

topology, 389
 generated by a metric, 393
 generated by family of functions,
 390
 generated by family of sets, 390
 induced by a metric, 393
 induced by another topology, 390
 inherited from a topological space,
 390
 metrizable, 393
 order, 393
 product, 392
 usual, 389
 weaker, 390
 total variation
 of complex measure, 119
 of signed measure, 117
 totally bounded space or set, 395
 trace
 of \mathcal{A} on C , 36
 of μ on C , 36
 translate, 25
 translation-invariant measure, 25, 285
 triangle inequality, 85, 86
 truncated random variable, 323
 Tychonoff's theorem, 392

U

unconditionally convergent series, 91
 uniform convergence of distribution
 functions, 326
 uniform distribution, 310
 uniform norm, 85
 uniformly
 absolutely continuous, 129
 dense family of functions, 392
 integrable, 129
 uniformly continuous
 function, 386
 left, 281
 right, 281
 unimodular, 294
 universal set, 253
 universally measurable set, 264
 upcrossing, 349
 inequality, 350

upper
 bound, 379
 derivate, 166
 integral, 67
 semicontinuous, 175
 sum, 67
 Urysohn's lemma, 184

V
 vanishes
 at $-\infty$, function that, 133
 at infinity, function that, 199
 variance, 309
 variation
 of a complex measure, 118
 of a signed measure, 117
 of a vector-valued measure, 404
 of F over $[a, b]$, 133
 vector
 lattice, 227
 space, 383
 vector-valued measure, 404
 version of a conditional expectation, 342
 Vitali
 covering, 164
 covering theorem, 164
 volume, 14

W

weak
 convergence, 140
 topology, 274
 weak law of large numbers, 320
 weak-* topology, 274
 Weierstrass approximation theorem, 325
 Wiener measure, 356
 existence, 357–361
 Wilson, Trevor, 419
 word, 421

