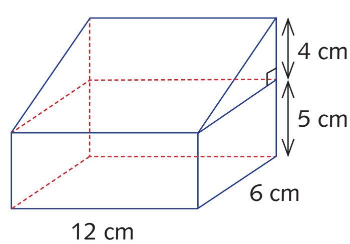

8 A child's pool is in the shape of a cylinder with radius 1.2 m and depth 20 cm. Calculate the volume of the pool in litres, correct to the nearest litre.

9 Convert each of the following measurements into the units shown in brackets.

a 14 cm^2 (mm^2) **b** 1600 mm^2 (cm^2) **c** 2.1 m^2 (cm^2)

d 46 000 cm^2 (m^2) **e** 64 000 m^2 (km^2) **f** 8000 m^2 (ha)

g 41000 mm^2 (m^2) **h** 2.6 km^2 (cm^2)

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10 Convert the following measurements into the units shown in brackets.

a 410 mm^3 (cm^3)
b 0.2 cm^3 (mm^3)
c 64 000 cm^3 (m^3)
d 8000 cm^3 (litres)
e 8 600 000 mL (m^3)
f 0.006 m^3 (mL)
g 9 000 000 000 000 m^3 (km^3)
h 0.26 km^3 (m^3)
i 4.1 km^3 (L)
j 8.6 m^3 (mm^3)

Chapter 17: Quadratic equations

1 Solve:

a $x(x - 3) = 0$
b $3x(2x + 1) = 0$
c $(3m + 1)(2m - 5) = 0$
d $(4p + 3)(5p - 7) = 0$

2 Solve:

a $x^2 - 5x = 0$
b $2x^2 - 3x = 0$
c $8x - x^2 = 0$
d $x^2 - 16 = 0$
e $4x^2 - 9 = 0$
f $2x^2 - 72 = 0$
g $3x^2 - 75 = 0$
h $x^2 + 7x + 6 = 0$
i $x^2 + 9x + 18 = 0$
j $x^2 - 14x + 33 = 0$
k $x^2 = 6x - 8$
l $-x^2 + 3x + 40 = 0$
m $x^2 + 5x = 14$
n $-2x^2 + 7x + 15 = 0$
o $6x^2 + 7x + 2 = 0$
p $4x^2 + 17x - 15 = 0$
q $6x^2 - 11x - 2 = 0$
r $15x^2 = x + 2$

3 Solve:

a $x^2 = 8x - 12$
b $x^2 = 3x + 28$
c $x(x - 1) = 12$
d $x(x + 2) = 15$
e $(x + 2)(x + 3) = 2$
f $(x + 1)(x - 3) = 21$
g $x = \dfrac{12}{x - 4}$
h $x = \dfrac{18}{x + 7}$
i $x = \dfrac{10}{x} - 3$
j $x = \dfrac{12}{x} + 4$

4 One more than a number is the same as 42 divided by the number. What can the number be?

5 Twenty-one less than a number is equal to 100 divided by the number. What can the number be?

6 The product of a number and four less than the number is equal to 16 more than twice the number. Find the number.

7 Draw up a table of values for each of the following and use your table to plot the graph. In each case, state the x-intercepts.

a $y = x^2 - x - 6, \quad -3 \le x \le 4$
b $y = x^2 + x - 2, \quad -3 \le x \le 2$
c $y = x^2 + 3x - 4, \quad -5 \le x \le 2$
d $y = 3 - 2x - x^2, \quad -4 \le x \le 2$

8 Solve the equations.

a $x^2 - 7 = 0$
b $2x^2 - 16 = 0$
c $8 - x^2 = 0$
d $30 - 6x^2 = 0$

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9 Solve each equation by completing the square.

a $x^2 + 2x - 2 = 0$ **b** $x^2 - 6x + 6 = 0$ **c** $x^2 + 4x + 2 = 0$

d $x^2 + 3x + 1 = 0$ **e** $x^2 - 5x + 3 = 0$ **f** $x^2 - x - 1 = 0$

10 Sketch the parabolas.

a $y = (x - 1)^2$ **b** $y = (x + 2)^2 - 1$ **c** $y = x^2 + 3x + 3$

Chapter 18: Rates and direct proportion

1 Drew runs at 8 m/s for 12 seconds. How far does he run?

2 a Convert 180 km/h into metres per second.

b Convert 12 m/s to kilometres per hour.

