xponential Brownian motion with drift] Let $x, m \in \mathbb{R}$ and $\sigma > 0$ and we set $Y_t = xe^{mt+\sigma W_t}$. The process Y is called an exponential Brownian motion with drift m and variance σ^2. We first notice that the process Y is adapted to the filtration $\mathcal{F}$. Next,

$$\mathbb{E}[Y_t] = xe^{mt}\mathbb{E}(e^{\sigma W_t}) = xe^{mt}M_{W_t}(\sigma) = xe^{(m+\sigma^2/2)t}.$$

We want to show that the process Y is an $\mathcal{F}$-martingale if and only if $m = -\sigma^2/2$. Indeed, for $s \leq t$,

$$\begin{aligned} \mathbb{E}[xe^{mt+\sigma W_t}|\mathcal{F}_s] &= \mathbb{E}[xe^{ms+m(t-s)+\sigma W_s+\sigma(W_t-W_s)}|\mathcal{F}_s] \\ &= Y_s\mathbb{E}[e^{m(t-s)+\sigma(W_t-W_s)}] \\ &= Y_s e^{m(t-s)+\sigma^2(t-s)/2} \end{aligned}$$

and the claim follows.
+end$_s rc$
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