Z

Zaanen, A. C., 226
 zero-dimensional space, 251
 Zorn's lemma, 377

Index of notation

Symbols

$[-\infty, +\infty]$, 380

A

$\mathcal{A} \times \mathcal{B}$, 143

aB, Ba , 280

\mathcal{A}_C , 36

$(A I)$, 163

A^o , 389

\mathcal{A}_* , 264

B

B^{-1} , 280

$\mathcal{B}_0(X)$, 197

BC , 280

$\mathcal{B}(\mathbb{C})$, 74

$\mathcal{B}(\mathbb{R})$, 4

$\mathcal{B}(\mathbb{R}^d)$, 4

$\mathcal{B}(X)$, 189

$B(X, \mathcal{A}, \mathbb{C})$, 120

$B(X, \mathcal{A}, \mathbb{R})$, 120

$B(x, r)$, 385

C

c (cardinality of the continuum), 376

c_0 , 89

$C_0^{\mathbb{C}}(X)$, 199

$C_0(X)$, 199

$C[a, b]$, 85

$C(X)$, 188, 197

D

Δ (symmetric difference of sets), 373

Δ (modular function), 294

δ_x , 8

\det , 156

$\text{diam}(A)$, 393

$\text{diff}(A)$, 28

$D\mu$, 166

$\overline{D}\mu$, 166

$\underline{D}\mu$, 166

$\frac{dv}{d\mu}$, 126

E

$e(C)$, 164

$e(U)$, 166

$E(X)$, 308

E_x , 144

$E(X \mathcal{B})$, 342

$E(X Y = y_j)$, 341

E^y , 144

F

\mathcal{F} , 5

$\langle f \rangle$, 96

$F'(x_0)$, 157

f^+ , 46

f^- , 46

$\mathcal{F}_\alpha(X)$, 255

\check{f} , 285

$[f, g]$, 229

$f \prec U$, 192

$f \vee g$, 43

$f \wedge g$, 43

F_μ , 19

\mathcal{F}_σ , 5

$f * g$, 153, 298

$\{\mathcal{F}_n\}_{n=0}^\infty$, 345

$\{\mathcal{F}_t\}_{t \in T}$, 345

| | | | |
|---------------------------------|----------------------|---|--------|
| \mathcal{F}_τ | 354 | $L^p(X, \mathcal{A}, \mu)$ | 96 |
| f_x | 144, 285 | \mathcal{L}^1 | 56 |
| ${}_x f$ | 285 | $L^1(G)$ | 300 |
| $f: X \rightarrow Y$ | 374 | $\mathcal{L}^1(X, \mathcal{A}, \mu, E)$ | 400 |
| f^y | 144 | $\mathcal{L}^1(X, \mathcal{A}, \mu, \mathbb{R})$ | 56 |
| | | ℓ^2 | 90 |
| | | λ | 18 |
| G | | λ_d | 18 |
| \mathcal{G} | 5 | λ^* | 13, 14 |
| G_3 | 418 | λ_d^* | 14 |
| $\mathcal{G}_\alpha(X)$ | 255 | L_\bullet | 233 |
| $g \cdot x$ | 417 | L^\bullet | 233 |
| $GL(d, \mathbb{R})$ | 284 | $l(f, \mathcal{P})$ | 67 |
| $\text{gr}(f)$ | 244 | $\lim_n f_n$ | 43 |
| \mathcal{G}_δ | 5 | $\liminf_n f_n$ | 43 |
| | | $\liminf_n x_n$ | 381 |
| | | $\underline{\lim}_n x_n$ | 381 |
| H | | $\lim_n s_n$ | 86 |
| $(H) \int_a^b f$ | 430 | $\limsup_n f_n$ | 43 |
| $(H) \int_a^b f(x) dx$ | 430 | $\limsup_n x_n$ | 381 |
| | | $\overline{\lim}_n x_n$ | 381 |
| | | ℓ^∞ | 89 |
| I | | $(L) \int_a^b f$ | 56 |
| \mathcal{I} | 229, 243 | $(L) \int_a^b f(x) dx$ | 56 |
| $\Im(f)$ | 75 | $\mathcal{L}^p(X, \mathcal{A}, \mu, \mathbb{C})$ | 91 |
| $\inf(A)$ | 381 | $\mathcal{L}^p(X, \mathcal{A}, \mu, \mathbb{R})$ | 91 |
| $\inf_n f_n$ | 43 | $\mathcal{L}^\infty(X, \mathcal{A}, \mu, \mathbb{C})$ | 92 |
| $\int f d\mu$ | 53, 55, 56, 120, 399 | $\mathcal{L}^\infty(X, \mathcal{A}, \mu, \mathbb{R})$ | 92 |
| $\int f(x) \mu(dx)$ | 56 | | |
| $\int f(x) d\mu(x)$ | 56 | | |
| $\int_A f d\mu$ | 56 | | |
| $\int_a^b f$ | 56, 68 | | |
| $\int_a^b f(x) dx$ | 56, 68 | | |
| $\int_a^b f$ | 67 | | |
| $\overline{\int_a^b} f$ | 67 | | |
| (\cdot, \cdot) | 89 | | |
| $\Im(z)$ | 382 | | |
| | | | |
| J | | | |
| J_F | 158 | | |
| | | | |
| K | | | |
| $\mathcal{K}^\mathbb{C}(X)$ | 184 | | |
| $\mathcal{K}(X)$ | 184 | | |
| | | | |
| L | | | |
| L_1 | 234 | | |
| $\underline{L}(f)$ | 234 | | |
| $\overline{L}(f)$ | 234 | | |
| $L^\infty(X, \mathcal{A}, \mu)$ | 96 | | |

