

is enough to observe that a difference of two continuous local martingales is also a continuous local martingale, and a difference of two continuous processes of finite variation also follows a continuous process of finite variation. In our case, we have $M - \widetilde{M} = \widetilde{A} - A$ and $M_0 - \widetilde{M}_0 = \widetilde{A}_0 - A_0 = 0$.

Consequently, in view of (vi) of Theorem 5.1.1, any continuous local martingale of finite variation starting at 0 at time 0 is a null process. We therefore conclude that $M_t = \widetilde{M}_t$ and $A_t = \widetilde{A}_t$ for every $t \in [0, T]$.

Theorem 0.0.1.