Scalar Valued Functions

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Abstract

Whilst the author believes the raison d'être of this manuscript is obvious, they do not believe that the scope is.

The *Theory of Functions* is rich and central to Mathematics. As such, we limit our scope here to definitions and graphs of univariate functions $f : \mathbb{R} \to \mathbb{R}$. Whilst we include common equalities between different functions - say circular and exponential - what you will not find here are derivations of any sort. You will **not** find proofs **nor** set theoretic discussions of "jectivities", binary relations, etc. Furthermore there is a purposeful lack of rigour in this / catalogue/; theorems are asserted as is, with no warranty and no proof. Finally, analytic concerns of limits and convergence are also dutifuly ignored.

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1. Elementary

These such functions are continuous on their domains and include taking **sums**, **products**, **roots** and **compositions** of finitely many algebraic or transcendental functions.

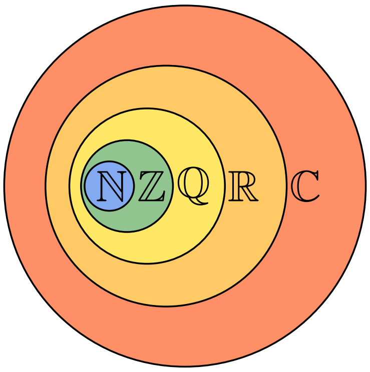
1.1. Algebraic

1.1.1. Polynomials

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_0 = \sum_{k=0}^{n} a_k x^k \quad (1)$$

1.1.2. Rational

Much in the same way that $\mathbb{Q}$ is defined as any element $\frac{a}{b}$ where $a, b \in \mathbb{Z}$:



a function f is called a rational function if it can be written in the form:

$$f(x) = \frac{P(x)}{Q(x)} \quad (2)$$

where $P(x)$ and $Q(x)$ are polynomial functions of x and Q is not the zero function.

1.1.3. Power

Note that $\sqrt{x}$ is not a polynomial because $\sqrt{x} = x^{\frac{1}{2}}$ and $\frac{1}{2} \notin \mathbb{Z}$.

1.2. Transcendental

These are the analytic functions that **do not** satisfy a polynomial equation.

1.2.1. Exponential

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n \quad (3)$$

furthermore,

$$\exp(x) = \lim_{n \to \infty} \left(1 + \frac{x}{n}\right)^n \tag{4}$$

graphically we have:

and by Euler's identity we have:

$$e^{i\theta} = \cos(\theta) + i\sin(\theta) \tag{5}$$

which relates the "circular" functions cosine and sine with the "exponential" □

1.2.2. Logarithm

setting $y = e^x$ and swapping variables: $x = e^y \implies y = \ln(x)$. as such the logarithm and exponential functions are inverses of each other.

1.2.3. Trigonometric

1.2.4. Inverse Trig

1.2.5. Reciprocal Trig

1.2.6. Hyperbolic

1.2.7. Inverse Hyper

1.2.8. Reciprocal Hyper

1.2.9. Factorial

$x!$ and $\frac{1}{x!}$

2. Non-Elementary

2.1. Transcendental

2.1.1. Gamma

2.1.2. Beta

2.1.3. Riemann Zeta

2.1.4. Error

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\Pi}} \int_0^x e^{(-t)^2} \, \mathrm{d}t \tag{6}$$

2.1.5. Tetration

2.1.6. Elliptic Integrals

2.1.7. Trigonometric Integrals

$$\operatorname{Si}(x) = \int_0^x \frac{\sin(t)}{t} \, \mathrm{d}t \tag{7}$$

$$\operatorname{si}(x) = -\int_\infty^x \frac{\sin(t)}{t} \, \mathrm{d}t \tag{8}$$

$$\operatorname{Si} - \operatorname{si} = \frac{\Pi}{2} = \int_0^\infty \frac{\sin(t)}{t} \, \mathrm{d}t \tag{9}$$

label as Dirichlet's integral

2.1.8. Fresnel

$$\mathrm{S}(x) = \int_0^x \sin(t^2) \, \mathrm{d}t, \ \mathrm{C}(x) = \int_0^x \cos(t^2) \, \mathrm{d}t \tag{10}$$

2.2. Algebraic

2.2.1. Bessel

2.2.2. Hypergeometric

$$\mathrm{B}_0 + \mathrm{B}_1 z + \mathrm{B}_2 z^2 + \ldots = \sum_{n \geq 0} \mathrm{B}_n z^n \tag{11}$$

where the ratio of successive coefficients is a rational function of n:

$$\frac{\mathrm{B}_{n+1}}{\mathrm{B}_n} = \frac{A(n)}{B(n)} \tag{12}$$

3. Discontinuous

3.1. Absolute Value

3.2. Step

3.2.1. Heaviside

3.2.2. Floor

3.2.3. Ceiling

3.2.4. Square Wave