right Haar measure, 285
rising sum lemma, 164

S

Saks–Henstock lemma
for Henstock–Kurzweil integral, 433
for McShane integral, 437
sample point, 307
Schröder–Bernstein theorem, 261, 376, 418
second countable topological space, 391
second moment, 309
sections, 144
semimetric, 86
seminorm, 85
separability of L^p , 102
separable space, 86, 390
separated σ -algebra or measurable space, 270
separation
of points
 by a family of functions, 392
 by a family of sets, 270
of sets
 by Borel sets, 257
 by open sets, 182
theorem for analytic sets, 257
set
 Lebesgue measurable but not Borel measurable, 48
 not Lebesgue measurable, 27–29
 theory, basic concepts, 373–378
 σ -algebra, 2
 generated by a collection of sets, 3, 270
 σ -algebra or measurable space
 countably generated, 102, 270
 countably separated, 271
 separated, 270
 σ -compact topological space, 183
 σ -field, 2
 σ -finite
 measure or measurable space, 9
 set, 9
 σ -ring, 228
signed measure, 114
simple function, 42, 397
simulation
 of normal random variables, 319
 of random variables, 317
singular part of a measure, 130
singularity of measures, 130
Souslin space, 274
special orthogonal group, 418
standard deviation, 309

standard measurable space, 270
step function, 101, 102
stochastic process, 345
 continuous-time, 345
 discrete-time, 345
Stone's condition, 227
Stone, M. H., 226
Stone–Weierstrass theorem, 392
stopping time, 345
strong law of large numbers, 322
 derived from martingale convergence theorem, 356
strongly
 integrable function, 399
 measurable function, 397
subcover, 386, 391
submartingale, 347
subspace
 of a topological space, 390
 of a vector space, 383
summable, 234
summable function, 56
sup norm, 85
supermartingale, 347
support
 of a function, 184
 of a measure, 207
supremum, 379
surjection, surjective function, 375
symmetric
 difference, 373
 set, 280

T

tag, 70, 429
tagged partition, 70, 429
 δ -fine, 430
 subordinate to δ , 430
tagged subpartition, 432
 δ -fine, 433
 subordinate to δ , 433
tail
 σ -algebra, 321
 event, 321
three series theorem, 327
Tietze extension theorem, 188
tightness, 335
 uniform, 335
Tonelli's theorem, 147
topological
 dual space, 106
 group, 279
 space, 389

normed
 linear space, 85
 vector space, 85
 nowhere dense set, 395

O

one-point compactification, 185
 open
 ball, 385, 393
 cover, 386, 391
 neighborhood, 389
 set, 385, 389
 optional time, 345
 order topology, 393
 ordinal numbers
 spaces of, 189
 orthogonal, 91
 group, 284
 matrix, 284, 418
 operator, 426
 vectors, 426
 orthonormal basis, 426
 outer measure, 12, 33
 outer regularity, 190

P

paradoxical, 420
 parallelogram law, 90
 partial order, 377
 partially ordered set, 377
 partition, 67, 377, 429
 path, 357
 π -system, 37
 point
 of density, 169
 of dispersion, 169
 point at infinity, 185
 point mass, 8
 Poisson distribution, 319
 polar coordinates, 162
 Polish space, 239
 positive
 linear functional, 107, 181, 192, 202
 measure, 114
 part, 46, 117
 set, 115
 power set, 374
 probability, 307
 space, 307
 product
 measure, 145
 of Borel σ -algebras, 219–220, 243

of countably many probability measures,
 365
 of indexed family of sets, 375
 of infinite sequence of measurable spaces,
 243
 of sets, 375
 of uncountably many probability measures,
 370
 σ -algebra, 143, 243
 topology, 392

Q

quotient space, 96

R

Radon measure, 215–218
 Radon–Nikodym
 derivative, 126
 theorem, 123, 125, 129, 404
 random variable, 308
 continuous, 309
 discrete, 309
 real-valued, 308
 real
 numbers, field of, 379
 part, 74–75, 382
 rectangle with measurable sides, 143
 reduced word, 421
 refinement, 67
 regular
 Borel measure, 190
 Borel product, 222
 finite signed or complex measure, 200
 measure, 23, 34, 189
 regularity
 of finite Borel measures on Polish spaces,
 245
 of finite Borel measures on \mathbb{R}^d , 34
 of finite Borel measures on Souslin spaces,
 275
 of Lebesgue measure, 23
 relation, 376
 relatively compact set, 263
 reverse martingale, 356
 Riemann
 integrability, 67, 430
 integral, 68, 430
 sum, 70, 429, 433
 Riesz Representation Theorem, 192
 Riesz, F, 164
 right
 uniformly continuous, 281