3 a A plane travels 2000 km in 2 hours 30 minutes. What is the average speed of the plane?

b A car travels 200 km in 1 hour 50 minutes. What is the average speed of the car? Give your answer in kilometres per hour (km/h).

4 A car is travelling at 80 km/h.

a What is the formula for the distance d (in kilometres) travelled by the car in t hours?

b What is the gradient of the straight-line graph of d against t?

5 Write each of the following in words.

a $x \propto z$ **b** $y \propto x^3$ **c** $p \propto \sqrt{n}$ **d** $x^3 \propto z^4$

6 In each of the following $q \propto p$. Find the constant of proportionality and complete the table.

a

p	2	4		9
q	14		35	

b

p	1	4	7	
q		2		8

c

p	2	6	8	
q		8		24

7 Given $y \propto x^3$ and $y = 40$ when $x = 2$, find the formula for y in terms of x and the exact value of:

a y when $x = 10$ **b** y when $x = 3.5$ **c** x when $y = 135$

8 Given $d \propto t^2$ and $d = 18.75$ when $t = 5$, find the formula for d in terms of t and the exact value of:

a d when $t = 9$ **b** d when $t = 12$ **c** t when $d = 48$

9 The surface area of a sphere varies as the square of the radius. If the surface area of a spherical ball of radius 7 cm is 196π cm^2, find the corresponding surface area of a sphere of radius 3.5 cm.

10 Given that $q \propto p^3$, what is the effect on q when p is doubled?

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Chapter 19: Statistics

1 In a class of 20 students, 8 travel to school by train, 6 catch a bus, 4 are driven to school and 2 walk. Represent this information in a column graph.

2 Twenty students were asked how long they spend doing homework each night. Their responses, in minutes, are given below:

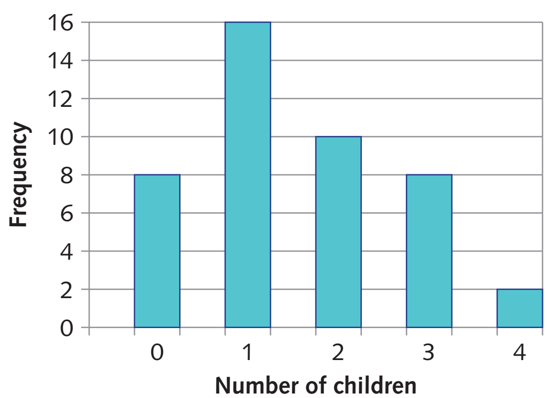
36 42 51 31 35 28 36 34 37 45

36 31 27 38 39 44 45 40 37 44

a Represent this information in a stem-and-leaf plot.

b How many students spent more than 40 minutes on homework?

3 The column graph below shows the number of children in families that completed a survey.



a How many families were surveyed?

b How many of the families have more than two children?

c What percentage of the families have no child or one child, correct to the nearest 1%?

d In total, how many children are there?

4 Over the course of a year, Ken obtained the following test results in Mathematics.

46 55 44 62 75 83 75 68 49 59 72 75 61

a Calculate the range for Ken's test results.

b Calculate the mean of his results, correct to 1 decimal place.

c If Ken does one more test and raises his average to 64, what was his mark on that test?

5 The average of 7 numbers is 12. An 8th number is added, and the average becomes 12.5. What number was added?

6 The average of 8 numbers is 24. What is the average of the same 8 numbers plus the two numbers 22 and 32?

7 The times, in seconds, it takes for nine students to run 100 m are:

12.6 14.2 13.1 12.9 15.1 14.6 13.7 14.2 14.8

a Calculate the mean, correct to 1 decimal place.

b Calculate the median.

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8 In an AFL football season, the goals scored by a player week by week are:

2 5 1 3 2 4 2 3 5 2 0 1 0 4 6 2 3 5 4 2 1 3

a Identify the mode in this data set.

b Calculate the mean number of goals scored in a week.

c Calculate the median number of goals scored in a week.

d In how many weeks did the player score more goals than the mean?

