Examples 0.0.1 (Itô's integral for a deterministic integrand). For a deterministic function f, we have

$$X_t = \int_0^t f(s)\,dW_s \sim \mathcal{N}\left(0, \int_0^t f^2(s)\,ds\right).$$

The computation of $\mathbb{E}(X_t)$ and $\mathrm{Var}(X_t)$ follows directly from Theorem 4.1.1. Normality follows from the fact that X_t is (the limit of) the sum of independent normal r.v.s:

$$\int_a^t f(s)\,dW_s = \lim_{n\to\infty} \sum_{j=0}^{n-1} f(t_j)(W_{t_{j+1}} - W_{t_j}).$$