| | |
|---------------------------------------|---------------------------------|
| $\mu * \nu$, 301 | S |
| μ_X or μ , 309 | $\text{sgn}(z)$, 108 |
| $M(X, \mathcal{A}, \mathbb{C})$, 119 | $\sigma(\mathcal{F})$, 3 |
| $M(X, \mathcal{A}, \mathbb{R})$, 119 | σ_X^2 , 309 |
| | σ_X , 309 |
| | $\sigma(X_i, i \in I)$, 309 |
| N | $\sigma(X_1, X_2, \dots)$, 309 |
| n , 243 | $SO(d)$, 418 |
| N , 243 | $\sum_\alpha X_\alpha$, 241 |
| $\mathcal{N}(n_1, \dots, n_k)$, 243 | $\sup(A)$, 381 |
| $N(0, 1)$, 311 | $\sup_n f_n$, 43 |
| $N(\mu, \sigma^2)$, 311 | $\text{supp}(f)$, 184 |
| v_a , 130 | $\text{supp}(\mu)$, 207 |
| v_f , 204 | |
| $v \ll \mu$, 122, 125 | |
| v_s , 130 | |
| $v_1 * v_2$, 314 | |

| | |
|--------------|--------------------------|
| O | U |
| $O(d)$, 284 | $u(f, \mathcal{P})$, 67 |

| | |
|-------------------------------|-----------------------|
| P | V |
| $P(A B)$, 340 | $\text{var}(X)$, 309 |
| ϕ_μ , 331 | V_\bullet , 233 |
| $\prod_n \mathcal{A}_n$, 243 | V^\bullet , 233 |
| $\mathcal{P}(X)$, 374 | $V_F[a, b]$, 133 |
| | V^* , 106 |

| | |
|--|--------------------|
| R | X |
| $\overline{\mathbb{R}}$, 380 | $x \vee y$, 380 |
| $\Re(f)$, 75 | $x \wedge y$, 380 |
| $\mathcal{R}(f, \mathcal{P})$, 70, 429, 433 | |
| $(R) \int_a^b f$, 68 | |
| $(R) \int_a^b f(x) dx$, 68 | |
| $\Re(z)$, 382 | Z |
| | \bar{z} , 108 |

Index

A

\mathcal{A} -measurable
function, 42
set, 2
a.e., 50
a.e. [μ], 50
a.s., 319
absolute continuity, 122
for functions from \mathbb{R} to \mathbb{R} , 135
for signed and complex measures, 125
uniform, 129
absolutely continuous part of a measure, 130
absolutely convergent series, 88
act, 417
action, 417
adapted, 345
algebra, 300
Banach, 300
of functions, 392
of sets, 1
algebraic dual space, 106
almost everywhere, 50
almost everywhere differentiability
of finite Borel measures, 167
of functions of finite variation, 171
of monotone functions, 171
almost surely, 319
analytic
measurable space, 270
set, 248
measurability, 262
that is not a Borel set, 254
ancestor, 419
approximate identity, 305
atom of a σ -algebra, 272
axiom of choice, 27, 377

B

Baire
category theorem, 395
measure, 197
set, 197
 σ -algebra, 197, 226
Banach
algebra, 300
space, 87
Banach–Tarski paradox, 417, 419
base
for a family of neighborhoods, 280
for a topological space, 390
basis
Hamel, 30
Beppo Levi's theorem, 62
Bernoulli distribution, 315
bijection, bijective function, 375
binary expansion, 315–316, 382
binomial distribution, 318
Blackwell's theorem on analytic measurable spaces, 272
Bochner
integrable function, 399
integral, 399
Borel
function, 42
isomorphism, 259
measurability of the image of a Borel set
under an injective Borel function, 260
measurable function, 42, 189, 397
measure, 11, 189
product, regular, 222
 σ -algebra, 4, 189
subsets, 4, 189
Borel–Cantelli lemmas, 320

linear (*cont.*)

