Remark 0.0.1. (equivalent conditions for discrete time martingales) For a discrete time process, it suffices to show

- $\mathbb{E}(M_{n+1}|\mathcal{F}_n) = M_n$ for all $n = 0, 1, \ldots, N-1$, or
- $\mathbb{E}(M_N|\mathcal{F}_n) = M_n$ for all $n = 0, 1, \ldots, N$.

Other ways to prove a martingale in continuous case include:

- Lemma 2.2.1: A sub- or supermartingale with constant expectation.
- $\mathbb{E}(M_T|\mathcal{F}_t) = M_t$ for all $t \in [0, T]$.
- Corollary 3.3.2: $M^2 - \langle M \rangle$ is an $\mathcal{F}$-martingale if M is a continuous, square integrable $\mathcal{F}$-martingale.
- Theorem 4.1.1: $I(\gamma)$ for $\gamma \in L_P^2(W)$ is an $\mathcal{F}$-martingale.
- Theorem 4.2.1: $I(\gamma)$ for $\gamma \in L_P(W)$ is an $\mathcal{F}$-local martingale.
- Itô's lemma (Theorem 4.3.1) proves a continuous semimartingale when g is sufficiently smooth.