Lemma 0.0.1 (quadratic variation of Ito processes)**.** *Consider an Ito process denoted by* $dX_t = \alpha_t \, dt + \beta_t \cdot dW_t$.

(i) The quadratic variation of the Ito process is $\langle X \rangle_t = \int_0^t \beta_u^2 \, du$.

(ii) The quadratic covariation (aka cross-variation) of two Ito processes $X_t = \int_0^t \alpha_u \, dW_u$ *and* $Y_t = \int_0^t \beta_u \, dW_u$ *is*

$$\langle X, Y \rangle_t = \int_0^t \alpha_u \beta_u \, du.$$

(iii) ***(polarisation formula)*** *A more general version of (ii) gives the quadratic covariation of two continuous semimartingales. If* $X^i = X_0^i + M^i + A^i$ *and* $X^j = X_0^j + M^j + A^j$ *are in* $\mathcal{S}^c(\mathbb{P})$, *then*

$$\langle X^i, X^j \rangle = \langle M^i, M^j \rangle = \frac{1}{2} \left(\langle M^i + M^j \rangle - \langle M^i \rangle - \langle M^j \rangle \right) = \frac{1}{4} \left(\langle M^i + M^j \rangle - \langle M^i - M^j \rangle \right).$$