 lifting, 406

 operator, 105

 order, 377

 transformation, 105

locally

 almost everywhere, 92

 null set, 92

locally compact

 group, 279

 topological space, 182

lower

 bound, 379

 derivate, 166

 integral, 67

 semicontinuous, 175, 209

 sum, 67

Lusin space, 274

Lusin's theorem, 208

M

martingale, 346

 convergence theorem, 348

 related to differentiation, 347–348, 353

 relative to $\{\mathcal{F}_n\}$, 346

 reverse, 356

maximum, 43, 380

McShane

 integrability, 436

 integral, 436

 equivalent to Lebesgue integral, 437–438

mean value theorem, 387

measurability of analytic sets, 262

measurable

 function, 42, 73, 235

 set, 15

 space, 8

 analytic, 270

 standard, 270

measure, 7

 absolutely continuous, 122, 125

 Borel, 11

 complex, 118

 continuous, 11

 countably additive, 7

 counting, 8

 discrete, 11

 finitely additive, 7, 111

 Haar, 285

 inner, 33

 on (X, \mathcal{A}) or on X , 8

outer, 33

 positive, 114

 product, 145

 Radon, 215–218

 space, 8

 translation-invariant, 25, 285

mesh, 70, 429

metric, 86, 393

 space, 86, 393

metrizability

 of second countable compact Hausdorff spaces, 186

 of second countable locally compact Hausdorff spaces, 187

metrizable space, 393

metrize, 393

minimum, 43, 380

Minkowski's inequality, 94

modular function, 294

monotone class, 40

 theorem, 40

monotone convergence theorem, 61

 for conditional expectations, 344

 for Henstock–Kurzweil integral, 434

 for McShane integral, 437

monotonicity, 13

μ -a.e., 50

μ -almost everywhere, 50

 on E , 50

μ -integrable function, 56

μ -measurable, 31

μ -negligible, 30

μ -null, 30

μ^* -measurable sets, 15, 212–218

N

negative

 part, 46, 117

 set, 115

neighborhood, open, 389

net, 305

non-measurable set, 27–29

non-regular Borel measure, 197

norm, 84, 385

 associated to an inner product, 90

 of a linear operator, 106

 of a partition, 70, 429

normal

 distribution, 310

 number, 325

 random variable, 311

 to base b , 325

normal topological space, 183

increasing sequence of sets, 5
 independent events, 312
 random variables, 312
 and product measures, 313
 σ -algebras, 312
 index set, 375
 indicator function, 331, 375
 infimum, 379
 infinitely often, 320
 injection, injective function, 375
 inner measure, 33
 inner product, 89
 space, 90
 inner regularity, 190
 integrable function, 56, 399
 over A , 56
 uniformly, 129
 integral
 basic properties, 53–60
 Bochner, 399
 convergence theorems, 61–64
 definition, 53–56
 improper, 71
 of f over A , 56
 with respect to finite signed or complex measure, 120
 integration by parts, 151, 173
 interior of a set, 389
 intermediate value theorem, 387
 interval, d -dimensional, 14
 inverse Fourier transform, 334
 inverse image of a set, 374
 irrational numbers, set of
 as a Polish space, 243
 homeomorphic to \mathcal{N} , 255
 isometric isomorphism, 106
 isometry, 106
 isomorphic, 259
 isomorphism
 Borel, 259
 isometric, 106
 measurable, 259
 of groups, 384
 theorem for Borel sets, 259, 261
 iterated integrals, 147

J
 Jacobian, 158
 Jensen's inequality, 98
 for conditional expectations, 354

Jordan decomposition
 of complex measure, 118
 of signed measure, 117
 theorem, 117

K
 kernel, 65
 Kindler, J., 226
 Kolmogorov's consistency theorem, 368
 inequality, 322
 zero-one law, 321

