Theorem 0.0.1 (Ito's Lemma, multidimensional). *Suppose that g is a function of class $C^2(\mathbb{R}^k, \mathbb{R})$, i.e. g is twice continuously differentiable, and that X a k-dimensional Ito process. Then*

$$\begin{aligned} dg(X_t) &= \sum_{i=1}^{k} g_{x_i}(X_t)\, dX_t^i + \frac{1}{2} \sum_{i,j=1}^{k} g_{x_i x_j}(X_t) \beta_t^i \cdot \beta_t^j \, dt \\ &= \sum_{i=1}^{k} g_{x_i}(X_t) \alpha_t^i dt + \sum_{i=1}^{k} g_{x_i}(X_t) \beta_t^i \cdot dW_t + \frac{1}{2} \sum_{i,j=1}^{k} g_{x_i x_j}(X_t) \beta_t^i \cdot \beta_t^j \, dt. \end{aligned}$$