9 The numbers of Tasmanian devils that crossed a particular road over a number of weeks are recorded in the table below.

Week number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Number Of Tasmanian devils	24	41	44	26	27	38	35	39	40	41	23	46	33	36	38	41	24	22	31

a Draw a stem-and-leaf plot of the data.

b Find the mean, median and range of this data.

10 The heights of 15 children, to the nearest centimetre, are given below.

87 96 94 102 105 89 96 92 110 104 99 107 95 107

a Represent this information on a stem-and-leaf plot.

b Represent this information on a histogram using the classes:

i 85–89, 90–94, 95–99, 100–104, 105–109, 110–114

ii 80–89, 90–99, 100–109, 110–119

20B Problem-solving

1 Marbles of several different colours are placed in a bag. Sarah randomly draws one of the marbles from the bag, notes its colour and then replaces the marble.

a After doing this 10 times, her results are as follows.

Colour	Red	Blue	Yellow
Number of marbles	2	4	4

Decide whether each of the following statements is definitely true, could be true or is definitely false.

i There are 10 marbles in the bag.

ii There are twice as many blue marbles in the bag as there are red marbles.

iii There are equal numbers of blue and yellow marbles in the bag.

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iv The bag contains only 2 marbles and both are red.

v The bag contains marbles coloured red, blue or yellow only.

b Repeat part **a** if the results after Sarah has drawn 1000 marbles are as follows.

Colour	Red	Blue	Yellow
Number of marbles	201	701	98

2 Goran walks from O to A, 10 km away, on a true bearing of 300°T. He then walks 24 km from A to B on a true bearing of 30°T.

a Draw a diagram to represent Goran's walk and include the given information.

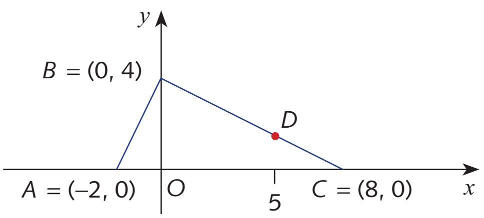
b State the size of $\angle OAB$.

c Calculate the distance between his start and finish points (that is, OB).

d Calculate the size of $\angle AOB$, correct to 2 decimal places.

e Hence calculate the bearing of O from B correct to the nearest degree.

3 Points A, B, C have coordinates as shown in the diagram. D is the point on line BC with an x-coordinate of 5.



a i Calculate the gradient of line AB.

ii Show that the equation of the line BC is $y = -\frac{1}{2}x + 4$. Find the coordinates of D.

iii Calculate the ratios $\frac{OB}{OA}$ and $\frac{OC}{OB}$.

iv Explain why ΔAOB is similar to ΔBOC.

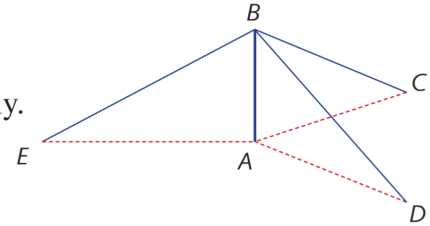
b Let the size of $\angle OAB = \alpha$.

i In terms of α, what is the size of $\angle OBA$?

ii In ΔOBC, find the size of $\angle OBC$ in terms of α.

iii Find the size of $\angle ABC$.

4 Students at an adventure camp erect a vertical mast AB, as shown in the diagram. BC, BD and BE are straight wires that are fixed to the ground at C, D and E respectively.



a In ΔEBA, $\angle EAB$ is a right angle and $\angle BEA = 30°$. If $EB = 40$ m, use trigonometry to show that the height of the mast AB is 20 m.

b In ΔEBA, use Pythagoras' theorem to find the length EA, giving your answer in simplest surd form.

c In ΔCBA, $\angle CAB$ is a right angle and the length AC is 11.547 m. Find the size of $\angle BCA$, giving your answer correct to the nearest degree.

d In ΔDBA, $\angle DAB$ is a right angle and $\angle BDA = 40°$. Calculate the length AD, giving your answer as a number of metres correct to 3 decimal places.

e Calculate the combined distance ($EA + CA + DA$), giving your answer correct to 2 decimal places.

