Proof. The only if part is straightforward. We prove only the converse.

Suppose M is a submartingale with constant expectation. Let $0 \leq u \leq t \leq T$. Then $\mathbb{E}[M_t|\mathcal{F}_u] - M_u \geq 0$ and

$$\mathbb{E}[\mathbb{E}[M_t|\mathcal{F}_u] - M_u] = \mathbb{E}[M_t] - \mathbb{E}[M_u] = 0$$

by assumption. A non-negative random variable X with $\mathbb{E}(X) = 0$ almost surely, as the Markov inequality implies $\mathbb{P}(X \geq 2^{-n}) \leq 2^n \mathbb{E}(X) = 0$. This implies that $\mathbb{E}[M_t|\mathcal{F}_u] = M_u$ almost surely, i.e. M is an $\mathcal{F}$-martingale. $\blacksquare$