**Theorem 0.0.1** (Doob decomposition theorem)**.** *A discrete time submartingale $X = (X_n)_{n=0,1,...,N}$ can be decomposed into*

$$X_n = M_n + A_n$$

*where $M$ is a martingale and $A$ is a predictable, increasing process with $A_0 = 0$. Similarly, a supermartingale can be decomposed into a martingale and a decreasing process. This decomposition is almost surely unique.*