Examples 0.0.1 (Counterexample of the converse). We give an example where X and Y are modifications of each other, but not distinguishable. Consider the space $([0,1], \mathcal{B}([0,1]), \mathbb{P})$, where $\mathbb{P}$ is the Lebesgue measure, i.e. $\mathbb{P}([a,b]) = b - a$. Let $(Y_t)_{t\in[0,1]}$ denote a constant process given by $Y_t = 0$ and $(X_t)_{t\in[0,1]}$ given by

$$X_t(\omega) = \begin{cases} 1 & \text{if } t = \omega, \\ 0 & \text{if } t \neq \omega. \end{cases}$$

For a fixed ω, the trajectories $X_t(\omega)$ and $Y_t(\omega)$ differs only at the point $t = \omega$. To see that X and Y are modifications of each other, for every $t \in [0,1]$, we have

$$\{\omega : X_t(\omega) = Y_t(\omega)\} = \{\omega : \omega \neq t\} = \Omega \setminus \{\omega : \omega = t\}$$

This shows that $\mathbb{P}(X_t = Y_t) = 1 - \mathbb{P}(\{t\}) = 1$. Here the process X is not a right continuous process (càdlàg), therefore one cannot conclude that X and Y are indistinguishable. In fact, we see that

$$\{\omega : X_t(\omega) = Y_t(\omega),\ \text{for all } t \in [0,1]\} = \bigcap_{t\in[0,1]} \{\omega : X_t(\omega) = Y_t(\omega)\} = \emptyset$$

since the complement is given by

$$\bigcup_{t\in[0,1]} \{\omega : X_t(\omega) \neq Y_t(\omega)\} = \bigcup_{t\in[0,1]} \{\omega : \omega = t\} = \Omega$$

which means $(X_t)_{t\in[0,1]}$ and $(Y_t)_{t\in[0,1]}$ cannot be indistinguishable.