Theorem 0.0.1 (properties of Ito's integrals)**.** *Let* $\gamma \in L^2_{\mathbb{P}}(W)$*, i.e.* $\mathbb{E}\left[\int_0^T |\gamma_u|^2\,du\right] < \infty$*. Then:*

(i) ***(martingale)*** $I(\gamma)$ *is a continuous, square integrable* $\mathcal{F}$*-martingale.*

(ii) ***(linearity)*** *For constants* a *and* b*,* $I_t(a\gamma + b\eta) = aI_t(\gamma) + bI_t(\eta)$*.*

(iii) ***(Ito isometry)*** $\mathbb{E}[I_t^2(\gamma)] = \mathbb{E}\left[\int_0^t \gamma_u^2\,du\right]$.

(iv) ***(quadratic variation)*** *The quadratic variation accumulated up to time* t *by the Ito's integral is*

$$\langle I(\gamma)\rangle_t = \int_0^t \gamma_u^2\,du.$$

Furthermore, Theorem 3.3.3 says that $I^2(\gamma) - \langle I(\gamma)\rangle$ *is an* $\mathcal{F}$*-martingale. Corollary 3.3.2 implies that*

$$\mathrm{Var}(I_t(\gamma)) = \mathbb{E}[I_t^2(\gamma)] = \mathbb{E}[\langle I(\gamma)\rangle_t].$$

(v) ***(local property)*** *For any* $\mathcal{F}$*-stopping time* τ*,* $I(\gamma \mathbf{1}_{[0,\tau]}) = I_\tau(\gamma)$*.*