L
 L -almost everywhere, 235
 L -measurable, 235
 L -negligible, 235
 L -null, 235
 L -summable, 234
 L^1 -bounded set, 129
 Lebesgue density theorem, 169
 integrability, 56
 integral, 56
 measurable function, 42
 measurable set, 15
 measure, 18
 outer measure, 13, 14
 point, 174
 set, 174
 Lebesgue decomposition
 of a measure, 130
 theorem, 130
 left uniformly continuous, 281
 left Haar measure, 285
 Lévy's metric, 339
 lifting
 linear, 406
 of \mathcal{L}^∞ , 405
 of a σ -algebra, 413
 limit
 inferior, 381
 of a sequence in \mathbb{R}^d , 386
 of a sequence in \mathbb{R} , 381
 of a sequence in a metric space, 86, 394
 point, 385
 superior, 381
 line segment, 384
 linear function, 383
 functional, 106

bound for a linear operator, 106
 bounded
 linear operator, 106
 set, 385, 393
 variation, function of, 133
 Bourbaki's treatment of integration, 215–218
 Brownian motion, 357
 existence, 357–363
 nowhere differentiable paths, 361

C
 C^1 function, 158
 Cantor
 function, 48–49, 52, 130, 137, 178
 set, 26–27, 47–49
 singular function, 48
 Cantor's nested set theorem, 394
 capacitable, 266
 capacity, 266
 cardinality, 375
 of the continuum, 376
 Cartesian product, 375
 Cauchy criterion
 for Henstock–Kurzweil integrability, 431
 for McShane integrability, 437
 Cauchy sequence, 86, 394
 Cauchy–Schwarz inequality, 90
 central limit theorem, 338
 chain rule, 157
 change of variable, 155–162
 characteristic function, 331, 375
 continuity of, 331
 derivatives of, 332
 of binomial distribution, 338
 of convolution, 333
 of normal distribution, 333
 of Poisson distribution, 339
 of uniform distribution, 339
 uniform continuity of, 339
 characterization
 of absolutely continuous functions, 135–137, 173
 of compact metric spaces, 395
 classification of Borel sets, 255–257
 closed
 ball, 393
 set, 385, 389
 closure
 of a set, 385, 389
 under an operation, 1
 compact
 group, 279

set, 387, 391
 topological space, 391
 complete
 measure or measure space, 30
 metric space, 87, 394
 ordered field, 379
 completeness
 of $C[a, b]$, 87
 of L^p , 99
 of $C_0(X)$ and $C_0^c(X)$, 199
 of $M(X, \mathcal{A}, \mathbb{R})$ and $M(X, \mathcal{A}, \mathbb{C})$, 119
 completion of a σ -algebra or measure, 31
 complex
 conjugate, 108
 measure, 118
 numbers, 74, 382
 valued functions, 74
 concentration of a measure on a set, 130
 condensation point, 252
 conditional expectation of X given \mathcal{B} , 341
 basic properties, 342–344
 conditional expectation of X , given that $Y = y_j$, 341
 conditional probability of A given B , 340
 conjugate
 complex, 108
 exponents, 93, 108
 space, 106
 construction of random variables, 316, 365
 continuous
 function, 386, 390
 linear operator, 105
 continuous measure, 11
 continuum hypothesis, 376
 convergence
 almost everywhere, 80
 almost sure, 319
 almost uniform, 82
 in L^p -norm, 96
 in p^{th} mean, 96
 in distribution, 328
 and characteristic functions, 337
 in mean, 82
 in measure, 79
 in probability, 319
 in \mathbb{R}^d , 386
 of binomial distribution to Poisson, 339
 of random series, 355
 weak, 140, 328
 convergent
 sequence in a metric space, 86, 394
 sequence in \mathbb{R}^d , 386
 convergent series, 88

