

Corollary 0.0.1. (from Theorem 1.3.1 and Theorem 1.1.1) A function $X : \Omega \rightarrow \mathbb{R}$ is a random variable if and only if $\{\omega : X(\omega) \leq a\} = X^{-1}((-\infty, a]) \in \mathcal{A}$ for all $a \in \mathbb{Q}$.