Definition 0.0.1 (continuous semimartingale)**.** A real-valued, continuous, $\mathcal{F}$-adapted process X is called a (real-valued) **continuous semimartingale** if it admits a (canonical) decomposition

$$X_t = X_0 + M_t + A_t, \quad \forall t \in [0, T],$$

where X_0, M and A satisfy:

(i) X_0 is an $\mathcal{F}_0$-measurable random variable.

(ii) M is a continuous local martingale with $M_0 = 0$.

(iii) A is a continuous process whose almost all sample paths are of finite variation on the interval $[0, T]$ with $A_0 = 0$.

We denote by $\mathcal{S}^c(\mathbb{P})$ the class of all real-valued continuous semimartingales on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$. A continuous semimartingale is a continuous local martingale if and only if the process A in its canonical decomposition $X = X_0 + M + A$ vanishes, that is, is indistinguishable from the null process.