- converse to strong law of large numbers, 324
- convex
 - function, 98
 - set, 384
- convolution, 153, 298, 301, 314
- coset, 96, 283
- countability of the set of rational numbers, 376
- countable
 - additivity, 7, 113, 118
 - set, 375
 - subadditivity, 9, 13
- countably
 - additive, 7
 - measure, 7
 - generated σ -algebra or measurable space, 102, 270
 - separated σ -algebra or measurable space, 271
 - subadditive, 13
- counting measure, 8
- Cousin's lemma, 430
- covering
 - Vitali, 164
- cross sections, 267–270
- cube
 - closed, 164
 - half open, 24
 - open, 166
- D**
 - d -system, 37
 - Daniell, P. J., 226
 - De Morgan's laws, 374
 - decreasing sequence of sets, 5
 - defined piecewise, 418
 - dense set, 86, 390
 - density
 - of a distribution, 310
 - of a random variable, 310
 - density in L^p of subspace determined by
 - continuous functions, 101
 - simple functions, 100
 - step functions, 101
 - density of $\mathcal{H}(X)$ in $C_0(X)$, 199
 - derivate
 - lower, 166
 - upper, 166
 - derivative, 157, 166
 - determinant
 - of a linear operator, 156
 - of a matrix, 155
 - determines, 100
- diameter of a set, 393
- differentiable, 157, 166
- Dini's theorem, 227
- directed
 - set, 305
 - upward, 209
- discrete
 - measure, 11
 - topological space, 182
 - topology, 182
- disjoint union
 - of sets, 241
 - of topological spaces, 241
- distance between a point and a set, 393
- distribution, 308
 - joint, 308
- distribution function, 309
 - cumulative, 309
- dominated convergence theorem, 63, 400
 - for conditional expectations, 344
- Doob's martingale convergence theorem, 348
- double series, 151, 154
- dual
 - of L^1 , 214, 303
 - of L^1 , example, 140, 215
 - of L^p , 108, 138–140
 - of $C_0(X)$ or of $C_0^{\mathbb{C}}(X)$, 201
 - space, 106
- dyadic rational, 382
- Dynkin class, 37
- E**
 - Egoroff's theorem, 81
 - elementary
 - integral, 227
 - outcome, 307
 - empirical distribution function, 326
 - enumeration, 375
 - equidecomposable, 418
 - equivalence
 - classes of functions, 96
 - of McShane and Lebesgue integrals, 437–438
 - relation, 376
 - essentially bounded function, 92
 - event, 307
 - existence
 - of sequences of independent random variables, 317, 365
 - expectation, 308
 - expected value, 308
 - experiment, 307

extended
 real numbers, 380
 real-valued function, 46
 extremal subset, 407
 extreme point, 407

F
 Fatou's lemma, 63
 field, 379
 of sets, 2
 ordered, 379
 filtration, 345
 finer, 67
 finite
 additivity, 7, 113
 intersection property, 391
 measure or measurable space, 9
 signed measure, 114
 variation, function of, 133

finitely additive, 7
 measure, 7, 111

Fourier
 inversion formula, 334
 transform, 331

free group, 421

freely generated, 421

freely tagged partition, 436
 δ -fine, 436

F_σ , 5, 183

Fubini's theorem
 on iterated integrals, 148, 224
 on the differentiation of series, 171

function
 continuous, 386, 390
 lower semicontinuous, 175, 209
 uniformly continuous, 386
 upper semicontinuous, 175

G
 G -equidecomposable, 418

G -paradoxical, 420

gambling, 347, 348

gauge, 430

Gaussian
 distribution, 310

random variable, 311

G_δ , 5, 183

general linear group, 284

generalized Riemann integral, 429

generated, 421

freely, 421

Glivenko–Cantelli theorem, 326
 graph of a function, 244
 group, 384
 abelian, 384
 commutative, 384
 compact, 279
 general linear, 284
 locally compact, 279
 orthogonal, 284
 topological, 279

H

Haar measure, 285
 examples, 285, 292, 297
 existence, 286
 left, 285
 right, 285
 uniqueness, 290

Hahn decomposition, 116
 theorem, 116

Hahn–Banach theorem, 401

Hamel basis, 30

has

a finite expected value, 308
 an expected value, 308

Hausdorff space, 391

Heine–Borel theorem, 387

Henstock–Kurzweil

integrability, 430, 432
 of characteristic function of rationals, 432
 integral, 429, 430, 432
 extension of Lebesgue integral, 434–435

Hilbert space, 90

Hölder's inequality, 93

homeomorphic, 390

homeomorphism, 390

homogeneity, 85

homomorphism of groups, 384

I

I -capacitable, 266

i.i.d., 320

i.o., 320

ideal, 302, 414

identically distributed, 320

identification of functions that agree (locally)
 almost everywhere, 96

image of a set, 374

imaginary part, 74–75, 382