

Definition 0.0.1 (σ -algebra). A family of subsets $\mathcal{A} \subset 2^\Omega$ is called a σ -algebra if it satisfies

(i) $\Omega \in \mathcal{A}$, or $\emptyset \in \mathcal{A}$.

(ii) Closure under complement: If $A \in \mathcal{A}$ then $A^c \in \mathcal{A}$.

(iii) Closure under countable unions and intersections: If a sequence of sets $(A_i)_{i \geq 1}$ belongs to \mathcal{A} , then $\bigcup_{i=1}^{\infty} A_i$ (or $\bigcap_{i=1}^{\infty} A_i$, by De Morgan's law¹) also belongs to \mathcal{A} .

The smallest possible σ -algebra is the trivial σ -algebra: $\{\emptyset, \Omega\}$. The largest possible σ -algebra is 2^Ω . Between the two, we have e.g. for a 6-faced dice, $\mathcal{A} = \{\{1\}, \{2, 3, 4, 5, 6\}, \emptyset, \Omega\}$.

¹ $\bigcap_{i=1}^{\infty} A_i = (\bigcup_{i=1}^{\infty} A_i^c)^c$.