

Theorem 0.0.1 (quadratic variation of semimartingales). *If $X = X_0 + M + A \in \mathcal{S}^c(\mathbb{P})$, then the quadratic variation $\langle X \rangle = \langle M \rangle$. More generally, if $X^i = X_0^i + M^i + A^i$ and $X^j = X_0^j + M^j + A^j$ are in $\mathcal{S}^c(\mathbb{P})$, then*

$$\langle X^i, X^j \rangle = \langle M^i, M^j \rangle = \frac{1}{2} (\langle M^i + M^j \rangle - \langle M^i \rangle - \langle M^j \rangle) = \frac{1}{4} (\langle M^i + M^j \rangle - \langle M^i - M^j \rangle).$$