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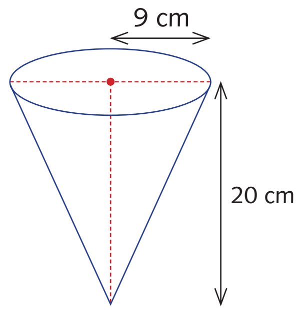
5 In this question you will need to use the following formula:

volume of a cone, $V = \frac{1}{3}\pi r^2 h$

A right circular cone has height 20 cm and radius 9 cm, as shown in the diagram.



a Water is poured into the inverted cone until the depth of the water is 14 cm.

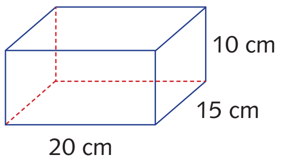
i If r centimetres is the radius of the water's surface, use similar triangles to calculate the value of r.

ii Use your answer to calculate the area of the water's surface, giving your answer in decimal form correct to 2 decimal places.

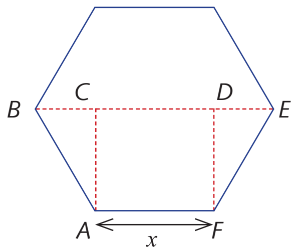
b The cone is now filled right to the top with water. What is the volume of the water in the cone, correct to the nearest 0.1 cm^3?

c The water is then poured from the cone into the rectangular tank shown.

Calculate the depth of water in the tank, correct to 2 decimal places.



6 A gardener is building a vegetable garden in the shape of a regular hexagon. In the given diagram $ACDF$ is a rectangle. Let x metres be the length AF.



a Find the size of $\angle BAC$.

b Find the following in terms of x.

i BC **ii** AC

c Show that the area (A m^2) of the hexagonal vegetable garden is given by $A = \frac{3\sqrt{3}x^2}{2}$.

d Find, correct to 2 decimal places, the area (A m^2) if $x = 2.5$ m.

To fit in the gardener's backyard, the distance between opposite sides of the vegetable garden must be less than or equal to 7 m.

e Find the largest value of x, correct to 2 decimal places, such that the vegetable garden, perhaps rotated, will fit in the gardener's backyard.

f Using this value, find correct to 2 decimal places the largest possible area of the vegetable garden.

7 **a** A card is drawn at random from a pack of 52 playing cards. Find the probability that the card will be:

i a Heart **ii** a Queen **iii** not a Queen

iv a Queen of Hearts **v** a Queen or a Heart

A new experiment involves drawing two cards from a pack of 52 playing cards. A card is drawn at random and the suit is noted – Diamond (D), Heart (H), Spade (S), Club (C). The card is returned to the pack and the cards are shuffled. A second card is drawn at random from the pack and its suit is also noted.

b Draw an array that lists all the possible outcomes, specifically, all the possible suit pairs that can be randomly drawn.

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c List the set of outcomes in which at least one Heart is obtained.

d Find the probability that the following is obtained:

i at least one Heart

ii exactly one Heart

iii a Heart and a Spade (in any order)

iv two cards of the same suit

e The experiment described above (involving drawing two cards with replacement) is repeated 100 times and the number of pairs that have the same suit are counted. On average, how many pairs would you expect to have the same suit?

8 Substance A (initially 30 g) evaporates at a rate of $1\frac{1}{2}$ g per minute while substance B (initially 20 g) evaporates at a rate of $\frac{2}{3}$ g per minute. y_A and y_B grams are the amounts of substances A and B respectively at time t minutes after the substances begin to evaporate.

a Write down formulas for y_A and y_B in terms of t.

b Find the time it takes for substance A to evaporate to 0 g.

c After how long do both substances have the same weight?

Consider now that substance A initially weighs a grams and evaporates at a rate of b grams per minute, and substance B initially weighs c grams and decays at a rate of d grams per minute.

d Write down rules for y_A and y_B in terms of t.

e Find in terms of a, b, c and d:

i the time when substance A and B are of the same weight

ii the weight of each substance A and B when they are of the same weight, expressing your answer as a fraction.

