Theorem 0.0.1 (probability measure $\mathbb{P}$ is continuous along monotone sequences of events)**.** *If* $\mathbb{P} : \mathcal{A} \to [0,1]$ *is a probability measure, then*

- *If* $(A_i)_{i\geq 1} \in \mathcal{A}$ *is increasing, i.e.* $A_1 \subseteq A_2 \subseteq A_3 \subseteq \cdots$ *with* $\lim_{i\to\infty} A_i = \bigcup_{i=1}^{\infty} A_i = A$*, denoted* $A_i \uparrow A$*, then*
$$\lim_{i\to\infty} \mathbb{P}(A_i) = \mathbb{P}\left(\lim_{i\to\infty} A_i\right) = \mathbb{P}(A).$$
- *If* $(A_i)_{i\geq 1} \in \mathcal{A}$ *is decreasing, i.e.* $A_1 \supseteq A_2 \supseteq A_3 \supseteq \cdots$ *with* $\lim_{i\to\infty} A_i = \bigcap_{i=1}^{\infty} A_i = A$*, denoted* $A_i \downarrow A$*, then*
$$\lim_{i\to\infty} \mathbb{P}(A_i) = \mathbb{P}\left(\lim_{i\to\infty} A_i\right) = \mathbb{P}(A).$$