Definition 0.0.1 (filtration, $\mathcal{F}$-adapted process, natural filtration)**.**

- A **filtration** $\mathcal{F} = (\mathcal{F}_t)_{t \in [0,T]}$ is an increasing family of σ-algebras, that is, $\mathcal{F}_u \subseteq \mathcal{F}_t$ for any $0 \leq u \leq t \leq T$.
- A stochastic process $X = (X_t)_{t \in [0,T]}$ defined on $(\Omega, \mathcal{F}, \mathbb{P})$ is **$\mathcal{F}$-adapted** if for any $t \in [0,T]$, the random variable X_t is $\mathcal{F}_t$-measurable, i.e. for any $x \in \mathbb{Q}$, the event $\{X_t \leq x\} \in \mathcal{F}_t$ (Corollary 1.3.1).
- The **natural filtration** of X (or the filtration generated by X) is defined as $\mathcal{F}^X = (\mathcal{F}_t^X)_{t \in [0,T]}$ where $\mathcal{F}_t^X = \sigma(X_u, u \leq t)$. Any stochastic process X is, by definition, adapted to its natural filtration.