9 The length of a rectangular lawn is 3 m shorter than twice the width. There is a path around the lawn 0.75 m wide. Let the width of the lawn be x metres.

a Express the length of the lawn in terms of x.

b Express the area of the lawn in terms of x.

c i Show that the area A cm^2 of the path in terms of x is $A = \frac{9(2x-1)}{4}$.

ii If the area of the path is 20.25 m^2, find x.

iii Find the area of the lawn using the value of x obtained in part **ii**.

d If the area of the lawn is 90 m^2, find the corresponding value of x.

10 A jumbo jet does a return trip from Perth to Melbourne flying on the same course. On the trip from Perth to Melbourne, there is a tailwind and the jet averages 700 km/h. On the return trip, the jet averages 560 km/h The total travel time is 11 hours, including a two-hour stop in Melbourne. Let d kilometres be the distance between Melbourne and Perth.

a Express the time for the trip from Perth to Melbourne in terms of d.

b Express the time for the trip from Melbourne to Perth in terms of d.

c Express the total time for the return trip from Perth to Melbourne in terms of d.

d Hence calculate the total distance travelled.

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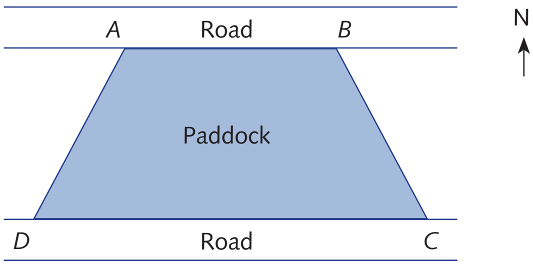
11 Jan travels at an average speed of x km/h and Len travels at an average speed of $(x + 4)$ km/h on a trip of 120 km.

a How long does it take Jan to complete the trip?

b How long does it take Len to complete the trip?

c Jan actually takes $1\frac{1}{2}$ hours longer than Len to complete the trip. Find Jan's average speed.

12 A farmer owns a paddock marked $ABCD$ in the diagram, which is located between two east–west roads. A is north-east of D. The fence AB is 1200 m, the fence AD is 1500 m and the fence from DC is 3200 m.



Calculate:

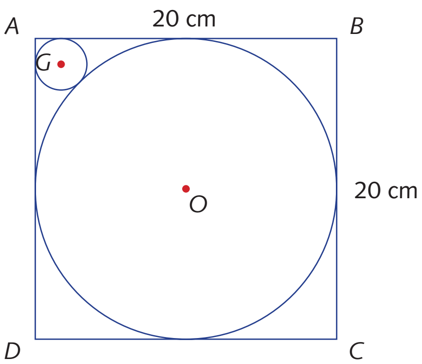
a the distance between the roads, to the nearest metre

b the area of the paddock in hectares, to the nearest hectare

c the bearing of the corner post B from the corner post C, correct to the nearest degree

13 A circle of radius 10 cm and centre O is placed inside the square $ABCD$ of side length 20 cm. A small circle with centre G is placed between a corner of the square and the circle so that it touches the circle and the two sides of the square.

Note: You may assume that the points O, G and A lie on the same line.



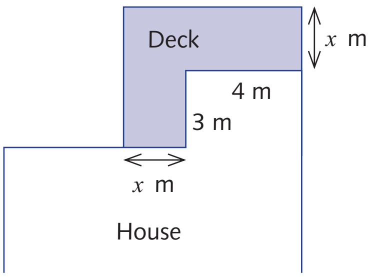
a Find the exact length of OA.

b If the radius of the small circle is r centimetres, find the length of GA in terms of r.

c Using $OA = OG + GA$, find OA in terms of r.

d Find the exact value of r.

14 A timber deck is built at the back of a house, as shown in the diagram. The measurements along the existing walls are 3 m and 4 m, as shown. The width of the deck is x metres, as shown.



a Express the area A m^2 of the deck in terms of x.

b If $A = 30$, find x.

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